Testing the New Keynesian Phillips curve through Vector Autoregressive models: Results from the Euro area

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January 2005

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This version: January 2007.

Abstract

This paper addresses the issue of testing the 'hybrid' New Keynesian Phillips Curve (NKPC) through Vector Autoregressive (VAR) systems and likelihood methods, giving special emphasis to the case where variables are non stationary. The idea is to use a VAR for both the inflation rate and the explanatory variable(s) to approximate the dynamics of the system and derive testable restrictions. Attention is focused on the 'inexact' formulation of the NKPC. Empirical results over the period 1971-1998 show that the NKPC is far from being a 'good first approximation' of inflation dynamics in the Euro area.

Keywords: Inflation dynamics, Forecast model, New Keynesian Phillips Curve, Forward-looking behavior, VAR expectations.

J.E.L. Classification: C32, C52, E31, E32.

*I wish to thank Christopher Bowdler, Søren Johansen, David Hendry, Keshab Bhattarai, Massimo Franchi and three anonymous referees for helpful comments and suggestions on earlier versions of the paper. I am responsible for all errors. Paper presented at the "*First Italian Congress of Econometrics and Empirical Economics*", Venice 24-25 January 2005, "*The Cointegrated VAR model: Methods and Applications*", Copenhagen, 18-20 June 2006, and the "*61st European Meeting of the Econometric Society*", Vienna, 24-28 August 2006. Partial financial support from Italian MIUR grants ex-60% is gratefully acknowledged.

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1 Introduction

The Phillips curve plays a central role in our understanding of business cycles and the management of monetary policy. Several of the New Keynesian models of inflation dynamics, including the models of staggered contracts of Taylor (1979) and Calvo (1983), and the quadratic price adjustment cost model of Rothemberg (1982), have a common formulation that is similar to the expectations-augmented Phillips curve of Friedman and Phelps (Roberts, 1995). The empirical literature on the so-called New Keynesian Phillips curve (NKPC) has expanded rapidly without consensus on the role of forward-looking components in inflation dynamics.

The NKPC we discuss in the present paper can be regarded as the aggregate supply equation of 'miniature' dynamic stochastic general equilibrium (DSGE) policy models, derived under the hypothesis of intertemporal micro-optimizing households and firms. Typically these models include a forward-looking IS curve, the NKPC and an interest rate rule, see King (2000) and Henry and Pagan (2004). Obviously, considering the NKPC as a single-equation model or as part of a 'trinity' DSGE model affects the way the empirical analysis is tackled, i.e. 'limited' versus 'full' information methods.

The recent success of the NKPC can be attributed to the papers by Galí and Gertler (1999) and Galí et al. (2001), where by the use of limited information methods, the so-called 'hybrid' version of the Phillips curve is found to provide 'good first approximation' of inflation in the US and Euro area, see also Sbordone (2002) and Galí et al. (2005). In Galì et al. (2001) real unit labor costs are used to proxy real marginal costs as opposed to the use of the output gap, and this choice is regarded as a crucial fact underlying the empirical success of the NKPC. On the other hand, the results on the United States obtained by Fuhrer and Moore (1995), Fuhrer (1997) and Rudd and Whelan (2005*a*, 2005*b*, 2006) by full information methods and the output gap as the driving variable seem to undermine the role of forward-looking components as relevant causes of inflation.

The use of the NKPC as a consensus model of inflation dynamics seems to disregard the idea that there exists many sources of price growth, see e.g. Hendry (2001). Furthermore, aside from the subtle question of disentangling empirically between forward and backward-looking behavior (e.g. Ericsson and Hendry, 1999), when aggregate data are used as for the Euro area, the aggregation process might blur the actual single-agent behavioral relations connecting prices and other macroeconomic variables at the country level. Nonetheless, as the NKPC is presently the leading model of inflation dynamics, the issue of testing its empirical validity is a challenge deserving of attention.

This paper contributes to the empirical literature by addressing the econometric investigation of the NKPC through Vector Autoregressive (VAR) systems, giving special emphasis to the case where variables are non stationary.

VARs are extensively used to proxy agent's expectations (Brayton et al., 1997) and to estimate and test the NKPC, see e.g. Fuhrer and Moore (1995), Fuhrer (1997), Sbordone (2002, 2005), Rudd and Whelan (2005*a*, 2005*b*, 2006) and Kurmann (2006). The common practise is to refer to stationary systems which are exploited to approximate the dynamics of the forcing variables in the present value formulation of the model.[1] This allows to derive a set of (testable) cross-equation restrictions. However, when the roots of the VAR are close to the unit circle, test statistics based on standard asymptotic theory and the typical sample lengths of macroeconomic analysis may suffer of large size distortion and power losses, see e.g. Johansen (2006). The idea of the present paper is that it may be convenient, from the point of view of reliable asymptotic inference, to recognize that (aggregate) time-series might be approximated as non stationary integrated of order one (I(1)) processes. Although theory at the individual (firm) level is based on stationary variables, we argue that non-stationarity may stem from the aggregation of sectoral and regional/national Phillips curves.

The method we use in the paper is inspired by Sargent's (1979) VAR-based analysis of Euler equations, and generalizes to some extent the likelihood-based estimation and testing strategy set out in Johansen and Swensen (1999) and Fanelli (2002, 2006) for forward-looking models with I(1) variables. The idea is to nest the NKPC within a dynamic system (the VAR) serving as agents' forecast model. The VAR, including inflation and its driving variable(s), can be reparameterized in Vector Equilibrium Correction (VEqC) form when time-series are non stationary; in turn, the VEqC can be opportunely mapped into a stationary representation that facilitates the derivation of cross-equation restrictions with the NKPC. Two models are then considered: a restricted system embodying the cross-equation restrictions implied by the forward-looking

[1] Throughout we shall use the terms 'explanatory variable', 'forcing variable' and 'driving variable' interchangeably.

model, and an unrestricted system representing agents' reduced form (statistical model). The log-likelihoods of the two systems can be compared to assess whether the NKPC is supported by the data. If the model is not rejected, consistent estimates of the structural parameters can be recovered from the restricted model.

The paper focuses on the 'inexact' version of the NKPC, i.e. on a formulation of the forward-looking model of inflation dynamics that incorporates an exogenous disturbance term modelled as a martingale difference sequence (MDS), intended to capture (unexplained) transitory deviations from the theory. Aside from studies based on 'miniature' DSGE models (e.g. Lindé, 2005), to our knowledge Bårdsen et al. (2004) and Kurmann (2006) are existing examples where the investigation of the 'inexact' NKPC is addressed. We show how the econometric analysis of the 'inexact' NKPC can be extended to the case where agents' forecast model is a non stationary VAR.

The proposed method is applied to quarterly inflation dynamics in the Euro area over the period 1971-1998. We use the same data set as in Galì et al. (2001) and two proxies of firms' marginal costs: the wage share and the output gap, where the latter is measured as in Fagan et al. (2001), i.e. with potential output expressed as a constant returns to scale Cobb-Douglas production function and neutral technical progress. In line with the conclusions of Bårdsen et al. (2004) based on the encompassing principle, our results on the Euro area suggest that the hybrid formulation of the NKPC suffers from 'missing dynamics', in the sense we explain in the paper.

The paper is organized as follows. Section 2 introduces the hybrid NKPC and Section 3 addresses the empirical issue of non stationarity. Section 4 sketches the VAR-based investigation of the 'inexact' NKPC. Section 5 summarizes empirical results for the Euro area over the period 1971-1998, and Section 6 contains some concluding remarks. Technical details are outlined in the Appendix.

