The Extended Atkinson Family and Changes in the Expenditure Distribution: Spain 1973/74-2003

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The Extended Atkinson Family and Changes in the Expenditure Distribution: Spain 1973/74-2003

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This paper emphasizes the properties of a family of inequality measures which extends the Atkinson indices and is axiomatically characterized by a multiplicative decomposition property where the within-group component is a generalized weighted mean with weights summing exactly to 1. This family contains canonical forms of all aggregative inequality measures, each bounded above by 1, has a useful and intuitive geometric interpretation and provides an alternative dominance criterion for ordering distributions in terms of inequality.

Taking the Spanish Household Budget Surveys (HBS) for 1973/74, 1980/81, and 1990/91 and the more recent Continuous HBS for 2003, we show the advantages and possibilities of this extended family in regard to completing and detailing information in studies of inequality focussing on the tails of the distribution and on the changes in the distribution when the population is partitioned into population subgroups.

Keywords: inequality measurement, Atkinson indices.
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Introduction

Apart from the Gini coefficient, two families of relative inequality indices have been widely used in the literature for measuring inequality in a population: the Generalised Entropy Family (henceforth, the GEF) (among others, Bourguignon, 1979; Shorrocks, 1980; 1984; Cowell, 1980; Cowell and Kuga, 1981a;b; Russell, 1985; and the Atkinson Family (henceforth, the AF) Atkinson, 1970).

Specific properties of each family have afforded each one a certain following. Two properties are the main attractions of the GEF. Firstly, every aggregative inequality measure may be generated as an increasing transformation of a member of this family (Shorrocks, 1984). Secondly, the cardinalization of these indices enables decomposing overall inequality in a population split into groups as the sum of the between- and within-group inequality terms. In fact this decomposition property characterises the members of this family (Bourguignon, 1979; Shorrocks, 1980; Cowell, 1980; and Russell, 1985).

On the other hand, the AF, as is well-known, has been derived from a normative approach and is based on social value judgements. As a consequence, the cardinalization of each member of this family can be interpreted in terms of the welfare waste because of the inequality. Moreover, the AF is ordinally equivalent to one subset of the GEF. Lasso de la Vega and Urrutia (2008) provide a natural extension of the AF, which they called the Extended Atkinson Family (henceforth, the EAF), so that each member of the GEF can be increasingly transformed into one of this extended family. Hence, the EAF contains alternative canonical forms of all aggregative measures, each bounded above by 1. The family has a useful and intuitive geometric interpretation. In addition, every measure in this class fulfils a multiplicative decomposition which has the following features:

i. the between-group component is, according to the traditional approach, the equality level of a hypothetical distribution in which each person's income is replaced by the mean income of his/her group;

ii. the within-group component is a generalised weighted mean of the group equality levels, where the weights depend on their aggregated characteristics and their sum is 1;

iii. overall equality is the product of the within- and between-group equality terms.

In fact, the EAF is essentially the only class of continuous multiplicatively decomposable measures (Lasso de la Vega and Urrutia, 2008).[2]

Following the traditional approach in this paper, we show that the new measures of the EAF can also be interpreted from a normative approach in terms of welfare waste, in a way similar to the interpretation given to the traditional AF.

Using the EAF, we take a new and original approach to ranking income distributions and measuring inequality in Spain over the period 1973/74-2003. Lorenz dominance and second-order stochastic dominance are considered as appropriate procedures for deciding whether one distribution is unambiguously less unequal than another, as long as one subscribes to the principle of transfers. When the Lorenz curves intersect, we propose the use of this family as a tool for ordering distributions in terms of inequality in the class of aggregative measures. Moreover, this procedure not only complements the information given by the Lorenz curve, but also provides a neater representation at the tails, and since smoothing is not required, as, for example, in nonparametric density estimations, the picture that emerges at the extremes of the distribution is not distorted by statistical procedures. Thus, this approach allows us to pay particular attention to different parts of the distribution, while studying income inequality. Next, we study trends in inequality. Since such a family is bounded for every parameter value, with bounds that can be interpreted in terms of the tails of the distribution, we can see immediately whether the evolution of inequality is driven by movements at the bottom or at the top of the distribution. Finally, an analysis in terms of subgroups of the population is also conducted, focusing on how changes in between- and within-group equality affect overall equality.

The rest of the paper is structured as follows. The next section presents the notation and the basic definitions used in the paper. In the following section, we introduce the single parameter family of inequality measures which extends the AF and discuss its properties. Next, the issues are illustrated in the empirical application mentioned. A final section offers some concluding remarks.

Notation and Definitions

We consider a population consisting of $n \geq 2$ individuals. Individual i's income is denoted by $y_i \in R_{++} = (0, \infty)$, $i = 1,2,\ldots,n$. An income distribution is represented by a vector $\mathbf{y} = (y_1, y_2, ..., y_n) \in R^n_{++}$. We let $D = \bigcup_{n=1}^{\infty} R^n_{++}$ represent the set of all finite dimensional income distributions and denote the mean and population size of any $\mathbf{y} \in D$ by $m(\mathbf{y})$ and $n(\mathbf{y})$, (or m and n if there is no room for confusion) respectively.

We say that distribution $\mathbf{x} \in D$ is a *permutation* of $\mathbf{y} \in D$, if $\mathbf{x} = \prod \mathbf{y}$ for some permutation matrix P; that $\mathbf{x}$ is an *m-replication* of $\mathbf{y}$, if $\mathbf{x} = (\mathbf{y},\mathbf{y},\ldots,\mathbf{y})$ and $n(\mathbf{x}) = m.n(\mathbf{y})$ for some positive integer m; and that $\mathbf{x}$ is obtained from $\mathbf{y}$ by a *progressive transfer*, if, for some i and j with $x_i \leq x_j$ we have x_i - $y_i = y_j - x_j > 0$, while for all $k \neq i, j$ we have $x_k = y_k$. We use the vector $\bar{\mathbf{y}}$ to signify the equalised version of $\mathbf{y}$, defined by $n(\bar{\mathbf{y}}) = n(\mathbf{y})$ and $\bar{y}_i = \mu(\mathbf{y})$ for all $i = 1,2,\ldots,n(\mathbf{y})$.

In this paper, we assume that a relative inequality measure I is a real-valued continuous function $I : D \rightarrow R$, which fulfils the following properties:

Property I. *Symmetry.* $I(\mathbf{x}) = I(\mathbf{y})$, whenever $\mathbf{x}$ is a *permutation* of $\mathbf{y}$.

Property II. *Pigou-Dalton Transfers Principle.* $I(\mathbf{x}) < I(\mathbf{y})$, whenever $\mathbf{x}$ is obtained from $\mathbf{y}$ by a *progressive transfer*.

Property III. *Normalisation.* $I(\bar{\mathbf{y}}) = 0$ for all $\mathbf{y} \in D$. Otherwise, $I(\mathbf{y}) > 0$.

