Homogenous Agent Wage-Posting Model with Wage Dispersion

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ABSTRACT
In the paper we test a homogenous agent version of the Montgomery's (1991) non-cooperative wage posting model. The inclusion of intrinsic costs, related to the uncertainty when changing the alternative agents are already using, alters the outcome of the model in two respects: firstly, it significantly prolongs the convergence-time to the equilibrium, and, more importantly, it may lead to the wage dispersion, irrespective of equally-productive-agent proposition, something not present in the model of Montgomery.

JEL Classification: C7, C15, D8, J3.
Keywords: Job-search model, wage posting, wage dispersion, numerical optimization

INTRODUCTION

Why do some workers earn higher wages than others for doing similar jobs? In the paper we discuss the issue along a simple homogenous-agent model of Montgomery (1991). Employers in the model announce their wage posts in a non-cooperative game and workers apply to the vacancies. All bargaining power is given to employers while workers only direct their job search according to their preferences. All job-postings offer the same expected benefits to applicants, should the latter apply. However, this condition does not prevent employers from posting different wages. The expected benefits as assumed by the model are namely a self-correcting mechanism, because vacancies that offer lower wages attract fewer applicants and are thus more easily accessible than those offering higher wages, keeping expected benefits to the applicants equal.

Although employers[1] prefer such alternatives as to attain the highest outcome and, thus, tend to choose them, they face intrinsic costs, related to the uncertainty when changing the alternative they are already using. As a consequence, employers are not prone of changing their current alternatives, especially when it is expected that the benefit of adopting a new alternative would be quite small. Rubinstein (1998) defines such behavior by the tradeoff between complexity and efficiency of alternatives where agents (employers in our case) prefer efficient and simple alternatives.[2]

We test the effects of such intrinsic costs by introducing a stochastic factor, a “noise”, into the original Montgomery model that affects the decision-making (wage policy) of employers. Then, even the homogeneous-agent model may lead to wage dispersion, something that is not possible in the original Montgomery model, but has been modeled within the framework of Burdett and Judd (1983).

Paper proceeds as follows. The model is developed in the chapter following the introductory remarks. The third chapter brings numerical simulations with a basic description of the numerical algorithm reflecting the computer code used for simulations. Results with graphs and explanation are presented in the fourth chapter. The last chapter concludes.

THE MODEL

The model resembles a simple 2×2 case from Montgomery (1991). Suppose that the labor market consists of two identical workers and two identically productive employers each having only one vacancy. Then both employers non-cooperatively post wages w as to maximize their profits π and each worker is allowed to make only one job application.

In a non-cooperative game-setting as proposed by Montgomery workers apply to the first employer with probability p and to the second employer with probability

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[1] This is true in general but our terminology (and application) in the paper is confined solely to decisions taken by employers.

[2] For the literature review on the role of behavioral studies on the decision-making, see Hirshleifer (2001).

$(1-p)$.[3] If both agents apply to the same vacancy, then the employer randomly chooses one applicant while leaving the other unemployed. The following output matrix applies:

	1	2
1	$\frac{1}{2}w_1 / \frac{1}{2}w_1$	w_1 / w_2
2	w_2 / w_1	$\frac{1}{2}w_2 / \frac{1}{2}w_2$

The game has three Nash equilibria $\left\{(w_1, w_2),(w_2, w_1),\left(\left(\frac{1}{2}w_1, \frac{1}{2}w_2\right),\left(\frac{1}{2}w_1, \frac{1}{2}w_2\right)\right)\right\}$. In the first case worker 1 applies to employer 1 and worker 2 applies to employer 2, in the second worker 1 applies to employer 2 and worker 2 applies to employer 1, and a mixed strategy, where both workers apply to the both employers with probability 0.5. Solving for p in a mixed Nash equilibrium yields:

$$p = \frac{2w_1 - w_2}{w_1 + w_2} \qquad (1).$$

Employers may increase probability of getting application to the vacancy by offering higher wage.[4] However, higher wages attract more job applications thus causing a reduced probability for a particular worker to getting higher-paid job.

