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Abstract

This paper presents a general equilibrium model that is consistent with recent empirical evidence showing that the U.S. price level and inflation are much more responsive to aggregate technology shocks than to monetary policy shocks. The model of this paper builds on recent work by Mackowiak and Wiederholt (2009), who show that models of endogenous attention allocation deliver prices to be more responsive to more volatile shocks as, everything else being equal, firms pay relatively more attention to more volatile shocks. In fact, according to the U.S. data, aggregate technology shocks are more volatile than monetary policy shocks inducing in this paper, firms to pay more attention to the former than to the latter. However, most important, this work adds to the literature by showing that the ability of the model of this paper to account for observed price dynamics crucially depends on monetary policy. In particular, this paper shows how interest rate feedback rules affect the incentives faced by firms in allocating attention. A policy rate responding more actively to expected inflation and output fluctuations induces firms to pay relatively more attention to more volatile shocks. This new mechanism of transmission of monetary policy helps rationalizing the observed behavior of prices in response to technology and monetary policy shocks, and implies novel predictions about the impact of changes in Taylor rules coefficients on economic fluctuations.

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1 Introduction

Recent empirical work on nominal price adjustment has shown that the U.S. aggregate price level and inflation are much more responsive to aggregate technology shocks, such as innovation in total factor productivity, than to monetary policy shocks, such as unexpected innovations in the Federal Funds rate.[1] Standard models of sticky prices have a hard time explaining the different behavior of the price level and inflation in response to these two aggregate shocks.[2] Indeed, one of the central issues in modern macroeconomics is understanding how firms set their prices in response to different aggregate shocks. This is an important task for monetary policy analysis and implementation. Understanding the transmission of technology and monetary policy shocks is particularly relevant as these shocks account together for a large fraction of business cycle fluctuations.[3]

I present a model that is consistent with the empirical evidence that prices respond much more quickly to aggregate technology shocks than to monetary policy shocks. I show that this response pattern arises naturally in a framework based on imperfect information with an endogenous choice of information structure similar to Sims [24]. In this model, firms will optimally choose to allocate more attention to those particular shocks that, in expectations, most reduce profits when prices are not adjusted properly. The more attention firms pay to a type of shock, the faster they respond to it.

This is a result that has been emphasized in the seminal paper by Mackowiak and Wiederholt [18], where these authors have shown that firms pay more attention to sector specific shocks than to aggregate nominal shocks roughly because the former are much more volatile than the latter. So, at first sight, this result would directly translate to a framework with aggregate technology and monetary policy shocks:

[1] See Altig, Christiano, Eichenbaum and Linde [2] and Paciello [21]. Figure 1 at the end of the paper plots inflation and price level responses estimated by Paciello [21].

[2] See Dupor, Han and Tsai [10].

[3] See, for intance, Smets and Wouters (2007).

since in the U.S. aggregate technology shocks are more volatile than monetary policy shocks, everything else being equal, firms allocate more attention to the former than to latter, inducing faster price responses to technology shocks.[4]

However, most important, I show that this not the whole story. In a standard general equilibrium model, for given shock volatilities, there are two important channels that may amplify or reduce differences in attention allocation across different types of shocks. These channels relate to monetary policy and real rigidities. Both channels influence the attention allocation decision by changing the incentives faced by firms in allocating attention. In particular, the monetary policy channel has not been studied in the literature.

I show that, when monetary policy follows a simple interest rate feedback rule, such as a Taylor rule, a policy responding more to expected inflation and output fluctuations increases complementarity in attention allocation. This higher complementarity induce firms to pay more attention to the same variables that other firms pay more attention to, amplifying the difference in price responsiveness to technology and monetary policy shocks. Under the benchmark calibration of the model, monetary policy activism substantially contributes to magnifying the impact of different shock volatilities onto attention allocation decision. This amplification helps to rationalize the observed difference in price responsiveness to technology and monetary policy shocks.

Moreover, these results unveil a novel mechanism of transmission of monetary policy to the economy: monetary policy affects price responsiveness through its feedback on the attention allocation decision. This mechanism introduces an asymmetry in the way changes in coefficients of the Taylor rule influence price responsiveness to different shocks. When, for instance, coefficients on expected inflation and output fluctuations increase, the new equilibrium is characterized by a larger fraction of

[4] Figure 2 at the end of the paper plots the growth rate in total factor productivity and the change in the FedFunds rate from 1960 to 2007. Other authors have estimated the volatility of technology and monetary policy shocks within DSGE models. See, for instance, Smets and Wouters [25].

attention paid to the most volatile shocks, and a smaller fraction paid to the least volatile ones. As a consequence the change in policy, everything else being equal, this channel of transmission causes price variability to reduce relatively less conditional on the most volatile shocks, and more conditional on the least volatile ones.

In addition, this paper adds to the literature by deriving a closed form solution to the static linear-quadratic version of the general equilibrium model. This solution yields valuable economic insights on the feedback from the different structural parameters of the model to the attention allocation decision, and allows to fully capture the interaction between monetary policy, real rigidities and complementarity in attention allocation.

The results of this paper are obtained within a standard general equilibrium framework with a representative household, monopolistically competitive firms and a central bank that sets the nominal interest rate according to a Taylor-type policy rule. In this model, prices respond more to the realizations of shocks about which firms are better informed. Technology shocks are aggregate innovations to labor productivity, while monetary policy shocks are temporary deviations of the nominal interest rate from the monetary policy rule. The only friction introduced in this framework is that firms might not be well informed about the realizations of the shocks when changing their prices. The information structure of the economy is modeled along the lines of Mackowiak and Wiederholt [18]. There is a limit on the total attention a firm can pay to the different shocks. This limit introduces a trade-off in the allocation of attention.

This paper relates to the large literature studying price setting decisions under incomplete information. Incomplete information theories have been popular in accounting for the sluggish price adjustment in response to monetary policy shocks. Behind these theories there is the assumption that firms only pay attention to a relatively small number of economic indicators. With imprecise information about aggregate conditions, prices respond with delay to changes in nominal spending. This simple idea was first proposed by Phelps [22] and formalized by Lucas [16]. More recently Woodford [26], Mankiw and Reis [17], and Sims [24], have renewed attention to imperfect information and limited information processing as sources of inertial prices. In particular, Woodford has used an incomplete information model to explain the sluggish response of prices to aggregate nominal shocks. According to Woodford [26], such a framework could deliver prices responding more to aggregate supply shocks than to nominal demand shocks, if firms were relatively more informed about the former than they were about the latter. However, he leaves open the question of why firms should choose to be relatively more informed about some types of shocks.

Sims [24] and Mackowiak and Wiederholt [18] study the endogenous optimal choice of the information structure. In particular, Mackowiak and Wiederholt [18] focus on the differential response of prices to aggregate nominal shocks versus idiosyncratic shocks in a framework with limited information-processing capabilities, and with an exogenous process for nominal spending. In parallel and independent work Mackowiak and Wiederholt [19] have extended their previous analysis to study business cycle dynamics under rational inattention in a DSGE model. Similarly to this paper, these authors find that this class of models generates prices and inflation to be more responsive to aggregate technology shocks than to monetary policy shocks. However, the two papers are complements on other important dimensions. In particular, while Mackowiak and Wiederholt [19] focus more on the interaction between attention allocation decision by firms and real rigidities originating from imperfectly informed households, this paper studies more in detail the role of monetary policy. Monetary policy proves crucial for inflation and price level responsiveness, affecting directly the attention allocation decision. Moreover, this paper provides a closed form solution to the general equilibrium of the static model.

This paper also relates to the work by Branch, Carlson, Evans and McGough [7]. These authors have studied a model of endogenous inattention, where monetary policy activism influences the overall information acquisition rate of firms. This paper contributes to this literature in studying the way monetary policy influences economic dynamics through a new margin related to the allocation of given information processing capability across different types of information.

Finally, within the imperfect information literature, Hellwig and Veldkamp [13] have recently emphasized the interaction of strategic complementarity in price setting with endogenous information acquisition by firms. Relative to these authors, this paper further shows how the interaction of strategic complementarity in price setting and endogenous information acquisition depends on monetary policy activism.

The paper is organized as follows. Section 2 introduces the model. Section 3 describes a static solution of the model. Section 4 discusses a dynamic extension of the model. Section 5 assesses robustness of results. Section 6 concludes.

2 The model

Apart from the information structure, this paper studies a standard general equilibrium model of incomplete nominal adjustment with monopolistic firms along the lines of Blanchard and Kiyotaki [6]. The information structure of firms is modeled along the lines of Mackowiak and Wiederholt [18]. Time is discrete and infinite. There is a measure 1 of different intermediate goods, indexed by $i \in [0, 1]$, each produced by a monopolistic firm using labor as the only input into production. Intermediate goods are aggregated into a final good by a perfectly competitive final good sector through a Dixit-Stiglitz technology with constant returns to scale. On the consumption side, there is an infinitely-lived representative household with preferences defined over consumption and labor supply in each period. Financial markets are complete and financial assets are in zero initial supply. For simplicity it is assumed that the representative household takes its decisions under perfect information. The monetary authority controls the risk free nominal interest rate according to a given monetary policy rule. There are two sources of uncertainty in the economy: the first is related to realizations of aggregate technology shocks to labor productivity and the second is associated to unexpected deviations of the nominal interest rate from the monetary policy rule.

Household Preferences: The representative household's preferences over sequences of the final good consumption and labor supply $\{C_{t+\tau}, L_{t+\tau}\}_{\tau=0}^{\infty}$ are given by

$$U_t = E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left(\log C_{t+\tau} - L_{t+\tau}\right), \tag{1}$$

where $\beta \in (0, 1)$ is the discount factor, and $\mathrm{E}_t(\cdot)$ denotes the household's expectations conditional on the realizations of all variables up to period t. The household has complete information. The household's objective is to maximize (1) subject to its sequence of flow budget constraints, for $\tau = 0, 1, ...$

$$P_{t+\tau} C_{t+\tau} + E_{t+\tau} \left[Q_{t+\tau,t+\tau+1} S_{t+\tau+1}\right] = W_{t+\tau} L_{t+\tau} + S_{t+\tau} + D_{t+\tau}, \tag{2}$$

where $S_{t+\tau}$ denotes the nominal value of the state-contingent asset in period $t + \tau$, $Q_{t+\tau,t+\tau+1}$ represents the period $t + \tau$ price of one unit of currency to be delivered in a particular state of period $t + \tau + 1$, $P_{t+\tau}$ is the price of the final consumption good, $W_{t+\tau}$ the nominal wage rate, and $D_{t+\tau}$ the aggregate profits of the corporate sector rebated to the household. The household is subject to a borrowing constraint that prevents engaging in Ponzi schemes,

$$S_{t+\tau+1} \geq - \sum_{T=t+\tau+1}^{\infty} E_{t+\tau+1} \left[Q_{t+\tau+1,T} \left(W_T L_T + D_T\right)\right] \tag{3}$$

with certainty, and in each state of the world that may be reached in period $t+\tau+1$, where $Q_{t+\tau,T} = \prod_{s=t+\tau+1}^{T} Q_{s-1,s}$.

The assumption of complete financial markets ensures the existence of a risk-free portfolio in period t paying a nominal interest rate R_t in period $t + 1$.

Monetary Policy: It is assumed that the monetary authority controls the nominal interest rate according to a Taylor-type policy rule,

$$\frac{R_t}{\bar{R}} = \left(E_t \frac{\Pi_{t+1}}{\bar{\Pi}}\right)^{\phi_\pi} \left(\frac{C_t}{C_t^*}\right)^{\phi_c} e^{\varepsilon_{r,t}}, \tag{4}$$

where ϕ_π and ϕ_c are parameters, $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$ is inflation, and C_t^* is the level of potential consumption, defined as the level of consumption that would hold in the frictionless economy with perfect information; $\varepsilon_{r,t}$ is an *iid* and normally distributed monetary policy disturbance, $\varepsilon_{r,t} \backsim N\left(0, \sigma_r^2\right)$; $\bar{\Pi}$ and $\bar{R}$ are inflation and the nominal interest rate in the non-stochastic steady state. The policy rule given by (4) is appealing both on theoretical and empirical grounds. Approximate (and in some cases exact) forms of this rule are optimal for a central bank that has a quadratic loss function in deviations of inflation and output from their respective targets in a generic macro model with price inertia.[5] On the empirical side, a number of authors have emphasized that policy rules like (4) provide reasonable good descriptions of the way major central banks behave, at least in recent years.[6] Later in the paper, I will extend the analysis to allow for inertia in nominal interest rates.