Property IV. *Replication Invariance.* $I(\mathbf{x}) = I(\mathbf{y})$, whenever $\mathbf{x}$ is a *replication* of $\mathbf{y}$.

Property V. *Scale Invariance.* $I(\lambda\mathbf{y}) = I(\mathbf{y})$ for all $\lambda > 0$.

We define the equality index as $E(\mathbf{y}) = 1 - I(\mathbf{y})$.

Suppose that the population of n individuals is split into $J \geq 2$ mutually exclusive subgroups with income distribution $\mathbf{y}^j = (y_1^j, y_2^j, ..., y_{n_j}^j)$, mean incomes $\mu_j = \mu(\mathbf{y}^j)$, and population sizes $n_j = n(\mathbf{y}^j)$ for all $j = 1,2,\ldots,J$. Let inequality and equality in group j be written $I_j = I(\mathbf{y}^j)$ and $E_j = E(\mathbf{y}^j)$. Let p_j and s_j be the respective population and income shares of the subgroup j.

Shorrocks (1984) introduced the following property for any partitioning of the population into exhaustive and disjoint subgroups:

Property VI. *Aggregative Principle.* (Shorrocks, 1984) An inequality index I will be said to be aggregative if there exits an "aggregator" function Q such that

$$I(\mathbf{x},\mathbf{y}) = Q\left(I(\mathbf{x}), I(\mathbf{y}), \mu(\mathbf{x}), \mu(\mathbf{y}), n(\mathbf{x}), n(\mathbf{y})\right)$$

for all $\mathbf{x},\mathbf{y} \in D$, where $Q(.)$ is continuous and strictly increasing in its first two arguments.

Finally, in the sequel p-order means will play a role. We use $m_p(\mathbf{y})$ to represent the mean of the order p, i.e.,:

$$m_p(\mathbf{y}) = \begin{cases} \left(\frac{1}{n}\sum_{i=1}^{n} y_i^p\right)^{\frac{1}{p}}, & p \in R, p \neq 0 \\ \left(\prod_{i=1}^{n} y_i\right)^{\frac{1}{n}}, & p = 0, \end{cases}$$

where, in particular, $m_1(\mathbf{y})$ is the arithmetic mean, $m(\mathbf{y})$, and $m_0(\mathbf{y})$ is the geometric mean. The mapping $p \rightarrow m_p$ is a non-decreasing, continuous function on all of R. The limiting case at one extreme is as $p \rightarrow -\infty$, giving $m_p(\mathbf{y}) \rightarrow \min\{y_i\}_{i=1}^n$. At the other extreme, as $p \rightarrow \infty$, giving $m_p(\mathbf{y}) \rightarrow \max\{y_i\}_{i=1}^n$. Moreover, for a given p, m_p is non decreasing in the vector $\mathbf{y}$ components, and concave for $p \leq 1$ and convex for $p \geq 1$.[3]

The Extended Atkinson Family

As already mentioned, two families of relative inequality measures are widely used in the literature. The GEF is given by:

$$I_{\alpha}^{GE}(\mathbf{y}) = \begin{cases} \frac{1}{n} \cdot \frac{1}{\alpha^2 - \alpha} \sum_{i=1}^{n} \left[\left(\frac{y_i}{\mu} \right)^{\alpha} - 1 \right], & \alpha \in R, \alpha \neq 0, 1 \\ -\frac{1}{n} \sum_{i=1}^{n} \log \frac{y_i}{\mu} & , \alpha = 0 \\ \frac{1}{n} \sum_{i=1}^{n} \frac{y_i}{\mu} \log \frac{y_i}{\mu} & , \alpha = 1^4 \end{cases} \tag{1}$$

The GEF includes the mean logarithmic deviation, when α is equal to 0 and the entropy inequality measure, when α is equal to 1, both introduced by Theil (1967) and known as Theil's two indices.

A relevant result of Shorrocks (1984) identifies the whole class of aggregative inequality indices as just increasing transformations of the members of the GEF. He establishes that any measure fulfils the Aggregative Principle if and only if it is an increasing transformation of one of the GEF, and therefore they are ordinally equivalent, in the sense that both of them order distributions in terms of inequality in the same way. Somehow the members of the GEF become canonical forms for all aggregative inequality measures.

Moreover, the members of this family are the only additively decomposable measures, in the sense that overall inequality can be decomposed as the sum of the between component and a weighted sum of the subgroup inequality levels (Bourguignon, 1979; Shorrocks, 1980; Cowell, 1980; and Russell, 1985).[5]

The other prominent class of inequality measures is the AF (Atkinson, 1970),[6] given by

$$I_{\alpha}^{A}(\mathbf{y}) = \begin{cases} 1 - \left[\frac{1}{n} \sum_{i=1}^{n} \left(\frac{y_i}{\mu} \right)^{\alpha} \right]^{\frac{1}{\alpha}}, & \alpha < 1, \alpha \neq 0 \\ 1 - \prod_{i=1}^{n} \left(\frac{y_i}{\mu} \right)^{\frac{1}{n}}, & \alpha = 0^7 \end{cases} \tag{2}$$

Firstly, it is noteworthy that, whereas the GEF is defined for all real values of the α parameter, (i.e., it has two tails), the AF makes sense only for parameter values less than 1, (i.e., it has only one tail). It may be interesting to note that, as is well-known, the AF is ordinally equivalent to one tail of the GEF, that with the same values for the α parameter.

Two questions arise almost simultaneously. On the one hand, what happens with the other tail? On the other hand, if they order distributions in the same way, what is the contribution of the AF to measuring inequality?

Definitely, two drawbacks to the AF with respect to the GEF may be argued. Firstly, it is not possible to generate all the aggregative indices from them. Secondly, in empirical applications when the population is split into groups, they do not allow us to know and analyse the contribution to the inequality of the between- and within-group components.

By contrast, the main advantage of the measures of the AF rests on the facts that they have been derived from a normative approach and the underlying social welfare functions have been axiomatically characterised (Atkinson, 1970). As a consequence, because it is normatively significant, the numerical value of any Atkinson index has clear meaning: it is associated to the Equally Distributed Equivalent (henceforth, EDE) income, a particular cardinalization of the social welfare function. The EDE income is the income level that, if equally distributed among the population, would give us the same level of social welfare as the present distribution. Hence, once the inequality aversion level has been chosen through the α parameter, any Atkinson index is the percentage of total income that could be discarded if the remaining amount were distributed equally. An index value of 0.4, for instance, means that, if incomes were equally distributed, 40 per cent of total income would be wasted, in the sense that we would need only 60 per cent of the present total income to achieve the same level of social welfare.

