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Kazumasa, Oguro and Shoichiro, Yuyama

Policy Research Institute, Ministry of Finance, Japan

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A Study on Financial Deficit and Declining Birthrate

— From the Viewpoint of "Children as a Social Security Revenue Source" —-

Kazumasa Oguro[1]

Shoichiro Yuyama[2]

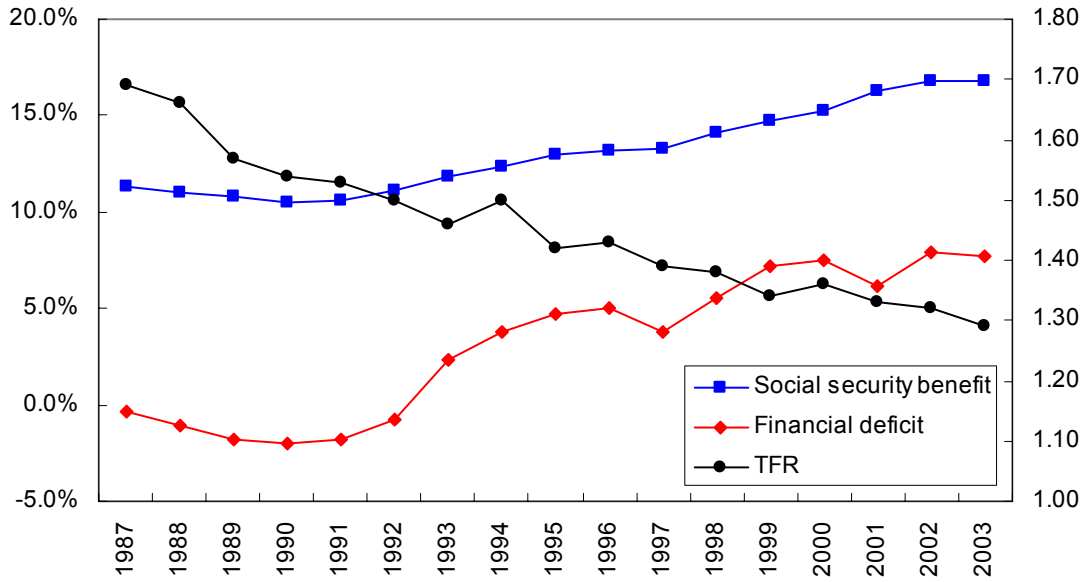
Chapter 0. Introduction

1. "Children as a Social Security Revenue Source" and Financial Deficit

With a declining birthrate and an aging population, Japan is increasingly finding itself in deeper financial difficulties. Recognizing these conditions, the government is now examining a framework for maintaining fiscal sustainability, mainly at the Council of Economic and Fiscal Policy.

The predominant cause of such financial difficulties is the existence of "financial deficit" which has persevered for several years. In the background, however, it has also been largely influenced by the increasing social security cost[3]. In actual fact, social security benefit made up 11.3% of the annual GDP in 1987, rising to 16.8% in 2003, which was an increase of about 1.5 times. On the other hand, the financial balance (relative to GDP) of the general government account accrued a 0.3% surplus in 1987, but a 7.7% deficit in 2003.

[Figure 1] Change in Social Security Benefit, Financial Deficit (Relative to GDP) and Total



(Data) The National Institute of Population and Social Security Research data, OECD Outlook, Cabinet

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[1] Senior Economist, Policy Research Institute, Ministry of Finance, Japan

[2] Planning Chief for International Affairs, Policy Research Institute, Ministry of Finance, Japan

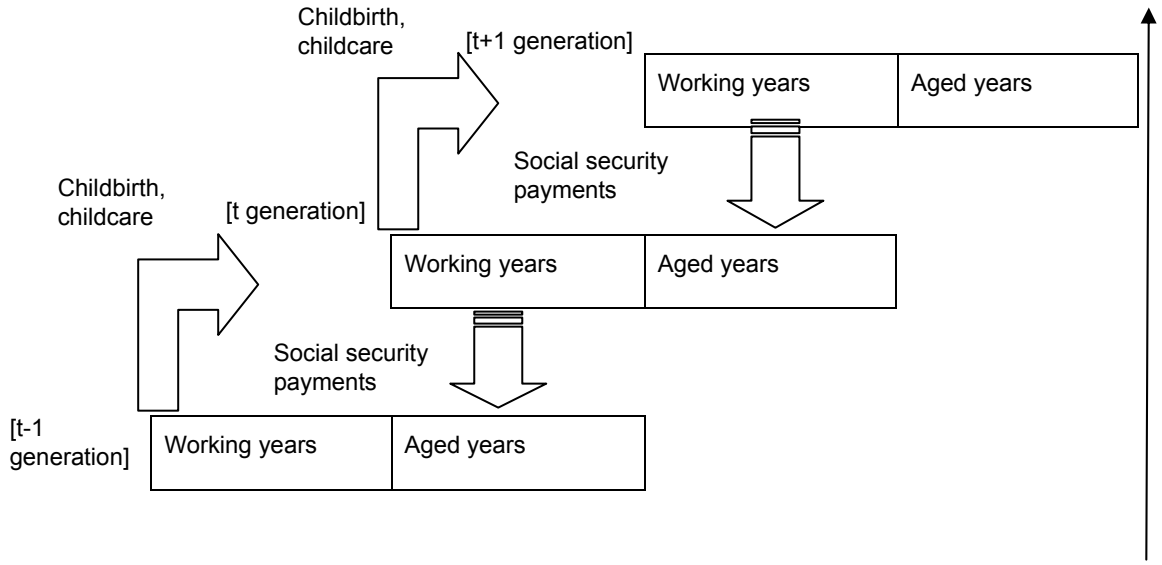
[3] According to the data presented by Minister Tanigaki to the Council of Economic and Fiscal Policy on March 7, 2006, using FY1990 as the basis for comparison, an increase of about 380 trillion yen in the remaining straight government bond debt from FY1990 to FY2006 was shown, and about 25%, or about 90 trillion yen, was purely social security expenditures.

Incidentally, many of the social security systems Japan and other countries have adopted are based on an "assessment-based method", including pension insurance systems. This means that many social security systems in foreign countries are also based on the "reliance between generations," that is, between the older generation and the working generation.

Then, what resources are securing the sustainability of this "reliance between generations?" It is of course children. Needless to say, the working generation is the children whom the old generation bore and raised. To the old generation, this working generation is now paying money into the social security fund (social security payment) in the form of taxes and social insurance premiums which support the social security system. Similarly, when the working generation turns into the old generation, the children whom the working generation bore and raised become the new working generation and support the social security system as their parents did. Therefore, children can be regarded as "a social security revenue source."

With this line of thinking, under the social security system, each generation bears and raises their children as a social security revenue source while concurrently making social security payments during their working years to aid their parents' generation. In their own aged years, when they are not able to earn such a level of income, they will live on the social security payments from their children's generation (see Figure 2).