Final Good Producers: The final consumption good is produced by a large number of perfectly informed producers through a constant return to scale technology given by

$$C_t = \left[\int_0^1 \left(c_{i,t}\right)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \tag{5}$$

where $\theta > 1$ is the demand elasticity parameter. The demand for intermediate good i follows from profits maximization by final good producers and it is given by

$$c_{i,t} = c\left(p_{i,t}\right) = C_t \left(\frac{p_{i,t}}{P_t}\right)^{-\theta}. \tag{6}$$

It follows from $(5) - (6)$ that the final good price P_t is given by the Dixit-Stiglitz

[5] See, e.g., Woodford [27].
[6] See, e.g., Orphanides [20].

aggregator

$$P_t = \left[\int_0^1 (p_{i,t})^{1-\theta} \, di \right]^{\frac{1}{1-\theta}}. \tag{7}$$

Intermediate Good Producers: Each intermediate good is produced by a single monopolistic firm using labor as the only input into production, according to a technology with decreasing returns to scale given by

$$c_{i,t} = e^{\varepsilon_{a,t}} L_{i,t}^{\alpha}, \tag{8}$$

where $\varepsilon_{a,t}$ is an *iid* and normally distributed technology innovation to aggregate labor productivity, $\varepsilon_{a,t} \backsim N(0, \sigma_a^2)$, and $\alpha \in [0,1]$ determines the returns to scale in production, corresponding for instance to the presence of a firm-specific factor that is costly to adjust at short horizons. Firm i's nominal profits are given by

$$\pi_{i,t} = p_{i,t} c(p_{i,t}) - W_t L_{i,t}. \tag{9}$$

By substituting (8) into (9), nominal profits can be expressed as a function of firm i's prices

$$\pi_{i,t} = \pi(p_{i,t}) = p_{i,t} c(p_{i,t}) - W_t \left(\frac{c(p_{i,t})}{e^{\varepsilon_{a,t}}} \right)^{\frac{1}{\alpha}}. \tag{10}$$

Given (6) and (10), the first-order condition for profit-maximization under perfect information implies[7]

$$\log(p_{i,t}^*) = \alpha\xi \log\left(\frac{1}{\alpha} \frac{\theta}{\theta - 1} \right) + \log(P_t) + \xi (\log(C_t) - \varepsilon_{a,t}), \tag{11}$$

where $p_{i,t}^*$ denotes the profit-maximizing price, and ξ is the degree of real rigidity,

[7] Notice that, in deriving (11), I have used the fact that $\frac{W_t}{P_t} = C_t$ from the household's intratemporal Euler condition.

given by

$$\xi \equiv \frac{1}{\alpha + \theta (1 - \alpha)}. \tag{12}$$

Limited information processing capabilities: In the spirit of the rational inattention literature, information on realizations of all economic variables is assumed to be equally available, but intermediate good producers have limited information processing capabilities: they cannot attend perfectly to all available information. This idea is formalized following Sims [24] by modelling limited attention as a constraint on information flow. Intermediate good producers decide how to use the available information flow, and in particular how to attend to the different shocks that affect the optimal price decision. Similarly to Mackowiak and Wiederholt [18], it is assumed that information about technology and monetary policy shocks is processed independently and that the noise in the decision is independent across firms. The last assumption accords well with the idea that the constraint is the decision-makers limited attention rather than the availability of information. Firms decide how to allocate their attention in period zero by maximizing the discounted sum of profits from future activity, $E_0 \sum_{t=1}^{\infty} Q_{0,t}\pi_{i,t}$.[8] In order to have an analytical solution to the attention allocation problem, this paper considers a second order Taylor expansion of the discounted sum of future profits around the non-stochastic steady state, in deviation from the discounted value of profits under the profit-maximizing behavior. This quadratic approximation is given by

$$-\lambda \sum_{t=1}^{\infty} \beta^t E_0 \left[\left(\log\left(p_{i,t}\right) - \log\left(p_{i,t}^*\right) \right)^2 \right], \tag{13}$$

where $\lambda \equiv \frac{1}{2}\bar{C}\left(\theta^2\left(\frac{1}{\alpha} - 1\right) + \theta\left(2 - \frac{1}{\alpha}\right) - 1\right) > 0$ is a constant and $\bar{C}$ is the level of consumption in the non-stochastic steady state.[9] Given (13) and the assumption

[8] In the static equilibrium of this model this assumption is irrelevant as the attention allocation choice is time-consistent.

[9] See appendix A1 for the derivation. In a similar framework Maćkoviak and Wiederholt [18] show that solving the attention allocation problem through the quadratic approximation of the objective delivers accurate results when the amount of information processed per period (i.e. κ as

of independent information processing about the two types of shocks, the attention allocation problem of intermediate good producer i reads

$$\max_{\{s_{ai,t},\ s_{ri,t}\}} -\lambda \sum_{t=1}^{\infty} \beta^t E_0 \left[\left(\log\left(p_{i,t}\right) - \log\left(p_{i,t}^*\right) \right)^2 \right], \tag{14}$$

subject to the information flow constraint

$$I\left(\left\{\varepsilon_{a,t}, \varepsilon_{r,t}\right\}; \left\{s_{ai,t}, s_{ri,t}\right\}\right) \leq \kappa, \tag{15}$$

and to the optimal price setting behavior conditional on the information available at each period,

$$\log\left(p_{i,t}\right) = E\left[\log\left(p_{i,t}^*\right) \mid s_{ai}^t, s_{ri}^t\right], \tag{16}$$

where $s_{ai}^t = \{s_{ai,1},\ s_{ai,2}, ..., s_{ai,t}\}$ and $s_{ri}^t = \{s_{ri,1},\ s_{ri,2}, ..., s_{ri,t}\}$ represent the realization of the signal processes about technology and monetary policy shocks respectively up to period t. The parameter κ indexes firm's total attention. In practice, if κ is finite, the information flow constraint prevents decision makers from choosing $p_{i,t} = p_{i,t}^*$ in each period and state of the world. The operator I measures measures the average amount of information contained in the signal processes $\{s_{ai,t}, s_{ri,t}\}$ about the realizations of the fundamental shocks of the economy, and viceversa.[10]

For simplicity, this paper considers signals taking the form of fundamental shock plus noise,

$$s_{ai,t} = \varepsilon_{a,t} + u_{ai,t}, \quad u_{ai,t} \sim N\left(0, \upsilon_{ai}^2\right), \tag{17}$$

$$s_{ri,t} = \varepsilon_{r,t} + u_{ri,t}, \quad u_{ri,t} \sim N\left(0, \upsilon_{ri}^2\right), \tag{18}$$

where $u_{ai,t}$ and $u_{ri,t}$ are iid errors with standard deviations υ_{ai} and υ_{ri}.[11] This signal

definied later) is large enough so that the actual pricing behavior is not very different from the profit-maximizing one.

[10] For a definition of the operator I see Appendix A2.

[11] It is possible to show that, in the static equilibrium of this model, the optimal signal structure

structure, together with constraint (15), implies a trade-off in the attention allocation across the two types of shocks: if a firm pays more attention to one type of shock (i.e. chooses the corresponding signal process to be relatively more informative), it necessarily has to pay less attention to the other type of shock. While the assumption that firms process information independently about technology and monetary policy shocks is probably extreme, it has the important advantage of introducing an endogenous information choice into an otherwise standard general equilibrium framework while keeping the model tractable enough to allow for a closed form solution. This solution provides valuable information on the interaction between the different components of the model. In Section 5 I will show that main results of the paper are robust to other signal structures where the independence assumption is removed. In particular, I show that results of the paper about the interaction of monetary policy activism and attention allocation still hold when firms are allowed, to some extent, to process information jointly about the two types of shocks.

Equilibrium Definition: Definition 1 describes stationary equilibria in which all the endogenous variables of the economy can be expressed as functions of the realizations of the fundamental shocks $\{\varepsilon_{a,t}\}$ and $\{\varepsilon_{r,t}\}$. In what follows the notation $X_t(\cdot)$ reads $X\left(\{\varepsilon_{a,\tau}\}_{\tau=0}^{t}, \{\varepsilon_{r,\tau}\}_{\tau=0}^{t}\right)$.

Definition 1 *A stationary equilibrium is a set of functions,* $C_t(\cdot)$, $L_t(\cdot)$, $S_t(\cdot)$, $P_t(\cdot)$, $W_t(\cdot)$, $Q_{t,t+1}(\cdot)$, $p^*_{i,t}(\cdot)$, $p_{i,t}(\cdot)$, $s_{ai,t}(\cdot)$ *and* $s_{ri,t}(\cdot)$ *such that:*

(i) $\{C_t(\cdot), L_t(\cdot), S_t(\cdot)\}$ *maximizes* (1) *subject to* (2) *and* (3);

(ii) $P_t(\cdot)$ *satisfies* (7);

(iii) υ_{ai} *and* υ_{ri} *maximize* (14) *subject to* $(15)-(16)$ *and* $(17)-(18)$;

(iv) $p^*_{i,t}(\cdot)$ *satisfies* (11);

(v) $p_{i,t}(\cdot)$ *satisfies* (16);

(vi) each intermediate good producer i *satisfies the incoming demand at* $p_{i,t}(\cdot)$;

(vii) all other markets clear.

in $(14)-(16)$ is of the form $(17)-(18)$. Appendix C contains more details.

3 The static equilibrium

The model is solved through a log-linearization of the first order conditions characterizing the equilibrium of the economy in a neighbor of the non-stochastic steady state. In what follows $\hat{X}_t \equiv \log X_t - \log \bar{X}$ denotes the value of X_t in log-deviations from the non-stochastic steady state. Lemma 1 describes the non-stochastic steady state.

Lemma 1 *For a given normalization of $\bar{P}$, there exists a unique non-stochastic steady state in which $\bar{L} = \alpha\frac{\theta-1}{\theta}$, $\bar{C} = \bar{L}^{\alpha}$, $\bar{W} = \bar{P}\bar{C}$, $\bar{R} = \frac{1}{\beta}$, $\bar{p}_i = \bar{p}_i^* = \bar{P}$.*

Proof: See appendix B.

Solving for the equilibrium of this economy requires solving for a fixed point. In fact, the attention allocation problem in $(14) - (16)$ depends on the stochastic process for the profit-maximizing price, $\hat{p}_{i,t}^*$, which in turn depends on the stochastic process for the price level, $\hat{P}_t$. The latter is an average over all intermediate good prices and therefore depends itself on the solution to the attention allocation problem of firms. Proposition 1 describes the equilibrium dynamics of $\hat{P}_t$ and $\hat{C}_t$.

Proposition 1 *There exists a static equilibrium in which the equilibrium dynamics of economic variables in log-deviations from the non-stochastic steady state in period t are given by a set of linear functions of $\varepsilon_{a,t}$ and $\varepsilon_{r,t}$. In this equilibrium, the price level and consumption are given by*

$$\hat{P}_t = -\frac{\xi}{1+\phi_c}\left(\gamma_a\varepsilon_{a,t} + \gamma_r\varepsilon_{r,t}\right), \tag{19}$$

$$\hat{C}_t = -\frac{1-\phi_\pi}{1+\phi_c}\hat{P}_t - \frac{\phi_c}{1+\phi_c}\varepsilon_{a,t} - \frac{1}{1+\phi_c}\varepsilon_{r,t}, \tag{20}$$

where γ_a *and* γ_r *are coefficients given by*

$$(\gamma_a;\ \gamma_r) = \begin{cases} (\bar{\gamma};\ 0) & if \quad \sigma > \bar{\sigma} \\ \left(\Gamma(\sigma);\ \Gamma\left(\frac{1}{\sigma}\right)\right) & if \quad \frac{1}{\bar{\sigma}} \leq \sigma \leq \bar{\sigma} \\ (0;\ \bar{\gamma}) & if \quad \sigma < \frac{1}{\bar{\sigma}} \end{cases}, \tag{21}$$

while the coefficients σ, ϕ, $\bar{\gamma}$, $\bar{\sigma}$, *and the function* $\Gamma(\cdot)$ *are given by*

$$\sigma \equiv \frac{\sigma_a}{\sigma_r}, \tag{22}$$

$$\phi \equiv \frac{1-\phi_\pi}{1+\phi_c}, \tag{23}$$

$$\bar{\gamma} = \frac{1-2^{-2\kappa}}{1-(1-\phi\xi)\left(1-2^{-2\kappa}\right)}, \tag{24}$$

$$\Gamma(x) = \frac{\phi\xi + 2^{-2\kappa}(1-\phi\xi) - 2^{-\kappa}\frac{1}{x}}{(\phi\xi)^2 - 2^{-2\kappa}(1-\phi\xi)^2}, \tag{25}$$

$$\bar{\sigma} = \min\left\{2^{\kappa}\frac{\xi\phi}{1-\xi\phi}; 2^{\kappa}\phi\xi + 2^{-\kappa}(1-\phi\xi)\right\}. \tag{26}$$

Proof: See Appendix C.

