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# Genuine Savings with Adjustment Costs

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## Abstract

In this paper, we consider how genuine savings would be altered if the adjustment costs of capitals are taken into account in the stylized capital-resource model. It is shown that, in order to derive the modified genuine savings, through shadow prices, the original genuine savings have to be divided by the marginal adjustment costs of the capital in question. This implies that economies with volatile savings harbor hidden costs even if they are judged as sustainable by conventional genuine savings indicators.

## 1 Introduction

Debates about genuine savings have focused attention on the trend an economy's path is likely to follow. Since this line of research was initiated by Pearce and Atkinson (1993), judging sustainability of nations by measuring genuine savings has been the norm<sup>1</sup>. Among the large body of literature, Hamilton and Clemens (1999) was the first to demonstrate comprehensively that some seemingly well-performing economies have reduced their genuine savings from which current and future generations obtain consumption. Using the same data set, Arrow et al.

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<sup>1</sup>The concept of genuine savings has been christened another couple of names. While inclusive capital and comprehensive wealth (Dasgupta 2009) are appealing, we opt for the adjective 'genuine' in the present paper because our focus here is not on the totality of various capitals in question but on the 'genuineness' of the index net of associated social costs.

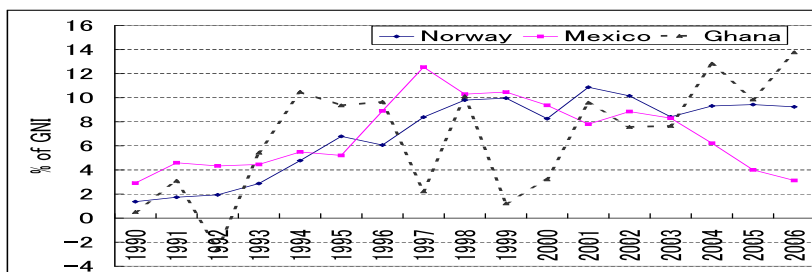


Figure 1: Genuine savings of Norway, Mexico and Ghana from 1990-2006. *Source: World Bank (2008).*

(2004) have shown that some countries in sub-saharan Africa and Indian subcontinent exhibit negative genuine savings for the past couple of decades.

A closer look at those data, however, suggests that even within economies that have been judged as sustainable, some are highly volatile in genuine savings while others are not (Sato and Samreth, 2008). Figure 1 provides a comparison of genuine savings of Norway, Mexico and Ghana, whose annual genuine savings are again similar (7.02, 6.88 and 6.71, respectively). They are all resource dependent economies to a certain extent (oil for Norway and Mexico, and mineral resources for Ghana), successful in recording positive genuine savings consecutively for over a decade, but it seems clear that the behavior of Ghana's genuine savings has been quite volatile compared to the other two nations.

All these might lead to a conjecture that economies with volatile genuine savings are worse off than those with relatively stable genuine savings if aggregate genuine savings within a given period are the same<sup>2</sup>. A straightforward way to treat this issue would be to introduce adjustment costs of capital into genuine sav-

<sup>2</sup>When stocks enter the utility function, this point becomes even more significant, but here we ignore this possibility.

ings<sup>3</sup>. The idea here is that the economy needs ‘some getting used to’ when there happens a large change in the level of capitals. Examples that pop to mind include the ability of management when a firm grows rapidly, or institutions to manage common property resources (a class of natural capital) after a massive reduction in its size. Indeed, there is already a heap of research about the adjustment costs of conventional capital, which we briefly overview in the next section. Given that the recent developments in the green accounting research include capturing investment in natural capital and human capital as well, it is fair to say that we should also account for the adjustment costs of those nonconventional capitals. Specifically relevant for resource economies is how to capture “disinvesting” in exhaustible resources, but this can be formulated in virtually the same manner as negative investment into conventional capital, hence we incorporate the same concept into the change of natural capital. It also could be that the volatility of genuine savings in exhaustible resource economies are largely accounted for by resource price volatilities. In that case as well, the economy is considered to entail large hidden costs in adapting to new level of capitals.

