Irrational Bias in Inflation Forecasts

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Abstract

This paper investigates the issue of rational expectations using inflation forecasts from the Survey of Professional Forecasters (SPF) and the Green Book. We provide an alternative test of rational expectations hypothesis by measuring the degree of persistence of potential systematic mistakes. The test is obtained by solving a signal extraction problem that distinguishes between systematic and non-systematic forecast errors. The findings indicate highly persistent systematic mistakes, which are driven by the inefficient use of available information, and reject the rational expectations hypothesis. The estimated time-varying bias can be used to improve the SPF and Green Book inflation forecast performance by at least 13.4%. This paper also documents evidence that the real interest rate plays a crucial role in explaining the level of bias that leads to under- and over predictions of actual inflation.

JEL Classification: D84,E31,E37

Key words: Inflation Expectations, Bias, Forecasts, Rational Expectations.
1 Introduction

“Undoubtedly, the state of inflation expectations greatly influences actual inflation and thus the central bank’s ability to achieve price stability.”—Bernanke 2007–

Inflation forecasts play an important role in determining the interest rate, thereby affecting other key macroeconomic variables. Although the accuracy of these forecasts is of great importance, forecasters have made mistakes in predicting future inflation rate. In particular, there was a tendency to under-predict actual inflation before 1980 and after 2000, but over-predict it between 1980 and 2000. Although these mistakes have been repeated for the past four decades, the literature does not provide a satisfactory explanation for systematic mistakes (biases) in the observed inflation expectations (Capistrán and Timmermann 2009). In response to this challenge, this paper provides a method to verify the potential existence and source of systematic mistakes in inflation forecasts. In addition, it introduces a method to reduce these errors. We also propose a new test for rational expectations, which is closely related to the existence of systematic mistakes.

To dates, there is still an ongoing debate about rational expectations. Romer and Romer (2000) and Ball and Croushore (2003) provide evidence in favor of the rational inflation expectations hypothesis. On the other hand, several other studies such as Croushore (1996), Roberts (1997), Tomas (1999) and Mehra (2002), among others, present evidence against rational expectations such as bias, inefficient use of available information, and serial correlation in observed inflation forecasts of the SPF and the Livingston Survey. In response to these empirical results, several studies attempt to rationalize evidence that can be viewed as a rejection of the rational expectations hypothesis. Mankiw, Reis and Wolfers (2003) try to explain forecast errors and bias based on the assumption that economic agents update information infrequently. Capistrán (2008) uses an asymmetric loss function in forecasting to rationalize bias in the Green Book inflation forecasts. Andolfatto, Hendry and Moran (2008) show that bias and serial correlation in forecast errors can be generated by perfectly rational agents who solve a signal extraction problem.

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1We investigate inflation forecasts from the Survey of Professional Forecasters (SPF) and the Green Book. The Survey of Professional Forecasters initiated by the American Statistical Association and the National Bureau of Economic Research are currently conducted by the Federal Reserve Bank of Philadelphia. The Green Book is prepared for each meeting of the Federal Open Market Committee (FOMC).
to distinguish between persistent and transitory shifts in inflation targets. Other studies explain bias using strategic behavior of forecasters (e.g., Ehrbeck and Waldmann 1996, Laster, Bennett and Geoum 1999).

In order to examine the issue of rational expectations, we propose a new test of rational expectations that differs from the one commonly used in the literature, which is quite sensitive to sample size because the bias is assumed to be constant (e.g., Tomas 1999, Romer and Romer 2000). An alternative test of the rational expectations hypothesis assumes that the potential bias is time-varying. The test measures the degree of persistence in systematic mistakes obtained by solving a signal extraction problem that distinguishes between systematic and non-systematic forecast errors. Our findings indicate that systematic mistakes are highly persistent and predictable. Moreover, systematic mistakes are mostly driven by the inefficient use of available information, thus rejecting the rational expectations hypothesis.