[Figure 2]



Moreover, according to this line of thinking, each generation rationally chooses the number of children to have during their working years depending on the level of social security payments necessary for comfort in their aged years. Interestingly, however, the total fertility rate (TFR) of Japan, for example, leans towards a consistently declining trend and the

overall number of children is decreasing (See Figure 1). How can this phenomenon be explained most rationally from an economic point of view?

To this question, this paper presents a hypothesis, as one of possible explanations, that the supplementing of social security benefits by the financial deficit also acts as a factor to encourage the decline in birthrate.

Even if the number of childbirths as a social security revenue source decreases, if that decrease is covered by financial deficit and the burden is put off for later generations to deal with, then the incentive to bear and raise children as a social security revenue source might be lowered. Such a possibility is not dismissible. That is, "Isn't there a possibility that the supplement to social security benefit by financial deficit dilutes the value of children as a social security revenue source, and facilitates a decline in birthrate?" This is our hypothesis.

2. Previous Studies

With previous studies on childbirth decision making, Becker (1960, 1991) and many others have been accumulating hypotheses and demonstrations. They point out that the main cause of declining birthrate is a trend towards getting married later or not at all[4]. Also in Japan, a tendency to marry late or not at all can be clearly seen as in Figure 3. Various hypotheses on the cause of this tendency have also been presented and demonstrated from an economic point of view. However, not many of them consider the relationship between financial deficit and declining birthrate.

[Figure 3] Change in Percentage of Unmarried People and the Average Age for First Marriage

			1975	2003
Percentage of unmarried people	25 to 29 years old	Man	48.3%	69.3%
		Woman	20.9%	54.0%
	40 to 44 years old	Man	3.7%	18.4%
		Woman	5.0%	8.6%
Average age for first marriage		Man	27.0 years old	29.4 years old
		Woman	24.7 years old	27.6 years old

(Data) Statistics Bureau, Ministry of Internal Affairs and Communications, "National Census Report"

First of all, why are people not worrying much about their aged years where they have a limited ability to earn money, even though the trend towards late or no marriage continues? Usually, if the trend towards late marriage or none at all continues and the number of children as a social security revenue source decreases, then the social security benefit payments will be

[4] See Date and Shimizutani (2004) "Why Has Japan's Fertility Rate Declined: An Empirical Literature Survey with Emphasis on Policy Implications" (ESRI Discussion Paper Series No.94), and others.

in a general decline. Despite this, no problem has surfaced, yet. The underlying reason may be the existence of the financial deficit. That is to say, even if children as a social security revenue source decrease in number, if social security benefit is supplemented by financial deficit and if the burden can be postponed to the later generations, then even if the birthrate further declines, the problem will not surface for a while.

As discussed above, we cannot deny the possibility that the supplement to social security benefit by financial deficit may have a certain influence on the number of births as a social security revenue source.

3. Composition of this Paper

Therefore, in this paper, in Chapter 1, we first construct a model of overlapping generations from a macro perspective that the supplement to social security benefit by financial deficit may have a certain influence on the number of births as a social security revenue source. In Chapter 2, we demonstrate and analyze this influence. And, in Chapter 3 we consider the number of births as a social security revenue source from a micro perspective, and in the final Chapter 4, we offer a summary and policy consideration.

Chapter 1. Theoretical Model

1. Construction of a Theoretical Model

First, we construct a theoretical model to analyze what influence financial deficit has on the number of births as a social security revenue source.

To simplify the argument, we assume that each generation lives the two periods of working years and aged years based on the Lifecycle Hypothesis, and that every generation is homogeneous. We also assume that the interest rate is zero, the income in aged years is zero, and the working generation faces the following budget restraint at each time point t:

$$Y_t + (1-\alpha)\Delta B_t = Cy_t + e \cdot n_t + T_t + S_t + B_t \quad \text{..... (1)}$$

where, Y_t is the working generation's income, Cy_t is the working generation's consumption, e is the cost of a child, n_t is the number of births, T_t is the social security payments to the old generation, S_t is the working generation's savings, and B_t is public bond accepted (including refunding bond). And, $(1-\alpha)\Delta B_t$ on the left side of (1) expresses that the resources acquired from the working generation by public bond (financial deficit: $\Delta B_t = B_t - B_{t-1}$) are distributed to the working generation at a ratio of $(1-\alpha)$.

On the other hand, we assume that the old generation faces the following budget restraint at each time point t:

$$T_t + S_{t-1} + B_{t-1} + \alpha\Delta B_t = Co_t \quad \text{..... (2)}$$

where, T_t is the social security payments from the working generation, S_{t-1} is the old generation's savings during working years, B_{t-1} is redemption of the public bond accepted during working years, Co_t is the old generation's consumption. And, $\alpha\Delta B_t$ on the left side of (2) expresses that the resources acquired from the working generation by public bond (financial deficit) are distributed to the old generation at a ratio of α, that is, supplementing social security benefit by financial deficit.

To incorporate a mechanism that each generation's children become the social security revenue source for their parents' generation, we assume that the social security payment is expressed as follows with positive constant τ:

$$T_t = \tau n_{t-1} \quad \text{..... (3)}$$

Incidentally, financial deficit cannot be sustained forever, so we introduce an equation of long-term financial balance. That is, if at time point t, a financial reconstruction of σB_{t-1} is

done with probability p, and a financial deficit by D occurs with probability (1-p), then the following equation holds, with E being the expected value:

$$E(\Delta B_t) = -p\sigma B_{t-1} + (1-p)D$$

By further taking the expected value of the equation above, we can consequently obtain the following difference equation with respect to public bond balance B_t:

$$E(\Delta B_t) = -p\sigma E(B_{t-1}) + (1-p)D \quad \text{..... (4)}$$

Now, let's get into the crux of the model. Let us assume that Y and S (zero in this paper) are constant, and that each generation is selfish and maximizes its utility function $U=Cy_t \cdot Co_{t+1}$. Then we can obtain the expected value of the number of births n_t as follows (for derivation, see Supplement):

$$E(n_t) = \frac{Y - \frac{1-p}{p\sigma} D\left(\alpha + \frac{e}{\tau}\right)}{2e + \tau} + \frac{\left\{\left(1 + p\sigma\frac{e}{\tau}\right)\alpha + e/\tau\right)(1 - p\sigma) + (\alpha - 1)p\sigma\right\}\left\{B_0 - \frac{1-p}{p\sigma} D\right\}}{2e(1-p\sigma) + \tau}(1-p\sigma)^t \quad \text{..... (5)}$$

In this equation (5), the initial time point t=0 is arbitrary. Therefore, without losing generality, we can let $B_0=0$, and we can see that the financial deficit pressure, D, effectively lowers the expected value of the number of births, $E(n_t)$. We can also see that the expected value of the number of births settles to a certain value as $t \to \infty$, because we introduced a mechanism to balance finance in the long term (equation (4)).

