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WORKER FLOWS AND FIRM DYNAMICS IN A LABOR MARKET MODEL*

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Abstract

In this paper we build an integrated framework of the labor market in which worker replacement, job creation and job destruction are decided simultaneously at the firm level, providing a rigorous instrument for the analysis of worker flows. The main features of the model are uncertainty related to worker \times firm match quality and search frictions. Worker flow components are decided as firms learn about the quality of their matches. A negative correlation between replacement and job creation arises from this mechanism. The model also provides several implications for firm dynamics, which are all confirmed by related empirical papers.

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1 INTRODUCTION

Recent literature on employment dynamics established two empirical regularities. The first is the large heterogeneity in employment growth even across similar firms. Davis and Haltiwanger (1999) surveyed several papers on this topic and showed that a significant share of the total variance in employment growth is due to the within group component.¹ The second fact is that job flows and worker replacement occur simultaneously at the firm level, both with significant magnitudes. For instance, Davis and Haltiwanger (1999)² report job flow rates between 15% and 24%, which represent between 41% and 56% of total worker flow rates for annual data of three developed countries.³

The main goal of the present paper is to elaborate a theoretical framework that encompasses both of these facts, providing a rigorous instrument for the analysis of worker flows. The framework developed in this paper also has other interesting worker flows predictions that can be used to assess how close the theory matches reality. The model suggests that job creation and replacement should be negatively related to each other, which finds empirical support in a companion paper, Corseuil (2009). It also suggests that replacement is negatively related to both wage level and wage growth.

It turns out that the way we model the labor market also generates implications for firm dynamics, even employing the simplest representation of the product market. We offer the following predictions, all confirmed by results

¹At least 80% according to table 5 in Davis and Haltiwanger (1999) which reports results for 13 countries.

²See table 9 in their paper.

³See similar numbers in Hamermesh et al. (1996), Davis and Haltiwanger (1998), Albaek and Sorensen (1998), Abowd et al. (1999), and Burgess et al. (2000).

on firm dynamics surveyed in Sutton (1997); Caves (1998); and Bartelsman and Doms (2000):

- Homogeneous firms (in terms of observable characteristics such as industry, size and age) attain distinct performance levels measured either as productivity at the end of the first period or job creation in the second period.
- The higher a firm's initial productivity level, the higher its employment growth.
- Job creation for surviving firms in the second period is negatively correlated with labor force size at the end of the first period.
- The correlation between job creation and initial size mentioned above tends not to be valid for the largest firms.
- This relationship approaches Gibrat's Law if we consider the overall population of firms.
- Firms that in the first period are relatively more productive tend to keep this position in the second period.
- Smaller firms have lower probability of surviving than larger firms.
- The variance of employment growth is larger for smaller firms than for larger firms.

The driving force behind our model is a learning mechanism taking place inside the firm about the quality of its matches with employees. This match

quality is assumed to be a non-observable variable, *a priori*, but eventually will be learned by the firm, which might then seek to improve the overall match composition by trying to form new high quality matches either as part of a worker replacement or a job creation process. We develop a particular formulation for a turnover model which adds search friction to the uncertainty faced by firms concerning the quality of matches. Such complex hiring decisions are mainly responsible for broadening the scope of relevant facts beyond worker flows to firm dynamics.

Most predictions in our paper on firm dynamics are shared with other papers such as Ericson and Pakes (1995); and Klette and Kortum (2004). However, the present paper does not rely on modeling a complex production process with strategic investment decisions taking place, as is the case in those papers. They address investment decisions on R&D or quality improvement, which is more suitable for a high-tech environment. Alternatively, the scope of our model encompasses any firm, from those with the most primitive to those with the most complex production process. To the best of our knowledge, no similar framework is available in the literature. The originality of the model also applies to the worker flow facts encompassed among its predictions.

It is worth mentioning that Pries (2004); Pries and Rogerson (2005); Munasingue (2005); Moscarini (2005); and Burgess and Turon (2005) followed a similar strategy, combining turnover models with models of search friction. However, in all those papers, firm size is fixed to one job position, which makes them silent on firm dynamics. We relax that restriction in the present paper. Finally, the interaction between firms' strategic decisions on the labor

market and implications beyond employment dynamics towards firm dynamics is also addressed by Morita (2008). However, his focus is on training decisions as opposed to hiring decisions.

The remainder of this paper is outlined as follows:

We start with a simpler version of the model, which is the object of the next two sections. In section 2 we introduce a simpler version of the model set-up, where jobs are destroyed exogenously, firms set wages, and are allowed to hire at most one worker per period. The solution for this simpler version of the model is derived in section 3. After getting used to the notation and mechanics of the model, we relax the restriction on hiring in section 4. Sections 5 and 6 are devoted to even more general versions of the model where firms bargain wages with workers and jobs are destroyed endogenously. In each of these sections, we discuss how abolishing the restrictive assumptions enlarges the set of model predictions. The final section summarizes the main contributions.

2 THE BARE-BONES MODEL ASSUMPTIONS

2.1 Timing and Information

- *Timing*

Time is discrete and has two distinct periods, $t \in \{1, 2\}$. Agents' discount rate is ρ . Decisions are made only at the beginning of each time period, when search and matching processes occur instantaneously. Any new information becomes available at the end of the first period.

- *Match quality*

There are two types of worker \times firm matches: high quality (+) and low quality (-). They are defined according to the productivity levels, which are 1 and θ respectively.

- *Learning*

Any new worker \times firm combination has, *a priori*, the probability s of attaining the higher level of productivity.

The true value for the quality of the match is revealed to the current employer at the end of the first period.

2.2 Labor Supply

- *Worker population*

Worker population is fixed and denoted by L .

- *Search activity*

Only unemployed workers search for jobs (no on-the-job searches occur).

- *Worker decision process*

If an offer is made, the unemployed worker decides whether or not to accept it. When accepting, (s)he becomes employed earning wage w , while b is the reward if (s)he remains unemployed.

2.3 Labor Demand

- *Size limitation*

Firms can post at most one vacancy per period, so the maximum employment level is one worker in the first period, and two workers in the second period.

- *Firm's population*

Any firm can enter the market in either time period, as long a start-up lump sum cost ν is paid.

- *Job destruction*

Throughout t_1 , firms may be hit, with exogenous probability (η), by a negative productivity shock, forcing them to leave the market at the end of this period.

- *Production technology*

All firms produce the same good using the same technology, based solely on the employment level of each type of match.⁴

$$f(\ell_t^+, \ell_t^-) = \ell_t^+ + \theta \cdot \ell_t^-.$$

- *Product market*

Firms are price takers and goods are sold at a fixed price (p).

⁴The following properties for technology apply: i) constant marginal productivity for both types of workers (1 for high type and θ for low type) and ii) constant returns to scale. Note that under these circumstances the first period production level corresponds to the productivity of the single match, if there is a match.

2.4 Match Technology

- *Firm's search activity and wage setting*

Firms post vacancies and set wages on the spot market.

- *Match probability*

An aggregate matching function $m(U_t^b, V_t)$ with constant returns to scale is the key determinant of the probability of a firm filling a vacancy, which is represented by $M(U_t^b, V_t)/V_t = m(U_t^b/V_t) = m(\gamma_t)$. The inputs correspond to the total number of job-seeking workers at the beginning of each period (U_t^b) and the aggregate number of vacancies (V_t). The ratio U_t^b/V_t hereafter is denoted by γ_t .

- *Vacancy cost in the replacement process*

A search component of vacancy cost (c^{rep}) is paid when vacancies are posted. This is the only cost paid by the firm when the vacancy is caused by a replacement process.

- *Vacancy costs for new jobs*

Vacancies caused by the creation of new job positions incur a higher cost (c^{jc}) due to an incremental component (c) as described by the equation below:

$$c^{jc} = c^{rep} + c.$$

The interpretation of c will be discussed in the next section.

- *No vacancy storage*

If a vacancy is not filled in the first period, the firm has to pay the associated cost if it decides to post the vacancy again in the second period.

2.5 Critical Comments on Selected Assumptions

Most of the assumptions listed above are used frequently in search models of the labor market. What follows is a brief overview of some of the present paper's key assumptions about which there is no consensus in the literature.

