Auctioning with Aspirations: Keep Them Low (Enough)

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Abstract

In an auction with a buy price, a seller offers bidders the opportunity to forgo competing in an auction by transacting immediately at a pre-specified fixed price. If a seller has aspirations in the form of a reference price that depends upon the auction's reserve price and buy price, she does best to keep her aspirations sufficiently low by designing a no-reserve auction with a buy price low enough that some bidder types would exercise it with positive probability in equilibrium. The seller is indifferent between the auction component of her mechanism being a first- or second-price auction.

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1 Introduction

From the growth of auctions on the Internet emerged a new selling mechanism, an auction with a buy price. In an auction with a buy price, a seller offers bidders the opportunity to forgo competing in an auction by transacting immediately at a pre-specified fixed price. The "Buy-It-Now" feature of industry leader eBay is a leading example of an auction with a buy price.[1] In the first quarter of 2009, eBay's sales (Gross Merchandise Volume) were $12.8 billion, 49% of which were through buy price transactions.[2] Thus, auctions with buy prices are popular among buyers and sellers in online auctions and are economically significant.

That auctions with buy prices are so popular poses somewhat of a puzzle from the vantage point of standard auction theory. Namely, when demand is uncertain and markets are thin, the market price is unclear and auctioning dominates fixed price selling from a seller's perspective (Milgrom, 1989; Wang, 1993). Thus, when a seller has an auction mechanism at her disposal online, the option to augment her auction with a fixed price seems unnecessary. A growing theoretical literature studies this "hybrid" mechanism and seeks to understand under what conditions a seller would benefit from augmenting her auction with a buy price. Existing rationales for auctions with buy prices include: Bidder or seller risk aversion (Budish & Takeyama, 2001; Hidvégi, Wang, & Whinston, 2006; Mathews & Katzman, 2006; Reynolds & Wooders, 2009); bidder or seller impatience (Mathews, 2004); auction transaction costs for bidders (Wang, Montgomery, & Srinivasan, 2008); seller competition and multi-unit demands (Kirkegaard & Overgaard, 2008); price discrimination when the set of bidder types is not connected (Bose & Daripa, 2009); and bidder reference-dependence (Shunda, 2009).

This paper contributes to this literature by exploring a model of auctioning with aspirations. I model a seller with aspirations as one who evaluates outcomes in relation to some fixed reference outcome; such a seller has reference-dependent preferences over revenue (see, e.g., Kahneman and Tversky (1979) and Köszegi and Rabin (2006) for general models of reference-dependence). Thus, a seller with aspirations cares about revenue and (possibly very slightly) how her sales price compares to a reference price. I demonstrate that a risk-neutral seller with aspirations does best to conduct her sale via a no-reserve auction with a buy price in contrast to a pure auction and is indifferent between the auction component of her mechanism being a first- or second-price auction.

[1] The "Buy Price" on Yahoo!, "Take-It" on Amazon, and "uBuy It" on uBid auctions are other examples of online auctions with buy prices.

[2] Figures are from http://investor.ebay.com/index.cfm.

2 Model

A seller conducts the sale of a single unit of an indivisible good for which she derives value $v_0 = 0$ through an auction with a buy price B^* and, if no bidder accepts the buy price, holds a second-price sealed-bid auction with reserve price r. The auction's buy price is "temporary" in that it disappears if no bidder is willing to exercise it.[3] There are $n \geq 2$ risk-neutral bidders, each of whose valuation v_i is private information and is an independent draw from the common distribution F that is continuous on its support $[0, \overline{v}]$ with a density f that is finite on its support and bounded away from zero. A bidder with valuation v_i who wins the good at a price p earns surplus of $v_i - p$ and earns 0 otherwise. The number of bidders, buy price, reserve price, and the distribution of valuations are common knowledge. Thus, the model is within the symmetric independent private values framework. To demonstrate that a seller would set a buy price that some bidder types would exercise with positive probability in equilibrium, I additionally require that $f' \equiv \frac{df}{dv}$ exists and is finite on its support.

The novel feature of the model is the addition of a seller with aspirations. Such a seller cares about revenue and (possibly very slightly) how her sales price compares to a fixed reference price (which one can conceptualize as her aspired selling price).[4] Thus, if the good sells for price $x \geq 0$, the seller's utility is $x + \delta(x - \sigma)$. The seller's reference price is σ and δ is a small positive number. I assume that the seller's reference price is $\sigma = \mu r + (1 - \mu)B^*$ with $\mu \in (0, 1)$. Thus, a seller with aspirations cares about revenue and experiences a utility (disutility) of $\delta(x - \sigma)$ if $x > (<)\sigma$. Setting $\delta = 0$ reproduces the standard model of a risk-neutral seller as a special case.