2. A Proposition Obtained from the Model Solution

From this equation (5), we can see that the following important proposition holds.

[Proposition]

When each generation is selfish, if supplement α to social security benefit by financial deficit becomes zero or pressure D of financial deficit becomes zero, then the expected value of the number of births increases.

This proposition is rational. Because, if the supplement α to social security benefit by financial deficit or the pressure D of financial deficit is zero, then the incentive to bear and raise children as a social security revenue source does not decrease. In this case, each generation of people should rationally choose during working years the number of births necessary as a social security revenue source during their aged years, from the viewpoint of maximizing their own utility.

On the other hand, if neither the supplement α to social security benefit by financial deficit nor the pressure D of financial deficit are zero, then the incentive to bear and raise children as a social security revenue source decreases. As a result, each generation of people should rationally choose a number less than the number of births in the case where the supplement α to social security benefit by financial deficit or the pressure D of financial deficit is zero, as the number of births necessary as a social security revenue source during their aged years, from the viewpoint of maximizing their own utility.

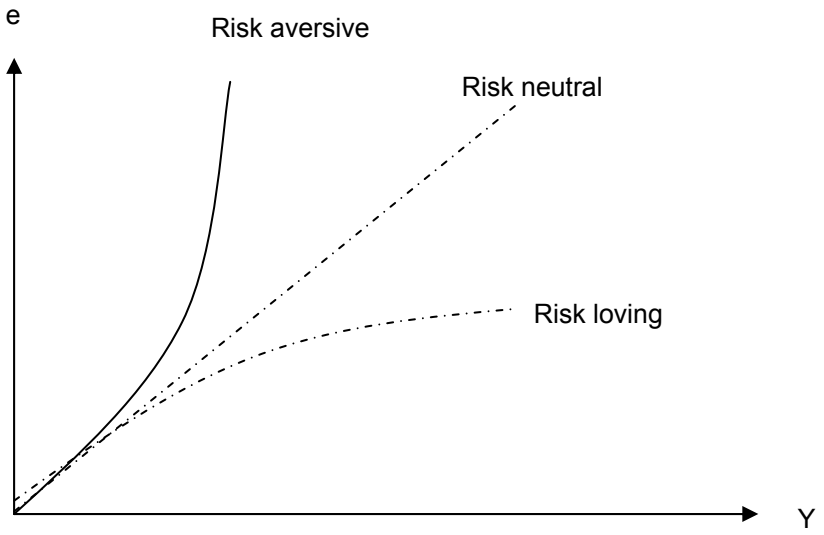
Therefore, if the supplement α to social security benefit by financial deficit or the pressure D of financial deficit becomes zero, then it becomes a rational choice for each generation to increase the number of births as a social security revenue source, to maximize their own utility.

3. Model Improvement and Notes

Equation (5) needs some notes. With respect to the relation between birthrate and income level, equation (5) has a positive correlation. However, the results of existing demonstration studies show that a negative correlation is seen no matter what data is used in the cross-country time-series. Therefore, equation (5) seems contradictory. However, in this paper we have not yet made it clear what the cost of a child, e, and the coefficient of social security payment, τ, are. In general, it would be natural to think that the cost of a child, e, and the coefficient of social security payment, τ, are linear functions of the working generation's income, Y. Because, in a country where the working generation's income Y is high, the child cost e and the social security payment coefficient τ are high; and in a country where Y is low, e and τ are low. However, while the social security payment coefficient τ is determined by the government exogenously, the child cost e has a possibility to be determined by each generation endogenously, including not only child care expenses but also the child's quality of life. In this case, there is a possibility that the child cost e is not a linear function of the working generation's income Y. For example, in a high-income country, if a female university graduate quits her job for childbirth reasons, her lost income is high. Therefore, the woman's opportunity cost for childbirth and child care is high. And, it can be said that this opportunity cost is by itself a risk when choosing childbirth and child care. Therefore, if each generation is "risk aversive" to the choice of childbirth and child care, then the child cost e contains this opportunity cost, and it can be seen as obeying the following equation (6) (see Figure 4):

$e=e_0 \cdot Y^{\Theta}$ （where, $\Theta>1$） ..... (6)

[Figure 4]



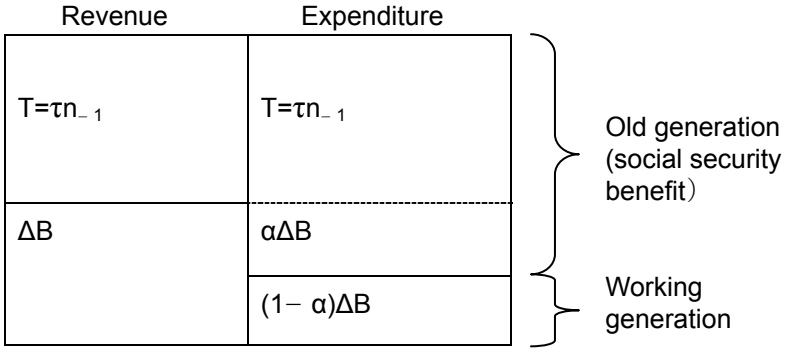
Here, by letting $\tau=\tau_0 \cdot Y$, we can arrange equation (5) as follows:

$$E(n_t) = \frac{1-\frac{1-p}{p\sigma}D\left(\frac{\alpha}{Y}+\frac{e_0}{\tau_0}Y^{\Theta-2}\right)}{2e_0Y^{\Theta-1}+\tau_0} - \frac{\{(1+p\sigma e_0/\tau_0 Y^{\Theta-1})\alpha + e_0/\tau_0 Y^{\Theta-1})(1-p\sigma)+(\alpha-1)p\sigma\}D/Y}{2e_0(1-p\sigma)Y^{\Theta-1}+\tau_0}(1-p\sigma)^t$$

..... (7)

Since the first term of this equation (7) is largely a decreasing function with respect to Y, it no longer contradicts the existing demonstration studies on birthrate and income level based on the cross-country time-series. Therefore, for the remainder of this paper, our discussion will be based on equation (7), which is an improvement of equation (5).

In this model, the government's revenue is "tax and social insurance premium + financial deficit (public bond issuance income)" (T+ΔB). Of this expenditure (T+ΔB), (1− α) ΔB is distributed to the working generation, and the remaining (T+αΔB) is distributed to the old generation as social security benefit.



Here, if α is zero, then the public bond issuance income (= financial deficit ΔB) acquired from the working generation is only redistributed to the working generation as it is. That is, it is just redistributed within the working generation. This is obvious also when we see equation (1) + equation (2): If α=0, then ΔB is canceled by the life-long budget restraint of each generation, and financial deficit does not influence each generation's choice on the number of births. However, in Japan for example, the real flow of funds to the social security system is complicated, partly because of the existence of state contribution from the general account. Therefore, it would be difficult to control the supplement α to social security expenses by financial deficit to zero, if the government has a financial deficit.