The assumption of match-specific productivity levels is present in Jovanovic (1979) and other articles that use his framework to address worker turnover. It contrasts with the assumption on workers' intrinsic productivity used by Gibbons and Katz (1991) among others. Both share the successful prediction of a negative hazard rate, but they generate different predictions concerning welfare loss of displaced workers. Welfare loss of displaced workers due to adverse selection arises only in an environment where productivity is intrinsic to the worker. Empirical papers testing this prediction are not conclusive. While the results in Gibbons and Katz (1991) favor predictions related to workers' intrinsic productivity, those in Grund (1999); and Song (2007) reject them.

The assumption of a higher vacancy cost for job creation relative to replacement is also employed in Pries and Rogerson (2005). It can be rationalized combining two assumptions. First, that any worker needs a unit of capital to produce, which should be bought by the firm before hiring occurs.

Second, this unit is fully depreciated if not used, and has no depreciation at all if it is used. Alternatively, we can define this extra cost as a maintenance cost for the capital while it is not in use.

Finally, the assumptions on exogenous job destruction, wage setting and size limitation are not discussed here because each of them will be the focus of specific sections of this paper devoted to generalizations of the framework that comes from relaxing those assumptions.

3 THE BARE-BONES MODEL SOLUTION

3.1 Labor Demand Policy

For the sake of simplifying the exposition of the solution, we begin by discussing labor demand assuming that firms have enough bargaining power to extract all the rent from their workers, setting wages at the same value of what workers gain from unemployment ($w = b$). Later in this section, we show that this can result from the combination of no on-the-job searches and the productivity level being specific to the match.

Under these circumstances, the combination of constant returns to scale with the hiring limit makes labor demand trivial for start-ups. Those firms which decide to enter the market in the first period will post one vacancy. The same cannot be said for incumbent firms in the second period. Firms in this situation also benefit from posting another vacancy, but they have to choose between two alternatives: i) post a vacancy to replace the worker hired in the first period, or ii) post a vacancy to create a new job position, while keeping the worker hired in the first period. We will analyze this situation

in detail according to the type of match formed in the first period.

3.1.1 Incumbent firms with a high quality match

We show in this section that a firm which learns it holds a high quality match will prefer to create a new job position with the single vacancy rather than to replace its worker hired in the first period. The expected profit related to the latter choice (r) can be written as:

$$E_{\xi_2} E_{\mu_2} [\pi_2(r \mid \xi_1 = 1)] = \{p \cdot [s + \theta \cdot (1 - s)] - b\} \cdot m(\gamma_2) - c^{rep},$$

where the first term corresponds to the profit component related to the production from a match with the expected quality value, taking into account the chance that no match can be formed. The last component is the hiring cost that needs to be paid anyway. If the firm decides to choose the job creation alternative (jc), the analogous equation is the following:

$$E_{\xi_2} E_{\mu_2} [\pi_2(jc \mid \xi_1 = 1)] = \{p \cdot [s + \theta \cdot (1 - s)] - b\} \cdot m(\gamma_2) - c^{jc} + (p - b).$$

Apart from changing the hiring cost component depending on the reason for hiring, this expression has an extra term corresponding to the profit component related to the production of the worker hired in the first period. One can see that in order for a firm to choose the latter alternative, the following condition should hold:

$$p - b > c.$$

But it is easy to see that the condition does hold since:

$$p - b > \{p \cdot [s + \theta \cdot (1 - s)] - b\} \cdot m(\gamma_2) = c^{jc} > c,$$

where the equality above comes from the zero profit condition for start-ups associated with the free entry assumption.⁵

3.1.2 Incumbent firms with a low quality match

The expected profits for a firm that learns it holds a low quality match will be the following in case of deciding in favor of replacement or job creation, respectively:

$$E_{\xi_2} E_{\mu_2} [\pi_2(r \mid \xi_1 = \theta)] = \{p \cdot [s + \theta \cdot (1 - s)] - b\} \cdot m(\gamma_2) - c^{rep};$$

and

$$E_{\xi_2} E_{\mu_2} [\pi_2(jc \mid \xi_1 = \theta)] = \{p \cdot [s + \theta \cdot (1 - s)] - b\} \cdot m(\gamma_2) - c^{jc} + p \cdot \theta - b.$$

The optimal choice now depends on the comparison between $p \cdot \theta - b$ and c . We will focus on the more interesting case where:

⁵This same condition implies that in equilibrium the profit associated with creating a new job position will be the same as that associated with keeping only the first period worker (not posting a vacancy).

$$0 < p \cdot \theta - b < c$$

In this case, the optimal choice is to fire the worker and post a vacancy to replace him. The condition above seems reasonable, since it says that the rent extracted from the low quality match is positive, but not too large so that keeping the worker is not the best alternative.

3.2 Model Overview and Micro Implications

The labor market represented in our model evolves in the following way. Initially, firms post a vacancy with the corresponding wage. The match quality will vary among firms which succeed in hiring a worker. Firms may then decide to open a new vacancy either to replace the incumbent employee or to create a new position. Displaced workers will enter the job seekers' pool together with unemployed workers and those coming from firms that have left the market.

It turns out that firms' decisions on replacement and job creation depend on the quality of their matches, where low quality matches lead to replacement and high quality matches to job creation. These heterogeneous decisions lead to the following implications related to the dynamics on size and productivity, which constitute established facts in the empirical literature where these issues are analyzed:⁶

- Firms whose observable characteristics are homogeneous (industry, size

⁶See for instance Bartelsman and Doms (2000); and Davis and Haltiwanger (1999) who report results concerning productivity and job flows respectively.

and age) attain distinct performance levels measured either as productivity at the end of the first period or job creation in the second period; and

- The higher a firm's initial productivity level, the higher its employment growth.

A new finding of the model is the negative correlation between the number of workers replaced and the number of jobs created in the second period. However, the limit of only one vacancy per period imposes this negative correlation, since firms are forced to choose only one of these actions. Our goal in the next section is to build a theoretical framework where firms may create new job positions and replace workers simultaneously. In addition, we show that the negative correlation between these variables still holds when we allow firms to choose freely the number of vacancies posted in each period. Moreover, this less restrictive environment also will provide richer implications concerning firm dynamics. Before moving to the next section we dedicate the remainder of this section to discussing two issues that were previously unaddressed: wage determination and model closure.

3.3 Wage and Labor Supply Determination

The object of this section is to provide more details on labor supply and wage determination. In each of the two time periods, these variables are determined through a game involving firms and workers as described below.

3.3.1 First period

Firms move first, offering wages that maximize their expected profit conditioned on their beliefs about workers' acceptance policy. Equilibrium occurs when the workers' acceptance policy for the observed wage offer profile corresponds to the firms' beliefs.

In equilibrium, firms will offer constant and homogeneous wages equivalent to the unemployment benefit ($w_t = b$) and any unemployed worker will accept such an offer if (s)he gets one.

It is easy to see that this is a Nash equilibrium. Given search frictions, workers would accept any wage offer greater than or equal to b . Conditioned on this acceptance policy, firms maximize their profits by offering b .

3.3.2 Second period

In the second period firms are involved in two games.

The first, played with workers who were unemployed in the first period, is described in the same way as in the first period. In this game it is easy to see that the same results apply.

The other game is played with incumbent workers who have just revealed their productivity levels to the current employer. The existence of two types of matches is a relevant departure point from the usual assumptions present in search models.

Nevertheless, the assumptions of no on-the-job search and match specific productivity, as opposed to worker specific productivity, imply that the outside option to join the job seeking pool of workers is the same for any worker

in the second period and also the same one that prevailed in the first period.⁷ Therefore, this game is equivalent to the previous one and, once more, the equilibrium is $w = b$.

One can see this solution as a particular case of a wage bargain framework where the firms extract all the rent. Cooper et al. (2007), for example, formulate the wage setting process this way.