3 Results

The model I describe in the previous section induces a two-stage game of incomplete information among the bidders and seller. Given the seller's choice of B^* and r, bidders decide whether or not to exercise the buy price in stage one and, if no bidder exercised the buy price, submit a bid to the auction in stage two. Ties are broken randomly. To analyze the model, I search for a symmetric Bayes-Nash equilibrium.

Since a standard second-price sealed-bid auction occurs if no bidder exercises the buy price, it is well-known that each bidder has a weakly dominant strategy to bid according to

[3] The "Buy-It-Now" price is a leading example of a temporary buy price. Buy prices on Yahoo! and Amazon auctions are examples of "permanent" buy prices available for the auction's entire duration. For analyses of auctions with permanent buy prices, see Hidvégi et al. (2006) and Reynolds and Wooders (2009).

[4] My conceptualization of reference-dependence follows closely Rosenkranz and Schmitz (2007) who introduce a similar formulation into auction theory via bidder utilities.

$\beta_S(v_i) = v_i$ if $v_i \geq r$ and to not participate otherwise.

Suppose a bidder exercises the buy price if and only if $v_i \geq v^*$ and rejects it otherwise. For a bidder with $v_i \geq r$, expected surplus from exercising the buy price given that rivals with $v \geq v^*$ exercise it is

$$\begin{aligned}\pi^B(v_i, v^*) &= \sum_{k=0}^{n-1} \binom{n-1}{k} F(v^*)^{n-k-1}(1-F(v^*))^k \frac{1}{1+k}(v_i - B^*) \\ &= \left(\frac{1-F(v^*)^n}{n(1-F(v^*))}\right)(v_i - B^*). \qquad (1)\end{aligned}$$

On the other hand, for such a bidder, expected surplus from the auction is

$$\begin{aligned}\pi^A(v_i, v^*) &= (v_i - r)F(r)^{n-1} + \int_r^{\min\{v_i,v^*\}} (v_i - x)(n-1)F(x)^{n-2}f(x)dx \\ &= (v_i - \min\{v_i, v^*\})F(\min\{v_i, v^*\})^{n-1} + \int_r^{\min\{v_i,v^*\}} F(x)^{n-1}dx \qquad (2)\end{aligned}$$

(where the second equality follows from integration by parts).

Following Mathews and Katzman (2006), define as $B(v_i, r)$ the buy price that makes a bidder of type v_i indifferent between the buy price and the auction. This threshold buy price solves

$$\int_r^{v_i} F(x)^{n-1}dx = \left(\frac{1-F(v_i)^n}{n(1-F(v_i))}\right)(v_i - B(v_i, r)). \qquad (3)$$

Straightforward differentiation reveals that $B(v_i, r)$ is increasing in both v_i and r. Define v^* as the unique $v \in [0, \overline{v}]$ such that $B(v^*, r) = B^*$ and define $v^* = \overline{v}$ if $B(v, r) < B^*$ for all $v \in [0, \overline{v}]$. I obtain the following result on bidder behavior.

Proposition 1. *There exists a symmetric equilibrium in which a bidder of type v_i accepts the buy price if and only if $B^* < B(v_i, r)$ and submits a bid of $\beta_S(v_i) = v_i$ in the auction. This is the unique symmetric equilibrium.*

Proof. Equilibrium bidding according to $\beta_S(v_i) = v_i$ follows from a standard dominance argument.

A necessary condition for v^* to be part of an equilibrium is that $\pi^A(v^*, v^*) = \pi^B(v^*, v^*)$. Since v^* satisfies $B(v^*, r) = B^*$, it is immediate that $v_i = v^*$ equates (1) and (2). Note that

