The Maastricht Criteria and Optimal Monetary and Fiscal Policy Mix for the EMU Accession Countries

Lipinska, Anna

Universitat Autònoma de Barcelona

June 2008

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Anna Lipińska†

May 2008

Abstract

The Maastricht convergence criteria set constraints on both monetary and fiscal policies in the EMU Accession Countries. This paper uses a DSGE model of a two sector small open economy with distortionary taxes to address the following question: How do the Maastricht convergence criteria modify an optimal monetary and fiscal policy mix in an economy facing domestic and external shocks?

We find that targets of the unconstrained optimal monetary and fiscal policy are similar to those of the optimal monetary policy alone. The constrained policy is characterised by additional elements that penalize fluctuations of monetary and fiscal variables around the new targets which are different from the steady state of the unconstrained optimal monetary policy.

Under the chosen parameterization (which aims to reflect the Czech Republic economy) the optimal monetary and fiscal policy violates three Maastricht criteria: on the CPI inflation rate, the nominal interest rate and deficit to GDP ratio. Both the stabilization component and deterministic component of the constrained policy are different from the unconstrained optimal policy. Since monetary criteria play a dominant role in affecting the stabilization process of the constrained policy, CPI inflation and the nominal interest are characterised by a smaller variability (than under the unconstrained policy) at the expense of a higher variability of deficit to GDP ratio. The constrained policy is characterised by a deflationary bias which results in targeting the CPI inflation rate and the nominal interest rate that are lower by 1.3% (in annual terms) than the CPI inflation rate and the nominal interest rate in the countries taken as a reference. The constrained policy is also characterised by targeting surplus to GDP ratio at around 3.7%. As a result the policy constrained by the Maastricht convergence criteria induces additional welfare costs that amount to 60% of the initial deadweight loss associated with the optimal policy.

JEL Classification: F41, E52, E58, E61.

Keywords: Optimal monetary and fiscal policy, Maastricht convergence criteria, EMU accession countries

*I would like to thank Kosuke Aoki for his excellent supervision and encouragement. I also thank Gianluca Benigno for helpful comments and suggestions. This work was undertaken as part of my International Doctorate in Economic Analysis, Universitat Autonoma de Barcelona, Spain, and was written during my stay at the Centre for Economic Performance, London School of Economics and Political Science, United Kingdom. It was not connected in any way to the Bank of England's research program, and therefore does not necessarily reflect the views of that institution.

†Bank of England, Monetary Analysis, International Economic Analysis Division, Threadneedle Street, London EC2R 8AH, United Kingdom; e-mail: Anna.Lipinska@bankofengland.co.uk.

1 The introduction

Monetary policy has been used as the main stabilization tool in many countries. Interestingly, EMU accession countries, on their way to EMU, face the Maastricht criteria that put serious constraints on their monetary policies (as it was analyzed in Lipińska (2008)). Specifically, the countries should achieve a high and durable degree of price stability, which is, in quantitative terms, reflected in low inflation rates and low long term interest rates. Additionally, nominal exchange rates of the EMU accession countries versus the euro should stay within normal fluctuation margins. At the same time, fiscal policy that could be seen as an additional stabilization tool is bound by the restrictions imposed in the Stability and Growth Pact. Accordingly, these countries should be characterized by a sustainable government financial position which is defined in terms of upper limits on deficit to GDP ratio and debt to GDP ratio. At the moment many EMU accession countries do not satisfy some of the criteria.[1]

In this context a number of questions arise: Can fiscal policy serve as an additional stabilization tool in the presence of restrictions set on monetary policy? How does this ability of fiscal policy to mitigate business cycles change when faced with fiscal constraints as well? In general, what should be the optimal monetary and fiscal policy that satisfies all the Maastricht criteria? And finally, which criteria: fiscal or monetary ones put stronger constraints on stabilization macroeconomic policies?

To this purpose, we develop a DSGE model of a small open economy with nominal rigidities, distortionary taxation and government debt exposed to both domestic and external shocks. The model is an extension of the framework constructed by Lipińska (2008) where fiscal policy does not issue any debt and taxes are assumed to be lump sum. Importantly, as in Lipińska (2008) the production structure is composed of two sectors: nontraded and home traded sector. In that way, we want to take into account recent empirical literature both on OECD and EMU Accession countries that highlights the role of sector specific shocks in explaining international business cycle fluctuations (see e.g. Canzoneri et al. (1999), Marimon and Zilibotti (1998), Mihaljek and Klau (2004)). Finally, following Benigno and Woodford (2003) and Benigno and De Paoli (2006) monetary and fiscal policy is conducted in a fully coordinated way by a single policy maker.

In this framework we characterize the optimal monetary and fiscal policy from a timeless perspective (Woodford (2003)). As in Lipińska (2008), we derive the micro founded loss function using the second order approximation methodology developed by Rotemberg and Woodford (1997) and Benigno and Woodford (2005). We find that the optimal monetary and fiscal policy (unconstrained policy) should not only target inflation rates in the domestic sectors and aggregate output fluctuations but also domestic and international terms of trade. Subsequently, we present how the loss function changes when the monetary and fiscal policy is constrained by the Maastricht convergence criteria. We derive the optimal monetary and fiscal policy that satisfies all the Maastricht convergence criteria (constrained policy). Importantly, the Maastricht convergence criteria are not easily implementable in our model. Here we take an advantage of the methodology developed by Rotemberg and Woodford (1997, 1999) for the analysis of the zero bound problem and adapted by Lipińska (2008) for the analysis of the monetary criteria. This method enables us to verify whether a given criterion is satisfied by only computing first and second moments of a variable for which the criterion is set.

[1] Lipińska (2008) reports that Bulgaria, Estonia, Hungary, Latvia, Lithuania, Romania and Slovakia fail to fulfill the CPI inflation rate criterion (see Figures A.2 and A.3 in Appendix A). Moreover, Hungary and Romania also violate the nominal interest rate criterion (see Figures A.4 and A.5 in Appendix A). Moreover, the nominal exchange rate fluctuations of Polish Zloty, Slovakian Koruna and Romanian Lei versus the euro exceed the band set by the nominal exchange rate criterion (see Figure A.6 in Appendix A). Additionally, deficit to GDP criterion is not satisfied by the Czech Republic, Hungary, Poland and Slovakia (see Figure A.7 in Appendix A). Finally, only Hungary is characterised by an excessive debt to GDP ratio according to the limits set by the Maastricht criteria.

Under the chosen parameterization (which aims to reflect the Czech Republic economy) the optimal monetary and fiscal policy violates three Maastricht convergence criteria: on the CPI inflation rate, the nominal interest rate and deficit to GDP ratio. Both the stabilization component and deterministic component of the constrained policy are different from the unconstrained optimal policy. The constrained policy leads to a smaller variability of the CPI inflation and the nominal interest rate and at the same time higher variability of deficit to GDP ratio. This reflects an active stabilization role of fiscal policy in the presence of direct constraints on monetary instrument. Moreover, as in Lipińska (2008) the constrained policy is characterized by a deflationary bias which results in targeting the CPI inflation rate and the nominal interest rate that are lower by 1.3% (in annual terms) than the CPI inflation rate and the nominal interest rate in the countries taken as a reference. Importantly, the constrained policy is also characterized by targeting surplus to GDP ratio at around 3.7%. This result is determined by a relative dominance of the monetary criteria over the fiscal ones in affecting the stabilization process. Accordingly the constrained policymaker uses actively fiscal instruments and in order to comply with all the criteria has to assign a relatively high surplus to GDP ratio target. As a result, the policy constrained by the Maastricht convergence criteria induces additional welfare costs that amount to 60% of the initial deadweight loss associated with the optimal unconstrained policy. These welfare costs have their origin in conflicting interest of monetary and fiscal criteria and also relatively poor performance of fiscal policy as an additional stabilization tool.

The literature on the macroeconomic policies in the EMU Accession countries concentrated so far on the analysis of the monetary criteria and their impact on the appropriate choice of the monetary regime. In particular, Devereux and Lane (2003), Ferreira (2006), Laxton and Pesenti (2003) and Natalucci and Ravenna (2007) study this issue in a framework of open economy DSGE models.[2]

The issue of a proper design of fiscal policy in the EMU Accession countries has not been studied up till now. However, theoretical literature addressed already the problem of a joint optimal monetary and fiscal policy both in the closed, open economy and monetary union environment. Schmitt-Grohe and Uribe (2004) and Benigno and Woodford (2003) study the design of optimal monetary and fiscal policy in the closed economy environment, while Benigno and De Paoli (2006) derive the optimal monetary and fiscal policy for a small open economy. These papers find that variations in fiscal instruments should serve the same objectives as those in the optimal monetary policy design, i.e. stabilization of inflation and the output gap that measures total distortion of the level of economic activity. Ferrero (2005), Gali and Monacelli (2008) and Pappa and Vassilatos (2005) are examples of papers that study optimal monetary and fiscal policy in a monetary union. In particular, Ferrero (2005) shows that regional fiscal policies that respond to a measure of real activity perform better in terms of welfare than balanced budget rules. Gali and Monacelli (2005) find that the lack of regional monetary instrument generates a stabilization role for regional fiscal policies. Interestingly, Pappa and Vassilatos (2006) examine how general fiscal rules that are designed to satisfy fiscal criteria affect macroeconomic stability and welfare in a two-region monetary union. They find that some flexibility in compliance with fiscal criteria can be welfare improving.

We take advantage of these theoretical studies and characterize an optimal monetary and fiscal policy mix in a model which tries to reflect some of the characteristics of the EMU Accession countries. Then we analyze the effects of the Maastricht criteria on the optimal policies. In that way we can set guidelines on the way monetary and fiscal policy should be conducted in the EMU Accession countries.

The rest of the paper is organized as follows: the next section describes the model. Section 3 explains

[2] Lipińska (2008) provides a detailed discussion of both empirical and theoretical papers on the monetary policy in the EMU Accession Countries.

derivation of the optimal monetary and fiscal policy. Section 4 presents the way we reformulate the Maastricht convergence criteria in order to implement them into our framework. Section 5 is dedicated to derivation of the optimal policy constrained by the Maastricht convergence criteria. Section 6 compares the unconstrained optimal monetary and fiscal policy with the optimal monetary and fiscal policy constrained by the Maastricht convergence criteria under the chosen parameterization of the model. Section 7 concludes.

2 The model

Our modelling framework is based on a two-sector SOE model of Lipińska (2008) and one-sector SOE models of De Paoli (2004) and Benigno and De Paoli (2006). Following De Paoli (2004), we model a small open economy as the limiting case of a two-country problem, i.e. where the size of the small open economy is set to zero. We consider two highly integrated economies where asset markets are complete. In each of the economies, there are two goods sectors: nontraded goods and traded goods. Each of the sectors (domestic and foreign) features heterogeneity of goods and monopolistic competition. Labour is the only factor of production and is mobile between sectors in each country and immobile between countries. We assume the existence of home bias in consumption which, in turn, depends on the relative size of the economy and its degree of openness. Although the law of one price holds, existence of home bias leads to deviations from the purchasing power parity.

As far as the monetary and fiscal policy is concerned we follow Benigno and Woodford (2003) and Benigno and De Paoli (2006) and assume that the policies are conducted in a fully coordinated way by a single policymaker. The role for monetary policy arises through the introduction of monopolistic competition and price rigidities with staggered Calvo contracts in all goods sectors. The model features complete pass-through as prices are set in the producer's currency. We abstract from any monetary frictions by assuming cashless limiting economies.[3] The fiscal policy issues a one period nominal non-state contingent debt which is financed only through the distortionary revenue taxes collected in both domestic sectors. On the contrary to Lipińska (2008) the lump sum taxes are not available.

Finally, the stochastic environment of the small open economy is characterized by asymmetric productivity shocks originating in both domestic sectors, preference shocks, foreign consumption shocks and government expenditure shocks.

2.1 Households

The world economy consists of a continuum of agents of unit mass: $[0, n)$ belonging to a small country (home) and $[n, 1]$ belonging to the rest of the world, i.e. the euro area (foreign). There are two types of differentiated goods produced in each country: traded and nontraded goods. Home traded goods are indexed on the interval $[0, n)$ and foreign traded goods on the interval $[n, 1]$, respectively. The same applies to nontraded goods. In order to simplify the exposition of the model, we explain in detail only the structure and dynamics of the domestic economy. Thus, from now on, we assume the size of the domestic economy to be zero, i.e. $n \to 0$.

Households are assumed to live infinitely and behave according to the permanent income hypothesis. They can choose between three types of goods: nontraded, domestic traded and foreign traded goods. C_t^i represents consumption at period t of a consumer i and L_t^i constitutes his labour supply. Each agent i maximizes the following utility function:[4]

[3] See Woodford (2003).

[4] In general, we assume U to be twice differentiable, increasing and concave in C_t and V to be twice differentiable, increasing and convex in L_t.

$$\max E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[U\left(C_t^i, B_t\right) - V\left(L_t^i\right) \right] \right\}, \tag{1}$$

where E_{t_0} denotes the expectation conditional on the information set at date t_0, β is the intertemporal discount factor and $0 < \beta < 1$, $U(\cdot)$ stands for flows of utility from consumption and $V(\cdot)$ represents flows of disutility from supplying labour.[5] C is a composite consumption index. We define consumers' preferences over the composite consumption index C_t of traded goods ($C_{T,t}$) (domestically produced and foreign ones) and nontraded goods ($C_{N,t}$):

$$C_t \equiv \left[\mu^{\frac{1}{\phi}} C_{N,t}^{\frac{\phi-1}{\phi}} + (1-\mu)^{\frac{1}{\phi}} C_{T,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}, \tag{2}$$

where $\phi > 0$ is the elasticity of substitution between traded and nontraded goods and $\mu \in [0,1]$ is the share of the nontraded goods in overall consumption. Traded good consumption is a composite of the domestically produced traded goods (C_H) and foreign produced traded goods (C_F):

$$C_{T,t} \equiv \left[(1-\lambda)^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + \lambda^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{3}$$

where $\theta > 0$ is the elasticity of substitution between home traded and foreign traded goods, and λ is the degree of openness of the small open economy ($\lambda \in [0,1]$).[6] Finally, C_j (where $j = N$, H, F) are consumption sub-indices of the continuum of differentiated goods:

$$C_{j,t} \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n c_t(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}, \tag{4}$$

where $\sigma > 1$ represents elasticity of substitution between differentiated goods in each of the sectors. Based on the above presented preferences, we derive consumption-based price indices expressed in the units of currency of the domestic country:

$$P_t \equiv \left[\mu P_{N,t}^{1-\phi} + (1-\mu) P_{T,t}^{1-\phi} \right]^{\frac{1}{1-\phi}}, \tag{5}$$

$$P_{T,t} \equiv \left[\nu P_{H,t}^{1-\theta} + (1-\nu) P_{F,t}^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{6}$$

with

$$P_{j,t} \equiv \left[\left(\frac{1}{n}\right) \int_0^n p_t(j)^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}}. \tag{7}$$

[5] We assume specific functional forms of consumption utility $U\left(C_t^i\right)$, and disutility from labour $V\left(L_t^i\right)$: $U\left(C_t^i\right) \equiv \frac{\left(C_t^i\right)^{1-\rho} B_t^{\rho}}{1-\rho}$, $V\left(L_t^i\right) \equiv \varphi_l \frac{\left(L_t^i\right)^{1+\eta}}{1+\eta}$ with ρ ($\rho > 0$), the inverse of the intertemporal elasticity of substitution in consumption and η ($\eta \geq 0$), the inverse of labour supply elasticity and B_t , preference shock.

[6] Following de Paoli (2004) and Sutherland (2002), we assume home bias (ν) of the domestic households to be a function of the relative size of the home economy with respect to the foreign one (n) and its degree of openness (λ) such that $(1-\nu) = (1-n)\lambda$ where $\lambda \in [0,1]$. Importantly, the higher is the degree of openness, the smaller is the degree of home bias. Since $n \to 0$, we obtain that $\nu = 1 - \lambda$.

Although we assume the law of one price in the traded sector (i.e. $p(h) = Sp^*(h)$ and $p(f) = Sp^*(f)$ where S is the nominal exchange rate), both the existence of the nontraded goods and the assumed home bias cause deviations from purchasing power parity, i.e. $P \neq SP^*$. The real exchange rate can be defined in the following manner: $RS \equiv \frac{SP^*}{P}$. Moreover, we define the international terms of trade as $T \equiv \frac{P_F}{P_H}$ and the ratio of nontraded to traded goods' prices (domestic terms of trade) as $T^d \equiv \frac{P_N}{P_T}$.

From consumer preferences, we can derive total demand for the generic goods – n (home nontraded ones), h (home traded ones), f (foreign traded ones):

$$y^d(n) = \left[\frac{p(n)}{P_N}\right]^{-\sigma} \left[\frac{P_N}{P}\right]^{-\phi} \mu(C+G), \tag{8}$$

$$y^d(h) = \left[\frac{p(h)}{P_H}\right]^{-\sigma} \left[\frac{P_H}{P_T}\right]^{-\theta} (1-\lambda)(C_T+G_T) + \left[\frac{p^*(h)}{P^*_H}\right]^{-\sigma} \left(\frac{P^*_H}{P^*_T}\right)^{-\theta^*} \lambda(C^*_T+G^*_T), \tag{9}$$

$$y^d(f) = \left[\frac{p^*(f)}{P^*_F}\right]^{-\sigma} \left[\frac{P^*_F}{P^*_T}\right]^{-\theta} (C^*_T+G^*_T) \tag{10}$$

where variables with an asterisk represent the foreign equivalents of the domestic variables. Moreover, G and G^* denote exogenous aggregate government expenditures which have the same composition as the private consumption. Accordingly, G_T and G^*_T denote government expenditure in the tradable sector. Importantly, since the domestic economy is a small open economy, demand for foreign traded goods does not depend on domestic demand. However, at the same time, demand for domestic traded goods depends on foreign demand.

Households get disutility from supplying labour to all firms present in each country. Each individual supplies labour to both sectors, i.e. the traded and the nontraded sector:

$$L^i_t = L^{i,H}_t + L^{i,N}_t. \tag{11}$$

We assume that consumers have access to a complete set of securities-contingent claims traded internationally. Each household faces the following budget constraint:

$$P_t C^i_t + E_t\{Q_{t,t+1} D_{t+1}\} \leq D_t + W^i_{H,t} L^i_{H,t} + W^i_{N,t} L^i_{N,t} + \frac{\int_0^n \Pi^i_{N,t} di}{n} + \frac{\int_0^n \Pi^i_{H,t} di}{n}, \tag{12}$$

where at date t, D_{t+1} is nominal payoff of the portfolio held at the end of period (t), $Q_{t,t+1}$ is the stochastic discount factor for one-period ahead nominal payoffs relevant to the domestic household, $\Pi_{H,t}$ and $\Pi_{N,t}$ are nominal profits from the domestic firms. Moreover, consumers face no Ponzi game restriction.

The short-term interest rate (R_t) is defined as the price of the portfolio which delivers one unit of currency in each contingency that occurs in the next period:[7]

$$R^{-1}_t = E_t\{Q_{t,t+1}\}. \tag{13}$$

The maximization problem of any household consists of maximizing the discounted stream of utility (1) subject to the budget constraint (12) in order to determine the optimal path of the consumption index, the labour index and contingent claims at all times. The solution to the household decision problem gives a set

[7] Following the literature, we assume one period to be one quarter.

of first-order conditions.[8] Optimization of the portfolio holdings leads to the following Euler equations for the domestic economy:

$$U_C(C_t, B_t) = \beta E_t \left\{ U_C(C_{t+1}, B_{t+1}) Q_{t,t+1}^{-1} \frac{P_t}{P_{t+1}} \right\}. \tag{14}$$

There is a perfect sharing in this setting, meaning that marginal rates of consumption in nominal terms are equalized between countries in all states and at all times.[9] Subsequently, appropriately choosing the distribution of initial wealth, we obtain the risk sharing condition:

$$\frac{U_C(C_t, B_t)}{U_C(C_t^*, B_t^*)} = \upsilon \frac{P_t}{S_t P_t^*} = \upsilon RS_t^{-1}, \tag{15}$$

where $\upsilon > 0$ and depends on the initial wealth distribution. The risk sharing condition implies that the real exchange rate is equal to the marginal rate of substitution between domestic and foreign consumption.

The optimality condition for labour supply in the domestic economy is the following:

$$\frac{W_t^k}{P_t} = \frac{V_L(L_t)}{U_C(C_t, B_t)}, \tag{16}$$

where W^k is the nominal wage of the representative consumer in sector k $(k = H, N)$.[10] So the real wage is equal to the marginal rate of substitution between labour and consumption.

2.2 Firms

All firms are owned by consumers. Both traded and nontraded sectors are monopolistically competitive. The production function is linear in labour which is the only input. Consequently, its functional form for firm i in sector k $(k = N, H)$ is the following:

$$Y_{k,t}(i) = A_t^k L_t^k(i). \tag{17}$$

Price is set according to the Calvo (1983) pricing scheme. In each period, a fraction of firms $(1-\alpha_k)$ decides its price, thus maximizing the future expected profits. The maximization problem of any firm in sector k at time t_0 is given by:

$$\max_{P_{k,t_0}(i)} E_{t_0} \sum_{t=t_o}^{\infty} (\alpha_k)^s Q_{t_0,t} \left[(1-\tau_{R,t}^k) P_{k,t_0}(i) - MC_t^k(i) \right] Y_{k,t_0:t}^d(i)$$

$$\textit{subject to } Y_{k,t_0:t}^d(i) = \left(\frac{P_{k,t_0}(i)}{P_{k,t}} \right)^{-\sigma} Y_{k,t}, \tag{18}$$

where $Y_{k,t_0:t}^d(i)$ is demand for the individual good in sector k produced by producer i at time t conditional on keeping the price $P_{k,t_0}(i)$ fixed at the level chosen at time t_0, $MC_t^k = \frac{W_t^k(i)}{A_t^k}$ is the nominal marginal cost in sector k at time t, and $\tau_{k,t}$ are revenue taxes in sector k.

[8] We here suppress subscript i as we assume that in equilibrium, all agents are identical. Therefore, we represent optimality conditions for a representative agent.

[9] We have to point out here that although the assumption of complete markets conveniently simplifies the model, it neglects a possibility of wealth effects in response to different shocks (Benigno (2001)).

[10] Notice that wages are equalized between sectors inside each of the economies, due to perfect labour mobility and perfect competition in the labour market.

Given this setup, the price index in sector k evolves according to the following law of motion:

$$(P_{k,t})^{1-\sigma} = \alpha_k (P_{k,t-1})^{1-\sigma} + (1-\alpha_k)(P_{k,t_0}(i))^{1-\sigma}. \tag{19}$$

2.3 Government budget constraint

The government issues a nominal, non-state contingent debt denominated in domestic currency and taxes the revenue income of firms in the nontraded sector at rate $\tau_{N,t}$ and also in the home traded sector at rate $\tau_{H,t}$. The revenues are spent on government expenditures (G_t) and interest payments on outstanding nominal debt. We assume that there are no seigniorage revenues.