The rest of the paper is organized as follows. In the next section, we introduce standard adjustment costs of capital. We extend this idea to the case of natural capital in Section 3. Section 4 set the dynamic optimization problem, the result of which is applied to our measure of genuine savings in a perfect economy. Section 5 provides some back-of-the-envelope exercise on genuine savings data as well as second-order approximation, on which we consider empirical implications. The last section concludes and suggests some future research.

## 2 Adjustment Costs of Manufactured Capital

In this section, we describe how adjustment costs are taken into consideration for the case of man-made capital in a standard manner. Let the well-being of an economy be

$$\int_0^{\infty} U(C_t)e^{-\delta t} dt, \tag{1}$$

where  $U(C_t)$  stands for the instantaneous utility derived from the consumption at  $t$ ,  $C_t$ , by a representative agent, and  $\delta$  is a social discount rate of utility.

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<sup>3</sup>Another, perhaps more straightforward, way is to include stochastic elements and analyze the effect of variance in the context of uncertain consumption paths à la Weitzman (2007).

Specifically, the dynamics of the manufactured capital is described as

$$\dot{K}_t = \psi(I_t, K_t) \quad (2)$$

$$I_t = F(K_t, R_t) - C_t. \quad (3)$$

Here  $K_t$  stands for the traditional manufactured capital like machinery and equipment, and the economy is assumed to invest  $I_t$  in this stock, which however is subject to the adjustment cost, and  $R_t$  is the resource use rate at time  $t$ . The increase in the real capital  $\psi(K_t, I_t)$  is characterized by

$$\psi_I \equiv \partial\psi/\partial I_t > 0, \quad \partial^2\psi/\partial I_t^2 < 0, \quad (4)$$

$$\psi_K \equiv \partial\psi/\partial K_t < 0, \quad \partial^2\psi/\partial K_t^2 < 0. \quad (5)$$

In this setting,  $\dot{K}_t \leq \psi(I_t, K_t)$  is a convex set, so that  $I$  units of investment does not lead to the increase in the real capital of that amount generally. The more the economy seeks to invest at a time, the more costly it takes to adjust. Consequently,  $\psi$  is increasing but concave in  $I$ <sup>4</sup>.

When investment is negative, it is customary to assume that the installation function  $\psi$  declines sharply. This is intuitive considering the discount which a secondhand capital is subject to in the market. The installation function<sup>5</sup>  $\psi$  is also a decreasing function of  $K$ , since it is natural to assume the cost of installing another unit of manufactured capital increases as its size rises.

Following Lucas (1967) and Uzawa (1969), we take it that adjustment costs are internal rather than external, meaning that those costs are not caused by economy-wide factor price changes but triggered by requiring more elaborate institutions to handle the new level of capital stocks. Examples include the ability of management when a firm grows rapidly, or institutions to manage common property resources (a class of natural capital) after a massive reduction in its size. Indeed, Uzawa (1969) advances his argument after presupposing “the administrative, managerial, and other abilities which are required by the firm in the process of growth and expansion” (p.640). Interpreted this way, the institution as a resource allocation mechanism (Dasgupta 2009) is reflected in the installation function  $\psi$ . Despite the possibility that resource allocation mechanisms co-evolve with

<sup>4</sup>Cooper and Haltiwanger (2005) find that non-convex as well as convex adjustment costs fit their data at the plant level well, but at the same time mention that those non-convexities are less relevant at the macro, as opposed to micro, level. Considering our purpose here, it is not implausible to assume convex adjustment costs.

<sup>5</sup>This terminology is due to Hayashi (1982). For a textbook treatment of the subject, see Romer (2001). His formulation is based on subtraction of the cost of investment from the profits.

the capital, the cost of installation monotonically increases with the level of the stock for the sake of simplicity. Consequently, a firm tries to optimize based on its long-term costs and profits a marginal investment would provide.

While the above argument is well-known in the literature, we need some justification for the case of non-conventional capitals, on which we now elaborate.