2 The New Keynesian Phillips curve

The hybrid formulation of the NKPC reads as a Linear Rational Expectations (LRE) model where the inflation rate depends on the expected future value of inflation rate, lagged inflation and a set or a single driving variable. Following Gali et al. (1999) and Galì et al. (2001), this equation can be formulated, in its 'final' structural form, as

$$\pi_t = \gamma_f E_t \pi_{t+1} + \gamma_b \pi_{t-1} + \lambda' x_t + u_t \tag{1}$$

where π_t is the inflation rate at time t, x_t the vector of explanatory variable(s), $E_t\pi_{t+1}$ is the expected value at time t of the inflation rate prevailing at time $t+1$, u_t a disturbance term that we discuss in Section 4 and γ_f, γ_b and λ are structural parameters, with λ a scalar or vector, depending on the dimension of x_t. Expectations are conditional upon the information set available at time t, i.e. $E_t\pi_{t+1} = E(\pi_{t+1} \mid \mathcal{F}_t)$.

In most recurrent specifications, x_t is a single driving variable (so λ is a scalar), usually a proxy of demand pressure such as the output gap or the unemployment rate, or a measure of firm's real marginal costs, such as real unit labor costs. In small-open economy versions of the NKPC x_t is a vector incorporating unit labor costs and e.g. the price of imported intermediate goods, see e.g. Petursson (1998) and Batini et al. (2005).

The equation (1) can be derived through several routes within the New Keynesian paradigm, see e.g. Roberts (1995). Galì et al. (2001) refer to the RE staggered-contracting model of Calvo (1983). In general, $\gamma_f \geq 0$, $\gamma_b \geq 0$, $\lambda > 0$ and $\gamma_b + \gamma_f \leq 1$, and in the Calvo model γ_f, γ_b and λ can be directly associated with other 'deep' structural parameters related to firms' discount factor, the fraction of backward-looking firms and the average time over which prices are kept fixed, see Galì and Gertler (1999).

From the policy point of view the NKPC implies that a fully credible disinflation implies a positive sacrifice ratio which increases with the fraction of backward-looking firms. On the other hand if $\gamma_b = 0$ the purely forward-looking NKPC entails that a fully credible disinflation has no output costs. The inclusion of lagged inflation terms in the base 'pure forward-looking' version of the model ($\gamma_b = 0$) can be also motivated by referring to models with two (or more) period overlapping wage contracts as in e.g. Fuhrer and Moore (1995).[2] In this paper we shall refer to the hybrid formulation (1) of the model as the NKPC,

[2] In practise the inclusion of lags of inflation in the baseline model allows to overcome the 'jump' dynamics that the non-hybrid specification would entail, making hard a reconciliation among observed inflation patterns and the way central banks react to supply shocks. Policy implications are different if one appeals to the standard ($\gamma_b = 0$) or hybrid formulation ($\gamma_b \neq 0$) of the NKPC: according to the former monetary policy can drive a positive rate of inflation to zero with virtually no loss of output and employment ("disinflation without recession"); in the latter disinflation experiments can not be accompanied by low sacrifice ratios.

except where indicated.

The NKPC can be investigated as a single-equation model, leaving the variable(s) in x_t in reduced form, or as the pricing equation of a prototypical 'small scale' DSGE model of monetary policy. For instance, suppose that $x_t \equiv y_t$ in (1) represents the output gap and that the model for y_t is given by the following forward-looking IS schedule

$$y_t = \varphi_1 E_t y_{t+1} + (1 - \varphi_1) y_{t-1} - \varphi_2 (i_t - E_t \pi_{t+1}) + u_t^d \tag{2}$$

which can be derived from a representative agent intertemporal utility maximizer with external habit persistence. In (2) φ_i, $i = 1, 2$ are structural parameters, i_t is a short term nominal interest rate and u_t^d is a demand shock. The model given by (1) and (2) can be closed by specifying an interest rate rule of the form

$$i_t = \bar{i} + \zeta_1 (E_t \pi_{t+1} - \pi^*) + \zeta_2 y_t + u_t^m \tag{3}$$

where $\bar{i}$ is the long run equilibrium nominal interest rate, π^* is the target inflation rate, ζ_i, $i = 1, 2$ are the parameters and in particular $\zeta_1 > 1$ indicates an active stabilization policy (Clarida et al., 1999), and u_t^m is the 'unsystematic portion' of monetary policy.

The DSGE prototype system (1), (2) and (3) generates a great deal of debate in monetary policy, e.g. King (2000) and Henry and Pagan (2004). This paper focuses on the estimation of (1).

3 Addressing the empirical analysis

Suppose one wants to investigate empirically, for a given developed economy, the NKPC (1). Bårdsen et al. (2004), Mavroeidis (2004, 2005) and Nason and Smith (2005) show that the empirical analysis of the NKPC can be hardly carried out within a single-equation framework, without any concern on the process generating explanatory variables. The literature on LRE models shows that the dynamic specification of the x_t variable(s) is crucial for the identification of the parameters of the NKPC, even when these are thought to be exogenously given, see e.g. Pesaran (1987), Chap. 6. Thus one can consider either structural or reduced form equations for x_t, and then apply 'full information' techniques.

This section focuses on a specific issue characterizing the econometric analysis of (1): the non stationarity of variables. The estimation of the NKPC is usually carried out by treating inflation and its forcing variable(s) as realization of stationary processes, without any concern on the analysis of misspecification and the properties of estimates; Petursson (1998), Bårdsen et al. (2004), Juselius (2006) and Boug et al. (2006) are remarkable exceptions. Using US quarterly data Furher and Moore (1995), Section 2, recognize the empirical relevance of unit roots but do not estimate the forward-looking model of inflation dynamics under the I(1) hypothesis. Such a limited attention to the empirical implications of non stationarity finds its motivation in the underlying theory, which is intrinsically built on mean-reverting variables, and in the observation that DSGE models are obtained as linearized approximations of non-linear models around some steady-state.

Yet, while theory is formulated at the single agent level, estimation is usually based on aggregate data. Aggregation may have both theoretical and empirical consequences. For instance, as shown in e.g. Hughes Hallet (2000), the aggregation of sectoral, regional/national Phillips curves may yield an inflation-unemployment trade-off that is not vertical in the long run, despite the 'individual' curves being vertical. On the other hand, by aggregating simple, possibly dependent, dynamic micro-relationships, the resulting aggregate series might display univariate long-memory, and obey integrated, or infinite length transfer function relationships, as detailed in e.g. Granger (1980). In line with these considerations, O'Reilly and Whelan (2005) and Batini (2006) both find that the persistence of Euro area inflation is very close to one.

Whether e.g. inflation is best described as an highly persistent stationary or unit root process has a number of economic and empirical implications that we do not address in the present paper.[3] We refer to e.g. Culver and Papell (1997) and references therein for a detailed investigation. Likewise, though the output gap is conceptually a stationary variable, there is no guarantee that methods based on e.g. the Hodrick-Prescott (HP) filter, or on regressions of output on deterministic terms actually deliver stationary time-series. Computing e.g. the log labour income share 'in deviation from the steady state' by removing some constant from the corresponding time-series does not guarantee that the resulting variable is actually stationary. Moreover, test statistics based on standard

[3] In principle, theoretical arguments can be advocated to emphasize that unit roots are not conceptually tenable within the class of DSGE models. For instance, it can be argued that to the extent that monetary policy targets inflation at a low level to keep economic activity near capacity, a unit root in inflation does not make sense. As we stress throughout the paper, a unit root must not be interpreted as an 'intrinsic' property of a given variable but as a statistical approximation useful for inference.

asymptotic theory and the typical sample lengths of macroeconomic analysis may suffer large size distortion and power losses when the roots of the characteristic equation of the system are close to the unit circle. Johansen (2006) shows that if in DSGE models one insists that a root very close to one is a stationary root, then many more observations than what is usually available for conducting inference on steady state values are needed. Hence, fixing the number of unit roots of the system when there exist a sound suspect that variables might be driven by stochastic trends, can relieve the small sample issues characterizing inference. This paper shows how the econometric analysis of the NKPC can be addressed in these circumstances.[4]

4 Testing the NKPC

As with many other economic theories, the NKPC specifies a relationship involving future expectations (forecasts) of a set of variables. This relationship implies a set of restrictions which may be tested, along the lines of Sargent (1979) and Baillie (1989), against some general unrestricted dynamic model for $Y_t = (\pi_t, x_t')'$ such as a VAR representing agents' forecast system.