Government debt, D_t, expressed in nominal terms, follows the law of motion:

$$D_t = R_{t-1} D_{t-1} - P_t sr_t \tag{20}$$

where sr_t is the real primary budget surplus:

$$sr_t = \tau_{N,t} p_{N,t} Y_{N,t} + \tau_{H,t} p_{H,t} Y_{H,t} - G_t \tag{21}$$

and $p_{N,t} \equiv \frac{P_{N,t}}{P_t}$ and $p_{H,t} \equiv \frac{P_{H,t}}{P_t}$ denote relative prices. We define:

$$d_t \equiv \frac{D_t R_t}{P_t} \tag{22}$$

in order to rewrite the government budget constraint as:

$$d_t = d_{t-1} \frac{R_t}{\Pi_t} - R_t sr_t. \tag{23}$$

The rational-expectations equilibrium requires that the expected path of government surpluses must satisfy an intertemporal solvency condition :

$$\frac{d_{t_0-1}}{\Pi_{t_0}} U_C(C_{t_0}, B_{t_0}) = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} U_C(C_t, B_t) sr_t. \tag{24}$$

in each state of the world that may be realized at date t_0. This condition restricts the possible paths that may be chosen for the tax rates $\{\tau_{N,t}, \tau_{H,t}\}$. Moreover, monetary policy can affect this condition by influencing inflation in period t_0 and also affecting the discount factors in subsequent periods. This condition is derived from the household optimization condition (14) and law of motion of debt (23). As discussed in Woodford (2001) this condition serves as one of the constraints in choosing an optimal plan among possible rational-expectations equilibria.

2.4 Monetary and fiscal policy

A role for the macroeconomic policy arises due to existing nominal and real rigidities in the economy: price stickiness (together with monopolistic competition), home bias and the nontraded good sector, which lead to deviations from PPP. The policy maker has three instruments: two fiscal ones - revenue tax rates in both domestic sectors and a monetary one - nominal interest rate. The system is therefore closed by defining appropriate monetary and fiscal policy rules for the domestic economy.

We approximate the model around a steady state in which exogenous shocks take constant values. Moreover steady state inflation is zero and tax rates are chosen in such a way in order to maximize welfare of the agents. The loglinearized version of the model is available in the Appendix.

3 The optimal fiscal and monetary policy

Since our model is microfounded the optimal policy is defined as the policy that maximizes welfare of society subject to the structural equations of an economy.

We use a linear quadratic approach (Rotemberg and Woodford (1997, 1999)) and define the optimal monetary policy problem as a minimization problem of the quadratic loss function subject to the loglinearized structural equations (presented in the Appendix). First, we present the welfare measure derived through a second-order Taylor approximation of equation (1):

$$W_{t_0} = U_C \overline{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [z_v' \widehat{v}_t - \frac{1}{2} \widehat{v}_t' Z_v \widehat{v}_t - \widehat{v}_t' Z_\xi \widehat{\xi}_t] + tip + O(3), \tag{25}$$

where $\widehat{v}_t' = \left[\begin{array}{ccccc} \widehat{C}_t & \widehat{Y}_{N,t} & \widehat{Y}_{H,t} & \widehat{\pi}_{N,t} & \widehat{\pi}_{H,t} \end{array} \right]$; $\widehat{\xi}_t' = \left[\begin{array}{ccccc} \widehat{A}_{N,t} & \widehat{A}_{H,t} & \widehat{B}_t & \widehat{C}_t^* & \widehat{G}_t \end{array} \right]$;

$z_v' = \left[\begin{array}{ccccc} 1 & -s_{CY_N} & -s_{CY_H} & 0 & 0 \end{array} \right]$ with $s_{CY_N} \equiv \frac{\overline{\omega} \overline{Y_N}}{\overline{C}}$ - steady state share of nontraded labour income in domestic consumption, $s_{CY_H} \equiv \frac{\overline{w} \overline{Y_H}}{\overline{C}}$ - steady state share of home traded labour income in domestic consumption, and matrices Z_v, Z_ξ are defined in Appendix B; *tip* stands for *terms independent of policy* and $O(3)$ includes terms that are of a higher order than the second in the deviations of variables from their steady state values.

Notice that the welfare measure (25) contains the linear terms in aggregate consumption and sector outputs. These linear terms originate from different distortions in the economy. First, monopolistic competition together with distortionary revenue taxes in both domestic sectors lead to inefficient levels of sector outputs and also an inefficient level of aggregate output. Second, since government spends its revenues on government expenditures domestic consumption and aggregate output are not equalized. Third, openness of the economy can also result in trade imbalances, i.e. domestic consumption can be different from the aggregate output. Importantly, their composition depends on the domestic and international terms of trade. Finally, similarly to De Paoli (2007) and Lipinska (2008) there exists an international terms of trade externality that creates an incentive for policy to generate a welfare improving real exchange rate appreciation, i.e. disutility from labour decreases without an accompanying decrease in the utility of consumption.

The presence of linear terms in the welfare measure (25) means that we cannot determine the optimal policy, up to first order, using the welfare measure subject to the structural equations (presented in the Appendix) that are only accurate to first order. Following the method proposed by Benigno and Woodford (2005) and Benigno and Benigno (2005), we substitute the linear terms in the approximated welfare function (25) by using a second order approximation to some of the structural conditions.[11] As a result, we obtain the fully quadratic loss function which can be represented as a function of aggregate output ($\widehat{Y}_t$), domestic and international terms of trade ($\widehat{T}_t^d$, $\widehat{T}_t$), domestic sector inflation rates ($\widehat{\pi}_{H,t}$, $\widehat{\pi}_{N,t}$) and revenue tax rates in the nontraded and home traded sector ($\widehat{\tau}_{N,t}$, $\widehat{\tau}_{H,t}$). Its general expression is given below:

[11] Details of the derivation can be found in Appendix B.

$$\min L_{t_0} = U_C\overline{C}\sum_{t=t_0}^{\infty}\beta^{t-t_0}[\frac{1}{2}\Phi_y(\widehat{Y}_t-\widehat{Y}_t^T)^2+\frac{1}{2}\Phi_{T^d}(\widehat{T}_t^d-\widehat{T}_t^{dT})^2+\frac{1}{2}\Phi_T(\widehat{T}_t-\widehat{T}_t^T)^2+ \quad (26)$$

$$\frac{1}{2}\Phi_{\tau_N}(\widehat{\tau}_{N,t}-\widehat{\tau}_{N,t}^T)^2+\frac{1}{2}\Phi_{\tau_H}(\widehat{\tau}_{H,t}-\widehat{\tau}_{H,t}^T)^2+\Phi_{YT^d}\widehat{Y}_t\widehat{T}_t^d+\Phi_{YT}\widehat{Y}_t\widehat{T}_t+\Phi_{T^dT}\widehat{T}_t^d\widehat{T}_t+ \quad (27)$$

$$+\widehat{\tau}_{N,t}(\Phi_{Y\tau_N}\widehat{Y}_t+\Phi_{T\tau_N}\widehat{T}_t+\Phi_{T^d\tau_N}\widehat{T}_t^d)+\widehat{\tau}_{H,t}(\Phi_{Y\tau_H}\widehat{Y}_t+\Phi_{T\tau_H}\widehat{T}_t+\Phi_{T^d\tau_H}\widehat{T}_t^d)+ \quad (28)$$

$$+\frac{1}{2}\Phi_{\pi_N}\widehat{\pi}_{N,t}^2+\frac{1}{2}\Phi_{\pi_H}\widehat{\pi}_{H,t}^2]+tip+O(3) \quad (29)$$

where $\widehat{Y}_t^T$, $\widehat{T}_t^{d^T}$, $\widehat{T}_t^T$, $\widehat{\tau}_{N,t}^T$, $\widehat{\tau}_{H,t}^T$ are target variables which are functions of the stochastic shocks and, in general, are different from the flexible price equilibrium processes of aggregate output, domestic terms of trade and international terms of trade.[12] The term *tip* stands for *terms independent of policy*. The coefficients Φ_Y, Φ_{T^d}, Φ_T, Φ_{τ_N}, Φ_{τ_H}, Φ_{TT^d}, Φ_{YT^d}, Φ_{YT}, Φ_{π_H}, Φ_{π_N}, $\Phi_{Y\tau_N}$, $\Phi_{T\tau_N}$, $\Phi_{T^d\tau_N}$, $\Phi_{Y\tau_H}$, $\Phi_{T\tau_H}$, $\Phi_{T^d\tau_H}$, are functions of the structural parameters of the model.

Similarly to Lipińska (2008) our loss function generalizes previous studies regarding optimal monetary policy characterization in both closed economy environments (Aoki (2001), Benigno (2004), Rotemberg and Woodford(1997)) and open economy environments (Gali and Monacelli (2005), De Paoli (2007)). Moreover this loss function is also related to the literature on optimal monetary and fiscal policy in the sticky price environment (Benigno and Woodford (2003), Benigno and De Paoli (2007), Schmitt - Grohe and Uribe (2003)). As in Benigno and Woodford (2003) and Benigno and De Paoli (2007) we obtain that variations in distortionary taxation should be chosen to serve the same objectives as those emphasized in the literature on monetary stabilization policy. Interestingly, our loss function also involves some stabilization of taxes.[13]

To simplify the exposition of the optimal plan we reduce number of variables to a set of eight domestic variables which determine the loss function (26), i.e. $\widehat{Y}_t$, $\widehat{T}_t^d$, $\widehat{T}_t$, $\widehat{\pi}_{H,t}$, $\widehat{\pi}_{N,t}$, $\widehat{\tau}_{N,t}$, $\widehat{\tau}_{H,t}$, $\widehat{d}_t$. In Appendix we present the structural equations of the two-sector small open closed economy in terms of these variables. Finally, the policy maker following the optimal plan under commitment chooses $\{\widehat{Y}_t,\widehat{T}_t^d,\widehat{T}_t,\widehat{\pi}_{H,t},\widehat{\pi}_{N,t},\widehat{\tau}_{N,t},\widehat{\tau}_{H,t},\widehat{d}_t\}_{t=t_0}^{\infty}$ in order to minimize the loss function (26) subject to the constraints (237)–(241), given the initial conditions on nonpredetermined variables: $\widehat{Y}_{t_0},\widehat{T}_{t_0}^d,\widehat{T}_{t_0},\widehat{\pi}_{H,t_0},\widehat{\pi}_{N,t_0},\widehat{\tau}_{N,t_0},\widehat{\tau}_{H,t_0},\widehat{d}_{t_0}$. In accordance with the definition of the optimal plan from a timeless perspective (see Woodford (2003), p.538) we derive the first-order conditions of the problem for all $t \geq t_0$(we present them in Appendix in order not to overload the main text, see equations (244)–(255)). Equations that represent first order conditions (244)–(255) and constraints (237)–(241) fully characterize behaviour of the economy under the optimal policy.

4 The Maastricht convergence criteria – a reinterpretation

The Maastricht convergence criteria have a nonlinear nature as they set specific bounds on both monetary and fiscal variables. Subsequently, derivation of the optimal policy constrained by the Maastricht criteria would involve solving a nonlinear optimization problem that requires computationally demanding techniques. On the other hand, as already emphasized by Lipińska (2008) the linear quadratic approach has two important

[12] As previously shown in papers by Gali and Monacelli (2005) and De Paoli (2007), in the small open economy framework the target variables will be identical to the flexible price allocations only in some special cases, i.e. an efficient steady state, no markup shocks, no expenditure switching effect (i.e. $\rho\theta = 1$) and no trade imbalances. Moreover, as shown by Benigno and Woodford (2003) a non-zero steady state share of government expenditures in output affects the target variables.

[13] As it is going to be seen in our numerical analysis, this stabilisation feature is insignificant in quantitative terms.

advantages: an analytical and intuitive expression for the loss function and also easy to check second-order conditions for local optimality of the derived policy. As a result, following Lipińska (2008) we reformulate the Maastricht criteria in order to introduce them as additional constraints faced by the optimal policy in the linear quadratic approach.

We consider three monetary criteria regarding CPI inflation rate, nominal interest rate and nominal exchange rate and also one fiscal criterion that sets an upper bound on deficit to GDP. We neglect the debt to GDP criterion as almost all the EMU accession countries are characterized by moderate debt to GDP ratios (see Figure (10.2)) that are smaller than the upper limit of 60% quoted in the Stability and Growth Pact. Moreover, in the past failure in compliance with debt to GDP criterion (on the contrary to the deficit to GDP criterion) was not treated as an obstacle to enter to the EMU (e.g. case of Belgium or Italy). This is in accordance with the stipulates of the Maastricht Treaty Article (see Appendix).

Subsequently, we summarize the criteria (described in the introduction) by the following inequalities:

- CPI aggregate inflation criterion

$$\pi_t^A - \pi_t^{A,*} \leq B_\pi, \tag{30}$$

  where $B_\pi = 1.5\%$, π_t^A is annual CPI aggregate inflation in the domestic economy, $\pi_t^{A,*}$ is the average of the annual CPI aggregate inflations in the three lowest inflation countries of the European Union.

- nominal interest rate criterion

$$R_t^L - R_t^{L,A*} \leq C_R \tag{31}$$

  where $C_R = 2\%$, R_t^L is the annul interest rate for ten-year government bond in the domestic economy, $R_t^{L,A*}$ is the average of the annual interest rates for ten-year government bonds in the three countries of the European Union with the lowest inflation rates.

- nominal exchange rate criterion

$$(1 - D_S)S \leq S_t \leq (1 + D_S)S, \tag{32}$$

  where $D_S = 15\%$ and S is the central parity between euro and the domestic currency and S_t is the nominal exchange rate.

- deficit to GDP criterion

$$df_t \leq F_{df} \tag{33}$$

  where $F_{df} = 3\%$ and df_t - annual deficit to GDP. In our framework deficit is defined as a sum of interest payments on outstanding debt minus the primary surplus that consists of tax revenues and government expenditures.

We decide to impose a number of adjustments on the original form of the Maastricht criteria. First, these adjustments originate from the structure of the model which assumes that there are two countries in the world and dynamics are explained in quarters.[14] As a result, we assume that the variables $\pi_t^{A,*}$ and $R_t^{L,A*}$, respectively, represent foreign aggregate inflation and the foreign nominal interest rate, i.e. $\widehat{\pi}_t^*$, $\widehat{R}_t^*$ (which are proxied to be the euro area variables). Subsequently, all the four criteria are reformulated in quarterly terms. We change appropriately upper bounds regarding the CPI inflation rate and the nominal interest rate, i.e. $B_\pi \equiv ((1.015)^{0.25} - 1)$, $C_R \equiv ((1.02)^{0.25} - 1)$. Additionally, we define the central parity of the nominal exchange rate as the steady state value of the nominal exchange rate ($S = \overline{S}$). Moreover, taking into account the evidence

[14] A detailed explanation regarding the reformulation of monetary criteria can be found in Lipinska (2008).

on the predominance of domestic shocks in the EMU accession countries (see Fidrmuc and Kirhonen (2003)) we assume that foreign economy is in the steady state (i.e. foreign inflation and foreign nominal interest rate $(\widehat{\pi}_t^*, \widehat{R}_t^*)$ are zero).[15]

Second, using the method proposed by Rotemberg and Woodford (1997, 1999) and Woodford (2003) we approximate the Maastricht criteria in order to implement them into the linear quadratic framework. The authors propose to approximate the zero bound constraint for the nominal interest rate by restricting the mean of the nominal interest rate to be at least k standard deviations higher than the theoretical lower bound, where k is a sufficiently large number to prevent frequent violation of the original constraint. The main advantage of this alternative constraint over the original one is that it is much less computationally demanding and it only requires computation of the first and second moments of the nominal interest rate.

Similarly to Woodford (2003), we redefine the criteria using discounted averages in order to conform with the welfare measure used in our framework. Let us remark that the average value of any variable (x_t) is defined as the discounted sum of the conditional expectations, i.e.:

$$\widehat{m}(x_t) \equiv E_{t_0} \sum_{t=t_0}^{\infty} \beta^t x_t. \tag{34}$$

Accordingly, its variance is defined by:

$$\widehat{var}(x_t) \equiv E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (x_t - \widehat{m}(x_t))^2. \tag{35}$$

Below, we show the reformulated Maastricht convergence criteria.[16] Each criterion is presented as a set of two inequalities:

- CPI aggregate inflation criterion:

$$(1-\beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_\pi - \widehat{\pi}_t) \geq 0, \tag{36}$$

$$(1-\beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_\pi - \widehat{\pi}_t)^2 \leq K \left((1-\beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_\pi - \widehat{\pi}_t) \right)^2 ; \tag{37}$$

- nominal interest rate criterion:

$$(1-\beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (C_R - \widehat{R}_t) \geq 0 \tag{38}$$

$$(1-\beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (C_R - \widehat{R}_t)^2 \leq K \left((1-\beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (C_R - \widehat{R}_t) \right)^2 \tag{39}$$

- nominal exchange rate criterion must be decomposed into two systems of the inequalities, i.e. the upper bound and the lower bound:

[15] Lipinska (2008) discusses the consequences of relaxing this assumption (e.g. a departure from the steady state of the foreign economy or a suboptimal foreign monetary policy) for the nature of optimal policy constrained by the Maastricht criteria and the associated welfare loss.

[16] The detailed derivation of the Maastricht convergence criteria can be found in Appendix B.

– upper bound

$$(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(D_S-\widehat{S}_t)\geq 0 \tag{40}$$

$$(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(D_S-\widehat{S}_t)^2\leq K\left((1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(D_S-\widehat{S}_t)\right)^2 \tag{41}$$

– lower bound

$$(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(D_S+\widehat{S}_t)\geq 0 \tag{42}$$

$$(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(D_S+\widehat{S}_t)^2\leq K\left((1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(D_S+\widehat{S}_t)\right)^2 \tag{43}$$

- deficit to GDP criterion:

$$(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(F_{df}-\widehat{df}_t)\geq 0, \tag{44}$$

$$(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(F_{df}-\widehat{df}_t)^2\leq K\left((1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(F_{df}-\widehat{df}_t)\right)^2; \tag{45}$$

where $K=1+k^{-2}$ and $D_S=15\%$, $B_\pi=(1.015)^{0.25}-1$, $C_R=(1.02)^{0.25}-1$, $F_{df}=3\%$ and $k=1.96$.

The first inequality means that the average values of the CPI inflation rate, the nominal interest rate, the nominal exchange rate and deficit to GDP, respectively, should not exceed the bounds, B_π, C_R and D_S and F_{df}. The second inequality further restrains fluctuations in the Maastricht variables by setting an upper bound on their variances. This upper bound depends on the average values of the Maastricht variables and the bounds, B_π, C_R, D_S and F_{df}. Importantly, it also depends on parameter K which guarantees that the original constraints on the Maastricht variables ((30)–(33)) are satisfied with a high probability. Under a normality assumption, by setting $K=1+1.96^{-2}$, we obtain that fulfillment of inequalities (36)–(45) guarantees that each of the original constraints should be met with a probability of 95%.

Summing up, the set of inequalities (36)–(45) represent the Maastricht convergence criteria in our model.

5 Optimal policy constrained by the Maastricht criteria

Following Woodford (2003) and Lipinska (2008) we present the loss function of the optimal policy constrained by the Maastricht convergence criteria summarized by inequalities (36)–(43) (constrained optimal policy). The loss function of the constrained optimal policy is augmented by the new elements which describe fluctuations in monetary and fiscal variables, i.e.: CPI aggregate inflation, the nominal interest rate, the nominal exchange rate and deficit to GDP ratio.

We state a proposition which can be seen as an extension of the Proposition 1 in Lipińska (2008):[17]

[17] Both propositions are based on Proposition 6.9 (p.428) in Woodford (2003).

Proposition 1 *Consider the problem of minimizing an expected discounted sum of quadratic losses:*

$$E_{t_0}\left\{(1-\beta)\sum_{t=t_0}^{\infty}\beta^t L_t\right\}, \tag{46}$$

subject to (36) - (45). Let $m_{1,\pi}$, $m_{1,R}$, $m_{1,S}^U$, $m_{1,S}^L$, $m_{1,df}$ *be the discounted average values of* $(B_\pi-\widehat{\pi}_t)$, $(C_R-\widehat{R}_t)$, $(D_S-\widehat{S}_t)$, $(D_S+\widehat{S}_t)$, $(F_{df}-\widehat{df}_t)$ *and* $m_{2,\pi}$, $m_{2,R}$, $m_{2,S}^U$, $m_{2,S}^L$, $m_{2,df}$ *be the discounted means of* $(B_\pi-\widehat{\pi}_t)^2$, $(C_R-\widehat{R}_t)^2$, $(D_S-\widehat{S}_t)^2$, $(D_S+\widehat{S}_t)^2$, $(F_{df}+\widehat{df}_t)^2$ *associated with the optimal policy. Then, the optimal policy also minimizes a modified discounted loss criterion of the form (46) with* L_t *replaced by:*

$$\widetilde{L}_t \equiv L_t+\Phi_\pi(\pi^T-\widehat{\pi}_t)^2+\Phi_R(R^T-\widehat{R}_t)^2+\Phi_{S,U}(S^{T,U}-\widehat{S}_t)^2+\Phi_{S,L}(S^{T,L}-\widehat{S}_t)^2+\frac{1}{2}\Phi_{df}(df^T-df_t)^2, \tag{47}$$

under constraints represented by the structural equations of an economy. Importantly, $\Phi_\pi\geq 0$, $\Phi_R\geq 0$, $\Phi_{S,U}\geq 0$, $\Phi_{S,L}\geq 0$, $\Phi_{df}\geq 0$ *and take strictly positive values if and only if the respective constraints (37), (39), (41), (43), (45) are binding. Moreover, if the constraints are binding, the corresponding target values* π^T, R^T, $S^{T,U}$, $S^{T,L}$, df^T *satisfy the following relations:*

$$\pi^T = B_\pi - Km_{1,\pi} < 0 \tag{48}$$

$$R^T = C_R - Km_{1,R} < 0 \tag{49}$$

$$S^{T,U} = D_S - Km_{1,S} < 0 \tag{50}$$

$$S^{T,L} = -D_S + Km_{1,S} > 0 \tag{51}$$

$$df^T = F_{df} - Km_{1,df} < 0. \tag{52}$$

Proof can be found in Appendix B.

In the presence of binding constraints, the optimal policy constrained by the Maastricht criteria do not only lead to smaller variances of the Maastricht variables, it also assigns target values for these variables that are different from the deterministic steady state of the optimal policy. These targets reflect precautionary motive of the constrained policy.[18] In other words, the policy maker needs a buffer when it faces inequality constraints.

As in Lipińska (2008) if the monetary constraints on the CPI inflation or the nominal interest rate are binding, the constrained policy maker sets targets on these variables that are lower than their foreign equivalents. As far as the nominal exchange rate is concerned, depending on which: appreciation or depreciation constraint is binding, constrained policy maker will target, respectively, a more depreciated or more appreciated nominal exchange rate. Finally, when deficit to GDP criterion is binding, the constrained policy maker will target surplus to GDP.

6 Numerical exercise

This section characterizes the optimal monetary and fiscal policy for the economy bound to satisfy the Maastricht convergence criteria. First, we characterize the unconstrained optimal monetary and fiscal policy and control whether such a policy violates any of the Maastricht convergence criteria. Second, we characterize the optimal policy which is only constrained by monetary criteria or fiscal criterion. We analyze how the loss functions are augmented and also the stabilization pattern of the constrained policies. Finally, we describe the optimal policy constrained by all the criteria. We identify which criteria are in quantitative terms important in shaping the constrained policy and also compare the welfare losses among the constrained and unconstrained policy.

[18] Similarly, Woodford (2003) shows that a policy maker constrained by the zero bound on the nominal interest rate targets a positive rate of the nominal interest rate.

6.1 Parameterization

Our calibration follows to a great extent the previous analysis by Lipińska (2008) and also literature on the EMU accession countries (i.e. Laxton and Pesenti (2003) and Natalucci and Ravenna (2007)). We calibrate the model to match the moments of the variables for the Czech Republic economy.

