A small open economy model for Nigeria: a BVAR-DSGE approach

Olayeni, Olaolu Richard

Obafemi Awolowo University, Ile-Ife, Nigeria

8 June 2009

A small open economy model for Nigeria: a BVAR-DSGE(λ) approach

Ọláyẹni Ọláolú Richard
Department of Economics, Obafemi Awolowo University, Ile-Ife, Nigeria
Tel:234(0)8059856698; email:rolayeni@oauife.edu.ng

July 8, 2009

Abstract

Motivated by the way a small open economy should react to business cycles, we have estimated a small open economy (SOE) model for Nigeria. This is with a view to understanding how the Nigerian economy should be managed in the face of a cycle such as the current global meltdown. Our SOE model is used to generate dummy observation priors for the VAR in line with the BVAR-DSGE(λ) technique. We consider four monetary policy rules and estimate each of the resulting models using DYNARE 4.0.2. We find that the Central Bank of Nigeria (CBN) places little weight on the exchange rate behaviour in reacting to the cycles, resulting in overshooting and persistence in the exchange rate but strongly reacts to the behaviour of inflation and, to a lesser degree, of output, output gap or its growth following the shocks.

We conclude that it will be important for the CBN to pursue a guided exchange rate policy by actively responding to the exchange rate movement to avoid overshooting and persistence, that the terms of trade must be endogenize and that there is scope for the CBN to learn from past policy outcome by building a much stronger feedback.

***Keywords*: BVAR-DSGE(λ), SOE, Nigeria**
***JEL classification numbers*: C11,C13,C69**

1 Introduction

The way a small open economy should react to business cycles is a current issue in the macroeconomic literature. The importance of this issue has been brought to the fore consequent upon the current global meltdown. For a developing economy like Nigeria, this suggests a serious concern for the policymakers. Following the slides in the price of crude oil and the concomitant depreciation of the naira, the way the Nigerian economy should react to cycles has dominated the discussions in the political and academic circles. From October 2008 until March 2009, the immediate past Governor of the Central Bank of Nigeria, Prof. Soludo, frequently appeared before the National Assembly (NA) to explain issues bordering on exchange rates and other macroeconomic issues. In fact, the NA had tacitly called for reintroduction of the fixed exchange system, while the CBN Governor had evaluated the cost involved in choosing that system in the face of the current economic recession. Indeed, pronouncements have been made and denied, and the issues are far from over on which direction the economy should follow. This paper intends to contribute to the discussions by empirically examining some of the issues relating to the movements in the exchange rates, output level, terms of trade and inflation rate during the cycles in Nigeria.

In line with the current tools used in analyzing open economy issues, this paper tilts toward the small open economy (SOE) model. Galí and Monacelli's (2005) SOE model or its simplification by Lubik and Schorfheide (2005) has become standard and vastly used in the literature. This model is a variant of the dynamic stochastic general equilibrium (DSGE) model and derives from the first principle by explicitly modelling the household and firm behaviour as well as the monetary authority's reaction function while at same time incorporating the foreign economy. Thus, it is robust against the well-known Lucas (1976) critique, and a good model for policy analysis. The version of the model adopted for this exercise is Lubik and Schorfheide's (2006) which has been applied by other authors for other economies. One of the unique strands of this paper, therefore, is the explicit modelling of the optimal choices made by economic agents in Nigeria. Central banks around the world overwhelmed by the helicopter's view of this modelling approach are already building versions of the DSGE models and it will not be out of place to reckon with Nigeria.

It should be mentioned that the DSGE model sparingly shares resemblance with the Computable General Equilibrium (CGE). However, the CGE is strictly deterministic and shirks under stochastic issues. Another problem with the CGE is that it is prone to calibration errors as most of the parameters and "stylized ratios" have to be subjectively constructed before simulation can start. On the other hand, the DSGE is amenable to estimation using in particular the Bayesian approach—and superiority of the Bayesian approach is well-documented. This paper further demonstrates the use of a current estimation technique in the toolkit of the macroeconomic analysts—the BVAR-DSGE(λ).

The BVAR-DSGE(λ) was proposed by Del Negro and Schorfheide (2004) and extended by Del Negro et al (2007). In the BVAR-DSGE(λ), the posterior distribution is derived from combining a VAR likelihood function with the DSGE priors—the dummy observation priors—akin to the Minnesota-styled priors in the BVAR. Provided that a DSGE model used in generating the artificial data is consistent and features key areas of interest in an economic system, DSGE priors can be trusted and confidently used. A number of papers have explored the relevance of this approach, mostly for the developed and transition economies. Studies using this approach for developing economies are still few and far between. In what follows we bridge this gap by using the approach to study the Nigerian macroeconomy within the small open economy (SOE) model.

The rest of this paper is organized as follows. Section 2 summarizes our SOE model while section 3 discusses the BVAR-DSGE(λ) approach. Sections 4 analyzes the Nigerian data and discusses the results. Section 5 concludes.

2 Model in brief

Seminal paper by Galí and Monacelli (2005) or its simplification by Lubik and Schorfheide (2005) is an important reference point in modelling and analyzing the impact of globalization on the macroeconomic performance of the domestic economy. Its building blocks are the open economy IS equation describing the evolution of the output level, a new Phillips curve describing the dynamics of inflation, the exchange rate equation relating the economy's term of trade and the monetary policy rule featuring the feedback mechanism. The model vastly draws on the growing new open economy model (NOEM) to reach most of its conclusions. The previous studies that have used this model include Smets and Wouters (2005), Liu (2005) and Lees *et al* (2007).

The agents in this model are the households who have preference over leisure and consumption with external habit formation, a continuum of monopolistically competitive firms in a continuum of countries producing differentiated goods and setting prices according to the Calvo staggered pricing scheme as well as the government that uses Taylor's rule with feedback mechanism. The firms are owned by the households both in the home and foreign economies. A typical firm in the home economy uses a linear technology to produce differentiated goods. Prices are inflexible because the firms periodically review the prices according to the Calvo (1983) staggered contract scheme where a fraction of firms set their prices optimally each period, and the remaining simply index their prices to the past inflation rate. The scheme gives rise to a hybrid of forward- and backward-looking behaviour among the firms. All the agents are characterized through their optimality conditions.

Linearized model equations. The model analyzed consists of a set of linearized equations so that each variable in the model is in percentage deviation from its steady state value. That is, $x_t = (X_t - X)/X$ is in log-deviation and X is the steady state value. The variables with asterisks are the foreign variables. The open economy model we shall be estimating for Nigeria is Lubik and Schorfheide's (2007). As in other variants of the SOE model, there are basically four key equations and some set of exogenous variables.

The IS equation. The first of these equations is the open economy IS equation linking the log deviation of domestic output level, y_t, to the ex ante real interest rate, $R_t - \pi_{t+1}$, the log of productivity shock, z_t, the terms of trade, q_t, and the foreign output level, y_t^*. The IS equation so described is structurally given by

$$\begin{aligned} y_t = E_t y_{t+1} &- (\tau + \alpha(2-\alpha)(1-\tau))(R_t - E_t\pi_{t+1}) - \rho_z z_t \\ &- \alpha(\tau + \alpha(2-\alpha)(1-\tau))E_t \Delta q_{t+1} + \alpha(2-\alpha)\frac{1-\tau}{\tau}E_t(y_{t+1}^* - y_t^*) \end{aligned} \tag{1}$$

or, by the AR(1) exogenous processes to be specified shortly, is given by

$$\begin{aligned} y_t = E_t y_{t+1} &- (\tau + \alpha(2-\alpha)(1-\tau))(R_t - E_t\pi_{t+1}) - \rho_z z_t \\ &- \alpha(\tau + \alpha(2-\alpha)(1-\tau))(\rho_q - 1)q_t + \alpha(2-\alpha)\frac{1-\tau}{\tau}(\rho_y - 1)y_t^* \end{aligned} \tag{2}$$

where α measures the extent of openness, τ the elasticity of substitution and ρ_z is the coefficient of autocorrelation in the world-wide technology shock. A closed economy model is obtained when the openness parameter α is set to zero. In that case,

$$y_t = E_t y_{t+1} - \tau(R_t - \pi_{t+1}) - \rho_z z_t$$

The above reveals that there are two channels in this model through which demand in the domestic economy can be influenced from abroad:

- the increased purchasing power of the rest of the world; and
- the changes in the terms of trade.