Therefore, in the next chapter we will do demonstration analysis on what influence the number of births α receives from financial deficit, based on equation (7) and also by using other countries' data.

Chapter 2. Demonstration Analysis

1. Demonstration Analysis by a Time-Series Panel Analysis of 17 Countries

Here, based on the equation (7) derived in the previous chapter, we do demonstration analysis on the assumption that the total fertility rate (TFR) is a function of per capita real GDP (GDPp) and per capita real financial balance (FBp). That is, we assume that the total fertility rate TFR obeys the following linear function and estimate whether the coefficient of per capita real financial balance FBp is significantly positive in sign:

$$TFR_t = g(FBp_t, GDPp_t)$$
$$= \beta_2 \cdot FBp_t + \beta_1 \cdot GDPp_t + \beta_0 \quad \text{..... (8)}$$

Figure 5 shows the result of a time series panel analysis of data from 1987 to 2003 of the TFR, FBp, and GDPp of 17 countries: Japan, USA, UK, Germany, France, Canada, Australia, Austria, Belgium, Denmark, Hungary, Italy, Netherlands, Norway, Spain, Sweden and Switzerland.

[Figure 5]

	Coefficient (t-value)
FBp	0.003082 (2.052231)*
GDPp	-0.861547 (-10.13564)*
Constant	2.525063 (27.52024)*
Ajusted-R^2	0.990465

Note) TFR data is from the National Institute of Population and Social Security Research; FBp and GDPp data is from OECD Outlook, etc. The analysis method is Panel EGLS (Cross-section weights)・fixed effect. GDPp is standardized to 1987=1, and FBp is obtained by dividing each year's real financial balance (relative to real GDP) by population (standardized to 1987=1). * indicates 5% significance level.

Figure 5 shows that the coefficient of per capita real financial balance (FBp) has a significantly positive sign at the 5% significance level. Therefore, when finance is in deficit, FBp is negative, so TFR decreases in equation (8). That is, this estimation result indicates a possibility that financial deficit has a negative influence on the number of childbirths. The coefficient of per capita real GDP (GDPp) also has a significantly negative sign, which is consistent with equation (7).

2. Notes

This estimation result needs several notes.

The first is on the size of the t-value of the constant term. This t-value is the largest, so we cannot deny the possibility that there may be other factors explaining the TFR, such as child cost, other than FBp and GDPp. However, in this paper we have not furthered our analysis on this matter, because there is some restriction on these countries' data about child cost. On this point, we need further study.

The second is how to treat the supplement to social security benefit by financial deficit. Japan and most of the 17 countries mentioned above are operating their social security systems in forms close to the assessment method for annuity insurance and others. However, when financial deficit occurs, some countries adopt the Golden Rule[5], and others do not. Therefore, it is possible that limited countries can use financial deficit as a revenue source for social security benefit. On this point, we need to understand complicated flows of resources for each country's social security system. However, each country has adopted its own unique financial system, social security system, and intergovernmental financial system between the national government and local governments. The work is not easy. Therefore, in this paper, we have not furthered our analysis on this matter. On this point, we also need further study.

The third is how much we can eliminate the possibility of an apparent correlation between birthrate and financial deficit. That is, there is a possibility that the decreasing birthrate and the increasing financial deficit are occurring concurrently. It is difficult to eliminate the possibility that another third factor is influencing both of them. On this point, further study is required.

Nevertheless, if we assume the theoretical model in Chapter 1 and assume that every country is supplementing social security benefit by financial deficit to a certain extent, then we can say that the estimation result of equation (8) suggests a possibility that the financial deficit has a negative influence on the number of births. Therefore, in the next chapter we consider the number of births as a social security revenue source, from a micro perspective.

[5] Government account is divided into current accounts and capital accounts, and public bond issuance is principally limited to borrowing for the capital account for investment purposes. This rule has been adopted by the UK and other countries.

Chapter 3. Micro Perspective Consideration

1. Comparison Between Actual TFR and Theoretical Value

Figure 6 is a graph for comparison between the theoretical value estimated in Figure 5 and Japan's actual TFR (for other countries, see Figure 9).

[Figure 6]

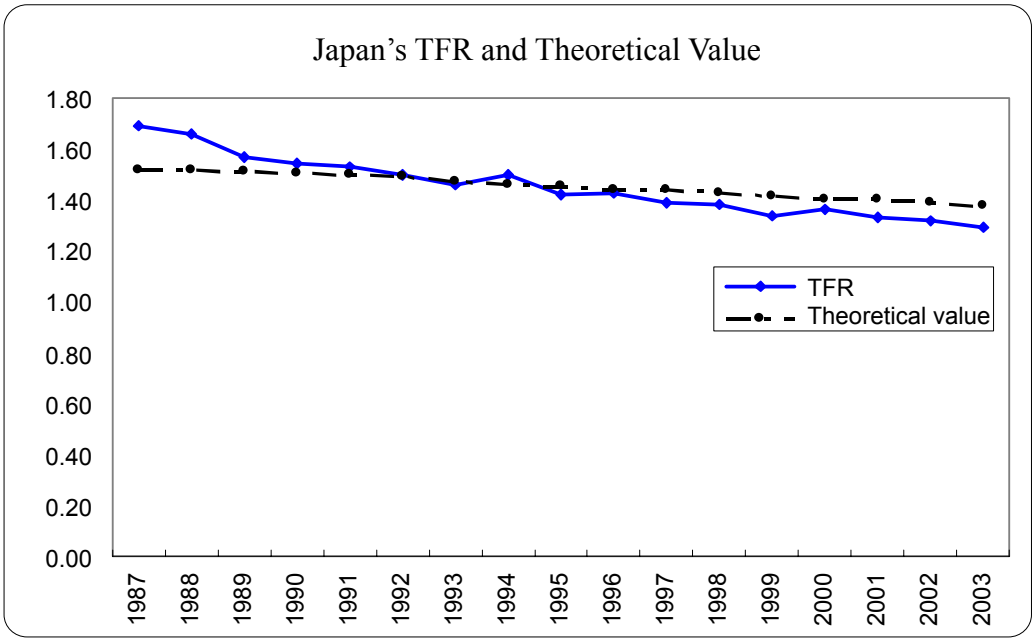


Figure 6 shows that Japan's actual TFR has been lower than the theoretical value of TFR since 1995. The demonstration analysis in the previous chapter suggested a possibility that financial deficit may negatively influence the number of births. Besides this, some other factor is likely to be influencing it. Therefore, we will briefly consider how "the number of births as a social security revenue source" is influenced when each household chooses its number of births in the knowledge of other households' number of births, micro perspectively, from a viewpoint that children are a social security revenue source.