The equilibrium responses of prices to the two shocks depend on relative volatility, σ, on the degree of real rigidity, ξ, on the average quantity of information processed per period, κ, and on ϕ. The parameter ϕ has an important economic meaning, as it indexes relative monetary policy aggressiveness on expected inflation and output-gap. The smaller ϕ, the more aggressive policy on expected inflation or output-gap.

The function $\Gamma(\cdot)$ determines the equilibrium price level responsiveness to a given shock as a function of relative volatility of that shock. The function $\Gamma(\cdot)$ is increasing in its argument for values of $\sigma \in (\frac{1}{\bar{\sigma}}, \bar{\sigma})$. Therefore, the equilibrium price level is more responsive to relatively more volatile shocks.

Moreover, the slope of $\Gamma(\cdot)$ with respect to its argument depends on $\phi\xi$ and κ : the smaller $\phi\xi$ and κ, the larger the impact of a change in relative volatility, σ, on price level responsiveness to the two shocks.

Let's define relative price responsiveness to the two types of shocks as $\gamma \equiv \frac{\gamma_a}{\gamma_r}$, where γ_a and γ_r are given by (21). If $\gamma > 1$ prices are relatively more responsive to technology shocks than to monetary policy shocks and viceversa.

Proposition 2 *At an interior solution of the attention allocation problem in* (28),

1. *Relative price responsiveness, γ, is strictly increasing in relative standard deviation of technology shocks, σ.*

2. *If $\sigma > 1$ $(\sigma < 1)$, relative price responsiveness to technology shocks, γ, is strictly decreasing (increasing) in the degree of real rigidity, ξ, in the degree of relative monetary policy aggressiveness, ϕ, and in the upper bound on information flow, κ.*

Proof: See appendix D.

For illustrative purposes, in Figure 3 I plot values of γ as a function of $\phi\xi$ and σ, for a given value of κ. For instance, if $\sigma = 2$ and $\phi\xi = 0.5$, price responsiveness to technology shocks is only about fifty percent larger than to monetary policy shocks; if, instead, $\sigma = 2$ and $\phi\xi = 0.3$, price responsiveness to technology shocks becomes four times as large as price responsiveness to monetary policy shocks. If $\phi\xi$ is further decreased, the model delivers a corner solution where prices respond only to technology shocks. Therefore, in this example, relatively more aggressive monetary policy on expected inflation and output-gap (i.e. lower ϕ), or higher real rigidity (i.e. lower ξ), significantly magnify differences in price responsiveness.

Next sections discusses more in detail the way monetary policy and the other structural parameters affect equilibrium price level responsiveness through the endogenous attention allocation decision.

3.1 Equilibrium attention allocation

The equilibrium price responsiveness in $(21) - (26)$ depends on the equilibrium attention allocation by firms. In fact, the more informative signals $(17) - (18)$ are, the more responsive prices are to each shock. How informative is each type of signal is determined endogenously through the attention allocation decision. This section describes the properties of the equilibrium attention allocation.

Solving the attention allocation problem implies choosing the precision of signals $(17)-(18)$ so to maximize (14) subject to $(15)-(16)$. The attention allocation problem depends on the equilibrium dynamics of the profit-maximizing price. These dynamics, in deviations from the non-stochastic steady state, are obtained by substituting (20) into (11),

$$\hat{p}_{it}^{*} = (1 - \xi\phi)\,\hat{P}_t - \frac{\xi}{1+\phi_c}\left(\varepsilon_{a,t} + \varepsilon_{r,t}\right) \tag{27}$$

where the equilibrium dynamics of $\hat{P}_t$ are given by $(21)-(26)$. The coefficient $\xi\phi$ can be interpreted as the degree of strategic complementarity in price setting: the smaller $\xi\phi$, the larger the feedback from the price level to profit-maximizing prices. Given that attention allocation decision depends on the dynamics of $\hat{p}_{it}^{*}$, and the price level, $\hat{P}_t$, depends on the average allocation of attention of firms in the economy, the coefficient $\xi\phi$ also represents the degree of complementarity in attention allocation: the smaller $\xi\phi$, the larger the feedback from average attention allocation to to profit-maximizing prices and, therefore, to firm's allocation of attention decision.

According to the objective of the attention allocation problem, for given dynamics of $\hat{p}_{it}^{*}$, the firms faces a smaller loss in profits at lower values of the mean square error in price setting. Given the average amount of information processed per period, κ, the mean square error in price setting is larger, the larger the volatility of the shocks and the larger the responsiveness of $\hat{p}_{it}^{*}$ to the shocks. Firms can reduce the mean square error due to a particular shock by allocating relative more attention to it. Therefore, firms have incentives to allocate a larger fraction of κ to the type of shock that is either more volatile or induces a larger responsiveness of the profit-maximizing price.

Proposition 3 *In equilibrium, the optimal attention allocation is such that signal* *precision to each type of shock is given by*

$$\left(\frac{\sigma_a^2}{\sigma_a^2+\upsilon_{ai}^{2*}};\frac{\sigma_r^2}{\sigma_r^2+\upsilon_{ri}^{2*}}\right)=\begin{cases}\left(1-2^{-2\kappa};\ 0\right) & if\ \sigma>\bar{\sigma}\\ \left(1-\frac{2^{-\kappa}}{\omega\sigma};\ 1-2^{-\kappa}\omega\sigma\right) & if\ \frac{1}{\bar{\sigma}}<\sigma<\bar{\sigma}\\ \left(0;\ 1-2^{-2\kappa}\right) & if\ \sigma<\frac{1}{\bar{\sigma}}\end{cases} \tag{28}$$

where ω represents profit-maximizing price responsiveness to technology shocks relative to monetary policy shocks,

$$\omega\equiv\frac{(1-\xi\phi)\,\gamma_a+1}{(1-\xi\phi)\,\gamma_r+1}. \tag{29}$$

Proof: See Appendix C.

Firms allocate relatively more attention to technology shocks than to monetary policy shocks either because technology shocks are more volatile, i.e. $\sigma>1$, or because they have a larger impact on the profit-maximizing price than monetary policy shocks, i.e. $\omega>1$. However, while shock volatilities are exogenous to the model, profit-maximizing price responsiveness is not. It depends on the responsiveness of the price level to the different shocks, i.e. γ_a and γ_r. In particular, by substituting (21) into (29) it is possible to derive ω as a function only of the structural parameters of the model,

$$\omega=\frac{\phi\xi-\frac{1}{\sigma}2^{-\kappa}\,(1-\phi\xi)}{\phi\xi-\sigma2^{-\kappa}\,(1-\phi\xi)}. \tag{30}$$

It follows from (28) and (30) that shock volatilities affect the attention allocation through two channels. First, as discussed above, for given profit-maximizing price responsiveness to shocks, more attention is paid to more volatile shocks. Second, shock volatilities influence the attention allocation problem through relative profit-maximizing responsiveness, ω: since more volatile shocks receive relatively more attention by all firms, they also have a higher associated price level responsiveness; the feedback effect from price level responsiveness to the profit-maximizing price responsiveness affects the attention allocation decision. Whether this feedback reinforces or reduces the impact of differences in volatilities of shocks on the attention allocation decision depends on the degree of complementarity in attention allocation, $\phi\xi$. It is at this stage that parameters of the interest rate feedback rule affect the attention allocation decision.

In the case of positive complementarity in attention allocation, $\phi\xi < 1$, if intermediate good producer i's competitors are more responsive to a type of shock, then it is more worthwhile for intermediate good producer i to pay attention to that shock. In this case, the feedback effect reinforces the impact of different volatilities on attention allocation; in contrast, in the case $\phi\xi > 1$, if intermediate good producer i's competitors are more responsive to a type shock, then it is less worthwhile for intermediate good producer i to pay attention to that shock. In this case, the feedback effect reduces the impact of different volatilities.

3.1.1 Discussion of results

This section provides a more informal discussion of results about the interaction of real rigidities, monetary policy and complementarity in attention allocation. Economic intuition can be gained from the profit-maximizing price equation (11), where $\log(p_{it}^{*})$ depends on the price level, P_t, and on the the output-gap, $\frac{C_t}{e^{\varepsilon_{a,t}}}$. It follows from (11) that the partial elasticity of the profit-maximizing price with respect to the price level is equal to one, while it is equal to ξ with respect to the output-gap. Therefore, for given price level and output-gap dynamics, the smaller ξ, the relatively larger the weight of the price level in profit-maximizing price dynamics. Higher real rigidities imply relatively higher feedback from the price level to profit-maximizing prices. Therefore, through the price level, the allocation of attention decision by other firms becomes relatively more important for the individual firm decision.

In order to understand how monetary policy interacts with complementarities, we need to understand the way monetary policy interacts with output-gap dynamics. In the policy rule (4) , an increase in both ϕ_π and ϕ_c reduces the fluctuations in output-gap to all shocks. For given price level responsiveness, the smaller responsiveness of the output-gap to shocks induces the price level to be relatively more important for profit-maximizing price dynamics. Of course, in equilibrium, the increase in ϕ_π and ϕ_c also affects price level responsiveness, but it does so through averaging over prices set by firms, which depends on the feedback from the price level to the profit-maximizing price. Therefore, a monetary policy that *lean against the wind* increases the feedback effect from the price level to the profit-maximizing price, increasing complementarity in attention allocation and, therefore, amplifying the difference in price responsiveness.

4 The dynamic extension

The simple general equilibrium model analyzed sofar has provided valuable economic insights on the role of monetary policy, and other structural parameters, in determining price responsiveness to technology and monetary policy shocks. This section extends such a model to a more dynamic framework in order to study price and inflation impulse responses to persistent innovations.

In particular, let's assume that innovations to labor productivity in (8) depend on the following exogenous processes,

$$\varepsilon_{a,t} = \rho_a \varepsilon_{a,t-1} + \eta_{a,t} \tag{31}$$

where $\eta_{a,t}$ is normal and iid, $\eta_{a,t} \backsim N\left(0, \sigma_a^2\right)$. Let's also assume that there is inertia in nominal interest rates so that the dynamics of R_t are given by

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\rho_r} \left[\left(E_t \frac{\Pi_{t+1}}{\bar{\Pi}}\right)^{\phi_\pi} \left(\frac{C_t}{C_t^*}\right)^{\phi_c} e^{\varepsilon_{r,t}}\right]^{1-\rho_r}, \tag{32}$$

The rest of the economy is unchanged from previous sections. While this basic model lacks many features of standard business cycle models, such as physical capital accumulation, it is able to generate quite rich dynamics of price and inflation impulse responses to the two types of shocks.[12]

4.1 Model calibration

It is not possible to solve the model analytically so I use numerical methods.[13] I drew on the business cycle literature for the values of the preference parameter, θ, of output elasticity to labor, α, and discount factor, β. In particular, similarly to Golosov and Lucas [12], the demand elasticity parameter θ is set equal to 7, while the parameter α is set equal to 0.64, to match the average labor share of output in the U.S. This implies a degree of real rigidity $\xi = 0.32$.[14] The discount factor β is set to $\beta = 0.993$, so to have an annual nominal interest rate in steady state equal to 3 percent.

Monetary policy parameters, ϕ_π and ϕ_c, are set equal to estimates of (32) on the U.S. data from 1979 to 2007, corresponding to the terms of Volcker and Greenspan at the helm of the Federal Reserve.[15] Given these estimates, I set $\phi_\pi = 2$, $\phi_c = 0.21$ and $\rho_r = 0.71$. The volatility of the monetary policy shock is set equal to the standard deviation of the residual in the estimation of (32) , implying $\sigma_r = 0.0018$.