The discount factor, β, equals 0.99, which implies an annual interest rate of around four percent. The coefficient of risk aversion in consumer preferences is set to 2 as in Stockman and Tesar (1995) to get an intertemporal elasticity of substitution equal to 0.5. Inverse of the labour supply elasticity (η) is chosen to be 4 following the micro data evidence and also a small open economy model of Gali and Monacelli (2003). The elasticity of substitution between nontradable and tradable consumption, ϕ, is set to 0.5 as in Stockman and Tesar (1994) and the elasticity of substitution between home and foreign tradable consumption, θ, is set to 1.5 (as in Chari et al. (2002) and Smets and Wouters (2004)). The elasticity of substitution between differentiated goods, σ, is equal to 10, which together with the revenue tax of 0.19[19] implies a markup of 1.37.[20]

The share of nontradable consumption in the aggregate consumption basket, μ, is assumed to be 0.42, while the share of foreign tradable consumption in the tradable consumption basket, λ, is assumed to be 0.4. These values correspond to the weights in CPI reported for the Czech Republic over the period 2000–2005.[21] The steady state shares of the government expenditure to GDP (d_G) and also debt to GDP (b_D) correspond to the average values for the Czech republic economy over the period 1995-2006 and are set to 0.2 and 1.6 respectively. As far as the foreign economy is concerned, we set the share of nontradable consumption in the foreign aggregate consumption basket, μ^*, to be 0.6, which is consistent with the value chosen by Benigno and Thoenisen (2003) regarding the structure of euro area consumption. Finally, the steady state share of debt to GDP in the foreign economy is assumed to be 2.4 which reflects an average debt to GDP ratio in the euro area for the period 1995-2005. Taking government expenditure to GDP and debt to GDP shares as given we obtain that the steady state revenue tax rate should be 19,3%.[22]

Following Natalucci and Ravenna (2007), we set the degree of price rigidity in the nontraded sector, α_N, to 0.85. The degree of price rigidity in the traded sector, α_H, is slightly smaller and equals 0.8. These values are somewhat higher than the values reported in the micro and macro studies for the euro area countries.[23] Still, Natalucci and Ravenna (2007) justify them by a high share of the government regulated prices in the EMU accession countries.

All shocks that constitute the stochastic environment of the small open economy follow the AR(1) process. The parameters of the shocks are chosen to match the historical moments of the variables (see Table B.3 in Appendix B). Similarly to Natalucci and Ravenna (2007) and Laxton and Pesenti (2003), the productivity shocks in both domestic sectors are characterized by a strong persistence parameter equal to 0.85. Standard deviations of the productivity shocks are set to 1.6% (nontraded sector) and 1.8% (traded sector). These values roughly

[19] This value is calculated from the steady state budget constraint. Debt to GDP and steady state share of government expenditures to GDP are taken from the data on the Czech Republic economy.

[20] Martins et al. (1996) estimate the average markup for manufacturing sectors at around 1.2 in most OECD countries over the period 1980-1992. Some studies (Morrison (1994), Domowitz et al (1988)) suggest that the plausible estimates range between 1.2 and 1.7.

[21] Source: Eurostat.

[22] Note that the steady state which we present in the numerical exercise and in the Appendix differs from the steady state used for calibration in two aspects: revenue tax rates can differ between the domestic sectors and are chosen in order to maximize welfare of the domestic consumers.

[23] Stahl (2004) estimates that the average duration between price adjustment in the manufacturing sector is nine months (which corresponds to the degree of price rigidity: 0.67). On the other hand, Gali et al (2001) and Benigno and Lopez-Salido (2003) estimate the aggregate supply relations for the European countries and find the overall degree of price rigidity for these countries to be 0.78.

reflect the values chosen by Natalucci and Ravenna (2007), 1.8% (nontraded sector) and 2% (traded sector). Moreover, the productivity shocks are strongly correlated, their correlation coefficient is set to 0.7.[24] All other shocks are independent of each other. Parameters defining the preference shock are, 0.72% (standard deviation) and 0.95 (persistence parameter). These values are similar to the values chosen by Laxton and Pesenti (2003), 0.4% (standard deviation) and 0.7 (persistence parameter). Parameters of the foreign consumption shock are estimated using quarterly data on aggregate consumption in the euro area over the period 1990-2005 (source: Eurostat). The standard deviation of the foreign consumption shock is equal to 0.23% and its persistence parameter is 0.85. Similarly, parameters of the domestic government expenditure shock are estimated based on quarterly data on the final consumption of general government in the Czech Republic over the period 1995-2006. The standard deviation of the domestic government expenditure shock is equal 2% and its persistence parameter is 0.5.

Following Natalucci and Ravenna (2007), we parametrize the monetary policy rule, i.e. the nominal interest rate follows the rule described by: $\widehat{R}_t = 0.9\widehat{R}_{t-1} + 0.1(\widehat{\pi}_t + 0.2\widehat{Y}_t + 0.3\widehat{S}_t) + \widehat{\varepsilon}_{R,t}$, where $\widehat{\varepsilon}_{R,t}$ is the monetary policy innovation with a standard deviation equal to 0.44%. Such a parametrization of the monetary policy rule enables us to closely match the historical moments of the Czech economy. As far as the fiscal policy is concerned, we choose a fiscal rule described in Duarte and Wolman (2003). The rule takes a form of tax rate adjustment to debt to GDP dynamics: $\tau_t = \tau_{t-1} + \alpha_{b,\tau}(b_t - \bar{b}) + \alpha_{\Delta b,\tau}(b_t - b_{t-1})$. Parameters of the rule are taken from Mitchell et al (2002) and are set to $\alpha_{b,\tau} = 0.04/16$, $\alpha_{\Delta b,\tau} = 0.3/4$.

We summarize all parameters described above in Table B.1 (Structural parameters) and Table B.2 (Stochastic environment) in Appendix B. Moreover Table 3 (Matching the moments) in Appendix B compares the model moments with the historical moments for the Czech Republic economy.

6.2 The unconstrained optimal policy

The optimal monetary and fiscal policy is characterized around the optimal steady state. The optimal steady state is defined as a steady state in which revenue taxes in all the sectors are chosen to maximize welfare of the economies given exogenous share of government expenditure to GDP and debt to GDP. The optimal tax rates in the domestic sectors are equal respectively, in the nontraded sector: $\overline{\tau_N} = -0.38$ and in the home traded sector: $\overline{\tau_H} = 0.57$. The implied tax base (total tax revenue to GDP ratio) is equal to 21%.

As in Lipińska (2008) we obtain that the highest penalty coefficient is assigned to fluctuations in the nontradable sector inflation and home tradable inflation (see $Table$ 1). The optimal monetary and fiscal policy is mainly concerned with stabilization of the domestic inflation, which is in line with core inflation targeting argument (Aoki (2001)) and also literature on the optimal monetary and fiscal policy (e.g. Benigno and Woodford (2003), Benigno and De Paoli (2006)). Moreover, the policy faces also trade-offs between stabilizing the output gap and sector inflation which is reflected in the positive values of the penalty coefficients associated with fluctuations of domestic and international terms of trade.

Similarly to the previous literature on optimal taxation (among others Barro (1979), Aiyagari et al (2002), Schmitt-Grohe and Uribe (2004), Benigno and Woodford (2003)) we find that the optimal policy does not stabilize taxes and debt which implies their nonstationary behaviour. We decide to induce stationarity into the model in order to be able to characterize properly the constrained policy. As already analyzed by Woodford (2003) and Lipińska (2008), the constrained policy is characterized by two components: stabilization one (a coefficient that penalizes fluctuations of a variable of interest) and deterministic one (a target value for a variable of interest). While the stabilization component affects the way policy responds to the shocks, the

[24] Empirical evidence shows that productivity shocks are highly persistent and positively correlated (see Backus et al (1992)).

deterministic component affects the steady state of the optimal policy and therefore also discounted means of the variables. Importantly, due to the nonstationarity of debt and taxes the steady state of the policy would not exist. In order to induce stationarity we add a new element to the original loss function which penalizes debt fluctuations.[25] Value of the coefficient (ϕ_d) is chosen to be quantitatively small in order not to affect dynamics of the model, i.e. $\phi_d = 10^{-4}$.[26]

Table 1: Values of the loss function coefficients

Φ_{π_N}	Φ_{π_H}	Φ_Y	Φ_{T^d}	Φ_T	Φ_{TT^d}	Φ_{YT^d}	Φ_{YT}
123.14	35.29	4.26	0.2	0.17	0.02	-0.58	-0.32

In order to understand nature of the optimal policy we investigate how the optimal policy responds to the shocks. Based on variance decomposition of the Maastricht variables (presented in *Table* 2) we choose to analyze the impulse responses to a nontraded productivity shock.

Table 2: Variance decomposition under the unconstrained policy

	shocks:				
The Maastricht variables:	A_N	A_H	B	C^*	G
CPI inflation	86%	4%	5%	2%	3%
nominal interest rate	87%	4%	1%	1%	7%
nominal exchange rate	79%	3%	15%	2%	1%
deficit to GDP	70%	1%	20%	1%	10%
debt to GDP	74%	1%	21%	1%	3%

Similarly to Lipińska (2008), monetary instrument of the optimal policy - the nominal interest rate decreases in response to a positive nontraded productivity shock. This stabilizes the deflationary pressures in the domestic nontraded sector and at the same time supports increase in aggregate output. As a result, the nominal exchange rate depreciates (in accordance with the uncovered interest rate parity condition). Interestingly, fiscal component of the policy is characterized by a countercyclical behaviour. Such a behaviour of taxes has its origin in a specific structure of the economy, i.e. openness and two domestic sectors. First of all, as already studied by Benigno and De Paoli (2006) an open economy nature of the economy gives the optimal policy maker an incentive to use taxes in the countercyclical way thanks to existence of terms of trade externality.[27] By setting higher taxes in the sector where the shock occurred the optimal policy maker can engineer a welfare-improving real exchange rate appreciation. Secondly, two sector structure creates important trade-offs for the optimal policy maker. This trade-off was already studied by Gali and Monacelli (2005) in a model of monetary union, where monetary and fiscal policy are set optimally under full coordination. In their model, each country's fiscal authority faces a trade-off between stabilization of domestic inflation as opposed to output and fiscal gap. Since the cost of inflation is higher than of the changes in distortionary taxation the optimal policy maker allows for fluctuations in the fiscal instruments.[28] As a result, in our model revenue taxes in the nontraded sector rise in order to

[25] This additional element is a bit ad-hoc although it is motivated by an idea of model stationarization by Schmitt-Grohe and Uribe (2003). Alternatively, if one assumes that government debt is denominated in foreign currencies introduction of portfolio adjustment costs (presented by Schmitt-Grohe and Uribe (2003)) would also stationarize the model.

[26] For the purposes of sensitivity analysis we also present the results for $\phi_d = 10^{-5}$ and also for the unconstrained policy $\phi_d = 0$.

[27] This incentive is present under an assumption of the substitutability between home and foreign goods.

[28] Notice however that on the contrary to our model Gali and Monacelli (2005) study a demand side fiscal instrument, i.e. government expenditures.

stabilize the nontraded output and deflation in the nontraded sector. At the same time, revenue taxes in the home traded sector decrease to stabilize the home traded output and inflationary pressures in this sector.

Consequently, as in Gali and Monacelli (2005) domestic inflation stabilizes. Moreover, output increases in both domestic sectors. Finally, since the overall tax revenues rise deficit to GDP and debt to GDP decrease.

$[Figure(1)\ about\ here]$

Let us investigate now which Maastricht criteria are not satisfied by the optimal policy. Under the optimal policy means of all the variables are zero so the reinterpreted Maastricht criteria can be reduced to the constraints that set upper bounds on the variances of the Maastricht variables, i.e.:

$$\widehat{var}(\widehat{\pi}_t) \leq (K-1)B_{\pi}^2 \tag{53}$$

$$\widehat{var}(\widehat{R}_t) \leq (K-1)C_R^2 \tag{54}$$

$$\widehat{var}(\widehat{S}_t) \leq (K-1)D_S^2 \tag{55}$$

$$\widehat{var}(\widehat{df}_t) \leq (K-1)F_{df}^2 \tag{56}$$

where $\widehat{var}(x_t)$ with $x_t = \widehat{\pi}_t$, $\widehat{R}_t$, $\widehat{S}_t$, $\widehat{df}_t$ is defined by (35).

In the table below ($Table$ 3) we present the variances of the Maastricht variables together with the upper bounds implied by the Maastricht criteria. We also show variance of debt to GDP and a respective bound for this variables in accordance with the limit set out in the Maastricht Treaty. Let us note that although variances of debt and deficit to GDP ratio do depend on the chosen value of the coefficient ϕ_d variances of other Maastricht variables do not.

Table 3: Variances of the Maastricht variables under optimal policy

ϕ_d	$\widehat{\pi}_t$	$\widehat{R}_t$	$\widehat{S}_t$	$\widehat{df}_t$	$\widehat{b}_t$
0	0.5808	0.4173	22.3785	4.1889	2392.8873
ϕ_d	$\widehat{\pi}_t$	$\widehat{R}_t$	$\widehat{S}_t$	$\widehat{df}_t$	$\widehat{b}_t$
10^{-4}	0.5808	0.4088	22.8337	3.3054	702.2339
ϕ_d	$\widehat{\pi}_t$	$\widehat{R}_t$	$\widehat{S}_t$	$\widehat{df}_t$	$\widehat{b}_t$
10^{-5}	0.5808	0.4156	22.5169	4.0013	1758.4676
bound	0.0356	0.0651	58.5693	2.3428	650.7705
probability[29]	0.69	0.78	1	0.92	0.99
criterion	*violated*	*violated*	*satisfied*	*violated*	*satisfied*[30]

note: Variances and bounds are multiplied by 100^2 (in $(\%)^2$)

[29] Since debt follows a near nonstationary and also very persistent process we perform a Monte Carlo simulation exercise in which we simulate our model for $T = 50$ periods and repeat this simulation $J = 1000$ times. Based on this, we can calculate the average probabilities, for each of the Maastricht variables, of compliance with the criteria. We assume that a given criterion is not satisfied if the probability for a given variable is lower than 95% (which is in accordance with the parameter k).

[30] satisfied on the basis of the value of the simulated average probability.

The optimal unconstrained policy does not satisfy three of the Maastricht criteria: the CPI inflation criterion, the nominal interest rate criterion and the deficit to GDP criterion. As a result, the loss function of optimal policy that satisfies the Maastricht criteria has to have some additional elements.

6.3 The constrained optimal policy

6.3.1 The optimal policy constrained by monetary criteria

Now we analyze the policy constrained only by monetary criteria: CPI inflation rate and the nominal interest rate. In particular, we examine whether in the presence of the monetary criteria fiscal policy can act as an additional stabilization tool.

First, we present parameters of the loss function associated with this constrained policy. The loss function takes the following form:

$$\widetilde{L}_t^m = L_t^s + \frac{1}{2}\phi_\pi(\pi^T - \widehat{\pi}_t)^2 + \frac{1}{2}\phi_R(R^T - \widehat{R}_t)^2 \tag{57}$$

where $\phi_\pi > 0$, $\phi_R > 0$ and $\pi^T < 0$, $R^T < 0$. Similarly to the policy constrained only by the fiscal criterion, values of parameters $(\phi_\pi, \phi_R, \pi^T, R^T)$ can be obtained from the solution to the minimization problem of the loss function L_t^s constrained by structural equations and also the monetary constraints. *Table* 4 provides the specific values for all the parameters for two different values of the penalty coefficient on debt fluctuations. It appears that values of the parameters of the constrained policy by monetary criteria do not depend to a great extent on the degree of debt stabilization (ϕ_d). Importantly, values of the penalty coefficients on the nominal interest rate and CPI inflation rate are of the same magnitude as the penalty coefficients of the domestic inflation rates in the original loss function. Deterministic component of the constrained policy tells us that the policy maker constrained by monetary criteria should target CPI inflation rate and the nominal interest rate that are 0.8% *p.a.* lower than in the countries of reference.

Table 4: The policy constrained by monetary criteria
Parameters of the augmented loss function

ϕ_d	Φ_π	Φ_R	π^T (in %)	R^T (in %)
10^{-4}	36	31.1	-0.2082	-0.2331
10^{-5}	35.62	31.12	-0.2005	-0.2250

Second, we show moments of the Maastricht variables under the policy constrained by monetary criteria (see Table 5). As far as discounted means are concerned, negative targets of the CPI inflation rate and the nominal interest rate lead to negative means in all the Maastricht variables, except for the mean of debt to GDP. A higher mean of debt to GDP results from a higher mean of surplus to GDP and higher means of revenue taxes (to be seen later in the analysis of the impulse responses). Variances of the nominal variables: CPI inflation rate, the nominal interest rate and nominal exchange rate are lower than under the optimal unconstrained policy. However, this smaller variability of nominal variables is accompanied by much higher variability of the fiscal variables: deficit to GDP and debt to GDP. Compliance with monetary criteria restricts usage of the nominal interest rate as a stabilization tool and requires stronger movements in taxes. These fiscal instruments have a direct impact on domestic inflation rates and also dampen changes in the aggregate output when responding to shocks. Subsequently, surplus to GDP is characterized by much higher variance and so does deficit to GDP and debt to GDP.

Table 5: Moments of the Maastricht variables
The policy constrained by monetary criteria

ϕ_d		$\widehat{\pi}_t$	$\widehat{R}_t$	$\widehat{S}_t$	$\widehat{df}_t$	$\widehat{b}_t$
10^{-4}	mean (in %)	-0.0667	-0.0688	-6.5996	-0.0969	1.5020
	variance	0.0836	0.0501	15.7296	15.9482	1419.7149
ϕ_d		$\widehat{\pi}_t$	$\widehat{R}_t$	$\widehat{S}_t$	$\widehat{df}_t$	$\widehat{b}_t$
10^{-5}	mean (in %)	-0.0641	-0.0662	-6.3404	-0.0760	2.5511
	variance	0.0828	0.0495	15.2364	17.1314	3137.7899
criterion		*satisfied*	*satisfied*	*satisfied*	*violated*	*violated*

note: Variances are multiplied by 100^2 (in $(\%)^2$)

Third, we analyze how the policy constrained by monetary criteria differs from the optimal unconstrained policy in the stabilization process of an economy hit by a shock. We choose the shock that explains the most of variability of the Maastricht variables (see $Table$ 2 on variance decomposition).

[$Figure$(3) *about here*]

The policy constrained by monetary criteria aims at stabilizing CPI inflation and restricts the nominal interest rate movements. Accordingly, the monetary policy increases nominal interest rate on impact. Thanks to this, nominal exchange rate depreciates by less dampening the inflationary impact of the import sector on the aggregate CPI. However such a contractionary behaviour of the monetary policy leads to stronger deflationary pressures in the domestic sector. The domestic deflation is partly stabilized by the fiscal component of the constrained policy which is more countercyclical than the unconstrained policy, i.e. revenue taxes rise in both domestic sectors. This leads to a much stronger decrease in deficit to GDP, debt to GDP and also dampened increase in domestic aggregate output in comparison with the unconstrained policy.

6.3.2 The optimal policy constrained by fiscal criterion

Let us now present the constrained policy by the fiscal criterion: deficit to GDP criterion. We concentrate on how the fiscal criteria affect the ability of fiscal policy to stabilize business cycle fluctuations. The loss function of the policy constrained by the deficit to GDP criterion can be represented in the following way:

$$\widetilde{L}_t^f = L_t^s + \frac{1}{2}\phi_{df}(df^T - df_t)^2 \tag{58}$$

where $L_t^s = L_t + \phi_d \widehat{d}_t$ and $\phi_{df} > 0$ and $df^T < 0$. The solution to the minimization problem of the loss function L_t^s constrained by structural equations and also the constraint on deficit to GDP gives us values for the parameters ϕ_{df} and df^T. We present these values in $Table$ 6 for two different values of the coefficient on debt stabilization.

Table 6: The policy constrained by deficit to GDP criterion Parameters of the augmented loss function		
ϕ_d	ϕ_{df}	df^T (%)
10^{-4}	0.0304	-1.58
10^{-5}	0.0219	-6

Values of the penalty coefficient on the deficit to GDP fluctuations are small in comparison with penalty coefficients associated with the variables present in the loss function. At the same time, deterministic component of the constrained policy involves targeting surplus to GDP equal to 1.6%. The sensitivity analysis reveals that parameters of the augmented loss function do depend on the chosen value of ϕ_d. However the general pattern of the constrained policy is the same. The goal of complying with the deficit to GDP criterion is achieved rather through deterministic component than the stabilization one.

Let us now check how the optimal policy constrained by deficit to GDP criterion affects compliance of the monetary criteria. In $Table$ 7 we present the means and variances of all the Maastricht variables and also report whether each of the criteria is satisfied (based on the inequalities (30)–(33)).

Table 7: Moments of the Maastricht variables The policy constrained by deficit to GDP criterion						
ϕ_d		$\widehat{\pi}_t$	$\widehat{R}_t$	$\widehat{S}_t$	$\widehat{df}_t$	$\widehat{b}_t$
10^{-4}	mean (in %)	$-4 * 10^{-4}$	$2 * 10^{-6}$	-0.0447	-0.0586	-3.6984
	variance	0.5802	0.3951	23.7452	2.4352	582.6315
ϕ_d		$\widehat{\pi}_t$	$\widehat{R}_t$	$\widehat{S}_t$	$\widehat{df}_t$	$\widehat{b}_t$
10^{-5}	mean (in %)	-0.0001	0.0044	-0.0301	-0.4855	-30.6188
	variance	0.5801	0.4030	23.0651	3.1624	1473.1135
	criterion	*violated*	*violated*	*satisfied*	*satisfied*	*satisfied*
	note: Variances are multiplied by 100^2 (in $(\%)^2$)					

Although the effects of a nonzero target for deficit to GDP are quantitatively small we can see that a negative target for deficit to GDP results in smaller discounted means of inflation, the nominal exchange rate and (by definition) also of debt to GDP. On the other hand, mean of the nominal interest rate (and also of aggregate output) increases as a result of the smaller means of revenue taxes (to be seen later when analyzing the impulse responses). Moreover, a smaller variance in the deficit to GDP triggers smaller variances of the CPI inflation and the nominal interest rate. At the same time, variance of the nominal exchange rate increases (this is in line with a higher variance of aggregate output - to be later seen in the analysis of impulse responses).

In order to understand how the nature of the policy constrained by deficit to GDP criterion differs from the optimal unconstrained policy, we analyze how both policies respond to the shocks. As previously, we concentrate on impulse responses to a positive nontraded productivity shock.

[*Figure*(2) *about here*]

The policy constrained by fiscal criterion restricts fluctuations of deficit to GDP. Accordingly, the fiscal component of the constrained policy has a more procyclical nature than the unconstrained policy, i.e. nontraded taxes increase by less and at the same time home traded taxes decrease by more. Interestingly, the monetary policy component of the constrained policy is more contractionary than under the unconstrained policy. Nominal interest rate decreases by less on impact than under the unconstrained policy leading to a smaller decline in debt interest payments. As a result, deficit to GDP decreases by less (surplus to GDP increases by less) and so does the debt to GDP. Moreover, the constrained policy is characterized by higher on impact nominal exchange rate depreciation than under the unconstrained policy. This is consistent with a slightly higher aggregate output (due to lower taxes). Finally, a higher nominal exchange rate depreciation leads to a higher on impact CPI inflation under the constrained policy.

