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In this paper we test for deterministic chaos (i.e., nonlinear deterministic processes which look random) in seven Mont Belview, Texas hydrocarbon markets, using monthly data from 1985:1 to 1996:12—the markets are those of ethane, propane, normal butane, iso-butane, naptha, crude oil, and natural gas. In doing so, we use the Lyapunov exponent estimator of Nychka, Ellner, Gallant, and McCaffrey (1992). We conclude that there is evidence consistent with a chaotic nonlinear generation process in all five natural gas liquids markets.

I. INTRODUCTION

In recent years, interest in deterministic chaos (i.e., nonlinear deterministic processes which look random) has increased tremendously and the literature is still growing. Besides its obvious intellectual appeal, chaos represents a radical change of perspective in the explanation of fluctuations observed in economic and financial time series. In this view, the fluctuations and irregularities observed in such series receive an endogenous explanation and are traced back to the strong nonlinear deterministic structure that can pervade the economic system. Moreover, if chaos can be shown to exist, the implication would be that (nonlinearity-based) prediction is possible (at least in the short run and provided the actual generating mechanism is known exactly). Prediction, however, over long periods is all but impossible, due to the "sensitive dependence on initial conditions" property of chaos.

Until recently, chaotic dynamics had been studied almost exclusively by theoreticians. However, theorizing might be viewed (by economists) as empty if there is no evidence of chaos in macroeconomic and financial time series.

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Therefore, a number of researchers have recently focused on testing for nonlinearity in general and chaos in particular in economic and financial time series, with encouraging results, especially in the case of financial time series. For example, Scheinkman and LeBaron (1989) studied United States weekly returns on the Center for Research in Security Prices (CRSP) value-weighted index, and found rather strong evidence of nonlinearity and some evidence of chaos. Some similar results have been obtained by Frank and Stengos (1989), investigating daily prices for gold and silver. More recently, Serletis and Gogas (1997) test for chaos in seven East European black-market exchange rates and find evidence consistent with a chaotic nonlinear generation process in two out of the seven series—the Russian ruble and East German mark. Barnett and Serletis (1999) provide a state-of-the-art review of this literature.

In this paper we test for deterministic chaos in North American hydrocarbon markets. In doing so, we use monthly data, from 1985:1 to 1996:12, on Mont Belview, Texas ethane (C2), propane (C3), normal butane (nC4), iso-butane (iC4), naptha (C5), crude oil, and natural gas prices. In the last decade, the North American hydrocarbon industry has seen a dramatic transformation from a highly regulated environment to one which is more market-driven, and this transition has led to the emergence of different markets (especially for natural gas and natural gas liquids) throughout North America—see Serletis (1997), for example, for more details. However, capacity constraints seem to be distorting these markets raising the possibility of chaotic prices behavior, arising from within the structure of these markets.

The paper is organized along the following lines. Section II provides some background regarding North American hydrocarbon markets. Section III discusses some basic data facts and investigates the univariate time series properties of Belview hydrocarbon prices, interpreting the results in terms of the permanent/temporary nature of shocks. Section IV provides a description of the key features of the Nychka et al. (1992) Lyapunov exponent estimator, focusing explicit attention on the test's ability to detect chaos. Section V presents the results of the chaos tests and the final section concludes with some suggestions for potentially useful future empirical research.

II. BACKGROUND

The raw natural gas that comes from wells consists mainly of methane (C_1). However, it also contains various quantities of other heavier hydrocarbons such as ethane (C_2), propane (C_3), butane (C_4), and pentane plus (C_5^+)—the subscripts correspond to the number of carbon atoms that the respective gas molecule contains. Moreover, butane can take one of two forms (isomers), normal butane (nC_4) and isobutane (iC_4). These heavier products (with respect to methane) are collectively known as natural gas liquids (NGLs), with C_3 and C_4 often referred to as liquefied petroleum gases (LPGs).

NGLs are extracted from raw natural gas in mixed streams. For example, a C_2^+ stream contains C_2, C_3, C_4, and C_5 while a C_3^+ stream contains all of the above except C_2. In fact, some liquids extraction from raw natural gas is necessary in order to meet minimum (gas) pipeline quality specifications. Also, the majority of the C_3^+ is removed from raw natural gas to prevent condensation of these liquids in gas pipelines. Of course, the amount of processing depends on how 'wet' or 'dry' the raw gas is—gas that is rich in NGLs is referred to as 'wet,' whereas gas with a lower than average NGL content is referred to as 'dry' or 'lean.'

Liquids production depends on raw natural gas production, which depends on geographic distribution across basins. In the last decade, the North American natural gas industry has seen a dramatic transformation from a highly regulated industry to one which is more market-driven. The transition to a less regulated, more market-oriented environment has led to the emergence of different spot markets throughout North America. In particular, producing area spot markets have emerged in Alberta, British Columbia, Rocky Mountain, Anadarko, San Juan, Permian, South Texas, and Louisiana basins. Moreover, production sites, pipelines and storage services are more accessible today, thereby ensuring that changes in market demand and supply are reflected in prices on spot, futures, and swaps markets.

Liquids markets, however, have their own dynamics. For example, the fuels do not compete at any of the major burnertips and what has been done to restructure the North American natural gas business has little to do with liquids markets. Capacity constraints, however, that distort North American natural gas markets impact production of natural gas and thus processed liquids. For example, the development of spot markets for natural gas and of storage facilities has had an effect on propane markets, especially the use of propane for peaking and enriching of lean gas streams. Also, on the demand side, there is not a large consumer market for liquids in the United States and Canada, in the sense that liquids are not a primary domestic or commercial fuel, like they are in other countries.