The parameters of the exogenous productivity process are obtained from fitting an AR(1) process to the detrended logarithm of U.S. total factor productivity estimated by Fernald [11] from 1979 to 2007.[16] Therefore, I set $\rho_a = 0.7$ and σ_a to match the estimated standard deviation of innovations in the AR(1) process for total factor productivity, equal to 0.006.[17] Finally, similarly to Mackowiak and Wiederholt [18], I

[12] In a previuos version of this paper (available on the author's web site) I have solved a model with capital accumulation, investment adjustment costs and habit formation. While the computational burden increases, results of this paper are robust to these different assumptions.

[13] See Appendix E for detalis.

[14] Notice that this is a conservative calibration of ξ. In the new-Keynesian literature the parameter ξ is often set at lower values. For instance, Woodford [27] suggest values of ξ between 0.1 and 0.15.

[15] Estimates have been obtained applying GMM techniques, as suggested by Clarida, Gali and Gerlter [8]. I refer to these authors for more details on the estimation technique. Data on expected inflation has been obtained from the Survey of Professional Forecasters available on-line at the Philadelphia FED.

[16] Fernald [11] estimates TFP in the U.S. with a Solow residual approach, adjusting for labor hoarding and capital utilization.

[17] Figure 2 plots US TFP growth rate and changes in the Federal Funds rate.

set $\kappa = 3$. This is a conservative calibration for κ, as in equilibrium firms face a very small loss from not being perfectly informed about technology and monetary policy shocks. Such a loss is in the order of 0.1 percent of steady state revenues.

4.2 Impulse responses

In the first column of Figure 4, I plot the impulse responses of inflation and price level to technology and monetary policy shocks. The model correctly predicts inflation and the price level to be substantially more responsive to technology shocks than to monetary policy shocks. In fact, firms allocate 78 percent of information processing capabilities, κ, to technology shocks and only 22 percent to monetary policy shocks. As a consequence, they are on average more informed about realizations of aggregate technology shocks, justifying the asymmetry in inflation and prices behavior in response to the two shocks seen in the data.

As benchmark of comparison, in the second column of Figure 4, I plot impulse responses of inflation and the price level under the assumption that the friction in price setting is not imperfect information but rather nominal rigidities. In particular, I consider a standard Calvo-type model of price setting under perfect information, where firms have an exogenous probability δ of not changing their prices in any given period. In this model, the dynamics of inflation in log-deviation from the non-stochastic steady state are given by[18]

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{(1-\delta\beta)(1-\delta)}{\delta} \xi \left(\hat{C}_t - \varepsilon_{a,t} \right). \tag{33}$$

I calibrate δ to 0.3 as estimated by Bils and Klenow [5] on U.S. data. Comparing the Calvo model to the rational inattention model we see that: i) inflation and the price level display similar inertia to monetary policy shocks in the two models; ii) inflation and price level respond much faster to technology shocks under rational inattention

[18] See Woodford [27] for a derivation.

than under Calvo. More specifically, under Calvo, the price level and inflation display identical dynamics in response to technology and monetary policy shocks. Intuitively, in both models of price setting, the underlying framework is such that the mapping from the two types of shocks to the profit-maximizing price is the same. However, differently from the rational inattention model, in the Calvo model the friction in price setting is also identical across the two shocks. The latter is roughly responsible for the different predictions of the Calvo model.

However, these results do not mean that Calvo models of price setting always imply inflation to respond the same way to technology and monetary policy shocks. In fact, it is possible to build a model where inflation responds differently to the two shocks, by allowing for a different mapping from shocks to profit-maximizing prices. However, other authors have shown that matching inflation responses to technology and monetary policy shocks in these models is, at least, challenging.[19]

The advantage of the model presented in this paper is that it does not need to rely on specific assumptions about the way technology and monetary policy shocks transmits to profit-maximizing prices in order to explain the different behavior of inflation, but only relies on endogenous attention allocation decisions by firms.

4.3 Interest rate feedback rule and endogenous attention allocation

The numerical implementation in the previous section has shown that the model of rational inattention successfully accounts for the different behavior of inflation in response to technology and monetary policy shocks. From the closed form solution to the static model of section 3 we have learned that this results depends on two main ingredients: i) technology shocks need to be more volatile than monetary policy shocks; ii) together with real rigidities, the weights the interest rate feedback rule assigns to expected inflation and output stabilization directly affect the attention

[19] See Dupor et al. [10], Altig et. al. [2] and Paciello [21].

allocation decision by firms through complementarity in attention allocation.

This section answers the following question: how important is monetary policy activism in explaining the different behavior of inflation to technology and monetary policy shocks under rational inattention? In order to answer this question, I do the following counterfactual exercise: I solve the model under the assumption of inactive monetary policy, i.e. $\phi_\pi \to 1$ and $\phi_c \to 0$, while the remaining structural parameters are unchanged from the benchmark calibration.[20]

In the first column of Figure 5, I plot impulse responses of inflation and price level to technology and monetary policy shocks under the counterfactual monetary policy. As we can see, inflation and price level respond much more similarly to the two shocks than under the benchmark calibration. In particular, the allocation of attention to technology shocks drops from 78 percent of κ under the benchmark calibration, to 65 percent of κ, under the counterfactual policy. As a consequence, attention allocation to monetary policy shocks rises from 22 percent to 35 percent of κ.

Therefore, according to the model of this paper, the *active* interest rate feedback rule estimated in the data has amplified substantially the impact of differentials in shock volatilities on differentials in inflation responsiveness to technology and monetary policy shocks. In this sense, monetary policy is as important as shock volatilities in explaining observed inflation responsiveness.

4.3.1 Discussion on impact of monetary policy on economic fluctuations

Several authors have recently studied optimal monetary policy in models of imperfect information.[21] While studying optimal policy is beyond the scope of this paper, the paper yields novel predictions on the impact of a change in the coefficients of the

[20] One could also allow for κ to respond to the change in in monetary policy. While this is realistic, it has been studied by Branch et al. [7] in a framework with endogenous inattention, and I refer to these authors for a discussion. This paper looks at another margin, working through

[21] For instance, Adam [1] has studied optimal monetary policy under imperfect information, but wihtout attention allocation decision. Lorenzoni [15], Angelitos and La'O [3] and Angelitos and Pavan [4] have recently studied optimal monetary policy in frameworks with imperfect information, where the monetary policy instruments may affect information dipsersion.

Taylor rule on the economy when compared to more standard models of sticky prices. In particular, this paper has shown that monetary policy affects the economy trough a novel channel related to the attention allocation decision.

When monetary authority changes the coefficients of the Taylor rule on expected inflation and output, it affects the economy trough two channels. The first channel is a standard one, taking place also in models of nominal rigidities: for given information structure, a nominal interest rate responding more (less) to expected inflation and output fluctuations accommodates technology shocks and offsets monetary policy shocks more (less); this reduces (increases) output-gap fluctuations, causing a smaller (larger) variability of prices to both types of shocks. The second channel is novel: by affecting the degree of complementarity in attention allocation, a more (less) active policy induces firms to pay more (less) attention to the most volatile shocks and less (more) to the least volatile ones.

Tables 1 and 2 report standard deviations of inflation and output-gap respectively, computed conditional on technology and monetary policy shocks, under both active and inactive policies.[22]

Table 1: volatility of quarter-on-quarter inflation conditional on technology and monetary policy shocks

	Rational Inattention Model		Calvo Model	
	Active Policy	Inactive Policy	Active Policy	Inactive Policy
TECH	0.57	0.52	0.08	0.17
MP	0.16	0.38	0.08	0.17

[22] The active policy is the benchmark calibration: $\phi_\pi = 2$, $\phi_c = 0.25$. The inactive policy is $\phi_\pi \to 1$, $\phi_c \to 0$.

Each statistic is scaled by the standard deviation of the corresponding shock. Equivalently, these statistics refer to shocks with unit standard deviations.

Table 2: volatility of quarterly output-gap conditional on technology and monetary policy shocks

	Rational Inattention Model		Calvo Model	
	Active Policy	Inactive Policy	Active Policy	Inactive Policy
TECH	0.57	0.54	0.11	0.20
MP	0.19	0.31	0.11	0.20

In the rational inattention model, going from the inactive to the active monetary policy causes little impact on inflation and output-gap variability conditional on technology shocks. In contrast, conditional on monetary policy shocks, inflation and output-gap variability get reduced by about a half by the monetary policy activism. This asymmetry is due to the fact that monetary policy activism causes higher fraction of attention allocated to technology shocks, making firms more informed on these shocks. This worsens monetary authority power to stabilize the economy conditional on these shocks, so that, despite the more aggressive policy, inflation and output-gap variabilities are not reduced. In contrast, monetary policy activism causes lower fraction of attention allocated to monetary policy shocks. This improves monetary authority power to stabilize the economy conditional on these shocks, so that the more aggressive policy has a larger impact on inflation and output-gap variabilities to monetary policy shocks.

These results contrast with the predictions from the Calvo model: there monetary policy activism has similar effects on output-gap and inflation variability conditional on technology and monetary policy shocks, as the frequency of price setting is exogenous to the model.[23] Exploring further the consequences for optimal monetary policy of the link between monetary policy and information acquisition decisions by firms is

[23] Notice that volatility of inflation and output-gap are generally lower under Calvo. This is due to the conservative calibration of κ under rational inattention, i.e. to the low amount of frictions assumed in the model. I see this as a plus: decreasing κ would further increase asymmetry in price responsiveness.

in the author's view an important avenue for future research.

Finally, one could also allow for κ to respond endogenously to the changes in the monetary policy rule. The endogeneity of information acquisition rate has already been studied by Branch et al. [7] in a slightly different framework with endogenous inattention. This paper focuses instead on the attention allocation margin, as this is the margin that allows to explain the different behavior of prices in response to technology and monetary policy shocks. Intuitively, adding the extra-margin of Branch et al. would reinforce results: for a given marginal cost of an additional unit of κ, as monetary policy gets more active, nominal variability decreases, inducing an endogenous decrease in κ; the decrease in κ causes relative differences in attention allocation and price responsiveness to increase even more. Therefore, allowing for endogenous κ would further amplify the effect of changes in monetary policy parameters on the attention allocation.

5 Robustness analysis

This section investigates to what extent results from the model of section 2 are robust to different set of assumptions about information channels. The insights from these exercises reinforce the results obtained in the previous sections.

5.1 Removing the independency assumption on information processing

So far this paper has assumed that attending to technology and monetary policy shocks are separate activities. Hellwig and Venkateswaran [13] show that, by allowing for a signal process that contains information on two types of shocks, it is possible that firms respond relatively fast to a given type of shock, despite this shock is relatively not very volatile. Therefore, let's consider the case in which signals provide information of both types of shocks, similarly to Hellwig and Venkateswaran [13], but where the volatility of the noise in these signals is endogenous. In particular, let's consider a signal structure suggested by Mackowiak and Wiederholt [18],

$$s_{ai,t} = \hat{p}^*_{at} + \psi \hat{p}^*_{rt} + u_{ai,t}, \quad u_{ai,t} \sim N\left(0, \upsilon^2_{ai}\right), \tag{34}$$

$$s_{ri,t} = \hat{p}^*_{rt} + \psi \hat{p}^*_{at} + u_{ri,t}, \quad u_{ri,t} \sim N\left(0, \upsilon^2_{ri}\right), \tag{35}$$

where $\hat{p}^*_{ait}$ and $\hat{p}^*_{rit}$ are linear combinations of ε_{at} and ε_{rt}, representing the profit-maximizing responses to technology and monetary policy shocks, so that from (27) I have that $\hat{p}^*_{at} = \hat{p}^*_{at} + \hat{p}^*_{rt}$.[24] The coefficient ψ is a constant, indexing the information content of each signal about the two types of shocks: if $0 < \psi < 1$, signal $s_{ai,t}$ is relatively more informative about profit-maximizing responses to technology shocks than to monetary policy shocks.

The firm will now choose υ_{ai} and υ_{ri} to maximize (14) subject to $(15)-(16)$, given the signal structure in $(34) - (35)$. If technology shocks are relatively more volatile than monetary policy shocks, the optimal attention allocation is such that firms pay relatively more attention to the signal providing relatively more information on technology shocks. As $\psi \to 0$ or $\psi \to \infty$ the solution converges to the solution presented in Section 2. Only if the decision-maker can attend directly to a sufficient statistic concerning the profit-maximizing price ($\psi = 1$) the price responds to monetary policy shocks in the same way as to aggregate shocks.