The New Keynesian Phillips curve. The second important equation in this model is the open economy version of the New Keynesian Phillips curve that has become a standard medium of analyzing inflation dynamics. This equation like the IS equation has been derived from the first principle, and the primary building blocks of this equation is the Calvo staggered pricing scheme. It is given by

$$\pi_t = \beta E_t \pi_{t+1} + \alpha\beta E_t q_{t+1} - \alpha q_t + \frac{\kappa}{[\tau + \alpha(2-\alpha)(1-\tau)]}(y_t - \bar{y}_t) \tag{3}$$

where

$$\bar{y}_t = -\alpha(2-\alpha)\frac{1-\tau}{\tau}y_t^*$$

is the flexible-price consistent output level that obtains in the absence of technology shocks so that $y_t - \bar{y}_t$ is the output gap and $\beta \equiv \exp(-\bar{r}/400)$.

The ToT-exchange rate equation. The third equation relates the changes in nominal exchange rate with the changes in the terms of trade as well as the domestic and foreign inflation. This equation is given by

$$\Delta e_t = \pi_t - (1-\alpha)\Delta q_t - \pi_t^* \tag{4}$$

which allows us to study the competitiveness in the economy against the rest of the world.

Exogenous processes. The exogenous processes are defined for the foreign output, y_t^*, the terms of trade, q_t, the worldwide technology shocks, z_t, and the foreign inflation, π_t^* respectively as:

$$y_t^* = \rho_y y_{t-1}^* + \nu_t^{y^*} \tag{5}$$

$$q_t = \rho_q q_{t-1} + \nu_t^q \tag{6}$$

$$z_t = \rho_z z_{t-1} + \nu_t^z \tag{7}$$

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \nu_t^{\pi^*} \tag{8}$$

By this specification, we pin down the small open economy as a system affected by foreign and worldwide data-generating processes but which has little or no perceptible influence on the rest of the world. It is in this sense that we interpret our SOE model.

The monetary reaction function. The last equation describes the monetary rule embarked upon in the economy. This is in the form of the well-documented Taylor rule. There can very many variants of this rule, some with feedback mechanisms and others with and without forcing variables. In some cases, it is of interest whether the delayed response between these forcing variables and the interest rate be accounted for. We shall be estimating the following rules:

$$\begin{aligned}
&\text{Rule 1}: \quad R_t = \rho_R R_{t-1} + (1-\rho_R)[\psi_1 \pi_t + \psi_2 y_t + \psi_3 e_t] + \nu_t^R \\
&\text{Rule 2}: \quad R_t = \rho_R R_{t-1} + (1-\rho_R)[\psi_1 \pi_t + \psi_2 (y_t - \bar{y}_t) + \psi_3 e_t] + \nu_t^R \\
&\text{Rule 3}: \quad R_t = \rho_R R_{t-1} + (1-\rho_R)[\psi_1 \pi_t + \psi_2 (y_t - y_{t-1} + z_t) + \psi_3 e_t] + \nu_t^R \\
&\text{Rule 4}: \quad R_t = \rho_R R_{t-1} + (1-\rho_R)[\psi_1 E_t \pi_{t+1} + \psi_2 (y_t - \bar{y}_t) + \psi_3 e_t] + \nu_t^R
\end{aligned} \tag{9}$$

Model solution. The model above can be compactly written as

$$E_t[\mathcal{F}_\gamma(\Upsilon_{t+1}, \Upsilon_t, \Upsilon_{t-1}, \xi_t)] = 0 \tag{10}$$

where

$$\gamma = [\alpha, \bar{r}, \kappa, \tau, \psi_1, \psi_2, \psi_3, \rho_R, \rho_q, \rho_z, \rho_{y^*}, \rho_{\pi^*}, \sigma_R, \sigma_q, \sigma_z, \sigma_{y^*}, \sigma_{\pi^*}]'$$

and

$$\xi_t = [\nu_t^z, \nu_t^q, \nu_t^R, \nu_t^{y^*}, \nu_t^{\pi^*}]'$$

The solution to Equation (10) is a unique, stable and invariant stochastic difference equation given by

$$\Upsilon_t = \mathcal{T}_\gamma(\Upsilon_{t-1}, \xi_t) \tag{11}$$

Equation (11) is the transition equation. To link this solution to the data, we require the observation equation:

$$W_t = \mathcal{D}\Upsilon_t + \eta_t \tag{12}$$

These two equations form the state-space representation of the model to be estimated. Matrix $\mathcal{D}$ selects a subset of Υ_t for which we have data. The vector of such data is given by W_t. The term η_t is the measurement error. This specification allows the use of the Kalman filter to obtain data for the unobserved variables by recursively updating the system using our state-space representation of the model (see Hamilton, 1994).

3 Empirical methodology: BVAR-DSGE(λ)

The BVAR-DSGE(λ) method was introduced by Del Negro and Schorfheide (2004), extended by Del Negro et al (2004) and has been applied by a number of authors[1]. The BVAR-DSGE(λ), also called DSGE-VAR(λ), is an amalgam of the DSGE and the VAR models with a Bayesian method of estimation.

Unlike the Bayesian VAR (BVAR), where the so-called Minnesota priors are used to tilt the estimates toward random walks in the parameter space, the BVAR-DSGE(λ) model uses the artificial data generated from the DSGE to tilt the estimates toward the region of the parameter space. This should give a better bargain in that the DSGE models are theoretically motivated. Example is our SOE model previously derived. In other words, the BVAR-DSGE(λ) model strives to strike a balance between the statistical representation (VAR) and the economic requirement (DSGE). The hyperparameter governing how this should be done is the λ interpreted as the weight placed on both the VAR and the DSGE parts of the BVAR-DSGE(λ). One may follow either of two views on how the optimal balance can be attained:

- restrict the VAR parameters as in Del Negro and Schorfheide (2004)
- relax the DSGE restrictions as in Del Negro et al (2007)

[1]The burgeoning literature includes Del Negro and Schorfheide (2004) for the US, Lees et al (2007) for New Zealand, Del Negro et al (2007) for Euro countries, Adjemian et al (2008) for the US, Silveira (2008) for Brazil and Hodge et al (2008) for Australia.

Basically, the goal of the BVAR-DSGE(λ) is the construction of the dummy observation priors akin to the Minnesota-style priors in the BVAR (Sims and Zha, 2006). These dummy observation priors are then used to weigh the VAR likelihood function in order to derive the posterior distribution. Consider a reduced-form p-order vector autoregressive, VAR(p), process:

$$y_t = \Phi_0 + \Phi_1 y_{t-1} + \cdots + \Phi_p y_{t-p} + u_t \tag{13}$$

where y_t is $n \times 1$. By defining $\Phi = [\Phi_0', \Phi_1', \ldots, \Phi_p']'$ and an $np \times 1$ vector $\tilde{y}_t = [1, y_{t-1}', \ldots, y_{t-p}']'$,

$$Y = X\Phi + U \tag{14}$$

where $u_t \sim \mathcal{N}(0, \Sigma)$ and $Y = [y_1', \ldots, y_T']'$, $X = [\tilde{y}_1', \ldots, \tilde{y}_T']'$ with $U = [u_1', \ldots, u_T']'$. The VAR likelihood function is then given as

$$\begin{aligned} p(Y|\Phi, \Sigma) \propto |\Sigma|^{-\frac{T}{2}} \times \\ \exp\left(-\frac{1}{2}\text{tr}\left[\Sigma^{-1}(Y'Y - \Phi'X'Y - Y'X\Phi + \Phi'X'X\Phi)\right]\right) \end{aligned} \tag{15}$$

The next task is the construction of the dummy observation priors $p(\Phi, \Sigma|\gamma)$. Del Negro and Schorfheide (2004) assumed $T^*(= \lambda T)$ artificial observations (Y^*, X^*) generated from the DSGE model. For that process, the likelihood function is assumed to be normal so that the first two moments are all that are needed to adequately describe this distribution.