3.4 Model Closure

As mentioned earlier, the zero profit condition for start-ups related to the entry cost assumption imposes the following restriction on parameter values in the second period:

$$\{p \cdot [s + \theta \cdot (1 - s)] - b\} \cdot m(\gamma_2) = c^{jc} + \nu.$$

From this relationship comes the solution for the probability of filling a vacancy, $m(\gamma_2)$. The analogous zero profit condition for start-ups in the first period is a bit more cumbersome, since it also addresses the present value of the expected profit in the second period, as shown below:

$$m(\gamma_1) \cdot \{s \cdot p \cdot (1 - \theta) + (p \cdot \theta - b) - \frac{c^{jc}}{m(\gamma_1)} - \frac{\nu}{m(\gamma_1)} + \rho' \cdot s \cdot m(\gamma_2) \cdot (p - b) + \rho' \cdot (1 - s) \cdot c\} = 0.$$

However it can be seen that the expression above is just a function of $m(\gamma_1)$ and $m(\gamma_2)$. We can then derive the value of the first parameter,

⁷As in the first period the job seeker pool is considered by the firms as a homogeneous group (any element has probability s to take part in a highly productive match).

provided that we previously have solved the value for the second parameter. The fact that we have assumed that $m(\gamma_t)$ is a monotonic transformation of γ_t means that we can recover the values of γ_1 and γ_2 . These parameters are the ratio between the number of job seeking workers and the number of vacancies at the beginning of the respective periods. In the first period, all workers are job seekers, which sets $U_1^b = L$.

The last step for the closure of the model in the first period is to obtain the value of V_1 , the aggregate vacancy level in the first period, from γ_1 and $U_1^b = L$. Note that in this version of the model, V_1 also corresponds to the number of firms posting vacancies at the beginning of the first period.⁸ The aggregate vacancy level in the second period, V_2 , can be obtained in the same way. However the number of job seeking workers in the beginning of the second period, U_2^b , does not correspond to L anymore. In the second period we have to address the inflows and outflows from unemployment as shown below:

$$U_2^b = U_1 + JD + R = (L - M_1) + \eta \cdot M_1 + (1 - \eta) \cdot (1 - s) \cdot M_1,$$

where M_1 stands for the number of matches formed in the first period, which corresponds to $m(\gamma_1) \cdot V_1$. It is easy too see that U_2^b depends on quantities that have their values already determined above. Then we can plug U_2^b into $m(\gamma_2)$ to get the value of V_2 . The number of vacancies posted in the second period by incumbent firms is $(1 - \eta) \cdot m(\gamma_1) \cdot V_1$, while the remaining vacancies

⁸This is not the same as the number of operating firms in the first period due to search frictions.

will be posted by start-ups.

4 THE MODEL WITH ENDOGENOUS VACANCY LEVELS

The aim of this section is to generalize the theoretical framework to allow firms to choose freely the number of vacancies posted. Obviously this procedure will make the analysis of the labor demand policy more cumbersome. On the other hand, wage and labor supply determinations remain unchanged. For this version of the model, we still rely on most of the assumptions stated previously. Those that need to be amended are restated below.

4.1 Assumptions

- *Size limitation II*

Firms can hire any number of workers in either time period.

- *Match probability II*

The probability of a firm filling ℓ_t of its v_t open vacancies is independent of the number of posted vacancies, and given by $m(\gamma_t)$ as before.

- *Vacancy cost in the replacement process II*

The search component of vacancy cost is convex. For the sake of convenience, we will use the following quadratic parametrization, $c^{rep}(v_t) = a \cdot v^2$, with $a > 0$ defining the curvature of this cost function.

- *Vacancy cost for new jobs II*

A vacancy due to the creation of a new job position has an extra linear component to be paid. The following equation summarizes the relation between the two cost components: $c^{jc}(v_t) = c^{rep}(v_t) + c \cdot v$, with $c > 0$.

The assumption of convex vacancy cost deserves comment. It is shared with Bertola and Caballero (1994); Bertola and Garibaldi (2001); Garibaldi (2006); and Garibaldi and Moen (2008). This is a critical assumption for our results on firm dynamics. Although it contrasts with the standard assumption of linear costs present in most unemployment matching models, it is an assumption critical to our results on firm dynamics. Not only do Garibaldi and Moen (2008) provide theoretical justification for this assumption, there also is empirical evidence supporting the convex shape. Yashiv (2000) points out that a structural estimation of the standard matching model performs better with convex than with linear specification for the hiring cost. Cooper et al. (2004) claim that a quadratic shape with a disruption component fits the data better than the alternatives considered, including the linear case.⁹ An important difference to be noted in these two papers is that the first uses gross hirings and separations as a measure of labor adjustment while the second uses net employment growth. Hence, the intercept in Cooper et al. (2004) may be related to the fact that firms may pay adjustment costs even when they do not grow, as long as they hire workers to replace others.

⁹Cooper et al. (2004) stress that the quadratic component is close to zero, but its inclusion does improve the fit to the data.

4.2 Labor Demand

The problem of choosing the vacancy level raises the level of complexity of labor demand policy in both periods. In the second period, firms now have to address two choices, which are: 1) how much to spend on improving the quality of the matches; and 2) how much to spend on increasing the number of jobs. The first choice consists in choosing the number of workers to be replaced, while the second consists of choosing the number of vacancies allocated to job creation. In the first period, firms now face a dynamic environment, in that their first period decisions on vacancy level will affect their available choices in the second period. Therefore, the solution to the firm problem will be stated below in the form of backward induction.

4.2.1 Start-up in the second period

Employment policy for start-up firms in the second period is the simplest case to be considered since the choice set for these firms is not influenced by past decisions. In this case, firms just choose the number of vacancies to post (v_2). The profit of a firm which hires ℓ_2^+ and ℓ_2^- when posting v_2 vacancies is represented by the following equation:

$$\pi_2(v_2 | \ell_2^+, \ell_2^-) = p \cdot (\ell_2^+ + \theta \cdot \ell_2^-) - b \cdot \ell_2 - c^{je}(v_2) - \nu.$$

However ℓ_2^+ and ℓ_2^- are not under the firm's control. They are defined by the success rate in filling vacancies (μ_2) and the proportion of high quality matches (ξ_2). These random variables have expected values s and $m(\gamma_2)$ respectively. Therefore, the firm's objective function to be maximized is the

expected profit concerning these two variables, as described by:

$$E_{\xi_2} E_{\mu_2} [\pi_2(v_2)] = \{p \cdot [s + \theta \cdot (1 - s)] - b\} \cdot m(\gamma_2) \cdot v_2 - a \cdot v_2^2 - c \cdot v_2 - \nu.$$

The convex shape of the cost function guarantees that there is only one maximum associated with an optimal vacancy level, to which we will refer as v_2^{st} . It is easy to see that

$$v_2^{st} = \frac{\{p \cdot [s + \theta \cdot (1 - s)] - b\} \cdot m(\gamma_2) - c}{2 \cdot a}.$$

The only endogenous parameter on the right side of this equation is $m(\gamma_2)$, which is determined by the free entry condition.¹⁰

4.2.2 Incumbent firms in the second period

Employment policy in the second period is more complex for incumbent firms. Between the first and second periods, they decide on the number of workers to be replaced (r) and the number of new job positions to be created (jc), having been informed about first period labor force size (ℓ_1) and the quality of the matches (ξ_1). We can write in the following way the second period profit of a firm which hires ℓ_2^+ and ℓ_2^- , when posting r vacancies devoted to replace workers in low quality matches and jc vacancies to create new job positions:

¹⁰There is a unique value for this parameter that is associated with zero profit for start-ups in the second period.

$$\begin{aligned} \pi_2(r, jc \mid \ell_2^+, \ell_2^-, \ell_1^+, \ell_1^-) &= p \cdot (\ell_2^+ + \theta \cdot \ell_2^-) - b \cdot \ell_2 - \\ & p \cdot (\ell_1^+ + \theta \cdot \ell_1^-) - b \cdot \ell_1 - [(p \cdot \theta - b) \cdot r] + [c^{jc}(jc + r) - c \cdot r]. \end{aligned}$$

The terms on the right side in the first row refer to the profit component derived from the production associated to workers hired in the second period. The second row starts with the profit component derived from the production associated to workers hired in the first period, discounting the related production of those workers replaced in the second period. The last term corresponds to vacancy costs taking into account that some of these may be allocated to replacement while the remaining to job creation. All the other terms have straightforward interpretations.

The equation above uses the following facts:

- Any displacement of workers not related to a replacement process is not considered since both types of matches are profitable;¹¹ and
- Firms will never replace any worker in a high quality match, since no new worker can be more profitable.