In deriving the restrictions and testing the model, however, a relevant issue is whether $u_t = 0$ in (1) or not, i.e. whether the NKPC is specified as an 'exact' or 'inexact' LRE model. Abstracting from contributions based on 'small scale' DSGE models of the form (1)-(3), empirical investigations of the NKPC through 'full-information' methods have been typically carried out with respect to the 'exact' NKPC, see e.g. Furher and Moore (1995), Furher (1997), Sbordone (2002, 2005), Ruud and Whelan (2005*a*, 2005*b*, 2006). However, aside from the myriad of possible economic interpretations that one can give to a non zero u_t in (1), the specification of the NKPC with $u_t \neq 0$ is more appealing as such a disturbance term can be regarded as a quantity capturing temporary (unexplained) deviations from the theory. A convenient way to characterize this type of model uncertainty is to assume either that u_t obeys a MDS with respect to the information set $\mathcal{F}_t$, i.e. $E(u_t \mid \mathcal{F}_{t-1}) = 0$, or that u_t is an iid process. Bårdsen et al. (2004) and Kurmann (2006) take an explicit stand

[4]Once one accepts the idea that the time-series involved in the empirical analysis of the NKPC (1) might be approximated as unit root processes, even with 'limited information' GMM methods and 'strong' instruments, the use of conventional asymptotic chi-squared criteria for testing hypotheses on $(\gamma_f, \gamma_b, \lambda)$ requires caution, especially if π_t and x_t are not cointegrated, see Kitamura and Phillips (1997).

on the 'inexact' NKPC, however, whereas the former recognize that Euro area inflation dynamics resembles the behaviour of a unit root process, the latter treats variables as stationary series.[5]

The econometric analysis of the 'inexact' NKPC through VAR models has received limited attention. For this reason in this section we discuss how the predictions of the 'inexact' NKPC can be addressed in the non stationary framework.

To approximate agents' expectation generating system, we consider the $p \times 1$ vector $Y_t = (\pi_t,\, x_t')'$, $p = (q+1)$, where x_t can be a scalar ($q = 1$) or a vector ($q \geq 2$) of explanatory variables, and the VAR(k) representation

$$Y_t = A_1 Y_{t-1} + \ldots + A_k Y_{t-k} + \mu_0 + \mu_d D_t + \varepsilon_t \tag{4}$$

where A_1, ..., A_k are $p \times p$ matrices of parameters, k is the lag length, Y_{-p}, ..., Y_{-1}, Y_0, are given, μ_0 is a $p \times 1$ constant, D_t is a $d \times 1$ vector containing deterministic terms (linear trend, seasonal dummies, intervention dummies and so on) and μ_d the corresponding $p \times d$ matrix of parameters. Moreover, $\varepsilon_t \sim N(0,$ $\Omega)$ is a $p \times 1$ MDS with respect to the sigma-field $I_t = \sigma\left\{Y_t,\ Y_{t-1},\ ...,\ Y_1\right\} \subseteq \mathcal{F}_t$, and it is assumed that the parameters $(A_1, \ldots, A_k, \mu_0, \mu_d, \Omega)$ are time invariant and that the roots of the characteristic equation associated with the VAR

$$\det(A(z)) = \det(I_p - A_1 z - A_2 z^2 - ... - A_k z^k) = 0 \tag{5}$$

are such that $\mid z \mid > 1$ or $z = 1$. Finally, we maintain that the VAR lag length is $k \geq 2$, since as shown in Fanelli (2002) and Mavroedis (2004), $k \geq 2$ is a necessary condition for the identification of the structural parameters of Euler equations of the form (1).

The VAR(k) (4) can be written in Vector Equilibrium Correction (VEqC) form

$$\Delta Y_t = \Pi Y_{t-1} + \Phi_1 \Delta Y_{t-1} + \ldots + \Phi_{k-1} \Delta Y_{t-k+1} + \mu_0 + \mu_d D_t + \varepsilon_t \tag{6}$$

where $\Pi = -(I_p - \sum_{i=1}^{k} A_i)$ is the long run impact matrix, and $\Phi_j = -\sum_{i=j+1}^{k} A_i$, $j = 1, ..., k-1$. When there are exactly $p - r$ unit roots in the system, $rank(\Pi) = r$, $0 < r < p$, in (6), and $\Pi = \alpha\beta'$, with α and β two $p \times r$ full rank matrices, whose meaning is detailed in Johansen (1996).

[5] Differently from previous likelihood-based findings on the US economy, Kurmann (2006) shows that results coincide by and large with Gali and Gertler's (1999) GMM estimates, confirming that conditional on marginal cost being (correctly) measured by labor income share, forward-looking behavior is an important feature of price setting.

Using simple algebra, the NKPC (1) can be expressed in error-correction form

$$\Delta\pi_t = \psi E_t \Delta\pi_{t+1} + \omega z_t + u_t^* \tag{7}$$

where, provided that $\gamma_b + \gamma_f < 1$, $z_t = (\pi_t - \xi' \; x_t)$, $\xi = \frac{\lambda}{1-\gamma_f-\gamma_b}$, $\psi = \frac{\gamma_f}{\gamma_b}$, $\omega = \left(\frac{\gamma_f+\gamma_b-1}{\gamma_b}\right)$ and $u_t^* = \gamma_b u_t$. Observe that in the parameterization (7) z_t reads as the driving variable of the acceleration rate. Interestingly, if π_t and x_t are generated by I(1) processes, it turns out that z_t must be stationary for (7) to be a balanced model.[6] Apparently (7) involves only two parameters, ψ and ω, which in turn depend on γ_f and γ_b; however, from the definitions above it turns out that the third structural parameter, λ, is embedded in the definition of z_t. Hence, given an estimate of ξ, $\widehat{\xi}$ (see below), and γ_f and γ_b, λ is automatically determined by $\lambda = (1-\gamma_f-\gamma_b)\widehat{\xi}$.

By conditioning both sites of (7) with respect to I_{t-1}, using the law of iterated expectations and exploiting the MDS property of u_t (u_t^*) yields the relation

$$E(\Delta\pi_t \mid I_{t-1}) = \psi E(\Delta\pi_{t+1} \mid I_{t-1}) + \omega E(z_t \mid I_{t-1}) \tag{8}$$

which can be used to derive cross-equation restrictions once expectations are replaced by the corresponding VEqC-based forecasts stemming from (6). Therefore, using the companion form representation of the system and incorporating the restriction $z_t = (\pi_t - \xi' \; x_t) = \beta' Y_t$ (implying that the cointegration rank is $r = 1$), it is possible to retrieve a set of non-linear restrictions between the VEqC and the NKPC.

In the Appendix we describe a procedure for deriving the cross-equation restrictions between (6) and (7), based on a particular representation of the VEqC (6) with $\Pi = \alpha\beta'$. More precisely, we show that for given cointegration rank r and cointegration matrix β, the VEqC (6) can be represented as a stable VAR(k) of the form

$$W_t = B_1 W_{t-1} + ... + B_k W_{t-k} + \mu^0 + \mu_d^0 D_t + \varepsilon_t^0 \tag{9}$$

where the $p \times 1$ vector W_t is defined as

$$W_t = \begin{pmatrix} \beta' Y_t \\ v' \Delta Y_t \end{pmatrix} \equiv \begin{pmatrix} W_{1t} \\ W_{2t} \end{pmatrix} \begin{matrix} r \times 1 \\ (p-r) \times 1 \end{matrix}, \tag{10}$$

[6] Observe that $\gamma_f + \gamma_b = 1$ is not consistent with a NKPC where π_t and x_t are cointegrated. It can be easily proved, however, that $\gamma_f + \gamma_b = 1$ is consistent with the presence of unit roots in the system.

v is a $p \times (p-r)$ matrix such that $\det(v'\beta_\perp) \neq 0$, $\beta_\perp$ is the orthogonal complement of β (Johansen, 1996), and B_i, $i = 1, ..., k$, μ^0, μ_d^0 and ε_t^0 are defined (and constrained) opportunely.