Define the population auto-covariance matrices for the DSGE model $\Gamma_{YY}(\gamma) = E(Y^{*\prime}Y^*)$, $\Gamma_{YX}(\gamma) = E(Y^{*\prime}X^*)$, $\Gamma_{XY}(\gamma) = E(X^{*\prime}Y^*)$, and $\Gamma_{XX}(\gamma) = E(X^{*\prime}X^*)$ under the assumption of covariance-stationary observables. The sample auto-covariance matrices are therefore $\lambda T\Gamma_{YY}(\gamma)$, $\lambda T\Gamma_{YX}(\gamma)$, $\lambda T\Gamma_{XY}(\gamma)$ and $\lambda T\Gamma_{XX}(\gamma)$. Del Negro and Schorfheide (2004) combined this likelihood function with diffuse priors, $p(\Phi, \Sigma) \propto |\Sigma|^{-\frac{n+1}{2}}$, to yield the dummy observation prior density given as:

$$\begin{aligned} p(\Phi, \Sigma|\gamma) \propto |\Sigma|^{-\frac{\lambda T+n+1}{2}} \times \\ \exp\left(-\frac{1}{2}\text{tr}\left[\Sigma^{-1}(Y^* - X^*\Phi(\gamma))'(Y^* - X^*\Phi(\gamma))\right]\right) \end{aligned} \tag{16}$$

where

$$\begin{aligned} (Y^* - X^*\Phi(\gamma))'(Y^* - X^*\Phi(\gamma)) = \\ \lambda T\left[\Gamma_{YY}(\gamma) - \Phi(\gamma)'\Gamma_{XY}(\gamma) - \Gamma_{YX}(\gamma)\Phi(\gamma) + \Phi(\gamma)'\Gamma_{XX}(\gamma)\Phi(\gamma)\right] \end{aligned}$$

To center the prior density above, they used a linear projection from the DSGE model to the VAR model:

$$\Phi^*(\gamma) = \Gamma_{XX}^{-1}(\gamma)\Gamma_{XY}(\gamma) \tag{17}$$

$$\Sigma^*(\gamma) = \Gamma_{YY}(\gamma) - \Gamma_{XY}(\gamma)\Gamma_{XX}^{-1}(\gamma)\Gamma_{XY}(\gamma) \tag{18}$$

In other words, conditional on the model parameters γ and λ, the priors for the VAR parameters are:

$$\Phi|\Sigma, \gamma, \lambda \sim \mathcal{N}\left(\Phi^*(\gamma), \frac{1}{\lambda T}\left[\Sigma^{-1} \otimes \Gamma_{XX}(\gamma)\right]^{-1}\right) \tag{19}$$

$$\Sigma|\gamma, \lambda \sim \mathcal{IW}\left(\lambda T \Sigma^*(\gamma), \lambda T - k - n\right) \tag{20}$$

with $\mathcal{IW}$ denoting the inverted Wishart distribution. In order to ensure that the priors are proper, $\Gamma_{XX}(\gamma)$ must be nonsingular while $\lambda \geq \frac{k+n}{T}$. To perfectly ensure that this is the case, Del Negro et al (2004) restricted λ to the following interval $\lambda \in [(k+n)/T, \infty]$. Defined hierarchically, the posterior probability is

$$p_\lambda(\Phi, \Sigma, \gamma|Y) = \frac{p(Y|\Phi, \Sigma, \gamma) p_\lambda(\Phi, \Sigma|\gamma) p(\gamma)}{p(Y)} \tag{21}$$

where $p(Y|\Phi, \Sigma, \gamma)$ is the VAR likelihood function defined in Equation (15), $p(\Phi, \Sigma|\gamma)$ is the dummy observation prior density defined in Equation (16) and $p(\gamma)$ is the joint prior density for the DSGE deep parameters; and

$$p(Y) = \int_{\gamma \in \Gamma} p(Y|\Phi, \Sigma, \gamma) p_\lambda(\Phi, \Sigma|\gamma) p(\gamma) d\gamma$$

is the marginal density, which does not affect the distribution under investigation because it is independent of the parameters of interest. Hence, we have

$$p_\lambda(\Phi, \Sigma, \gamma|Y) \propto p(Y|\Phi, \Sigma, \gamma) p_\lambda(\Phi, \Sigma|\gamma) p(\gamma) \tag{22}$$

In other terms, Del Negro and Schorfheide (2004) showed that conditional on γ and λ the preceding posterior distribution is Wishart-Normal:

$$\Sigma|\Phi, \gamma, \lambda, Y \sim \mathcal{IW}\left((\lambda+1)T\hat{\Sigma}(\gamma, \lambda), (1+\lambda)T - nk - n\right) \tag{23}$$

$$\text{vec}(\Phi)|\gamma, \lambda, Y \sim \mathcal{N}\left(\text{vec}(\hat{\Phi}(\gamma, \lambda)), \Sigma \otimes [\lambda T \Gamma_{XX}(\gamma) + X'X]^{-1}\right) \tag{24}$$

where vec(.) is the vectorization operator and

$$\hat{\Phi}(\gamma, \lambda) = (\lambda \Gamma_{XX}(\gamma) + X'X)^{-1}(\lambda T \Gamma_{XY}(\gamma) + X'Y)$$

and

$$\hat{\Sigma}(\gamma, \lambda) = \frac{1}{(1+\lambda)T}\left[\lambda T \Gamma_{YY}(\gamma) + Y'Y - (\lambda T \Gamma_{YX}(\gamma) + Y'X)\hat{\Phi}(\gamma, \lambda)\right]$$

Del Negro et al (2004) factorized the posterior distribution into the conditional density for the VAR parameters given as $p_\lambda(\Phi, \Sigma|Y, \gamma)$ given the DSGE parameters and the marginal density for the DSGE parameters γ given as $p_\lambda(\gamma|Y)$. To assess the fit of the DSGE, they studied the posterior distribution of the hyperparameters λ. In their paper, they considered a finite number of grids $\Lambda = [\lambda_1, \cdots, \lambda_q]$ with $\lambda_1 = (n+k)/T$ and $\lambda_q = \infty$. The distribution of this hyperparameter is given by

$$p_\lambda(Y) = \int p_\lambda(\Phi, \Sigma, \gamma|Y) d(\Phi, \Sigma, \gamma) \tag{25}$$

where

$$p_\lambda(\Phi, \Sigma, \gamma|Y) = p_\lambda(\Phi, \Sigma|Y, \gamma) p_\lambda(\gamma|Y)$$

the mode of which is given by

$$\hat{\lambda} = \text{argmax}_{\lambda \in \Lambda} p_\lambda(Y) \tag{26}$$

In DYNARE 4.0.2, officially available at its website[2], the implementation of this procedure is quite different. Rather than examining the posterior over the grids, which is the approach taken by Lees *et al* (2007), $\hat{\lambda}$ is estimated as part of the structural parameters of the DSGE model as in Adjemian *et al* (2008). This is the approach we follow in this paper.

4 Data and empirical results

4.1 Data

The study employs quarterly data obtained from the International Financial Statistics (IFS) for the period 1986:I to 2004:IV. All the data except those for the terms of trade were obtained from this source. Annual data for the terms of trade were obtained from the Food and Agriculture Organization's (FAO) website[3] and disaggregated using Guerrero's AR method zipped with LeSage's Matlab-based Econometrics Toolbox[4]. The terms of trade series are a proxy because they were computed from the price indices for agriculture imports and exports. The starting period for this study is chosen based on the fact that prior to the period Nigeria operated a fixed exchange regime. In our model there are five exogenous variables. We thus make use of the following five-series vector of observable data:

$$W_t = \begin{bmatrix} R_t^o \\ \pi_t^o \\ g(y_t^o) \\ q_t^o \\ e_t^o \end{bmatrix} = \begin{bmatrix} 4R_t \\ \pi_t \\ y_t - y_{t-1} + z_t \\ q_t \\ e_t \end{bmatrix} \tag{27}$$

where $g(y_t^o)$ refers to the growth rate of output and ^{o} indicates observed data.

[2] http://www.cepremap.cnrs.fr/dynare

[3] http://www.fao.org/corp/statistics/en.

[4] http://www.spatial-econometrics.com

4.2 Empirical results

Structural analysis. In Table 1 in the appendix, we present the priors used for the study. Some of these priors are standard in the literature while some others are chosen to reflect the features of the Nigerian economy. The modal values of the posterior distributions for the structural parameters of our SOE model are given in Table 2 and the means of the posterior distributions in Table 3. Our analysis is however based on the mean values of these distributions. Figures 5 to 8 in the appendix show the prior (grey) as well as the posterior (black) distributions for the structural parameters in our model.