Our objective in this study is not to examine how the North American hydrocarbon markets are linked together, but to test for deterministic chaos in North American hydrocarbon markets, using Mont Belview, Texas spot prices. One of the most interesting aspects of Belview prices is that they are 'marker' prices for traders from many countries. For example, liquids traders at Petrobras, Brazil's national oil company, use Belview in all of their trading formulas. Moreover, international trading activity is important in the formation of liquids prices at Belview. Brazil, for example, is a huge importer of liquids from the United States (and elsewhere), and liquids constitute almost 80% of domestic fuel use in Brazil (and about 90% in Mexico), suggesting that liquids prices at Belview have more to do with trading factors overseas than with North America.

In what follows, we turn to a discussion of some basic facts and to an investigation of the univariate time series properties of Belview hydrocarbon prices. In Section IV, we consider univariate statistical tests for nonlinearity and chaos that have been recently motivated by the mathematics of deterministic nonlinear dynamical systems.

III. BASIC FACTS AND INTEGRATION TESTS

One interesting feature of Belview hydrocarbon prices is the contemporaneous correlation between these prices. These correlations are reported in Table 1 for log levels and in Table 2 for first differences of log levels. To determine whether these correlations are statistically significant, Pindyck and Rotemberg (1990) is followed and a likelihood ratio test of the hypotheses that the correlation matrices are equal to the identity matrix is performed. The test statistic is

$$-2\ln(|R|^{N/2})$$

where $|R|$ is the determinant of the correlation matrix and N is the number of observations. This test statistic is distributed as χ^2 with $0.5q(q-1)$ degrees of freedom, where q is the number of series.

Table 1. Contemporaneous Correlations Between Logged Prices

	C2	C3	nC4	iC4	C5	Crude oil	Natural gas
C2	1						
C3	0.767	1					
nC4	0.686	0.906	1				
iC4	0.588	0.821	0.923	1			
C5	0.611	0.766	0.869	0.928	1		
Crude oil	0.547	0.701	0.823	0.890	0.956	1	
Natural gas	0.431	0.437	0.396	0.278	0.289	0.266	1
			$\chi^2(21)$ = 1353.50				

Note: Monthly data: 1985:1 - 1996:12

Table 2.
Contemporaneous Correlations Between Differenced (logged) Prices

	C2	C3	nC4	iC4	C5	Crude oil	Natural gas
C2	1						
C3	0.785	1					
nC4	0.702	0.811	1				
iC4	0.617	0.725	0.828	1			
C5	0.646	0.708	0.777	0.803	1		
Crude oil	0.582	0.621	0.701	0.703	0.862	1	
Natural gas	0.222	0.172	0.121	0.011	0.005	0.035	1
			$\chi^2(21)$ = 849.57				

Note: Monthly data: 1985:2 - 1996:12

The test statistic is 1353.50 with a p-value of 0.000 for the logged hydrocarbon prices in Table 1, suggesting that the hypothesis that Belview hydrocarbon prices are uncorrelated in log levels is rejected. Turning now to Table 2, we see that the test statistic is 849.57 with a p-value of 0.000 for the first differences of the logged prices. Clearly, the null hypothesis that these prices are uncorrelated in first differences of log levels is also rejected.

The first step in testing for nonlinearity and chaos is to test for the presence of a stochastic trend (a unit root) in the autoregressive representation of each individual series. Nelson and Plosser (1982) argue that most macroeconomic and financial time series have a unit root (a stochastic trend), and describe this property as one of being "difference stationary" (DS) so that the first difference of a time series is stationary. An alternative "trend stationary" model (TS) has been found to be less appropriate.

In what follows we test the null hypothesis of a stochastic trend against the trend-stationary alternative by estimating by ordinary least-squares (OLS) the following augmented Dickey-Fuller (ADF) type regression (see Dickey and Fuller, 1981).

$$\Delta \log y_t = a_0 + a_2 t + \gamma \log y_{t-1} + \sum_{j=1}^{k} b_j \Delta \log y_{t-j} + \varepsilon_t \qquad (1)$$

where Δ is the difference operator. The k extra regressors in (1) are added to eliminate possible nuisance parameter dependencies in the limit distributions of the test statistics caused by temporal dependencies in the disturbances. The optimal lag length (that is, k) is taken to be the one selected by the Akaike information criterion (AIC) plus 2—see Pantula et al. (1994) for details regarding the advantages of this rule for choosing the number of augmenting lags in equation (1).

Table 3 presents the results. The first column of Table 3 gives the optimal value of k in equation (1), based on the AIC plus 2 rule, for each price series. This identifies k to be 3 for C2, nC4, iC4, and C5, 4 for C3, 5 for crude oil, and 10 for natural gas. The t-statistics for the null hypothesis $\gamma = 0$ in equation (1) are given under τ_τ in Table 3. Under the null hypothesis that $\gamma = 0$, the appropriate critical value of τ_τ at the 5% level (with 100 observations) is -3.45—see Fuller (1976, Table 8.5.2). Hence, the null hypothesis of a unit root cannot be rejected for all series.