$\frac{d\pi^A}{dv_i} = F(\min\{v_i, v^*\})^{n-1} \leq F(v^*)^{n-1}$. Therefore,

$$\begin{aligned}\frac{d\pi^A}{dv_i} - \frac{d\pi^B}{dv_i} &\leq F(v^*)^{n-1} - \frac{1-F(v^*)^n}{n(1-F(v^*))} \\ &= \frac{-(1-nF(v^*)^{n-1}+(n-1)F(v^*)^n)}{n(1-F(v^*))} \\ &< 0\end{aligned}$$

because $1 - nF(v^*)^{n-1} + (n-1)F(v^*)^n > 0$ for $F(v^*) < 1$. Thus, $\frac{d\pi^B}{dv_i} > \frac{d\pi^A}{dv_i}$. Since $\pi^A(v_i, v^*) = \pi^B(v_i, v^*)$ at $v_i = v^*$, we must have $\pi^A(v_i, v^*) > \pi^B(v_i, v^*)$ for $v_i < v^*$ and $\pi^B(v_i, v^*) > \pi^A(v_i, v^*)$ for $v_i > v^*$.

To see that this symmetric equilibrium is unique, note that because $B(v_i, r)$ is increasing in v_i, the existence of some $\hat{v} \neq v^*$ satisfying $B(\hat{v}, r) = B^*$ would lead to a contradiction. □

Denote the highest valuation x and the second-highest valuation y. Given the bidder behavior Proposition 1 describes, the seller's expected utility takes the form

$$\begin{aligned}U(r, v^*) &= F(r)^n(0+\delta(0-\sigma)) + \int_r^{v^*}\int_0^r (r+\delta(r-\sigma))n(n-1)F(y)^{n-2}f(y)dyf(x)dx \\ &\quad + \int_r^{v^*}\int_r^x (y+\delta(y-\sigma))n(n-1)F(y)^{n-2}f(y)dyf(x)dx \\ &\quad + (1-F(v^*)^n)(B(v^*,r)+\delta(B(v^*,r)-\sigma)) \\ &= -F(r)^n\delta\sigma + nF(r)^{n-1}(F(v^*)-F(r))(r+\delta(r-\sigma)) \\ &\quad + \int_r^{v^*}(y+\delta(y-\sigma))n(n-1)F(y)^{n-2}f(y)(F(v^*)-F(y))dy \\ &\quad + (1-F(v^*)^n)(B(v^*,r)+\delta(B(v^*,r)-\sigma))\end{aligned} \tag{4}$$

(where the second equality follows from exchanging the order of integration of the second double integral). Because $B(v_i, r)$ is increasing in v_i, we can analyze the seller's choice of B^* as her choice of v^*. Maximizing $U(r, v^*)$ with respect to $r \in [0, \overline{v}]$ and $v^* \in [0, \overline{v}]$ yields the following result.

Proposition 2. *Given bidders' equilibrium strategies, a seller with aspirations maximizes her expected utility by conducting a no-reserve auction with a buy price low enough that some bidder types would exercise it with positive probability in equilibrium.*

Proof. Differentiating $U(r, v^*)$ yields

$$\begin{aligned}\frac{\partial U}{\partial r} &= -nF(r)^{n-1}f(r)(1+\delta)r + nF(r)^{n-1}F(v^*)(1+\delta) - nF(r)^n(1+\delta) \\ &\quad +(1-F(v^*)^n)(1+\delta)\frac{\partial B}{\partial r} - \delta\mu - \delta(1-\mu)\frac{\partial B}{\partial r}\end{aligned}$$

and

$$\begin{aligned}\frac{\partial U}{\partial v^*} &= nF(v^*)^{n-1}f(v^*)(1+\delta)(v^* - B(v^*, r)) - (1+\delta)\int_r^{v^*} nF(y)^{n-1}f(v^*)dy \\ &\quad +(1-F(v^*)^n)(1+\delta)\frac{\partial B}{\partial v^*} - \delta(1-\mu)\frac{\partial B}{\partial v^*}.\end{aligned}$$

Because $\partial B/\partial r = n(1-F(v^*))F(r)^{n-1}/(1-F(v^*)^n)$,

$$\frac{\partial U}{\partial r} = -nF(r)^{n-1}f(r)(1+\delta)r - \delta\mu - \frac{\delta(1-\mu)n(1-F(v^*))F(r)^{n-1}}{1-F(v^*)^n} < 0$$

for all $v^* \in [0, \overline{v}]$. Therefore, it is optimal to set $r = 0$, a no-reserve auction.