As mentioned earlier, the objective function to be maximized by the firms is the expected profit regarding the random variables ξ_2 and μ_2 , which can be described as below:¹²

¹¹This comes from the fact that $p \cdot \theta > b$.

¹²We use the facts that $\ell_1^+ = \xi_1 \cdot \ell_1$ and $\ell_1^- = (1 - \xi_1) \cdot \ell_1$.

$$E_{\xi_2} E_{\mu_2} [\pi_2(r, jc | \xi_1, \ell_1)] = \{p \cdot [s + \theta \cdot (1 - s)] - b\} \cdot m(\gamma_2) \cdot (jc + r) - \\ c^{jc}(jc + r) + p \cdot [\xi_1 \cdot \ell_1 + \theta \cdot (1 - \xi_1) \cdot \ell_1] - b \cdot \ell_1 + [c - (p \cdot \theta - b)] \cdot r.$$

Note that the number of replacements should be no greater than the number of low quality matches, therefore, the incumbent firms' problem can be stated as:

$$\begin{aligned} \max_{r, jc} \quad & E_{\xi_2} E_{\mu_2} [\pi_2(r, jc | \xi_1, \ell_1)] \\ \text{s.t.} \quad & r \leq \ell_1^- \end{aligned} \tag{1}$$

The application of the Kuhn-Tucker conditions for this problem is shown in Appendix A. The optimal number of worker replacements and job creation shown in figure 1 are defined as:

$$r^* = \begin{cases} \ell_1^-, & \ell_1^- < \bar{\ell}^- \\ \bar{\ell}^-, & \ell_1^- \geq \bar{\ell}^- \end{cases} \tag{2}$$

and

$$jc^* = \begin{cases} \underline{\ell}^- - \ell_1^-, & \ell_1^- < \underline{\ell}^- \\ 0, & \ell_1^- \geq \underline{\ell}^-; \end{cases} \tag{3}$$

where:

$$\underline{\ell}^- = \frac{z_2}{2 \cdot a}$$

and

$$\overline{\ell}^- = \frac{z_2 + k}{2 \cdot a};$$

with z_2 and k defined as:

$$z_2 = \{p \cdot [s + \theta \cdot (1 - s)] - b\} \cdot m(\gamma_2) - c$$

and

$$k = [c - (p \cdot \theta - b)].$$

Note that in the previous section we have mentioned that we would consider the case where $(p \cdot \theta - b) < c$, which in turn guarantees that $\underline{\ell}^- < \overline{\ell}^-$.

There are two important features to be stressed in these expressions. First, the solution for the choice variables is guided by the number of low quality matches. So, the heterogeneity of the latter variable across firms implies heterogeneous choices of replacement and job creation as well. Second, when the levels of job creation and replacement are compared across firms, it turns out that they are negatively related to each other at the ratio of one-to-one for some region below a threshold level of ℓ_1^- and they then become unrelated to each other for higher values of ℓ_1^- .

4.2.3 Start-ups in the first period

We are now ready to tackle the problem that firms face in the first period using the results derived above. It will be useful to denote the value function of the expected profit at the beginning of the second period, conditioned on realizations of ξ_1 and ℓ_1 as:

$$E_{\xi_2} E_{\mu_2} [\pi_2(r^*, jc^*, \xi_1, \ell_1)]$$

This represents the expected profit for a firm that behaves optimally in the second period, having hired ℓ_1 workers in the first period, among which the share of good matches turns out to be ξ_1 .

Note that the values of ξ_1 and ℓ_1 are not known at the beginning of the first period. At this time firms may infer these values conditioned on the vacancy level chosen (v_1).

Denoting as ρ' the composed discount rate that takes into account the survival probability ($\rho' = \rho \cdot (1 - \eta)$), the first period decision can be represented as

$$\max_{v_1} E_1[\pi(v_1)] = \max_{v_1} \{E_1[\pi_1(v_1)] + \rho' \cdot E_1[\pi_2^* | v_1]\}. \quad (4)$$

The first term in the right hand side can be written as

$$\begin{aligned}
E_\xi E_{\mu_1}[\pi_1(v_1)] &= \\
&\{p_1 \cdot [s + \theta \cdot (1 - s)] - b\} \cdot m(\gamma_1) \cdot v_1 - c^{jc}(v_1) - \nu = \\
&(z_1 - c) \cdot v_1 - a \cdot (v_1)^2 - \nu,
\end{aligned}$$

where

$$z_1 = \{p_1 \cdot [s + \theta \cdot (1 - s)] - b\} \cdot m(\gamma_1).$$

As shown in Appendix B, $E_1[\pi_2^* | v_1]$ is defined as

$$E_1[\pi_2^* | v_1] = z_1 \cdot v_1 - \nu + \begin{cases} (z_2 + k) \cdot \bar{\ell}^- - a \cdot (\bar{\ell}^-)^2, \\ v_1 > \bar{v}_1 \\ \\ (z_2 + k) \cdot (1 - s) \cdot m(\gamma_1) \cdot v_1 - a \cdot E_1[(\ell_1^-)^2 | v_1], \\ \underline{v}_1 < v_1 < \bar{v}_1 \\ \\ z_2 \cdot \underline{\ell}^- + k \cdot (1 - s) \cdot m(\gamma_1) \cdot v_1 - a \cdot (\underline{\ell}^-)^2, \\ v_1 < \underline{v}_1. \end{cases}$$

We can now define inter-temporal profit as

$$E_1[\pi_1(v_1) + \rho' \cdot E_1[\pi_2^* | v_1]] = [1 + \rho'] \cdot z_1 \cdot v_1 - a \cdot (v_1)^2 - c \cdot v_1 + \quad (5)$$

$$\rho' \cdot \begin{cases} (z_2 + k) \cdot \bar{\ell}^- - a \cdot (\bar{\ell}^-)^2, & v_1 > \bar{v}_1 \\ -a \cdot E_1[(\ell_1^-)^2 | v_1] + (z_2 + k) \cdot (1 - s) \cdot m(\gamma_1) \cdot v_1, & \underline{v}_1 < v_1 < \bar{v}_1 \\ z_2 \cdot \underline{\ell}^- - a \cdot (\underline{\ell}^-)^2 + k \cdot (1 - s) \cdot m(\gamma_1) \cdot v_1, & v_1 < \underline{v}_1. \end{cases}$$

The equation above combines quadratic and linear terms in v_1 . This guarantees that local maxima are well defined. Nevertheless, the uniqueness of a global maximum depends on the values imposed for the parameters of the model. Figure 2 aids in understanding the above expression. In Appendix C we show that global maximum uniqueness is attained for a sufficiently large s .

Note that even when firms post the same number of vacancies, they will differ in two aspects at the end of the first period: i) labor force size and ii) labor force composition. These are consequences of the stochastic environment in which hiring occurs.¹³

¹³Another dimension in which firms will differ at this stage is productivity as an obvious consequence of the heterogeneities described previously. Firms' average productivity can be expressed as $\xi_1 + \theta \cdot (1 - \xi_1)$.

4.3 Updates in Model Overview and Micro Predictions

One of the main goals of the model developed above was to reproduce the fact that some firms replace workers and create new jobs simultaneously. The mechanism of firm dynamics that comes from the enriched set of assumptions used in this version of the model generates other predictions which can be used to check the pertinence of the model.

The dynamic mechanism of this version of the model is very similar to the one used for the simpler version described earlier. The difference is that the driving force here is the number of low quality matches formed in the first period. Those firms with low levels of ℓ^- create jobs and replace fewer workers. Those firms with high levels of ℓ^- focus their hiring efforts on the replacement process and create relatively fewer jobs or even do not create any jobs at all.

Bearing in mind that μ_t and ξ_t are independent, the initial level of low quality matches is positively correlated with the initial labor force size. As a consequence of this result we predict that:

- Job creation in the second period is negatively correlated to labor force size at the end of the first period.

This corresponds to the correlation between size and growth for surviving firms pointed by Sutton (1997). The author writes: “...the proportional rate of growth of a firm (or plant) conditional on survival is decreasing in size.”¹⁴

Note that in our model there is a threshold initial level of ℓ^- above which the vacancy level does not depend on this variable anymore. Therefore we

¹⁴op. cit., page 46.

can add the following remark to the prediction above:

- The relationship between job creation and initial size pointed out before tends not to be valid for the largest firms.

