A note on positive semi-definiteness of some non-pearsonian correlation matrices

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I. Introduction: A correlation matrix, $\mathbf{R}$, is a real and symmetric $m \times m$ matrix such that $-1 \leq r_{ij} \in \mathbb{R} \leq 1$; $i, j = 1, 2, \ldots, m$. Moreover, $r_{ii} = 1$. The Pearsonian (or the product moment) correlation coefficient, e.g. $r_{12}$ (between two variates, say $x_1$ and $x_2$, each in $n$ observations), is given by the formula:

$$r(x_1, x_2) = \frac{\text{cov}(x_1, x_2)}{\sqrt{\text{var}(x_1) \cdot \text{var}(x_2)}}$$

where, $\bar{x}_a = \frac{1}{n} \sum_{k=1}^{n} x_{ka}$; $\text{cov}(x_1, x_2) = \frac{1}{n} \sum_{k=1}^{n} x_{k1} x_{k2} - \bar{x}_{1}^2 \bar{x}_{2}^2$ and $\text{var}(x_a) = \text{cov}(x_a, x_a)$; $a = 1, 2$.

A little of algebra also gives us the identity:

$$r(x_1, x_2) = (1/4) \frac{\var(x_1 + x_2) - \var(x_1 - x_2)}{\sqrt{\var(x_1) \cdot \var(x_2)}}$$

The Pearsonian correlation matrix is necessarily a positive semi-definite matrix (meaning that all its eigenvalues are non-negative) since it is the quadratic form of a real matrix, $X(n, m)$. It also implies that if $\mathbf{R}$ is not a semi-positive matrix, then $X(n, m)$ is not a real matrix.

II. Robust Measures of Correlation: The Pearsonian coefficient of correlation as a measure of association between two variates is highly prone to the deleterious effects of outlier observations (data). Statisticians have proposed a number of formulas, other than the one that obtains Pearson’s coefficient of correlation, that are considered to be less affected by errors of observation, perturbation or presence of outliers in the data. Some of them transform the variables, say $x_1$ and $x_2$, into $z_1 = \phi_1(x_1)$ and $z_2 = \phi_2(x_2)$, where $\phi_a(x_a)$ is a linear (or nonlinear) monotonic (order-preserving) rule of transformation or mapping of $x_a$ to $z_a$. Then, $r(z_1, z_2)$ is obtained by the appropriate formula and it is considered as a robust measure of $r(x_1, x_2)$. Some others use different measures of central tendency, dispersion and co-variation, such as median for mean, mean deviation for standard deviation and so on. In what follows, we present a few formulas of obtaining different types of correlation efficient.

II.1. Spearman’s Rank Correlation Coefficient: If $x_1$ and $x_2$ are two variables, both in $n$ observations, and $z_1 = \pi(x_1)$ and $z_2 = \pi(x_2)$ are their rank numerals with $\pi(.)$ as the rank-ordering rule, then the Pearson’s formula applied on $(z_1, z_2)$ obtains the Spearman’s correlation coefficient (Spearman, 1904). There is a simpler (but less general) formula that obtains rank correlation coefficient, given as:

$$\rho(x_1, x_2) = r(z_1, z_2) = 1 - 6 \sum_{k=1}^{n} \left( \frac{z_{k1} - z_{k2}}{n(n-1)} \right)^2$$

... (3)
II.2. Signum Correlation Coefficient: Let \( c_1 \) and \( c_2 \) be the measures of central tendency or location (such as arithmetic mean or median) of \( x_1 \) and \( x_2 \) respectively. We transform them to \( z_{ka} = (x_{ka} - c_a) / |x_{ka} - c_a| \) if \( |x_{ka} - c_a| > 0 \), else \( z_{ka} = 1 \). Then, \( r(z_1, z_2) \) is the signum correlation coefficient (Blomqvist, 1950; Shevlyakov, 1997). Due to the special nature of transformation, we have
\[
\rho(z_1, z_2) = \frac{\text{cov}(z_1, z_2)}{\text{var}(z_1) \text{var}(z_2)} = \frac{1}{n} \sum_{i=1}^{n} z_{i1}^* z_{i2}^* \quad \ldots \quad (4)
\]
In this study we will use median as a measure of central tendency to obtain signum correlation coefficients.