A filled vacancy by the employer i produces a product v_i with employer i maximizing its profit (decision) function:

$$\max_{w_i} \pi_i = (v_i - w_i)\cdot\left(1-(1-p(w_i))^2\right) \text{ subject to } v_i \in [0,1], w_i \in (0,1) \qquad (2).$$

Second expression in (2) depicts the probability that the employer i receives at least one job application. Using (1) for p_i and inserting it into the profit-maximization equation (2), reduces the maximization problem:

$$\max_{w_i} \pi_i = (v_i - w_i)\cdot\left(\frac{3w_{-i}(2w_i - w_{-i})}{(w_i + w_{-i})^2}\right), \text{ given } v_i, w_{-i}(\bullet) \qquad (3).$$

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[3] As workers are assumed to be homogeneous they must apply with the same probability to particular vacancy if they are to maximize their expected benefit.

[4] That is to be expected in the case of differently-productive employers, where an empty vacancy of the more productive employer is more costly. Strictly positive first derivatives: $\partial p / \partial w_1 = \frac{3w_2}{(w_1 + w_2)^2}$ and $\partial(1-p)/\partial w_2 = \frac{3w_1}{(w_1 + w_2)^2}$ prove that higher wage is associated with higher job-application probability.

Solving (3) for both employers gives us their reaction functions, that is their best responses to each other's wage posting strategy:

$$R_1(w_2) = \frac{w_2(w_2 + 4v_1)}{5w_2 + 2v_1} \quad (4),$$

$$R_2(w_1) = \frac{w_1(w_1 + 4v_2)}{5w_1 + 2v_2} \quad (5).$$

Wage posted by a particular employer depends on the wage posted by other employer and employer's productivity level. The symmetry of both employers' reaction functions implies, theoretically, that equally productive employers should offer the same wage rates.

SIMULATIONS

A brief outline of the algorithm applied to solving the model is as follows. The model is populated with two employers i each of whom solves his optimization problem as given in (3). All games are iterated forward in time for $t = 1, 2, ..., 100$. As given in (3) each employer conditions its wage selection in time $t+1$ according to other employer's selection in time t simultaneously. To follow the algorithm the optimization problem could be rewritten as:[5]

$$w_{i,t+1} = \arg\max_{w_{i,t+1}^k} \pi_{i,t+1} = \left(v_i - w_{i,t+1}^k\right) \cdot \left(\frac{3w_{-i,t}\left(2w_{i,t+1}^k - w_{-i,t}\right)}{\left(w_{i,t+1}^k + w_{-i,t}\right)^2} \right), \text{ given } v_i, w_{-i,t}(\bullet) \quad (3a).$$

The decision function to be maximized in each period t is continuous, concave, and differentiable on the defined convex set $D \in (0,1)$ thus guarantying a maximum. Decision-making algorithm is based on the line search optimization and works as follows:

Initialization: choose initial $w_{i,t}^0$ and other parameter values

Step 1: evaluate maximization function $\pi_{i,t}\left(w_{i,t}^k\right)$.

Step 2: $w_{i,t}^{k+1} = w_{i,t}^k + step; step = 1E-7$.

Step 3: if $\pi_{i,t}\left(w_{i,t}^{k+1}\right) - \pi_{i,t}\left(w_{i,t}^k\right) > 0$, go to step 1, else go to step 4.

Step 4: quit iteration and report $w_{i,t}^k$ as the optimum wage posted by employer i in time t, $w_{i,t}^*$.

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[5] $-i$ stands for the not i employer.

After the $w^*_{i,t}$ is calculated, the simulation process continues in $t+1$ until the convergence of the optimal wage time-path: $w^*_{i,t+1} - w^*_{i,t} < 0$.[6]

To prove that the convergence path exists consider the maximization problem (3a). Symmetry implies that both homogenous employers offer $w_{i,t} = w_{-i,t}$ for all t. Say that $w_{i,t}$ and $w_{i,t+1}$ are optimum wages at time t and $t+1$, then we could rewrite the convergence condition to: $w_{i,t+1} - w_{i,t} = \left(\left(v_i - w_{i,t+1}\right) \cdot \left(\frac{3w_{-i,t}\left(2w_{i,t+1} - w_{-i,t}\right)}{\left(w_{i,t+1} + w_{-i,t}\right)^2} \right) - w_{-i,t} \right)$ or:

$w_{i,t+1} - w_{i,t} = w_{-i,t} \left(\frac{3\left(v_i - w_{i,t+1}\right)\left(2w_{i,t+1} - w_{-i,t}\right)}{\left(w_{i,t+1} + w_{-i,t}\right)^2} - 1 \right)$, corroborating a continuous, concave and differentiable function. QED

To allow for the intrinsic costs, the logistic (Fermi) probability function is used as a mechanism influencing employers' wage-setting process:

$$F\left(w_{i,t+1} \leftarrow w_{i,t}\right) = \left(1 + \exp\left[\left(w_{i,t} - w_{i,t+1}\right)\kappa^{-1}\right]\right)^{-1} \qquad (6).$$

Probability that the wage is regularly updated in each period t is a function of wage differential and the susceptibility to the "noise" parameter, $\kappa \in (0,1]$. The smaller the κ the larger the probability that employer follows its optimal strategy and vice versa. The rule to adopt a new wage in each period becomes: $\left(ran < F\left(w_{i,t+1} \leftarrow w_{i,t}\right)\right)$ then $w_{i,t+1} = w_{i,t+1}$, else $w_{i,t+1} = w_{i,t}$. $ran \in (0,1)$ is an i.i.d. random number.

Simulations are performed through the entire space of κ (step 0.25). Results are averaged over 50 realizations.

RESULTS

Initial parameter values are set as follows: the productivity level $v_1 = v_2 = 0.1$, wage rates $w_{1,0} = w_{2,0} = 0.0001$. Simulation results are depicted on Figure 1. The figure depicts the iteration process of the wage policy under the entire definition space of κ as stated above, plus for $\kappa = 0.001$.

[6] In the sequel, we omit asterisks for the sake of clearance.

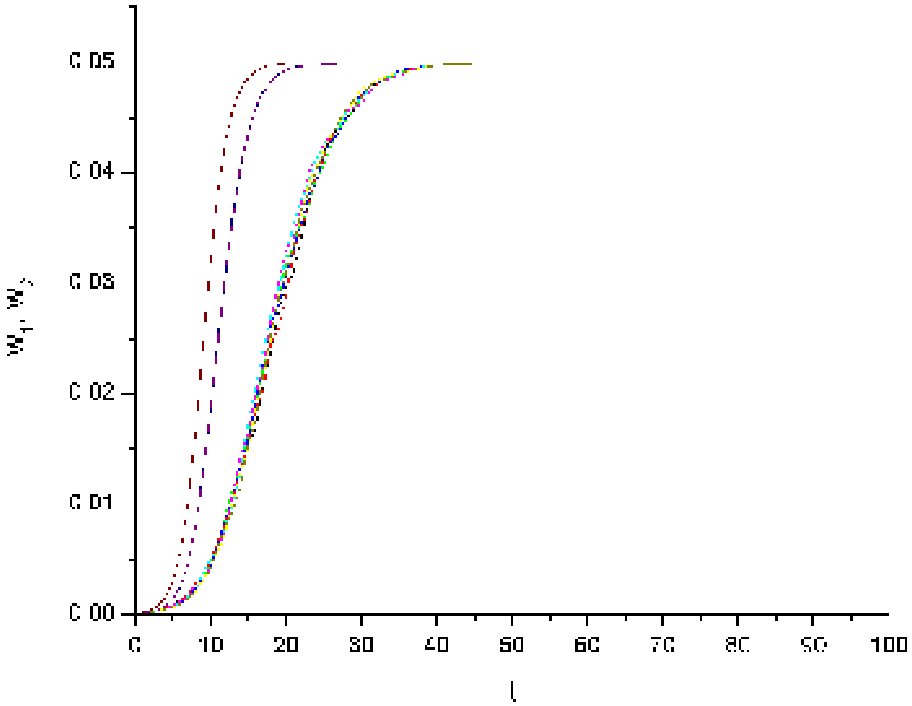

Figure 1: Optimal wage time-path

In the end all values converge to the analytical solution $w_1 = w_2 = 0.05$. Wage policy of the original Montgomery model without a "noise" is depicted on the far leftist curve. The curve is alike for both employers. This is as expected considering the fact that both are equally productive, share the same optimization problem and are not subject of any "noise".