Basic properties of the Extended Atkinson Family

Now we focus on the EAF given by (Lasso de la Vega and Urrutia, 2008):

$$I_\alpha(\mathbf{y}) = \begin{cases} 1-\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i}{\mu}\right)^{\alpha}\right]^{\frac{1}{\alpha}} & ,\alpha<1, \alpha\neq 0 \\ 1-\left(\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i}{\mu}\right)^{\alpha}\right]^{\frac{1}{\alpha}}\right)^{-1}, & \alpha>1 \\ 1-\prod_{i=1}^{n}\left(\frac{y_i}{\mu}\right)^{\frac{1}{n}}, & \alpha=0 \\ 1-\prod_{i=1}^{n}\left(\frac{\mu}{y_i}\right)^{\frac{y_i}{n\mu}}, & \alpha=1^{8} \end{cases} \tag{3}$$

This family is actually a single parameter class of relative inequality measures defined for all $\alpha \in R$; it has two tails, one of which, when $\alpha < 1$, is the AF. Every member of this family meets the Aggregative Principle and is bounded above by 1. Hence, after Shorrocks (1984), each member of this family is an increasing transformation of one member of the GEF, and not only that but the converse is also true. In other words, whereas the AF is ordinally equivalent to one tail of the GEF, this family is ordinally equivalent to the whole GEF. As a result, now it is possible to generate all the aggregative indices as increasing transformations of the members of this family, which can be used as alternative canonical forms of all aggregative inequality measures.

On the other hand, each member of this family fulfils a multiplicative decomposition. In fact, the following theorem shows that the measures of the EAF permit an alternative decomposition into the between- and the within-group equality components.[9]

***Theorem**: Consider any exhaustive collection of mutually exclusive population subgroups* $j = 1,2,...,J$. *For each* $\alpha \in R$, $I_\alpha(\mathbf{y})$ *verifies the following multiplicative* *decomposition in terms of equality indices:*

$$E_\alpha(\mathbf{y}) = \begin{cases} \left(\sum_{j=1}^{J}\omega_j\left[E_\alpha(\mathbf{y}^j)\right]^{\alpha}\right)^{\frac{1}{\alpha}} E_\alpha\left(\bar{\mathbf{y}}^1,\bar{\mathbf{y}}^2,\ldots,\bar{\mathbf{y}}^J\right), & \alpha<1, \alpha\neq 0 \\ \left(\sum_{j=1}^{J}\omega_j\left[E_\alpha(\mathbf{y}^j)\right]^{-\alpha}\right)^{-\frac{1}{\alpha}} E_\alpha\left(\bar{\mathbf{y}}^1,\bar{\mathbf{y}}^2,\ldots,\bar{\mathbf{y}}^J\right), & \alpha>1 \\ \prod_{j=1}^{J}\left[E_\alpha(\mathbf{y}^j)\right]^{\omega_j} E_\alpha\left(\bar{\mathbf{y}}^1,\bar{\mathbf{y}}^2,\ldots,\bar{\mathbf{y}}^J\right), & \alpha=0,1 \end{cases} \quad (4)$$

with weights $\omega_j = \frac{p_j^{1-\alpha}s_j^{\alpha}}{\sum_{j=1}^{J}p_j^{1-\alpha}s_j^{\alpha}}$ for all α.

The second term on the right-hand side of these equations is the equality level of a hypothetical distribution in which each person's income is replaced by the mean income of his/her subgroup and may be considered the between-group equality component according to the traditional approach.

With respect to the first term on the right-hand side of these equations, there are three possibilities:

i. when α is less than 1 and different from 0, the term is the α-order (weighted) mean of the group equalities;

ii. when α is greater than 1, it is the (-α)-order (weighted) mean of the group equalities;

iii. when α is equal to 0 or to 1, it is the geometric (weighted) mean of the group equalities.

In all cases, this term summarizes equality within the population subgroups and may be considered the within-group equality component.

The decomposition coefficients for these indices are functions of the subgroup means and population sizes, and their sum is equal to 1. In fact, they are the same as in the additive decomposition fulfilled by the GEF but normalised. The fact that the sum of these decomposition coefficients equals 1 is a very important characteristic, if we want to attach a meaning to the numerical value of the within-group component. In a situation in which the equality level in all the groups is exactly the same, the within-group equality is exactly that value, only if the sum of the decomposition coefficients is equal to 1, which seems reasonable. For the GEF, only the decomposition coefficients corresponding to Theil's two indices fulfil this property. As a consequence, even if the group inequality levels in every group are exactly the same, the traditional within-group component, except for Theil's two indices, is bound to be different from that common value.

Summing up, the Theorem shows that for each α parameter value, the overall equality as measured by the EAF can be decomposed as the product of the between-group equality component multiplied by the within-group equality component.[10]

Moreover, the multiplicative decomposition allows us to evaluate the impact of marginal changes from a given group on overall equality. Indeed the multiplicative decomposition of these indices can be transformed through the logarithmic transformation, so that it is additive in logs. Denoting as $E_{W\alpha}$ (**y**) the within-group component in Equations 4, namely, the first term on the right-hand side of these equations, and $E_{B\alpha}$ (**y**) the between-group component in Equations 4, namely, $E_{B\alpha} = E_{\alpha}\left(\bar{\mathbf{y}}^1, \bar{\mathbf{y}}^2, \ldots, \bar{\mathbf{y}}^J\right)$, and then taking logs in Equations 4, we find that log$E$ α (**y**) = log$E_{W\alpha}$(**y**) + log$E_{B\alpha}$ (**y**). So, using the fact that a percentage change in x, $\%\Delta x$, can be written as $\%\Delta x \approx 100.\ \Delta$ log(x) for small changes in x we find that:

$$\%\Delta E_{\alpha}(\mathrm{y}) \approx \%\Delta E_{W\alpha}(\mathrm{y}) + \%\Delta E_{B\alpha}(\mathrm{y}) \tag{5}$$

Equation 5 shows that the overall percentage of change in equality can be expressed as the sum of the percentage changes in the within- and the between-group components. Note that this analysis cannot be carried out with additive decompositions, where the available decompositions for changes rely on approximations (Theil and Sorooshian, 1979).

Now, the two initial drawbacks of the AF with respect to the GEF have been got over. On the one hand, we have derived another tail which allows us to extend the AF and generate all the aggregative indices as increasing transformations of the new family. On the other hand, multiplicative decomposition enables us to analyse the contribution of the changes of the between- and within-group equality to overall equality.

Some more properties of the inequality measures belonging to the new tail

Although the two tails in the EAF have similar characteristics, there are significant differences between them. There is therefore a need for a more detailed analysis of the new measures of this family.

To begin with, the Atkinson tail, i.e., when α is less than 1, satisfies the transfer sensitivity principle, according to Shorrocks and Foster (1987), which ensures that more weight in the inequality assessment is attached to transfers taking place lower down in the distribution, whereas the tail we have derived contains specific measures sensitive to high incomes. Therefore, this new tail, attaching greater importance to income changes among the more affluent people, seems to have a fundamental weakness. Here, with the following example, it is illustrated that, far from it, they actually complete the information about the inequality income comparisons.