6.3.3 The optimal policy constrained by all the Maastricht criteria

Having analyzed the impact of monetary and fiscal criteria separately on the optimal policy, we turn to the characterization of the optimal policy that complies at the same time with the monetary and fiscal criteria. In particular, we analyze which criteria: put more constraints on the optimal policy.

Similarly to previous sections, we present the parameters of such a policy, its moments and also response of the constrained policy to a positive nontraded productivity shock. Apart from that, we analyze welfare losses associated with the constrained policy and compare them with the loss of the optimal unconstrained policy. We also analyze which criteria: monetary or fiscal contribute the most to the generated loss under the constrained policy.

The loss function of the policy constrained by fiscal criterion: deficit to GDP and the monetary criteria: CPI inflation and the nominal interest rate can be represented in the following form:

$$\widetilde{L}_t = L_t^s + \frac{1}{2}\phi_\pi(\pi^T - \widehat{\pi}_t)^2 + \frac{1}{2}\phi_R(R^T - \widehat{R}_t)^2 + \frac{1}{2}\phi_{df}(df^T - df_t)^2 \tag{59}$$

where $\phi_\pi > 0$, $\phi_R > 0$, $\phi_{df} > 0$ and $\pi^T < 0$, $R^T < 0$, $df^T < 0$. Values of the parameters of such a constrained policy are obtained from the solution to the minimization problem of the loss function (L_t^s) constrained by the structural equations and fiscal and monetary criteria. As can be seen in *Table* 8, penalty coefficients of all the variables of interest are higher than under the policies that are only constrained by fiscal or monetary criteria. This feature reflects conflicting targets of each of the constrained stabilization policies. As far as targets are concerned we detect significant differences for deficit to GDP. As previously, we observe that although targets of deficit to GDP and debt do depend on the chosen value of the coefficient ϕ_d values of the targets of the monetary variables are not so much sensitive.

Table 8: The policy constrained by all the criteria
Parameters of the augmented loss function

ϕ_d	Φ_π	Φ_R	Φ_{df}	π^T (in %)	R^T (in %)	df^T (in %)
10^{-4}	36	39	0.26	-0.3601	-0.3684	-3.7143
10^{-5}	38	39	0.06	-0.2045	-0.1884	-20.5072

In *Table* 9 we show moments of the Maastricht variables under the optimal policy constrained by monetary and fiscal criteria. As in the case of the policy constrained by monetary criteria, nominal variables have negative means. Moreover, a negative target of deficit to GDP results in negative means of deficit to GDP and also debt to GDP (the negative effect of deficit to GDP on the mean of debt to GDP is stronger than the positive effect of CPI inflation and the nominal interest rate). Variances of the CPI inflation and the nominal interest rate are not significantly higher than under the policy constrained only by monetary criteria. On the other hand, variance of deficit to GDP is much higher than under the policy constrained only by the fiscal criterion. Variance of the nominal exchange rate is also a bit higher than under the policy constrained by monetary criteria.

Importantly, under chosen reinterpretation of the Maastricht criteria, the nominal exchange rate does not satisfy the lower (appreciation) bound of the Maastricht criteria. As we know, nominal exchange rate movements depend on the nominal interest rate behaviour through the uncovered interest rate parity. But since the nominal exchange rate has a nonstationary character a smaller variance of the nominal interest rate actually increases persistence of the nominal exchange rate movements (in the extreme situation when nominal interest rate does not change the nominal exchange rate jumps to the new level on impact and does not change for subsequent periods). As a result, variance of the nominal exchange rate can overvalue variability of the nominal exchange rate. That is why, since the variance of the nominal interest rate is 6 times smaller under the constrained optimal policy than under the unconstrained optimal policy, we assume that the nominal exchange rate criterion is satisfied under the optimal policy constrained by all the criteria.

Table 9: Moments of the Maastricht variables
Optimal constrained policy

ϕ_d		$\widehat{\pi}_t$	$\widehat{R}_t$	$\widehat{S}_t$	$\widehat{df}_t$	$\widehat{b}_t$
10^{-4}	mean (in %)	-0.1235	-0.1228	-12.2950	-1.0508	-53.6591
	variance	0.1012	0.0629	20.7093	4.2714	520.4854
ϕ_d		$\widehat{\pi}_t$	$\widehat{R}_t$	$\widehat{S}_t$	$\widehat{df}_t$	$\widehat{b}_t$
10^{-5}	mean (in %)	-0.0720	-0.0621	-7.3238	-3.7156	-227.5543
	variance	0.0852	0.0485	17.5175	11.7396	2178.8886
criterion		*satisfied*	*satisfied*	*satisfied*[31]	*satisfied*	*satisfied*
note: Variances are multiplied by 100^2 (in $(\%)^2$)						

Now we compare the impulse responses of the Maastricht variables to a positive nontraded productivity shock under the policy constrained by all criteria with the unconstrained optimal policy. The fiscal component of the constrained policy has a more countercyclical nature than the unconstrained policy in the first quarters, i.e. taxes in the nontraded sector increase by more than under the unconstrained policy, while taxes in the home traded sector decrease by less than under the unconstrained policy. This reflects a higher importance of the monetary criteria over the fiscal criterion. Still, changes in taxes are not as pronounced as under the policy constrained only by monetary criteria. That is why aggregate output increases by more and deficit to GDP decreases by less than under the policy constrained only by monetary criteria. Finally, the monetary component of the constrained policy features a contractionary behaviour as the policy constrained only by monetary criteria, i.e. nominal interest rate increases on impact to prevent an increase in CPI inflation.

[*Figure*(4) *about here*]

Now, we analyze the welfare losses associated with each policy. In *Table* 10 we report the expected discounted welfare losses for the policies constrained by fiscal criterion alone, by monetary criteria alone and by all the criteria. Importantly, values of the losses are not very much sensitive to the chosen value of debt stabilization coefficient.

[31] in accordance with the discussion above.

Compliance with the fiscal criterion does not induce substantial welfare losses. The welfare cost associated with such a policy is equal to 0.15% of the optimal policy loss. On the other hand, compliance with the monetary criteria generates additional welfare loss that amounts to 43% of the optimal policy loss. This result reflects the fact fiscal policy performs relatively poorly as an additional stabilization tool. The welfare losses come mainly from a higher variability of the domestic inflation rates. Obligation to satisfy both monetary and fiscal criteria involves more welfare costs. An active use of revenue taxes is limited to meet the bound on deficit to GDP variability. As a result, the policy constrained by monetary and fiscal criteria produces an additional welfare cost equal to 60% of the optimal policy loss.

Table 10: Welfare losses for the unconstrained and constrained optimal policy

ϕ^d	UOP	COP-deficit to GDP	COP-monetary criteria	COP-all
0	6.6683		-	
10^{-4}	6.7242	6.7310	9.5872	10.6250
10^{-5}	6.6786	6.6877	9.4945	9.9510

note: Losses are multiplied by 100^2 (in $(\%)^2$)

7 Conclusions

This paper studies the optimal monetary and fiscal policy constrained by the Maastricht convergence criteria in a small open economy exposed to domestic and external shocks. We develop a DSGE model of a small open economy with nominal rigidities and distortionary taxation.

First, we characterize the optimal monetary and fiscal policy from a timeless perspective using the linear quadratic approach. We find that the optimal monetary and fiscal policy (unconstrained policy) should not only target inflation rates in the domestic sectors and aggregate output fluctuations but also domestic and international terms of trade. Second, we analyze how the monetary and fiscal criteria affect the general properties of the optimal policy. We show that the policy constrained by the Maastricht criteria differs from the unconstrained policy along two dimensions: the stochastic and deterministic one. As expected, the constrained policy restricts fluctuations of the Maastricht variables. Moreover, using a precautionary motive such a policy also changes deterministic targets of the Maastricht variables in order to create an additional buffer.

Finally, we also perform a numerical exercise in which we parameterize our model to match the variability of the Czech Republic economy. We find that the optimal monetary and fiscal policy violates three Maastricht convergence criteria: on the CPI inflation rate, the nominal interest rate and deficit to GDP ratio. Similarly to Lipińska (2008) the constrained policy leads to a smaller variability of the CPI inflation and the nominal interest rate. Moreover, the policy is characterized by a deflationary bias which results in targeting the CPI inflation rate and the nominal interest rate that are lower by 1.3% (in annual terms) than the CPI inflation rate and the nominal interest rate in the countries taken as a reference.

Importantly, the constrained policy induces a higher variability of deficit to GDP ratio than under the unconstrained policy. This reflects the fact that monetary criteria play a dominant role in affecting the stabilization process of the constrained policy. Fiscal policy actively uses its instruments to stabilize the economy in the presence of direct constraints on monetary instruments. At the same time, it has to assign a relatively high surplus to GDP ratio target in order to comply with all the criteria. Still, the welfare costs of the constrained policy are quite substantial and amount to 60% of the initial deadweight loss associated with the optimal unconstrained policy.

8 References

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9 Appendix A

9.1 Figures - Comparison of different policies

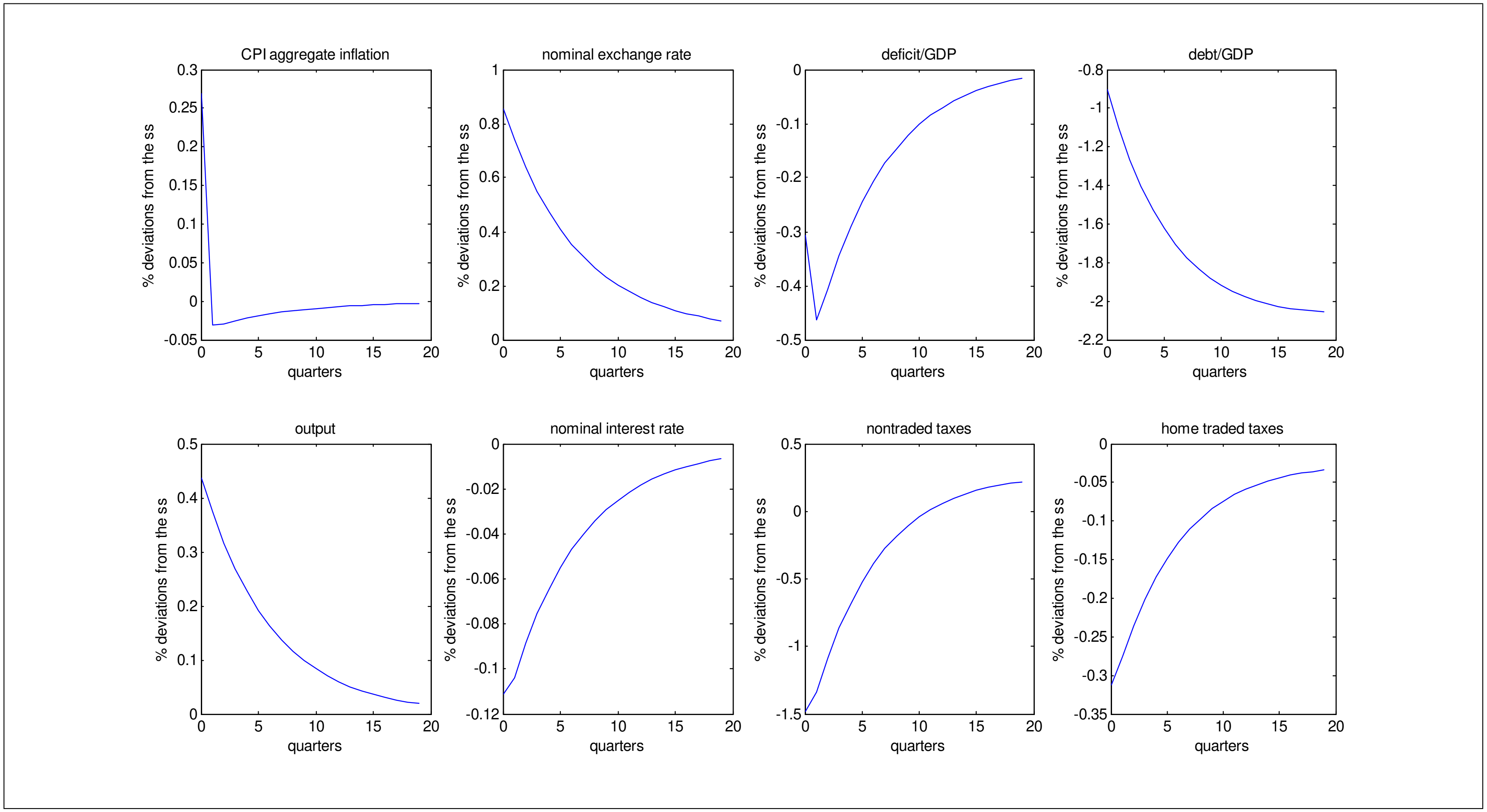

Figure 1: Impulse responses to a positive domestic nontraded productivity shock - unconstrained optimal policy

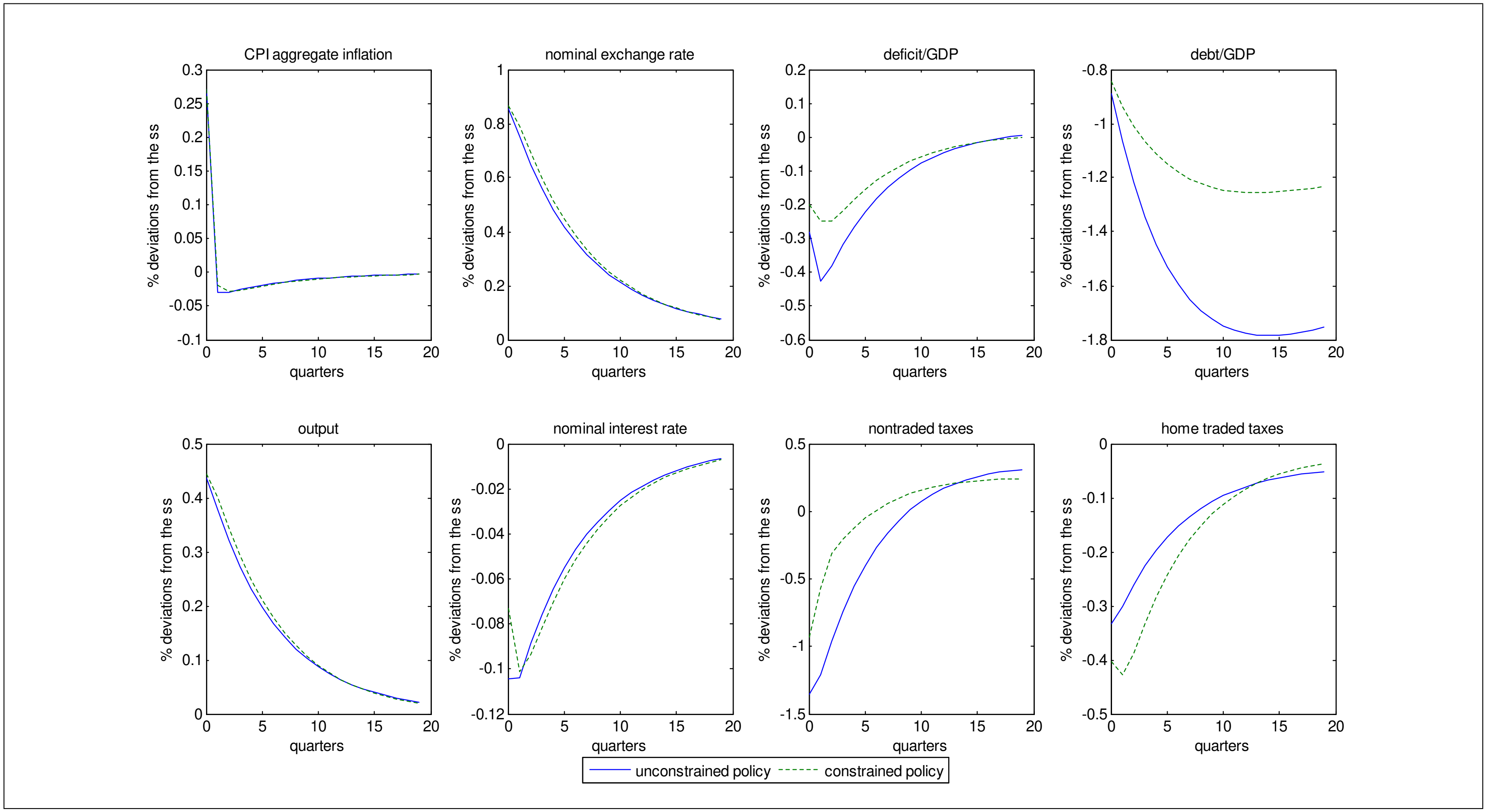

Figure 2: Impulse responses to a positive domestic nontraded productivity shock - fiscal criterion

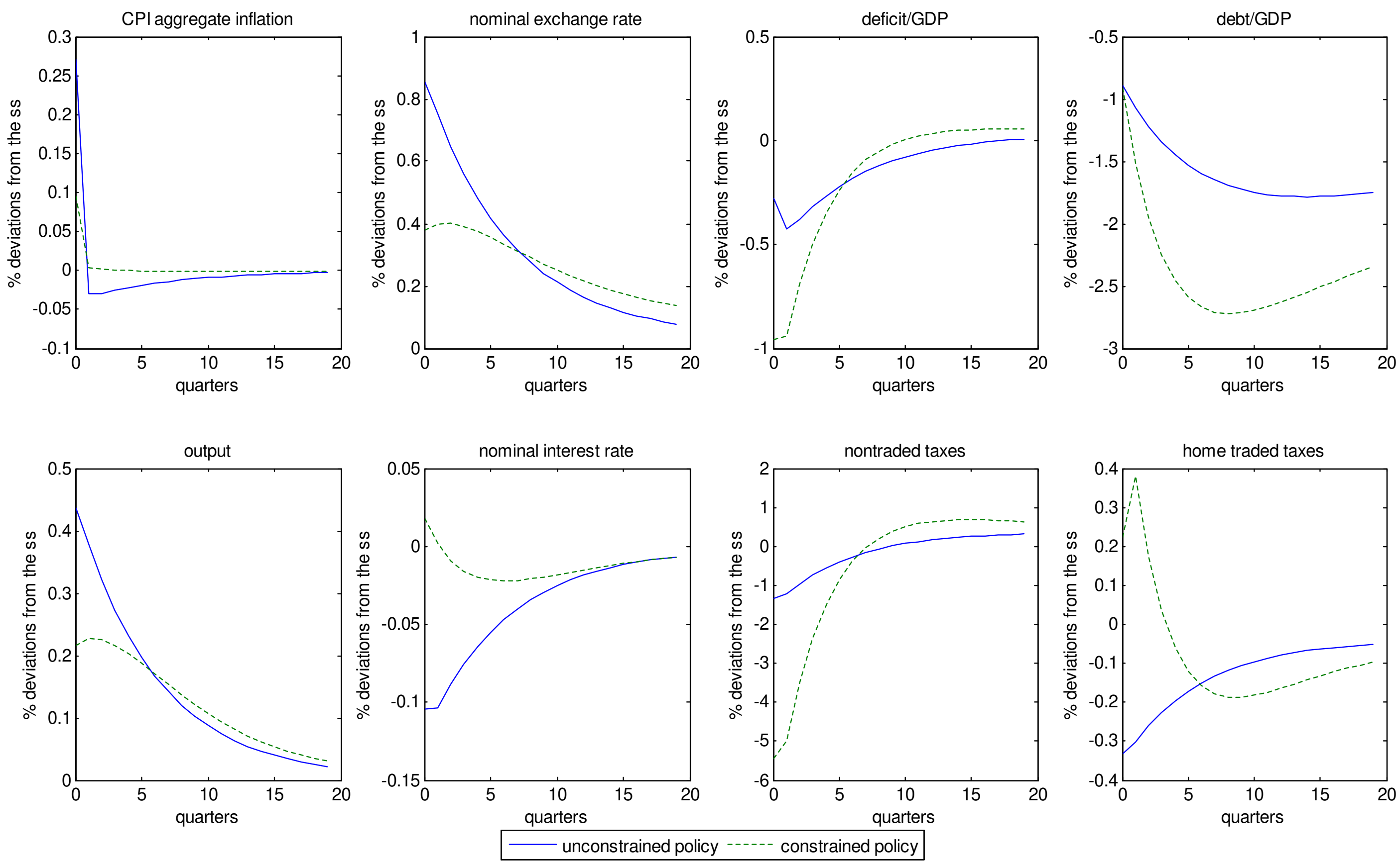

Figure 3: Impulse responses to a positive domestic nontraded productivity shock - monetary criteria

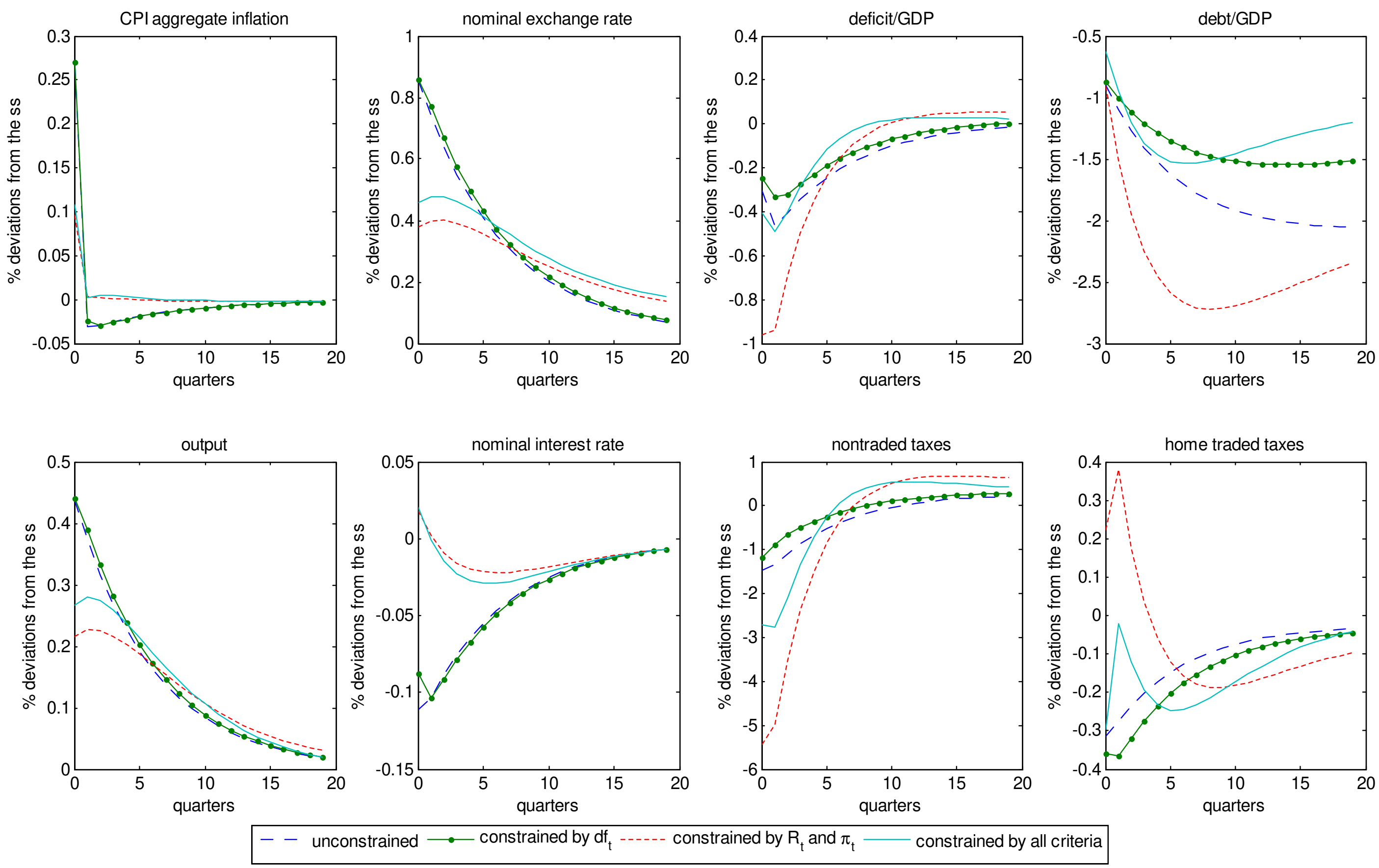

Figure 4: Impulse responses to a positive domestic nontraded productivity shock - comparison of different policies

10 Appendix B

10.1 The Convergence Criteria in the Maastricht Treaty

The Article 109j(1) of the Maastricht Treaty lays down the following monetary criteria as a prerequisite for entering the EMU:

- *the achievement of a high degree of price stability* which means that a Member State (of the EU) has a sustainable price performance and an average rate of inflation (the Consumer Price Index (CPI) inflation), observed over a period of one year before the examination, which does not exceed that of the three best performing Member States in terms of price stability by more than 1.5% points (the CPI inflation rate criterion);
- *the durability of the convergence ... reflected in the long term interest rate levels* which means that, over a period of one year before the examination, a Member State has an average nominal long-term interest rate that does not exceed that of the three best performing Member States in terms of price stability by more than 2% points (the nominal interest rate criterion);
- *the observance of the normal fluctuation margins provided for by the Exchange Rate Mechanism of the European Monetary System* (±15% bound around the central parity), for at least two years, without devaluing against the currency of any other Member State (the nominal exchange rate criterion).

