General correcting formula of forecasting?

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A general correcting formula of forecasting (as a framework for long-use and standardized forecasts) is proposed. The formula provides new forecasting resources and areas of application including economic forecasting.

Contents

Introduction

This paper presents in English, develops and generalizes the results of Harin (2008) and a part of the results of Harin (2009). From other works of the item see, e.g., Tsay (2008), Kasa (2000).

1. Approaches

1.1. The principle of uncertain future

General principle of uncertain future:

"A future event contains an uncertainty"

Specific principle of uncertain future:

"The estimation of the probability of a future event should (manifestly) contain an uncertainty"

or,

$$\boldsymbol{P_{estimated} \approx P_{estimated\ mean} \pm \Delta P}$$

Development and applications of the principle see in (Harin 2005 – Harin 2009).

1.1.1. The first consequence of the principle

1.1.1.1. A far analogy. Vibrations near a rigid wall

Suppose an electro-drill or any similar device, e.g., sewing-machine, vibrosieve, machine-gun, electric hammer etc. which (when working) can vibrate quickly. Presume the device has rigid flank sides and vibrates with the amplitude of, say, 1 mm.

Can we approach a flank side of the non-working drill (or of the device) to a rigid wall or ledge:

A) as close as at the distance, say, 0.1 mm;

B) tightly?

Certainly. Both A) and B).

And now turn the drill (the device) on. What will be the distance from the rigid wall to the working drill? Vibrations will repulse, shift the drill from the wall.

Due to the vibrations:

A) the distance from the drill to the wall will be more than 0.1 mm;

B) the gap, rupture will arise between the drill and the wall.

1.1.1.2. An example. Aiming firing at a target

General conditions

Suppose a hypothetic transportable testing stand, arrangement for testing the quality of rifles, guns, cartridges etc. To avoid human errors, the arrangement is made in the form of a standing man, a rifle is fasten onto the arrangement and the aiming is performed automatically. Suppose firing errors are minimized and are much less than one point of the target.

Suppose the arrangement is placed near a railway or Metro. The vibrations of the ground increase firing errors up to, say, 2 points. For the sake of simplicity, assume the target is strongly elongated in one of directions. So, the consideration is reduced to one-dimensional and uniform (without effects of curvature) case. Suppose the points are located in the scale from "0" to "10": "9", "8", "7" etc. are located after "10". Before "0" there is the blank space which is equivalent to "0".

Suppose following dispersion takes place: one shot =exact; one shot =+2 points; one shot =-2 points.

If the aiming is performed at, say, "7", the mean result is the same as the aiming value. The result is $(7+9+5)/3=7$.

A) The shift from the bounds to the middle of the target scale

If the aiming is performed at "9", one bullet should hit beyond the bound "10" at "11", but really hits at "9". The result is $(9+9+7)/3=25/3=8⅓$. One bullet, instead of "11", hits "9", i.e. 2 less than the aiming value. The mean result is shifted from the bound (from "10") to the middle (to ~"5") of the scale by 2/3 points.

If the aiming is performed at "1", one bullet should hit beyond the bound "0" at "-1", but really hits at the blank space which is equivalent to "0". The result is $(1+3+0)/3=1⅓$. One bullet, instead of "-1", hits "0", i.e. 1 more than the aiming value. The mean result is shifted from the bound (from "0") to the middle (to ~"5") of the scale by 1/3 points.

A) The dispersion causes the shifts of the mean results from the bounds to the middle of the target scale.

B) The ruptures in the target scale

If the aiming is performed at the bound of the target scale "10", one bullet should hit beyond the bound "10" at "12", but really hits at "8". The result is $(10+8+8)/3=26/3=8⅔$. One bullet, instead of "12", hits "8", i.e. 4 less than the aiming value. The rupture between the mean result and the bound "10" of the scale is 1⅓ points.

If the aiming is performed at the bound of the scale "0", one bullet should hit beyond the bound "0" at "-2", but really hits at the blank space which is equivalent to "0". The result is $(0+2+0)/3=2/3$. One bullet, instead of "-2", hits "0", i.e. 2 more than the aiming value. The rupture between the mean result and the bound "0" of the scale is 2/3 points.

B) The dispersion causes the ruptures near the bounds of the target scale.