The attractive feature of the representation (9)-(10) is that for $r = 1$ and $\beta = (1, -\xi')'$,[7] and for a suitable choice of v, the conditional expectations entering (8) can be easily computed, and the procedure for estimating and testing the NKPC can be set out along the lines of Campbell and Shiller (1987). In particular, using the system (9)-(10) to compute the forecasts $E(\Delta\pi_t \mid I_{t-1})$, $E(\Delta\pi_{t+1} \mid I_{t-1})$ and $E(z_t \mid I_{t-1})$ in (8), yields the following set of cross-equation restrictions

$$g_\pi' B(I_{pk} - \psi B) - \omega g_z' B = 0_{pk}' \tag{11}$$

where B is the companion matrix of (9), and g_π and g_z are two (known) selection vectors (see Appendix for details). Moreover, using the definitions of ψ and ω, the cross-equation restrictions (11) amount to

$$g_\pi' \gamma_b B(I_{pk} - \gamma_f B) - g_z'(\gamma_f + \gamma_b - 1)B = 0_{pk}'. \tag{12}$$

It can be shown that the VAR(k) (9) is locally identifiable under the cross-equation restrictions (11) ((12)), with a given number of overidentifying constraints, see Appendix. Hence, once β is fixed at its super-consistent estimate, the system (9)-(10) can be estimated both unrestrictedly, and subject to the constraints, and LR tests for the 'inexact' NKPC can be computed.

It is worth noting that the restrictions (11) ((12)) hold trivially also when the 'exact' formulation of the NKPC (1) is considered. To see this, set $u_t = 0$ in (1) and thus $u_t^* = 0$ in (7): it can be realized that the relation (8) still holds, so that (11) ((12)) can be also regarded as a 'weaker' set of constraints that the statistical model must embody for the 'exact' NKPC to hold.

A natural question here is: which economic interpretation can we attach to the stationary 'disequilibrium' $z_t = (\pi_t - \xi' x_t)$, i.e. to the fact that inflation and the selected proxy of firms' real marginal costs follow the same stochastic trend ? We simply argue that (7) reads a convenient empirical representation of the NKPC when aggregate variables behave as cointegrated processes. If at the individual (firm) level it can be hardly expected that π_t and x_t are I(1) and cointegrated, a common stochastic trend between inflation and e.g. the wage

[7] As shown in Section 5, $z_t = \beta' Y_t$ may also include a constant when μ_0 in (6) is restricted to lie in the cointegration space, see Johansen (1996).

share might result from the process of data aggregation. Section 5 shows that this possibility is not at odds with Euro area data over the period 1971-1998.

5 Results from the Euro area

Using Euro area data, Bårdsen et al. (2004) investigate the 'inexact' version of the NKPC, and using an encompassing framework conclude convincingly that the forward-looking model of inflation dynamics is almost indistinguishable from a standard dynamic mark-up equation. These authors recognize that Euro area inflation resembles the dynamics of a unit root process over the sample they analyze, but do not implement a VAR-based approach to (1). This section fills the gap by applying the method discussed in Section 4 and in the Appendix.

We consider quarterly data on the Euro area covering the period from 1971 up to 1998 and refer to Fagan et al. (2001) for a detailed analysis and definition of variables.[8] The empirical analysis is based on four VARs: two bivariate systems of the form $Y_t = (\pi_t, x_{1t})'$, with x_{1t} proxied by the wage share (ws_t) and the output gap ($\widetilde{y}_t$) respectively, and two systems of the form $Y_t = (\pi_t, x_{1t}, x_{2t})'$, with x_{1t} defined as before, and with $x_{2t} \equiv i_t$ representing a short term nominal interest rate.[9] The inflation rate, π_t, is calculated as in Galì et al. (2001), i.e. as the growth rate over a quarterly basis of the log of the implicit GDP deflator p_t, i.e. $\pi_t = p_t - p_{t-4}$. The output gap is defined as the deviations of real GDP from potential output, measured as a constant returns to scale Cobb-Douglas production function and neutral technical progress, see Fagan et al. (2001). Mnemonics and series definitions are listed in Table 1.

Each VAR has been estimated over the period 1971:1 - 1998:2 ($T = 110$ observations) with the deterministic part given by a constant and a dummy taking value 1 at the fourth quarter of 1974 in correspondence of the oil shock, and zero elsewhere.[10] The VAR lag length has been selected by combining standard information criteria (AIC, SC, HQ) with residual-based diagnostic tests; in all cases a VAR(5) seems to describe the dynamics of the system

[8] We consider the data release up to 1998 in order to compare results with Galì et al. (2001) and Bårdsen et al. (2004)

[9] As argued in Fuhrer and Moore (1995), the short-term nominal rate is closely linked to real output and thus can be essential to forming expectations of output and inflation

[10] Computations were performed with PcGive 10.0 (Hendry and Doornik, 2001). The VAR involving the output gap was estimated over a shorter sample because of data availability, see Table 1.

sufficiently well.

Preliminary results are summarized in the tables 2 through 5. Each table reports the LR trace test for cointegration rank,[11] the highest eigenvalues of the estimated VAR companion matrix, and the estimated long run relationships and corresponding adjustment coefficients (when cointegration is detected). The tests for cointegration rank (unit roots) highlight that none of the estimated VARs can be reasonably treated as stationary system over the period 1971:1-1998:2. This evidence is also supported by the estimated highest roots of VARs' companion matrices, which are all very close to one. Given the relatively small span covered by the data, we can argue that treating variables as stationary might have in this case detrimental effects on both size and power of the test of cross-equation restrictions implied by the NKPC.

Table 3 shows that by including the nominal interest rate in the system, a single cointegrating relation between π_t and ws_t (not involving i_t) is clearly supported by the data.[12] Surprisingly, a cointegrating relation is also found between π_t and $\widetilde{y}_t$, irrespective of whether the short term nominal interest rate is included or not in the system (Table 4, Table 5). From the statistical point of view it is not surprising, in light of the discussion of Section 3, that the chosen measure of the output gap is perceived to be I(1) over the 1971-1998 sample. From the economic point of view the result can be motivated by referring to Hughes Hallet (2000) who shows that a non-vertical Phillips curve may follow from the aggregation of the underlying (national, regional, sectoral) curves, especially in view of the structural differences and mismatch between supply and demand characterizing the labour markets of European countries.

The investigation of the 'inexact' hybrid NKPC (1) is summarized in Table 6. Also in this case we have considered four VARs of the form (9)-(10): $W_t = (z_t , \Delta\pi_t)'$ and $W_t = (z_t , \Delta\pi_t , \Delta i_t)'$, with $z_t = \widehat{\beta}' Y_t = (\pi_t - \widehat{\xi} x_t)$ defined as in Table 3 (wage share model, $x_t \equiv ws_t$) and in Table 5 (output gap model, $x_t \equiv \widetilde{y}_t$), respectively. The empirical assessment of the NKPC is carried out along the lines discusses in Section 4 and in the Appendix, and is based on the cross-equation restrictions that the forward-looking model of inflation dynamics (7) imposes on the four VARs. In particular, the LR statistics in the last column

[11] The test was carried out by restricting the constant to lie in the cointegration space to rule out deterministic linear trends in the variables (Johansen, 1996).