Importantly, the Maastricht Treaty also imposes the criterion on the fiscal policy, i.e. the sustainability of the government financial position which refers to a government budgetary position without an excessive deficit (Article 104c(6) of the Maastricht Treaty). The treaty stipulates: *The sustainability of the government financial position will be apparent from having achieved a government budgetary position without a deficit that is excessive.* In practice, the European Commission sets out two criteria:

- the annual government deficit: the ratio of the annual government deficit to gross domestic product must not exceed 3% at the end of the proceeding financial year. If this is not the case, the ratio must have declined substantially and continuously and reached a level close to 3% (interpretation in trend terms according to Article 104(2)) or alternatively, must remain close to 3% while representing only an exceptional and temporary excess
- government debt: the ratio of gross government debt to GDP must not exceed 60% at the end of the preceding financial year. If this is not the case, the ratio must have sufficiently diminished and must be approaching the reference value at a satisfactory pace (interpretation in trend terms according to Article 104(2)).

10.2 Data on EMU Accession Countries

We present figures and data regarding the EMU accession countries. All the data were collected from the Eurostat database and the European Commission webpage.

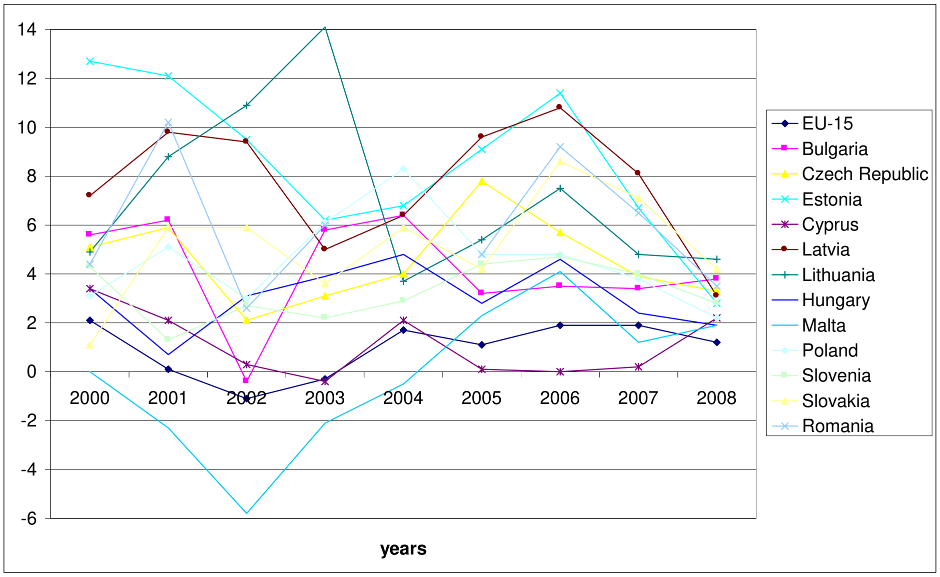

Figure B.1.: Total annual labour productivity growth in the EMU accession countries and the EU 15 (annual rates in %) for the period 2000 - 2008. Values for 2007 and 2008 are forecasts.

Table B.1: Structure of the EMU accession countries

countries	share of nontradables in consumption*	share of imports in GDP#
Czech Republic	42%	68%
Estonia	39%	86%
Hungary	44%	71%
Latvia	37%	55%
Lithuania	33%	58%
Poland	37%	35%
Slovenia	49%	59%
Slovakia	41%	78%
average in the EU - 15	51%	63%
note: *– average value for the period 2000 - 2005; #– average value for the period 2000 - 2007		

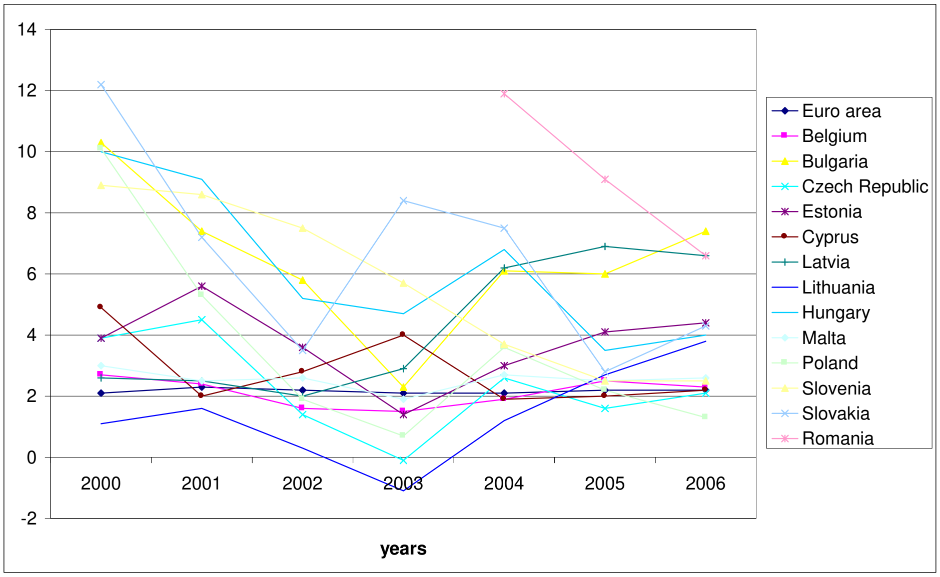

Figure B.2: CPI inflation in the EMU accession countries and the EU - 15 in 2000 - 2006 (annual % rates). For the purpose of clarity, CPI inflation rates in Romania are reported only for 2004 - 2006.

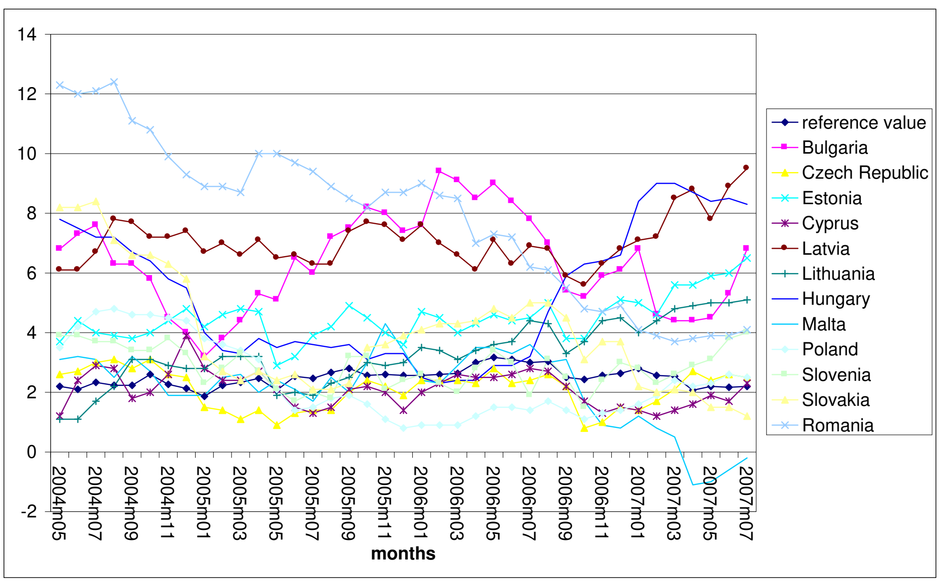

Figure B.3: CPI inflation rates in the EMU accession countries since their accession to the EU (annual rates in %)

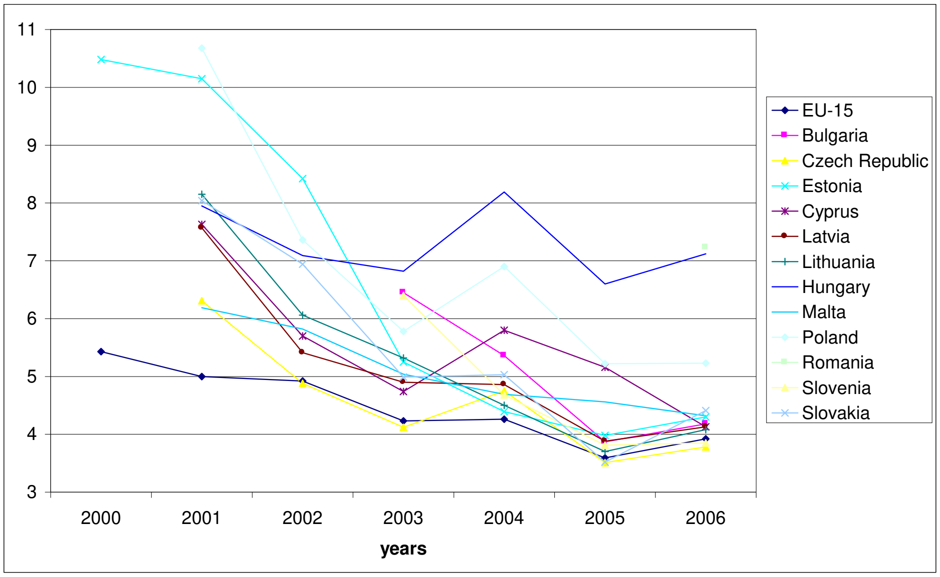

Figure B.4: EMU convergence criterion bond yields for the EMU accession countries and the euro area in 2001 - 2006 (annual % rates)

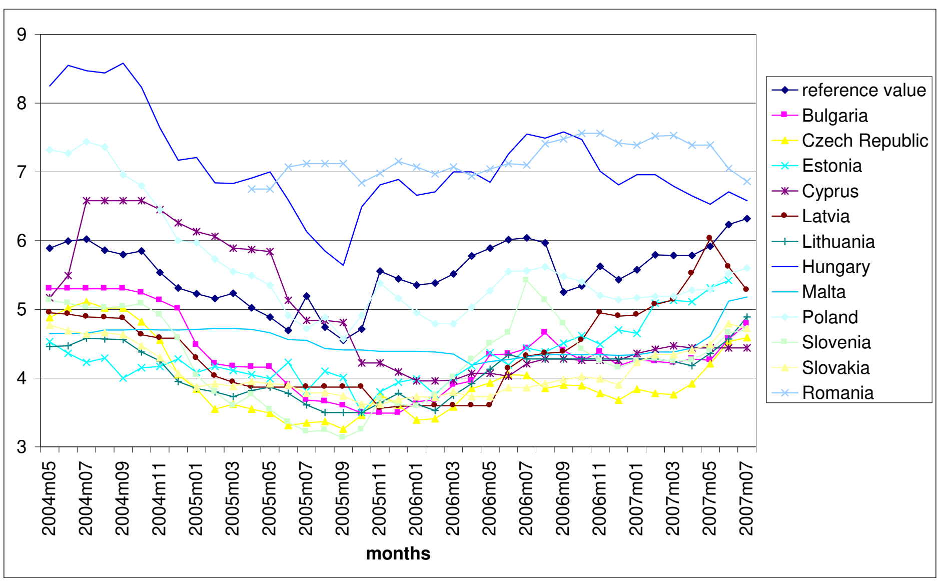

Figure B.5: EMU convergence criterion bond yields for the EMU accession countries since their accession to the EU (annual rates in %)

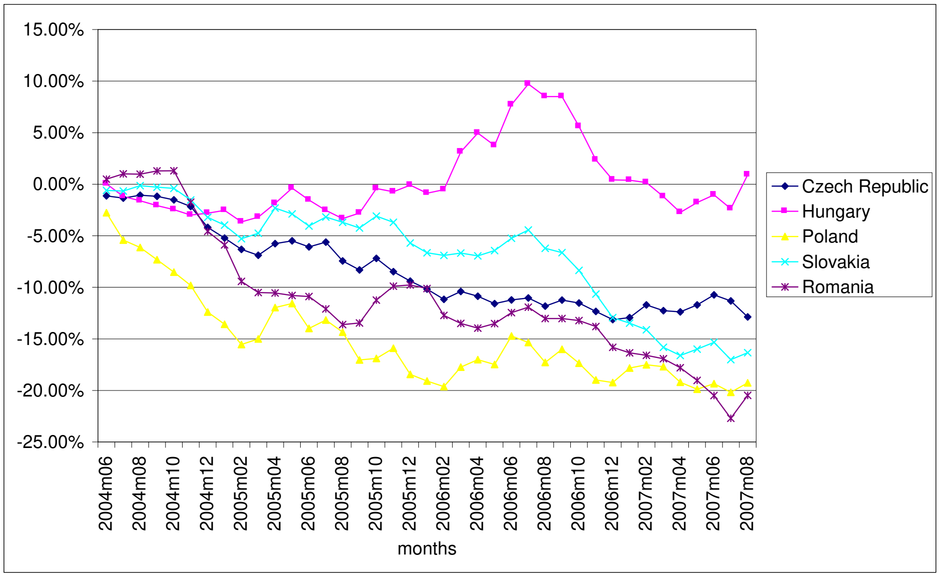

Figure B.6: Nominal exchange rate fluctuations vs. euro of the EMU accession countries since the accession to the EU (average monthly changes since the EU accession date)

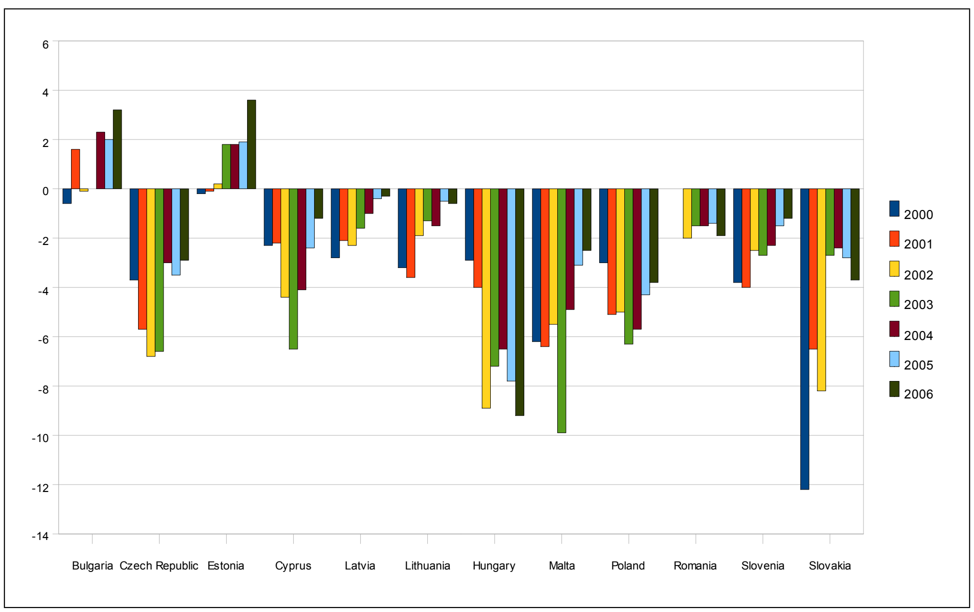

Figure B.7: Deficit to GDP ratio in the EMU Accession Countries in 2000 - 2005 (annual % rates)

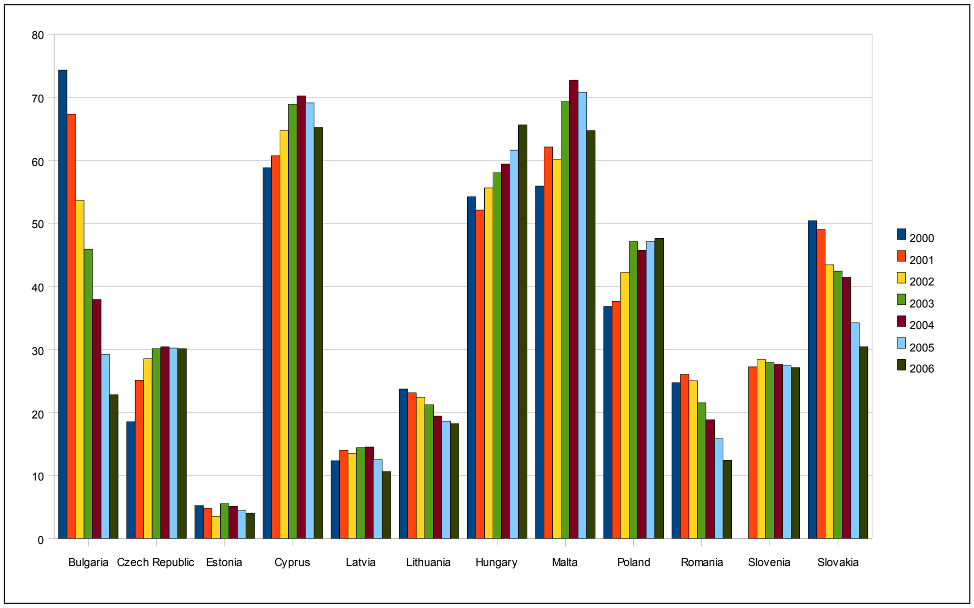

Figure B.8: Debt to GDP ratio in the EMU Accession Countries in 1998 - 2005 (annual % rates)

10.3 Characteristics of the model

10.3.1 Parameterization

We present values of the structural parameters and also values of the stochastic parameters chosen in the numerical exercise.

Table B.1: Structural parameters		
The parameter definition	value of the parameter	
inverse of the intertemporal elasticity of substitution	ρ	2
inverse of the labour supply elasticity	η	4
discount factor	β	0.99
intratemporal elasticity between variety of the goods	σ	10
elasticity of substitution between home and foreign tradables	θ	1.5
elasticity of substitution between tradables and nontradables	ϕ	0.5
share of nontradables	μ	0.42
degree of openness	λ	0.4
price rigidity in the nontradable sector	α_N	0.85
price rigidity in the home tradable sector	α_H	0.8
steady state share of taxes in the nontradable sector	τ_N	0.19
steady state share of taxes in the tradable sector	τ_H	0.19
steady state share of government expenditure in GDP	d_G	0.2
steady state debt to GDP ratio	b_D	1.6
Foreign economy:		
steady state share of government expenditure in GDP	d_G^*	0.2
share of nontradables	μ^*	0.6
steady state debt to GDP ratio	b_D^*	2.4

Table B.2: Stochastic environment		
shocks	autoregressive parameter	standard deviation (in %)
nontradable productivity (A_N)	0.85	1.6
tradable productivity (A_H)	0.85	1.8
preference (B)	0.95	0.72
foreign consumption (C^*)	0.85	0.23
government expenditure (G)	0.8	2
$corr(\widehat{A}_{N,t}, \widehat{A}_{H,t}) = 0.7$ where $corr$ - correlation coefficient		

Note: The policy rule is calibrated following Natalucci and Ravenna (2007): $\widehat{R}_t = 0.9\widehat{R}_{t-1} + 0.1(\widehat{\pi}_t + 0.2\widehat{Y}_t + 0.3\widehat{S}_t) + \widehat{\varepsilon}_{R,t}$, where $SD(\widehat{\varepsilon}_{R,t}) = 0.44$.

Table B.3: Matching the moments		
	Statistics	
Standard deviation in %	Model	Historical
Output:	1.79	1.68
nontraded sector	1.77	1.56
traded sector	3.25	4.32
Consumption	2.08	1.93
Nominal interest rate	0.51	0.52
Nominal exchange rate	2.83	2.59
Real exchange rate	2.36	3.62
CPI inflation rate:	0.81	0.91
nontraded sector	0.58	0.97
traded sector	0.92	0.74

Note: The model moments are theoretical. As far as the historical statistics are concerned our data sample for the Czech Republic is 1995:1 - 2006:2. CPI inflation rate in the traded and nontraded sector data sample is 2000:1 - 2006:2. All series are logged (except for interest and inflation rates) and Hodrick - Prescott filtered. Rates of change are quarterly. All data were collected from the Eurostat webpage. Data are seasonally adjusted where appropriate. We present the detailed data series. Output: Gross value added (GVA) at 1995 constant prices in national currency. Traded output is an aggregate of sectoral GVA for: Agriculture; Hunting; Forestry and Fishing; Total industry (excluding construction). Nontraded output is an aggregate of sectoral GVA for: Wholesale and retail trade, repair of motor vehicles, motorcycles and personal household goods; Hotels and restaurants; Transport, storage and communication; Financial intermediation, real estate, renting and business activities. Consumption: Final consumption expenditure of households at 1995 constant prices in national currency. Nominal interest rate: three months T - bill interest rate. Nominal exchange rate: Bilateral Koruny/euro exchange rate (quarterly average). Real exchange rate: CPI based real effective exchange rate (6 trading partners, quarterly average). CPI inflation rate: Harmonised Index of Consumer Prices (HICP). CPI inflation rate in the nontraded sector: HICP - Services. CPI inflation in the traded sector: HICP - Goods.

10.3.2 Steady state characterization

We define a deterministic steady state with zero inflation rate. We present a small open economy as the limiting case of a two country model, i.e. $n = 0$ and $\nu = 1 - \lambda$. All variables in the steady state are denoted with a bar. All the shocks take the constant values, in particular: $\overline{A_N} = \overline{A_H} = 1$, $\overline{B} = 1$.[32] The discount factors are:

$$Q_{t_0,t} = Q^*_{t_0,t} = \beta^{t-t_0} \tag{60}$$

We assume that the the levels of debt to GDP ratio in both domestic and foreign economy (d_G, d^*_G) are exogenously given. We characterise the steady state with optimal tax rates, i.e. tax rates that maximise welfare given an exogenous level of debt to GDP ratio. Foreign variables are taken as given.