Our empirical result shows that the feedback, measured by ρ_R, is quite low in all the rules considered in this study, ranging from 0.1410 in Rule 3 to 0.2058 in Rule 1. This reveals that the CBN has not been strongly accommodating the historical trend of its policy. In other words, the CBN has been acting almost "memorylessly " during the cycles, showing that the state of the economy in the immediate past is not important in determining the current state of the economy.

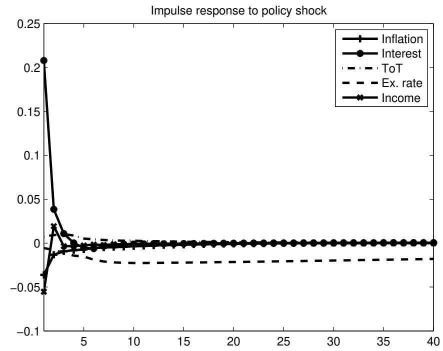
Also the design of the policy rule is not significantly responsive to exchange rate, the coefficient on e_t ranging from 0.0135 in Rule 4 to 0.0163 in Rule 1. Compared to its response to inflation, the CBN's response to exchange rate is only 92 basis points in Rule 1, 77 basis points in Rule 2, 90 basis points in Rule 3 and 74 basis points in Rule 4. The same values for income in Rule 1 are 11.8%, for output gap in Rule 2 is 17.9%, for output growth in Rule 3 is 64.4% and for output gap in Rule 4 is 31.2%. The estimated rules are given by

$$\begin{aligned}
\text{Rule 1}: &\quad R_t = 0.2058R_{t-1} + 1.7803\pi_t + 0.2097y_t + 0.0163e_t \\
\text{Rule 2}: &\quad R_t = 0.1638R_{t-1} + 1.9560\pi_t + 0.3495(y_t - \bar{y}_t) + 0.0151e_t \\
\text{Rule 3}: &\quad R_t = 0.1410R_{t-1} + 1.6842\pi_t + 1.0838(y_t - y_{t-1} + z_t) + 0.0152e_t \\
\text{Rule 4}: &\quad R_t = 0.1605R_{t-1} + 1.8189E_t\pi_{t+1} + 0.5678(y_t - \bar{y}_t) + 0.0135e_t
\end{aligned} \tag{28}$$

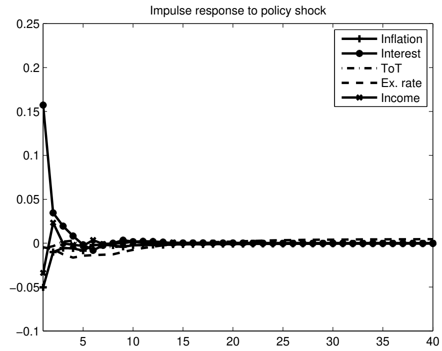
Impulse response analysis. Figure 1 shows the impulse response functions for the positive policy shock for all the policy rules. Under Rule 1, as the shock hits the system generating a contractionary effect inflation drops from its steady state value on impact but goes back by the fifth quarter. In the case of inflation, the contractionary effect manifests immediately and by the next quarter it goes back with a small overshooting effect which dies out by the third quarter. The terms of trade, however, is robustified against this policy shock, while the exchange rate, having got the impulse, departs from its steady state value, although marginally, and within the previewed quarters could not go back to it.

The contractionary effect of the positive policy shock under Rule 2 also follows that

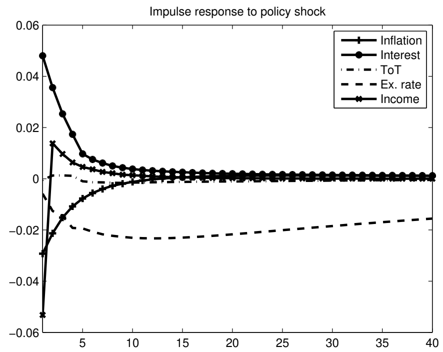
Figure 1: BVAR-DSGE(λ) impulse response functions for policy shock



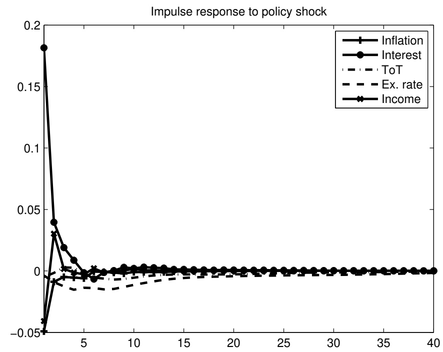
(a) Rule 1



(b) Rule 2



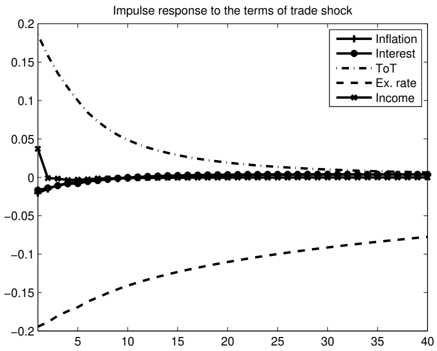
(c) Rule 3



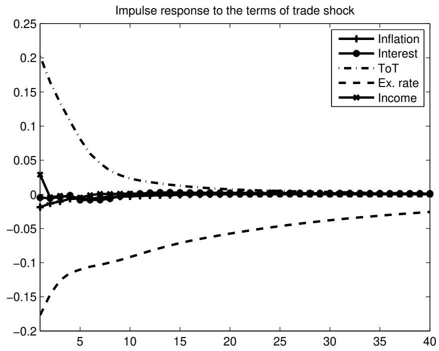
(d) Rule 4

of Rule 1 except that under this rule the exchange rate also goes back to its steady state by the twelfth quarter. When the authority is targeting the growth rate instead of income or its gap, as shown in Rule 3, a positive policy shock has the following implications. The contractionary effect is not as large as in the two previous rules on impact. Moreover, inflation gravitates back to equilibrium at a slower rate than does income. While inflation makes it back to equilibrium around the twelfth quarter, income does so in the next quarter following the impact, with noticeable overshooting. Rule 4 too follows the same pattern as other rules. Unlike Rules 1 and 3, and in line with Rule 2, the exchange rate goes back to equilibrium after it has depreciated until the twentieth quarter. This is indeed a long time. However, the depreciation is very much marginal. In comparing these rules however caution must be exercised: The scales are not the same so that elasticities are not the same. In Rule 3, the graph is much more magnified than in Rules 1, 2 and 4. Figure 2 presents the impulse response functions for the positive terms of trade shock for all the rules. Under Rule 1, this shock feeds into exchange rate causing an approximately opposite movement in exchange rate. It also affects both inflation and income causing imperceptible movements in them. Expansionary policy

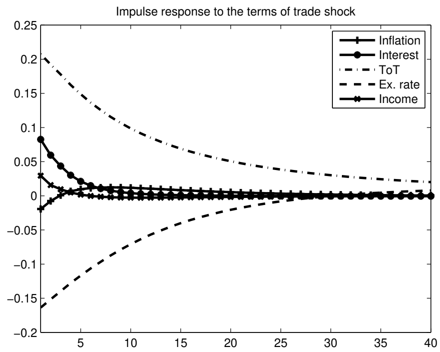
Figure 2: BVAR-DSGE(λ) impulse response functions for the terms of trade shock