Consider the income distribution **y**=: (1, 3, 5, 11, 30). Assume that 4 additional units of income have to be distributed among the individuals of the society and assume that we increase the lowest income by one and the highest income by three, so that the new distribution is **y'**=: (2, 3, 5, 11, 33). For while increasing the lowest income reduces inequality and increasing the highest income increases inequality, the magnitude of these two effects is not clear.[11] Perhaps not everybody would agree that this sharing of the extra income has been made in a way that reduces inequality. Nevertheless, this is the conclusion derived from using all the members of the AF. As regards most of the members of the other tail in the extended family, for $\alpha > 1.3$, the result is the opposite and in this case the conclusive verdict is that inequality has risen. It is clear that in this case, the AF, on its own, does not enable us to order these two distributions according to all inequality perceptions and that the information provided by this family can be completed using the new one.

In other words, it seems universally accepted that for measuring inequality we should focus on the situation of the worst-off and in consequence make use of measures sensitive to transfers among the poorest. This is not, however, the whole truth. This example directs attention towards the implications of using only measures attaching more weight to the lowest incomes, without taking into consideration the injustice of extremely high incomes. In this sense, the EAF, which contains specific measures sensitive to both high and low incomes may be of great interest, if we are really concerned about reducing inequality and improving the lowest incomes.

On the other hand, this new tail seems to have another noteworthy disadvantage. Whereas the AF has been derived from a normative approach, the new tail has been derived from an axiomatic framework. Since it is always appealing for an inequality measure to be consistent with reasonable social evaluation functions, we propose to follow the conventional approach (among others, Sen, 1973 and Blackorby and Donaldson, 1978) to associate every relative inequality index, I, to a social welfare function $W : D \rightarrow R$, according to the standard following expression:

$$W(\mathrm{y}) = \mu(\mathrm{y})\,(1 - I(\mathrm{y}))\,, \tag{6}$$

where $W(\mathbf{y})$, according to Blackorby and Donaldson (1978), is a continuous, symmetric, S-concave and homothetic function. For all the measures in the extended family, this value is always positive and less than the mean income. It can be considered the mean income adjusted by the inequality. Now, following the Atkinson framework, we call this cardinalization of the social evaluation, the EDE income, i.e., the income level that, if equally distributed, would give the same level of social welfare as the present distribution. In other words,

$$W(\mathbf{y}) = EDE(\mathbf{y}) = \mu(\mathbf{y})\,(1 - I(\mathbf{y})) \tag{7}$$

From Equation 7, we get as usual:

$$I(\mathbf{y}) = 1 - \frac{EDE(\mathbf{y})}{\mu(\mathbf{y})} \tag{8}$$

Therefore, the meaning of the numerical value of any index of the extended family according to Equation 8 is the same as that of the AF: the percentage of total income that is wasted if the remainder were equally distributed.

Before finishing this section, we would like to investigate a little further the social evaluation given in Equation 6 for the new measures in the extended family. Whereas the social welfare evaluation for the members of the new tail is always an increasing function of the mean income and a decreasing function of the inequality, properties basically assumed in the context of the social welfare, they are not always increasing in the individual incomes. As a consequence, an extra amount of income will always produce an increase in the mean income, but, as regards inequality, it may increase or decrease depending on who gets the extra income. In fact, if the receivers are the richest people, inequality is bound to increase. In this context, there exist situations in which welfare is going to decrease. Consequently the social welfare functions corresponding to the members of the new tail can not be specified from a utilitarian framework, in the sense that they can not be written as a weighted sum of the individual utility functions.

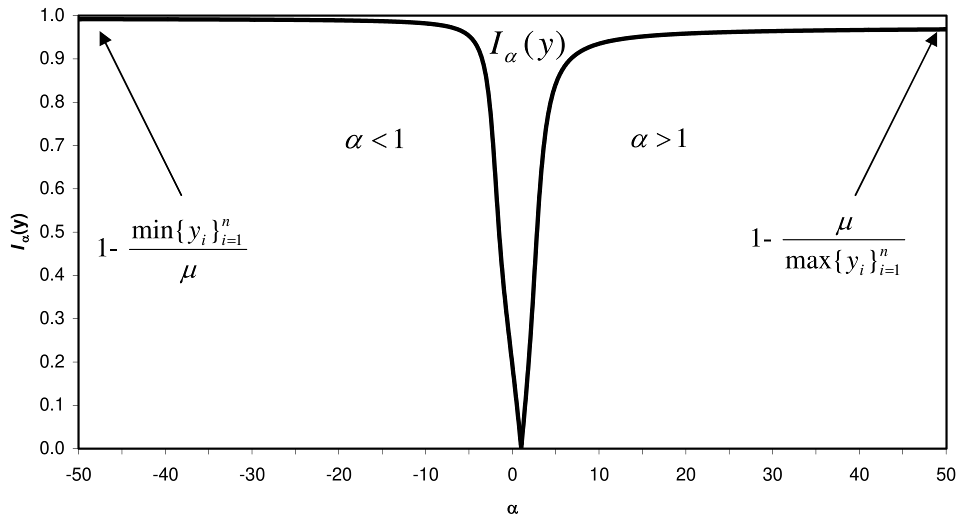
We are, however, by no means worried by this fact. On the contrary, we think that this new tail, more sensitive to transfers between the rich people, is better able to capture the intuitions underlying the criticisms to the utilitarian approach, as Sen (1973) stressed: "It is fairly restrictive to think that social welfare is a sum of individual welfare components", and that one might feel that "the social value of the welfare of the individuals should depend crucially on the levels of welfare of others". In conclusion, we are able to choose specific measures among the members of the EAF, which allows us to answer the main question, which is "whether one person's utility enters into another person's utility in a positive way or in a negative way",... since "surprisingly not everyone is a Paretian" (Podder, 1998).

α *(y)-curve dominance*

Another interesting characteristic of the EAF is that the inequality value $I_\alpha(\mathbf{y})$ varies continuously as a function of the α parameter at any given income distribution $\mathbf{y}$, except for $\alpha = 1$, leading to the I_α(y)-curve of Figure 1.[12] As already mentioned, the α parameter is a measure of the degree of relative sensitivity to transfers at different income levels. As α increases $I_\alpha(\mathbf{y})$ becomes more sensitive to transfers at the upper end of the distribution. The limiting case at one extreme is as $\alpha \to \infty$, giving $I_\alpha(\mathbf{y}) \to 1 - \frac{\mu}{\max\{y_i\}_{i=1}^n}$, which only considers the income of the richest income recipient. By contrast, as α decreases, this family becomes more sensitive to transfers at the lower end of the distribution. The limiting case at the other extreme is as $\alpha \to$ -∞, giving $I_\alpha(\mathbf{y}) \to 1 - \frac{\min\{y_i\}_{i=1}^n}{\mu}$, which only takes account of the income of the poorest income recipient.

Since different inequality indices embrace different perceptions of inequality, they may rank pairs of distributions in contradictory ways, and therefore we are not

Figure 1
I_α(y) **as a function of the** α **parameter**



to place too much reliance on a single inequality measure. It is certainly better to establish conditions which characterise unanimous agreement among the inequality indices in a given class.