The optimal tax rates in our small open economy can be derived from the following constrained optimization problem:

[32] Foreign consumption is derived from the steady state relations of the foreign economy.

$$\max_{\overline{C},\overline{Y_N},\overline{Y_H},\overline{p_N},\overline{p_H},\overline{p_T},\overline{RS},\overline{\tau_N},\overline{\tau_H}} (U(\overline{C}) - V(\overline{Y_N} + \overline{Y_H})) \tag{61}$$

subject to:

- goods' market clearing conditions:

$$\overline{Y_N} = \overline{p_N}^{-\phi}\mu(\overline{C} + \overline{G}), \tag{62}$$

$$\overline{Y_H} = \overline{p_H}^{-\theta}\overline{p_T}^{\theta-\phi}(1-\lambda)(1-\mu)(\overline{C} + \overline{G}) + \lambda(1-\mu^*)\overline{p_H}^{-\theta}\overline{RS}^{\theta}\overline{p_F^*}^{\theta-\phi}(\overline{C^*} + \overline{G^*}), \tag{63}$$

- relative prices:

$$1 = \mu\overline{p_N}^{1-\phi} + (1-\mu)\overline{p_T}^{1-\phi}, \tag{64}$$

$$1 = (1-\lambda)\overline{p_H}^{1-\theta} + \lambda\overline{p_F^*}^{1-\theta}\overline{RS}^{1-\theta}, \tag{65}$$

- risk sharing condition:

$$\overline{C}^{-\rho} = \overline{RS}^{-1}\overline{C^*}^{-\rho}, \tag{66}$$

- labour market clearing:

$$\overline{p_N} = \frac{\sigma}{\sigma-1}\frac{1}{1-\overline{\tau_N}}(\overline{Y_N} + \overline{Y_H})^{\eta}\overline{C}^{\rho}, \tag{67}$$

$$\overline{p_H} = \frac{\sigma}{\sigma-1}\frac{1}{1-\overline{\tau_H}}(\overline{Y_N} + \overline{Y_H})^{\eta}\overline{C}^{\rho}, \tag{68}$$

  where domestic real wage is equal to the marginal rate of substitution between labour and consumption, i.e. $\overline{\omega} \equiv (\overline{Y_N} + \overline{Y_H})^{\eta}\overline{C}^{\rho}$.

  One can define the markups in both sectors: $\overline{\mu_N} \equiv \frac{\sigma}{\sigma-1}\frac{1}{1-\overline{\tau_N}}$; $\overline{\mu_H} \equiv \frac{\sigma}{\sigma-1}\frac{1}{1-\overline{\tau_H}}$.

- government budget constraint:

$$(\beta^{-1}-1)b_D(\overline{p_N}\overline{Y_N} + \overline{p_H}\overline{Y_H}) = \overline{\tau_N p_N}\overline{Y_N} + \overline{\tau_H p_H}\overline{Y_H} - d_G\overline{Y}. \tag{69}$$

Similarly, we present the constrained maximisation problem that solves for the foreign variables:

$$\max_{\overline{C},\overline{Y_N^*},\overline{Y_F^*},\overline{\tau_N^*},\overline{\tau_H^*},\overline{p_N^*},\overline{p_F^*}} U(\overline{C^*}) - V(\overline{L^*}) \tag{70}$$

subject to:

$$\overline{Y_N^*} = \overline{p_N^*}^{-\phi}\mu^*(\overline{C^*} + \overline{G^*}), \tag{71}$$

$$\overline{Y_F^*} = (1-\mu^*)\overline{p_F^*}^{-\phi}(\overline{C^*} + \overline{G^*}), \tag{72}$$

$$1 = \mu^*\overline{p_N^*}^{1-\phi} + (1-\mu^*)\overline{p_F^*}^{1-\phi}, \tag{73}$$

$$\overline{p_N^*} = \frac{\sigma}{\sigma-1}\frac{1}{1-\overline{\tau_N^*}}(\overline{Y_N^*} + \overline{Y_F^*})^{\eta}\overline{C^*}^{\rho}, \tag{74}$$

$$\overline{p_F^*} = \frac{\sigma}{\sigma-1}\frac{1}{1-\overline{\tau_F^*}}(\overline{Y_N^*} + \overline{Y_F^*})^{\eta}\overline{C^*}^{\rho}, \tag{75}$$

$$(\beta^{-1}-1)b_D^*(\overline{p_N^* Y_N^*} + \overline{p_F^* Y_F^*}) = \overline{\tau_N^* p_N^* Y_N^*} + \overline{\tau_F^* p_F^* Y_F^*} - d_G^*\overline{C^*}. \tag{76}$$

Note that for calibration purposes we derive a steady state in which tax rates are equal in both sectors. The steady state values of domestic variables $(\overline{C}, \overline{Y_N}, \overline{Y_H}, \overline{p_N}, \overline{p_H}, \overline{p_T}, \overline{RS}, \overline{\tau})$ are a solution to the following system of equations:

$$\begin{cases} \overline{Y_N} = \overline{p_N}^{-\phi}\mu(\overline{C}+\overline{G}) \\ \overline{Y_H} = \overline{p_H}^{-\theta}\overline{p_T}^{\theta-\phi}(1-\lambda)(1-\mu)(\overline{C}+\overline{G}) + \lambda(1-\mu^*)\overline{p_H}^{-\theta}\overline{RS}^{\theta}\overline{p_F^*}^{\theta-\phi}(\overline{C^*}+\overline{G^*}) \\ 1 = \mu\overline{p_N}^{1-\phi} + (1-\mu)\overline{p_T}^{1-\phi} \\ 1 = (1-\lambda)\overline{p_H}^{1-\theta} + \lambda\overline{p_F^*}^{1-\theta}\overline{RS}^{1-\theta} \\ \overline{C}^{-\rho} = \overline{RS}^{-1}\overline{C^*}^{-\rho} \\ \overline{p_N} = \frac{\sigma}{\sigma-1}\frac{1}{1-\overline{\tau}}(\overline{Y_N}+\overline{Y_H})^{\eta}\overline{C}^{\rho} \\ \overline{p_H} = \frac{\sigma}{\sigma-1}\frac{1}{1-\overline{\tau}}(\overline{Y_N}+\overline{Y_H})^{\eta}\overline{C}^{\rho} \\ (\beta^{-1}-1)b_D(\overline{p_N}\overline{Y_N}+\overline{p_H}\overline{Y_H}) = \overline{\tau_N p_N}\overline{Y_N} + \overline{\tau_H p_H}\overline{Y_H} - d_G\overline{Y}. \end{cases} \tag{77}$$

10.3.3 A loglinearized version of the model

We present a system of the equilibrium conditions for the small open economy in the loglinear form, which is derived through the first-order approximation around the deterministic steady state with zero inflation described above. Here, we characterize the dynamic features of this model where the variables with a hat stand for the log deviations from the steady state. Variables with an asterisk represent the foreign equivalents of the domestic variables.

The supply-side of the economy is given by two Phillips curves, one for the nontraded and one for the domestic traded sector, respectively, which are derived from (18):

$$\widehat{\pi}_{N,t} = k_N(\rho\widehat{C}_t + \eta\widehat{L}_t - \widehat{A}_{N,t} - \rho\widehat{B}_t - \widehat{p}_{N,t} + \omega_N\widehat{\tau}_{N,t}) + \beta\widehat{\pi}_{N,t+1}, \tag{78}$$

$$\widehat{\pi}_{H,t} = k_H(\rho\widehat{C}_t + \eta\widehat{L}_t - \widehat{A}_{H,t} - \rho\widehat{B}_t - \widehat{p}_{H,t} + \omega_H\widehat{\tau}_{H,t}) + \beta\widehat{\pi}_{H,t+1} \tag{79}$$

where $\widehat{p}_{N,t} \equiv \ln(\frac{P_{N,t}}{P_t})$, $\widehat{p}_{H,t} \equiv \ln(\frac{P_{H,t}}{P_t})$, $\widehat{\pi}_{N,t} \equiv \ln(\frac{P_{N,t}}{P_{N,t-1}})$, $\widehat{\pi}_{H,t} \equiv \ln(\frac{P_{H,t}}{P_{H,t-1}})$, $k_N \equiv \frac{(1-\alpha_N)(1-\alpha_N\beta)}{\alpha_N}$, $k_H \equiv \frac{(1-\alpha_H)(1-\alpha_H\beta)}{\alpha_H}$, $\omega_N \equiv \frac{\overline{\tau_N}}{1-\overline{\tau_N}}$, $\omega_H \equiv \frac{\overline{\tau_H}}{1-\overline{\tau_H}}$ and aggregate labour supply $(\widehat{L}_t)$ is defined through the labour market clearing condition ((11), (17)):

$$\widehat{L}_t = \widetilde{d}_{Y_N}(\widehat{Y}_{N,t} - \widehat{A}_{N,t}) + \widetilde{d}_{Y_H}(\widehat{Y}_{H,t} - \widehat{A}_{H,t}), \tag{80}$$

where $\widetilde{d}_{Y_N} \equiv \frac{\overline{Y}_N}{\overline{Y}_N+\overline{Y}_H}$, $\widetilde{d}_{Y_H} \equiv \frac{\overline{Y}_H}{\overline{Y}_N+\overline{Y}_H}$ are steady state ratios.

It is worth underlining that inflation dynamics in both domestic sectors do not only depend on the real marginal costs in a given sector, but also on the relative prices of goods. In particular, a higher relative price of goods in one sector in relation to other goods induces a substitution away effect and leads to deflationary pressures in this sector.

The demand side of the small open economy is represented by the market clearing conditions in both nontraded and domestic traded sectors ((8), (9)):

$$\widehat{Y}_{N,t} = d_{CN}\widehat{C}_t - \phi\widehat{p}_{N,t} + d_{GN}\widehat{G}_t, \tag{81}$$

$$\widehat{Y}_{H,t} = d_{CH}\widehat{C}_t - \theta\widehat{p}_{H,t} + b(\phi-\theta)d_{CH}\widehat{T}_t^d + (1-d_H)\theta\widehat{RS}_t + d_{C^*H}\widehat{C}_t^* + b^*(\phi-\theta)d_{C^*H}\widehat{T}_t^{d*} + d_{GH}\widehat{G}_t \tag{82}$$

where $d_{CN} \equiv \overline{p_N}^{-\phi}\mu\frac{\overline{C}}{\overline{Y}_N}$, $d_{GN} \equiv \overline{p_N}^{-\phi}\mu\frac{\overline{G}}{\overline{Y}_N}$, $d_{CH} \equiv (1-\lambda)(1-\mu)\frac{\overline{C}}{\overline{Y_H}}\overline{p}_H^{-\theta}\overline{p}_T^{\theta-\phi}$, $d_{C^*H} \equiv \lambda(1-\mu^*)\frac{\overline{C}^*}{\overline{Y_H}}\overline{p_H}^{-\theta}\overline{RS}^{\theta}\overline{p_F^*}^{\theta-\phi}$, $d_{GH} \equiv (1-\lambda)(1-\mu)\frac{\overline{G}}{\overline{Y_H}}\overline{p}_H^{-\theta}\overline{p}_T^{\theta-\phi}$, $b \equiv \mu(\overline{p_N})^{1-\phi}$, $b^* \equiv \mu^*(\overline{p_N^*})^{1-\phi}$ are steady state ratios. Additionally, we define aggregate output as the sum of sector outputs:

$$\widehat{Y}_t = d_{YN}(\widehat{p}_{N,t} + \widehat{Y}_{N,t}) + d_{YH}(\widehat{p}_{H,t} + \widehat{Y}_{H,t}), \tag{83}$$

where $d_{YN} \equiv \frac{\overline{p_N}\,\overline{Y_N}}{\overline{Y}}$ and $d_{YH} \equiv \frac{\overline{p_H}\,\overline{Y_H}}{\overline{Y}}$ are steady state ratios.

The complete asset market assumption (15) gives us the following risk sharing condition:

$$\widehat{C}_t = \widehat{B}_t + \frac{1}{\rho}\widehat{RS}_t + \widehat{C}_t^* - \widehat{B}_t^*. \tag{84}$$

From the definition of price indices ((5), (6)), we obtain the following relations between relative prices, terms of trade, domestic terms of trade and real exchange rate:

$$(a-1)\widehat{p}_{H,t} = b\widehat{T_t^d} + a\widehat{RS_t} - b^* a\widehat{T_t^{d*}}, \tag{85}$$

$$\widehat{p}_{N,t} = (1-b)\widehat{T_t^d}, \tag{86}$$

$$\widehat{p}_{H,t} = -b\widehat{T_t^d} - a\widehat{T}_t, \tag{87}$$

where $a \equiv \lambda\left(\frac{\overline{RS}\,\overline{p_F^*}}{\overline{p_T}}\right)^{1-\theta}$ is the steady state ratio. We also derive the laws of motion for the international terms of trade and the domestic terms of trade from their definitions:

$$\widehat{T}_t = (\widehat{\pi}^*_{F,t} + \Delta\widehat{S}_t) - \widehat{\pi}_{H,t} + \widehat{T}_{t-1}, \tag{88}$$

$$\widehat{T}_t^d = \widehat{\pi}_{N,t} - \widehat{\pi}_{T,t} + \widehat{T}_{t-1}^d, \tag{89}$$

with $\widehat{\pi}_{T,t} \equiv \ln(\frac{P_{T,t}}{P_{T,t-1}})$, $\widehat{\pi}^*_{F,t} \equiv \ln(\frac{P^*_{F,t}}{P^*_{F,t-1}})$ and $\Delta\widehat{S}_t = \widehat{S}_t - \widehat{S}_{t-1}$. Tradable inflation ($\widehat{\pi}_{T,t}$) can be represented as:

$$\widehat{\pi}_{T,t} = \widehat{\pi}_{H,t} + a(\widehat{T}_t - \widehat{T}_{t-1}). \tag{90}$$

Dynamics of the government debt can be derived through the loglinerization of equation (23):

$$\widehat{d}_t = \frac{1}{\beta}(\widehat{d}_{t-1} + \widehat{R}_t - \widehat{\pi}_t) - d_{sr}(\widehat{R}_t + \widehat{sr}_t)$$

where $d_{sr} \equiv \frac{\overline{R}\,\overline{sr}}{\overline{d}}$ and real primary surplus ($\widehat{sr}_t$) evolves according to the loglinearised version of equation (21):

$$\widehat{sr}_t = s_{\tau_N}(\widehat{\tau}_{N,t} + \widehat{p}_{N,t} + \widehat{Y}_{N,t}) + s_{\tau_H}(\widehat{\tau}_{H,t} + \widehat{p}_{H,t} + \widehat{Y}_{H,t}) - s_G\widehat{G}_t$$

where $s_{\tau_N} \equiv \frac{\overline{\tau}_N\overline{p}_N\overline{Y}_N}{\overline{sr}}$; $s_{\tau_H} \equiv \frac{\overline{\tau}_H\overline{p}_H\overline{Y}_H}{\overline{sr}}$ and $s_G \equiv \frac{\overline{G}}{\overline{sr}}$.

Subsequently, an intertemporal government solvency condition has a following form:

$$\widehat{d}_{t-1} - \widehat{\pi}_t - \rho\widehat{C}_t + \rho\widehat{B}_t = (1-\beta)(-\rho\widehat{C}_t + \rho\widehat{B}_t + \widehat{sr}_t) + \beta E_t(\widehat{d}_t - \widehat{\pi}_{t+1} - \rho\widehat{C}_{t+1} + \rho\widehat{B}_{t+1}) \tag{91}$$

Additionally, we present equations defining monetary and fiscal variables that are constrained by the Maastricht criteria: the CPI inflation rate ($\widehat{\pi}_t$), the nominal interest rate ($\widehat{R}_t$) the nominal exchange rate ($\widehat{S}_t$), deficit to GDP ratio (df_t) and debt to GDP ratio (br_t). First, the nominal interest rate can be derived from the loglinearized version of the Euler condition (14):

$$\widehat{R}_t = \rho(\widehat{C}_{t+1} - \widehat{B}_{t+1}) - \rho(\widehat{C}_t - \widehat{B}_t) + \widehat{\pi}_{t+1}, \tag{92}$$

where $\widehat{\pi}_t \equiv \ln(\frac{P_t}{P_{t-1}})$. CPI aggregate inflation is a weighted sum of the sector inflation rates:

$$\widehat{\pi}_t = b\widehat{\pi}_{N,t} + (1-a)(1-b)\widehat{\pi}_{H,t} + a(1-b)\widehat{\pi}^*_{F,t} + a(1-b)(\widehat{S}_t - \widehat{S}_{t-1}). \tag{93}$$

Notice that CPI aggregate inflation does not only depend on the domestic sector inflation rates, but also on the foreign traded inflation rate and changes in the nominal exchange rate. For example, a nominal exchange rate depreciation puts an upward pressure on the CPI inflation rate.

The nominal exchange rate can be derived from the definition of the real exchange rate:

$$\widehat{S}_t = \widehat{S}_{t-1} + \widehat{\pi}_t - \widehat{\pi}^*_t + \widehat{RS}_t - \widehat{RS}_{t-1}. \tag{94}$$

The law of motion of the nominal exchange rate depends on the real exchange rate fluctuations and differences in the aggregate inflation rates between the home and the foreign economy. Additionally, by combining the international risk sharing condition (84) and Euler conditions for the domestic and foreign economy (92), we obtain a relation between the nominal interest rate and the nominal exchange rate:

$$\widehat{S}_t = \widehat{R}^*_t - \widehat{R}_t + \widehat{S}_{t+1}.$$

This equation represents a version of the uncovered interest rate parity, which implies that changes in the nominal exchange rate result from differences between the domestic and foreign monetary policy. Let us point out that although very intuitive, this equation does not constitute an independent equilibrium condition.

Deficit to GDP ratio depends on primary surplus and interest rate payments on debt:

$$df_t = \frac{\frac{d_{t-1}}{R_{t-1}}\pi_t(R_{t-1}-1) - sr_t}{Y_t}.$$

From definition steady state ratio of deficit to GDP ratio is zero. Therefore the loglinearised version of the above equation is:

$$\widetilde{df}_t = \frac{\overline{d}}{\overline{\overline{Y}}}(1-\beta)\widehat{d}_{t-1} + \frac{\overline{d}}{\overline{\overline{Y}}}(1-\beta)\widehat{\pi}_t + \beta\frac{\overline{d}}{\overline{\overline{Y}}}\widehat{R}_{t-1} - \frac{\overline{sr}}{\overline{\overline{Y}}}\widehat{sr}_t, \tag{95}$$

where $\widetilde{df}_t = df_t$.

Finally, debt to GDP evolves according to the following equation:

$$\widehat{br}_t = \widehat{d}_t - \widehat{Y}_t - \widehat{R}_t. \tag{96}$$

The system is closed by specifying a monetary and fiscal rule. In this paper, we derive the optimal monetary and fiscal policy rule which maximizes welfare of the society subject to the structural equations of the economy. The optimal rule is specified as a rule where the monetary and fiscal authority stabilizes the target variables in order to minimize the welfare loss of society and provide the most efficient allocation.[33]

[33] Giannoni and Woodford (2003) call these type of rules flexible inflation targeting rules.

Summing up, the dynamics of the small open economy are summarized by the following variables, $\widehat{\pi}_{N,t}$, $\widehat{\pi}_{H,t}$, $\widehat{C}_t$, $\widehat{L}_t$, $\widehat{Y}_{H,t}$, $\widehat{Y}_{N,t}$, $\widehat{p}_{N,t}$, $\widehat{p}_{H,t}$, $\widehat{Y}_t$, $\widehat{RS}_t$, $\widehat{T}_t^d$, $\widehat{T}_t$, $\widehat{S}_t$, $\widehat{\pi}_t$, $\widehat{R}_t$, $\widehat{\tau}_{N,t}$, $\widehat{\tau}_{H,t}$, $\widehat{d}_t$, $\widehat{df}_t$, $\widehat{br}_t$ which are determined by equations (78)–(96), given the evolution of the stochastic shocks $\widehat{A}_{N,t}$, $\widehat{A}_{H,t}$, $\widehat{B}_t$, $\widehat{G}_t$ and the foreign variables $\widehat{C}_t^*$, $\widehat{T}_t^{d^*}$, $\widehat{\pi}_t^*$, $\widehat{\pi}_{F,t}^*$.[34]

11 Appendix C

11.1 Quadratic representation of the optimal loss function

11.1.1 The second order approximation of the welfare function

We present a second order approximation to the welfare function (1):

$$W_{t_0} = U_C \overline{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [z_v' \widehat{v}_t - \frac{1}{2} \widehat{v}_t' Z_v \widehat{v}_t - \widehat{v}_t' Z_\xi \widehat{\xi}_t] + tip + O(3) \tag{97}$$

where $\widehat{v}_t' = \begin{bmatrix} \widehat{C}_t & \widehat{Y}_{N,t} & \widehat{Y}_{H,t} & \widehat{\pi}_{N,t} & \widehat{\pi}_{H,t} \end{bmatrix}$; $\widehat{\xi}_t' = \begin{bmatrix} \widehat{A}_{N,t} & \widehat{A}_{H,t} & \widehat{B}_t & \widehat{C}_t^* & \widehat{G}_t \end{bmatrix}$; *tip* stands for *terms independent of policy* and $O(3)$ includes terms that are of order higher than the second in the deviations of variables from their steady state values. The matrices z_v, Z_v, Z_ξ are defined below:

$$z_v' = \begin{bmatrix} 1 & -s_{CY_N} & -s_{CY_H} & 0 & 0 \end{bmatrix}, \tag{98}$$

$$Z_v = \begin{bmatrix} \rho - 1 & 0 & 0 & 0 & 0 \\ 0 & s_{CY_N}(1+\eta \widetilde{d}_{Y_N}) & \eta s_{CY_N} \widetilde{d}_{Y_H} & 0 & 0 \\ 0 & \eta s_{CY_N} \widetilde{d}_{Y_H} & s_{CY_H}(1+\eta \widetilde{d}_{Y_H}) & 0 & 0 \\ 0 & 0 & 0 & s_{CY_N}\frac{\sigma}{k_N} & 0 \\ 0 & 0 & 0 & 0 & s_{CY_H}\frac{\sigma}{k_H} \end{bmatrix}, \tag{99}$$

$$Z_\xi = \begin{bmatrix} 0 & 0 & -\rho & 0 \\ -s_{CY_N}(1+\eta \widetilde{d}_{Y_N}) & -\eta s_{CY_N} \widetilde{d}_{Y_H} & 0 & 0 \\ -\eta s_{CY_N} \widetilde{d}_{Y_H} & -s_{CY_H}(1+\eta \widetilde{d}_{Y_H}) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{100}$$

where $s_{CY_N} = \frac{\varpi \overline{Y_N}}{\overline{C}}$; $s_{CY_H} = \frac{\varpi \overline{Y_H}}{\overline{C}}$.

11.1.2 Elimination of the linear terms

This section describes in detail how we eliminate the linear terms in the second order approximation to the welfare function in order to obtain a quadratic loss function. Moreover we reduce the number of structural variables that represent the policy problem by appropriate substitutions.