How much ψ has to be different from 1 in order for prices to respond sufficiently stronger to technology shocks than to monetary policy shocks depends, among other things, on the degree of strategic complementarity in price setting, on monetary policy and on volatility of the two shocks. Figure 6 plots relative price responsiveness, γ, as

[24]

$$\begin{aligned} \hat{p}^*_{at} &= -\frac{\xi}{1+\phi_c}\left[\left(1-\xi\phi\right)\gamma_a + 1\right]\varepsilon_{a,t} \\ \hat{p}^*_{mt} &= -\frac{\xi}{1+\phi_c}\left[\left(1-\xi\phi\right)\gamma_m + 1\right]\varepsilon_{m,t} \end{aligned}$$

a function of ψ and $\xi\phi$, under a calibration for which technology shocks are relatively more volatile, $\sigma > 1$. Allowing for signals providing information on both types of shocks reduces differences in price responsiveness relative to the case of independent signals, for given parameterization of the model, but it is still the case that prices will respond relatively more to more volatile shocks, as the volatility in the signal noise is chosen optimally.

If signals provide information on both types of shocks, the impact of shock volatility differentials on price responsiveness differentials is weakened. This makes more crucial understanding the role played by strategic complementarity in price setting and monetary policy in magnifying the impact of volatilities differentials onto allocation of attention.

5.2 Allowing for signals on endogenous aggregate variables

An alternative assumption on the information structure of the private sector is to have firms processing information on the realizations of endogenous aggregate variables. Specifically, let's assume that each price setter can receive the following signals,

$$s_{i,t} = \begin{cases} \hat{C}_t + u^c_{i,t}, & u^c_{i,t} \sim N\left(0, \upsilon^2_c\right) \\ \hat{P}_t + u^p_{i,t}, & u^p_{i,t} \sim N\left(0, \upsilon^2_p\right) \\ \hat{R}_t + u^r_{i,t}, & u^r_{i,t} \sim N\left(0, \upsilon^2_r\right) \\ \hat{L}_t + u^l_{i,t}, & u^l_{i,t} \sim N\left(0, \upsilon^2_l\right) \end{cases}, \tag{36}$$

where $u^j_{i,t}$ is assumed to be *iid* across both time and individuals.[25] This signal structure conveys the idea that each firm processes information about realizations of variables that are usually available in the real world. Given that the price setter is interested in extracting information about the realization of the profit-maximizing price,

[25] I assume that these statistics contains no public noise. Information is therefore published and available with no error. The noise in the signals has to be interpreted exclusively as firm specific errors in processing the information.

$\hat{p}^*_{i,t}$, he will pay attention to the different signals accordingly. Differently from the signal-extraction literature, and in the spirit of the rational inattention literature, the price setter chooses the precision of the signals, $(\upsilon_c, \upsilon_p, \upsilon_r, \upsilon_l)$, to maximize the quadratic objective in (13), subject to the following constraint on the average amount of information processed per period,

$$I\left(\{\varepsilon_{a,t}, \varepsilon_{r,t}\}; \{s_{i,t}\}\right) \leq \kappa. \tag{37}$$

By choosing how precisely to acquire information about the different signals in (36), the price setter implicitly chooses to have its price responding more accurately to one of the two types of shocks. To understand why, let's focus on the signals on consumption and price level. The covariance between the profit-maximizing price and consumption, conditional on the realizations of technology shocks, has a negative sign: after a positive technology shock, the profit-maximizing price decreases while consumption increases. In contrast, the covariance between the profit-maximizing price and consumption, conditional on the realizations of monetary policy shocks, has a positive sign: after a positive monetary policy shock, both the profit-maximizing price and consumption decrease. If technology shocks have relatively larger volatility, $\sigma > 1$, then the covariance of the profit-maximizing price with consumption is negative, as such shocks account for a larger fraction of the overall covariance than monetary policy shocks. Therefore, if $\sigma > 1$, by responding to the arrival of information on consumption alone, the price setter responds with the *right* sign if the source of variation is a technology shock, but with the *wrong* sign if the source of variation is a monetary policy shock.

However, not all type of signals imply a *trade-off* in the sign of the response of prices to shocks. For example, if $\phi\xi < 1$, the price level is always positively correlated with the profit-maximizing price, independently from the type of shock. By paying more attention to the signal on the price level, the price setter responds with the *right* sign to both types of shocks. Figure 7 plots relative price responsiveness, γ, as a function of relative volatility of the two shocks for a given calibration of the other parameters. Similarly to the previous case, for given parameterization of the model, the difference in price responsiveness to the two types of shocks is smaller than in the benchmark model of section 2.

6 Concluding remarks

This paper has shown that a simple model of price setting under rational inattention and attention allocation naturally generates prices to be more responsive to aggregate technology shocks than to monetary policy shocks. In the model of this paper, firms have incentives to allocate more attention to technology shocks than to monetary policy shocks because the former are more volatile than the latter. However, a combination of relatively high real rigidity and aggressive monetary policy is needed to magnify the impact of different volatilities on relative price responsiveness. In particular, an interest rate feedback rule responding to expected inflation and output amplifies the effects of exogenous shock volatility differential on price responsiveness differential to the two shocks.

This paper has derived the channel through which parameters of the Taylor rule affect the attention allocation decision by firms. According to this channel, a monetary policy relatively more aggressive on inflation increases relative differences in price responsiveness to technology and monetary policy shocks by inducing firms to allocate more attention to the most volatile shock. This channel implies different predictions about the impact of a given policy rule on economic dynamics than more standard models of price rigidity.

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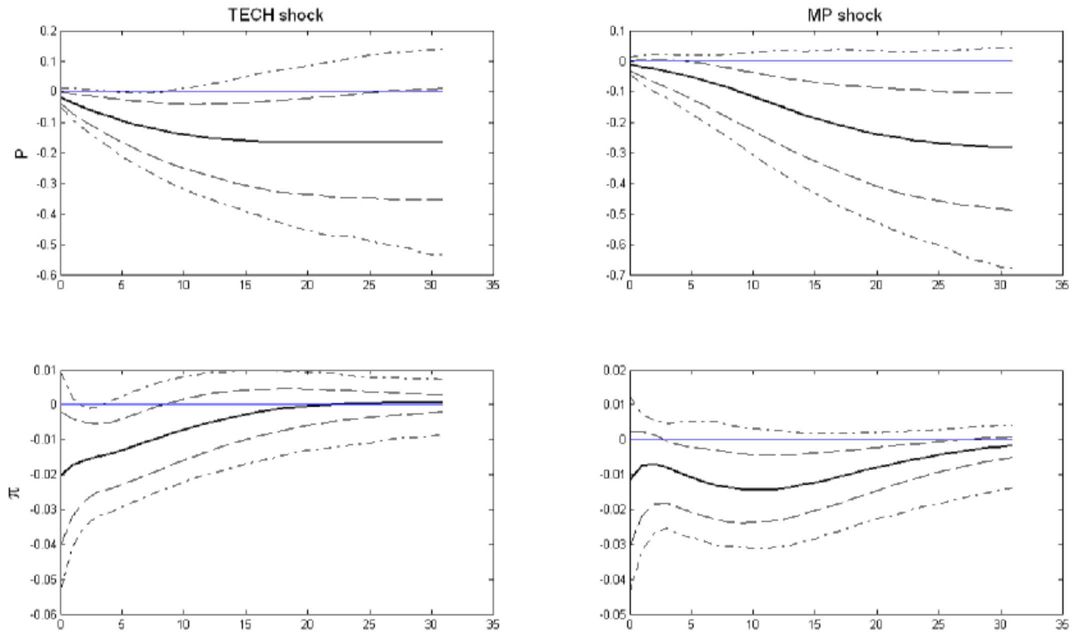

Figure 1: Responses of GDP deflator level (p) and inflation (π) to a one standard deviation shock to technology and monetary policy in the U.S. from 1960 t0 2007; source: Paciello (2009). Solid line is the median response, dotted lines are the 5^{th} , 16^{th} , 84^{th} and 95^{th} quantiles. Quarters are on the horizontal axis.

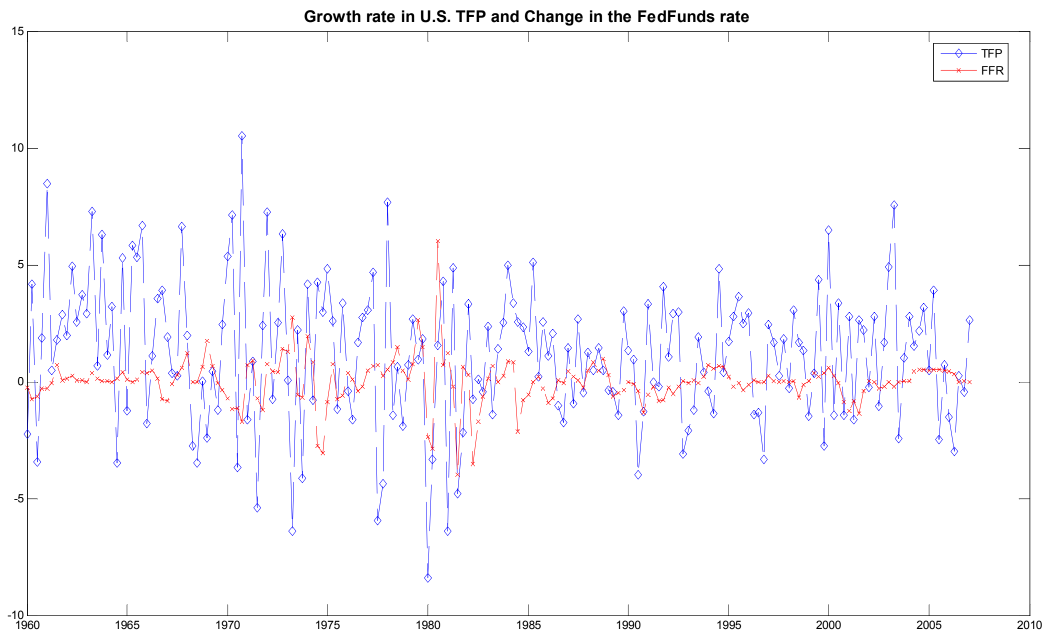

Figure 2: Growth rate in quarterly growth rate in U.S. TFP (annual basis) estimated by Fernald (2007) and change in the quarterly average of the FedFunds rate (annual basis).

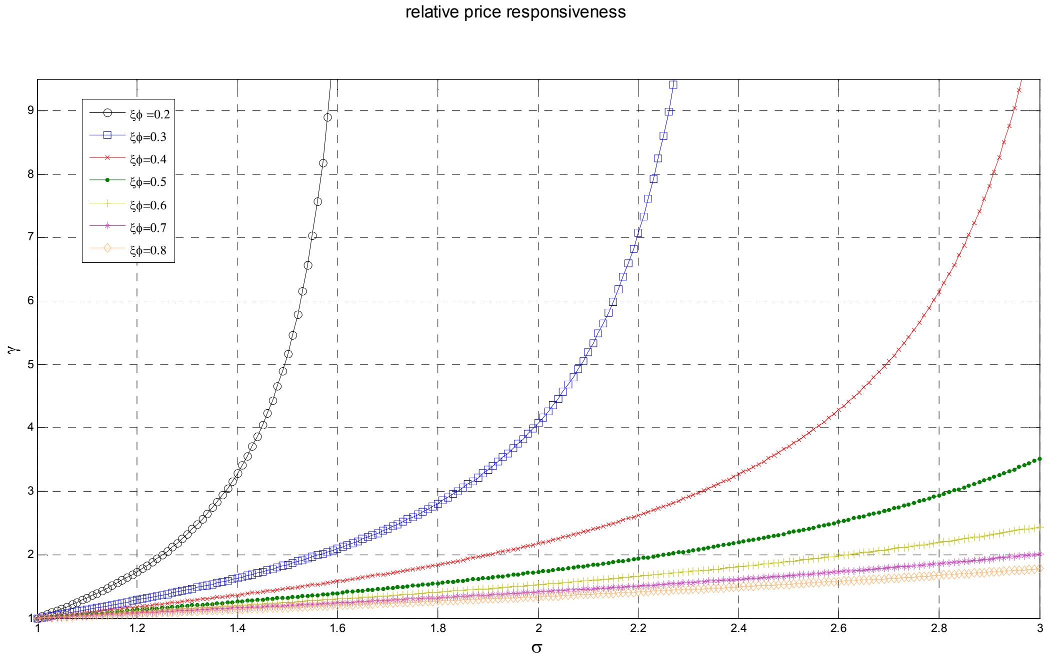

Figure 3 : Relative price responsiveness as a function of relative volatilities of shocks and complementarities; parameter $\kappa = 3$.