It is well known that the Lorenz dominance condition, popularised by Kolm (1969), Atkinson (1970), and Sen (1973), characterises unanimous agreement among all indices of relative inequality consistent with the Pigou-Dalton principle. All indices in this family unanimously agree on their inequality ordering of a pair of distributions, when their Lorenz curves do not cross. When the Lorenz curves intersect, unanimous agreement may also be achieved with various subsets of this family. In this respect, there exist in the literature some results for some classes of inequality indices. As regards the class of transfer sensitive inequality measures, two different criteria are provided to characterise unanimous agreement among these inequality measures. Whereas Shorrocks and Foster (1987) use third order stochastic dominance, Davies and Hoy (1995) provide a simple condition based on checking the variance of the distributions applied successively to every cumulated population share in which the Lorenz curves intersect.

A similar procedure to the Lorenz curves is provided in Lasso de la Vega and Urrutia (2008) to characterise unanimous agreement among the class of aggregative inequality measures, using the $I_\alpha(\mathbf{y})$-curves. When two distributions have non-intersecting $I_\alpha(\mathbf{y})$-curves and the curve of one distribution lies everywhere above the curve of the other, the former displays unambiguously more inequality as measured by the EAF. Since, however, the members of this family, as mentioned above, can be considered canonical forms of all aggregative inequality indices, this conclusion holds when inequality is measured by any aggregative inequality index. This unanimous ordering proves to be operationally meaningful, since the class of aggregative indices contains infinite elements, and looking for unanimity in this class involves an infinity of pairwise comparisons. Bearing in mind that the aggregative principle has often been invoked for measuring inequality in a population split into groups, $I_\alpha(\mathbf{y})$-curves provide a powerful tool in this respect.

Since the aggregative principle imposes an additional restriction to the Pigou-Dalton principle, conclusive rankings can be obtained in situations where the Lorenz dominance fails. Therefore, this procedure enables distributions whose Lorenz curves intersect to be conclusively ranked among the class of aggregative measures.

In another respect, in situations where the $I_\alpha(\mathbf{y})$-curves intersect, since the normative judgements associated with the values of α are explicit, some information may also be obtained. If the $I_\alpha(\mathbf{y})$-curve of one distribution crosses that of another from above on the left-hand side of the graph, the first distribution is more unequal than the second according to large negative values of α, which are sensitive to the incomes of the people who are the worst off. Conversely, when the curves intersect on the right-hand side, with α large but positive, the rule would be particularly sensitive to the incomes of the richest people. Therefore $I_\alpha(\mathbf{y})$-curves allow us to make inequality comparisons, giving a fuller description of differences in inequality.

Changes in the expenditure distribution: Spain 1973/74-2003

Let us use the foregoing approach to analyze the income distribution in Spain for the period 1973/74 to 2003 using expenditure as a proxy variable. We use the Household Budget Surveys (HBS) from 1973/74, 1980/81 and 1990/91, as well as the last available year from the Continuous Household Budget Survey, 2003, carried out by the Spanish Statistical Institute (*Instituto Nacional del Estadística* - INE).[13] All of them are representative at a regional level, namely, NUTS 2 regions (within the Nomenclature of Territorial Units for Statistics, regions with a minimum population of 800,000 and a maximum of 3 million)(see Appendix for the list of regions).[14] As the variable representative of the standard of living, we use *per capita* total expenditure,[15] defined as monetary expenditure plus non-monetary expenditure arising from self-consumption, self-supply, free meals, in-kind salary, and imputed rents for house ownership.[16] Total expenditure *per capita* is assigned to every person in the household, so even if the household is the basic statistical unit, person weights are applied to the calculations, since we are mainly concerned with the economic well-being of individuals. We have checked that our conclusions are mostly robust to economies of scale using the square root of the household size as the equivalence scale (Atkinson, Rainwater, and Smeeding, 1995), since the average household size has decreased from 3.7 members in 1973/74 to 3.0 in 2003.

Focusing on different parts of the income distribution

In Table 1, we present the generally accepted view about the evolution of inequality in the income distribution in Spain over the period of analysis (Goerlich and Mas, 2001; 2002; 2004; and Oliver-Alonso, Ramos and Raymond-Bara, 2001). We offer a plethora of inequality indices from the $I_{\alpha}^{GE}(\mathbf{y})$ and $I_{\alpha}(\mathbf{y})$ families, as well as the popular Gini (1912) index. Generally speaking all of them tell the same story. There is a continuous negative trend in inequality for the period 1973/74-1990/91 – the magnitude of the change depends, of course, on the particular index, but the general tendency seems to be clear – and a more or less stable distribution in the last period, 1990/91-2003. Contrary to what has happened in other developed countries, Spain has not experienced an increase in income inequality in recent years; instead the income distribution seems to have been stable in the last period. This is a robust fact from the examination of the data for the end of the 1990s and the beginning of this decade, using alternative definitions of income and equivalence scales (Aldás, Goerlich, and Mas, 2006; 2007).

Table 1
Inequality Indexes - Total Expenditure

	Gini	**Extended Atkinson** α						
		-2	**-1**	**0**	**0.5**	**1**	**2**	**3**
1973/74	0.340	0.476	0.323	0.177	0.094	0.185	0.202	0.411
	(0.002)	(0.015)	(0.003)	(0.003)	(0.002)	(0.003)	(0.008)	(0.022)
1980/81	0.333	0.462	0.317	0.171	0.090	0.174	0.185	0.370
	(0.002)	(0.011)	(0.004)	(0.002)	(0.001)	(0.003)	(0.005)	(0.020)
1990/91	0.316	0.417	0.284	0.154	0.081	0.159	0.169	0.336
	(0.003)	(0.012)	(0.004)	(0.003)	(0.002)	(0.004)	(0.007)	(0.020)
2003	0.317	0.367	0.269	0.153	0.083	0.166	0.184	0.364
	(0.003)	(0.006)	(0.004)	(0.003)	(0.002)	(0.003)	(0.006)	(0.011)
	Gini	**Generalized Entropy** α						
		-2	**-1**	**0**	**0.5**	**1**	**2**	**3**
1973/74	0.340	0.441	0.239	0.195	0.194	0.204	0.285	0.648
	(0.002)	(0.040)	(0.004)	(0.003)	(0.003)	(0.005)	(0.015)	(0.105)
1980/81	0.333	0.409	0.232	0.187	0.184	0.192	0.252	0.501
	(0.002)	(0.023)	(0.005)	(0.003)	(0.002)	(0.003)	(0.009)	(0.063)
1990/91	0.316	0.323	0.198	0.167	0.165	0.173	0.224	0.403
	(0.003)	(0.023)	(0.004)	(0.003)	(0.003)	(0.004)	(0.012)	(0.052)
2003	0.317	0.249	0.184	0.166	0.169	0.181	0.250	0.480
	(0.003)	(0.008)	(0.003)	(0.003)	(0.003)	(0.004)	(0.010)	(0.034)

Note: Simple bootstrap standard erros (Efron and Tibshirani (1993)) in brackets.
Source: HBS 1973/74, 1980/81, 1990/91 and 2003. Atkinson (1970) family corresponds to values a $<$ 1.