II.3. Bradley’s Absolute Correlation Coefficient: Bradley (1985) showed that if \( (u_k, v_k); k = 1, n \) are \( n \) pairs of values such that the variables \( u \) and \( v \) have the same median = 0 and the same mean deviation (from median) or \( (1/n) \sum_{k=1}^{n} |u_k| = (1/n) \sum_{k=1}^{n} |v_k| = d \neq 0 \), both of which conditions may be met by any pair of variables when suitably transformed, then the absolute correlation may be defined as
\[
\rho(u, v) = \left( \frac{1}{n} \sum_{k=1}^{n} (|u_k + v_k| - |u_k| - |v_k|) \right) / \left( \frac{1}{n} \sum_{k=1}^{n} (|u_k| + |v_k|) \right) \quad \ldots \quad (5)
\]

II.4. Shevlyakov Correlation Coefficient: Hampel et al. (1986) defined the median of absolute deviations (from median) as a measure of scale, \( s_H(x_a) = \text{median} |x_{ka} - \text{median}(x_{ka})| \); \( a = 1, 2 \) which is a very robust measure of deviation, and using this measure, Shevlyakov (1997) defined median correlation,
\[
r_{med} = \left[ \text{med}^2 |u| - \text{med}^2 |v| \right] \left[ \text{med}^2 |u| + \text{med}^2 |v| \right] \quad \ldots \quad (6)
\]
where \( u \) and \( v \) are given as \( u_k = (x_{k1} - \text{med}(x_1)) / s_H(x_1) + (x_{k2} - \text{med}(x_2)) / s_H(x_2) \) and \( v_k = (x_{k1} - \text{med}(x_1)) / s_H(x_1) - (x_{k2} - \text{med}(x_2)) / s_H(x_2) \); \( k = 1, 2, \ldots, n \).

III. Are Robust Correlation Matrices Positive Semi-definite? In this study we investigate into the question whether the correlation matrix, \( \mathcal{R} \), whose any element \( r_{ij} \) is a robust measure of correlation (obtained by the formulas such as Spearman’s, Blomqvist’s, Bradley’s or Shevlyakov’s), is positive semi-definite. We use the dataset given in Table-1 as the base data for our experiments.

We have carried out ten thousand experiments for each method (Spearman’s \( \rho \), Blomqvist’s signum, Bradley’s absolute \( r \), and Shevlyakov’s \( r_{med} \)) of computing robust correlation, \( r_{ij} \in \mathcal{R} \). In each experiment, the base data (Table-1) has been perturbed by a small (between -0.25 to 2.5) quantity generated randomly (and distributed uniformly) over \( n \) observations on all the eight variables. In each experiment, the correlation matrix, \( \mathcal{R} \), has been computed and its eigenvalues are obtained. If any (at least one) eigenvalue of the correlation matrix has been found to be negative, the occurrence has been counted as a failure (of the matrix being positive semi-definite), else it is counted as a success. Such three sets of experiments have been carried out with three different seeds for generating random numbers (for perturbation).

IV. The results, Discussion and Conclusion: Our findings reveal that while Spearman’s rho, Blomqvist’s signum, and Bradley’s absolute correlation formulas yield positive semi-definite correlation matrix (without any failure), the failure rate of Shevlyakov’s formula is very high (about 81 percent: more exactly 81.47%, 80.94% and 81.41% for the three random number seeds: 13317, 31921 and 17523,
respectively). Two sample correlation matrices (and their eigenvalues) are presented in Table-2.1 and Table-2.2. It is found that the smallest eigenvalue so often turns out to be negative.