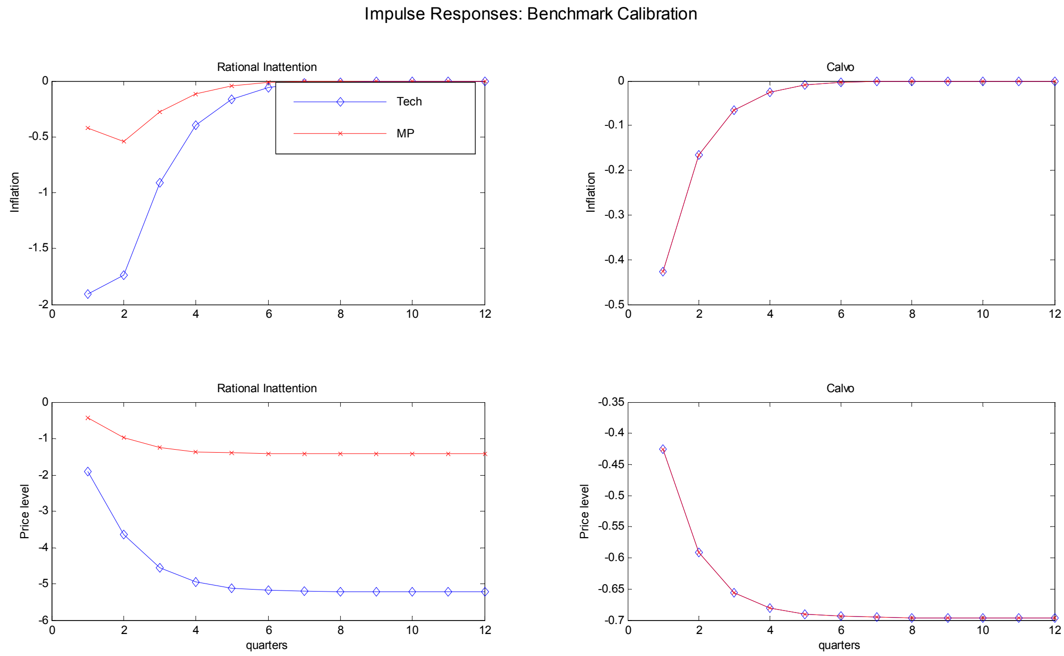

Figure 4: Inflation and price level impulse responses under the benchmark calibration.

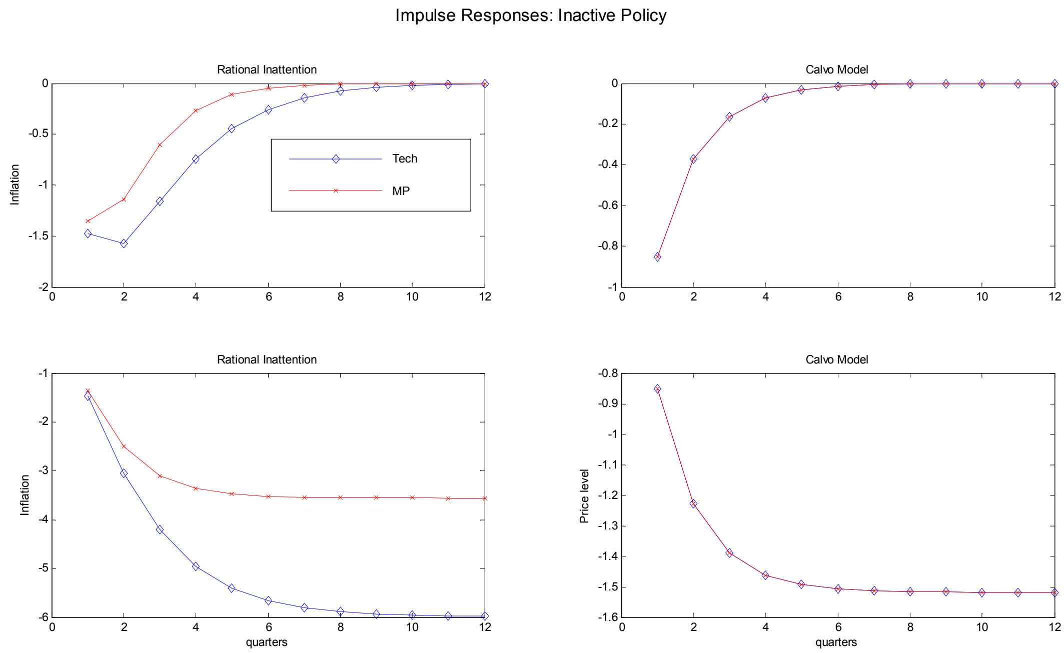

Figure 5: Inflation and price level impulse responses under inactive policy, $\phi_\pi \rightarrow 1$ and $\phi_c \rightarrow 0$.

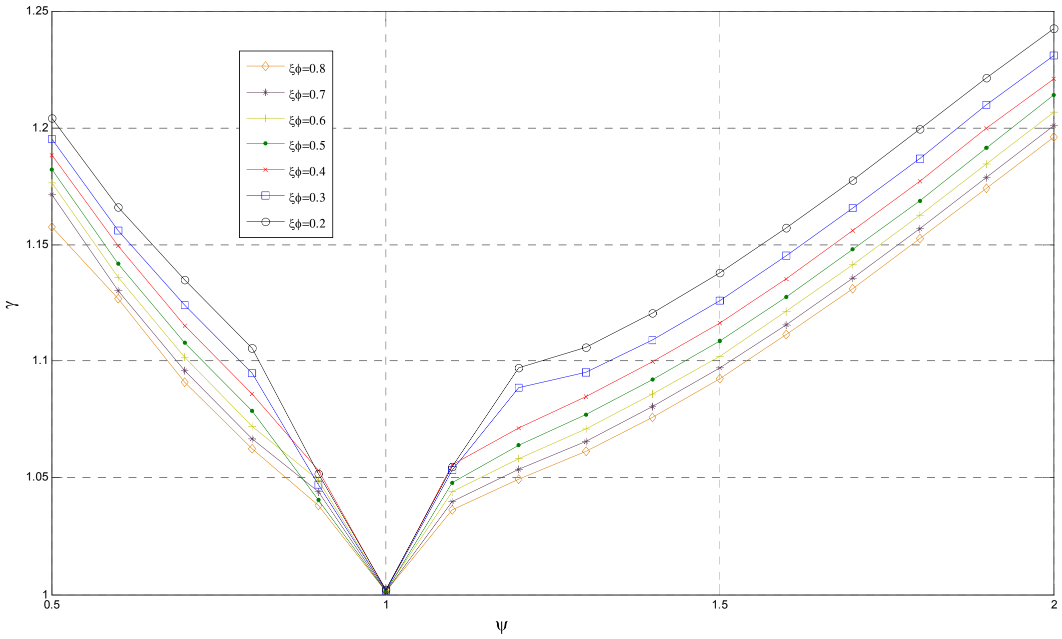

Figure 6 : Relative price responsiveness as a function of signal information content, ψ, and of strategic complementarities in price setting in model of section 5.1; parameters κ = 3, σ=2.

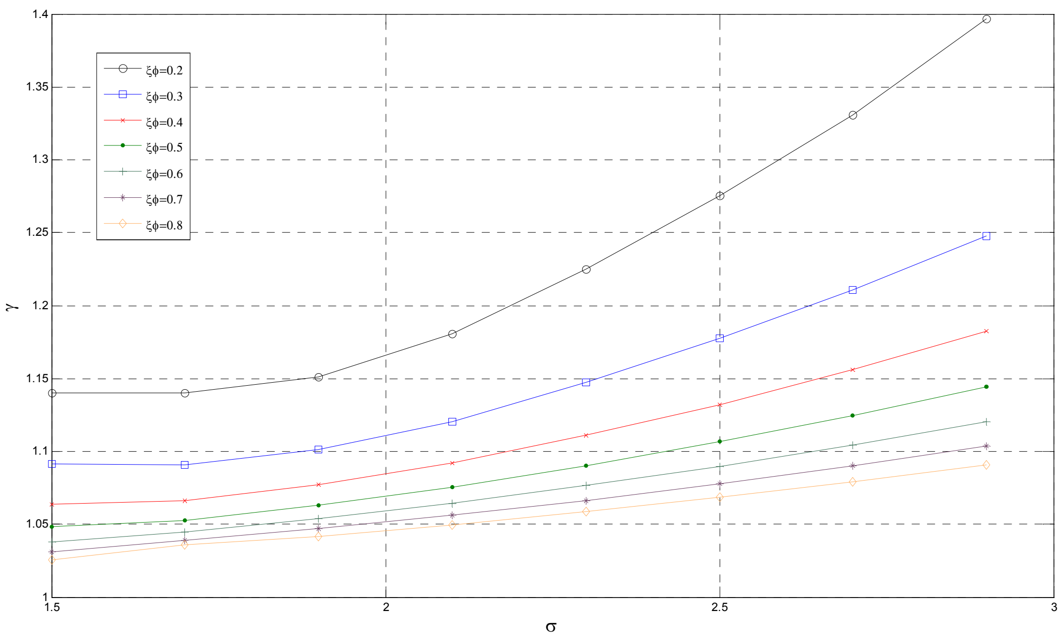

Figure 7 : Relative price responsiveness as a function of relative standard deviation,σ, and of strategic complementarities in price setting in model of section 5.1.1; parameter κ = 3.

Appendices

A.1 Derivation of attention allocation problem objective

The profit function of firm i at time t is given by

$$\pi_{i,t} = \pi\left(p_{i,t}, W_t, \varepsilon_{a,t}, C_t, P_t\right) = P_t C_t \left(\frac{p_{i,t}}{P_t}\right)^{1-\theta} - W_t \left(\frac{C_t \left(\frac{p_{i,t}}{P_t}\right)^{-\theta}}{e^{\varepsilon_{a,t}}}\right)^{\frac{1}{\alpha}}, \tag{38}$$

Profits can be expressed in terms of log-deviations from the non-stochastic steady state:

$$\begin{aligned} \pi_{i,t} &= \breve{\pi}\left(\hat{p}_{i,t}, \hat{W}_t, \varepsilon_{a,t}, \hat{C}_t, \hat{P}_t\right) \\ &\equiv \bar{C}\left[e^{\hat{P}_t + \hat{C}_t + (1-\theta)\left(\hat{p}_{i,t} - \hat{P}_t\right)} - \frac{\theta - 1}{\theta} \alpha e^{\hat{W}_t + \frac{1}{\alpha}\left(\hat{C}_t - \varepsilon_{a,t}\right) - \frac{\theta}{\alpha}\left(\hat{p}_{i,t} - \hat{P}_t\right)}\right], \end{aligned}$$

Firm i chooses the attention allocation so as to maximize the expected discounted sum of profits expressed in of log-deviations from the non-stochastic steady state,

$$\Pi_{i0} = E_0 \sum_{t=1}^{\infty} Q_{0,t} \pi_{i,t} = E_0 \left[\sum_{t=1}^{\infty} e^{\hat{Q}_{0,t}} \breve{\pi}\left(\hat{p}_{i,t}, \hat{W}_t, \varepsilon_{a,t}, \hat{C}_t, \hat{P}_t\right)\right]. \tag{39}$$