2. Micro Perspective Analysis on the Number of Births as a Social Security Revenue Source

In Chapter 1, we assumed that every generation is homogeneous at each time point t. Here, to do the micro perspective analysis mentioned above, we assume that the economy has only two periods of working years and aged years, and that there are two homogeneous households at each time point. And, to simplify the argument, we also assume that financial deficit and savings are zero.

Then, budget restraints in the working years and the aged years of household i (i = 1, 2) are as follows in equations (9) and (10):

$$Y_t^i = Cy_t^i + e \cdot n_t^i \quad \text{.....(9)}$$
$$T_{t+1}^I = Co_{t+1}^I \quad \text{.....(10)}$$

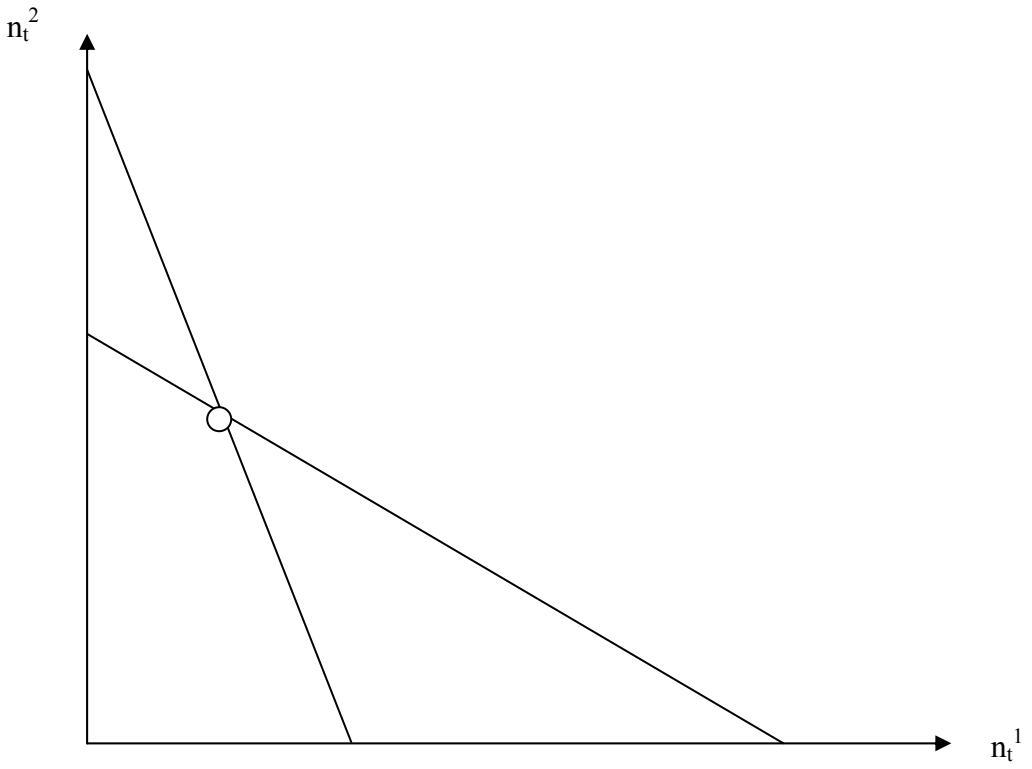
And, if we assume that social security payment T_t^i is irrelevant to the number of births each household bears and raises, that is, "the number of births as a social security revenue source", then the following equation holds, because the households are homogeneous ($Y_t^i = Y_t/2$):

$$T_{t+1}^i = \tau(\Sigma n_t^i)/2 \quad \text{.....(11)}$$

Here, if we assume that each household chooses its number of births in the knowledge of other household's number of births, under the restraint conditions of (9) to (11) above, to maximize the utility function $U^i = Cy_t^i \cdot Co_{t+1}^i$, then we can obtain the following result:

$$n_t^1 = (Y_t/2 - e \cdot n_t^2)/2e$$
$$n_t^2 = (Y_t/2 - e \cdot n_t^1)/2e \quad \text{..... (12)}$$

[Figure 7]



The Nash equilibrium solution to this system is $n_t^1 = n_t^2 = Y_t/6e$. Therefore, the sum of each household's number of births is (see Figure 7):

$$n_t^1 + n_t^2 = Y_t/3e \quad \text{..... (13)}$$

On the other hand, if the entire society's utility is $W=\Sigma U^i$, then the sum of each household's number of births that maximizes W using equations (9) to (11) as restraints can be obtained as follows:

$$n_t^1 + n_t^2 = Y_t/2e \quad \text{..... (14)}$$

This value is greater than (13).

Comparison between (13) and (14) suggests a possibility that each household's number of births may become lower than the number desirable for the entire society, if each household chooses its number of births in the knowledge of other households' number of births. This is a result caused by the assumption that social security payment T_t^i is irrelevant to "the number of births as a social security revenue source".

This means, there is a possibility that a kind of free ride problem may occur when each household chooses its number of births, because each one is choosing its number in the knowledge of other households' number of births. This happens because increasing one's consumption by decreasing the number of births as a social security revenue source for aged years would increase one's utility, if other households choose a slightly higher number of births for social security revenue source in aged years, during their working years. In other words, if social security payment T_t^i is irrelevant to "the number of births as a social security revenue source", then in from a micro perspective, there is a possibility that a kind of free ride problem may occur during working years, and the chosen number of births may be less than the number desired by society as a whole as a social security revenue for aged years. In fact, Japan's current social security system has a mechanism that gives annuity insurance benefits irrespective of the number of births. Therefore, we cannot deny the possibility that this is the factor making the actual TFR lower than the theoretical value of TFR in Figure 2.

3 A Case in Which Social Security Payment Increases Depending on Each Household's Number of Births

Now we consider how Nash equilibrium solution behaves when social security payment equation (11) is redesigned to increase depending on the number of births as follows. Here,

the second term $\tau k \cdot n_t^i$ expresses that social security benefit increases depending on the number of births[6].

$$T_{t+1}^i=\tau' (\Sigma n_t^i)/2+\tau' \cdot K \cdot n_t^i \quad \text{..... (15)}$$

Here, the condition for each household to maximize their utility function $U^i=Cy_t^i \cdot Co_{t+1}^i$ can obtain the following result from similar calculation:

$$n_t^1 =[Y_t/2 - e \cdot n_t^2/(1+K)]/2e$$
$$n_t^2 =[Y_t/2 - e \cdot n_t^1/(1+K)]/2e \quad \text{..... (16)}$$

The Nash solution to this is $n_t^1 = n_t^2 = Y_t/2〔2+1/(1+K)〕e$. Therefore, the sum of each household's number of births is:

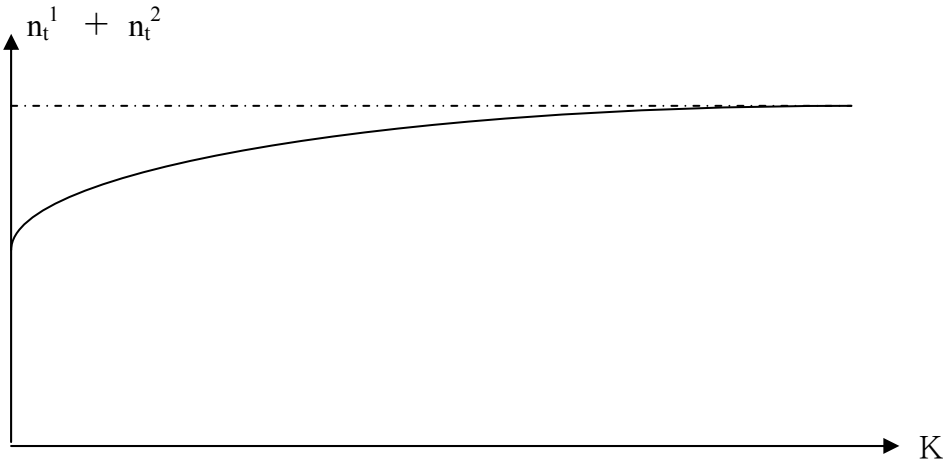
$$n_t^1 + n_t^2 = Y_t/[2+1/(1+K)]e \quad \text{..... (17)}$$

To make this equal to equation (14), we need to let $K\rightarrow\infty$ (see Figure 8). If we do this, while trying to balance benefit and burden, then $\tau'\rightarrow 0$ because $\tau'=\tau/(1+K)$. That would dismantle the frame of social security system itself, and go back to a system based on blood relation. Therefore, although letting $K\rightarrow\infty$ is unrealistic, it would be possible, to a certain extent, to lead society as a whole to a desirable number of births by setting up a certain positive value to K[7].

[6] The condition for the total benefits of equation (11) and equation (15) to be equal is $\tau'(\Sigma n_t^i) + \Sigma\tau' k \cdot n_t^i = \tau(\Sigma n_t^i)$. From this, we can obtain that $\tau'=\tau/(1+K)$.

[7] The social security system of Japan has been designed considering the social security system of Germany as a reference. Recognizing its ongoing decline in birthrate and its aging population, Germany is carrying out various social security system reforms. Among them, in public-nursing-care-insurance reform, although it is not the care grant itself, a method that provides a difference in insurance premiums according to the number of births is adopted. That is, although the Ruerup Report was negative about changing premium rates with the number of children, actually on and after January 1, 2005, 0.25% additional premiums are to be levied to insure those who have no child. In addition, federal constitution court judgment (2001.04.03) states that those insured who are supporting a child are burdened with the same premium rate as those insured without a child in care insurance, which is against the equality before the law which the basic law provides.

[Figure 8]



Chapter 4. Summary and Policy Consideration

As with the previous chapters, this paper considers the following: Considering a child as a social security revenue source, (1) From a macro perspective, what kind of influence does supplementing social security by financial deficit have on the birthrate? And (2) From a micro perspective, if social security payment is irrelevant to each household's number of births, what kind of influence does this have on the number of births for society as a whole?

As a result, for (1), a time-series panel analysis of 17 nations suggested that financial deficit might have a negative influence on the number of births. For (2), from the micro viewpoint, a simplified Nash equilibrium game suggested that in social security payments, if the social security benefit of each household is irrelevant to its number of births, it may have a negative influence to the desirable number of births for society as a whole.

For this reason, to dispose of these negative relations while standing on the viewpoint that a child is a social security revenue source, the following points are also worth consideration:

I. Lead the supplement α to social security benefit by financial deficit to zero, or pressure D of financial budget to zero; and

II. Introduce a mechanism to increase social security benefit such as annuity insurance according to each household's number of births, while balancing grant and burden.

However, this research that regards a child as a social security revenue source still has many problems. For example, in the model construction in Chapter 1, two investment outlets, "child as a social security revenue source" and "saving during working years," are assumed as objects of investment to prepare for each generation's aged years.

However, when the interest rate is not zero, which investment object is more rational to choose for each generation? This point is also an important problem to consider. However, this paper does not go that far in considering it. This also needs further research.

Finally, with the declining birthrate and aging population, advanced countries are now implementing various social security system reforms and measures against declining birthrate from a perspective of maintaining the sustainability of their social security systems. However, effects of the measures against declining birthrate will show up in the distant future; it takes time. For this reason, advanced countries, including Japan whose population has already begun decreasing, have to start social security system reforms immediately and increase their sustainability based on the above perspectives and the known population dynamics.[8] Solving this problem needs the understanding of many people and good political leadership. I would be pleased if this paper could contribute to their discussion.

[8] See Oguro and Morishita (2006) "Personal proposal of a model for the medical insurance system to improve the difference between generations, and its possibility — Overcoming of assessment system and funded system" PRI Discussion Paper Series 06A-02, etc.

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Supplement

By using Lagrange's method of undetermined multipliers assuming that S is zero, we can change the problem to solve it into the following form:

$$\text{Max } Cy_t \cdot Co_{t+1}$$
$$+ \lambda_t^1 (Cy_t + e \cdot n_t + \tau n_{t-1} + B_t - Y_t - (1-\alpha)\Delta B_t)$$
$$+ \lambda_t^2 (\tau n_t + B_t + \alpha \Delta B_{t+1} - Co_{t+1}) \quad \text{.....(A)}$$

From partial differentiation of this (A) with respect to Cy_t, Co_{t+1} and n_t , we can obtain the following:

$$Co_{t+1} = \lambda_t^1$$
$$Cy_{t+1} = -\lambda_t^2$$
$$e\lambda_t^1 + \tau\lambda_t^2 = 0 \quad \text{.....(B)}$$

By substituting this (B) into equations (1) and (2), eliminating Cy_t and Co_{t+1}, and taking the expected value E, we can obtain the following difference equation:

$$2e\,E(n_t) + \tau E(n_{t-1}) = Y - (\alpha + e/\tau)E(B_t) - (\alpha - 1)E(\Delta B_t) - e/\tau \cdot \alpha E(\Delta B_{t+1}) \quad \text{.....(c)}$$

Here, we denote $Y_t = Y$.