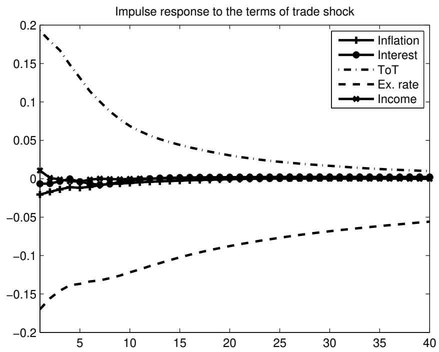
(a) Rule 1



(b) Rule 2



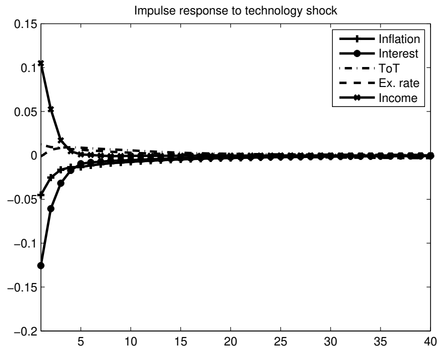
(c) Rule 3



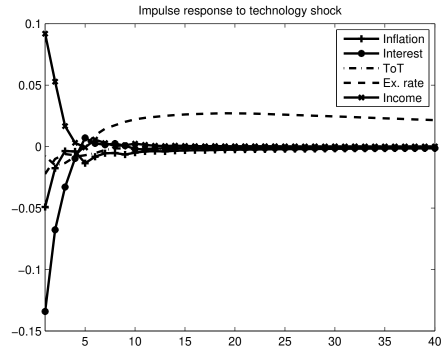
(d) Rule 4

is not reactionary enough to stem the depreciation of exchange rate but is enough to bring the income and inflation back to equilibrium. In particular, income reverts in the next quarter following the impact. Also inflation traces out the time path of the policy and reaches the steady state in just three quarters. Rule 2 is essentially similar to Rule 1 with no policy reaction at all, although there is a greater tendency for exchange rate to go back to equilibrium under Rule 2. In both cases, however, this tendency could not materialize in the forty-quarter period of preview. Under Rule 3, the terms of trade shock causes income and inflation to depart from the steady state on impact albeit very marginally. In this case, income decreases and inflation gets exacerbated. It also causes exchange rate to depreciate almost one-to-one. In response, the policy is more contractionary mediating through the trade-off between the income and inflation. This process of mediation is so smooth inflation and income move back to equilibrium gently. It is difficult to believe that monetary policy is reacting to the depreciation in exchange rate. Indeed after having reached equilibrium some time in the twenty-seventh quarter, exchange rate overshoots the equilibrium causing appreciation of the naira. Rule 4 generates quite the same responses as the other rules but the policy is not as

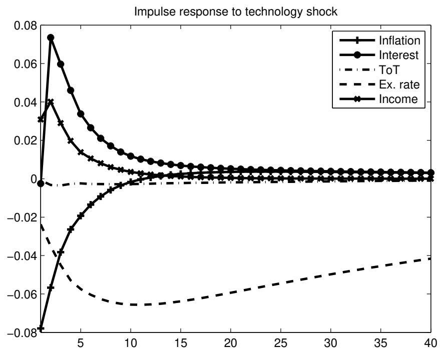
Figure 3: BVAR-DSGE(λ) impulse response functions for technology shock



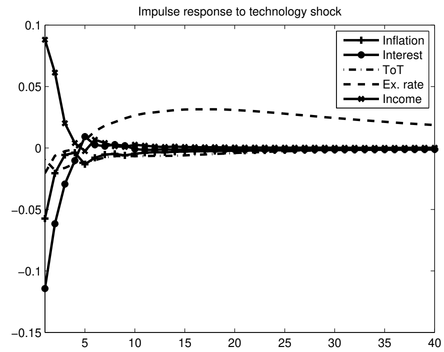
(a) Rule 1



(b) Rule 2



(c) Rule 3

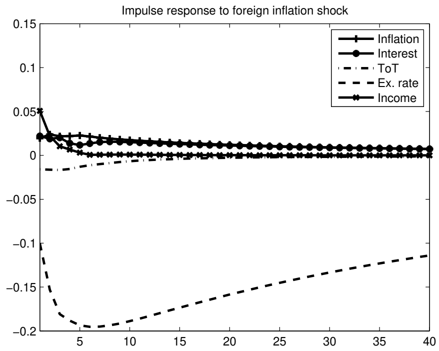


(d) Rule 4

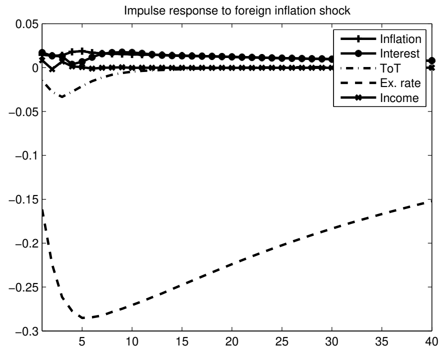
reactionary as in Rule 3 but in line with the first two rules.

Figure 3 shows the impulse response functions of the positive world-wide productivity shock, which on the impact feeds into the system through the IS equation and has positive relationship with income and negative relationship with inflation. As a result, under Rule 1, the economy overheats, and the policy reaction over the time is contractionary, trading off the relationship between inflation and income. Following this policy action, inflation and income move to their steady-state values over time, and by the tenth quarter they are both settled on the steady-state path. Both the terms of trade and exchange rate do not so much as depart from equilibrium even at the impact of the world-wide productivity shock. Under Rule 2 the pattern is not different, except that exchange rate after having departed from its steady-state overshoots for the rest of the period under preview. The pattern in Rule 3 is quite different. Under this rule, the authority is more willing to use an expansionary policy by responding more strongly to inflation than under the other rules. An insight for this can be obtained from Equation (28). Another different pattern of response is that of exchange rate. On impact, exchange rate depreciates with a wide amplitude. As a result, exchange rate is unable to

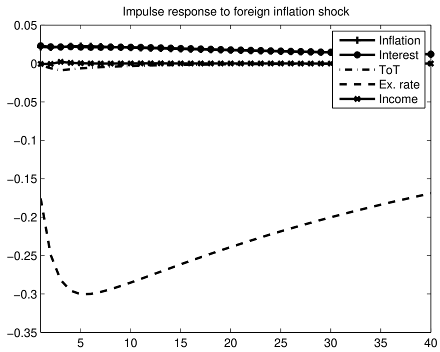
Figure 4: BVAR-DSGE(λ) impulse response functions for foreign inflation shock



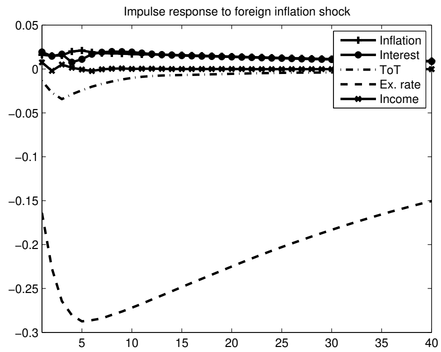
(a) Rule 1



(b) Rule 2



(c) Rule 3



(d) Rule 4

find its way back to equilibrium, displaying persistence on its path. It is clear that the authority has not been reacting to exchange rate movement. This also can be seen from the value of coefficient on exchange rate in Equation (28). Rule 4 is similar to Rule 2.

Finally, Figure 4 shows the impulse response function for the positive foreign inflation shock. This shock enters into the system primarily via the terms of trade equation, and given its small-size propagation effect, it only produces imperceptibly small variations in income, inflation and the terms of trade. Saving elasticities of the response functions [note that the scales are not the same], the pattern is the same under all the rules. Foreign inflation seems to have contributed to domestic inflation at the margin. In particular, foreign inflation causes variation in domestic inflation that is apparently permanent. Too, exchange rate depreciates supposedly enhancing the competitiveness of the domestic economy. It should be mentioned, however, that Nigeria is a member of OPEC and primarily engages in exportation of crude oil where it has no control over the oil price. I surmise that by and large the depreciation will only make import of consumption goods and key productive inputs expensive. In terms of the policy reaction, the authority does not react very much to changes in exchange rate.

5 Conclusion

Motivated by the need to understand the manner of response of the CBN to cycles, we have used a version of the dynamic stochastic general equilibrium (DSGE)—the so-called small open economy (SOE) model—which situates the Nigerian economy in global perspective to generate dummy observation priors complementing the VAR model in line with the BVAR-DSGE(λ) approach. We have examined four models, with the kind of monetary policy rules making the difference.

Our approach is very revealing in pointing out how the CBN reacts to cycles. In all the rules considered, the Central Bank of Nigeria (CBN) places little weight on the exchange rate behaviour in reacting to the cycles, resulting in overshooting and persistence by the exchange rate. This has the effect of diminishing the country's competitiveness and also raising the cost of imports. However, we find that the CBN strongly reacts to the behaviour of inflation and, to a lesser degree, to the behaviour of output, output gap or its growth following the shocks. This correspondingly means that when the economy is hit by the shocks, these variables are quickly brought back to equilibrium. The feedback is also very low, making the current policy focus less dependent on the previous policy focus. This smirks discontinuity in policy implementation.

We conclude that it will be important for the CBN to pursue a guided exchange rate policy by actively responding to the exchange rate movement. Exchange rate should not be left completely to the market as there is no self-correcting mechanism but overshooting and persistence. We also find that government needs to endogenize the behaviour of the terms of trade so as to have control over its behaviour. Otherwise, it often takes several quarters before the terms of trade is equilibriated following the impact. Lastly, there is scope for the CBN to learn from past policy outcome and it can build a much stronger feedback.