[12] Note, that π_t and ws_t seem to be cointegrated when the bivariate system $Y_t = (\pi_t, ws_t)'$ is estimated with a lower number of lags. This suggests that the outcome of the cointegration rank test in Table 2 might be effected by the selected lag length.

of Table 6 compare the log-likelihood of the unrestricted system with the log-likelihood of the system subject to (11) ((12)). The constrained estimation has been performed by setting γ_f and γ_b, and hence ψ and ω, within a grid of plausible values, and implementing quasi-Newton methods with the inverse Hessian approximated according to the BFGS update. The grid for γ_f and γ_b and λ has been constructed by considering the parameter range [0.1, 0.95], incremental value of 0.01 and the restrictions: $\gamma_f + \gamma_b < 1$, $0.03 \leq (1-\gamma_f - \gamma_b)\widehat{\xi} \leq 0.30$, where the latter is motivated by the necessity of considering, given the estimated $\widehat{\xi}$ in z_t, values of the structural parameter $\lambda = (1-\gamma_f-\gamma_b)\widehat{\xi}$ compatible with the Calvo set-up and with previous evidence.

Overall, Table 6 suggests that the 'inexact' NKPC for the Euro area data is sharply rejected over the period 1971:1-1998:2, though relatively high values of the forward-looking parameter γ_f, and relatively low values of the backward-looking parameter γ_b tend to be favoured in terms of likelihood.

6 Concluding remarks

In this paper we address the issue of testing the hybrid NKPC under VAR expectations, giving special emphasis to the case where variables are treated as realizations of non stationary, possibly cointegrated processes. The paper derives the cross-equation restrictions between agents' forecast system and the 'inexact' version of the NKPC. The estimation and testing procedure can be implemented with any existing econometric software.

Referring to their GMM estimates of the hybrid version of the NKPC for the Euro area, Galì et al. (2001), p. 1258, observe that: '*... it appears that the structural marginal cost based model can account for the inflation dynamics with relatively little reliance on arbitrary lags of inflation, as compared to the traditional Phillips curve* [...]'. The empirical evidence provided by this paper seems at odds with this claim for two reasons.

First, the persistence of variables over the period 1971-1998 appears consistent with that of unit roots cointegrated processes. This evidence is surprisingly overlooked, with few exceptions, in the literature on the NKPC, where the issue of non stationarity is usually dismissed as empirically irrelevant, and standard asymptotic inference is exploited regardless the actual persistence of time-series. The paper shows that the empirical assessment of the NKPC is more involved and controversial when the conventional 'highly persistent' stationary world is replaced by the unit root alternative. Secondly, the restrictions that the model (1) imposes on the model for the data are sharply rejected, irrespective of whether firms' real marginal costs are proxied by the wage share or the output gap.

The results obtained in this paper do not necessary imply that forward-looking behaviour is unimportant in modelling inflation in the Euro area. Additional lags (or leads) might be required in (1) to capture inflation persistence. More involved dynamic specifications of the NKPC can be motivated either empirically (Bårdsen et al. 2004), or by relying on sluggish intertemporal costs of adjustment (Price, 1992), Taylor-type contracting (Fuhrer, 1997), or sticky information models (Mankiw and Reis, 2002). Alternatively, the model might be augmented by other driving variables: for instance, using European data from the eighties onwards, Gerlach and Svensson (2003) show that also the real money gap (the difference between the real money stock and the long run equilibrium real money stock) plays a role in forecasting inflation. Quantifying the empirical relevance of these issues is the topic of ongoing research.

Appendix

In this Appendix we establish the link between the VEqC (6) and the VAR (9)-(10), and derive the cross-equation restrictions with the 'inexact' NKPC (7).

Paruolo (2003), Theorem 2, shows that given the I(1) cointegrated VEqC (6), the W_t vector defined in (10), that we report here for simplicity

$$W_t = \begin{pmatrix} \beta' Y_t \\ v'\Delta Y_t \end{pmatrix} \equiv \begin{pmatrix} W_{1t} \\ W_{2t} \end{pmatrix} \quad \begin{matrix} r \times 1 \\ (p-r) \times 1 \end{matrix} \tag{13}$$

admits the following VAR(k) representation:

$$B(L)W_t = \mu^0 + \mu_d^0 D_t + \varepsilon_t^0 \tag{14}$$

where μ^0 and μ_d^0 are function of μ_0, μ_d and $(\beta,v)'$, $\varepsilon_t^0 = (\beta,v)'\varepsilon_t$, $B(L) = I_p - \sum_{i=1}^{k} B_i L^i$ is a characteristic polynomial with B_i, $i = 1, ..., k-1$ $p \times p$ matrices of parameters, and with the roots of the characteristic equation, $\det[B(L)] = 0$, lying outside the unit circle. Furthermore, by partitioning parameters conformably with (13), the matrix B_k in (14) is restricted as

$$B_k = B_k^* \equiv \left[B_{w1,k} \,\vdots\, O \right] \quad p \times r \quad p \times (p-r) \tag{15}$$

where we have reported dimensions of sub-matrices alongside blocks.

Due to the super-consistency result, one can replace the cointegration parameters β ($\beta_\perp$) in (13)-(14) by the estimates $\widehat{\beta}$ ($\widehat{\beta}_\perp$) retrieved through cointegration methods, and treat $\widehat{\beta}$ ($\widehat{\beta}_\perp$) as the 'true' parameter value, see e.g. Johansen (1996). Clearly, when $r = 0$ (I(1) not cointegrated variables) the 'natural' choice in (13) is $v = I_p$, implying that the system (14) corresponds to a DVAR($k-1$) for $W_t \equiv W_{2t} = \Delta Y_t$; conversely, when $r = p$ (I(0) variables) and given the choice $\beta' = I_p$, the system (14) collapses to a VAR(k) for $W_t \equiv W_{1t} = Y_t$. If the NKPC with I(1) variables is supported by the data, one expects that $r = 1$ and $W_{1t} = z_t = \beta' Y_t = (\pi_t - \xi' x_t)$ in (14); however, also $r > 1$ may be consistent with the NKPC.[13]

The companion form representation of (13)-(14) is given by

$$\widetilde{W}_t = B\ \widetilde{W}_{t-1} + \widetilde{\varepsilon}_t^0 \tag{16}$$

where $\widetilde{W}_t = (W_t', \ ..., \ W_{t-k+1}')'$, $\widetilde{\varepsilon}_t^0 = (\mu^{0\prime} + D_t'\mu_d^{0\prime} + \varepsilon_t^{0\prime}, \ 0', \ ..., \ 0')'$ and the $pk \times pk$ companion matrix B defined accordingly, with the sub-matrix B_k subject to (15). VAR (VEqC) forecasts can be therefore computed, abstracting from deterministic terms,[14] using $E(\widetilde{W}_{t+j} \mid I_t) = B^j \widetilde{W}_t$. Let g_π and g_z be two selection vectors such that $g_\pi' \widetilde{W}_t = \Delta\pi_t$ and $g_z' \widetilde{W}_t = z_t$, where z_t corresponds to $\beta' Y_t \equiv W_{1t}$ if $r = 1$, or is an element of W_{1t} if $r > 1$; using these definitions, $E(\Delta\pi_t \mid I_{t-1}) = g_\pi' B \widetilde{W}_{t-1}$, $E(\Delta\pi_{t+1} \mid I_{t-1}) = g_\pi' B^2 \widetilde{W}_{t-1}$, and $E(z_t \mid I_{t-1}) = g_z' B \widetilde{W}_{t-1}$, so that the relation (8) of Section 4 can be written as

$$g_\pi' B \widetilde{W}_{t-1} = \psi g_\pi' B^2 \widetilde{W}_{t-1} + \omega g_z' B \widetilde{W}_{t-1}$$

and since the expression must hold a.s. for every $\widetilde{W}_{t-1}$, it must be the case that

$$g_\pi' B(I - \psi B) - \omega g_z' B = 0_{pk}' \tag{17}$$

as in (11).