The observed failure rate of Shevlyakov’s correlation matrix raises a question whether it can be used directly for further analysis of correlation matrices - without being approximated by the nearest positive semi-definite matrix (Mishra, 2008). While the product moment correlation coefficient (of Karl Pearson) is so much sensitive to the outliers, the robust nature of Shevlyakov’s correlation coefficient is attractive. But, unfortunately, its robustness goes along with its being extremely prone to non-positive semi-definiteness and unsuitability to multivariate analysis. In view of these findings, it is safe to use Spearman’s $\rho$, Blomqvist’s signum or Bradley’s absolute $r$ rather than Shevlyakov’s correlation coefficient for constructing correlation matrices for any further analysis.

### Table-1: Base Dataset for Computation of Robust Correlation Matrices by Different Methods

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### Table-2.1. Shevlyakov’s Robust Correlation matrix and its Eigenvalues (Sample-1)

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Eigenvalues (in descending order)

| 4.62370 | 1.17033 | 0.79635 | 0.61840 | 0.42434 | 0.16789 | 0.14943 | 0.04956 |
Table 2.2. Shevlyakov’s Robust Correlation matrix and its Eigenvalues (Sample-2)

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<td>$x_8$</td>
<td>0.45526</td>
<td>0.63320</td>
<td>0.43348</td>
<td>0.62314</td>
<td>0.41083</td>
<td>0.73863</td>
<td>0.69714</td>
<td>1.00000</td>
</tr>
</tbody>
</table>

Eigenvalues (in descending order)

|        | 4.67015 | 1.20149 | 0.83475 | 0.58967 | 0.42884 | 0.25716 | 0.13572 | **-0.11778** |

References

PROGRAM ROBUSTC  ! CHECK IF A ROBUST R MATRIX IS + SEMI-DEFINITE
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
PARAMETER (N=37, M=8, SCALEX=100, SCALEZ=0.7, NITER=1000)
C SCALEX SCALES GENERATED BASE 1ST VARIABLE DATA BETWEEN (0-SCALEX)
C SCALE2 GENERATES 2ND TO MTH VARIABLE SCALE*SCALEZ(+/-)RANDOM
C LARGER SCALEZ,SAY .9 GENERATES HIGHLY CORRELATED DATA & VICE VERSA
C TO GENERATE HIGHLY CORRELATED DATA SET SCALEZ LARGER, SAY 0.9
DIMENSION Z(N,M),X(N,M),R(M,M),RR(M,M),AV(M),SD(M),XX(N),U(N),V(N)
DIMENSION VR(M,M),WR(M,M),ZV(N,M)
COMMON /RNDM/ IU, IV
WRITE(*,*) 'FEED RANDOM NUMBER SEED'
WRITE(*,*) IU
WRITE(*,*) 'INPUT DATA: WHETHER GENERATE (0) OR READ FROM FILE (1)'
READ(*,*) IDAT
C IF,IDAT.NE.0 THEN
C READS DATA FROM FILE -----------------------------------------------
OPEN(7,FILE='NDAT1.TXT')
DO I=1,N
READ(I,*) ZV(I,J),J=1,M
WRITE(*,*) I, (ZV(I,J),J=1,M)
ENDDO
CLOSE(7)
ELSE
C GENERATES DATA ----------------------------------------------------
DO I=1,N
CALL RANDOM(RAND)
ZV(I,1)=RAND*SCALEX
DO J=2,M
CALL RANDOM(RAND)
ZV(I,J)=ZV(I,1)*SCALEZ+(RAND-0.5)*SCALEZ*2
ENDDO
ENDDO
ENDIF
C OPEN(7,FILE='CORIND.TXT')
WRITE(*,*) 'FEED CHOICE OF CORRELATION. ZERO (0) IS KARL PEARSON R'
WRITE(*,*) '(1): RANK CORRELATION, (2): SIGNUM CORRELATION'
WRITE(*,*) '(3): BRADLEY CORRELATION, (4) SHEVLYAKOV CORRELATION'
READ(*,*) NTYPE
ICHK=0 ! WILL COUNT THE NUMBER OF FAILURES
DO ITER=1,NITER !===============================================
DO I=1,N
DO J=1,M
CALL RANDOM(RAND)
Z(I,J)=-ZV(I,J)+(RAND-0.5D0)*0.5 &PURTURBATION (-0.25 TO 0.25)
ENDDO
ENDDO
DO J=1,M
DO I=1,N
X(I,J)=Z(I,J)
ENDDO
ENDDO
C IF(NTYPE.EQ.0) THEN
WRITE(*,*) 'KARL PEARSON CORRELATION MATRIX'
CALL PROD(X,N,M,AV,SD,R)
ENDIF
C IF(NTYPE.EQ.1) THEN
WRITE(*,*) 'SPEARMAN RANK CORRELATION MATRIX'
DO J=1,M
DO I=1,N
XX(I)=Z(I,J)
ENDDO
CALL RANK(XX,N) & RANK TRANSFORMATION OF X
DO I=1,N
X(I,J)=XX(I)
ENDO
ENDDO
CALL PROD(X,N,M,AV,SD,R)
WRITE(,*,'CORRELATION MATRIX')
DO J=1,M
WRITE(*,21)(R(J,JJ),JJ=1,M)
ENDDO
CALL EIGEN(R,VR,WR)
WRITE(*,'EIGENVALUES OF CORRELATION MATRIX')
DO J=1,M
WRITE(*,21)(WR(J,JJ),JJ=1,M)
ENDDO
JCHK=0
DO I=1,M
IF(WR(I,I).LT.0.AND.JCHK.EQ.0) THEN
ICHK=ICHK+1
JCHK=1
ENDIF
ENDDO