References

Adjemian, S., M. Darracq. Pariès and S. Moyen (2008) *Towards a Monetary Policy Evaluation*, European Central Bank, Working Paper Series No 942/ September.

Calvo, G., (1983) "Staggered Contracts in a Utility-Maximizing Framework." *Journal of Monetary Economics* 12, 383-398.

Da Silveira., M. A. C., (2008) "Using a Bayesian Approach to Estimate and Compare New Keynesian DSGE Models for the Brazilian Economy: the Role for Endogenous Persistence", RBE Rio de Janeiro v62 No1 pp 1-25 Jan-Mar.

Del Negro, M., and F. Schorfheide (2004) "Priors from General Equilibrium Models for VARs", *International Economic Review*, 45(2), pp 643-673.

Del Negro M., F. Schorheide, F. Smets and R. Wouters (2007) "On the Fit of New Keynesian Models", *Journal of Business and Economic Statistics*, 25(2), pp 123-143.

Galí, J., and T. Monacelli (2005) "Monetary Policy and Exchange Rate Volatility in a Small Open Economy", *Review of Economic Studies*, Vol. 72, pp 1165-1192.

Hamilton, J.D. (1994) *Time Series Analysis.* Princeton University Press. Princeton.

Hodge, A., T. Robinson and R. Stuart (2008) *A Small BVAR-DSGE for Forecasting the Australian Economy*, Reserve Bank of Australia, Research Discussion Paper 2008/04

Lees, K., T. Matheson and C. Smith (2007), *Open Economy DSGE-VAR Forecasting and Policy Analysis: Head to Head with the RBNZ Published Forecasts*, Reserve Bank of New Zealand Discussion Paper No 2007/01.

Liu, P., (2006) *A Small New Keynesian Model of the New Zealand Economy*, Reserve Bank of New Zealand, Discussion Paper 2006/03.

Lubik, T. A. and F. Schorfheide (2005) "Do Central Banks Responds to Exchange Rate Movements? A structural investigation", *Journal of Monetary Economics*, forthcoming, November

Lucas, R., (1976), "Econometric Policy Evaluation: A Critique", *Carnegie-Rochester Conference Series on Public Policy*, 1, pp 19-46.

Sims, C., and T. Zha (2006), "Were there regime switches in US monetary policy?" *American Economic Review*, 96(1) pp 54-81.

Smets, F., and R. Wouters (2005). "Comparing Shocks and Frictions in US and Euro Area Business Cycles: a Bayesian DSGE Approach". *Journal of Applied Econometrics*, 20(2) pp 161-183.

Appendix

Table 1: Priors for the model

Parameter	Density*	Prior mean	SD	Lower	Upper
α	$\mathcal{B}$	0.40	0.09	0.01	0.99
$\bar{r}$	$\mathcal{G}$	2.50	1.00	0.01	∞
κ	$\mathcal{G}$	0.50	0.25	0.01	∞
τ	$\mathcal{B}$	0.50	0.20	0.01	0.99
ψ_1	$\mathcal{G}$	0.50	0.20	0.01	∞
ψ_2	$\mathcal{G}$	0.25	0.13	0.01	∞
ψ_3	$\mathcal{G}$	0.25	0.13	0.01	∞
ρ_R	$\mathcal{B}$	0.65	0.20	0.01	0.99
ρ_q	$\mathcal{B}$	0.40	0.20	0.01	0.99
ρ_z	$\mathcal{B}$	0.20	0.05	0.01	0.99
ρ_{y^*}	$\mathcal{B}$	0.90	0.05	0.01	0.99
ρ_{π^*}	$\mathcal{B}$	0.80	0.10	0.01	0.99
σ_R	$\mathcal{IG}$	0.50	4.00	—	—
σ_q	$\mathcal{IG}$	1.50	4.00	—	—
σ_z	$\mathcal{IG}$	1.00	4.00	—	—
σ_{y^*}	$\mathcal{IG}$	1.50	4.00	—	—
σ_{π^*}	$\mathcal{IG}$	0.55	4.00	—	—
$\hat{\lambda}$	$\mathcal{U}$	5	—	0	10

* $\mathcal{B}$, $\mathcal{U}$, $\mathcal{G}$ and $\mathcal{IG}$ refer to Beta, Uniform, Gamma and Inverted-Gamma distributions respectively.

Table 2: Modal values of the structural parameters.

	Posterior mode			
Parameter	Rule 1	Rule 2	Rule 3	Rule 4
α	0.2139	0.2327	0.2580	0.2101
$\bar{r}$	2.0970	2.0986	2.1015	2.0984
κ	0.1023	0.2406	0.0258	0.1650
τ	0.7371	0.7213	0.7617	0.7037
ψ_1	2.0365	2.1495	1.8436	1.9902
ψ_2	0.2326	0.3593	1.2721	0.5217
ψ_3	0.0147	0.0154	0.0155	0.0146
ρ_R	0.1993	0.1313	0.1133	0.1574
ρ_q	0.8902	0.8002	0.9531	0.9116
ρ_z	0.3756	0.3225	0.0776	0.3407
ρ_{y^*}	0.8852	0.9468	0.8598	0.9470
ρ_{π^*}	0.4671	0.5187	0.4626	0.5005
σ_R	0.0742	0.0732	0.0811	0.0702
σ_q	0.2025	0.2003	0.2037	0.2002
σ_z	0.1537	0.1407	0.1685	0.1433
σ_{y^*}	0.5573	0.5750	0.6705	0.5792
σ_{π^*}	0.1699	0.1560	0.1780	0.1534
$\hat{\lambda}$	4.9843	1.6922	10.0000	1.6070

$\mathcal{B}$, $\mathcal{U}$, $\mathcal{G}$ and $\mathcal{IG}$ refer to beta, uniform, gamma and inverted gamma distributions respectively. The priors used are standard in the literature.

Table 3: Posteriors of the structural parameters

	Rule 1			Rule 2			Rule 3			Rule 4		
Parameter	* $P_m(\gamma)$	2.5%	97.5%	$P_m(\gamma)$	2.5%	97.5%	$P_m(\gamma)$	2.5%	97.5%	$P_m(\gamma)$	2.5%	97.5%
α	0.2015	0.0910	0.3034	0.2327	0.1310	0.3294	0.2447	0.1398	0.3508	0.2327	0.1164	0.3480
$\bar{r}$	2.4059	1.1015	3.6742	2.5230	0.9060	4.0889	2.4685	0.9417	3.9888	2.3450	0.9744	3.6667
κ	0.1235	0.0586	0.1897	0.2761	0.1736	0.3936	0.0372	0.0175	0.0538	0.2311	0.1117	0.3527
τ	0.6991	0.5569	0.8352	0.7352	0.6344	0.8347	0.7917	0.6966	0.8883	0.7895	0.6370	0.9242
ψ_1	2.2416	1.6771	2.7467	2.3391	1.8767	2.8791	1.9606	1.5389	2.3265	2.1667	1.6815	2.5948
ψ_2	0.2640	0.1251	0.4106	0.4180	0.1736	0.6774	1.2617	0.9606	1.6062	0.6763	0.2817	1.0198
ψ_3	0.0205	0.0113	0.0310	0.0181	0.0117	0.0252	0.0177	0.0117	0.0230	0.0174	0.0111	0.0241
ρ_R	0.2058	0.0751	0.3286	0.1638	0.0348	0.2661	0.1410	0.0395	0.2385	0.1605	0.0421	0.2624
ρ_q	0.8392	0.7216	0.9520	0.7675	0.6075	0.9232	0.9170	0.8636	0.9785	0.8947	0.8245	0.9663
ρ_z	0.3678	0.3123	0.4271	0.3251	0.2656	0.3827	0.0935	0.0568	0.1283	0.3392	0.2718	0.4077
ρ_{y^*}	0.8753	0.7977	0.9505	0.9267	0.8773	0.9704	0.8017	0.7174	0.8867	0.9316	0.8930	0.9802
ρ_{π^*}	0.4867	0.3281	0.6436	0.5227	0.3455	0.6966	0.4698	0.3183	0.6348	0.5191	0.2991	0.6920
σ_R	0.0781	0.0640	0.0910	0.0782	0.0637	0.0901	0.0816	0.0671	0.0974	0.0729	0.0604	0.0844
σ_q	0.2114	0.1863	0.2383	0.2175	0.1864	0.2501	0.2114	0.1863	0.2348	0.2099	0.1863	0.2321
σ_z	0.1647	0.1295	0.1926	0.1493	0.1230	0.1707	0.1653	0.1302	0.2003	0.1535	0.1243	0.1793
σ_{y^*}	0.6213	0.3431	0.8767	0.7019	0.3977	1.0106	0.8742	0.3910	1.3082	1.1977	0.4427	1.9288
σ_{π^*}	0.1783	0.1485	0.2102	0.1633	0.1311	0.1948	0.1801	0.1489	0.2087	0.1639	0.1262	0.2035
$\hat{\lambda}$	5.7911	2.5196	9.5091	1.9024	1.1341	2.6835	8.7561	8.0275	9.3389	2.1089	1.0911	3.3215

* $P_m(\gamma)$ stands for the posterior mean of γ. These results were based on 10,000 draws of which the initial 5,000 draws were discarded as burn-ins.