This is precisely how Caves (1998) puts the relationship between employment growth and initial size: “...mean growth rates of firms decrease with their initial sizes among small firms; for initially large firms growth rates and size are unrelated.”¹⁵

In terms of productivity, the model predicts that those firms in disadvantage (high ℓ_1^-) will make more effort to improve the composition concerning match quality. Hence, there is a catch-up process in terms of match quality composition which, in other words, means a catch-up in terms of productivity. It is not difficult to see that the catch-up is limited as described below:

- Firms that in the first period are relatively more productive tend to keep this relative position in the second period.

This prediction finds strong support, this time from Bartelsman and Doms (2000): “...highly productive firms today are more than likely to be highly productive firms tomorrow...”¹⁶.

5 WAGE BARGAINING

We will show that the problem that firms solve to determine their hiring policy under wage bargaining is extremely similar to the one postulated under

¹⁵op. cit., page 1976.

¹⁶op. cit., page 571.

the wage setting environment developed in the previous sections. The set-up is the same as the previous model except for the assumption on wage setting which can now be stated as:

- *Firm's search activity and wage setting II*

Firms post vacancies and bargain wages on the spot market, with bargaining power $1 - \beta$.

5.1 Model Solution: Wage Determination

Based on the last assumption, the wage determination can be easily developed. Bargaining will be conducted conditioned on the information available for match productivity. This information determines the rent to be split between firm and worker according to their respective bargaining power. Therefore we have three possible situations:¹⁷

- An ongoing match in the second period revealed to be of high quality, in which case the wage is defined as:

$$w^+ = \beta \cdot p + (1 - \beta) \cdot b;$$

- An ongoing match in the second period revealed to be of low quality, in which case the wage is defined as:

$$w^- = \beta \cdot p \cdot \theta + (1 - \beta) \cdot b;$$

¹⁷The derivations of the value for the wage given the value of the match productivity are straightforward and used in several papers relying on the search and match framework.

and;

- A newly formed match either in the first or second period in which case the productivity is not known and the expected value is $s + (1 - s) \cdot \theta$. Therefore the wage is:

$$w^e = \beta \cdot p \cdot [s + (1 - s) \cdot \theta] + (1 - \beta) \cdot b.$$

5.2 Model Solution: Labor Demand

5.2.1 Start-up in the second period

As developed in the previous section, start-ups in the second period choose the number of vacancies that maximize their expected profit, which in this case becomes

$$E_{\xi_2} E_{\mu_2} [\pi_2(v_2)] = \{p \cdot [s + \theta \cdot (1 - s)] - w^e\} \cdot m(\gamma_2) \cdot v_2 - a \cdot v_2^2 - c \cdot v_2 - \nu.$$

Using the value derived above for w^e , we have

$$E_{\xi_2} E_{\mu_2} [\pi_2(v_2)] = (1 - \beta) \cdot \{p \cdot [s + \theta \cdot (1 - s)] - b\} \cdot m(\gamma_2) \cdot v_2 - a \cdot v_2^2 - c \cdot v_2 - \nu.$$

It is easy to see that the only difference from the analogous definition in the wage setting version of the model is the pre-multiplication of $(1 - \beta)$ in the first term. This is also the case for the value of v_2^{st} , that can be derived

as:

$$v_2^{st} = \frac{(1 - \beta) \cdot \{p \cdot [s + \theta \cdot (1 - s)] - b\} \cdot m(\gamma_2) - c}{2 \cdot a}.$$

5.2.2 Incumbent firms in the second period

Again, the objective function to be maximized by firms is the expected profit regarding the random variables ξ_2 and μ_2 . Under wage bargaining, it can be described as:¹⁸

$$E_{\xi_2} E_{\mu_2} [\pi_2(r, jc | \xi_1, \ell_1)] = \{p \cdot [s + \theta \cdot (1 - s)] - w^e\} \cdot m(\gamma_2) \cdot (jc + r) - \\ c^{jc}(jc + r) + \ell_1 \cdot \{(p \cdot \xi_1 - w^+) + [p \cdot \theta \cdot (1 - \xi_1) - w^-]\} + [c - (p \cdot \theta - w^-)] \cdot r.$$

Using the values derived above for w^e , w^+ and w^- we have

$$E_{\xi_2} E_{\mu_2} [\pi_2(r, jc | \xi_1, \ell_1)] = (1 - \beta) \cdot \{p \cdot [s + \theta \cdot (1 - s)] - b\} \cdot m(\gamma_2) \cdot (jc + r) - \\ c^{jc}(jc + r) + (1 - \beta) \cdot \ell_1 \cdot [(p \cdot \xi_1 - b) + p \cdot \theta \cdot (1 - \xi_1) - b] + [c - (1 - \beta) \cdot (p \cdot \theta - b)] \cdot r.$$

As in previous sections, the firm labor demand decision can be solved through the following maximization problem:

¹⁸We use the facts that $\ell_1^+ = \xi_1 \cdot \ell_1$ and $\ell_1^- = (1 - \xi_1) \cdot \ell_1$.

$$\begin{aligned} \max_{r, jc} \quad & E_{\xi_2} E_{\mu_2} [\pi_2(r, jc \mid \xi_1, \ell_1)] \\ \text{s.t.} \quad & r \leq \underline{\ell}_1. \end{aligned} \tag{6}$$

The only difference from the analogous definition for the expected profit in the wage setting version of the model is the pre-multiplication of $(1 - \beta)$ in some terms. So the solution for this maximization problem will be analogous to the one developed for the wage setting version of the model, where both replacement level and job creation depend on $\underline{\ell}_1$ and the parameters of the model, as shown in figure 1. The only adaptation necessary for this version of the model is that the expressions for $\underline{\ell}_1$ and $\overline{\ell}_1$ include β :

$$\underline{\ell}' = \frac{z'_2}{2 \cdot a}$$

and

$$\overline{\ell}' = \frac{z'_2 + k'}{2 \cdot a}$$

where z'_2 and k' are defined as:

$$z'_2 = (1 - \beta) \cdot \{p \cdot [s + \theta \cdot (1 - s)] - b\} \cdot m(\gamma_2) - c$$

$$k' = [c - (1 - \beta) \cdot (p \cdot \theta - b)].$$

In Appendix D we show that this similarity between the wage bargaining

version of the model and the wage setting version depends on a necessary condition being met: that the probability of finding a job ($\frac{m(\gamma_2)}{\gamma_2}$) is not too high for a job seeking worker at the beginning of the second period.

5.3 Predictions on Wage Dynamics

The introduction of wage variation with the wage bargaining set-up developed above allows us to expand the scope of model predictions on worker flows. It is easy to realize the following relationship between wages and worker flows:

- At the individual level, the probability of remaining employed in the same firm is positively associated with the wage level in the second period and the wage growth between the two periods.

This prediction is confirmed by the findings of Topel and Ward (1992), a widely quoted empirical paper on the determinants of the mobility of (young) workers. As a consequence of this result, we may state that:

- At the firm level, the average wage level in the second period is negatively correlated to the replacement level.

This finds support in Galizzi and Lang (1998), which seems to be the only empirical paper relating wages and turnover at the firm level.

6 ENDOGENOUS JOB DESTRUCTION

6.1 Generalizing Incumbent Firms' Decisions

Until now we have assumed that all firms face a positive probability of leaving the market at the end of the first period due to an exogenous negative shock. Now we want to relax this assumption. We follow standard models of firm dynamics and allow firms to choose, at the end of the first period, whether to continue or stop their operations. In case of giving up, they are compensated with a scratch value (ψ) and their employees will join the group of workers searching for jobs between the end of the first period and the start of the second period. In case of continuing their activities, they choose replacement and job creation levels as described in the last section. The firm decision in the second period can be written then as

$$\begin{aligned} \max_{r, jc, \chi} \quad & \chi \cdot E_{\xi_2} E_{\mu_2} [\pi_2(r, jc \mid \xi_1, \ell_1)] + (1 - \chi) \cdot \psi & (7) \\ \text{s.t.} \quad & r \leq \ell_1^-, \end{aligned}$$

where χ values 1 when the firm decides to continue and 0 otherwise. We can see this problem as adding another layer to the decision process of the firm. After solving for the optimal levels of r and jc based on the expected profit associated with continuing their operations, firms compare this later magnitude with ψ to decide whether to continue or not.