C
IF(NTYPE.EQ.2) THEN
WRITE(*,'SIGNUM CORRELATION MATRIX')
DO J=1,M
DO JJ=1,M
R(J,JJ)=0.D0
DO I=1,N
CONC=X(I,J)*X(I,JJ)
R(J,JJ)=R(J,JJ)+CONC
ENDDO
R(J,JJ)=R(J,JJ)/N
ENDDO
ENDDO
DO J=1,M
WRITE(7,'(*) (R(J,JJ),JJ=1,M)
ENDDO
WRITE(*,'-------- SIGNUM CORRELATION MATRIX (PROPER) --------')
CALL PROD(X,N,M,AV,SD,R)
WRITE(*,'OVER')
WRITE(*,'CORRELATION MATRIX')
DO J=1,M
WRITE(*,21)(R(J,JJ),JJ=1,M)
ENDDO
CALL EIGEN(R,VR,WR)
WRITE(*,'EIGENVALUES OF CORRELATION MATRIX')
DO J=1,M
WRITE(*,21)(WR(J,JJ),JJ=1,M)
ENDDO
JCHK=0
DO I=1,M
IF(WR(I,I).LT.0.AND.JCHK.EQ.0) THEN
ICHK=ICHK+1
JCHK=1
ENDIF
ENDDO
ENDDO

IF (NTYPE.EQ.3) THEN
WRITE(*,*) 'BRADLEY CORRELATION MATRIX'
DO J=1,M
DO I=1,N
X(I,J)=0.D0
DEV=(Z(I,J)-AMED)
X(I,J)=DEV/AMDE

ENDDO

CALL MEDIAN(XX,N,AMED,AMDE)
DO I=1,N
S1=0.D0
S2=0.D0
DO I=1,N
S1=S1+DABS(X(I,J)+X(I,J)) - DABS(X(I,J)-X(I,J))
S2=S2+DABS(X(I,J))+ DABS(X(I,J))
ENDDO