In our methodological approach we obtain a time-varying systematic error (bias). The estimated time-varying bias allows us to correct systematic mistakes, which improves forecasting accuracy. Our results show that highly predictable bias provides room to improve forecasting performance by 21.3% for the SPF and by 16.0% for the Green Book. In addition, the time-varying bias facilitates the identification of predictable errors because unlike constant bias we can observe the dynamic relationship between the bias and the variables in the information set. We document evidence that the main source of bias from the SPF and the Green Book is attributed to the ex-post real interest rate available for forecasters.\(^2\) Our findings indicate that the real interest rate plays a crucial role in determining the level of bias that leads to under- and over predictions of actual inflation. We consider both the output gap and the labor share of income as potential driving sources of inflation, which also contribute to explaining the predictable systematic errors. Finally, we discuss the irreconcilability of our empirical findings with the rational expectations hypothesis as well as with the efforts to rationalize bias and serial correlation of forecast errors by Mankiw, Reis and Wolfers (2003), Andolfatto, Hendry and Moran (2008), Capistrán (2008) and Laster, Bennett and Geoum (1999).

The contents of the paper are as follows. In section 2, we discuss a prevalent way to test rationality and propose a more flexible alternative. We find that highly predictable

\(^2\)We employ lagged values of the real interest rate that is available to forecasters when inflation expectations are formed.
mistrakes cast doubt on the validity of the rational expectations hypothesis. In section 3, we investigate the source of bias and find that it is mostly explained by key macroeconomic variables. Section 4 concludes.

2 Predictable Mistakes and Irrational Bias

In order for inflation expectations to be rational, forecasts should be unbiased, that is, forecast errors should be distributed around zero. In addition, forecasters must use all available information in an efficient way, so that there is no predictable residual. A test for bias has often been carried out using the following equation:

\[ \pi_{t+1} = \alpha + \beta \hat{E}_t \pi_{t+1} + \varepsilon_{t+1} \]  (1)

where \( \pi_{t+1} \) and \( \hat{E}_t \pi_{t+1} \) denote the actual inflation rate and the observed inflation expectations, respectively. Lovell (1986), Tomas (1999), Romer and Romer (2000) and others examine whether the condition for unbiasedness, the joint test of \( \alpha = 0 \) and \( \beta = 1 \), is satisfied.

When the Wald test for \( \beta = 1 \) does not reject the null hypothesis, we can adopt an equivalent test for unbiasedness that can be carried out using the form:

\[ \pi_{t+1} - \hat{E}_t \pi_{t+1} = \alpha + \varepsilon_{t+1}. \]  (2)

The parameter \( \alpha \) is the mean value of forecast errors or bias. A significant estimate of \( \alpha \) is often reported in the literature as evidence against the rational expectations hypothesis (e.g., Mankiw, Reis and Wolfers 2003). Although the tests based on equation (1) or (2) have been often employed in the literature, the unbiasedness tests are quite problematic on the grounds that the joint test of \( \alpha = 0 \) and \( \beta = 1 \) in equation (1) or the test for \( \alpha = 0 \) in equation (2) is very sensitive to the sample period. While the use of longer sample often leads to the conclusion of unbiasedness (e.g., Romer and Romer 2000), using subsamples before and after 1980 tends to give rise to bias (e.g., Tomas 1999).

In order to address this issue, we employ an alternative method allowing the bias \( (\alpha) \) to vary over time. We set up a state-space model to decompose the forecast errors,

\(^3\)When the observed inflation expectations from the SPF and the Green Book are employed, we find that the Wald test for \( \beta = 1 \) does not reject the null hypothesis.
\( \pi_{t+1} - \hat{\pi}_{t+1} \), into a systematic term \((\alpha_{t+1})\) and a non-systematic term \((\varepsilon_{t+1})\) as follows:

\[
\pi_{t+1} - \hat{\pi}_{t+1} = \alpha_{t+1} + \varepsilon_{t+1} \tag{3}
\]

\[
\alpha_{t+1} = \phi \alpha_t + \nu_{t+1}. \tag{4}
\]