In what follows we show how the decision rule is defined. For the sake of simplicity we will work with the wage setting version of the model. First, it is

more convenient to write the value function of the second period continuation expected profit as a function of ℓ_1^- and ℓ_1^+ :

$$E_{\xi_2} E_{\mu_2} [\pi_2(r^*, jc^*, \ell_1^-, \ell_1^+)] = p \cdot [\ell_1^+ \theta \cdot \ell_1^-] - b \cdot (\ell_1^+ + \ell_1^-) \quad (8)$$

$$\begin{cases} z_2 \cdot \underline{\ell}^- + k \cdot \ell_1^- - a \cdot (\underline{\ell}^-)^2, & \ell_1^- < \underline{\ell}^- \\ (z_2 + k) \cdot \ell_1^- - a \cdot (\ell_1^-)^2, & \underline{\ell}^- < \ell_1^- < \bar{\ell}^- \\ (z_2 + k) \cdot \bar{\ell}^- - a \cdot (\bar{\ell}^-)^2, & \ell_1^- > \bar{\ell}^-. \end{cases}$$

The second step consists of defining the locus of points where

$$E_{\xi_2} E_{\mu_2} [\pi_2(jc^*, r^*, \ell_1^-, \ell_1^+)] = \psi.$$

Figure 3 shows the shape of this locus on the plane $\ell^- \times \ell^+$ (see the derivation of this shape in Appendix E). This line defines a border where points to the lower/left constitute the exit region, while points to the upper/right region constitute the stay region.

6.2 Further Implications on Firm Dynamics

The important thing to notice in figure 2 is the negative slope with absolute value lower than one. Some important implications follow from this decision rule. First, smaller firms tend to be closer to the border than larger firms, which means smaller firms have lower survival probabilities than larger

firms.¹⁹ It should be stressed though that this is not a deterministic relationship. For any firm in our “exit” region, but close enough to the border, one can always find another firm of the same size, or even smaller, located in our “stay” region.²⁰ The non-deterministic negative relationship between size and exit probability also features in the main theoretical frameworks of firm dynamics, such as Ericson and Pakes (1995); and Klette and Kortum (2004).

The fact that smaller firms are more inclined to leave the market provides two other implications which are frequently mentioned in the literature of firm dynamics. First, it makes the variance of employment growth larger for this group than for larger firms. Second, the relationship between initial size and employment growth for the overall firm population (including those withdrawing from the market) is closer to Gibrat’s Law, than when we restrict the population to surviving firms.²¹

7 SUMMARY AND CONCLUSIONS

In this paper we developed a labor market model where several facts on worker flow and firm dynamics were rationalized exclusively through employment decisions made by the firms. The main characteristics of the model were imperfect information about the quality of new matches and search frictions. When hiring workers, firms do not know whether there will be enough workers to fill the open vacancies and, once a new worker comes in, they do

¹⁹A smaller firm tends to be to the left and below a larger firm.

²⁰This is where the inclination lower than one is important.

²¹Gibrat’s Law says that firms’ employment growth rates are independent of size. Sutton (1997) discusses intensively the validity of this law.

not know the productivity level of that worker \times firm match.

In developing this model we built an integrated framework of the labor market in which replacement, job creation and job destruction were decided simultaneously at firm level. We have shown that several interesting micro implications come from this framework. These implications are not restricted to matters of worker flows, as they also encompass some facts on firm dynamics. Moreover, we showed that all the results derived for firm dynamics match empirical evidence documented by the related literature.

It should be stressed that other papers which offer comparable predictions on firm dynamics rely on some sort of investment decision composing a more elaborate production process. In our framework, the driving force for firm dynamics was the same as for worker flows, namely the learning process about match quality.

Desirable extensions of our framework would be an infinite horizon overlapping generation model to explore the macro implications of the model, and the effects of policy experiments. This framework would be able to predict coherently the effects of such experiments on several dimensions, such as job flows, unemployment flows, unemployment level, firms' size distribution and firms' productivity distribution. This provides valuable information for the welfare analysis of alternative policy interventions. For example, altering firing costs, social contributions or minimum wage potentially will affect all the above mentioned dimensions.

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Figure 1: Labor Demand for Incumbent Firms in the Second Period

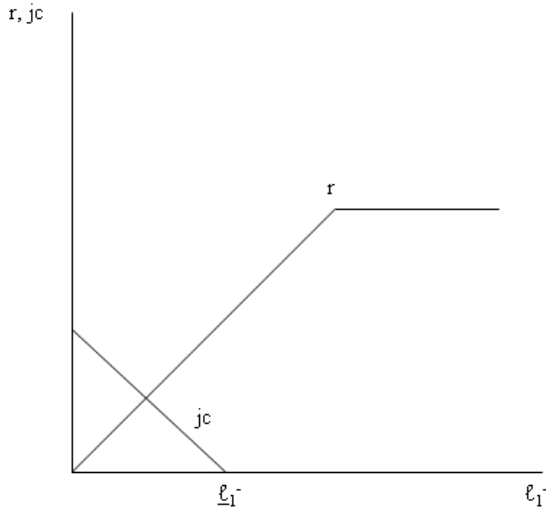


Figure 2: Labor Demand in the First Period

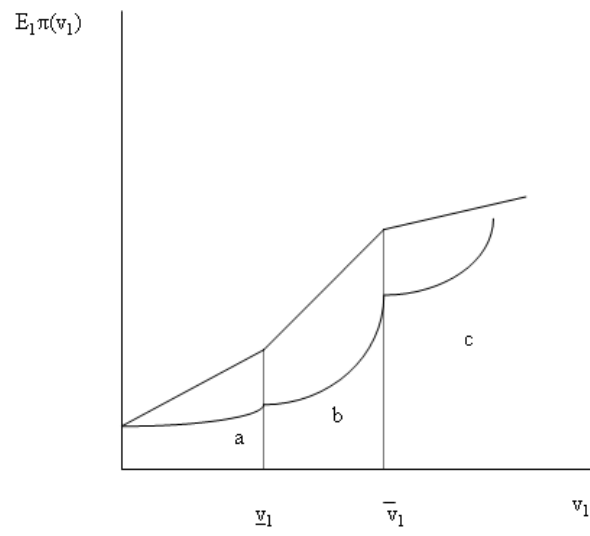
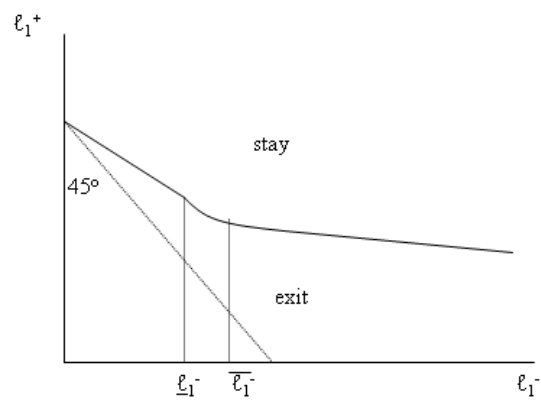


Figure 3: Exit Decision Rule



Appendices

A DERIVATION of r^* and jc^*

The incumbent firm solves the following problem in the interim between first and second periods:

$$\begin{aligned} \max_{r, jc} \quad & E_{\xi_2} E_{\mu_2} [\pi_2(r, jc \mid \xi_1, \ell_1)] \\ \text{s.t.} \quad & r \leq \ell_1^-. \end{aligned}$$

Another way of stating this problem, making the argument to be maximized an explicit function of r and jc , is the following:

$$\begin{aligned} \max_{r, jc} \quad & \{z_2 \cdot (jc + r) - a \cdot (jc + r)^2 + k \cdot r + p \cdot [\xi_1 \cdot \ell_1 + \theta \cdot (1 - \xi_1) \cdot \ell_1] - b \cdot \ell_1\} \\ \text{s.t.} \quad & r \leq \ell_1^- \end{aligned}$$

where z_2 and k correspond to:

$$z_2 = \{p \cdot [s + \theta \cdot (1 - s)] - b - c\} \cdot m(\gamma_2) - c$$

and

$$k = [c - (p \cdot \theta - b)].$$

Applying the standard Kuhn-Tucker conditions to this problem generate

three blocks of restrictions that need to be satisfied. The first is related to jc , the second to r , and the last to the lagrangian multiplier λ :

$$z_2 - 2 \cdot a \cdot (jc + r) \leq 0$$

$$jc \geq 0$$

$$[z_2 - 2 \cdot a \cdot (jc + r)] \cdot jc = 0$$

$$z_2 - 2 \cdot a \cdot (jc + r) + k - \lambda \leq 0$$

$$r \geq 0$$

$$[z_2 - 2 \cdot a \cdot (jc + r) + k - \lambda] \cdot r = 0$$

$$\ell_1^- - r \geq 0$$

$$\lambda \geq 0$$

$$(\ell_1^- - r) \cdot \lambda = 0$$

The following combinations of values for r and jc can occur:

- $jc > 0$ and $r = \ell_1^-$

According to the first block of equations above, if $jc > 0$ then

$$z_2 - 2 \cdot a \cdot (jc + r) = 0.$$

Plugging this into the second block we get $\lambda \geq k$, which implies a positive λ . When we take this to the third block, we get $r = \ell_1^-$. The value of jc can then be written as

$$jc = \frac{z_2}{2 \cdot a} - \ell_1^-.$$

Finally, in order to have $jc > 0$, the following condition must be satisfied:

$$\ell_1^- < \frac{z_2}{2 \cdot a}.$$

- $jc = 0$ and $r = \ell_1^- > 0$

According to the first block of equations above, if $jc = 0$ then

$$z_2 - 2 \cdot a \cdot r \leq 0,$$

which implies

$$\ell_1^- \geq \frac{z_2}{2 \cdot a}.$$

Concerning the restrictions on r , if $r > 0$, the second block implies

$$z_2 - 2 \cdot a \cdot r + k = \lambda.$$

On the other hand when we use $r = \ell_1^-$ in the third block, we get:

$$\lambda \geq 0$$

Putting these two conditions together we have:

$$\ell_1^- \leq \frac{z_2 + k}{2 \cdot a}.$$

So the combination of $jc = 0$ and $r = \ell_1^- > 0$ occurs when:

$$\frac{z_2}{2 \cdot a} \leq \ell_1^- \leq \frac{z_2 + k}{2 \cdot a}.$$

- $jc = 0$ and $r < \ell_1^-$

The relationships between z_2 and r derived above from $jc = 0$ and $r > 0$ remain valid. We repeat them below for convenience:

$$z_2 - 2 \cdot a \cdot r \leq 0$$

$$z_2 - 2 \cdot a \cdot r + k = \lambda.$$

However, when we take $r < \ell_1^-$ to the third block above we get $\lambda = 0$.

We can use this fact to recover the value of r from the equation above as

$$r = \frac{z_2 + k}{2 \cdot a},$$

which in turn implies that

$$\ell_1^- > \frac{z_2 + k}{2 \cdot a}.$$

B DERIVATION of $E_1[\pi_2^* | v_1]$

First, consider the following value function defined according to the optimal labor demand policy in the second period:

$$\pi_2(\xi_1, \ell_1, r^*, jc^*) = p \cdot [\xi_1 + \theta \cdot (1 - \xi_1)] - b \cdot \ell_1 + z_2 \cdot (jc^* + r^*) - a \cdot (jc^* + r^*)^2 + k \cdot r^*.$$

However, we have seen that r^* and jc^* are defined according to ℓ_1^- , which in turn can be expressed as $(1 - \xi_1) \cdot \ell_1$. So, plugging this definition of ℓ_1^- into the solution for replacement and job creation allows us to write the second period profit as

$$\pi_2^*(\xi_1, \ell_1) = \{p \cdot [\xi_1 + \theta \cdot (1 - \xi_1)] - b\} \cdot \ell_1 + \tag{9}$$

$$\left\{ \begin{array}{ll} (z_2 + k) \cdot \bar{\ell}^- - a \cdot (\bar{\ell}^-)^2, & \ell_1^- > \bar{\ell}^- \\ (z_2 + k) \cdot (1 - \xi_1) \cdot \ell_1 - a \cdot [(1 - \xi_1) \cdot \ell_1]^2, & \underline{\ell}^- < \ell_1^- < \bar{\ell}^- \\ z_2 \cdot \underline{\ell}^- - a \cdot (\underline{\ell}^-)^2 + k \cdot (1 - \xi_1) \cdot \ell_1, & \ell_1^- < \underline{\ell}^- \end{array} \right.$$

When firms are in the first period, they will look to the expected value of the second period profit conditioned on the first period vacancy level, which can be defined as below.

$$E_1[\pi_2^* | v_1] = z_1 \cdot v_1 + \tag{10}$$

$$\left\{ \begin{array}{l} (z_2 + k) \cdot \bar{\ell}^- - a \cdot (\bar{\ell}^-)^2, \\ (1 - s) \cdot m(\gamma_1) \cdot v_1 > \bar{\ell}^- \\ \\ (z_2 + k) \cdot (1 - s) \cdot m(\gamma_1) \cdot v_1 - a \cdot E_1[(\ell_1^-)^2 | v_1], \\ \underline{\ell}^- < (1 - s) \cdot m(\gamma_1) \cdot v_1 < \bar{\ell}^- \\ \\ z_2 \cdot \underline{\ell}^- - a \cdot (\underline{\ell}^-)^2 + k \cdot (1 - s) \cdot m(\gamma_1) \cdot v_1, \\ (1 - s) \cdot m(\gamma_1) \cdot v_1 < \underline{\ell}^-. \end{array} \right.$$

We have used the following facts: $\mu_1 \perp \xi_1$, $E[\xi_1 | v_1] = E[\xi_1] = s$ and $E[\ell_1 | v_1] = m(\gamma_1) \cdot v_1$.

It would be useful to redefine the inequalities above in terms of v_1 . It is easy to see that

$$\underline{v}_1 = \frac{\underline{\ell}^-}{(1 - s) \cdot m(\gamma_1)}$$

$$\bar{v}_1 = \frac{\bar{\ell}^-}{(1 - s) \cdot m(\gamma_1)}.$$

This completes our derivation for the second component of first period expected profit, which becomes:

$$E_1[\pi_2^* | v_1] = z_1 \cdot v_1 + \tag{11}$$

$$\left\{ \begin{array}{l} (z_2 + k) \cdot \bar{\ell}^- - a \cdot (\bar{\ell}^-)^2, \\ v_1 > \bar{v}_1 \\ \\ (z_2 + k) \cdot (1 - s) \cdot m(\gamma_1) \cdot v_1 - a \cdot E_1[(\ell_1^-)^2 | v_1], \\ \underline{v}_1 < v_1 < \bar{v}_1 \\ \\ z_2 \cdot \underline{\ell}^- - a \cdot (\underline{\ell}^-)^2 + k \cdot (1 - s) \cdot m(\gamma_1) \cdot v_1, \\ v_1 < \underline{v}_1. \end{array} \right.$$

C UNIQUENESS of v_1^*

The goal of this section is to describe sufficient conditions that guarantee a unique solution to the problem of choosing the vacancy level at the start of the first period. As discussed in the body of the paper, this choice is made in order to maximize the present value of the expected profit conditioned on behaving optimally in the second period given your choice of first period vacancy. The definition for the expected profit is

$$E_1[\pi_1(v_1) + \rho' \cdot E_1[\pi_2^* | v_1]] = [1 + \rho'] \cdot z_1 \cdot v_1 - a \cdot (v_1)^2 - c \cdot v_1 + \quad (12)$$

$$\rho' \cdot \begin{cases} (z_2 + k) \cdot \bar{\ell}^- - a \cdot (\bar{\ell}^-)^2, & v_1 > \bar{v}_1 \\ -a \cdot E_1[(\ell_1^-)^2 | v_1] + (z_2 + k) \cdot (1 - s) \cdot m(\gamma_1) \cdot v_1, & \underline{v}_1 < v_1 < \bar{v}_1 \\ z_2 \cdot \underline{\ell}^- - a \cdot (\underline{\ell}^-)^2 + k \cdot (1 - s) \cdot m(\gamma_1) \cdot v_1, & v_1 < \underline{v}_1. \end{cases}$$