The optimal monetary and fiscal policy solves the welfare maximization problem with the constraints given by the structural equations of the economy (their loglinearized versions are (78) - (91)). The matrix representation of the second order approximation to the welfare function is the following:

[34] For simplicity, we choose to consider only one type of external shocks, foreign consumption shocks ($\widehat{C}_t^*$). As a result, $\widehat{T}_t^{d^*}$, $\widehat{\pi}_t^*$, $\widehat{\pi}_{F,t}^*$ are assumed to be zero. Moreover, all shocks follow an AR(1) process with normally distributed innovations.

$$W = U_C \overline{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [z_x' \widehat{x}_t - \frac{1}{2} \widehat{x}_t' Z_x \widehat{x}_t - \widehat{x}_t' Z_\xi \widehat{\xi}_t] + tip + O(3). \tag{101}$$

Similarly we present a second order approximation to all the structural equations in the matrix form:

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [\begin{bmatrix} A_1 \widehat{x}_t \\ A_2 \widehat{x}_t \\ ... \\ A_{14} \widehat{x}_t \end{bmatrix} + \frac{1}{2} \begin{bmatrix} \widehat{x}_t{}' B_1 \widehat{x}_t \\ \widehat{x}_t{}' B_2 \widehat{x}_t \\ ... \\ \widehat{x}_t{}' B_{14} \widehat{x}_t \end{bmatrix} + \begin{bmatrix} \widehat{x}_t{}' C_1 \widehat{\xi}_t \\ \widehat{x}_t{}' C_2 \widehat{\xi}_t \\ ... \\ \widehat{x}_t{}' C_{14} \widehat{\xi}_t \end{bmatrix}] + tip + O(3) = 0 \tag{102}$$

with

$$\widehat{x}_t' = \begin{bmatrix} \widehat{Y}_t & \widehat{L}_t & \widehat{C}_t & \widehat{Y}_{N,t} & \widehat{Y}_{H,t} & \widehat{p}_{N,t} & \widehat{p}_{H,t} & \widehat{T}_t^d & \widehat{T}_t & \widehat{RS}_t & \widehat{\Delta S}_t & \widehat{\pi}_{H,t} & \widehat{\pi}_{N,t} & \widehat{\pi}_{T,t} & \widehat{\tau}_{N,t} & \widehat{\tau}_{H,t} \end{bmatrix}, \tag{103}$$

$$\widehat{\xi}_t' \equiv \begin{bmatrix} \widehat{A}_{N,t} & \widehat{A}_{H,t} & \widehat{B}_t & \widehat{C}_t^* & \widehat{G}_t \end{bmatrix}$$

where *tip* means terms independent of policy.

Following the methodology of Benigno and Woodford (2005) in order to eliminate the linear terms in the welfare function we solve the system of linear equations:

$$\zeta A = z_x^{'} \tag{104}$$

where $A_{(14\times16)} = \begin{bmatrix} A_1 \\ A_2 \\ ... \\ A_{14} \end{bmatrix}$;

$\zeta_{(1\times14)} = \begin{bmatrix} \zeta_1 & \zeta_2 & \zeta_3 & \zeta_4 & \zeta_5 & \zeta_6 & \zeta_7 & \zeta_8 & \zeta_9 & \zeta_{10} & \zeta_{11} & \zeta_{12} & \zeta_{13} & \zeta_{14} \end{bmatrix}$ and $z_{x(16\times1)}$.

As a result we obtain the loss function:

$$L_{t_0} = U_C \overline{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [\frac{1}{2} \widehat{x}_t' L_x \widehat{x}_t + \widehat{x}_t' L_\xi \widehat{\xi}_t] + tip + O(3) \tag{105}$$

where

$$L_x = Z_x + \zeta_1 B_1 + \zeta_2 B_2 + \zeta_4 B_4 + \zeta_6 B_6 + \zeta_9 B_9 + \zeta_{10} B_{10} + \zeta_{11} B_{11} + \zeta_{12} B_{12} + \zeta_{13} B_{13} + \zeta_{14} B_{14}, \tag{106}$$

$$L_\xi = Z_\xi + \zeta_1 C_1 + \zeta_2 C_2 + \zeta_4 C_4 + \zeta_6 C_6 + \zeta_{13} C_{13} + \zeta_{14} C_{14}. \tag{107}$$

11.1.3 Substitution of the variables

We want to represent the loss function (105) and also the whole model just in terms of the following variables:

$$\widehat{y}_t' = \begin{bmatrix} \widehat{Y}_t & \widehat{T}_t^d & \widehat{T}_t & \Delta\widehat{S}_t & \widehat{\pi}_{H,t} & \widehat{\pi}_{N,t} & \widehat{\pi}_{T,t} & \widehat{\tau}_{N,t} & \widehat{\tau}_{H,t} \end{bmatrix}. \tag{108}$$

In order to do this we define matrices $N_{x(16\times9)}$ and $N_{\xi(14\times6)}$ that map all the variables in the vector y_t' in the following way:

$$\widehat{x}_t = N_x \widehat{y}_t' + N_\xi' \widehat{\xi}_t \tag{109}$$

where:

$$N_x = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ l_Y & ltd & lt & 0 & 0 & 0 & 0 & 0 & 0 \\ c_Y & ctd & ct & 0 & 0 & 0 & 0 & 0 & 0 \\ yny & yntd & ynt & 0 & 0 & 0 & 0 & 0 & 0 \\ yhy & yhtd & yht & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & pntd & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & phtd & pht & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & rstd & rst & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \tag{110}$$

$$N_x = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ lan & lah & l_B & 0 & l_G \\ 0 & 0 & c_B & 0 & c_G \\ 0 & 0 & ynb & 0 & yng \\ 0 & 0 & yhb & 0 & yhg \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{111}$$

with parameters defined below:

$$
\begin{aligned}
pntd &= 1-b && (112)\\
phtd &= -b && (113)\\
pht &= -a && (114)\\
rstd &= -b && (115)\\
rst &= 1-a && (116)\\
yhc &= d_{CH}+d_{C^*H} && (117)\\
c_Y &= \frac{1}{d_{YN}d_{CN}+d_{YH}yhc} && (118)\\
c_B &= c_Y d_{C^*H}d_{YH} && (119)\\
c_G &= -c_Y(d_{YN}d_{GN}+d_{YH}d_{GH}) && (120)\\
ct &= c_Y(ad_{YH}(1-\theta)-d_{YH}(1-d_H)\theta(1-a)+\frac{1}{\rho}d_{YH}d_{C^*H}(1-a)) && (121)\\
ctd &= c_Y(d_{YN}(1-b)(\phi-1)+bd_{YH}(1-\theta)+b(\theta-\phi)d_Hd_{YH}+ && (122)\\
&\quad +(1-d_H)\theta d_{YH}b-d_{C^*H}\frac{1}{\rho}d_{YH}b) && (123)\\
yny &= c_Y d_{CN} && (124)\\
yntd &= d_{CN}ctd-\phi*(1-b) && (125)\\
ynb &= d_{CN}cb && (126)\\
yng &= d_{GN}+d_{CN}cg && (127)\\
ynt &= d_{CN}ct && (128)\\
yhy &= \frac{1-d_{YN}yny}{d_{YH}} && (129)\\
yhtd &= -\frac{d_{YN}yntd-(d_{YN}(1-b)-bd_{YH})}{d_{YH}} && (130)\\
yht &= -\frac{d_{YN}}{d_{YH}}ynt+a && (131)\\
yhb &= -\frac{d_{YN}}{d_{YH}}ynb && (132)\\
yhg &= -\frac{d_{YN}}{d_{YH}}yng && (133)\\
ltd &= \widetilde{d}_{YN}*yntd+\widetilde{d}_{YH}*yhtd && (134)\\
lt &= \widetilde{d}_{YN}*ynt+\widetilde{d}_{YH}*yht && (135)\\
lan &= -\widetilde{d}_{YN} && (136)\\
lab &= -\widetilde{d}_{YH} && (137)\\
l_B &= \widetilde{d}_{YN}*ynb+\widetilde{d}_{YH}*yhb && (138)\\
l_G &= \widetilde{d}_{YN}*yng+\widetilde{d}_{YH}*yhg && (139)\\
l_Y &= \widetilde{d}_{YN}*yny+\widetilde{d}_{YH}*yhy && (140)
\end{aligned}
$$

The loss function can be expressed now as:

$$L_{t_0} = U_C \overline{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [\frac{1}{2} \widehat{y}_t' L_y \widehat{y}_t + \widehat{y}_t' L_{\xi,y} \widehat{\xi}_t] + tip + O(3) \tag{141}$$

where:

$$L_y = N_x^{'} L_x N_x, \tag{142}$$
$$L_{\xi,y} = N_x^{'} L_x N_\xi + N_x^{'} L_\xi. \tag{143}$$

Since variables $[\Delta S_t, \pi_{T,t}]$ do not appear in the original welfare objective function and in the second order terms of the structural equations we can further reduce the set of the variables which appear in the loss function to:

$$\widehat{y}_t' = \left[\begin{array}{ccccccc} \widehat{Y}_t & \widehat{T}_t^d & \widehat{T}_t & \widehat{\pi}_{H,t} & \widehat{\pi}_{N,t} & \widehat{\tau}_{N,t} & \widehat{\tau}_{H,t} \end{array} \right]. \tag{144}$$

The final set of the structural equations which represent the constraints of the maximization problem is:

$$\widehat{\pi}_{N,t} = k_N \widehat{mc}_t^{N,r} + \beta \widehat{\pi}_{N,t+1}, \tag{145}$$

$$\widehat{\pi}_{H,t} = k_H \widehat{mc}_t^{H,r} + \beta \widehat{\pi}_{H,t+1}, \tag{146}$$

$$0 = n_Y \widehat{Y}_t + n_{T^d} \widehat{T}_t^d + n_T \widehat{T}_t + n_B \widehat{B}_t + n_G \widehat{G}_t - \widehat{C}_t^*, \tag{147}$$

$$\widehat{T}_t^d - \widehat{T}_{t-1}^d = \widehat{\pi}_{N,t} - \widehat{\pi}_{H,t} - a(\widehat{T}_t - \widehat{T}_{t-1}) \tag{148}$$

$$\widehat{d}_{t-1} = f_{\tau_N} \widehat{\tau}_{N,t} + f_{\tau_H} \widehat{\tau}_{H,t} + f_Y \widehat{Y}_t + f_{T^d} \widehat{T}_t^d + f_T \widehat{T}_t + f_{T^d(+1)} \widehat{T}_{t+1}^d + f_{T(+1)} \widehat{T}_{t+1} + f_{T(-1)} \widehat{T}_{t-1} + \tag{149}$$
$$+ f_{\pi_N} \widehat{\pi}_{N,t} + f_{\pi_H} \widehat{\pi}_{H,t} + f_{\pi_{N(+1)}} \widehat{\pi}_{N,t+1} + f_{\pi_{H(+1)}} \widehat{\pi}_{H,t+1} + \beta \widehat{d}_t + \tag{150}$$
$$+ f_G \widehat{G}_t + f_B \widehat{B}_t + f_{C^*} \widehat{C}_t^* - f_{C^*(+1)} \widehat{C}_{t+1}^* \tag{151}$$

where:

$$\widehat{mc}_t^{N,r} = m_{N,Y} \widehat{Y}_t + m_{N,T^d} \widehat{T}_t^d + m_{N,T} \widehat{T}_t + m_{N,\tau_N} \widehat{\tau}_{N,t} + \tag{152}$$
$$+ m_{N,A_N} \widehat{A}_{N,t} + m_{N,A_H} \widehat{A}_{H,t} + m_{N,B} \widehat{B}_t + m_{N,G} \widehat{G}_t \tag{153}$$
$$\widehat{mc}_t^{H,r} = m_{H,Y} \widehat{Y}_t + m_{H,T^d} \widehat{T}_t^d + m_{H,T} \widehat{T}_t + m_{H,\tau_H} \widehat{\tau}_{H,t} + \tag{154}$$
$$+ m_{H,A_N} \widehat{A}_{N,t} + m_{H,A_H} \widehat{A}_{H,t} + m_{H,B} \widehat{B}_t + m_{H,G} \widehat{G}_t \tag{155}$$

with:

$$
\begin{aligned}
m_{N,Y} &= c_Y\rho + l_Y\eta & (156)\\
m_{N,T^d} &= \rho * ctd + \eta * ltd - pntd & (157)\\
m_{N,T} &= \rho * ct + \eta * lt & (158)\\
m_{N,\tau_N} &= \omega_N & (159)\\
m_{N,A_N} &= -(1 + \eta * \widetilde{d}_{YN}) & (160)\\
m_{N,A_H} &= -\eta\widetilde{d}_{YH} & (161)\\
m_{N,B} &= \rho * (c_B - 1) + \eta * lb & (162)\\
m_{N,G} &= \rho c_G + \eta l_G & (163)
\end{aligned}
$$

$$
\begin{aligned}
m_{H,Y} &= c_Y\rho + l_Y\eta & (164)\\
m_{H,T^d} &= \rho * ctd + \eta * ltd - phtd & (165)\\
m_{H,T} &= \rho * ct + \eta * lt - pht & (166)\\
m_{H,\tau_H} &= \omega_H & (167)\\
m_{H,A_N} &= -\eta\widetilde{d}_{YN} & (168)\\
m_{H,A_H} &= -(1 + \eta * \widetilde{d}_{YH}) & (169)\\
m_{H,B} &= \rho * (c_B - 1) + \eta * l_B & (170)\\
m_{H,G} &= \rho c_G + \eta l_G & (171)
\end{aligned}
$$

$$
\begin{aligned}
n_Y &= c_Y & (172)\\
n_{T^d} &= ctd - \frac{1}{\rho} rstd & (173)\\
n_T &= ct - \frac{1}{\rho} rst & (174)\\
n_B &= cb - 1 & (175)\\
n_G &= cg & (176)
\end{aligned}
$$

$$
\begin{aligned}
f_{\tau_N} &= (1-\beta)s_{\tau_N} && (177)\\
f_{\tau_H} &= (1-\beta)s_{\tau_H} && (178)\\
f_Y &= (1-\beta)(s_{\tau_N}yny + s_{\tau_H}yhy) && (179)\\
f_{T^d} &= (1-\beta)(s_{\tau_N}(yny + pntd) + s_{\tau_H}(yhy + phtd)) + \beta rstd && (180)\\
f_{T^d(+1)} &= \beta rstd && (181)\\
f_T &= (1-\beta)(s_{\tau_N}yny + s_{\tau_H}(yhy + pht) + a(1-b)(1+\beta)) + \beta rst && (182)\\
f_{T(+1)} &= \beta(a(1-b) + rst) && (183)\\
f_{T(-1)} &= -a(1-b) && (184)\\
f_{\pi_N} &= b && (185)\\
f_{\pi_H} &= (1-b) && (186)\\
f_{\pi_N(+1)} &= \beta b && (187)\\
f_{\pi_H(+1)} &= \beta(1-b) && (188)\\
f_G &= (1-\beta)(s_{\tau_N}yng + s_{\tau_H}yhg) && (189)\\
f_B &= (1-\beta)(s_{\tau_N}ynb + s_{\tau_H}yhb) && (190)\\
f_{C^*} &= \rho\beta && (191)\\
f_{C^*(+1)} &= -\rho\beta && (192)
\end{aligned}
$$

Structural equations defining the Maastricht variables:

$$
\widehat{R}_t = b\widehat{\pi}_{N,t+1} + (1-b)\widehat{\pi}_{H,t+1} - \rho(1-cb)(\widehat{B}_{t+1} - \widehat{B}_t) + \rho(\widehat{Y}_{t+1} - \widehat{Y}_t) + \rho ctd(\widehat{T}^d_{t+1} - \widehat{T}^d_t) + (\rho ct + a(1-b))(\widehat{T}_{t+1} - \widehat{T}_t), \tag{193}
$$

$$
\widehat{\pi}_t = b\widehat{\pi}_{N,t} + (1-b)\widehat{\pi}_{H,t} + a(1-b)(\widehat{T}_t - \widehat{T}_{t-1}), \tag{194}
$$

$$
\widehat{S}_t = \widehat{S}_{t-1} + \widehat{\pi}_t + rstd(\widehat{T}^d_t - \widehat{T}^d_{t-1}) + rst(\widehat{T}_t - \widehat{T}_{t-1}), \tag{195}
$$

$$
\begin{aligned}
\widetilde{df}_t &= d_d\widehat{d}_{t-1} + d_\pi(b\widehat{\pi}_{N,t} + (1-b)\widehat{\pi}_{H,t} + a(1-b)(\widehat{T}_t - \widehat{T}_{t-1})) + d_R\widehat{R}_{t-1} + && (196)\\
&\quad + d_{sr}(sr_{\tau_N}\widehat{\tau}_{N,t} + sr_{\tau_H}\widehat{\tau}_{H,t} + sr_{T^d}\widehat{T}^d_t + sr_T\widehat{T}_t + sr_Y\widehat{Y}_t + sr_B\widehat{B}_t + sr_G\widehat{G}_t), && (197)
\end{aligned}
$$

$$
\widehat{br}_t = \widehat{d}_t - \widehat{Y}_t - \widehat{R}_t, \tag{198}
$$

where the parameters are defined below:

$$
\begin{aligned}
d_d &= \frac{\overline{d}}{\overline{\overline{Y}}}(1-\beta) \\
d_\pi &= \frac{\overline{d}}{\overline{\overline{Y}}}(1-\beta) && (199) \\
d_R &= \beta\frac{\overline{d}}{\overline{\overline{Y}}} && (200) \\
d_{sr} &= -\frac{\overline{sr}}{\overline{\overline{Y}}} && (201) \\
sr_{T^d} &= s_{\tau_N}(pntd + yntd) + s_{\tau_H}(phtd + yhtd) && (202) \\
sr_T &= s_{\tau_N} ynt + s_{\tau_H}(pht + yht) && (203) \\
sr_Y &= s_{\tau_N} yny + s_{\tau_H} yhy && (204) \\
sr_B &= s_{\tau_N} ynb + s_{\tau_H} yhb && (205) \\
sr_G &= s_{\tau_N} yng + s_{\tau_H} yhg - s_G && (206)
\end{aligned}
$$

11.2 Reinterpretation of the Maastricht convergence criteria

We show how to reinterpret each of the Maastricht criteria in order to be able to use the method of Rotemberg and Woodford (1997, 1999).

11.2.1 Exchange rate criterion

We reinterpret the criterion on the nominal exchange rate (32) into two inequalities given below:[35]

$$E\left(\widehat{S}_t\right) - k * SD(\widehat{S}_t) \geq -15\%, \tag{207}$$

$$E\left(\widehat{S}_t\right) + k * SD(\widehat{S}_t) \leq 15\%. \tag{208}$$

where k is large enough to prevent from violating the criterion (32) and SD refers to the standard deviation statistic.

These two inequalities can be represented as the following two sets of inequalities (to conform with the welfare measure we use discounted statistics):

$$\left\{\begin{array}{c} (1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t\left(\widehat{S}_t-(-15\%)\right)\geq 0 \\ (1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t\left(\widehat{S}_t-(-15\%)\right)^2\leq K\left((1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t\left(\widehat{S}_t-(-15\%)\right)\right)^2, \end{array}\right. \tag{209}$$

$$\left\{\begin{array}{c} (1-\beta)E_{t_0}\sum_{t=0}^{\infty}\beta^t\left(15\%-\widehat{S}_t\right)\leq 0 \\ (1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t\left(15\%-\widehat{S}_t\right)^2\leq K\left((1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t\left(15\%-\widehat{S}_t\right)\right)^2, \end{array}\right. \tag{210}$$

where $K = 1 + k^{-2}$.

[35] E stands for the expectation operator and SD stands for the standard deviation operator.

11.2.2 Inflation criterion

We redefine the condition (30). We assume that the average inflation in the domestic economy should be at least k standard deviations smaller than the average inflation in the foreign economy plus a margin summarized by B_π (where $B_\pi = \sqrt[4]{1,015} - 1$):

$$E(\widehat{\pi}_t) \leq E(\widehat{\pi}_t^*) + B_\pi - kSD(\widehat{\pi}_t) \tag{211}$$

where $\widehat{\pi}_t$, $\widehat{\pi}_t^*$ are treated as deviations from the zero inflation steady state in the domestic economy and the foreign one accordingly (i.e. $\overline{\pi} = \overline{\pi}^* = 0$) and k large enough to prevent from violating criterion (30). We assume that the foreign economy is in the steady state so $\widehat{\pi}_t^* = 0$ $\forall t$. As a result our restriction (211) becomes:

$$E(\widehat{\pi}_t) \leq B_\pi - kSD(\widehat{\pi}_t). \tag{212}$$

Since B_π is a constant we can use the following property of the variance: $Var(\widehat{\pi}_t) = Var(B_\pi - \widehat{\pi}_t)$. Our restriction becomes:

$$kSD(B_\pi - \widehat{\pi}_t) \leq E(B_\pi - \widehat{\pi}_t). \tag{213}$$

This restriction can be represented as a set of two restrictions:

$$(1-\beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \left(B_\pi - \pi_t\right) \geq 0, \tag{214}$$

$$(1-\beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \left(B_\pi - \widehat{\pi}_t\right)^2 \leq K \left((1-\beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \left(B_\pi - \widehat{\pi}_t\right) \right)^2. \tag{215}$$

11.2.3 Nominal interest rate criterion

Similarly to the criterion on the CPI aggregate inflation we interpret the inequality (31):

$$E(\widehat{R}_t) \leq E(\widehat{R}_t^*) + C_R - kSD(\widehat{R}_t) \tag{216}$$

where k is large enough to prevent from frequent violating the criterion (31) and $C_R = \sqrt[4]{1,02} - 1$. As in the case of the foreign inflation we assume that $\widehat{R}_t^* = 0$ $\forall t$. So the restriction (216) becomes:

$$kSD(C_R - \widehat{R}_t) \leq E(C_R - \widehat{R}_t). \tag{217}$$

This inequality can be represented as a set of two inequalities:

$$(1-\beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \left(C_R - \widehat{R}_t\right) \geq 0, \tag{218}$$

$$(1-\beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \left(C_R - \widehat{R}_t\right)^2 \leq K \left((1-\beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \left(C_R - \widehat{R}_t\right) \right)^2. \tag{219}$$

11.2.4 Deficit to GDP criterion

Finally we interpret the inequality (33) that summarizes deficit to GDP criterion:

$$E(\widehat{df}_t) \leq F_{df} - kSD(\widehat{df}_t) \tag{220}$$

where k is large enough to prevent from frequent violating the criterion (33) and $F_{df} = 3\%$. Subsequently, this inequality can be represented as a set of two inequalities:

$$(1-\beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \left(F_{df} - \widehat{df}_t \right) \geq 0, \tag{221}$$

$$(1-\beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \left(F_{df} - \widehat{df}_t \right)^2 \leq K \left((1-\beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \left(F_{df} - \widehat{df}_t \right) \right)^2. \tag{222}$$

11.3 The constrained loss function

We provide the proof of the Proposition 1 stated in the main text. Since all the sets of the constraints have a similar structure the proof concerns the optimal monetary policy with only one constraint on the CPI inflation rate. The proof is based on the proof of Proposition 6.9 in Woodford (2003).