### 3 Adjustment Costs of Natural Capital

In the current model we also assume that some classes of natural capital are exposed to adjustment costs. Let  $S_t$  denote an exhaustible or renewable resource at time  $t$  which grows according to

$$\dot{S}_t = \phi(J_t, S_t), \quad (6)$$

$$J_t = g(S_t) - R_t, \quad (7)$$

where  $J_t$  is the ‘investment’ in the resource stock,  $\phi(\cdot, \cdot)$  is its installation function,  $g(\cdot)$  is the growth function of the resource. The natural capital we currently consider can be extended to contain exhaustible resources like oil and natural gas, renewable resources like fisheries and forests, or assimilative capacities like atmosphere as a sink of greenhouse gases. As with the installation in the reproducible capital, the real increase in the resource capital is described as

$$\phi_J \equiv \partial\phi/\partial J_t > 0, \quad \partial^2\phi/\partial J_t^2 < 0, \quad (8)$$

$$\phi_S \equiv \partial\phi/\partial S_t < 0, \quad \partial^2\phi/\partial S_t^2 < 0, \quad (9)$$

so that a unit of ‘investing’ in the capital does not necessarily translate into one unit increase of the capital.

As mentioned above, it is customary to assume that the installation of reproducible capital sharply declines for negative values of investment. We have many reasons for this steepness of the installation function for disinvesting: when downsizing the existent land, property and buildings, firms are obliged to sell those depreciated assets in secondary markets of capital goods, so they are most likely to be discounted compared to their original acquisition costs. Hayashi (1982) argues that this steepness is due to irreversibility of investment per se, including the case of sunk costs.

It is not self-evident, however, that this logic should be directly applied to our parallel debate in natural capital. Let us make sure of this possibility. When the resource  $S_t$  is an exhaustible resource,  $g(\cdot)$  crumbles to zero and  $J_t$  is always

nonpositive. The marginal installation of ‘disinvesting’ in the resource stock is assumed to become costlier as the stock is depleted and as the economy tries to extract the resource massively at a time. This may be due to the diminishing returns to scale, or it could be due to the ecological stress the extraction might cause<sup>6</sup>.

One can proceed the argument even to include the idea of irreversibility of natural capital in the installation function. Where a natural capital, say a fishery or ecosystem at large, degrades so much by perturbation such as human intervention, the system can exhibit irreversibility, in which case the system does not recover its original state even if the perturbation comes to a halt. This can be represented by prohibitively high adjustment costs when disinvesting approaches a certain level.

## 4 Conditions for utilitarian optimum and genuine savings

### 4.1 General characteristics

Now that the dynamics of the current economy has been described, we shall go into the usual derivation of utilitarian optimum when the resource is exhaustible. Set the current-value Hamiltonian as

$$\mathcal{H} = U(C) + \lambda_K \psi(I_t, K_t) + \lambda_S \phi(J_t, S_t), \quad (10)$$

and the generalized Hamiltonian as

$$\mathcal{L} = \mathcal{H} + \mu_I (F(K_t, R_t) - C_t - I_t) + \mu_J (-R_t - J_t),$$

where  $\lambda_K$ ,  $\lambda_S$ ,  $\mu_I$  and  $\mu_J$  are the shadow prices associated, respectively, with  $K_t$ ,  $S_t$ ,  $I_t$  and  $J_t$ . We have assumed that the installation functions are convex, in which case we can swing the usual maximum principle into action<sup>7</sup>.

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<sup>6</sup>For an example of the former case, consider associated costs from enhanced oil recovery. Sustainable mining is an attempt to mitigate the installation cost of disinvesting in the resource, which represents the latter case.

<sup>7</sup>In reality individual installation function may not be convex. When the level of installation is proportionate to investment, a corner solution is obtained. When there is a sunk irreversible cost, bunching investment, rather than smoothing it, is likely to be optimal.