1.1.1.3. The first consequence of the principle

Suppose we wish to test a probability value P, which is very close (but not equal) to the bound P_{bound} of the probability scale, i.e. to 0% or 100%. The mean-square error, the uncertainty value of the estimation of P is ΔP (taking into account Novosyolov 2009). Let us examine two cases. Name them conditionally: certain ($P=P_{certain}$ and $\Delta P=\Delta P_{certain}$) and uncertain ($P=P_{uncertain}$ and $\Delta P=\Delta P_{uncertain}$). Suppose the certain case is an initial one and the uncertain case is a final one. Suppose for both cases the number of trials, tests, outcomes etc. is the same. So, the difference, change $\Delta P_{uncertain}$-$\Delta P_{certain}$ is defined only by the difference, change of noises, disturbances etc.

Suppose in the certain case the uncertainty value of the probability estimation is equal to 0 or is much less than the difference between the initial probability $P_{certain}$ and the bound of the probability scale P_{bound}

$$\Delta P_{certain} << |P_{bound} - P_{certain}|$$

Suppose in the uncertain case the uncertainty value of the probability estimation is more than the difference between the initial probability $P_{certain}$ and the bound of the probability scale P_{bound}

$$\Delta P_{uncertain} > |P_{bound} - P_{certain}|$$

A) Shifts in the probability scale

If, due to increasing of noises, the uncertainty ΔP of the probability estimation increases from initial $\Delta P_{certain} << |P_{bound} - P_{certain}|$ to final $\Delta P_{uncertain} > |P_{bound} - P_{certain}|$, then the probability will be shifted, "pushed away" by the noises from the bound to the middle of the probability scale.

Indeed, if, e.g., for initial $P_{certain}=99\%$, the uncertainty of the probability estimation increases from initial $\Delta P_{certain}<<1\%$ to final $\Delta P_{uncertain}=5\%$, then, evidently, the probability will be shifted from initial $P_{certain}=99\%$ to final $P_{uncertain}<99\%$. Similarly, if for initial $P_{certain}=1\%$, the uncertainty of the probability estimation increases from initial $\Delta P_{certain}<<1\%$ to final $\Delta P_{uncertain}=5\%$, then, evidently, the probability will be shifted from initial $P_{certain}=1\%$ to final $P_{uncertain}>1\%$.

A) The increasing of the probability estimation uncertainty (caused by the increasing of the noises) shifts the probability from the bounds to the middle of the probability scale. At high probabilities, the final probability $P_{uncertain}$ will be lower than the initial $P_{certain}$. At low probabilities, the final probability $P_{uncertain}$ will be higher (*without the influence of the second consequence of the principle) than the initial $P_{certain}$.

$\boldsymbol{P_{high\ uncertain} < P_{high\ certain}}$
$\boldsymbol{{}^*P_{low\ uncertain} > P_{low\ certain}}$

B) Ruptures in the probability scale

If the mean-square error, the uncertainty value $\Delta P_{certain}$ of the estimation of the probability P is equal to zero, then the probability $P_{certain}$ can be arbitrarily close to the bound P_{bound} of the probability scale. If, due to noises, the uncertainty value $\Delta P_{uncertain}$ of the estimation of the probability P is finite, then the probability $P_{uncertain}$ can not be closer to the bound P_{bound} of the probability scale than the finite quantity $\delta P_{uncertain}$ (see Harin 2009-2).

Indeed, if the uncertainty of the probability estimation increases from initial value $\Delta P_{certain}<<1\%$ to final value, e.g., $\Delta P_{uncertain}=5\%$, then, evidently, the probability $P_{uncertain}$ can not be closer to the bound P_{bound} of the probability scale than $\delta P_{uncertain}=0.5\%$. So, the probability can not be more than 99.5%. It can not be (*see the second consequence below) less than 0.5% also.

B) Due to noises, there will exist ruptures, gaps, forbidden bands in the probability scale. Or

$\boldsymbol{{}^*|P_{bound} - P_{uncertain}| \geq \delta P_{uncertain}}$

where

$\Delta P_{uncertain} \geq \delta P_{uncertain} \geq O(\Delta P_{uncertain}) \geq const > 0.$

Evidently, the statements A) and B) of the first consequence of the principle are true both for the present and future.