The same results are also got in the modified model when the "noise" parameter κ is set to almost zero. For, when $\lim_{\kappa \downarrow 0} F\left(w_{i,t+1} \leftarrow w_{i,t}\right) = 1$, irrespective of $w_{i,t}, w_{i,t+1}$ values, which entails $\left(ran < F\left(w_{i,t+1} \leftarrow w_{i,t}\right)\right)$. Then the wage policy is regularly updated.

Figure 1 reveals that a slight increase in the "noise" parameter value, like $\kappa = 0.001$, changes the behavior of the model. First, it doubles the convergence time. For instance, the original model converged in $t = 28$ (far leftist curve), while even for $\kappa = 0.001$ (second from the left) the model converged in $t = 45$ (for $\kappa = 0.25$ (among the bunch of curves on the right) the model converged in $t = 63$).

More importantly, the inclusion of "noise" parameter that allows an "out-of-equilibrium" decision-making might result in the wage dispersion even among homogenous agents, employers and workers. To show this, focus to the enlarged time interval $t = 15,...,20$ of simulation results for κ with the step 0.25 in Figure 2. Except for the original model, other wage curves $w_{1,t}$ and $w_{2,t}$ of employer 1 and 2 do not equal along the whole time interval, $t = 1,2,...,100$. The finding is similar to Burdett and Judd (1983) who show that the price (wage) dispersion may exist in the case of homogenous agents provided that some “noise” is introduced into the sequential search technology.

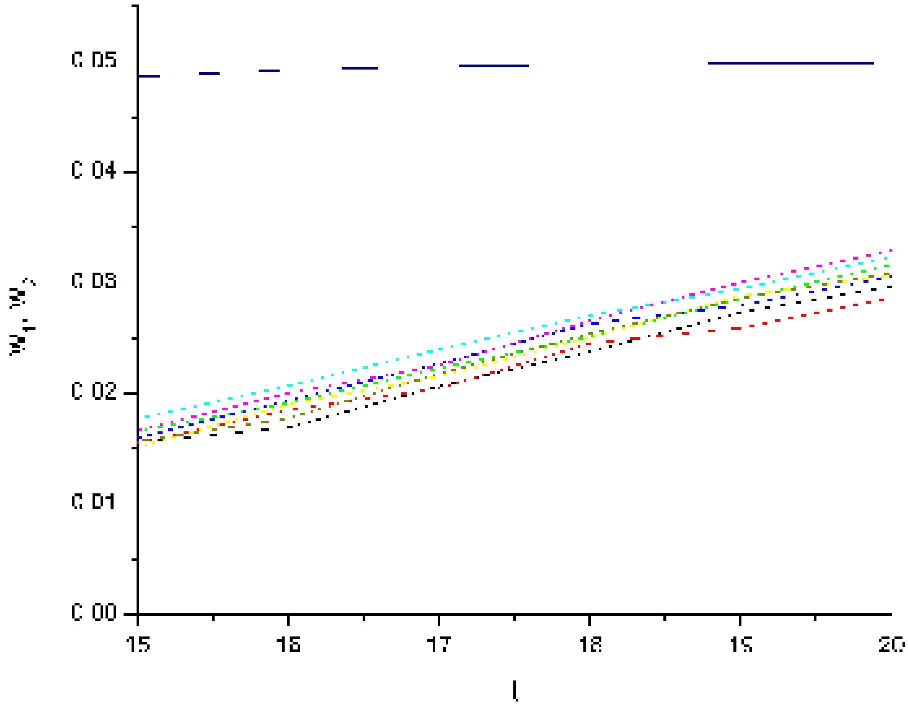

Figure 2: Optimal wage time-path and the wage dispersion

CONCLUSION

In the paper we test a 2×2 homogenous-agent version of the Montgomery model (1991) of the wage posting. In his paper Montgomery predicts that in such an environment both employers should offer equal wages due to the symmetry of their decision functions. However, our simulations reveal that it is also the homogeneous-agent model that could entail wage dispersion, provided that the “noise” factor is included. In addition, a “noise” factor also significantly prolongs the optimal-wage-path convergence time.

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