To see how things work in practice, suppose first, without loss of generality, that x_t in (1) is a scalar ($q = 1$, hence $p = 2$) and that π_t and x_t are cointegrated

[13] Of course, this may happen when x_t in (1) ((7)) is a vector. When $r > 1$ it is necessary to identify the 'additional' cointegrating relation(s); for instance, one might have a Fisher-type parity relation between i_t and π_t.

[14] For the sake of simplicity we ignore the role of deterministic components in the derivation of cross-equation restrictions. In general, however, it is possible to account for deterministic terms to the extent that deterministic components are also included in the forward-looking model; see e.g. Fanelli (2002) for an example in a related context.

with cointegrating vector $\beta = (1, -\xi)'$. This means that the cointegration rank in the VEqC is equal to $r = 1$, and that $W_{1t} = \beta' Y_t = (\pi_t - \xi\ x_t) = z_t \sim$I(0). Assume further that the cointegrating vector is fixed at its super-consistent estimate $\beta = \widehat{\beta} = (1, -\widehat{\xi})'$ and that k in (6) is equal to 2. Given $v = (1, 0)'$, it follows that $W_{2t} = v'\Delta Y_t = \Delta\pi_t$ $(\det(v'\beta_\perp) \neq 0)$ so that $W_t = (z_t, \Delta\pi_t)'$. The VAR (14)-(15) specializes in

$$\left(I_2 - \begin{bmatrix} b_{1,11} & b_{1,12} \\ b_{1,21} & b_{1,22} \end{bmatrix} L + \begin{bmatrix} b_{2,11} & 0 \\ b_{2,21} & 0 \end{bmatrix} L^2\right)\begin{pmatrix} z_t \\ \Delta\pi_t \end{pmatrix} = \begin{pmatrix} \mu_z^0 \\ \mu_\pi^0 \end{pmatrix} + \begin{pmatrix} \varepsilon_{zt}^0 \\ \varepsilon_{\pi t}^0 \end{pmatrix} \tag{18}$$

where L is the lag operator ($L^j Y_t = Y_{t-j}$), and $b_{i,jh}$ is the jh element of B_i, $i = 1, 2$. Observe that $b_{2,12} = 0$, $b_{2,22} = 0$ by construction because of (15), hence the total number of free parameters of the unrestricted system is $p^2k - p(p-r) = 8 - 2 = 6$. In this case, using simple algebra the cross-equation restrictions (17) can be written in explicit form as

$$b_{1,11} = \frac{b_{1,21}(1 - \psi b_{1,22}) - \psi b_{2,21}}{\omega + \psi b_{1,21}} \tag{19}$$

$$b_{1,12} = \frac{b_{1,22}(1 - \psi b_{1,22})}{\omega + \psi b_{1,21}} \tag{20}$$

$$b_{2,11} = \frac{b_{2,21}(1 - \psi b_{1,22})}{\omega + \psi b_{1,21}}. \tag{21}$$

Observe that (19)-(21) represent the unique mapping relating the parameters of the z_t-equation of the VAR (18) to the structural parameters (ψ, ω) and the remaining VAR coefficients on the other hand. Note also that the number of free parameters in the restricted system is $(p-r)[pk-(p-r)]+2 = (4-2+1)+2 = 5$, where 2 is the number of structural parameters of the NKPC. Hence, in this case the number of overidentifying restrictions is $p^2k - (p-r)[pk+r] - 2 = 6 - 5 = 1$.

To compute LR tests for the NKPC, the VAR (18) must be estimated by ML under the restrictions (19)-(21) and unrestrictedly. The unrestricted estimation is standard. The estimation under (19)-(21) requires numerical optimization methods. Kurmann (2006) recommends the simulated annealing algorithm. Nevertheless, since the range of values that γ_f and γ_b (hence ψ and ω) can take is bounded by construction (see Section 5 for an example), the maximization of the likelihood of the system under the restrictions (19)-(21) can be achieved by combining grid search techniques for ψ and ω (γ_f, γ_b) with quasi-Newton methods. Provided that the LR test for over-identifying restrictions does not reject the model, ML estimates of ψ and ω (γ_f and γ_b) can be recovered from the constrained VAR. An indirect ML estimate of λ is retrieved from the estimated cointegration relation (recall that $\beta' Y_t = (\pi_t - \xi\ x_t) = z_t$), by using $\lambda = (1 - \gamma_f - \gamma_b)\widehat{\xi}$.

The procedure works similarly if the VAR includes three or more variables. Suppose, that $x_t = (x_{1t},\, x_{2t})'$ ($q = 2$, $p = 3$) and that π_t and x_t are cointegrated with cointegrating vector $\beta = (1,\, -\xi')'$ ($r = 1$), so that $W_{1t} = \beta' Y_t = (\pi_t - \xi_1\ x_{1t} - \xi_2 x_{2t}) = z_t$ $\sim$I(0), where ξ_2 can be possibly zero. Assume further, to keep the algebra less involving, that the optimal number of lags in the VAR for $Y_t = (\pi_t, x_{1t}, x_{2t})'$ is two ($k = 2$). Defined $v = (e_1, e_3)'$, where e_i is a $p \times 1$ vector with 1 as his i-th element and zero elsewhere, it turns out that $W_{2t} = v' \Delta Y_t = (\Delta\pi_t, \Delta x_{2t})$ ($\det(v'\beta_\perp) \neq 0$) and $W_t = (z_t, \Delta\pi_t, \Delta x_{2t})'$, so that the VAR (14)-(15) now reads as

$$\left(I_3 - \begin{bmatrix} b_{1,11} & b_{1,12} & b_{1,13} \\ b_{1,21} & b_{1,22} & b_{1,23} \\ b_{1,31} & b_{1,32} & b_{1,33} \end{bmatrix} L + \begin{bmatrix} b_{2,11} & 0 & 0 \\ b_{2,21} & 0 & 0 \\ b_{2,31} & 0 & 0 \end{bmatrix} L^2 \right) \begin{pmatrix} z_t \\ \Delta\pi_t \\ \Delta x_{2t} \end{pmatrix}$$

$$= \begin{pmatrix} \mu_z^0 \\ \mu_\pi^0 \\ \mu_{x_2}^0 \end{pmatrix} + \begin{pmatrix} \varepsilon_{zt}^0 \\ \varepsilon_{\pi t}^0 \\ \varepsilon_{x_2 t}^0 \end{pmatrix}. \qquad (22)$$

Also in this case it is possible to re-formulate the relations in (17) in explicit form, such that the non zero parameters $b_{i,jh}$ associated with the z_t-equation of the VAR (22) depend on $(\psi,\, \omega)$ and the remaining non zero VAR coefficients, i.e.

$$\begin{aligned} b_{1,11} &= \frac{b_{1,21} - \psi[b_{2,21} + b_{1,22}(b_{1,21} + b_{1,31})]}{\omega + \psi b_{1,21}}, \\ b_{1,12} &= \frac{b_{1,22}(1 - \psi b_{1,22})}{\omega + \psi b_{1,21}} \quad , \quad b_{1,13} = \frac{b_{1,22} - \psi[b_{1,22}(b_{1,22} + b_{1,33})]}{\omega + \psi b_{1,21}} \\ b_{2,11} &= \frac{b_{2,21} - \psi b_{1,22}[b_{2,21} + b_{2,31}]}{\omega + \psi b_{1,21}}. \end{aligned}$$

Clearly, when in the VAR $k > 2$, the derivation of restriction is similar, albeit algebraically tedious.

References

[1] Baillie, R. T. (1989), Econometric tests of rationality and market efficiency, *Econometric Reviews* 8, 151-186.

[2] Bårdsen, G., Jansen, E. S. and Nymoen, R. (2004), Econometric evaluation of the New Keynesian Phillips curve, *Oxford Bulletin of Economics and Statistics* 66 (Supplement), 671-685.