Inequality indices balance in some way the different parts of the income distribution in order to provide a real number summarizing the whole distribution, and these parts may be pushing inequality in different directions. It may be of interest to try to look at both sides of the distribution and see how these affect the inequality index. This can be done quite easily by varying the α parameter, since this reflects the different weights attached by the index to different parts of the distribution. Even though this is true for both families, $I_\alpha^{GE}(\mathbf{y})$ and $I_\alpha(\mathbf{y})$, the bounded behaviour of $I_\alpha(\mathbf{y})$ makes the $I_\alpha(\mathbf{y})$-curves very handy for studying the tails of the distribution.

The tendencies shown in Table 1 are masking a rather different trend on either side of the distribution in the different periods covered by our study. The examination of the $I_\alpha(\mathbf{y})$-curves reveals this in a simple, graphical and intuitive form.

In Figure 2, we represent the $I_\alpha(\mathbf{y})$-curve for all the years covered and values of α in the range of -40 to +40. The EAF behaves as expected. Inequality is very high for large negative and positives values of α, but decreases steadily as α moves towards one and as the underlying inequality index becomes less and less sensitive to the extremes of the society.

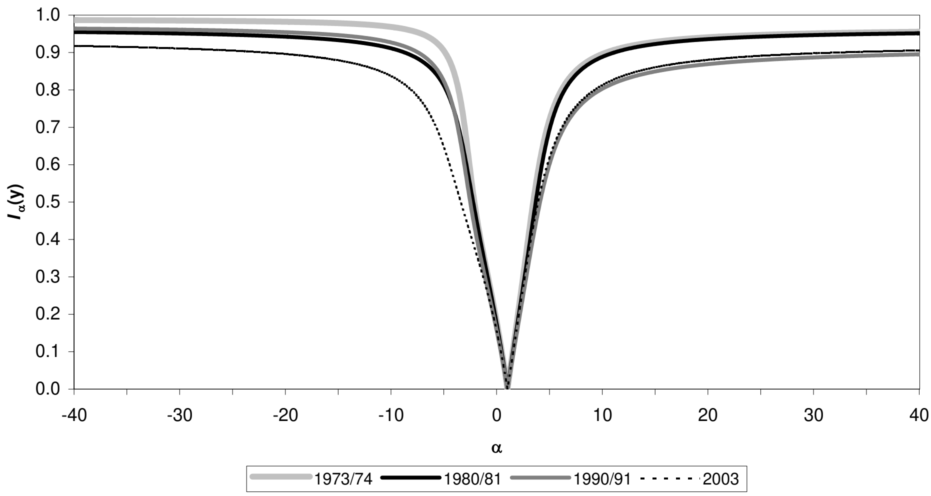
Another conclusion emerges clearly from Figure 2. For every value of the sensitivity parameter, the index value $I_\alpha(\mathbf{y})$ in 2003 is lower than its equivalent value in 1973/74, and thus the corresponding curves do not intersect. This suggests that there has been a redistribution of income from those who are better off to those who are worse off throughout Spain over this period, or that low incomes have grown faster than high incomes.[17]

This decrease in inequality is, however, not monotonic over the complete period at the different tails of the distribution. So the overall trend, shown in Table 1, is picking up conflicting tendencies at different parts of the income distribution. In particular, it is shown quite clearly how, between 1973/74 and 1980/81, inequality decreases according to negative values of α, indicating that the worse-off are approaching the mean income, the "middle class", whereas the change in the index in this period remains fairly small as α increases.

In the next period, from 1980/81 to 1990/91, just the opposite behaviour is detected. Inequality significantly decreases for positive values of α, so it is the richest part of the society that is approaching the average person, while for negative values of the parameter, the change in the index is small and in the direction of a slight increase in inequality. Hence the overall index, showing a reduction in inequality, is mainly picking up the distributional changes in the higher part of the distribution.

Eventually, for the last period the stability observed in the Gini, Theil, $I_1^{\mathrm{GE}}(\mathbf{y})$ and $I_0^{\mathrm{GE}}(\mathbf{y})$, or standard AF, $I_0(\mathbf{y})$, indices results from opposite tendencies at either side of the mean. We observe an important reduction in inequality for $\alpha < 1$, reflecting the catching-up of the poor, and a slight increase in inequality for $\alpha > 1$, which shows the tendency of the richest part of the society to move away from the "middle class". Note that this behaviour can be observed from Table 1, for values of α, such as -2 or 3, which move in opposite directions for either $I_\alpha(\mathbf{y})$ and

Figure 2
Inequality in Spain: I_α(y)-curves



$I_\alpha^{\mathrm{GE}}(\mathbf{y})$. So the apparent stability in the distribution in this last period is a result of conflicting tendencies at the different tails of the distribution.[18]

Overall, this is consistent with the view that, for the whole period, the reductions in inequality are not monotonic and come from different parts of the distribution.

Accounting for changes over time in income equality

Our next application involves the multiplicative decomposability property, as indicated in the Theorem above. This, however, involves switching the mind from the standard view of inequality to the complementary view of equality, $E(\mathbf{y}) = 1 - I(\mathbf{y})$.

Contrary to the additive decomposition of the GEF (Shorrocks, 1980), the decomposition shown in Equations 4 is not useful in a static context, but it can be very handy in a dynamic one, to account for changes in the equality index in terms of the changes inside the within- and between-group distributions.

As an example, consider two possible partitions of the population. In the first place, we consider a regional partition, in which we divide the population into to the 17 NUTS 2 regions, the political (*Comunidades Autónomas*) into which Spain is divided. So now the criterion is regional residence. In the static context, using I_α^{GE}(y), it is well known that the between-group term is much less important than the within-group term, and that this importance has been diminishing over the time of our period of study as a result of regional convergence (Goerlich and Mas, 2001). Hence, inequality is mainly in the personal rather than the regional distribution of income. In a dynamic context, it is not clear what the role of the demographic factors has been in altering the inequality and its contributions.

Secondly, we consider an educational partition according to the level of studies of the head of the family. We consider four groups: illiterates, those with a primary-school education, those with a secondary-school education, and those who have pursued university studies. The importance of these groups has changed a great deal in relative terms over the period of study. Access to higher education has increased the weight of the secondary-school and university-studies groups, while the other two groups have seen an important reduction. In this way we can examine how the changes in the distribution within each group have contributed to overall distributional changes.[19]

Before we present the numerical examples, let us consider a bit further the breakdown of the equality changes given by Equation 5, in the particular case in which $\alpha = 0$. For this value, the multiplicative decomposition of Equation 4 satisfies the path-independent property for the equality index, so the between- and the within-group components are independent (Foster and Shneyerov, 2000; and Lasso de la Vega and Urrutia, 2005) and the different contributions of the different groups to changes in overall equality are easy and intuitive to investigate.