We assume that \( \nu_t \sim N(0, \sigma_\nu) \) and \( \varepsilon_t \sim N(0, \sigma_\varepsilon) \) where \( \varepsilon_{t+1} \) is independent of \( \nu_{t+1} \). The time-varying bias \( \alpha_t \) follows an AR(1) process, which allows the model to capture systematic mistakes. The coefficient \( \phi \) measures the degree of persistence of potential systematic mistakes. Note that if \( \phi = 0 \), the bias turns out to be zero, which implies that forecasters do not make predictable mistakes. On the other hand, when the estimate of \( \phi \) is statistically different from zero, the null hypothesis of rationality is rejected due to the presence of predictable mistakes.

We estimate the unobserved bias by solving a signal extraction problem to disentangle between systematic and non-systematic mistakes. This simple approach has several advantages over assuming constant bias. First, the proposed test for rationality is not sensitive to the sample period. Second, a time-varying bias enables us to investigate the source of predictable mistakes because we can observe the dynamic relationship between the bias and variables in the information set. Finally, our method allows us to correct the bias, which serves to improve the accuracy of inflation forecasts.

Table 1 displays estimates of equation (3) and (4). The sample period ranges from 1968:4 to 2005:4 for the SPF and from 1968:4 to 2001:4 for the Green Book forecasts, which are available with a 5-year lag. The median inflation expectations of the SPF are used in this paper. The coefficient \( \phi \) is estimated to be 0.927 for the SPF and 0.916 for the Green Book, suggesting that biases are highly persistent. The significant estimates of \( \phi \) are inconsistent with the rational expectation hypothesis since biases from the SPF and the Green Book are highly predictable.

In order to evaluate the performance of inflation forecasts with and without bias-corrections, we use the root mean square error (RMSE) as a measure of accuracy for the observed inflation expectations and the bias-corrected inflation expectations. We define the latter as the sum of the observed inflation expectations and expected future bias,

\[\text{RMSE}_{\text{bias-corrected}} = \text{RMSE}_{\text{observed}} + \text{RMSE}_{\text{bias}}.\]

Table 1 shows that the RMSE of bias-corrected forecasts is lower than that of the observed forecasts, indicating that the bias correction improves the accuracy of inflation forecasts.
that is, $E_t \pi_{t+1} = \hat{E}_t \pi_{t+1} + E_t \alpha_{t+1}$. The predicted bias ($E_t \alpha_{t+1} = \phi \alpha_t$) is computed with available information at time $t$. Since current inflation may not be observed in real time, we also correct systematic mistakes after forecasting future bias with the information set available at $t - 1$. Expected future bias is predicted as $E_{t-1} \alpha_{t+1} = \phi^2 \alpha_{t-1}$ in real time.

Table 2 exhibits the RMSE for the observed inflation expectations and the bias-corrected inflation expectations that are computed with available information at $t$ and $t - 1$. We find that the correction of bias improves the accuracy of inflation forecasts.

[Insert Table 2 Here]

While the RMSE for the SPF is 1.359, the one for the Green Book is calculated to be 1.187. Correcting the biases, which are predicted with available information at time $t$, reduces the RMSEs for both forecasts, showing that bias plays a crucial role in improving the accuracy of forecasts. For the SPF, the RMSE decreases from 1.359 to 1.069 improving the accuracy by 21.3%. The Green Book forecast, which is known as one of the most accurate predictions, can also be improved by 16.0% with bias correction. When systematic mistakes are corrected using the predicted biases with available information at time $t - 1$, the RMSEs decrease from 1.359 to 1.141, 16.0%, for the SPF and from 1.187 to 1.028, 13.4%, for the Green Book. These results show that the RMSEs could be reduced by correcting predictable systematic mistakes.