The economy tries to maximize (1) subject to (2-9), whereby the first-order conditions for optimality are

$$\begin{aligned} U_C &= \lambda_K \psi_I, \\ \lambda_K \psi_I F_R &= \lambda_S \phi_J, \\ \psi_I F_K + \psi_K &= \delta - \dot{\lambda}_K / \lambda_K, \\ \phi_S &= \delta - \dot{\lambda}_S / \lambda_S. \end{aligned}$$

These are all too familiar except that the adjustment costs are considered in the optimum: when there is no adjustment costs,  $\psi_I = \phi_J = 1$ . So our version of the Hotelling rule is

$$\frac{\dot{F}_R}{F_R} + \phi_S - \dot{\phi}_J / \phi_J = \psi_I F_K + \psi_K - \dot{\psi}_I / \psi_I,$$

which ensures intertemporal efficiency when the adjustment costs are taken into consideration for both of the two stocks.

We can also confirm that the adjustment costs enter the Euler equation as follows:

$$\psi_I F_K + \psi_K - \dot{\psi}_I / \psi_I = \delta - \dot{U}_C / U_C.$$

Substituting the above optimality conditions, the current-value Hamiltonian (10) becomes

$$\mathcal{H} = U(C) + U_C \left[ \frac{\dot{K}}{\psi_I} + F_R \frac{\dot{S}}{\phi_J} \right].$$

Let the term in the above bracket denote  $G$ . We can show that the time derivative of  $\mathcal{H}$  equals  $\delta U_C G$  (Hamilton and Atkinson 2006), so that  $G$  turns out to be genuine savings for this economy when the numéraire is consumption. In other words, the traditional savings of each capital have to be divided by its marginal cost of adjustment. But why the move? Because of the presence of the adjustment costs, the resultant increase in reproducible capital is  $\dot{K}$ . Since we are in the model of a perfect economy, the shadow price of the capital should be multiplied by an inverse of  $\psi_I$  ( $< 1$ ). The same goes for the natural capital. Consequently, as with the shadow prices, it is the *relative* adjustment costs among the capitals that matters. To see this, if the numéraire is the real increase in reproducible capital, we can write the genuine savings as

$$\psi_I G = \dot{K} + F_R \frac{\psi_I}{\phi_J} \dot{S}. \quad (11)$$



It is also evident from the above expression that even when  $\psi_I = \phi_J$ , we do have to modify the traditional savings by the marginal adjustment costs of reproducible or natural capital. Even so, the interesting case is a general one where the marginal adjustment costs of capitals are not equal.

## 4.2 Homogeneous of degree one installation

To proceed from here, we shall consider a specific class of the installation function. For simplicity, assume the installation functions are homogeneous of degree one. In this case, it follows that  $\psi(I, K) = \psi_I I + \psi_K K$  and  $\phi(J, S) = \phi_J J + \phi_S S$ <sup>8</sup>. The genuine savings then are reduced to

$$G = (I + \frac{\psi_K}{\psi_I} K) + F_R(J + \frac{\phi_S}{\phi_J} S).$$

If we further assume the function as Cobb-Douglas,  $\psi(I, K) = AI^p K^q$  and  $\phi(J, S) = BJ^u S^v$ , where  $p + q < 1$  and  $u + v < 1$  and with  $A$  and  $B$  given, then the genuine savings are measured simply by

$$G = I/p + F_R J/u. \tag{12}$$

This identification is practically appealing, in that genuine savings with adjustment costs can be obtained by merely dividing the ‘share’ (denoted here as  $p$  and  $u$ ) of investment in the installation function for each of the investment of capitals being accounted for.

## 5 Some empirics of genuine savings with adjustment costs

### 5.1 Modified shadow prices

To get a sense of the orders of magnitude of the theoretical modification we have seen, we employ specific values for the marginal adjustment costs of investment and see its consequences in genuine savings. The equation we put in practice for our direct purpose is (12). For the values of  $p$  and  $u$ , the literature on which we can rely is scant. In a search for a convex or non-convex structure of adjustment costs,

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<sup>8</sup>A special example of this appears in Uzawa (2003).