Similar to Maćkowiak and Wiederholt [18], the value of the quadratic objective at the profit-maximizing behavior $\left\{\hat{p}^*_{i,t+\tau}\right\}_{\tau=0}^{\infty}$ is subtracted from the quadratic approximation of (39). The second-order Taylor approximation around the non-stochastic steady state of Π_{i0} is computed. It follows that

$$\begin{aligned} \Pi_{i0} &\propto \tilde{\Pi}_{i0} = E_0 \left[\sum_{t=1}^{\infty} e^{\hat{Q}_{0,t}} \left(\breve{\pi}\left(\hat{p}_{i,t}, \hat{W}_t, \varepsilon_{a,t}, \hat{C}_t, \hat{P}_t\right) - \breve{\pi}\left(\hat{p}^*_{i,t}, \hat{W}_t, \varepsilon_{a,t}, \hat{C}_t, \hat{P}_t\right)\right)\right] \\ &\approx E_0 \sum_{t=1}^{\infty} \beta^t [\breve{\pi}_1 \hat{p}_{i,t} + \frac{1}{2}\breve{\pi}_{11}\hat{p}_{i,t}^2 + \frac{1}{2}\breve{\pi}_{12}\hat{p}_{i,t}\hat{W}_t + \frac{1}{2}\breve{\pi}_{13}\hat{p}_{i,t}\varepsilon_{a,t} + \frac{1}{2}\breve{\pi}_{14}\hat{p}_{i,t}\hat{C}_t + \frac{1}{2}\breve{\pi}_{15}\hat{p}_{i,t}\hat{P}_t \\ &\quad -\breve{\pi}_1 \hat{p}^*_{i,t} - \frac{1}{2}\breve{\pi}_{11}\hat{p}_{i,t}^{*2} - \frac{1}{2}\breve{\pi}_{12}\hat{p}^*_{i,t}\hat{W}_t - \frac{1}{2}\breve{\pi}_{13}\hat{p}^*_{i,t}\varepsilon_{a,t} - \frac{1}{2}\breve{\pi}_{14}\hat{p}^*_{i,t}\hat{C}_t - \frac{1}{2}\breve{\pi}_{15}\hat{p}^*_{i,t}\hat{P}_t]. \end{aligned}$$

Using the fact profit-maximizing conditions imply that $\breve{\pi}_1 = 0$, and that $\hat{p}^*_{i,t} = -\frac{1}{\breve{\pi}_{11}}\frac{1}{2}\left(\breve{\pi}_{12}\hat{W}_t + \breve{\pi}_{13}\varepsilon_{a,t} + \breve{\pi}_{14}\hat{C}_t + \breve{\pi}_{15}\hat{P}_t\right)$, it follows that

$$\Pi_{i0} \propto -\frac{\breve{\pi}_{11}}{2}\sum_{t=1}^{\infty}\beta^t E_0\left[\left(\hat{p}_{i,t} - \hat{p}^*_{i,t}\right)^2\right].$$

Given that in the non-stochastic steady state $\bar{p}^*_i = \bar{p}_i$, it follows that

$$\Pi_{i0} \propto -\lambda\sum_{t=1}^{\infty}\beta^t E_0\left[\left(\log\left(p_{i,t}\right) - \log\left(p^*_{i,t}\right)\right)^2\right],$$

where $\lambda \equiv \frac{1}{2}\breve{\pi}_{11} = \frac{1}{2}\bar{C}\left((1-\theta)^2 - \frac{\theta-1}{\theta}\alpha\left(\frac{\theta}{\alpha}\right)^2\right) > 0$.

A.2 Definition of information flow operator

Following the rational inattention literature, the operator I is defined such that

$$I\left(\{\varepsilon_{a,t}, \varepsilon_{r,t}\}; \{s_{ai,t}, s_{ri,t}\}\right) = \lim_{T\to\infty}\frac{1}{T}\left[H(\varepsilon^T_a, \varepsilon^T_r) - H(\varepsilon^T_a, \varepsilon^T_r \mid s^T_a, s^T_r)\right], \quad (40)$$

where $H(\cdot)$ denotes the entropy of a vector of realizations of random variables[26], and ε^T_a denotes the vector of realizations associated to the stochastic process $\{\varepsilon_{a,t}\}$ up to time T, $\varepsilon^T_a = (\varepsilon_{ai0}, \varepsilon_{ai1}, ..., \varepsilon_{aiT})$; ε^T_r, s^T_a and s^T_r are defined similarly. The larger the entropy associated with a random vector, the larger the uncertainty about its realizations. The entropy of the random vector $\varepsilon^T_a = (\varepsilon_{ai0}, \varepsilon_{ai1}, ..., \varepsilon_{aiT})$ with density $f(\varepsilon_{ai0}, \varepsilon_{ai1}, ..., \varepsilon_{aiT})$ is defined as:

$$H\left(\varepsilon^T_a\right) = -\int_{-\infty}^{+\infty} f\left(\varepsilon^T_a\right)\log_2\left(f\left(\varepsilon^T_a\right)\right)d\varepsilon^T_a.$$

In the case in which the vector ε^T_a has a multivariate Gaussian distribution with matrix of variance-covariance Ω_T, the entropy is given by

[26] For a definition of entropy see Cover and Thomas (1991).

$$H\left(\varepsilon_a^T\right) = \frac{1}{2}\log_2\left(\left(2\pi e\right)^T \det\left(\Omega_T\right)\right).$$

From the properties of entropies and given the assumptions $\{\varepsilon_{a,t}\} \perp \{\varepsilon_{r,t}\}$, $\{s_{a,t}\} \perp \{s_{r,t}\}$, $\{\varepsilon_{a,t}\} \perp \{s_{r,t}\}$ and $\{s_{a,t}\} \perp \{\varepsilon_{r,t}\}$, it follows that

$$I\left(\{\varepsilon_{a,t}, \varepsilon_{r,t}\}; \{s_{ai,t}, s_{ri,t}\}\right) = I\left(\{\varepsilon_{a,t}\}; \{s_{ai,t}\}\right) + I\left(\{\varepsilon_{r,t}\}; \{s_{ri,t}\}\right).$$

For a proof see Cover and Thomas [9].

B.1 First order conditions

Define λ_t as the Lagrangian multiplier on (2). The first order conditions to the household's problem are given by

$$\begin{aligned} C_t^{-1} &= \lambda_t P_t, & (41)\\ Q_{t,t+1} &= \beta\frac{\lambda_{t+1}}{\lambda_t}, & (42)\\ 1 &= \lambda_t W_t, & (43) \end{aligned}$$

where (42) holds in each state of the world in t+1, and (2) holds in each period. For given P_t and W_t, this set of equations determines the equilibrium dynamics of C_t, $Q_{t,t+1}$, L_t, λ_t. The equilibrium condition into the labor market, $L_t = \int_0^1 L_{i,t}$, determines W_t. R_t is given by (4). The equilibrium condition for the risk-free portfolio,

$$R_t = E_t\left[\frac{1}{Q_{t,t+1}}\right], \qquad (44)$$

determines the equilibrium dynamics of P_t. Eq. (6) determines $c_{i,t}$. Eq. (11) determines $p_{i,t}^*$.

The equations determining the equilibrium $p_{i,t}$, $s_{ai,t}$ and $s_{ri,t}$ depends on the solution to the problem in $(14) - (16)$, and are derived below.

B.2 Proof of existence of non-stochastic steady state

Given the absence of uncertainty and homogeneity of firms it follows from (7) that $\bar{p}_i = \bar{p}_i^* = \bar{P}$. Given stationarity, it follows from (42) and (44) that $\bar{R} = \frac{1}{\beta}$. By substituting $\bar{p}_i^* = \bar{P}$ into (11) it follows that $\bar{L} = \alpha\frac{\theta-1}{\theta}$. Finally, from (8) it follows that $\bar{C} = \bar{L}^{\alpha}$.

C Proof of Proposition 1

I use the method of undetermined coefficients to show that (19) – (26) is an equilibrium.

(Step 1): Derivation of profit-maximizing responses conditional on each shock

By substituting (20) into (11) , it follows that

$$\hat{p}_{i,t}^* = (1 - \phi\xi)\,\hat{P}_t - \frac{\xi}{1+\phi_c}\left(\varepsilon_{a,t} + \varepsilon_{r,t}\right). \tag{45}$$

where $\phi \equiv \frac{1-\phi_\pi}{1+\phi_c}$. In addition, by substituting (19) into (45) , $\hat{p}_{i,t}^*$ can be expressed as the sum of two independent components, each depending on one of the two types of shocks, $\hat{p}_{i,t}^* = \hat{p}_{ai,t}^* + \hat{p}_{ri,t}^*$, where $\hat{p}_{ai,t}^*$ and $\hat{p}_{ri,t}^*$ are defined as

$$\hat{p}_{ai,t}^* \equiv -\omega\left(\gamma_a\right)\frac{\xi}{1+\phi_c}\varepsilon_{a,t}, \tag{46}$$

$$\hat{p}_{ri,t}^* \equiv -\omega\left(\gamma_r\right)\frac{\xi}{1+\phi_c}\varepsilon_{r,t}, \tag{47}$$

and where $\omega(\cdot)$ is a linear function of γ_a and γ_r,

$$\omega(x) = (1 - \phi\xi)\,x + 1. \tag{48}$$

(Step 2): Solving the attention allocation problem

Given (46) – (48) , it is possible to solve the attention allocation problem in (14) – (16) as a function of γ_a and γ_r. By substituting (46) – (48) into (16), and using the independence assumption, $s_{ai,t} \perp s_{ri,t}$, it is possible to express $\hat{p}_{i,t}$ as $\hat{p}_{i,t} = \hat{p}_{ai,t} + \hat{p}_{ri,t}$, where $\hat{p}_{ai,t} = E\left[\hat{p}^*_{ai,t} \mid s^t_{ai}\right]$ and $\hat{p}_{ri,t} = E\left[\hat{p}^*_{ri,t} \mid s^t_{ri}\right]$. Notice that in the static equilibrium of the model in section 2 conditional expectations coincide with unconditional ones. Therefore (14) can be expressed as

$$\begin{aligned}
-\lambda \sum_{t=1}^{\infty} \beta^t E_0\left[\left(\log\left(p_{i,t}\right) - \log\left(p^*_{i,t}\right)\right)^2\right] &= -\frac{\beta}{1-\beta}\lambda E\left[\left(\log\left(p_{i,t}\right) - \log\left(p^*_{i,t}\right)\right)^2\right] = \\
&= -\frac{\beta}{1-\beta}\lambda E\left[\left(\hat{p}_{i,t} - \hat{p}^*_{i,t}\right)^2\right] = \\
&= -\frac{\beta}{1-\beta}\lambda E\left[\left(\hat{p}_{ai,t} - \hat{p}^*_{ai,t}\right)^2\right] - \frac{\beta}{1-\beta}\lambda E\left[\left(\hat{p}_{ri,t} - \hat{p}^*_{ri,t}\right)^2\right]
\end{aligned}$$

where I have used $\bar{p}_i = \bar{p}^*_i$. Maćkowiak and Wiederholt ([18], p.21) proof that this problem can be expressed in terms of Gaussian signals on the fundamental shocks:

$$\begin{aligned}
&\max_{(\upsilon_{ai} \geq 0,\ \upsilon_{ri} \geq 0)} -\lambda \frac{\beta}{1-\beta} E\left[\left(\hat{p}_{ai,t} - \hat{p}^*_{ai,t}\right)^2 + \left(\hat{p}_{ri,t} - \hat{p}^*_{ri,t}\right)^2\right], \\
&s.t.
\end{aligned}$$

$$\begin{aligned}
i) &: \quad s_{ai,t} = \varepsilon_{a,t} + u_{ai,t}, \quad u_{ai,t} \sim N\left(0, \upsilon^2_{ai}\right), \\
ii) &: \quad s_{ri,t} = \varepsilon_{r,t} + u_{ri,t}, \quad u_{ri,t} \sim N\left(0, \upsilon^2_{ri}\right), \\
iii) &: \quad \hat{p}_{ai,t} = E\left[\hat{p}^*_{ai,t} \mid s_{ai,t}\right], \\
iv) &: \quad \hat{p}_{ri,t} = E\left[\hat{p}^*_{ri,t} \mid s_{ri,t}\right], \\
v) &: \quad I\left(\left\{\varepsilon_{a,t}, \varepsilon_{r,t}\right\}; \left\{s_{ai,t}, s_{ri,t}\right\}\right) \leq \kappa.
\end{aligned}$$

where $u_{ai,t}$ and $u_{ri,t}$ are idiosyncratic noise, iid across firms and time. From appendix A, and given the joint Gaussian distribution of $\varepsilon_{a,t}$ and $s_{ai,t}$, and of $\varepsilon_{r,t}$ and $s_{ri,t}$, it follows that

$$\begin{aligned}
I\left(\left\{\varepsilon_{a,t}, \varepsilon_{r,t}\right\}; \left\{s_{ai,t}, s_{ri,t}\right\}\right) &= I\left(\left\{\varepsilon_{a,t}\right\}; \left\{s_{ai,t}\right\}\right) + I\left(\left\{\varepsilon_{r,t}\right\}; \left\{s_{ri,t}\right\}\right) \\
&= \frac{1}{2}\log_2\left(1 + \frac{\sigma^2_a}{\upsilon^2_{ai}}\right) + \frac{1}{2}\log_2\left(1 + \frac{\sigma^2_r}{\upsilon^2_{ri}}\right).
\end{aligned}$$