[3] Batini, N. (2006), Euro area inflation persistence, *Empirical Economic* 31, 977 - 1002.

[4] Batini, N., Jackson, B., Nickell, S. (2005), An open-economy New Keynesian Phillips curve for the U.K., *Journal of Monetary Economics* 52, 1061-1071.

[5] Boug, P., Cappelen, Å., Swensen, A. R. (2006), The new Keynesian Phillips curve for a small open economy, *Statistics Norway, Discussion Paper* No. 460

[6] Brayton, F., Eileen, M., Reifschneider, Tinsley, P., Williams, J. (1997), The role of expectations in the FRB/US macroeconomic model, *Federal Reserve Bulletin* 83 (April), 227-245.

[7] Calvo, G. A. (1983), Staggered contracts in a utility-maximizing framework, *Journal of Monetary Economics* 12, 383-398.

[8] Campbell, J. Y. and Shiller, R. J. (1987), Cointegration and tests of present value models, *Journal of Political Economy* 95, 1062–1088.

[9] Clarida, R., Galí, J., Gertler, M. (1999), The science of monetary policy: a New Keynesian perspective, *Journal of Economic Literature* 37, 1661-1707.

[10] Culver, S. E., Papell, D.H. (1997), Is there a unit root in the inflation rate? Evidence from sequential break and panel data models, *Journal of Applied Econometrics* 12, 435-444.

[11] Ericcsson, N. R. and Hendry, D. F. (1999), Encompassing and rational expectations: How sequential corroboration can imply refutation, *Empirical Economics* 24, 1-21.

[12] Fagan, G., Henry, G. and Mestre, R. (2001), An area-wide model (awm) for the Euro area, *European Central Bank, Working Paper* No. 42.

[13] Fanelli, L. (2002), A new approach for estimating and testing the linear quadratic adjustment cost model under rational expectations and I(1) variables, *Journal of Economic Dynamics and Control* 26, 117-139.

[14] Fanelli, L. (2006), Dynamic adjustment cost models with forward-looking beahviour, *Econometrics Journal* 9, 23-47.

[15] Fuhrer, J. C. (1997), The (un)importance of forward-looking behavior in price specifications, *Journal of Money Credit and Banking* 29(3), 338-350.

[16] Fuhrer, J. and Moore, G. (1995), Inflation persistence, *Quarterly Journal of Economics* 110, 127-159.

[17] Galí, J. and Gertler, M. (1999), Inflation dynamics: a structural econometric analysis, *Journal of Monetary Economics* 44, 195-222.

[18] Galí, J., Gertler M. and Lopez-Salido, J.D. (2001), European inflation dynamics, *European Economic Review* 45, 1237-1270.

[19] Galí, J., Gertler, M. and López-Salido, J. D. (2005), Robustness of the estimates of the hybrid New Keynesian Phillips curve, *Journal of Monetary Economics* 52, 1107-1118.

[20] Gerlach, S. and Swenson, L. E. O. (2003), Money and inflation in the euro area: A case for monetary indicators?, *Journal of Monetary Economics* 50, 1649-1672.

[21] Granger, C.W.J. (1980), Long memory relationships and the aggregation of dynamic models, *Journal of Econometrics* 14, 227-238.

[22] Hendry, D. F. (1995), *Dynamic econometrics*, Oxford University Press, Oxford.

[23] Hendry, D. F. (2001), Modelling UK inflation, *Journal of Applied Econometrics* 16, 255-275.

[24] Hendry, D. F. and Doornik, J. (2001), E*mpirical econometric modelling using PcGive*, Timberlake Consultants, London.

[25] Henry, S.G.B., and Pagan, A.R. (2004), The econometrics of the New Keynesian policy model: Introduction, *Oxford Bulletin of Economics and Statistics* 66 (Supplement), 581-607.

[26] Hughes Hallet, A.J. (2000), Aggregate Phillips curves are not always vertical: heterogeneity and mismatch in multiregion or multisector economies, *Macroeconomic Dynamics* 4, 534-546.

[27] Johansen, S. (1996), *Likelihood-based inference in cointegrated vector autoregressive models*, Oxford University Press, Oxford.

[28] Johansen, S. (2006), Confronting the economic model with the data, in Colander, D. (ed): *Post Walrasian Macroeconomics*, Cambridge University Press, Cambridge, pp. 287-300.

[29] Johansen, S. and Swensen, A. R. (1999), Testing exact rational expectations in cointegrated vector autoregressive models, *Journal of Econometrics* 93, 73-91.

[30] Juselius, M. (2006), Testing the new Keynesian model on US and European data, paper presented at the 61st *European Meeting of the Econometric Society*, 24-28 August 2006.

[31] King, R.G. (2000), The new IS-LM model: language, logic, and limits, *Federal Reserve Bank of Richmond, Economic Quarterly* 86/3, 45-103.

[32] Kitamura, Y., Phillips, P.C.B. (1997), Fully modified IV, GIVE and GMM estimation with possibly non-stationary regressors and instruments, *Journal of Econometrics* 80, 85-123.

[33] Kurmann, A. (2006), Maximum likelihood estimation of dynamic stochastic theories with an application to New Keynesian pricing, *Journal of Economic Dynamics and Control*, forthcoming.

[34] Lindé, J. (2005), Estimating New Keynesian Phillips curves: A full information maximum likelihood approach, *Jounrnal of Monetary Economics* 52, 1135-1149.

[35] Mankiw, G. N. and Reis, R. (2002), Sticky information versus sticky prices: A proposal to replace the New Keynesian Phillips curve, *Quarterly Journal of Economics* 117, 1295-1328.

[36] Mavroeidis, S. (2004), Weak identification of forward-looking models in monetary economics, *Oxford Bulletin of Economics and Statistics* 66 (Supplement), 609-635.

[37] Mavroeidis, S. (2005), Identification issues in forward-looking models estimated by GMM, with an application to the Phillips curve, *Journal of Money Credit and Banking* 37, 421-448.

[38] Nason, J. M. and Smith, G. W. (2005), Identifying the New Keynesian Phillips curve, *Queen's Economic Department, Working Paper* No. 1026.

[39] O'Relly, G., Whelan, K. (2005), Has euro-area inflation persistence changed over time ?, *Review of Economics and Statistics* 87, 709-720.

[40] Paruolo, P. (2003), Common dynamics in I(1) systems, *Università dell'Insubria, Varese, Working Paper* 2003/33.

[41] Pesaran, M. H. (1987), *The limits to rational expectations*, London: Basil Blackwell.

[42] Petursson, T. G. (1998), Price determination and rational expectations, *International Journal of Finance and Economics* 3, 157-167.

[43] Price, S. (1992), Forward-looking price setting in UK manufacturing, *The Economic Journal* 102, 497-505.

[44] Roberts, J. M. (1995), New Keynesian economics and the Phillips curve, *Journal of Money, Credit, and Banking* 27 (4), 975-984.

[45] Rotemberg, J. J. (1982), Sticky prices in the United States, *Journal of Political Economy* 60, 1187-1211.

[46] Ruud, J. and Whelan, K. (2005*a*), Does labor's share drive inflation ?, *Journal of Money Credit and Banking* 37, 297-312.

[47] Ruud, J. and Whelan, K. (2005*b*), New tests of the New Keynesian Phillips curve, *Journal of Monetary Economics* 52, 1167-1181.

[48] Ruud, J. and Whelan, K. (2006), Can rational expectations sticky-price models explain inflation dynamics?, *American Economic Review* 96, 303-320.

[49] Sargent, T. J. (1979), A note on the maximum likelihood estimation of the rational expectations model of the term structure, *Journal of Monetary Economics* 5, 133-143.

[50] Sbordone, A. M. (2002), Prices and unit labor costs: a new test of price strickiness, *Journal of Monetary Economics* 49, 265-292.