Cooper and Haltiwanger (2005) estimate that the multiplier that represents the loss incurred by the adjustment is in the order of 0.3. For the production sector of exhaustible resources, Bovenberg and Goulder (2000) reckon that the output in the oil and gas industry should be multiplied by  $1.27[1 - (Z/450)^2]$  where  $Z$  represents cumulated production of oil. Even from these articles, it is difficult to construct figures for the coefficients in (12), but let us assume the three cases below. We also assume that the adjustment costs of human capital (calculated as education expenditure) are the same with those of physical capital. Since they are neither a rigorous estimate nor an educated guess, what we shall undergo now should be taken as a numerical exercise to grab a hint of the consequences of accounting for the relative adjustment costs.

- When there is no adjustment cost for physical capital but there is some adjustment cost for natural capital, genuine savings are bound to be worse. Set  $p = 1$  and  $u = 0.9$ , genuine savings for 2006 become worse by over 5 percentage points in some oil-exporting economies: Angola, Azerbaijan, Bolivia, Chad, Iran, Kazakhstan, and Nigeria.
- When there are adjustment costs for both of physical capital and natural capital, and assume that  $p = 0.7$  and  $u = 0.5$ , then the gap between genuine savings and our modified genuine savings are sometimes quite large. The nations for which the gap has recorded more than 50% are, in addition to the oil-exporting countries shown above, : Ecuador, Estonia, Russia, Syrian Republic, Venezuela, and Zambia. Moreover, in this case a large portion of each capital sinks as adjustment costs, the aggregate genuine savings all over the world shrink dramatically (Figure 2).
- Assume, in turn, that  $p = 0.7$  and  $u = 0.9$ . Some nations enjoy over a 15-percentage-point increase in genuine savings after this modification. Among them are Bhutan, China, Ireland, Mongolia, Morocco, Namibia, and Singapore. They sometimes depreciate environmental assets (especially China), but they all invest enough in conventional assets to recover those losses.

It all goes to show that we should focus on the relative adjustment costs among the score of capital assets. The second case of resource pessimism –in terms of adjustment costs– is worth attention, since the introduction of the costs would drive down already negative genuine savings for resource-depleting nations.