R(J,JJ)=S1/S2
ENDDO

IF (WR(I,I).LT.0. AND. JCHK.EQ.0) THEN
ICHK=ICHK+1
JCHK=1
ENDIF

ENDDO

ENDIF

IF (NTYPE.EQ.4) THEN
WRITE(*,*) 'SHEVLYAKOV CORRELATION MATRIX'
DO J=1,M
DO I=1,N
XX(I)=Z(I,J)
ENDDO

CALL MEDIAN(XX,N,AMED,AMDE)
DO I=1,N
XX(I)=DABS(Z(I,J)-AMED)
ENDDO

CALL MEDIAN(XX,N,HMED,HMED)
DO I=1,N
X(I,J)=(Z(I,J)-AMED)/HMED
ENDDO

ENDDO

DO J=1,M
DO JJ=1,M
R(J,JJ)=0.D0
ENDDO
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DO I=1,N
U(I)=DABS(X(I,J)+X(I,JJ))
V(I)=DABS(X(I,J)-X(I,JJ))
ENDDO
CALL MEDIAN(U,N,UMED,UMDEV)
CALL MEDIAN(V,N,VMED,VMDEV)
R(J,JJ)=(UMED**2-VMED**2)/(UMED**2+VMED**2)
ENDDO
WRITE(*,*)'CORRELATION MATRIX'
DO J=1,M
WRITE(*,21)(R(J,JJ),JJ=1,M)
ENDDO
CALL EIGEN(R,VR,WR)
WRITE(*,21)'EIGENVALUES OF CORRELATION MATRIX'
DO J=1,M
WRITE(*,21)(WR(J,JJ),JJ=1,M)
ENDDO
WRITE(*,*)'------------------------------------------------------'
DO I=1,M
IF(WR(I,I).LT.0.AND.JCHK.EQ.0) THEN
JCHK=JCHK+1
ENDIF
ENDDO
WRITE(*,*)'------------------------------------------------------'

SUBROUTINE PROD(X,N,M,AV,SD,R)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION X(N,M),R(M,M),AV(M),SD(M)
DO J=1,M
AV(J)=0.0D0
SD(J)=0.0D0
DO I=1,N
AV(J)=AV(J)+X(I,J)
SD(J)=SD(J)+X(I,J)**2
ENDDO
SD(J)=DSQRT((N*SD(J)-AV(J)*AV(J))/N**2)
AV(J)=AV(J)/N
ENDDO
DO J=1,M
DO JJ=1,M
R(J,JJ)=0.0D0
DO I=1,N
R(J,JJ)=R(J,JJ)+X(I,J)*X(I,JJ)
ENDDO
R(J,JJ)=(R(J,JJ)/N-AV(J)*AV(J))/(SD(J)*SD(JJ))
ENDDO
ENDDO
ENDDO
DO J=1,M
WRITE(*,*)R(J,JJ),JJ=1,M
ENDDO
WRITE(*,1)AV(J),J=1,M
WRITE(*,1)SD(J),J=1,M
1 FORMAT(8F9.5)
WRITE(*,*)'------------------------------------------------------'
RETURN
END
SUBROUTINE RANK(X,N)
!
PARAMETER (NMAX=1000)
DIMENSION X(N), SL(NMAX)
!
DO I=1,N
SL(I)=DFLOAT(I)
ENDDO
!
DO II=I+1,N
IF(X(I).LT.X(II)) THEN
  T=X(I)
  X(I)=X(II)
  X(II)=T
  SL(I)=SL(II)
  SL(II)=T
ENDIF
ENDDO
!
DO I=1,N-1
DO II=I+1,N
IF(SL(I).GT.SL(II)) THEN
  TEMP=SL(I)
  SL(I)=SL(II)
  SL(II)=TEMP
ENDIF
ENDDO
!
RETURN
!
END
!