Table 3. Unit Root Test Results

		Test statistics				
Series	k	τ_τ	$t(a_2)$	ϕ_3	τ_μ	Decision
C2	3	-3.09	1.66	5.48	-2.75	I(1)
C3	4	-2.59	2.43	4.64	-1.67	I(1)
nC4	3	-3.33	1.53	6.42	-3.13*	I(0)
iC4	3	-2.83	1.11	4.83	-2.82	I(1)
C5	3	-3.26	0.95	6.22	-3.32*	I(0)
Crude oil	5	-3.20	0.94	6.03	-3.23*	I(0)
Natural gas	10	-1.80	2.74	4.94	-1.16	I(1)

Notes: Monthly data, 1985:1 - 1996:12. All the series are in logs. An asterisk indicates rejection of the null hypothesis at the 5% significance level. τ_τ is the t-statistic for the null hypothesis $\gamma = 0$ in equation (1). Under the null hypothesis, the appropriate critical value of τ_τ at the 5% significance level (with 100 observations) is -3.45—see Fuller (1976, Table 8.5.2). $t(a_2)$ is the t-statistic for the presence of the time trend (i.e., the null hypothesis $a_2 = 0$) in equation (1), given the presence of a unit root. The appropriate 95% critical value for $t(a_2)$, given by Dickey and Fuller (1981), is 2.79. The ϕ_3 statistic tests the joint null $a_2 = \gamma = 0$ in equation (1). The 95% critical value, given by Dickey and Fuller (1981) is 6.49. Finally, τ_μ is the t-statistic for the null $\gamma = 0$ in equation (2). The appropriate 95% critical value of τ_μ is -2.89—see Dickey and Fuller (1976, Table 8.5.2).

Since the null hypothesis of a unit root hasn't been rejected, there is a question concerning the test's power in the presence of the deterministic part of the regression (i.e., $a_0 + a_2t$). In particular, one problem is that the presence of the additional estimated parameters reduces degrees of freedom and the power of the test—reduced power means that we will conclude that the process contains a unit root when, in fact, none is present. Another problem is that the appropriate statistic for testing $\gamma = 0$ depends on which regressors are included in the model.

Although we can never be sure of the actual data-generating process, here we follow the procedure suggested by Doldado et al. (1990) for testing for a unit root when the form of the data-generating process is unknown. In particular, since the null hypothesis of a unit root is not rejected, it is necessary to determine whether too many deterministic regressors are included in equation (1). We therefore test for the significance of the trend term in equation (1) under the null of a unit root, using the $t(a_2)$ statistic in Table 3. Under the null that $a_2 = 0$ given the presence of a unit root, the appropriate critical value of $t(a_2)$ at the 5% significance level is 2.79—see Dickey and Fuller (1981). Clearly, the null cannot be rejected, suggesting that the trend is not significant. The ϕ_3 statistic which tests the joint null hypothesis $a_2 = \gamma = 0$ reconfirms this result.

This means that we should estimate the model without the trend, i.e., in the following form

$$\Delta \log y_t = a_0 + \gamma y_{t-1} + \sum_{j=1}^{k} b_j \Delta \log y_{t-j} + \varepsilon_t \tag{2}$$

and test for the presence of a unit root using the τ_μ statistic. The results, reported in Table 3, indicate that the null hypothesis of a unit root is now rejected for nC4, C5, and crude oil. The remaining series do contain a unit root, based on this unit root testing procedure. Our decision regarding the univariate time series properties of these series is summarized in the last column of Table 3.

IV. TESTS FOR CHAOS

Recently, five highly regarded tests for nonlinearity or chaos (against various alternatives) have been introduced—see Barnett et al. (1995, 1997) for a detailed discussion. All five of the tests are purported to be useful with noisy data of moderate sample sizes. The tests are the Hinich (1982) bispectrum test, the BDS (Brock, Dechert, Scheinkman, and LeBaron, 1996) test, White's (1989) neural network test, Kaplan's (1994) test, and the Nychka, Ellner, Gallant, and McCaffrey (1992) dominant Lyapunov exponent estimator. Another very promising test [that is, similar in some respects to the Nychka, et al. (1992) test] has also been recently proposed by Gencay and Dechert (1992).

It is to be noted, however, that the Hinich bispectrum test, the BDS test, White's test, and Kaplan's test are currently in use for testing nonlinear dependence [whether chaotic (i.e., nonlinear deterministic) or stochastic], which is necessary but not sufficient for chaos. Only the Nychka et al. (1992) and the Gencay and Dechert (1992) tests are specifically focused on chaos as the null hypothesis. In what follows, we only apply the Lyapunov exponent estimator of Nychka et al. (1992). This is a Jacobian-based method involving the use of a neural net to estimate a map function by nonlinear least squares, and subsequently the use of the estimated map and the data to produce an estimate of the dominant Lyapunov exponent. We first describe this test, following Serletis and Gogas (1997).

We assume that the data $\{x_t\}$ are real-valued and are generated by a nonlinear autoregressive model of the form

$$x_t = f(x_{t-L}, x_{t-2L}, \dots, x_{t-mL}) + e_t \qquad (3)$$

where L is the time-delay parameter, m is the length of the autoregression, and e_t is a sequence of zero mean (and unknown constant variance) independent random variables. A state-space representation of (3) can be written as follows

$$\begin{pmatrix} x_t \\ x_{t-L} \\ \cdot \\ \cdot \\ \cdot \\ x_{t-mL+L} \end{pmatrix} = \begin{pmatrix} f(x_{t-L}, \dots, x_{t-mL}) \\ x_{t-L} \\ \cdot \\ \cdot \\ \cdot \\ x_{t-mL+L} \end{pmatrix} + \begin{pmatrix} e_t \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{pmatrix}$$

or equivalently,

$$X_t = F(X_{t-L}) + E_t \qquad (4)$$

where

$X_t = (x_t, x_{t-L}, \dots, x_{t-mL+L})^T$, $F(X_{t-L}) = f\big((x_{t-L}, \dots, x_{t-mL}), x_{t-L}, \dots, x_{t-mL+L}\big)^T$,

and $E_t = (e_t, 0, \dots, 0)^T$.