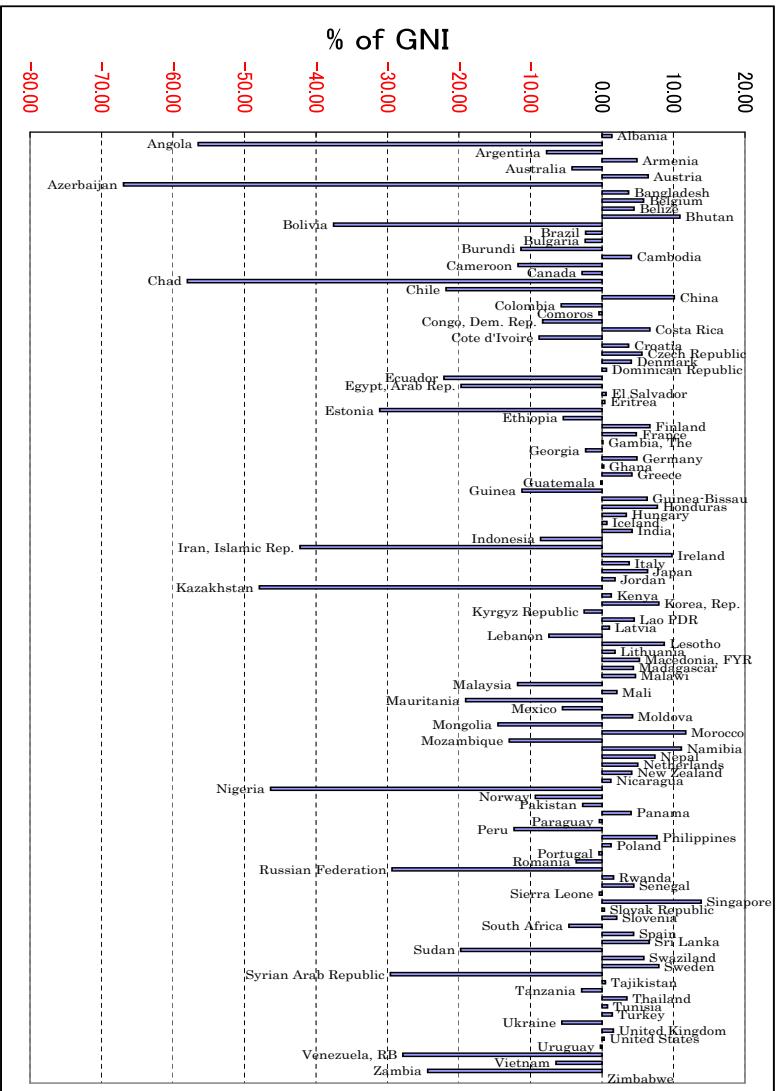


Figure 2: The gap between genuine savings and modified savings (the latter subtracted by the former) for 2006. For the latter, it is assumed that  $p = 0.7$  and  $u = 0.5$  for illustration. Source: The authors' calculation and World Bank (2008).

## 5.2 Approximation of installation function

To make the formulation applicable to data, let us approximate the installation function of natural capital. A second-order Taylor expansion of (6) yields

$$d\dot{S}_t = d\phi(J_t, S_t) \approx \frac{\partial\phi}{\partial J}(J - \bar{J}) + \frac{\partial\phi}{\partial S}(S - \bar{S}) + \frac{1}{2}\frac{\partial^2\phi}{\partial J^2}(J - \bar{J})^2 + \frac{1}{2}\frac{\partial^2\phi}{\partial S^2}(S - \bar{S})^2, \quad (13)$$

where  $\bar{J}$  and  $\bar{S}$  denote the means of investment and stock, respectively. This shows how investment in the natural capital can be measured. But from (8) and (9), the latter two terms with the sum of the squared residuals on the RHS are bound to be negative. It follows that larger variance due to volatile investment would translate into worse performance of genuine savings. With proper data set, we could directly analyze the performance of genuine savings by the above equation.

## 6 Conclusion

In this article, we have built in the concept of adjustment costs into a conventional argument of genuine savings or green national accounts.

Although our numerical examples in the last section are very crude in that we assumed only two capitals and that the marginal costs of adjustment stay the same, for all the nations under question. Also, the differences in adjustment costs of a variety of capitals needs to be explained. We could guess that, even if there is an ample investment in physical capital, say educational facilities like school, underinvesting in human capital like school teachers and general health of children would entail large adjustment costs to the economy in question. If this is the case, assuming cross derivatives of adjustment costs between reproducible capital and human capital would be more realistic<sup>9</sup>. In this way, explaining adjustment cost structures could lead to dynamics among a score of capitals. Another drawback of the analysis is, admittedly, the assumption of perfect economy. The present argument should be more significant in a setting of an imperfect economy along the line of Arrow et al. (2003).

Despite the above points, the underlying theory crystallizes some portions implicit in the idea of shadow prices, by extracting adjustment costs. Being defined as the change in the well-being brought about by an incremental change in some

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<sup>9</sup>Duczynski (2002) considers large adjustment costs for human capital and moderate adjustment costs for physical capital.

capital, the shadow price of a capital should include the class of adjustment costs discussed here. In the example of the modification of genuine savings under a set of assumptions for adjustment costs, we have seen that relatively large adjustment cost of natural capital worsens resource-rich nations with volatile genuine savings, whereas relatively large adjustment cost of physical capital might work as a stronger compensation for resource depletion.

Another light the analysis might shed is its prescriptiveness. Massive effort has been undertaken for the past decade toward constructing theories and plugging data to test genuine savings across nations, but they are largely *retrospective* studies regarding investment —although its returns are gained in the future—, and a big task among the required research is how one can prescribe how much an economy should invest in which capital on a *forward-looking* basis. Toward this, we should consider not just social return of investment of various capitals, but also adjustment costs of them. Elaborating the adjustment cost structures of a variety of capitals might lead to a better understanding of the economy's desired direction, since capitals with big social returns relative to its adjustment costs are prioritized for social investment.

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