1.1.2. The second consequence of the principle. Incompleteness of the present probability system of future events

The probability of an event, which is not forbidden by objective laws, is more than zero (in the microcosm even virtual events can occur that infringe the laws of conservation). Hence, at any real number of foreseen events, an unforeseen event with the probability more than zero will occur in any forecast or plan. Or

$$\sum P_{foreseen} + \sum P_{unforeseen} = 100\%$$
$$\sum P_{unforeseen} > 0\%$$

Hence

$$\boldsymbol{\sum P_{foreseen} < 100\%}$$

where and further

$\sum P_{foreseen}$ - the sum of estimates of probabilities of all foreseen events;
$\sum P_{unforeseen}$ - the sum of estimates of probabilities of all unforeseen events;

1.1.3. Examples of applications of the principle

In the probability theory the principle provides the statement of existence of ruptures in the probability scale near 0% and 100% due to noises and uncertainties (Harin 2009-2).

In economics for the Allais paradox (Allais 1953), the "fourfold pattern" paradox, risk aversion, loss aversion, overweighting of low probabilities, uniform explanation of choices for both gains and losses, the equity premium puzzle, etc (see, e.g., Di Mauro and Maffioletti 2004) the principle (Formation of ruptures in the probability scale) provides an uniform solution (Harin 2007). For the problems of the incompleteness of systems of preferences, ambiguity aversion, the Ellsberg paradox (Ellsberg 1961), etc the principle (Incompleteness of the probability system) provides uniform solution also (Harin 2007).

In the theory of complex systems the principle provides a possibility of infringement of division into groups of inconsistent events for future events (Karassev 2007).

1.2. Optimal frames of reference

From physics it is well known an event may be described in various frames of reference. Optimal choice of frame of reference is well known to be valuable. When one use various frames of reference, the expressions of transmission between various frames of reference are necessary.

1.3. Formula of forecasting as a framework for forecasts

An unforeseen event can modify an ideally forecasted phenomenon. Hence if a forecast is used after such unforeseen event the forecast should be corrected.

The correcting formula of forecasting represents the correction (in a sense, a framework for forecasts) which should be done for the forecast to be true after unforeseen events have been occurred. In general, corrections may involve corrections of errors and functions.

The correcting formula of forecasting may be also used as an adapting tool in addition to unified and standardized forecasts to take into account distinctive features of specific situations.

1.4. New forecasting resources and areas of application

At present a high-quality forecasting is an expensive work. In addition, in the case of unforeseen events, the forecast can considerably loose its value. Therefore at present, only government and large-size firms may order a high-quality forecasting.

Using the formula of forecasting may considerably increase the time of use of forecasts and, so, decrease the cost of forecasting for customers. Using of formula of forecasting may increase unification and standardization of forecasting and, so, decrease the cost of developing and revisions of forecasts. Eventually, these reasons can lead to wide expansion of forecasting into the areas of municipal needs, middle-size and small-size business and, even, to individual forecasting.

2. Development of the formula. Errors

2.1. Ideal initial circumstances

At ideal initial circumstances (that is at circumstances when foreseen errors may be neglected) for a function $F(t)$ at $t=t_0$, taking into account unforeseen events that may cause an error $\pm\Delta(t_0,t)$, we obtain the forecast of the function $F(t)$

$$F(t) \approx F_{base}(t_0,t) \times [1 \pm \Delta_{error,unforeseen}(t_0,t)]$$

or, omitting variables,

$$F \approx F_{base} \times [1 \pm \Delta_{error,unforeseen}]$$

where and further

$F(t)$ - the corrected forecast for the moment t: $t>t_{corr}>t_0$;

t_0 - the moment, the time when the forecast was created;

t_{corr} - the moment, the time when the forecast is corrected;

$F_{base}(t_0,t)$ - the base forecast;

$\Delta_{error,unforeseen}(t_0,t)$ - the forecast error which is caused by unforeseen events: $\Delta(t_0,t)=0$ at $t\le t_0$ and $\Delta(t_0,t)>0$ at $t>t_0$.

An averaged example: For an averaged case when $F(t)\sim Const$ and for $\Delta_{error,unforeseen}(t_0,t)\sim\theta(t-t_{possible})$ – step-function of an unforeseen event which can occur at a possible moment $t_{possible}>t_0$ with the probability $P_{unforeseen}$: $P_{unforeseen}\times(t-t_0)<<1$ we have:

$$F \approx F_{base}(t_0,t) \times [1 \pm \Delta_{error,unforeseen,linear} \times (t-t_0)]$$

- a linear (at the initial stage) increase of the error with the factor linear of the increase $\Delta_{error,unforeseen,linear}$.