Proposition 2 *Consider the problem of minimizing an expected discounted sum of quadratic losses:*

$$E_{t_0} \left\{ (1-\beta) \sum_{t=t_0}^{\infty} \beta^t L_t \right\} \tag{223}$$

subject to (36) - (37). Let $m_{1,\pi}$, $m_{2,\pi}$ be the discounted average values of $(B_\pi - \widehat{\pi}_t)$ and $(B_\pi - \widehat{\pi}_t)^2$ associated with the optimal policy. Then the optimal policy also minimizes a modified discounted loss criterion of the form (223) with L_t replaced by:

$$\widetilde{L}_t \equiv L_t + \Phi_\pi (\pi^T - \widehat{\pi}_t)^2 \tag{224}$$

under constraints represented by the structural equations. Importantly $\Phi_\pi \geq 0$ and takes strictly positive value if and only if the constraint (37) binds. Moreover if the constraint (37) binds the corresponding target value π^T is negative and given by the following relation:

$$\pi^T = B_\pi - K m_{1,\pi} < 0. \tag{225}$$

Proof. Let $m_{1,\pi}$ and $m_{2,\pi}$ be the discounted average values of $(B_\pi - \pi_t)$ and $(B_\pi - \pi_t)^2$ associated with the policy that solves the constrained optimization problem stated in the corollary. Let $m^*_{1,\pi}$ and $m^*_{2,\pi}$ be the values of these moments for the policy that minimizes (223) without additional constraints. Notice that since $m^*_{1,\pi} = B_\pi$ the constraint (36) does not bind.[36] We identify the deterministic component of policy, i.e. $m_{1,\pi}$ and also the stabilization component of policy which is: $m_{2,\pi} - (m_{1,\pi})^2$. Moreover we also conclude that $m_{1,\pi} \geq m^*_{1,\pi}$ since there is no advantage from choosing $m_{1,\pi}$ such that: $m_{1,\pi} < m^*_{1,\pi}$ - both constraints set only the lower bound on the value of $m_{1,\pi}$ for any value of the stabilization component of policy. If one chooses $m_{1,\pi}$ such that: $m_{1,\pi} > m^*_{1,\pi}$ then one can relax the constraint (37). So $m_{1,\pi} \geq m^*_{1,\pi}$. Based on the above discussion we formulate two alternative constraints to the constraints (36, 37):

$$(1-\beta)E_0 \sum_{t=0}^{\infty} \beta^t (B_\pi - \widehat{\pi}_t) \geq m_{1,\pi}, \tag{226}$$

[36] Means of all the variables under the unconstrained optimal policy are zero.

$$(1-\beta)E_0 \sum_{t=0}^{\infty} \beta^t \left(B_\pi - \widehat{\pi}_t\right)^2 \leq m_{2,\pi}. \tag{227}$$

Observe that any policy that satisfies the above constraints satisfies also the weaker constraints: (36, 37). Now we take advantage of the Kuhn – Tucker theorem: the policy that minimizes (223) subject to (226, 227) also minimizes the following loss criterion:

$$E_0 \left\{ (1-\beta) \sum_{t=0}^{\infty} \beta^t L_t \right\} - \mu_{1,\pi} E_0 \left\{ (1-\beta) \sum_{t=0}^{\infty} \beta^t \left(B_\pi - \widehat{\pi}_t\right) \right\} + \mu_{2,\pi} E_0 \left\{ (1-\beta) \sum_{t=0}^{\infty} \beta^t \left(B_\pi - \widehat{\pi}_t\right)^2 \right\} \tag{228}$$

where $\mu_{1,\pi}$ and $\mu_{2,\pi}$ are the Lagrange multipliers which are nonnegative. If (37) binds then we obtain the following relation between the multipliers:

$$\mu_{1,\pi} = 2Km_{1,\pi}\mu_{2,\pi} \tag{229}$$

since $m_{2,\pi} = Km_{1,\pi}^2$.

Rearranging the terms in (228) we can define the new loss function as:

$$\widetilde{L}_t \equiv L_t + \mu_{2,\pi} \left(\left(B_\pi - \widehat{\pi}_t\right) - \frac{\mu_{1,\pi}}{2\mu_{2,\pi}} \right)^2 \tag{230}$$

where the final term appears only when $\mu_{2,\pi} > 0$. Therefore $\Phi_\pi = \mu_{2,\pi} \geq 0$ and takes a strictly positive value only if (37) binds. Moreover for $\Phi_\pi > 0$ we have that:

$$\pi^T = B_\pi - \frac{\mu_{1,\pi}}{2\mu_{2.\pi}} = B_\pi - Km_{1,\pi}. \tag{231}$$

Notice that the target value for the CPI inflation is negative (since $K > 1$ and $m_{1,\pi} \geq B_\pi$):

$$\pi^T = B_\pi - Km_{1,\pi} < 0. \tag{232}$$

■

11.4 Unconstrained optimal monetary and fiscal policy

We derive the first order conditions for the unconstrained optimal monetary and fiscal policy.

$$\min L_{t_0} = U_C\overline{C} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [\frac{1}{2}\Phi_y(\widehat{Y}_t - \widehat{Y}_t^T)^2 + \frac{1}{2}\Phi_{T^d}(\widehat{T}_t^d - \widehat{T}_t^{dT})^2 + \frac{1}{2}\Phi_T(\widehat{T}_t - \widehat{T}_t^T)^2 + \tag{233}$$

$$\frac{1}{2}\Phi_{\tau_N}(\widehat{\tau}_{N,t} - \widehat{\tau}_{N,t}^T)^2 + \frac{1}{2}\Phi_{\tau_H}(\widehat{\tau}_{H,t} - \widehat{\tau}_{H,t}^T)^2 + \Phi_{YT^d}\widehat{Y}_t\widehat{T}_t^d + \Phi_{YT}\widehat{Y}_t\widehat{T}_t + \Phi_{T^dT}\widehat{T}_t^d\widehat{T}_t + \tag{234}$$

$$+\widehat{\tau}_{N,t}(\Phi_{Y\tau_N}\widehat{Y}_t + \Phi_{T\tau_N}\widehat{T}_t + \Phi_{T^d\tau_N}\widehat{T}_t^d) + \widehat{\tau}_{H,t}(\Phi_{Y\tau_H}\widehat{Y}_t + \Phi_{T\tau_H}\widehat{T}_t + \Phi_{T^d\tau_H}\widehat{T}_t^d) + \tag{235}$$

$$+\frac{1}{2}\Phi_{\pi_N}\widehat{\pi}_{N,t}^2 + \frac{1}{2}\Phi_{\pi_H}\widehat{\pi}_{H,t}^2] + tip + O(3) \tag{236}$$

subject to :

$$\widehat{\pi}_{N,t} = k_N(m_{N,Y}\widehat{Y}_t + m_{N,T^d}\widehat{T}_t^d + m_{N,T}\widehat{T}_t + m_{N,\tau_N}\widehat{\tau}_{N,t} + m_{N,A_N}\widehat{A}_{N,t} + m_{N,A_H}\widehat{A}_{H,t} + m_{N,B}\widehat{B}_t + m_{N,G}\widehat{G}_t) + \beta\widehat{\pi}_{N,t+1}, \tag{237}$$

$$\widehat{\pi}_{H,t} = k_H(m_{H,Y}\widehat{Y}_t + m_{H,T^d}\widehat{T}_t^d + m_{H,T}\widehat{T}_t + m_{H,\tau_H}\widehat{\tau}_{H,t} + m_{H,A_N}\widehat{A}_{N,t} + m_{H,A_H}\widehat{A}_{H,t} + m_{H,B}\widehat{B}_t + m_{H,G}\widehat{G}_t) + \beta\widehat{\pi}_{H,t+1}, \tag{238}$$

$$\widehat{C}_t^* = n_Y\widehat{Y}_t + n_{T^d}\widehat{T}_t^d + n_T\widehat{T}_t + n_B\widehat{B}_t + n_B\widehat{B}_t, \tag{239}$$

$$\widehat{T}_t^d - \widehat{T}_{t-1}^d = \widehat{\pi}_{N,t} - \widehat{\pi}_{H,t} - a(\widehat{T}_t - \widehat{T}_{t-1}), \tag{240}$$

$$\begin{aligned}
\widehat{d}_{t-1} &= f_{\tau_N}\widehat{\tau}_{N,t} + f_{\tau_H}\widehat{\tau}_{H,t} + f_Y\widehat{Y}_t + f_{T^d}\widehat{T}_t^d + f_T\widehat{T}_t + f_{T^d(+1)}\widehat{T}_{t+1}^d + f_{T^{(+1)}}\widehat{T}_{t+1} + f_{T^{(-1)}}\widehat{T}_{t-1} + && (241)\\
&\quad + f_{\pi_N}\widehat{\pi}_{N,t} + f_{\pi_H}\widehat{\pi}_{H,t} + f_{\pi_{N(+1)}}\widehat{\pi}_{N,t+1} + f_{\pi_{H(+1)}}\widehat{\pi}_{H,t+1} + \beta\widehat{d}_t + && (242)\\
&\quad + f_G\widehat{G}_t + f_B\widehat{B}_t + f_{C^*}\widehat{C}_t^* - f_{C^*(+1)}\widehat{C}_{t+1}^* && (243)
\end{aligned}$$

First order conditions of the minimization problem:

- wrt $\widehat{\pi}_{N,t}$:

$$0 = \Phi_{\pi_N}\widehat{\pi}_{N,t} + \gamma_{1,t} - \gamma_{1,t-1} - \gamma_{4,t} - f_{\pi_N}\gamma_{5,t} - \beta^{-1}f_{\pi_{N(+1)}}\gamma_{5,t-1}, \tag{244}$$

- wrt $\widehat{\pi}_{H,t}$:

$$0 = \Phi_{\pi_H}\widehat{\pi}_{H,t} + \gamma_{2,t} - \gamma_{2,t-1} + \gamma_{4,t} - f_{\pi_H}\gamma_{5,t} - \beta^{-1}f_{\pi_{H(+1)}}\gamma_{5,t-1}, \tag{245}$$

- wrt $\widehat{Y}_t$:

$$\begin{aligned}
0 &= \Phi_Y(\widehat{Y}_t - \widehat{Y}_t^T) + \Phi_{YT^d}\widehat{T}_t^d + \Phi_{YT}\widehat{T}_t - k_N m_{N,Y}\gamma_{1,t} && (246)\\
&\quad - k_H m_{H,Y}\gamma_{2,t} - n_Y\gamma_{3,t} - f_Y\gamma_{5,t}, && (247)
\end{aligned}$$

- wrt $\widehat{T}_t^d$:

$$\begin{aligned}
0 &= \Phi_{T^d}(\widehat{T}_t^d - \widehat{T}_t^{d^T}) + \Phi_{TT^d}\widehat{T}_t + \Phi_{YT^d}\widehat{Y}_t - k_N m_{N,T^d}\gamma_{1,t} - k_H m_{H,T^d}\gamma_{2,t} && (248)\\
&\quad - n_{T^d}\gamma_{3,t} + \gamma_{4,t} - \beta\gamma_{4,t+1} - f_{T^d}\gamma_{5,t} - \beta^{-1}f_{T^d(+1)}\gamma_{5,t-1}, && (249)
\end{aligned}$$

- wrt $\widehat{T}_t$:

$$\begin{aligned}
0 &= \Phi_T(\widehat{T}_t - \widehat{T}_t^T) + \Phi_{TT^d}\widehat{T}_t^d + \Phi_{YT}\widehat{Y}_t - k_N m_{N,T}\gamma_{1,t} - k_H m_{H,T}\gamma_{2,t} && (250)\\
&\quad - n_T\gamma_{3,t} + a\gamma_{4,t} - \beta a\gamma_{4,t+1} - f_T\gamma_{5,t} - \beta^{-1}f_{T^d(+1)}\gamma_{5,t-1} - \beta f_{T^{(-1)}}\gamma_{5,t+1}, && (251)
\end{aligned}$$

- wrt $\widehat{d}_t$:

$$0 = -\beta\gamma_{5,t} + \beta\gamma_{5,t+1}, \tag{252}$$

- wrt $\widehat{\tau}_{N,t}$:

$$\begin{aligned} 0 &= \Phi_{\tau_N}(\widehat{\tau}_{N,t} - \widehat{\tau}^T_{N,t}) + \Phi_{Y\tau_N}\widehat{Y}_t + \Phi_{T\tau_N}\widehat{T}_t + \Phi_{T^d\tau_N}\widehat{T}^d_t + && (253)\\ & -k_N m_{N,\tau_N}\gamma_{1,t} - f_{\tau_N}\gamma_{5,t}, && (254)\end{aligned}$$

- wrt $\widehat{\tau}_{H,t}$:

$$\begin{aligned} 0 &= \Phi_{\tau_H}(\widehat{\tau}_{H,t} - \widehat{\tau}^T_{H,t}) + \Phi_{Y\tau_H}\widehat{Y}_t + \Phi_{T\tau_H}\widehat{T}_t + \Phi_{T^d\tau_H}\widehat{T}^d_t + && (255)\\ & -k_H m_{H,\tau_H}\gamma_{2,t} - f_{\tau_H}\gamma_{5,t}. && (256)\end{aligned}$$

11.5 Constrained optimal monetary and fiscal policy

We derive the first order conditions for the optimal policy that satisfies the additional criteria on the nominal interest, the CPI aggregate inflation and deficit to GDP ratio.

$$\begin{aligned} \min L^c_{t_0} &= U_C\overline{C}\sum_{t=t_0}^{\infty}\beta^{t-t_0}[\frac{1}{2}\Phi_y(\widehat{Y}_t - \widehat{Y}^T_t)^2 + \frac{1}{2}\Phi_{T^d}(\widehat{T}^d_t - \widehat{T}^{dT}_t)^2 + \frac{1}{2}\Phi_T(\widehat{T}_t - \widehat{T}^T_t)^2 + && (257)\\ & \frac{1}{2}\Phi_{\tau_N}(\widehat{\tau}_{N,t} - \widehat{\tau}^T_{N,t})^2 + \frac{1}{2}\Phi_{\tau_H}(\widehat{\tau}_{H,t} - \widehat{\tau}^T_{H,t})^2 + \Phi_{YT^d}\widehat{Y}_t\widehat{T}^d_t + \Phi_{YT}\widehat{Y}_t\widehat{T}_t + \Phi_{T^dT}\widehat{T}^d_t\widehat{T}_t + && (258)\\ & +\widehat{\tau}_{N,t}(\Phi_{Y\tau_N}\widehat{Y}_t + \Phi_{T\tau_N}\widehat{T}_t + \Phi_{T^d\tau_N}\widehat{T}^d_t) + \widehat{\tau}_{H,t}(\Phi_{Y\tau_H}\widehat{Y}_t + \Phi_{T\tau_H}\widehat{T}_t + \Phi_{T^d\tau_H}\widehat{T}^d_t) + && (259)\\ & +\frac{1}{2}\Phi_{\pi_N}\widehat{\pi}^2_{N,t} + \frac{1}{2}\Phi_{\pi_H}\widehat{\pi}^2_{H,t} + && (260)\\ & +\frac{1}{2}\phi_\pi(\pi^T - \widehat{\pi}_t)^2 + \frac{1}{2}\phi_R(R^T - \widehat{R}_t)^2 + \frac{1}{2}\phi_{df}(df^T - df_t)^2] + tip + O(3) && (261)\end{aligned}$$

subject to :

$$\widehat{\pi}_{N,t} = k_N(m_{N,Y}\widehat{Y}_t + m_{N,T^d}\widehat{T}^d_t + m_{N,T}\widehat{T}_t + m_{N,\tau_N}\widehat{\tau}_{N,t} + m_{N,A_N}\widehat{A}_{N,t} + m_{N,A_H}\widehat{A}_{H,t} + m_{N,B}\widehat{B}_t + m_{N,G}\widehat{G}_t) + \beta\widehat{\pi}_{N,t+1}, \quad (262)$$

$$\widehat{\pi}_{H,t} = k_H(m_{H,Y}\widehat{Y}_t + m_{H,T^d}\widehat{T}^d_t + m_{H,T}\widehat{T}_t + m_{H,\tau_H}\widehat{\tau}_{H,t} + m_{H,A_N}\widehat{A}_{N,t} + m_{H,A_H}\widehat{A}_{H,t} + m_{H,B}\widehat{B}_t + m_{H,G}\widehat{G}_t) + \beta\widehat{\pi}_{H,t+1}, \quad (263)$$

$$\widehat{C}^*_t = n_Y\widehat{Y}_t + n_{T^d}\widehat{T}^d_t + n_T\widehat{T}_t + n_B\widehat{B}_t + n_B\widehat{B}_t, \quad (264)$$

$$\widehat{T}^d_t - \widehat{T}^d_{t-1} = \widehat{\pi}_{N,t} - \widehat{\pi}_{H,t} - a(\widehat{T}_t - \widehat{T}_{t-1}), \quad (265)$$

$$\begin{aligned} \widehat{d}_{t-1} &= f_{\tau_N}\widehat{\tau}_{N,t} + f_{\tau_H}\widehat{\tau}_{H,t} + f_Y\widehat{Y}_t + f_{T^d}\widehat{T}^d_t + f_T\widehat{T}_t + f_{T^d(+1)}\widehat{T}^d_{t+1} + f_{T^{(+1)}}\widehat{T}_{t+1} + f_{T^{(-1)}}\widehat{T}_{t-1} + && (266)\\ & +f_{\pi_N}\widehat{\pi}_{N,t} + f_{\pi_H}\widehat{\pi}_{H,t} + f_{\pi_{N(+1)}}\widehat{\pi}_{N,t+1} + f_{\pi_{H(+1)}}\widehat{\pi}_{H,t+1} + \beta\widehat{d}_t + && (267)\\ & +f_G\widehat{G}_t + f_B\widehat{B}_t + f_{C^*}\widehat{C}^*_t - f_{C^*(+1)}\widehat{C}^*_{t+1} && (268)\end{aligned}$$

$$\widehat{\pi}_t = b\widehat{\pi}_{N,t} + (1-b)\widehat{\pi}_{H,t} + a(1-b)(\widehat{T}_t - \widehat{T}_{t-1}), \tag{269}$$

$$\widehat{R}_t = b\widehat{\pi}_{N,t+1} + (1-b)\widehat{\pi}_{H,t+1} - \rho(1-cb)(\widehat{B}_{t+1} - \widehat{B}_t) + \rho(\widehat{Y}_{t+1} - \widehat{Y}_t) + \rho ctd(\widehat{T}^d_{t+1} - \widehat{T}^d_t) + (\rho ct + a(1-b))(\widehat{T}_{t+1} - \widehat{T}_t), \tag{270}$$

$$\widetilde{df}_t = d_d\widehat{d}_{t-1} + d_\pi(b\widehat{\pi}_{N,t} + (1-b)\widehat{\pi}_{H,t} + a(1-b)(\widehat{T}_t - \widehat{T}_{t-1})) + d_R\widehat{R}_{t-1} + \tag{271}$$

$$+d_{sr}(sr_{\tau_N}\widehat{\tau}_{N,t} + sr_{\tau_H}\widehat{\tau}_{H,t} + sr_{T^d}\widehat{T}^d_t + sr_T\widehat{T}_t + sr_Y\widehat{Y}_t + sr_B\widehat{B}_t + sr_G\widehat{G}_t), \tag{272}$$

First order conditions of the minimization problem:

- wrt $\widehat{\pi}_{N,t}$:

$$0 = \Phi_{\pi_N}\widehat{\pi}_{N,t} + \gamma_{1,t} - \gamma_{1,t-1} - \gamma_{4,t} - f_{\pi_N}\gamma_{5,t} - \beta^{-1}f_{\pi_{N(+1)}}\gamma_{5,t-1} \tag{273}$$

$$-b\gamma_{6,t} - \beta^{-1}b\gamma_{7,t} - bd_\pi\gamma_{8,t}, \tag{274}$$

- wrt $\widehat{\pi}_{H,t}$:

$$0 = \Phi_{\pi_H}\widehat{\pi}_{H,t} + \gamma_{2,t} - \gamma_{2,t-1} + \gamma_{4,t} - f_{\pi_H}\gamma_{5,t} - \beta^{-1}f_{\pi_{H(+1)}}\gamma_{5,t-1} \tag{275}$$

$$-(1-b)\gamma_{6,t} - \beta^{-1}(1-b)\gamma_{7,t} - (1-b)d_\pi\gamma_{8,t}, \tag{276}$$

- wrt $\widehat{Y}_t$:

$$0 = \Phi_Y(\widehat{Y}_t - \widehat{Y}^T_t) + \Phi_{YT^d}\widehat{T}^d_t + \Phi_{YT}\widehat{T}_t - k_N m_{N,Y}\gamma_{1,t} \tag{277}$$

$$-k_H m_{H,Y}\gamma_{2,t} - n_Y\gamma_{3,t} - f_Y\gamma_{5,t} + \tag{278}$$

$$+\rho\gamma_{7,t} - \rho\beta^{-1}\gamma_{7,t-1} - d_{sr}sr_Y\gamma_{8,t}, \tag{279}$$

- wrt $\widehat{T}^d_t$:

$$0 = \Phi_{T^d}(\widehat{T}^d_t - \widehat{T}^{d^T}_t) + \Phi_{TT^d}\widehat{T}_t + \Phi_{YT^d}\widehat{Y}_t - k_N m_{N,T^d}\gamma_{1,t} - k_H m_{H,T^d}\gamma_{2,t} \tag{280}$$

$$-n_{T^d}\gamma_{3,t} + \gamma_{4,t} - \beta\gamma_{4,t+1} - f_{T^d}\gamma_{5,t} - \beta^{-1}f_{T^d(+1)}\gamma_{5,t-1} \tag{281}$$

$$+\rho ctd\gamma_{7,t} - \beta^{-1}\rho ctd\gamma_{7,t-1} - d_{sr}sr_{T^d}\gamma_{8,t}, \tag{282}$$

- wrt $\widehat{T}_t$:

$$0 = \Phi_T(\widehat{T}_t - \widehat{T}^T_t) + \Phi_{TT^d}\widehat{T}^d_t + \Phi_{YT}\widehat{Y}_t - k_N m_{N,T}\gamma_{1,t} - k_H m_{H,T}\gamma_{2,t} \tag{283}$$

$$-n_T\gamma_{3,t} + a\gamma_{4,t} - \beta a\gamma_{4,t+1} - f_T\gamma_{5,t} - \beta^{-1}f_{T^d(+1)}\gamma_{5,t-1} - \beta f_{T^{(-1)}}\gamma_{5,t+1} \tag{284}$$

$$-a(1-b)\gamma_{6,t} + a(1-b)\beta\gamma_{6,t+1} + \tag{285}$$

$$(\rho ct + a(1-b))\gamma_{7,t} - (\rho ct + a(1-b))\beta^{-1}\gamma_{7,t-1}, \tag{286}$$

- wrt $\widehat{d}_t$:

$$0 = -\beta\gamma_{5,t} + \beta\gamma_{5,t+1} - \beta d_d\gamma_{8,t+1}, \tag{287}$$

- wrt $\widehat{\tau}_{N,t}$:

$$0 = \Phi_{\tau_N}(\widehat{\tau}_{N,t} - \widehat{\tau}^T_{N,t}) + \Phi_{Y\tau_N}\widehat{Y}_t + \Phi_{T\tau_N}\widehat{T}_t + \Phi_{T^d\tau_N}\widehat{T}^d_t + \tag{288}$$

$$-k_N m_{N,\tau_N}\gamma_{1,t} - f_{\tau_N}\gamma_{5,t} - d_{sr}sr_{\tau_N}\gamma_{8,t} \tag{289}$$

- wrt $\widehat{\tau}_{H,t}$

$$0 = \Phi_{\tau_H}(\widehat{\tau}_{H,t} - \widehat{\tau}^T_{H,t}) + \Phi_{Y\tau_H}\widehat{Y}_t + \Phi_{T\tau_H}\widehat{T}_t + \Phi_{T^d\tau_H}\widehat{T}^d_t + \tag{290}$$

$$-k_H m_{H,\tau_H}\gamma_{2,t} - f_{\tau_H}\gamma_{5,t} - d_{sr}sr_{\tau_H}\gamma_{8,t} \tag{291}$$

- wrt $\widehat{R}_t$:

$$0 = \Phi_R(\widehat{R}_t - R^T) + \gamma_{5,t} - d_R\beta\gamma_{8,t+1}, \tag{292}$$

- wrt $\widehat{\pi}_t$:

$$0 = \Phi_\pi(\widehat{\pi}_t - \pi^T) + \gamma_{6,t}, \tag{293}$$

- wrt $\widetilde{df}_t$:

$$0 = \Phi_{df}(\widetilde{df}_t - df^T) + \gamma_{8,t}. \tag{294}$$