The definition of the dominant Lyapunov exponent, λ, can be formulated more precisely as follows. Let X_0, $X'_0 \in R^m$ denote two "nearby" initial state vectors. After M iterations of model (4) with the same random shock we have (using a truncated Taylor approximation)

$$\| X_M - X_M' \| = \| F^M(X_0) - F^M(X_0') \| \approx \| (DF^M)_{X_0}(X_0 - X_0') \|$$

where F^M is the Mth iterate of F and $(DF^M)_{X_0}$ is the Jacobian matrix of F evaluated at X_0. By application of the chain rule for differentiation, it is possible to show that

$$\| X_M - X_M' \| \approx \| T_M(X_0 - X_0') \|$$

where $T_M = J_M J_{M-1} \ldots J_1$ and $J_t = (DF^M)_{X_t}$. Letting $v_1(M)$ denote the largest eigenvalue of $T_M^T T_M$ the formal definition of the dominant Lyapunov exponent, λ, is

$$\lambda = \lim_{M \to \infty} \frac{1}{2M} \ln |v_1(M)|$$

In this setting, λ gives the long-term rate of divergence or convergence between trajectories. A positive λ measures exponential divergence of two nearby trajectories [and is often used as a definition of chaos—see, for example, Deneckere and Pelikan (1986)], whereas a negative λ measures exponential convergence of two nearby trajectories.

In the next section we use the Nychka et al. (1992) Jacobian-based method and the LENNS program [see Ellner et al. (1992)] to estimate the dominant Lyapunov exponent. In particular we use a neural network model to estimate f by nonlinear least squares, and use the estimated map $\hat{f}$ and the data $\{x_t\}$ to produce an estimate of the dominant Lyapunov exponent. In doing so, we follow the protocol described in Nychka et al. (1992).

The predominant model in statistical research on neural nets is the single (hidden) layer feedforward network with a single output. In the present context it can be written as

$$\hat{f}(X_t,\theta) = \alpha + \sum_{j=1}^{k} \beta_j \psi(\omega_j + \gamma_j^T X_t)$$

where $X \in R^m$ is the input, ψ is a known (hidden) univariate nonlinear "activation function" [usually the logistic distribution function $\psi(u) = 1/(1 + \exp(-u))$—see, for example, Nychka et al. (1992) and Gencay and Dechert (1992)], $\theta = (\alpha,\beta,\omega,\gamma)$ is the parameter vector, and $\gamma_j = (\gamma_{1j}, \gamma_{2j}, \ldots, \gamma_{mj})^T$. $\beta \in R^k$ represents hidden unit weights and $\omega \in R^k$, $\gamma \in R^{kxm}$ represent input weights to the hidden units. k is the number of units in the hidden layer of the neural net. Notice that there are $[k(m+2)+1]$ free parameters in this model.

Given a data set of inputs and their associated outputs, the network parameter vector, θ, is fit by nonlinear least squares to formulate accurate map estimates. As appropriate values of L, m, and k, are unknown, LENNS selects the value of the triple (L,m,k) that minimizes the Bayesian Information Criterion (BIC)—see Schwartz (1978). Gallant and White (1992) have shown that we can then use $\hat{J}_t$, the estimate of the Jacobian matrix J_t obtained from the approximate map $\hat{f}$, as a nonparametric estimator of J_t. The estimate of the dominant Lyapunov exponent then is

$$\hat{\lambda} = \frac{1}{2N} \ln|\hat{\nu}_1(N)|$$

where $\hat{\nu}_1(N)$ is the largest eigenvalue of $T_N^T T_N$ and where $\hat{T}_N = \hat{J}_N \hat{J}_{N-1} \ldots \hat{J}_1$.

V. EMPIRICAL RESULTS

Before conducting nonlinear dynamical analysis the data must be rendered stationary, delinearized (by replacing the stationary data with residuals from an autoregression of the data) and transformed (if necessary). Since a stochastic trend has been confirmed for each of C2, C3, iC4, and natural gas, these series are rendered stationary by taking first differences of logarithms. In the case of C4, C5, and crude oil we use the logged series, since these are I(0). Also, since we are interested in nonlinear dependence, we remove any linear dependence in the stationary data by fitting the best possible linear model. In particular, we prefilter the stationary series by the following autoregression

$$z_t = b_0 + \sum_{j=1}^{q} b_j z_{t-j} + \varepsilon_t, \quad \varepsilon_t | I_{t-1} \sim N(0, w_0) \tag{5}$$

using for each series the number of lags, q, for which the Ljung-Box (1978) $Q(36)$ statistic is not significant at the 5% level. This identifies q to be 1 for C2 and nC4, 2 for C3, iC4, C5, and crude oil, and 3 for natural gas–see Table 4.