[51] Sbordone, A. M. (2005), Do expected future marginal costs drive inflation dynamics ?, *Journal of Monetary Economics* 52, 1183-1197.

[52] Taylor, J. B. (1979), Staggered contracts in a macro model, *American Economic Review* 69, 108-113.

A Tables

Mnemonic	Definition
p_t	log of the implicit GDP deflator
π_t	inflation rate: $p_t - p_{t-4}$
ws_t	log of deviations of real unit labor costs from a constant
$\widetilde{y}_t$	deviation of real GDP from potential output[a]
i_t	short term nominal interest rate

Table 1: Description of the data. Quarterly observations on the Euro area 1971:1 - 1998:2, see Fagan et al. (2001). NOTES; a=potential output is computed by constant-returns-to-scale Cobb-Douglas production function and neutral technical progress and the corresponding series starts at 1971:4.

VAR(5): $Y_t = (\pi_t \ , \ ws_t)'$, $x_t \equiv ws_t$		
Cointegration rank test		
$H_0 : r \leq j$	Trace	5% c.v.
j=0	11.08	19.96
j=1	1.70	9.24
highest roots: $0.9664 \pm 0.04316i$		

Table 2: LR test for cointegration rank over the period 1971:1-1998:2, and highest eigenvalues of VAR companion matrix. NOTES: the model includes an intervention dummy, see Section 5; 5% critical values for cointegration rank test are taken from Johansen (1996), Table 15.2; standard errors in parentheses; p-values in squared brackets.

VAR(5): $Y_t = (\pi_t \ , \ ws_t \ , \ i_t)'$, $\quad x_t \equiv (ws_t \ , \ i_t)'$

Cointegration rank test

$H_0 : r \leq j$	Trace	5% c.v.
j=0	41.83	34.91
j=1	16.34	19.96
j=2	3.99	9.24

highest roots: $0.9871 \pm 0.05232i$

Estimated cointegrating relation and adjustment coefficients

$$\widehat{\beta}' Y_t = \pi_t - \underset{(0.05)}{0.79} ws_t - \underset{(0.12)}{2.00}$$

$$\widehat{\alpha}' = (\underset{(0.03)}{0.08} \ , \ \underset{(0.05)}{0.19} \ , \ 0)' \qquad , \qquad \text{LR } \chi^2(2) = \underset{[0.61]}{0.996}\ ^a$$

Table 3: LR test for cointegration rank over the period 1971:1-1998:2, highest eigenvalues of VAR companion matrix and estimated cointegrating relation. NOTES: the model includes an intervention dummy, see Section 5; 5% critical values for cointegration rank test are taken from Johansen (1996), Table 15.2; a=LR test for the over-identifying restrictions; standard errors in parentheses; p-values in squared brackets.

VAR(5): $Y_t = (\pi_t\ ,\ \widetilde{y}_t)'\ \ ,\ \ x_t \equiv \widetilde{y}_t$		
Cointegration rank test		
$H_0 : r \leq j$	Trace	5% c.v.
j=0	35.95	19.96
j=1	0.39	9.24

highest roots: 0.9863, $0.6757 \pm 0.5889i$

Estimated cointegrating relation and adjustment coefficients

$$\widehat{\beta}' Y_t = \pi_t - \underset{(0.013)}{0.09}\, \widetilde{y}_t - \underset{(0.013)}{0.09}$$

$$\widehat{\alpha}' = (\underset{(0.004)}{-0.025}\ ,\ \underset{(0.72)}{1.31})'$$

Table 4: LR test for cointegration rank over the period 1973:1-1998:2, highest eigenvalues of VAR companion matrix, and estimated cointegration relation. NOTES: the model includes an intervention dummy, see Section 5; 5% critical values for cointegration rank test are taken from Johansen (1996), Table 15.2; standard errors in parentheses; p-values in squared brackets.

$$\text{VAR(5)}: \quad Y_t = (\pi_t \ , \widetilde{y}_t, \, i_t)' \ \ , \ \ x_t \equiv (\widetilde{y}_t, \, i_t)'$$

Cointegration rank test

$H_0 : r \leq j$	Trace	5% c.v.
j=0	36.28	34.91
j=1	11.61	19.96
j=2	5.23	9.24

highest roots: $0.9439 \pm 0.04188i$

Estimated cointegrating relation and adjustment coefficients

$$\widehat{\beta}' Y_t = \pi_t + \underset{(0.02)}{0.11} \widetilde{y}_t - \underset{(0.02)}{0.10} \quad , \quad \text{LR } \chi^2(1) = \underset{[0.80]}{0.06} \ ^{a}$$

$$\widehat{\alpha}' = (\underset{(0.004)}{-0.016} \ , \underset{(0.65)}{0.62} \ , \underset{(0.79)}{-2.36})'$$

Table 5: LR test for cointegration rank over the period 1973:1-1998:2, highest eigenvalues of VAR companion matrix and estimated cointegrating relation. NOTES: the model includes an intervention dummy, see Section 5; 5% critical values for cointegration rank test are taken from Johansen (1996), Table 15.2; a=LR test for the over-identifying restriction characterizing the cointegrating relation; standard errors in parentheses; p-values in squared brackets.

Testsof the “inexact” NKPC			
wage share model (see Table 3) : $z_t = \pi_t - 0.79 ws_t - 2.00$ $(\widehat{\xi} = 0.79)$			
VAR(5)	Unr. log-lik [a]	Restr. log-lik [b,c]	LR test
$W_t = \begin{pmatrix} z_t \\ \Delta\pi_t \end{pmatrix}$	880.02	870.21 $(\widehat{\gamma}_f$=0.71 , $\widehat{\gamma}_b$=0.19 , $\widehat{\lambda}$=0.079)	$\chi^2(7)$= 19.62 [0.0065]
$W_t = \begin{pmatrix} z_t \\ \Delta\pi_t \\ \Delta i_t \end{pmatrix}$	816.88	804.15 $(\widehat{\gamma}_f$=0.71 , $\widehat{\gamma}_b$=0.19 , $\widehat{\lambda}$=0.079)	$\chi^2(11)$= 25.46 [0.0078]
output gap model (see Table 5): $z_t = \pi_t - 0.11\widetilde{y}_t - 0.10$ $(\widehat{\xi} = 0.11)$			
VAR(5)	Unr. log-lik [a]	Restr. log-lik [b,c]	LR test
$W_t = \begin{pmatrix} z_t \\ \Delta\pi_t \end{pmatrix}$	629.42	606.53 $(\widehat{\gamma}_f$=0.80 , $\widehat{\gamma}_b$=0.07 , $\widehat{\lambda}$=0.03)	$\chi^2(7)$ = 45.78 [0.000]
$W_t = \begin{pmatrix} z_t \\ \Delta\pi_t \\ \Delta i_t \end{pmatrix}$	566.39	550.98 $(\widehat{\gamma}_f$=0.80 , $\widehat{\gamma}_b$=0.07 , $\widehat{\lambda}$=0.03)	$\chi^2(11)$ = 30.82 [0.001]

Table 6: LR tests of the "inexact" NKPC (Eq. (7)) in the Euro area over the period 1971:1-1998:2, see Section 4 and Appendix. NOTES: a= value of the log-likelihood of the VAR (9)-(10) (k=5 lags); b= value of the log-likelihood of the VAR (9)-(10) (k=5 lags) subject to the cross-equation restrictions (11); c = ψ and ω (and hence $(\gamma_f, \gamma_b, \lambda)$) are estimated through grid search as detailed in Section 5, and the mapping between (ψ, ω) and $(\gamma_f, \gamma_b, \lambda)$ is given by $\psi = (\gamma_f/\gamma_b)$, $\omega = (\gamma_f + \gamma_b - 1)/\gamma_b$, $\lambda = \widehat{\xi}(1 - \gamma_f - \gamma_b)$; p-values in squared brackets.