From i)-iv) and (46) − (48) it follows that

$$\begin{aligned}\hat{p}_{ai,t} &= -\frac{\sigma_a^2}{\sigma_a^2+\upsilon_{ai}^2}\omega\left(\gamma_a\right)s_{ai,t},\\ \hat{p}_{ri,t} &= -\frac{\sigma_r^2}{\sigma_r^2+\upsilon_{ri}^2}\omega\left(\gamma_r\right)s_{ri,t}.\end{aligned}$$

By substituting the results above into the objective function, the attention allocation problem reads

$$\max_{(\upsilon_{ai}\geq 0,\ \upsilon_{ri}\geq 0)} -\lambda\frac{\beta}{1-\beta}\left(\frac{1}{1+\frac{\sigma_a^2}{\upsilon_{ai}^2}}\left(\omega\left(\gamma_a\right)\sigma_a\right)^2+\frac{1}{1+\frac{\sigma_r^2}{\upsilon_{ri}^2}}\left(\omega\left(\gamma_r\right)\sigma_r\right)^2\right), \tag{49}$$

subject to the information flow constraint

$$\frac{1}{2}\log_2\left(1+\frac{\sigma_a^2}{\upsilon_{ai}^2}\right)+\frac{1}{2}\log_2\left(1+\frac{\sigma_r^2}{\upsilon_{ri}^2}\right)\leq\kappa. \tag{50}$$

The the first-order conditions to (49) − (50) imply

$$\left(\frac{\sigma_a^2}{\sigma_a^2+\upsilon_{ai}^{2*}};\frac{\sigma_r^2}{\sigma_r^2+\upsilon_{ri}^{2*}}\right)=\begin{cases}\left(1-2^{-2\kappa};\ 0\right) & if\ \omega\sigma>2^{\kappa}\\ \left(1-2^{-\kappa}\frac{1}{\omega\sigma};\ 1-2^{-\kappa}\omega\sigma\right) & if\ 2^{-\kappa}<\omega\sigma<2^{\kappa}\\ \left(0;\ 1-2^{-2\kappa}\right) & if\ \omega\sigma<2^{-\kappa}\end{cases} \tag{51}$$

where ω and σ are defined as

$$\begin{aligned}\omega &\equiv \frac{\omega\left(\gamma_a\right)}{\omega\left(\gamma_r\right)},\\ \sigma &\equiv \frac{\sigma_a}{\sigma_r}.\end{aligned}$$

(Step 3): Solving for undetermined coefficients γ_a and γ_r

It follows from optimal price setting behavior by firms that $\hat{p}_{ai,t}$ and $\hat{p}_{ri,t}$ are given by

$$
\begin{aligned}
\hat{p}_{ai,t} &= -\frac{\sigma_a^2}{\sigma_a^2+\upsilon_{ai}^{2*}}\omega\left(\gamma_a\right)\frac{\xi}{1+\phi_c}\left(\varepsilon_{a,t}+u_{ai,t}\right), \\
\hat{p}_{ri,t} &= -\frac{\sigma_r^2}{\sigma_r^2+\upsilon_{ri}^{2*}}\omega\left(\gamma_r\right)\frac{\xi}{1+\phi_c}\left(\varepsilon_{r,t}+u_{ri,t}\right),
\end{aligned}
$$

From the absence of ex-ante heterogeneity across firms, it follows that all firms make the same attention allocation decision: $\upsilon_{ai}^{2*}=\upsilon_a^{2*}$and $\upsilon_{ri}^{2*}=\upsilon_r^{2*}$ for all i. Using (19) it follows that

$$
\begin{aligned}
-\gamma_a\varepsilon_{a,t} &= \int_0^1 -\frac{\sigma_a^2}{\sigma_a^2+\upsilon_{ai}^{2*}}\omega\left(\gamma_a\right)\left[\varepsilon_{a,t}+u_{ai,t}\right]di = -\frac{\sigma_a^2}{\sigma_a^2+\upsilon_{ai}^{2*}}\omega\left(\gamma_a\right)\varepsilon_{a,t}, \\
-\gamma_r\varepsilon_{r,t} &= \int_0^1 -\frac{\sigma_r^2}{\sigma_r^2+\upsilon_{ri}^{2*}}\omega\left(\gamma_r\right)\left[\varepsilon_{r,t}+u_{ri,t}\right]di = -\frac{\sigma_r^2}{\sigma_r^2+\upsilon_{ri}^{2*}}\omega\left(\gamma_r\right)\varepsilon_{r,t},
\end{aligned}
$$

where the second equality follows from the assumption that errors in information processing are independent across firms, $\int_0^1 u_{ai,t}di=0$, $\int_0^1 u_{ri,t}di=0$. By substituting (51) in the equations above it follows that:

i) in the case of an interior solution to the attention allocation problem,

$$
\begin{aligned}
\gamma_a &= (\omega\left(\gamma_a\right)-2^{-\kappa}\omega\left(\gamma_r\right)\frac{1}{\sigma}), \\
\gamma_r &= (\omega\left(\gamma_r\right)-2^{-\kappa}\omega\left(\gamma_a\right)\sigma),
\end{aligned}
$$

by substituting (48) in the two equations above I can solve for the fixed point, obtaining

$$
\begin{aligned}
\gamma_a &= \Gamma\left(\sigma\right), \\
\gamma_r &= \Gamma\left(\frac{1}{\sigma}\right),
\end{aligned}
$$

where the function $\Gamma(\cdot)$ is given by

$$\Gamma(x) = \frac{2^{-2\kappa} + \xi\phi\left(1 - 2^{-2\kappa}\right) - 2^{-\kappa}\frac{1}{x}}{(\xi\phi)^2 - 2^{-2\kappa}\left(1 - \xi\phi\right)^2}.$$

ii) at the corner solution where attention is paid only to technology shocks it follows that

$$\begin{aligned}\gamma_a &= \frac{1 - 2^{-2\kappa}}{1 - \left(1 - \xi\phi\right)\left(1 - 2^{-2\kappa}\right)}, \\ \gamma_r &= 0.\end{aligned}$$

iii) similarly, at the corner solution where attention is paid only to monetary policy shocks it follows that

$$\begin{aligned}\gamma_a &= 0, \\ \gamma_r &= \frac{1 - 2^{-2\kappa}}{1 - \left(1 - \xi\phi\right)\left(1 - 2^{-2\kappa}\right)}.\end{aligned}$$

(Step 4): Derivation of $\bar{\sigma}$

I derive the interval for the values of σ for which there is an interior solution to the attention allocation problem in (49) – (50). An interior solution to the attention allocation problem, (i.e. signal to noise ratio positive and smaller than 1) requires:

$$\begin{aligned} i) \quad &: \quad 2^{-\kappa} \leq \omega\sigma \leq 2^{\kappa} \\ &\Longrightarrow \quad 2^{-\kappa}\frac{\omega\left(\gamma_r\right)}{\omega\left(\gamma_a\right)} \leq \sigma \\ &\Longrightarrow \quad 2^{\kappa}\frac{\omega\left(\gamma_r\right)}{\omega\left(\gamma_a\right)} \geq \sigma \end{aligned}$$

$$\Longrightarrow \frac{1}{2^{-\kappa} + \phi\xi\left(2^{\kappa} - 2^{-\kappa}\right)} \leq \sigma \leq 2^{-\kappa} + \phi\xi\left(2^{\kappa} - 2^{-\kappa}\right)$$

$$ii) \quad : \quad \omega \geq 0,$$

$$\implies 2^{-\kappa}\frac{\xi\phi}{1-\xi\phi} \leq \sigma \leq 2^{\kappa}\frac{\xi\phi}{1-\xi\phi}$$

Finally, let's define

$$\bar{\sigma} = \min\left\{2^{\kappa}\frac{\xi\phi}{1-\xi\phi}; 2^{\kappa}\phi\xi + 2^{-\kappa}(1-\phi\xi)\right\}$$

.

(Step 5): Solving for aggregate demand

It is left to show that, given $(21)-(26)$, (20) is also an equilibrium. From the Intertemporal Euler condition to the household's problem it follows that

$$-\mu\hat{C}_t = -\mu E_t\hat{C}_{t+1} + \hat{R}_t - E_t\hat{P}_{t+1} + \hat{P}_t. \tag{52}$$

From (4) it follows that

$$\hat{R}_t = \phi_\pi\left(E_t\hat{P}_{t+1} - \hat{P}_t\right) + \phi_c\left(E_t\hat{C}_{t+1} - E_t\hat{C}^*_{t+1}\right) + \phi_c\left(\hat{C}_t - \hat{C}^*_t\right) + \varepsilon_{r,t}. \tag{53}$$

From definition of $\hat{C}^*_t$ it follows that $\hat{C}^*_t = \varepsilon_{a,t}$. From $(19)-(20)$, and definition of a static equilibrium, it follows that $E_t\hat{C}_{t+1} = 0$ and $E_t\hat{P}_{t+1} = 0$. By substituting (53) and the results above into (52), and solving for $\hat{C}_t$, equation (20) is obtained.

D Proof of Proposition 4

At an interior solution, and for finite κ, the function $\Gamma(\cdot)$ is strictly decreasing:

$$\Gamma'(\cdot) = \frac{-2^{-\kappa}}{(\xi\phi)^2 - 2^{-2\kappa}(1-\xi\phi)^2} < 0.$$

Therefore, $\Gamma(\cdot)$ is decreasing in σ. From the definition of γ in (**??**), it immediately follows that γ is increasing in σ :

$$\frac{\partial\gamma}{\partial\sigma} = \frac{1}{\Gamma(\sigma)}\left(\Gamma'\left(\frac{1}{\sigma}\right) - \Gamma'(\sigma)\right) > 0.$$

The derivative of γ with respect to the degree of strategic complementarity in attention allocation is given by

$$\frac{\partial\gamma}{\partial(\xi\phi)} = \frac{1}{\Gamma(\sigma)}\left(\frac{\partial\Gamma\left(\frac{1}{\sigma}\right)}{\partial(\xi\phi)} - \frac{\partial\Gamma(\sigma)}{\partial(\xi\phi)}\right),$$

where

$$\frac{\partial\Gamma\left(\frac{1}{\sigma}\right)}{\partial(\xi\phi)} - \frac{\partial\Gamma(\sigma)}{\partial(\xi\phi)} = \frac{2^{-\kappa+1}\left(\xi\phi + 2^{-2\kappa}(1-\xi\phi)\right)}{\left((\xi\phi)^2 - 2^{-2\kappa}(1-\xi\phi)^2\right)^2}\left(\frac{1}{\sigma} - \sigma\right).$$

Therefore, it follows from above that

$$\begin{cases} \frac{\partial\gamma}{\partial(\xi\phi)} \leq 0 & \text{if } \sigma \geq 1 \\ \frac{\partial\gamma}{\partial(\xi\phi)} > 0 & \text{if } \sigma < 1 \end{cases}$$

E Solving the dynamic extension

The model is solved in two steps. In the first step, we approximate the dyamics of inflation in response to technology and monetary policy shocks in deviations from the non-stochastic steady-state as a function of two ARMA(2,2) processes,

$$\begin{aligned} \hat{\pi}_t &= \hat{\pi}_{a,t} + \hat{\pi}_{r,t}, \\ \hat{\pi}_{a,t} &= \phi_{a,1}\hat{\pi}_{a,t-1} + \phi_{a,2}\hat{\pi}_{a,t-2} + \vartheta_{a,0}\eta_{a,t} + \vartheta_{a,1}\eta_{a,t-1}, \\ \hat{\pi}_{r,t} &= \phi_{r,1}\hat{\pi}_{r,t-1} + \phi_{r,2}\hat{\pi}_{r,t-2} + \vartheta_{r,0}\eta_{r,t} + \vartheta_{r,1}\eta_{r,t-1}. \end{aligned}$$

We give a guess for the parameters of the two ARMA processes and solve the general equilibrium model with standard methods of undetermined coefficients, where we replace the equation defining inflation dynamics from firms'price setting behavior with the guess above.

In the second step, we solve for the attention allocation problem given dynamics of $\hat{\mathrm{p}}_t^*$ implied by step 1. The solution to the attention allocation problem gives the dynamics of $\hat{\pi}_{a,t}$ and $\hat{\pi}_{r,t}$. We approximate these dynamics with ARMA(2,2) processes as above, update the guess and start from step 1 until convergence. Notice that results are robust to ARMA(p,q) processes for q and p $>$ 2.