C     -----------------------------------------------------------------
SUBROUTINE MEDIAN(X,N,A,V) ! ------------------------------------
!
PARAMETER (NMAX=1000)
IMPLICIT DOUBLE PRECISION (A-H,O-Z)
DIMENSION X(N), Z(NMAX)
!
C     STORE X IN Z
DO I=1,N
  Z(I)=X(I)
ENDDO
!
C     ARRANGE Z IN AN ASCENDING ORDER
DO I=1,N-1
  DO II=I+1,N
    IF(Z(I).GT.Z(II)) THEN ! EXCHANGE
      TEMP=Z(I)
      Z(I)=Z(II)
      Z(II)=TEMP
    ENDIF
  ENDDO
ENDDO
!
K=(N+1)/2 ! K IS OBTAINED AS INT((N+1)/2.0D0)
A=(Z(K)+Z(N+1-K))/2.0D0 ! GIVES MEDIAN FOR ODD AS WELL AS EVEN N
!
C     FIND MEAN DEVIATION
V=0.0D0
DO I=1,N
  V=V+DABS(Z(I)-A) ! A IS MEDIAN
ENDDO
!
V=V/N ! V IS MEAN DEVIATION FROM MEDIAN
!
C WRITE(*,*)'MEDIAN=',A,' MEAN DEVIATION =',V
RETURN
SUBROUTINE EIGEN(A,V,W)

PARAMETER(N=8)

COMPUTES EIGENVALUES AND VECTORS OF A REAL SYMMETRIC MATRIX
A(N,N) =GIVEN REAL SYMMETRIC MATRIX WHOSE EIGENVALUES AND VECTORS
ARE BE FOUND. ITS ORDER IS N X N
W(N,N) CONTAINS EIGENVALUES IN ITS MAIN DIAGONAL. OTHER ELEMENTS=0
V(N,N) CONTAINS EIGENVECTORS

PROGRAM BY KRISNAMURTHY, EV & SEN (1976) COMPUTER-BASED NUMERICAL
ALGORITHMS. AFFILIATED EAST-WEST PRESS, NEW DELHI

DOUBLE PRECISION A(N,N),V(N,N),W(N,N),P(N)

DOUBLE PRECISION PMAX,EPLN,TAN,SIN,COS,AI,TT,TA,TB

DIMENSION MM(N)

--------------- INITIALISATION -------------------------------

DO I=1,N
DO J=1,N
V(I,J)=0.D0
W(I,J)=A(I,J)
ENDDO
ENDDO
P(I)=0.D0
ENDDO
PMAX=0.D0
EPLN=0.D0
TAN=0.D0
SIN=0.D0
COS=0.D0
AI=0.D0
TT=0
MM=1
EPLN=1.0D-100

IF(NN.NE.0) THEN
DO I=1,N
DO J=1,N
V(I,J)=0.D0
IF(I.EQ.J) V(I,J)=1.D0
ENDDO
ENDDO
ENDIF

IF(P(I).LE.DABS(A(I,J))) THEN
P(I)=DABS(A(I,J))
MM(I)=J
ENDIF
ENDDO
ENDDO

7 DO 8 I=1,MI
IF(I.LE.1) GOTO 10
IF(PMAX.GT.P(I)) GOTO 8
8 PMAX=P(I)
IP=I
JP=MM(I)
8 CONTINUE

P(EPLN+DABS(PMAX)*1.0D-09)

IF (PMAX.LE.EPLN) THEN
WRITE(*,*)'PMAX EPLN',PMAX, EPLN
PAUSE'CONVERGENCE CRITERION IS MET'
GO TO 12
ENDIF
NR=NR+1
TA = 2.0D0 * A(IP, JP)
TB = (DABS(A(IP, IP) - A(JP, JP)) +
1D5QRT((A(IP, IP) - A(JP, JP))**2 + 4.0D0 * A(IP, JP)**2))
TAN = TA / TB

IF(A(IP, IP) .LT. A(JP, JP)) TAN = -TAN
COS = 1.0D0 / DSQRT(1.0D0 + TAN**2)
SIN = TAN * COS
AI = A(IP, IP)
A(IP, IP) = (COS**2) * (AI + TAN * (2.0D0 * A(IP, JP) + TAN * A(JP, JP)))
A(JP, JP) = (COS**2) * (A(IP, JP) - TAN * (2.0D0 * A(IP, JP) - TAN * A(IP, JP)))
A(IP, JP) = 0.0D0