It should be noted, in general case even at ideal initial circumstances, a relative error, caused by unforeseen events, can be considerably more than 1.

Example. Hiroshima 1945

Suppose, in 1930-35 safety's calculation for underground factory, government bomb-proof shelter, etc was needed for the year 1945. The calculation should be based, e.g., on the forecast of maximal power of aircraft bomb for 1945.

Suppose, in 1930-35 the ideal forecast was made. The forecast should be based on maximal weight that bombing aircraft can lift. To 1945, due to the most optimistic forecasts, a bombing aircraft could lift a bombing weight much less than 20 tons and even less in trotyl equivalent. In 1945 Hiroshima was bombed by the 4-tons atomic bomb. But it was 20000 tons in trotyl equivalent.

The prerequisite of an atomic bomb (the division of uranium) was discovered in 1938. Naturally, in 1930-35 it was the unforeseen event.

So, in this case the relative error, caused by unforeseen event, is more than 1000 (more than 100000%).

2.2. Non-ideal initial circumstances

In a general case errors may be taken into account as

$$\Delta_{error,total} \equiv \Delta_{error} = \Delta_{error}(\delta_{error,foreseen}(t_0,t),\Delta_{error,unforeseen}(t_{corr},t))$$

where and further

$\Delta_{error,total} \equiv \Delta_{error}$ - the total relative error;
$\delta_{error,foreseen}(t_0,t)$ - the foreseen relative error;
$\Delta_{error,unforeseen}(t_{corr},t)$ - the foreseen relative error;
t_{corr} - the moment of correction of forecast;

3. Development of the formula. Functions

3.1. Additive and multiplicative functions

Corrections may be expressed in a form of additive functions:

$$F \approx F_{corr}(F_{base},\{\Phi_{addit,i}\},\Delta_{error})$$

$$F \approx [F_{base} + \sum_{i=1}^{I}\Phi_{addit,i}]\times[1 \pm \Delta_{error}]$$

where and further

F_{corr} - the correcting function;
$\{\Phi_{addit,i}\}$ - the set of additive functions;
$\sum\Phi_{addit,i}$ - the sum of additive functions;
$i,l,m,\ldots$ - indices in sets, sums and products;
$I,L,M,\ldots$ - maximal values of indices in sets, sums and products.

In a number of cases a form of multiplicative functions may be a more optimal choice of frame of reference such as:

$$F \approx F_{corr}(F_{base},\{\Phi_{multiplicat,m}\},\Delta_{error})$$

$$F \approx [F_{base} \times \prod_{m=1}^{M} K_{multiplicat,m}]\times[1 \pm \Delta_{error}]$$

where and further

$\{\Phi_{multiplicat,m}\}$ - the set of multiplicative functions;
$\prod K_{multiplicat,m}$ - the product of multiplicative functions.

3.2. Other functions

In a number of cases a form of other functions may be a more optimal choice of frame of reference such as:

$$F \approx F_{corr}(F_{base}, \{F_{special,i}\}, \{F_{foreseen,k}\}, \{\Phi_{unforeseen,l}\}, \Delta_{error})$$

or

$$F \approx F_{corr}(F_{base}, \{F_{special,i}\}, \{\Phi_{periodic,m}\}, \{\Phi_{internal,n}\}, \{\Phi_{external,r}\}, \Delta_{error})$$

and so on,

where and further

$\{F_{special,i}\}$	- the set of specializing, specifying, adapting, concretizing functions to specialize, specify unified and standardized forecasts to special, specific forecasting objects and situations;
$\{F_{foreseen,k}\}$	- the set of functions for foreseen corrections;
$\{\Phi_{unforeseen,l}\}$	- the set of functions for unforeseen corrections;
$\{\Phi_{periodic,m}\}$	- the set of periodic functions;
$\{\Phi_{internal,n}\}$	- the set of internal (relative to the object) functions;
$\{\Phi_{external,r}\}$	- the set of external (relative to the object) functions.