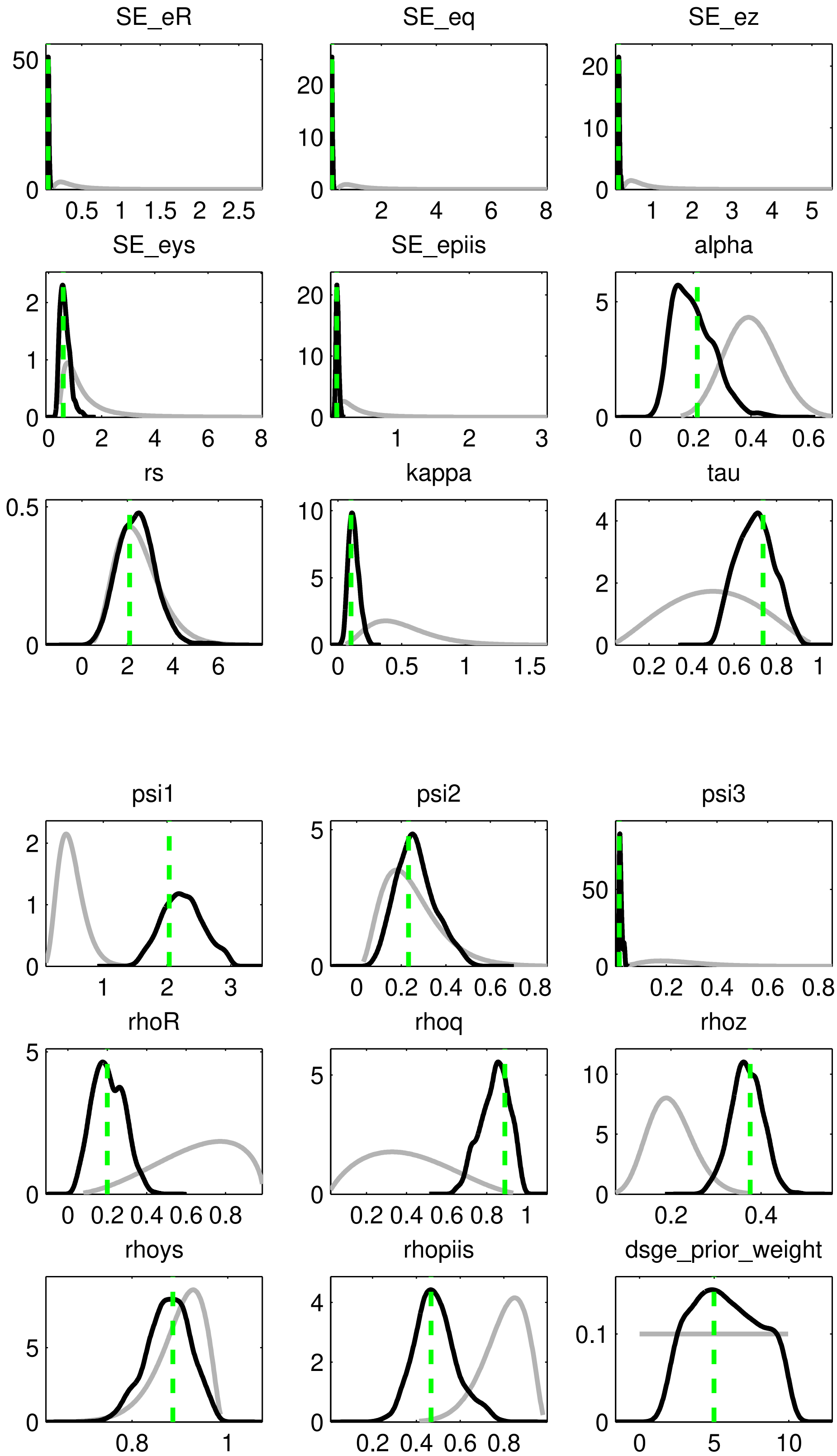


Figure 5: Posterior distribution for Rule 1.

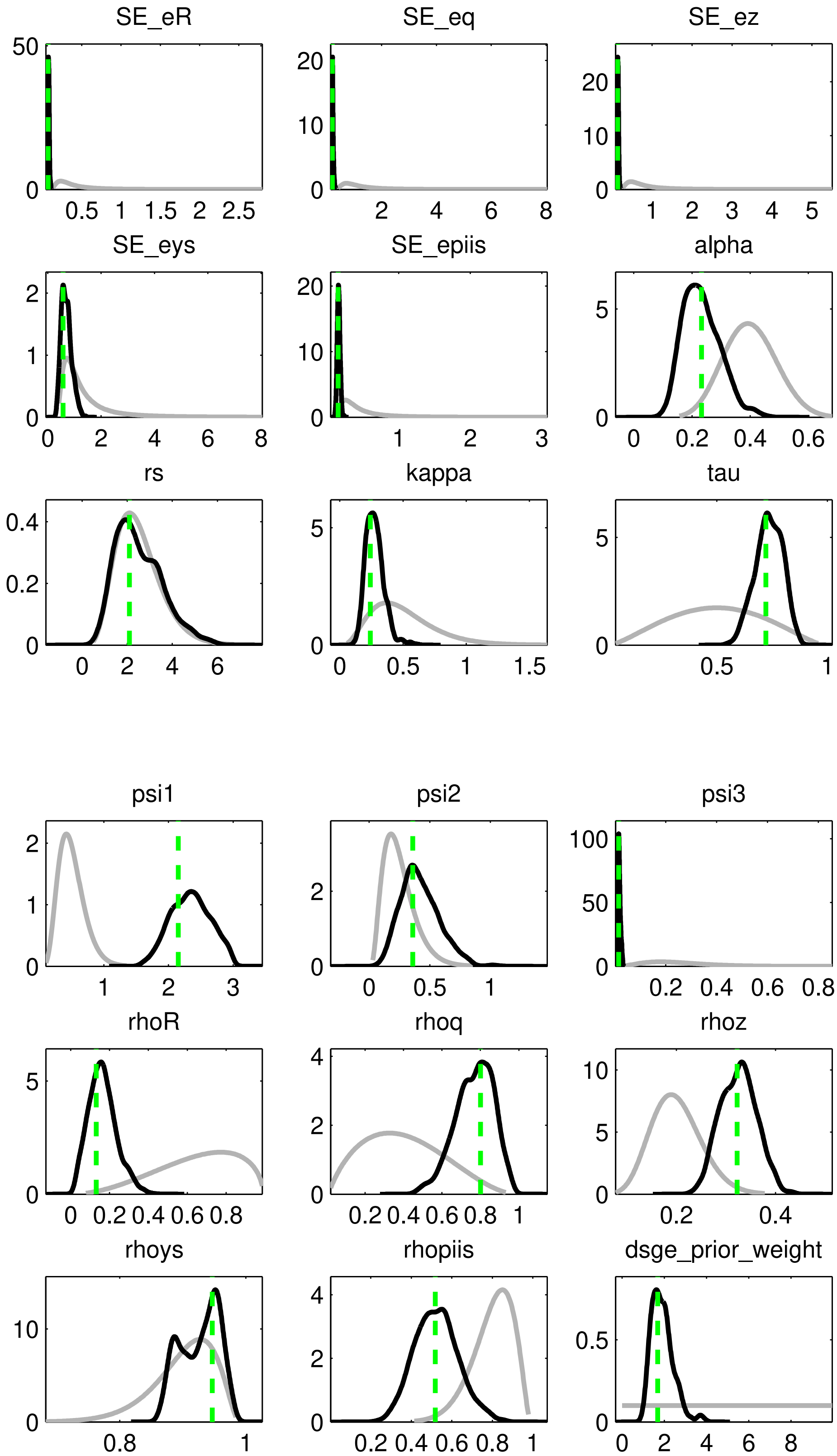


Figure 6: Posterior distribution for Rule 2.