IF(A(IP, IP) .GE. A(JP, JP)) GO TO 15
TT = A(IP, IP)
A(IP, IP) = A(JP, JP)
A(JP, JP) = TT

IF(SIN .GE. 0.0D0) GO TO 16
TT = COS
GO TO 17

TT = -COS
COS = DABS(SIN)
SIN = TT

DO 1 1 = I, MI
IF(I .LT. IP) 19, 18, 20
DO 1 20 IF(I .EQ. JP) GO TO 18
19 IF(MM(I) .EQ. IP) GO TO 21
20 IF(MM(I) .NE. JP) GO TO 18

K = MM(I)
TT = A(I, K)
A(I, K) = 0.0D0
MJ = I + 1
P(I) = 0.0D0

DO 22 J = MJ, N
21 IF(P(I) .GT. DABS(A(I, J))) GO TO 22
22 P(I) = DABS(A(I, J))

MM(I) = J
23 CONTINUE
A(I, K) = TT
24 CONTINUE

P(IP) = 0.0D0
P(JP) = 0.0D0

DO 23 I = 1, N
22 IF(I .LT. IP) 24, 23, 25
23 TT = A(I, IP)
A(I, IP) = COS * TT + SIN * A(I, JP)

IF(P(I) .GE. DABS(A(I, IP))) GO TO 26
25 P(I) = DABS(A(I, IP))
MM(I) = IP
26 A(I, JP) = -SIN * TT + COS * A(I, JP)

IF(P(I) .GE. DABS(A(I, JP))) GO TO 23
27 P(I) = DABS(A(I, JP))
MM(I) = JP
28 TT = A(IP, I)

A(IP, I) = COS * TT + SIN * A(I, JP)

IF(P(IP) .GE. DABS(A(IP, I))) GO TO 29
29 P(IP) = DABS(A(IP, I))
MM(IP) = I
30 A(IP, JP) = -TT * SIN + COS * A(I, JP)

IF(P(I) .GE. DABS(A(I, JP))) GO TO 23
31 P(I) = DABS(A(I, JP))

GO TO 30

TT = A(IP, I)
A(IP, I) = TT * COS + SIN * A(JP, I)

IF(P(IP) .GE. DABS(A(IP, I))) GO TO 31
32 P(IP) = DABS(A(IP, I))
robustc.f 8/8
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470:    MM(IP)=I
471:    31 A(JP,I)=TT*SIN+COS*A(JP,I)
472:    IF(P(JP).GE.DABS(A(JP,I))) GO TO 23
473:    P(JP)=DABS(A(JP,I))
474:    MM(JP)=I
475:    23 CONTINUE
476:    IF(NN.EQ.0) GOTO 7
477:    DO 32 I=1,N
478:    TT=V(I,IP)
479:    V(I,IP)=TT*COS+SIN*V(I,JP)
480:    V(I,JP)=TT*SIN+COS*V(I,JP)
481:    32 CONTINUE
482:    DO I=1,N
483:    P(I)=A(I,I)
484:    ENDDO
485:    DO I=1,N
486:    DO J=1,N
487:    A(I,J)=W(I,J)
488:    W(I,J)=0.DO
489:    ENDDO
490:    W(I,I)=P(I)
491:    ENDDO
492:    RETURN
493:    END
494:    C -----------------------------------------------------------------
495:    C RANDOM NUMBER GENERATOR (UNIFORM BETWEEN 0 AND 1 - BOTH EXCLUSIVE)
496:    C -----------------------------------------------------------------
497:    SUBROUTINE RANDOM(RAND)
498:    DOUBLE PRECISION RAND
499:    COMMON /RNDM/IU,IV
500:    IV=IU*65539
501:    IF(IV,LT,0) THEN
502:    IV=IV+2147483647+1
503:    ENDIF
504:    RAND=IV
505:    IU=IV
506:    RAND=RAND*0.4656613E-09
507:    RETURN
508:    END