3.3. Versions of the formula. Transformations

Versions of the formula

The most general form of the formula may be written, e.g., as:

$$F(t) \approx F_{corr}(F_{base}(t_0, t), \{\Delta_{error}(t_0, t_{corr}, t)_i\})$$

or, omitting variables and index,

$$F \approx F_{corr}(F_{base}, \{\Delta_{error}\})$$

More detailed (and on several lines)

$$F(t) \approx F_{corr}(F_{base}(t_0, t, \{X_{input,i}(t_0)\}), \{F_{special,k}(t_0, t, \{X_{input,l}(t_0)\})\},$$

$$\{F_{foreseen,m}(t_0, t, \{X_{input,n}(t_0)\}, \{X_{input,p}(t_{corr})\})\},$$

$$\{\Phi_{unforeseen,q}(t_{corr}, t, \{X_{input,r}(t_{corr})\})\},$$

$$\{\delta_{error,foreseen,s}(t_0, t)\}, \{\Delta_{error,unforeseen,t}(t_{corr}, t)\})$$

where and further

$\{X_{input,i}(t_0)\}$	- the set of input data at the moment t_0;
$\{X_{input,p}(t_{corr})\}$	- the set of input data at the moment t_{corr};

or, omitting variables and indexes,

$$F \approx F_{corr}(F_{base}, \{F_{special}\}, \{F_{foreseen}\}, \{\Phi_{unforeseen}\}, \{\Delta_{error}\})$$

For simple cases, when the sets may be substituted by their leading terms, or for cases, when a simplified description is needed, the formula may be written as

$$F \approx F_{corr}(F_{base}, F_{special}, F_{foreseen}, \Phi_{unforeseen}, \Delta_{error})$$

For a sufficiently general case

$$F \approx F_{corr}(F_{base}, \{F_{special}\}, \{F_{addit}\}, \{F_{multiplicat}\}, \{\Phi_{addit}\}, \{\Phi_{multiplicat}\}, \Delta_{error})$$

a particular form of the formula may be written

$$F \approx [F_{base} \times \prod_{i=1}^{I} K_{multiplicat,i} + \sum_{l=1}^{L} \Phi_{addi,l}] \times [1 \pm \Delta_{error}]$$

where and further

$\{F_{multiplicat}\}$	- the set of multiplicative foreseen functions;
$\{F_{addit}\}$	- the set of additive foreseen functions;
$\prod K_{multiplicat,m}$	- the product of multiplicative functions for specializing, foreseen and unforeseen corrections (coefficients);
$\sum \Phi_{addit,i}$	- the sum of additive functions for specializing, foreseen and unforeseen (absolute) corrections (the proportions between specializing, foreseen, unforeseen, multiplicative and additive functions are determined by the optimal choice of frame of reference);

or, if $F_{base} \times \prod K_{multiplicat,i} \neq 0$, preferentially for $F \sim F_{base}$, this may be written as

$$F \approx F_{base} \times [\prod_{i=1}^{I} (1 + k_{multiplicat,i})] \times [1 + \sum_{l=1}^{L} \varphi_{addi,l}] \times [1 \pm \Delta_{error}]$$

where and further

$1 + k_{multiplicat,i}$	- the multiplicative function for specializing, foreseen and unforeseen corrections;
$\varphi_{addit,l}$	- the additive function for specializing, foreseen and unforeseen (relative) corrections (normalized on $F_{base} \times \prod K_{multiplicat,i}$).

Transformations

Let us write transformations between versions

$$F \approx [F_{base} \times \prod_{i=1}^{I} K_{multiplicat,i} + \sum_{l=1}^{L} \Phi_{addit,l}] \times [1 \pm \Delta_{error}]$$

and

$$F \approx F_{base} \times [\prod_{i=1}^{I} (1 + k_{multiplicat,i})] \times [1 + \sum_{l=1}^{L} \varphi_{addit,l}] \times [1 \pm \Delta_{error}] .$$

For multiplicative functions

$$K_{multiplicat,i} = 1 + k_{multiplicat,i} .$$

For additive functions

$$\Phi_{addit,l} = \varphi_{addit,l} \times F_{base} \times [\prod_{i=1}^{I} (1 + k_{multiplicat,i})] .$$

Conclusions

It follows from the principle of uncertain future:

1) A forecast should manifestly contain errors' terms. A long-term forecast should manifestly contain unforeseen errors' terms (because the relative error, caused by an unforeseen event, can be much more than 100%).

2) A long-use forecast should contain correcting terms. These correcting terms may have the form of a framework for forecasts – a general correcting formula of forecasting.

The general correcting formula of forecasting may be used as a correcting tool for long-use forecasts and as an adapting tool in addition to unified and standardized forecasts to apply them to special objects or in special situations.