On the other hand, the long-term financial balance equation $E(\Delta B_t) = -p\sigma E(B_{t-1}) + (1-p)D$ is a first-order difference equation with respect to $E(B_t)$, so by letting $B_t = B_0$ at t=0, we can obtain the following:

$$E(B_t) = (1-p\sigma)^t \{B_0 - (1-p)D/p\sigma\} + (1-p)D/p\sigma \quad \text{.....(D)}$$

Then, by substituting this (D) into difference equation (C), it becomes a difference equation with respect to $E(n_t)$. By solving this, we can obtain the general solution as follows:

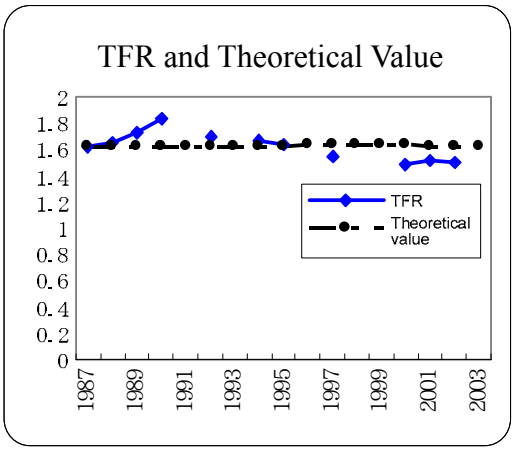
$$E(n_t) = \frac{Y - \frac{1-p}{p\sigma} D(\alpha + \frac{e}{\tau})}{2e + \tau}$$
$$+ \frac{\{((1 + p\sigma\frac{e}{\tau})\alpha + e/\tau)(1 - p\sigma) + (\alpha - 1)p\sigma\}\{B_0 - \frac{1-p}{p\sigma} D\}}{2e(1 - p\sigma) + \tau}(1 - p\sigma)^t$$
$$+ c\left(-\frac{\tau}{2e}\right)^t \quad \text{.....(E)}$$

Here, the third term of this equation (E) is a “vibrating solution” which vibrates at time points t and t+1. This is unnatural for this type of analysis, so removing it is rational. Thus, we obtain the following solution:

$$E(n_t) = \frac{Y - \frac{1-p}{p\sigma} D(\alpha + \frac{e}{\tau})}{2e + \tau}$$
$$+ \frac{\{((1 + p\sigma\frac{e}{\tau})\alpha + e/\tau)(1 - p\sigma) + (\alpha - 1)p\sigma\}\{B_0 - \frac{1-p}{p\sigma} D\}}{2e(1 - p\sigma) + \tau}(1 - p\sigma)^t$$

[Figure 9] Other Nations' Total Fertility Rate (TFR) and Theoretical Value (estimate)

Canada



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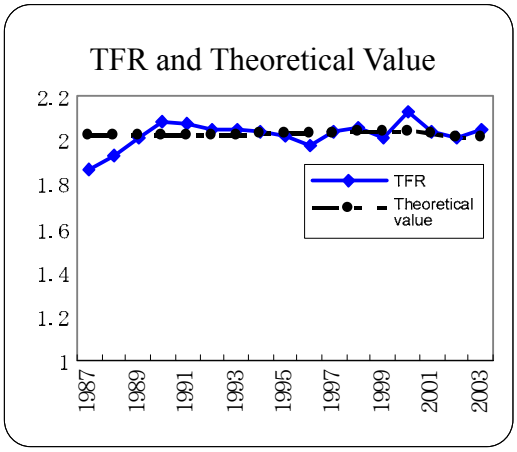

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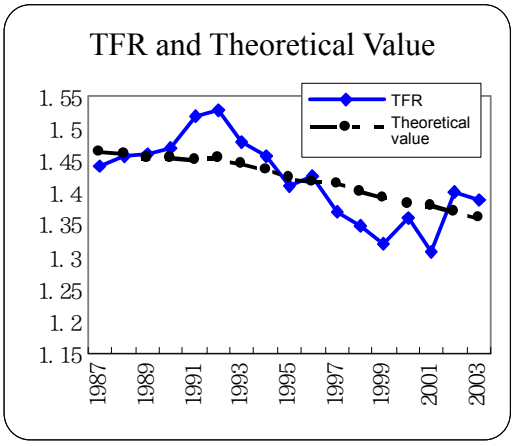

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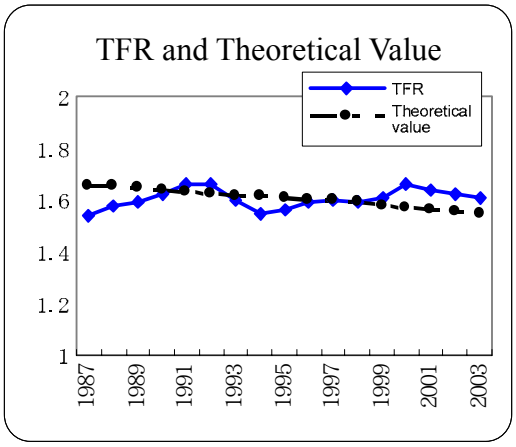
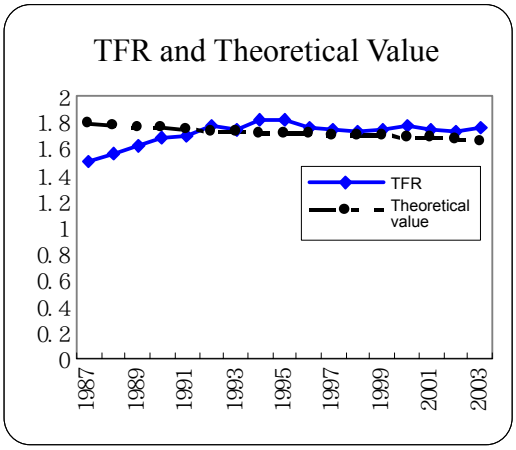


Denmark



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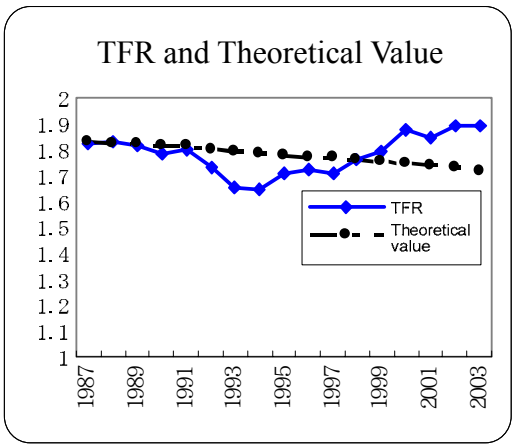

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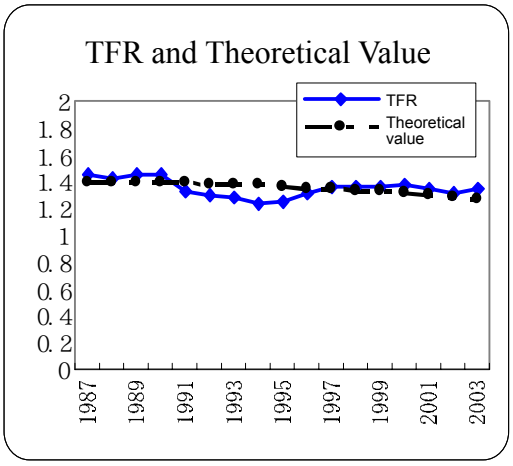