Table 4. Diagnostics of AR Models Under the Ljung-Box (1978) $Q(36)$ Test Statistic

$$z_t = b_0 + \sum_{j=1}^{q} b_j z_{t-j} + \varepsilon_t, \ \varepsilon_t \mid I_{t-1} \sim N(0, w_0)$$

		AR Error Term Diagnostics (p-values)		
Series	AR Lag, q	Q-statistic	ARCH	J-B
C2	1	0.532	0.025	0.000
C3	2	0.054	0.802	0.000
nC4	1	0.095	0.057	0.000
iC4	2	0.124	0.097	0.002
C5	2	0.840	0.030	0.000
Crude oil	2	0.639	0.049	0.000
Natural gas	3	0.098	0.035	0.000

Notes: The Q-statistic is distributed as a $\chi^2(36)$ on the null of no autocorrelation. ARCH is Engle's (1982) Autoregressive Conditional Heteroskedasticity (ARCH) test distributed as a $\chi^2(1)$ on the null of no ARCH. The Jarque-Bera test statistic is distributed as a $\chi^2(2)$ under the null hypothesis of normality.

Although the autocorrelation diagnostics in Table 4 indicate that the chosen AR models adequately remove linear dependence in the stationary data, the ARCH test suggests the presence of a time-varying variance (except in the case of C3). Since variance-nonlinearity could be generated by either a (stochastic) ARCH process or a deterministic process, in what follows we follow Serletis and Gogas (1997) and model the conditional variance (or predictable volatility) using Bollerslev's (1986) generalized autoregressive conditional heteroskedasticity (GARCH) model and Nelson's (1991) exponential GARCH (EGARCH) model. One important feature of what we are doing, however, is to present the results of a diagnostic test for checking the adequacy of these models and choose among the estimated GARCH and EGARCH models.

The GARCH model is a generalization of the pure ARCH model, originally due to Engle (1982) and is useful in detecting nonlinear patterns in variance while not destroying any signs of deterministic structural shifts in a model-see, for example, Lamoreux and Lastrapes (1990). Using the same AR structure as before we estimate the following GARCH(1,1) model

$$z_t = b_0 + \sum_{j=1}^{q} b_j z_{t-j} + \varepsilon_t, \quad \varepsilon_t | I_{t-1} \sim N(0, \sigma_t^2)$$
$$\sigma_t^2 = w_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{6}$$

where $N(0, \sigma_t^2)$ represents the normal distribution with mean zero and variance σ_t^2. Parameter estimates and diagnostic tests are given in Table 5. First, estimated coefficients of the ARCH term, α_1, and the GARCH term, β_1, are positive and (in general) significant at the 5% level. Also, the Q-test finds no linear dependence and the ARCH test finds no ARCH effects, suggesting that the lag structure of the conditional variance is correctly identified. However, the null hypothesis that $\alpha_1 + \beta_1 = 1$ cannot be rejected, suggesting the presence of integrated variances.

GARCH models assume that the conditional variance in equation (6) is a function only of the magnitude of the lagged residuals and not their signs—i.e., only the size, not the sign, of lagged residuals determines conditional variance. This assumption imposes important limitations on GARCH models. For example, these models are not well suited to capture the so-called "leverage effect." To meet these objections, we use Nelson's (1991) exponential GARCH (1,1), or EGARCH (1,1), also inspired by Engle's (1982) ARCH model, in which the conditional variance σ_t^2 depends on both the size and the sign of lagged residuals as follows

$$\log \sigma_t^2 = w_0 + \beta \log(\sigma_{t-1}^2) + \alpha \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| + \gamma \frac{\varepsilon_{t-1}}{\sigma_{t-1}} .$$

The log transformation ensures that σ_t^2 remains non-negative for all t. Clearly, the impact of the most recent residual is now exponential rather than quadratic.

Parameter estimates and diagnostic tests for the EGARCH (1,1) model are presented in Table 6. In general, the log likelihood for the EGARCH (1,1) model is higher than that for the GARCH (1,1) model, suggesting that the EGARCH model is superior to the GARCH model for these series.

Table 5. GARCH (1,1) Parameter Estimates and Error Term Diagnostics

$$z_t = b_0 + \sum_{j=1}^{q} b_j z_{t-j} + \varepsilon_t, \quad \varepsilon_t | I_{t-1} \sim N(0,\sigma_t^2), \quad \sigma_t^2 = w_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

		GARCH (1,1) Parameter Estimates			GARCH (1,1) Error Term Diagnostics (p-values)					
Series	AR Lag, q	w_0	α_1	β_1	Q-statistic	$Q(\varepsilon^2)$	ARCH	J-B	Log L	$\alpha_1 + \beta_1 = 1$
C2	1	0.000 (1.3)	0.151 (2.4)	0.759 (7.2)	0.877	0.994	0.985	0.000	283.978	0.234
C3	2	0.000 (1.1)	0.053 (0.6)	0.815 (4.5)	0.142	0.999	0.800	0.000	298.829	0.231
nC4	1	0.001 (1.2)	0.132 (1.2)	0.765 (4.2)	0.098	0.964	0.961	0.000	130.292	0.297
iC4	2	0.000 (0.9)	0.091 (0.9)	0.816 (4.2)	0.104	0.769	0.495	0.000	336.847	0.402
C5	2	0.001 (1.3)	0.098 (1.0)	0.676 (3.3)	0.733	0.998	0.928	0.000	159.809	0.147
Crude oil	2	0.001 (2.3)	0.467 (1.4)	0.449 (2.5)	0.064	0.992	0.588	0.013	177.956	0.680
Natural gas	3	0.000 (0.7)	0.920 (3.0)	0.547 (9.1)	0.000	0.997	0.982	0.000	-4.294	0.081

Notes: Numbers in parentheses next to the GARCH (1,1) parameter estimates are absolute t-ratios. The Q-statistic is distributed as a $\chi^2(36)$ on the null of no autocorrelation. The ARCH statistic is distributed as a $\chi^2(1)$ on the null of no ARCH. The Jarque-Bera test statistic is distributed as a $\chi^2(2)$ under the null hypothesis of normality.