Using of the correcting formula can provide new forecasting resources and areas of application, including economic forecasting, wide expansion of forecasting into the areas of municipal needs, middle-size and small-size business and, even, to individual forecasting.

The most general form of the formula may be written, e.g., as (omitting indices everywhere):

$$F(t) \approx F_{corr}(F_{base}(t_0, t), \{\Delta_{error}(t_0, t_{corr}, t)\})$$

or, omitting variables,

$$F \approx F_{corr}(F_{base}, \{\Delta_{error}\})$$

More detailed (and on several lines)

$$F(t) \approx F_{corr}(F_{base}(t_0, t, \{X_{input}(t_0)\}), \{F_{special}(t_0, t, \{X_{input}(t_0)\})\},$$
$$\{F_{foreseen}(t_0, t, \{X_{input}(t_0)\}, \{X_{input}(t_{corr})\})\},$$
$$\{\Phi_{unforeseen}(t_{corr}, t, \{X_{input}(t_{corr})\})\},$$
$$\{\delta_{error,foreseen}(t_0, t)\}, \{\Delta_{error,unforeseen}(t_{corr}, t)\})$$

or, omitting variables,

$$F \approx F_{corr}(F_{base}, \{F_{special}\}, \{F_{foreseen}\}, \{\Phi_{unforeseen}\}, \{\Delta_{error}\})$$

For a sufficiently general case

$$F \approx F_{corr}(F_{base}, \{F_{special}\}, \{F_{addit}\}, \{F_{multiplicat}\}, \{\Phi_{addit}\}, \{\Phi_{multiplicat}\}, \Delta_{error})$$

a particular form of the formula may be written

$$F \approx [F_{base} \times \prod K_{multiplicat} + \sum \Phi_{addit}] \times [1 \pm \Delta_{error}]$$

For cases when $F_{base} \times \prod K_{multiplicat,i} \neq 0$, preferentially for $F \sim F_{base}$, this may be written as:

$$F \approx F_{base} \times [\prod (1 + k_{multiplicat})] \times [1 + \sum \varphi_{addit}] \times [1 \pm \Delta_{error}],$$

References

Allais, M. (1953) “Le comportement de l'homme rationnel devant le risque: critique des postulats et axiomes de l'école Américaine” Econometrica 21, 503-46.

Di Mauro, C. and Maffioletti, A. (2004) “Attitudes to risk and attitudes to uncertainty: experimental evidence” Applied Economics, 36, 357-372.

Ellsberg, D. (1961) Risk, Ambiguity and the Savage Axioms. Quarterly Journal of Economics, 75, 643-669.

Harin, A. (2009-2) “About existance of ruptures in the probability scale. Calculation of ruptures’ values” accepted to Ninth International Scientific School "Modelling and Analysis of Safety and Risk in complex systems" 2009. (in Russian)

Harin, A. (2009) “To development of a correcting formula of forecasting for complex systems” accepted to Ninth International Scientific School "Modelling and Analysis of Safety and Risk in complex systems" 2009. (in Russian)

Harin, A. (2008) “To development of a general formula of forecasting” 51 scientific conference of MIPT “Modern problems of fundamental and applied sciences” 2008. (in Russian)

Harin, A. (2007) “ Principle of uncertain future, examples of its application in economics, potentials of its applications in theories of complex systems, in set theory, probability theory and logic” Seventh International Scientific School "Modelling and Analysis of Safety and Risk in complex systems" 2007. (in Russian)

Harin, A. (2005) “A new approach to solve old problems” Game Theory and Information from Economics Working Paper Archive at WUSTL, 0505005.

Hey, J. and Orme, C. (1994) “Investigating Generalizations of Expected Utility Theory Using Experimental Data” Econometrica, 62, 1291-1326.

Karassev, V. (2007) private communication.

Kasa, K. (2000) “A robust Hansen-Sargent prediction formula” Federal Reserve Bank of San Francisco 2000.

Novosyolov A. (2009) private communication.

Quiggin, J. (2005) “The precautionary principle in environmental policy and the theory of choice under uncertainty” No WPM05_3, Murray-Darling Program Working Papers from Risk and Sustainable Management Group, University of Queensland.

Tsay, R. (2008) “Lecture 7: Forecasting, Time Series Analysis” http://faculty.chicagobooth.edu/ruey.tsay/teaching/bs41910/lec7.pdf