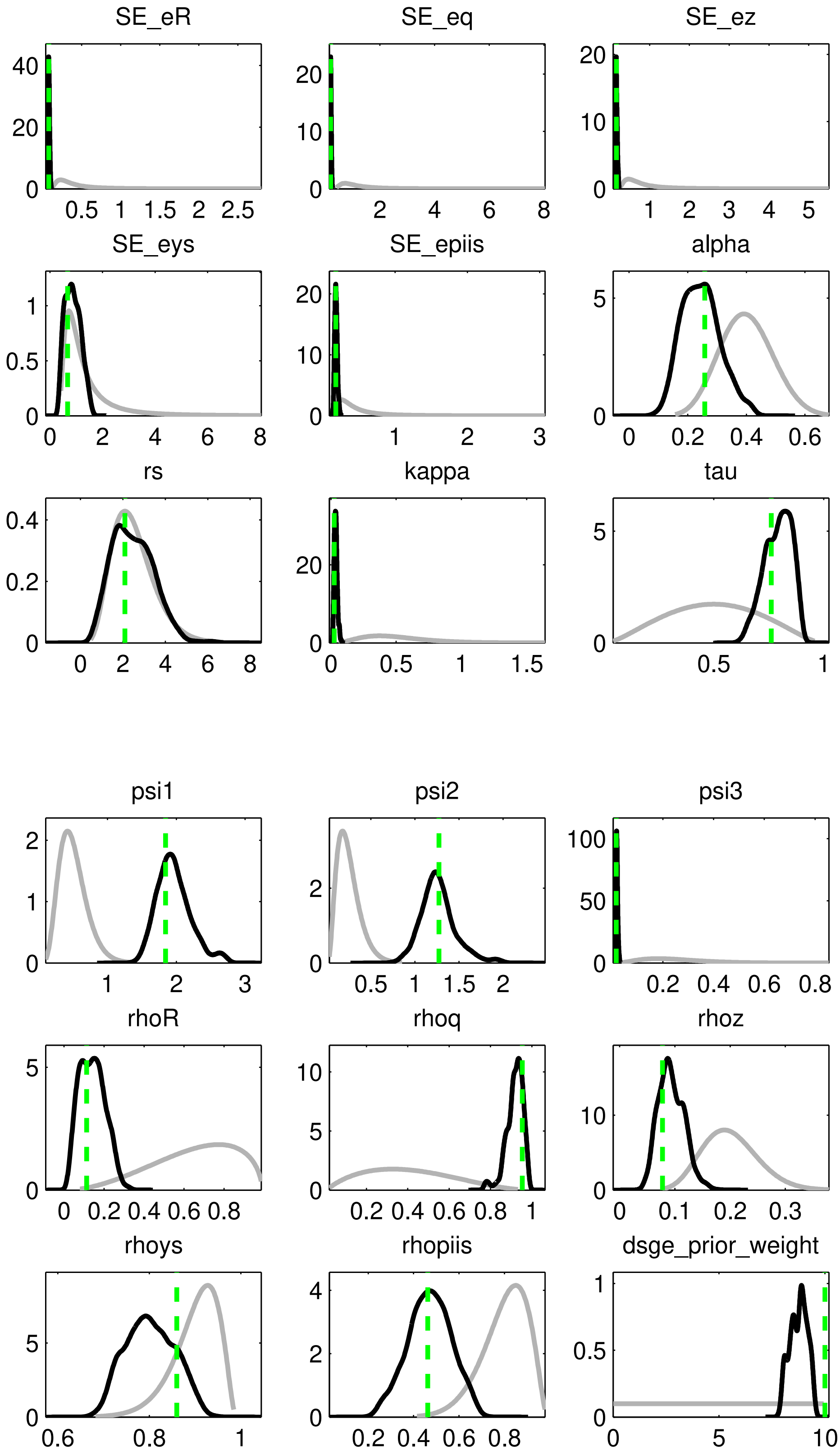


Figure 7: Posterior distribution for Rule 3.

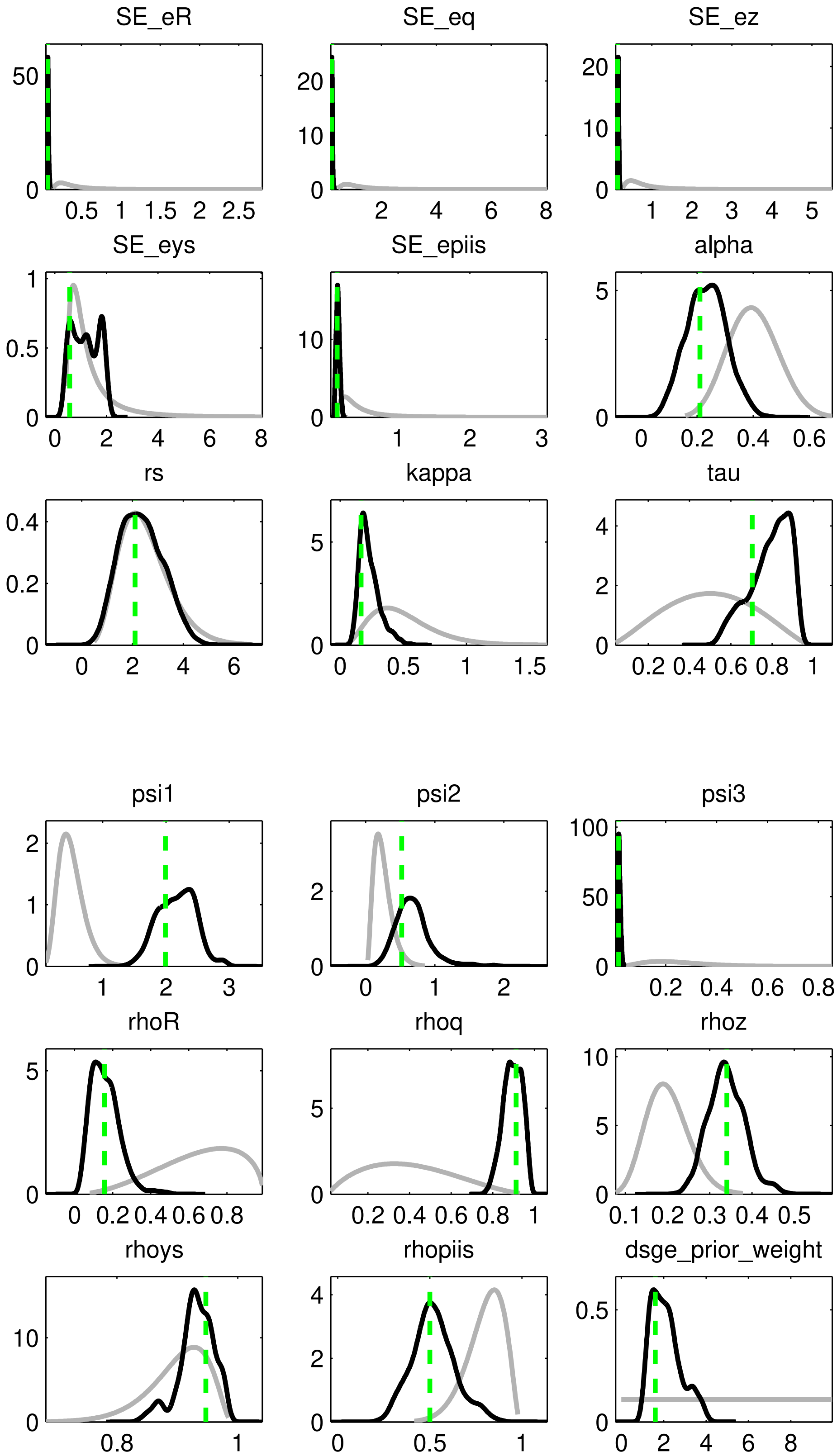


Figure 8: Posterior distribution for Rule 4.