Table 6. EGARCH (1,1) Parameter Estimates and Error Term Diagnostics

$$z_t = b_0 + \sum_{j=1}^{q} b_j z_{t-j} + \varepsilon_t, \quad \varepsilon_t \,|I_{t-1} \sim N(0,\sigma_t^2), \quad \log\sigma_t^2 = w_0 + \beta\log(\sigma_{t-1}^2) + \alpha\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| + \gamma\,\frac{\varepsilon_{t-1}}{\sigma_{t-1}}.$$

		EGARCH (1,1) Parameter Estimates				EGARCH (1,1) Error Term Diagnostics (p-values)					
Series	AR Lag, q	w_0	α	γ	β	Q-statistic	$Q(\varepsilon^2)$	ARCH	J-B	Log L	$\beta = 1$
C2	1	-1.068 (1.8)	0.347 (3.0)	-0.014 (0.1)	0.881 (10.9)	0.859	0.971	0.918	0.000	284.824	0.153
C3	2	-13.513(22.9)	-0.004 (0.0)	-0.239 (2.1)	-0.900 (18.5)	0.106	0.983	0.975	0.000	299.714	0.000
nC4	1	-1.083 (1.6)	0.290 (1.6)	0.085 (1.1)	0.809 (6.3)	0.096	0.947	0.937	0.000	130.201	0.139
iC4	2	-6.566 (2.3)	0.475 (2.7)	-0.119 (0.9)	0.185 (0.5)	0.036	0.339	0.799	0.000	337.415	0.030
C5	2	-1.225 (1.4)	0.265 (1.4)	-0.102 (1.1)	0.797 (5.2)	0.609	0.998	0.818	0.000	160.842	0.192
Crude oil	2	-1.220 (2.6)	0.589 (2.0)	-0.063 (0.5)	0.858 (12.9)	0.105	0.999	0.764	0.001	178.376	0.035
Natural gas	3	-1.543 (5.8)	1.271 (4.6)	-0.391 (2.5)	0.812 (26.1)	0.000	0.397	0.288	0.000	10.855	0.000

Notes: Numbers in parentheses next to the EGARCH (1,1) parameter estimates are absolute t-ratios. The Q-statistic is distributed as a $\chi^2(36)$ on the null of no autocorrelation. The ARCH statistic is distributed as a $\chi^2(1)$ on the null of no ARCH. The Jarque-Bera test statistic is distributed as a $\chi^2(2)$ under the null hypothesis of normality.

Table 7. Comparison of Predictive Power for the Conditional Variance of Belview Energy Prices

$$\hat{\varepsilon}_t^2 = b_0 + b_1 \hat{\sigma}_t^2 + \zeta_t$$

	GARCH (1,1) Results				EGARCH (1,1) Results			
Series	b_0	b_1	R^2	Q-statistic	b_0	b_1	R^2	Q-statistic
C2	0.000 (0.6)	0.803 (0.8)	0.064	0.512	0.000 (0.5)	0.823 (0.7)	0.064	0.558
C3	0.000 (0.1)	1.119 (0.2)	0.016	0.923	-0.000 (0.5)	1.302 (0.6)	0.046	0.877
nC4	0.002 (0.5)	0.796 (0.7)	0.045	0.977	0.002 (0.5)	0.818 (0.6)	0.045	0.960
iC4	0.000 (0.6)	0.692 (0.7)	0.019	0.631	0.000 (1.1)	0.590 (1.3)	0.026	0.670
C5	-0.001 (0.2)	1.126 (0.2)	0.031	0.998	-0.001 (0.3)	1.173 (0.4)	0.045	0.999
Crude oil	0.004 (2.2)	0.349 (4.5)	0.041	0.999	0.003 (1.5)	0.528 (2.5)	0.053	0.999
Natural gas	0.118 (0.8)	0.302 (13.3)	0.195	0.392	0.150 (1.5)	0.363 (48.3)	0.846	0.969

Notes: Absolute t-statistics for $b_0 = 0$ and $b_1 = 1$ are in parentheses. R^2 is the coefficient of determination. $Q(36)$ is the Ljung-Box statistic for 36 lags of the residual autocorrelation.

To investigate this further, and in order to choose between GARCH and EGARCH models, we present in Table 7 the results of a diagnostic test suggested by Kearns and Pagan (1993) for checking the adequacy of these models. The test involves the regression of $\hat{\varepsilon}_t^2$ against a constant and the estimated conditional variance $\hat{\sigma}_t^2$ The intercept of such a regression should be zero and the slope coefficient unity.

The insignificant $Q(36)$ statistic in Table 7 indicates that each of these models captures much of the persistence in actual volatility and the coefficient of determination indicates how well the estimated conditional variance predicts the actual variance and is used to compare the GARCH and EGARCH models. On the basis of these results, and a comparison between the log likelihood values in Tables 6 and 7, in what follows we test for chaos using the standardized EGARCH (1,1) residuals—the standardized residuals are defined as $\varepsilon_t / \hat{\sigma}_t$, where ε_t is the residual of the mean equation and $\hat{\sigma}_t^2$ its estimated (time-varying) variance.