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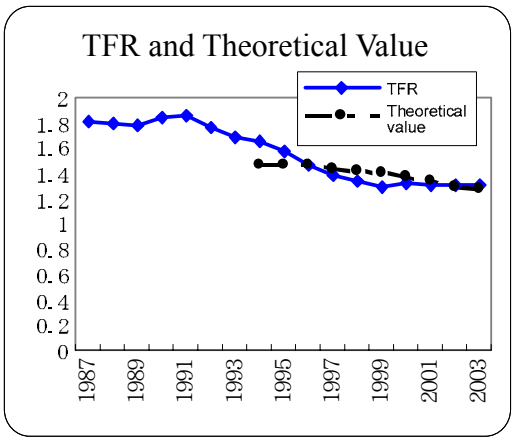

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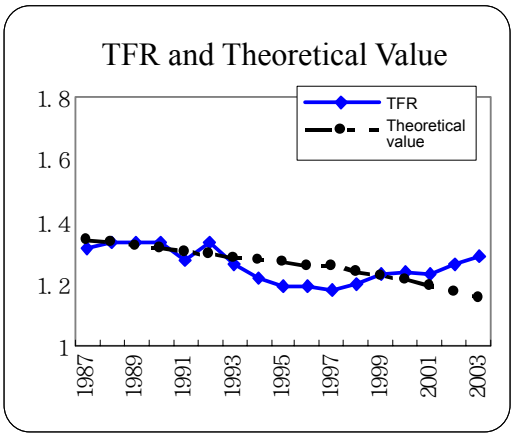

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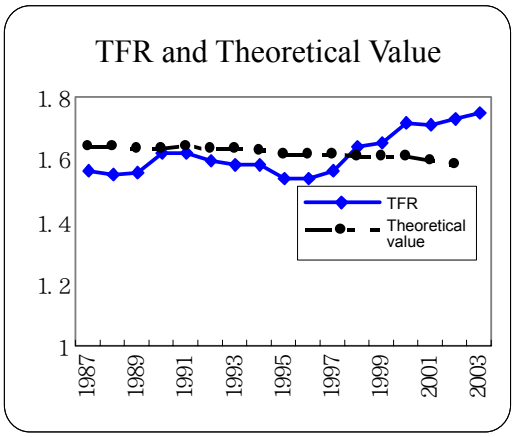

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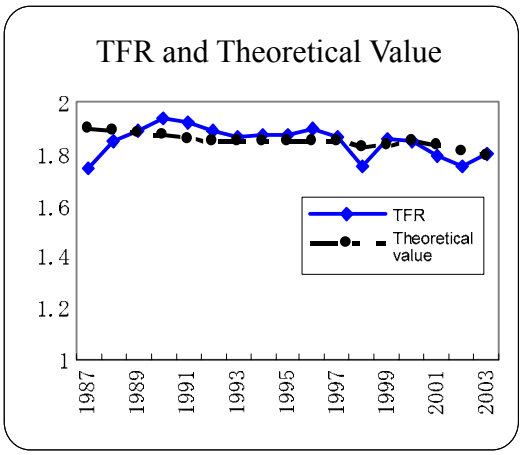

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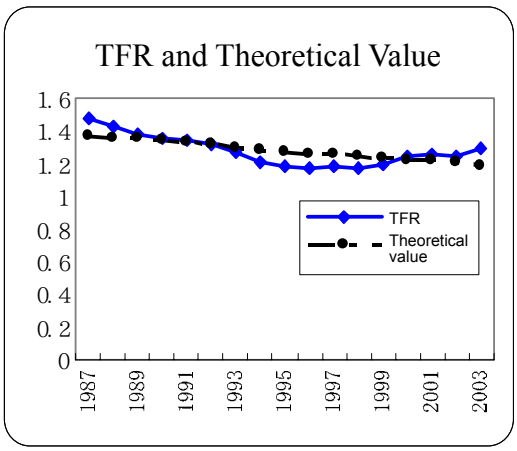

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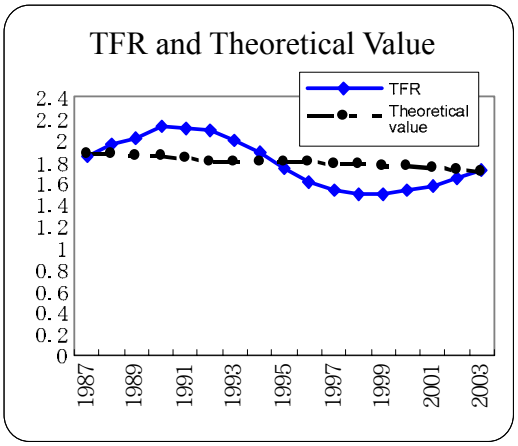

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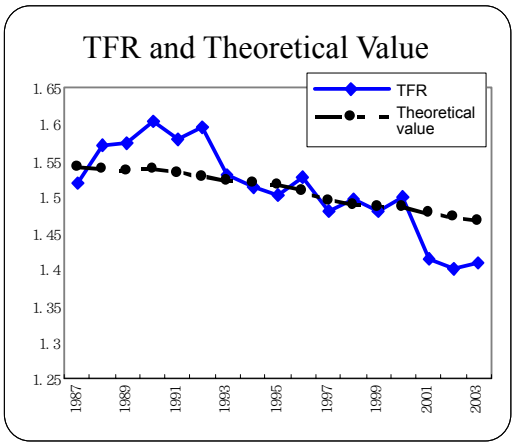

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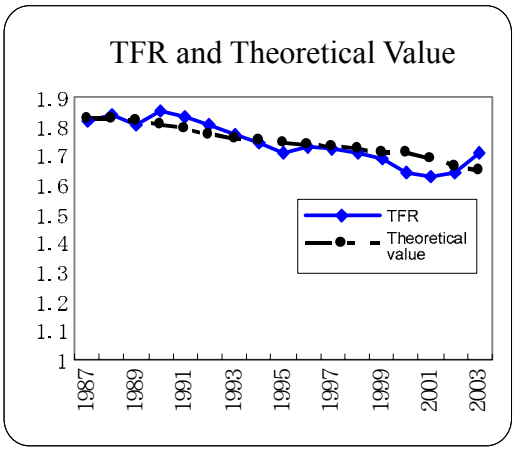
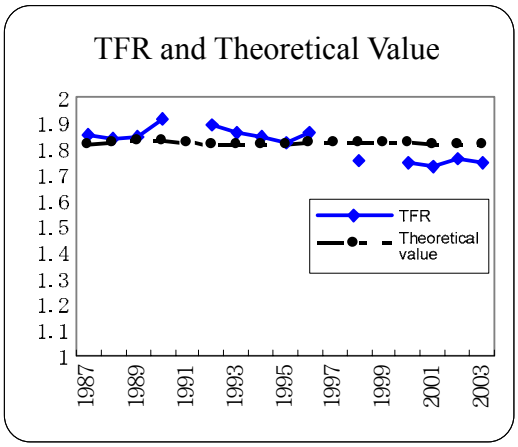
Australia