Thus, Equation 5 for $\alpha = 0$ can be rewritten alternatively as:

$$\Delta \log E_0(\mathbf{y}) = \Delta \log E_{\omega 0}(\mathbf{y}) + \Delta \log E_{B0}(\mathbf{y}),$$

but, given that from Equation 4, $\log E_{\omega 0}(\mathbf{y}) = \Sigma_{j=1}^{J} p_j \log E_0(\mathbf{y}^j)$, we can eventually write[20],

$$\begin{aligned} \Delta \log E_0(\mathbf{y}) &= \Delta\left(\sum_{j=1}^{J} p_j \log E_0(\mathbf{y}^j)\right) + \Delta \log E_{B0}(\mathbf{y}) \\ &= \sum_{j=1}^{J} \left(\Delta\left(p_j \log E_0(\mathbf{y}^j)\right)\right) + \Delta \log E_{B0}(\mathbf{y}) \end{aligned}, \tag{9}$$

which shows the particular contribution of a given group to changes in the overall index.

These contributions depend, however, on changes in both the within-group equalities, $E_0(\mathbf{y}^j)$, and their population shares, p_j. We suggest further decomposing the change in the within-group component, $\Delta \log E_{W0}(\mathbf{y})$, into both effects by means of a *shift-share* analysis. Using this, we can write

$$\begin{aligned} \Delta\left(\sum_{j=1}^{J} p_j \log E_0(\mathbf{y}^j)\right) &= \sum_{j=1}^{J} \Delta p_j \left[\frac{\log E_0(\mathbf{y}^j)_t + \log E_0(\mathbf{y}^j)_{t-1}}{2}\right] \\ &+ \sum_{j=1}^{J} \left[\frac{p_{j,t} + p_{j,t-1}}{2}\right] \Delta \log E_0(\mathbf{y}^j) \end{aligned}, \tag{10}$$

where the subscript$t$ means a point in time and the first term on the right hand side of the equal sign in Equation 10 represents the contribution to within-group changes due to changes in the population shares (*demographic factors*), and the second term represents the contribution due to changes in the equality (*distributional factor*). These could be further disaggregated for each individual group.

The decomposition in Equation 5 and the within-group contributions, in the regional partition for $\alpha = 0$ are presented in Table 2, both by groups and by factors, for the period 1973/74-2003. The corresponding equality indices are obtained directly from Table 1 and the breakdown Equation 5 shows that about 60 per cent of the change in the equality during the period can be attributed to the within contribution. This is quantitatively similar to the changes in the period 1973/74-1990/91. Hence, even if the static decomposition of $I_0^{GE}(\mathrm{y})$ in the between- and within-group terms shows that the latter is more important and has gained importance over the period (Goerlich and Mas, 2004), the improvement in distributional terms comes mostly from the within-group distribution. Moreover, the contribution of the between-group term, which is non-negligible, reflects regional convergence.

If we further disaggregate the within-group term by looking at the contribution of the different groups, we discover that there is no single regional experience. Some regions appear to have a negative contribution to improvements in the overall distribution, *Madrid*,[21] *Canarias* and *Comunidad Valenciana*. Others, like *Illes Balears*, *Cantabria*, *Cataluña*, *Región de Murcia*, *Navarra* and *La Rioja*, show no contribution, either negative or positive. For the rest of the cases, there does not seem to be a general pattern, with important contributions, either from the richer, e.g.,*País Vasco*, or the poorer regions, e.g.,*Castilla y León* or *Andalucía*.

Looking at the contribution of the different factors, through Equation 10, we see that the improvements in the within-group distribution are driven solely by distributional changes and not by demographic factors; so we have a truly distributional effect.[22]

The decomposition in Equation 5 and the within-group contributions, either by groups or by factors, in the educational partition for $\alpha = 0$ is offered in Table 3, for the period 1973/74-2003. In this case, we have a reversal in the contributions to the changes in the distribution. From the between-group components comes a slightly higher contribution, which indicates the major convergence of the different groups. The within-group contribution accounts, however, for almost 47 per cent of the change in equality. So, both of them are in fact quite important.[23]

Further disaggregating the within-group term in Equation 9 shows that the lower-educated groups have contributed positively to the improvement in the distribution, while the higher-educated groups have contributed negatively. Moreover, these are again pure distributional changes, with little aggregate effect from the changes in the composition of the two groups.

Table 2

	Regional Partition		
	Global	**External**	**Internal**
Changes 1973/74-2003	2.93%	1.19%	1.75%
		40.42%	**59.58%**
Internal decomposition			
by group			
Andalucía			0.54%
Aragón			0.24%
Asturias (Principado de)			0.10%
Balears (Illes)			-0.05%
Canarias			-0.10%
Cantabria			0.01%
Castilla y León			0.48%
Castilla - La Mancha			0.19%
Cataluña			0.03%
Comunidad Valenciana			-0.10%
Extremadura			0.25%
Galicia			0.21%
Madrid (Comunidad de)			-0.23%
Murcia (Región de)			-0.08%
Navarra (Comunidad Foral de)			0.02%
País Vasco			0.23%
Rioja (La)			0.01%
Total			**1.75%**
by factor			
Demographic			-0.03%
Distributional			1.78%
Total			**1.75%**

Note: When the economies of scale mentioned in the text are used, the global change amounts to a 3.5%, with an external contrinution of 30.6% and an internal contribution of 69.4%. The internal decomposition, either by groups of by factors, is essentially unchanged.
Source: HBS 1973/74 and 2003.

Concluding remarks

This paper highlights the properties and the underlying possibilities of the EAF, a class of inequality measures which is axiomatically characterized by an alternative multiplicative decomposition property. We have presented the main theoretical points and have illustrated the use of this family with an application from the HBS for 1973/74, 1980/81, 1990/91, and 2003.

Firstly, we have essentially used the class of measures to show how the negative trend in inequality over the 1973/74-2003 period can be identified as coming from different parts of the distribution in the different periods analyzed and how the apparently stable distribution in the recent years is the result of two opposite trends: an improvement in the distribution from the bottom and a worsening from the top.

Secondly, we have conducted a decomposition analysis that breaks down the

Table 3

	Educational Partition		
	Global	**External**	**Internal**
Changes 1973/74-2003	2.93%	1.56%	1.37%
		53.24%	**46.76%**
Internal decomposition			
by group			
Illiterate			2.75%
Primary School			5.11%
Secondary School			-4.58%
University Studies			-1.91%
Total			**1.37%**
by factor			
Demographic			-0.03%
Distributional			1.40%
Total			**1.37%**

Note: When the economies of scale mentioned in the text are used, the global change amounts to an 3.5%, with an external contrinution of 38.9% and an internal contribution of 61.1%. The internal decomposition, either by groups of by factors, is essentially unchanged.
Source: HBS 1973/74 and 2003.

change in the equality index into a between- and a within-group component. Using two alternative partitions of the population, we show how the level of equality is increasing, mostly through the within-group component in a regional partition, but mainly through the between-group component in an educational partition.