Table 8. The Nychka et al. (1992) BIC Selection of the Parameter Triple (*L,m,k*), the Value of the Minimized BIC, and the Dominant Lyapunov Exponent Point Estimate

Series	(*L,m,k*) Triple that Minimizes the BIC	Value of the Minimized BIC	Dominant Lyapunov Exponent Point Estimate
C2	(3,3,2)	1.447	0.056
C3	(2,7,2)	1.292	0.211
nC4	(1,7,2)	1.366	0.081
iC4	(2,6,2)	1.386	0.100
C5	(1,4,2)	1.362	0.068
Crude oil	(1,2,1)	1.427	-1.835
Natural gas	(2,8,1)	1.391	-0.063

Notes: Numbers in parentheses represent the BIC selection of the parameter triple, (*L,m,k*), where *L* is the time delay parameter, *m* is the number of lags in the autoregression and *k* is the number of units in the hidden layer of the neural net.

We now apply the Nychka et al. (1992) Lyapunov exponent test to the standardized residuals. The Bayesian Information Criterion (BIC) point estimates of the dominant Lyapunov exponent for each parameter triple (*L,m,k*) are displayed in Table 8 along with the respective optimized value of the BIC criterion. Clearly, all but two Lyapunov exponent point estimates are positive, supporting the conclusion that all Belview natural gas liquids prices have a chaotic nonlinear generating process.

Of course, the standard errors of the estimated dominant Lyapunov exponents are not known [there has not yet been any published research on the computation of a standard error for the Nychka et al. (1992) Lyapunov exponent estimate]. It is possible, however, to produce sensitivity plots that are informative about precision, as the ones in Figure 1. Figure 1 indicates the sensitivity of the dominant Lyapunov exponent estimate to variations in the parameters, by plotting the estimated dominant Lyapunov exponent for each setting of (L,m,k), where $L=1, 2, 3$, $m=1,\ldots,10$, and $k = 1, 2, 3$.

Figure 1. NEGM Sensitivity Plots

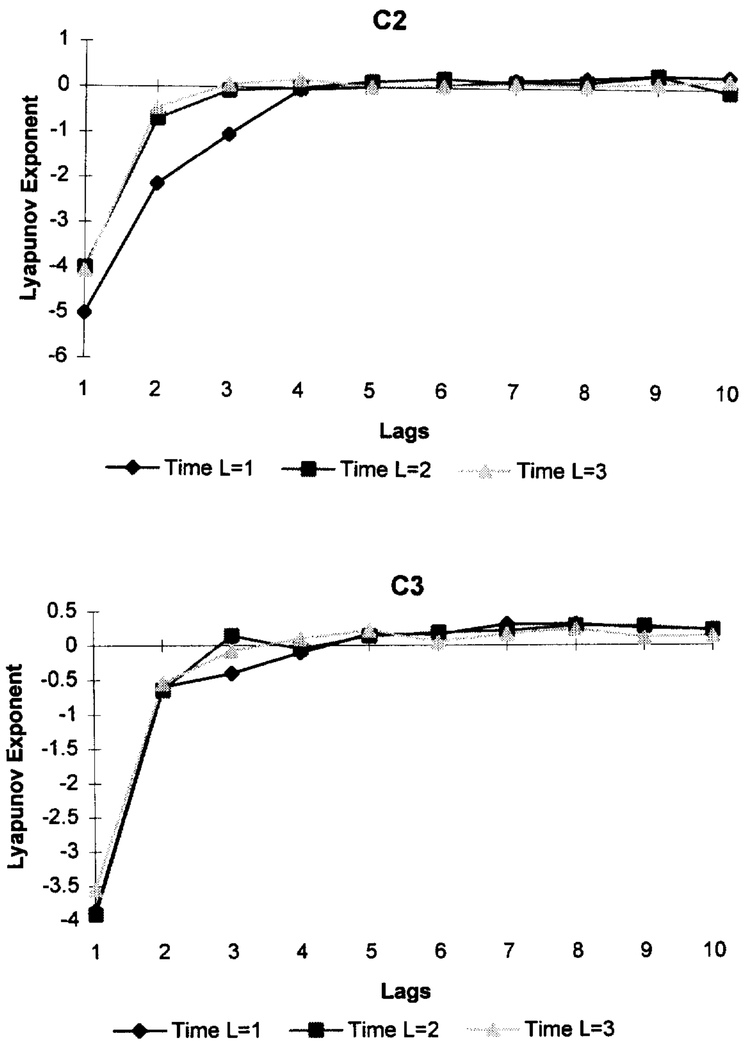
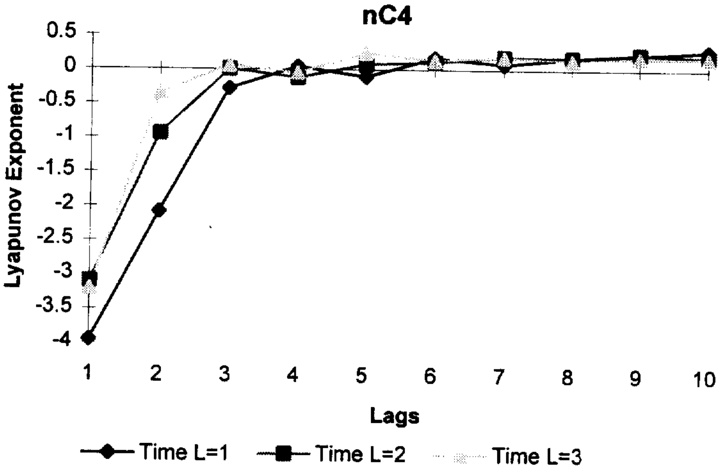


Figure 1. NEGM Sensitivity Plots (continued)