First, we disaggregate $E_1[\pi_2^*(v_1)]$ into linear and non-linear components, denoting them respectively by $Rmg(v_1)$ and $Cmg(v_1)$. They are illustrated in figure 2 which shows three different regions (a, b and c) delimited by \underline{v}_1 and \bar{v}_1 . Suppose the following restrictions hold:

$$Rmg^-(\underline{v}_1) < Cmg^-(\underline{v}_1)$$

$$Rmg^+(\underline{v}_1) < Cmg^+(\underline{v}_1)$$

$$Rmg^+(\bar{v}_1) < Cmg^+(\bar{v}_1)$$

That means the local maximum in region a is attained in the interior of its domain and the local maximum in regions b and c is attained at their respective left boundaries. When this is the case, the global maximum corresponds to the maximum in the first region, since the following relationship

holds:

$$\pi(v_a^*) > \pi(v_b^*) = \pi(\underline{v}_1) > \pi(\overline{v}_1) = \pi(v_c^*).$$

The first and second restrictions correspond to the ones shown below. The third one is not binding, since its requirement is guaranteed by the first restriction:

$$\begin{aligned} 2 \cdot a \cdot \underline{v}_1 &> (1 + \rho') \cdot z_1 - c + k \cdot \rho' \cdot (1 - s) \cdot m(\gamma_1) \\ 2 \cdot a \cdot \underline{v}_1 + \rho' \cdot a \cdot \frac{\partial E_1[(\ell_1^-)^2 | \underline{v}_1]}{\partial v} &> (1 + \rho') \cdot z_1 - c + (k + z_2) \cdot \rho' \cdot (1 - s) \cdot m(\gamma_1). \end{aligned}$$

Now consider the first inequality above. It can be rewritten as:

$$\frac{z_2}{(1 - s) \cdot m(\gamma_1)} > (1 + \rho') \cdot \frac{(z_2 + c) \cdot m(\gamma_1)}{m(\gamma_2)} - c + k \cdot \rho' \cdot (1 - s) \cdot m(\gamma_1),$$

using the definition for \underline{v}_1 and the fact that

$$z_1 = \frac{(z_2 + c) \cdot m(\gamma_1)}{m(\gamma_2)}.$$

In the remainder of this section we will assume $m(\gamma_1) \cong m(\gamma_2)$, for the sake of simplifying the notation. The argument to be developed does not depend on this assumption. After rearranging the terms we get

$$z_2 \cdot \frac{[1 - (1 + \rho') \cdot (1 - s)] \cdot m(\gamma_1)}{(1 - s) \cdot m(\gamma_1)} > \rho' \cdot c + k \cdot \rho' \cdot (1 - s) \cdot m(\gamma_1).$$

Finally, the fact that $\frac{\partial z_2}{\partial s} > 0$ guarantees that the first inequality holds when we increase s . The second inequality can be written as

$$\begin{aligned} z_2 \cdot \frac{[1 - (1 + \rho') \cdot (1 - s)] \cdot m(\gamma_1)}{(1 - s) \cdot m(\gamma_1)} - \rho' \cdot c + k \cdot \rho' \cdot (1 - s) \cdot m(\gamma_1) \\ > z_2 \cdot \rho' \cdot (1 - s) \cdot m(\gamma_1) - \rho' \cdot a \cdot \frac{\partial E_1[(\ell_1^-)^2 \mid \underline{v}_1]}{\partial v}. \end{aligned}$$

We have just shown that the left hand side (*LHS*) is greater than zero for a large enough s . The right hand side (*RHS*) can be restated as:

$$\begin{aligned} z_2 \cdot \rho' \cdot (1 - s) \cdot m(\gamma_1) - \frac{z_2 \cdot \rho' \cdot (1 - s)}{m(\gamma_1)} \\ - \frac{z_2 \cdot \rho' \cdot x}{m(\gamma_1) \cdot (1 - s)} \end{aligned}$$

where we have used the result below:

$$\begin{aligned} \frac{\partial E_1[(\ell_1^-)^2 \mid \underline{v}_1]}{\partial v} = 2 \cdot a \cdot \underline{v}_1 \cdot \\ \{(1 - s)^2 + \text{var}(1 - s) + [m(\gamma_2)]^2 + \text{var}[m(\gamma_2)]^2\} \end{aligned}$$

and have denoted $\{\text{var}(1 - s) + [m(\gamma_2)]^2 + \text{var}[m(\gamma_2)]^2\}$ as x . It is easy

to see that this *RHS* is negative for any s , which allows us to establish the following relationship:

$$LHS > 0 > RHS$$

Thus, a big enough s in an environment where $m(\gamma_1) = m(\gamma_2)$ suffices to guarantee a unique solution for v_1^* at a level lower than \underline{v}_1 .

D THE NECESSARY CONDITION THAT DRIVES THE SOLUTION OF THE WAGE BARGAINING VERSION OF THE MODEL

Consider a match that is revealed to be of low quality. At the beginning of the second period, the worker in this match may decide either to quit or to stay. The expected payoffs (*EP*) related to each of these two decisions, identified by superscripts, are the following:

$$EP^q = q \cdot \{p \cdot \beta \cdot [s + (1 - s) \cdot \theta] + (1 - \beta) \cdot b\} + (1 - q) \cdot b$$

and

$$EP^s = p \cdot \beta \cdot \theta + (1 - \beta) \cdot b,$$

where q represents the probability of a job seeking worker finding a job at the beginning of the second period, which is given by $\frac{m(\gamma_2)}{\gamma_2}$. Straightforward algebra can be used to show that workers will stay if:

$$f(q) < p \cdot \theta - b,$$

where $f(q)$ abbreviates $p \cdot q \cdot \theta + p \cdot q \cdot s \cdot (1 - \theta) - b \cdot q$. In what follows we will show that any value for q below a threshold will make such workers prefer to stay, as in the wage setting version of the model. First, note that the condition is attained when $q = 0$. Hence showing that $f'(q) > 0$ suffices to prove our claim. In fact, we have that

$$f'(q) = p \cdot [s + (1 - s) \cdot \theta] - b > 0.$$

Finally, the threshold value for q which makes the worker indifferent between quitting and staying is:

$$q^* = \frac{p \cdot \theta - b}{p \cdot [s + (1 - s) \cdot \theta] - b}.$$

E DERIVATION OF THE EXIT RULE

The main component of the exit rule consists of a line on the plane $\ell_1^- \times \ell_1^+$ which represents the locus of points where

$$E_{\xi_2} E_{\mu_2} [\pi_2(jc^*, r^* | \ell_1^-, \ell_1^+)] = \psi.$$

The shape of this line is given by the partial derivative of ℓ_1^+ with respect to ℓ_1^- keeping constant the value function above. That is, we need to compute:

$$\left. \frac{\partial \ell_1^+}{\partial \ell_1^-} \right|_{E_{\xi_2} E_{\mu_2} [\pi_2(jc^*, r^* | \ell_1^-, \ell_1^+)] = \psi}.$$

Straightforward derivations imply that:

$$\frac{\partial \ell_1^+}{\partial \ell_1^-} \Big|_{E_{\xi_2} E_{\mu_2} [\pi_2(jc^*, r^* | \ell_1^-, \ell_1^+)] = \psi} = \frac{1}{p-b} \left\{ \begin{array}{l} -[(p \cdot \theta - b) + k], \\ \underline{\ell}_1^- < \underline{\ell}^- \\ \\ 2 \cdot a \cdot \ell_1^- \cdot [(p \cdot \theta - b) + k + z_2], \\ \underline{\ell}^- < \ell_1^- < \overline{\ell}^- \\ \\ -(p \cdot \theta - b), \\ \ell_1^- < \overline{\ell}^-. \end{array} \right.$$

It is easy to see that the the first and last regions have negative and constant inclinations. In the intermediary region ($\underline{\ell}^- < \ell_1^- < \overline{\ell}^-$) the inclination raises monotonically with the level of ℓ_1^- . Plugging into the equation values for $\underline{\ell}^-$ and $\overline{\ell}^-$, one can see that the slope in this region varies monotonically from the value of the first region to the value of the last region.

The last thing to be noticed is that throughout all three regions, the inclination is always flatter than the 45 degrees line, as shown in figure 3. The result to prove this fact is related to the slope of the first region, which is higher than -1 . As this is the steepest region, all the others will also satisfy this condition.

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