Rank	**Name**	**Population**	**Percentage**
1	Andalusia	8.202.220	17,77%
2	Catalonia	7.364.078	15,95%
3	Madrid	6.271.638	13,59%
4	Valencia	5.029.601	10,90%
5	Galicia	2.784.169	6,03%
6	Castile and León	2.557.330	5,54%
7	Basque Country	2.157.112	4,67%
8	Canary Islands	2.075.968	4,50%
9	Castile-La Mancha	2.043.100	4,43%
10	Murcia	1.426.109	3,09%
11	Aragon	1.326.918	2,87%
12	Extremadura	1.097.744	2,38%
13	Asturias	1.080.138	2,34%
14	Balearic Islands	1.072.844	2,32%
15	Navarre	620.377	1,34%
16	Cantabria	582.138	1,26%
17	La Rioja	317.501	0,69%
18	Ceuta	77.389	0,17%
19	Melilla	71.448	0,15%
TOTAL		44,108,530	100%
87.38			

Notes

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[2] Blackorby, Donaldson and Auersperg (1981) present a multiplicative decomposition for the indices in the Atkinson-Kolm-Sen Family in terms of equality indices from a welfare-theory approach using subgroup Equally Distributed Equivalent (EDE) income levels to determine the between-group component of overall inequality. By contrast we retain the traditional "subgroup (arithmetic) mean income" approach to defining the between-group inequality.

[3] See, for example, Kendall and Stuart (1977), Magnus and Neudecker (1988), Bullen (2003), or Steele (2004).

[4] Note that for $\alpha \neq 0{,}1$, I_α^{GE} can be equivalently written as $I_\alpha^{GE}(\mathbf{y}) = \left[(m_\alpha/\mu)^\alpha - 1\right] \Big/ \left(\alpha^2 - \alpha\right)$, and for $\alpha = 0$, $I_0^{GE}(\mathbf{y}) = \log\left(\mu/m_0\right)$.

[5] This is true as long as we restrict ourselves to the arithmetic mean as the representative measure of the prosperity level in society. See Foster and Shneyerov (1999) for a generalization of the additive decomposition that relaxes this assumption.

[6] In the original formulation of Atkinson (1970) $A_{1-\varepsilon}(\mathbf{y}) = \begin{cases} 1 - \left[\frac{1}{n}\sum_{i=1}^n \left(y_i/\mu\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} & \varepsilon > 0, \varepsilon \neq 0 \\ 1 - \prod_{i=1}^n \left(y_i/\mu\right)^{1/n} & \varepsilon = 1 \end{cases}$, where $\varepsilon > 0$ represents the relative inequality aversion of the society, so in terms of (2) $\alpha = 1$ -ε and $\varepsilon >, 0$ implies $\alpha < 1$, i.e., $A_{1-\varepsilon}(\mathbf{y}) = I_\alpha^A(\mathbf{y})$.

[7] Note that for $\alpha \neq 0{,}1$, I_α^{GE} can be equivalently written as $I_\alpha^{GE}(\mathbf{y}) = \left[(m_\alpha/\mu)^\alpha - 1\right] \Big/ \left(\alpha^2 - \alpha\right)$, and for $\alpha = 0$, $I_0^{GE}(\mathbf{y}) = \log\left(\mu/m_0\right)$.

[8] Note that for $\alpha \neq 0{,}1$, I_α^{GE} can be equivalently written as $I_\alpha^{GE}(\mathbf{y}) = \left[(m_\alpha/\mu)^\alpha - 1\right] \Big/ \left(\alpha^2 - \alpha\right)$, and for $\alpha = 0$, $I_0^{GE}(\mathbf{y}) = \log\left(\mu/m_0\right)$.

[9] See Lasso de la Vega and Urrutia (2008) for the proof of this result. This extends the result in Lasso de la Vega and Urrutia (2003) for the AF. The focus on equality, instead of inequality, is not new; see Blackorby and Donaldson (1978), and Blackorby, Donaldson and Auersperg (1981)

[10] In addition, Lasso de la Vega and Urrutia (2008) give a key characterization of the members of the EAF as essentially the only continuous measures with such a weighted multiplicative decomposition, where weights can be general functions of the subgroup means and population sizes, summing exactly to 1.

[11] A deeper discussion about this issue can be found in Lambert and Lanza (2006).

[12] As $\alpha \to 1$, $I_\alpha(\mathbf{y})$ tends to a totally insensitive measure, whereas $I_1(\mathbf{y})$ is ordinally equivalent to what is commonly called the first Theil inequality index.

[13] The HBS from 1973/74, 1980/81 and 1990/91 are taken from the web of the Economics Department at University Carlos III of Madrid (http://www.eco.uc3m.es/investigacion/index.html#toc4). The HBS for 2003 was retrieved directly from the (*INE*)t's web (http://www.ine.es), and corresponds to merging the quarterly files of 2003 for the strong collaboration sub-sample. For methodological information see INE (1975; 1983; 1992; 1998).

[14] We exclude from the analysis the autonomous cities of Ceuta and Melilla, since they were excluded in the HBS of 1973/74.

[15] See Deaton and Zaidi (2002) for arguments in favour of using consumption for distributional purposes and Atkinson and Bourguignon (2000) for arguments in favour of using income. Essentially most of the arguments rely on consumption's being smoother than income, hence being a better proxy for permanent income.

[16] To be more specific "Transfers to other Households and Institutions" are excluded from the definition of expenditure in the HBS of 1980/81 and 1990/91, since they are not included either in the current Continuous HBS or in the HBS of 1973/74. Moreover, the HBS for 1990/91 includes a different valuation criterion for the non-monetary expenditures from the one used by (*INE*). See Arévalo, Cardelús, and Ruiz-Castillo (1998).

[17] This is also true for different equivalence scales (Aldás, Goerlich, and Mas, 2007). At $\alpha = 1$ we have a

single point in Figure 2, as can be seen from Equation 3, $1_{\alpha}(\mathbf{y})$. The corresponding values are given in Table 1.

[18] This result is consistent with the examination of the Lorenz ordinates, and also robust to previous years, e.g., 2002, and equivalence scales (Goerlich and Mas, 2004).

[19] Two other population partitions according to the gender or the age of the head of family are available from the authors upon request. In both of these cases, the between-group component plays almost no role, and most of the observed inequality is attributed to the within-group component.

[20] A similar analysis could be done for $\alpha = 1$ using income shares, s_j, instead of population shares, p_j. For other values of $\alpha \neq 0,1$, the contribution of each individual group to changes in the overall equality cannot be determined, as can be seen from Equation 4.

[21] The negative contribution from *Madrid* comes from the major increase in the population share over the period.

[22] All these results are robust to the equivalence scale considered; in fact, in this case, a slightly higher contribution to the change in equality from the within-group distribution is found.

[23] This reversal is not found for the equivalence scale considered, as the note in Table 3 indicates. This is the only difference with respect to the *per-capita* income case. Both contributions are important in this partition in any case.

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