Figure 1 displays the estimated biases and forecast errors from the SPF and the Green Book together. The biases and forecast errors from both show similar dynamics, implying there may be a common factor between the SPF and the Green Book.

[Insert Figure 1 Here]

In Figure 1, inflation forecasts from the SPF and the Green Book display a series of positive or negative biases over the whole sample period. While both forecasts tend to under-predict inflation before 1980, they tend to over-predict inflation in the 1980s and 1990s. Inflation is under-predicted again by these forecasts around the early 2000s. Several studies document that the bias could be explained by asymmetric loss functions used in forecasting, that is, that biased forecasts can be rationalized. In particular, Capistrán (2008) argues that the Fed may have different asymmetric loss functions before and after 1980. According to Capistrán (2008), the cost of having higher inflation than the targeted
was smaller than the cost of having lower inflation than targeted before 1980 and the opposite held after 1980. This argument may explain why the bias from the Green Book is positive in the pre-Volcker era and negative in the Volcker-Greenspan period. However, a question still remains because the bias from the SPF is similar to the one from the Green Book. This rationale cannot be applied to the bias from the SPF forecasts since there is no reason for commercial forecasters to have different asymmetric loss functions before and after 1980. Since both forecast biases from the SPF and the Green Book show quite similar dynamics, we may need a unified framework to explain those biases.

Other studies such as Ehrbeck and Waldmann (1996) and Laster, Bennette and Geoum (1999) examine whether biased forecasts arise from strategic behaviors between clients and forecasters. In particular, Laster, Bennette and Geoum (1999) develop a model in which forecasters’ wages depend on their accuracy and their ability to attract publicity by competing with all other forecasts. The incentive to make forecasts public compromises the accuracy of forecasts because the publicity can be obtained from differentiating their forecasts from the consensus (group mean forecasts as the most accurate), causing rational bias. The publicity “enhances forecasters’ reputation, credibility and name recognition among only occasional users, who are otherwise unfamiliar with most forecasters” (Laster, Bennette and Geoum 1999). In this respect, occasional clients play a crucial role in generating rational bias. However, these modeling approaches fail to explain the bias from the Green Book forecasts since the Fed cannot be classified as an occasional client who does not dedicate many of its resources to monitor macroeconomic forecasts. In addition, Fed’s forecasters do not have the incentive to make forecasts public because their forecasts are only available with a lag of 5 years. Thus, the models designed to explain bias from commercial forecasts fail to account for the Green Book forecasts’ bias.

Andolfatto, Hendry and Moran (2008) offer a different explanation of bias in line with Cukierman (1986). They set up a dynamic stochastic general equilibrium model in which agents cannot distinguish between permanent and transitory shifts in inflation targets. In their model, agents solve a signal extraction problem to disentangle the shifts, leading to correlated forecast errors and bias. Although rationalizing bias may explain one dimension of bias, our empirical results still cast doubt on their evidence for several reasons. First, our findings indicate that the bias is mostly explained by publicly available information.
at the time that forecasts are made. Second, their model may fail to account for why the Fed’s internal forecasts are biased in that the Fed’s forecasters could recognize permanent shifts in inflation targets through internal communication when they make predictions. Finally, their explanation is at odds with our empirical results in that bias can be still obtained from solving the signal extraction problem to disentangle the persistent and transitory forecast errors.

3 The Sources of Bias

This section examines the underlying source of bias in inflation forecasts. We test if the bias is predictable using available information at the time the forecasts are made. Let:

\[ \alpha_{t+1} = \mu + \psi I_{t-k} + \omega_{t+1} \]  

(5)

where \( I_{t-k} \) for \( k \in [1, 2, 3, 4] \) is the information set which includes variables such as labor’s share of income, the output gap (or the unemployment rate) and the ex-post real interest rate. Since these variables may be not available in real time, we consider past information sets to test for the robustness of our results. We use equation (5) to identify sources of the bias, which can be viewed as a test of efficient use of available information. Generally in the literature, the test of whether forecasters use information efficiently is conducted by running a regression of forecast errors on available information. We replace the forecast errors with the time-varying bias to examine what drives it as well as to determine whether forecasters use information efficiently.