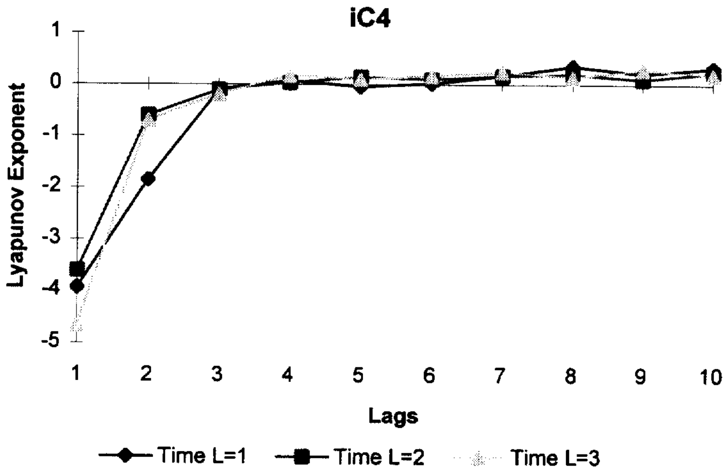
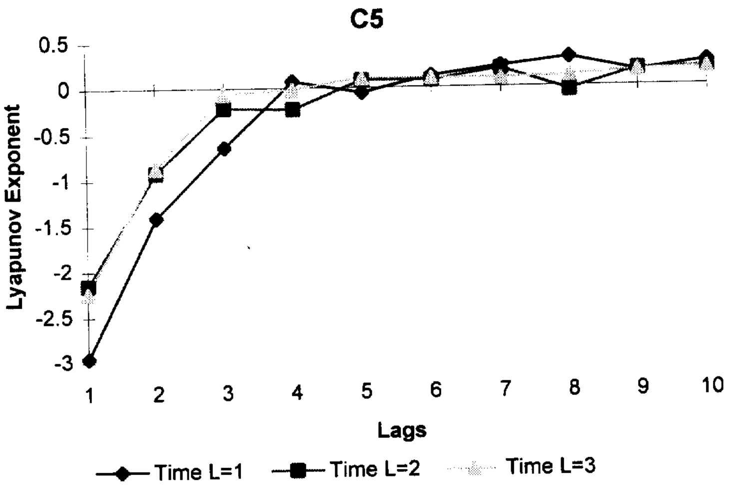


Figure 1. NEGM Sensitivity Plots (continued)



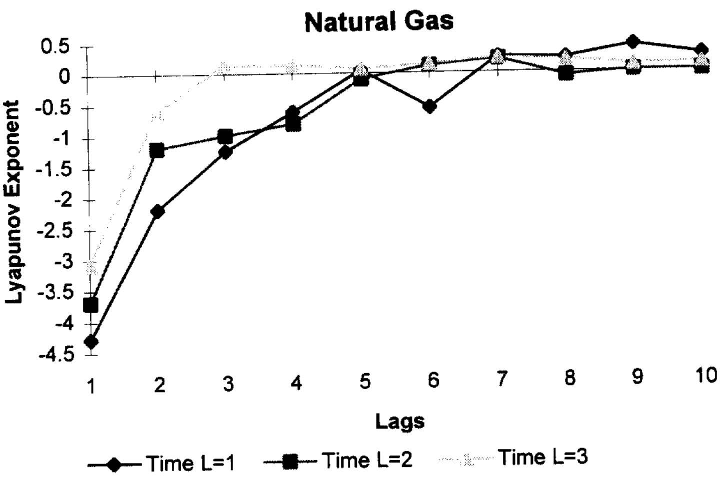
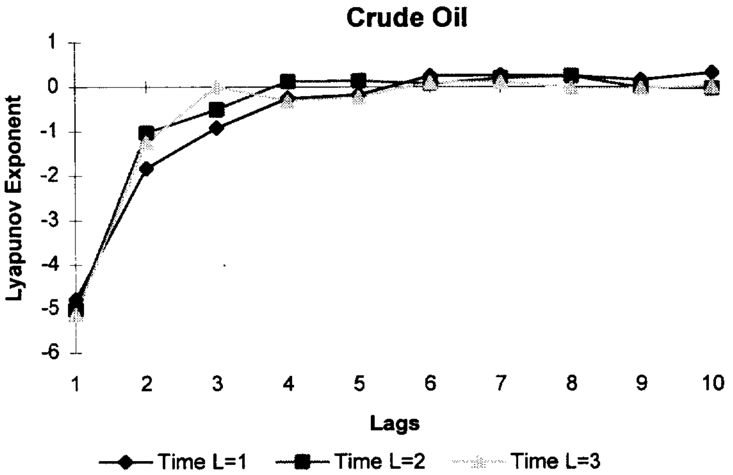


Figure 1. NEGM Sensitivity Plots (continued)



VI. CONCLUSION

We have provided results of nonlinear dynamical analysis of North American hydrocarbon prices using the Nychka et al. (1992) test for positivity of the dominant Lyapunov exponent. Before conducting such a nonlinear analysis, the data were rendered stationary and appropriately filtered, in order to remove any linear as well as nonlinear stochastic dependence.

We have found evidence of nonlinear chaotic dynamics in all five (C2, C3, nC4, iC4, and C5) Belview natural gas liquids markets. In principle, it should be possible to model (by means of differential/difference equations) the nonlinear chaos-generating mechanism and build a predictive model of North American natural gas liquids prices. This is an area for potentially productive future research that will undoubtedly improve our understanding of how North American NGLs prices change over time. See Barnett and Serletis (1999) for more insights regarding this line of research.

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