Note that if the seller sets $v^* = \overline{v}$, a buy price no bidder type would exercise with positive probability in equilibrium, we have

$$\begin{aligned}\left.\frac{\partial U}{\partial v^*}\right|_{v^*=\overline{v}} &= nf(\overline{v})(1+\delta)(\overline{v} - B(\overline{v}, r)) - (1+\delta)\int_r^{\overline{v}} nF(y)^{n-1}f(\overline{v})dy - \delta(1-\mu)\left.\frac{\partial B}{\partial v^*}\right|_{v^*=\overline{v}} \\ &= 0\end{aligned}$$

using (3) and the fact that $\partial B/\partial v^*|_{v^*=\overline{v}} = 0$. However, differentiating again and evaluating at $v^* = \overline{v}$, we have

$$\left.\frac{\partial^2 U}{\partial v^{*2}}\right|_{v^*=\overline{v}} = n(n-1)f(\overline{v})^2(1+\delta)(\overline{v} - B(\overline{v}, r)) + 2nf(\overline{v})(1+\delta) > 0$$

for all $r \in [0, \overline{v}]$ using again the fact that $\partial B/\partial v^*|_{v^*=\overline{v}} = 0$ and that $\partial^2 B/\partial v^{*2}\big|_{v^*=\overline{v}} = 0$. This implies that $\partial U/\partial v^* < 0$ to the left of $\overline{v}$ and, thus, that it is optimal to set $v^* < \overline{v}$, a buy price that some bidder types would exercise with positive probability in equilibrium. □

Finally, a seller with aspirations is indifferent between first- and second-price auction rules of her auction with a buy price.

Proposition 3. *First- and second-price auctions with a buy price are utility equivalent to a seller with aspirations.*

Proof. Consider a first-price sealed-bid auction with a buy price. If a bidder exercises the buy price if and only if $v_i \geq v^*$ and rejects it otherwise, expected surplus from exercising the buy price is as in (1). Since a standard first-price auction occurs if no bidder exercises the buy price, it is well-known that bidders with $v_i \geq r$ bid according to

$$\beta_F(v_i) = r\frac{F(r)^{n-1}}{F(v_i)^{n-1}} + \int_r^{v_i} \frac{(n-1)xF(x)^{n-2}f(x)}{F(v_i)^{n-1}}dx \tag{5}$$

in a symmetric equilibrium. Thus, expected surplus from the auction is

$$\begin{aligned} \pi^A(v_i, v^*) &= (v_i - \beta_F(\min\{v_i, v^*\}))F(\min\{v_i, v^*\})^{n-1} \\ &= (v_i - \min\{v_i, v^*\})F(\min\{v_i, v^*\})^{n-1} + \int_r^{\min\{v_i,v^*\}} F(x)^{n-1}dx, \end{aligned} \tag{6}$$

which, substituting (5) and again using integration by parts to obtain the second equality, is identical to (2). Therefore, a bidder's threshold buy price satisfies (3) as in a second-price auction with a buy price. Arguments similar to those in the proof of Proposition 1 establish the existence of a (unique) symmetric equilibrium in which a bidder with valuation v_i exercises the buy price if and only if $B^* < B(v_i, r)$ and bids according to (5) in the auction.

Given bidder behavior, the seller's expected utility takes the form

$$\begin{aligned} U(r, v^*) &= F(r)^n(0 + \delta(0 - \sigma)) + \int_r^{v^*} (\beta_F(x) + \delta(\beta_F(x) - \sigma))\, nF(x)^{n-1}f(x)dx \\ &\quad +(1 - F(v^*)^n)(B(v^*, r) + \delta(B(v^*, r) - \sigma)) \\ &= -F(r)^n\delta\sigma + nF(r)^{n-1}(F(v^*) - F(r))(r + \delta(r - \sigma)) \\ &\quad + \int_r^{v^*} (y + \delta(y - \sigma))n(n-1)F(y)^{n-2}f(y)(F(v^*) - F(y))dy \\ &\quad +(1 - F(v^*)^n)(B(v^*, r) + \delta(B(v^*, r) - \sigma)), \end{aligned} \tag{7}$$

which, substituting (5) and again exchanging the order of integration of the double integral to obtain the second equality, is identical to (4). □

4 Conclusion

I construct and study a model of an auction with a buy price for a seller with aspirations in the form of a reference price that depends upon the auction's reserve price and buy price. Such a seller does best to keep her aspirations sufficiently low by designing a no-reserve auction with a buy price low enough that some bidder types would exercise it with positive probability in equilibrium. Further, the seller is indifferent between the auction component of her mechanism being a first- or second-price auction.

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