The effective federal fund rate is employed to calculate the ex-post real interest rate. We employ the output gap measure provided by the Congressional Budget Office. Labor’s share of income is taken from non-farm business sector.

We consider the output gap and labor income share as potential driving forces of inflation. There is no consensus in the literature regarding the driving source of inflation. Gali and Gertler (1999) and Sbordone (2005) show that the new Keynesian Phillips curve fits the data well with labor’s share of income rather than the output gap. Gali and Gertler (1999) argue that labor income share measures the true economic activity better than the

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5 Details are explained in the following section.
6 Labor’s share of income is defined as (hours worked)*(nominal wage)/(price*output), which is available from the Bureau of Labor Statistics.
output gap on the grounds that the latter could suffer from considerable measurement error of potential output. On the other hand, Rudd and Whelan (2005) favor the output gap as a traditional driving source of inflation on the grounds that labor’s share of income shows counter-cyclical patterns. Given the pros and cons of each series, we include both series in the information set as the driving sources of inflation. The test of the efficient use of available information is described by the following equation:

$$\alpha_{t+1} = \mu + \delta_1 s_{t-k} + \delta_2 y_{t-k} + \theta_1 r_{t-k} + \theta_2 r_{t-k-1} + \theta_3 r_{t-k-2} + \theta_4 r_{t-k-3} + \theta_5 r_{t-k-4} + \omega_{t+1}. \quad (6)$$

where $k \in [1, 2, 3, 4]$, and $s_t, y_t$ and $r_t$ denote labor income share, the output gap, and the real interest rate, respectively. Table 3 reports estimates for $k \in [1, 2, 3, 4]$\textsuperscript{7}. We consider four different cases due to the possibility of data revision. We expect that minor revisions will occur in the explanatory variables with longer lags.

[Insert Table 3 Here]

We find that biases from the SPF and the Green Book are mostly explained by the output gap, labor’s share of income, and the real interest rate. The measure of goodness-of-fit, $R^2$, obtained is at least 0.60 across several specifications. The coefficients on the output gap and labor’s share of income are highly significant at the 5% significant level for all different cases.\textsuperscript{8} The ex-post real interest rate also plays a crucial role in accounting for bias. While the potential driving forces of inflation in the literature have a positive relationship with bias, the ex-post real interest rate is negatively related. These results imply that forecasters have under-reacted to changes in variables in the information set. For example, the positive coefficient on the output gap implies that forecasters tend to under-react to an increase (a decrease) in the output gap because realized inflation is higher (lower) than the predicted one. The coefficient on labor’s share of income could be interpreted in the same way. Similarly, the negative coefficient on the ex-post real interest rate suggests that the effect of a decrease (an increase) in the ex-post real interest rate on inflation is under-estimated because realized inflation tends to be higher (lower) than

\textsuperscript{7}In the case of $k=1$, the ex-post real interest rate at time $t-1$ may not be observed. However, our results are similar for $k \in [1, 2, 3, 4]$

\textsuperscript{8}Given the criticism regarding output gap for potential measurement error and data revision, we replace it with the unemployment rate, which is subject to minor measurement error and revision. We obtain very similar results for the unemployment rate with an even higher $R^2$. The results are available from the authors’ upon request.
predicted, causing higher (lower) forecast errors.\textsuperscript{9} We view these results as a rejection of the rational expectations hypothesis.

Mankiw, Reis and Wolfers (2003) attempt to rationalize bias and the inefficient use of available information in the observed inflation forecasts such as the SPF and Michigan Survey by assuming that agents may update information infrequently. As we discuss in the previous section, biases from the SPF and the Green Book show a similar pattern suggesting that it would be more appropriate to use a unified framework that is able to explain both biases. In this sense, their effort to rationalize bias and the inefficient use of available information on the basis of informational rigidities has to work for both forecasts. However, it is hard to reconcile that the Fed, which closely monitors the state of the economy, would collect information infrequently. We also cast doubts on the claim that informational rigidities faced by professional forecasters could lead to bias and the inefficient use of available information in the observed inflation expectations from the SPF. The data set we employ are publicly available without any costs to professional forecasters.

Finally, Table 3 suggests that the Fed’s forecasters may be more efficient than professional forecasters since only the first two lags of the real interest rate are significant in explaining bias from the Green Book. This finding is consistent with the results from Table 2, which shows that inflation forecasts from the SPF can be further improved than forecasts from the Green Book.

[Insert Figure 2 Here] [Insert Figure 3 Here]

Figures 2 and 3 plot biases and the fitted values for $k = 1$ to illustrate how well the fitted values provide a good prediction to biases from the SPF and the Green Book, respectively. The variables we employ account for biases reasonably well, fitting a series of negative and positive biases. The fitted values even predict two important turning points around 1980 and 2000 very closely. In both cases, the first turning point from a positive to a negative sign of the bias is consistent with the period that Volcker took office as a chairman of the Fed. In this respect, we further investigate the effect of the real interest rate on the bias.

[Insert Figure 4 Here]

\textsuperscript{9}See Ball and Croushore (2003), Mankiw, Reis and Wolfers (2003) for these interpretations.
Figure 4 displays negative the real interest rate, $-r_t$, and biases from the SPF and the Green Book to highlight their negative relationship. Biases move with negative the real interest rate ($-r_t$), implying that its level is closely connected with shifts in the real interest rate. In the pre-Volcker period, when the real interest rate was relatively low, forecasters underestimated its effect on the inflation rate, leading to under-prediction in inflation forecasts. In contrast to the pre-Volcker period, the real interest rate was relatively higher during the Volcker-Greenspan era, especially from 1980 to 2000. Again, the forecasts underestimated the negative effect of the high real interest rate on inflation, leading to over-prediction of future inflation expectations. Finally, since 2000, forecasters started to underestimate the effect of low real interest rate on inflation, further increasing the level of biases. Overall, findings indicate that the real interest rate plays a crucial role in explaining under- or over-prediction of inflation expectations, thus determining the level of biases. Since the output gap and labor income share display a cyclical pattern, it is less plausible that they account for the level of biases. Although we do not report here, residuals from a regression of biases on only the output gap, labor income share, and a constant, display very similar movements with the level of biases. This result shows that the potential driving forces of inflation cannot account for the level of biases. Overall, our findings indicate that biases in the SPF and the Green Book forecasts are mostly driven by the inefficient use of available information and cast doubt on the validity of the rational expectations hypothesis.

4 Conclusion

This paper contributes to the controversial literature that investigates the rational expectations hypothesis. We propose an alternative test of the rational expectations hypothesis in which the potential bias is time-varying. The test measures the degree of persistence in systematic mistakes obtained by solving a signal extraction problem that distinguishes between systematic and non-systematic forecast errors. Our findings indicate that systematic mistakes are highly persistent and predictable. Moreover, systematic mistakes are mostly driven by the inefficient use of available information; thus rejecting the rational expectations hypothesis. This paper also documents evidence that the ex-post real interest rate plays a crucial role in explaining the level of bias, illustrating the difficulty – even for the Fed’s forecasters – to predict the effect of real interest rate on inflation.
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Table 1: Estimation Results

<table>
<thead>
<tr>
<th>Inflation Forecasts</th>
<th>$\phi$</th>
<th>$\sigma_\epsilon$</th>
<th>$\sigma_\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPF</td>
<td>0.927**</td>
<td>0.793**</td>
<td>0.135</td>
</tr>
<tr>
<td></td>
<td>(0.043)</td>
<td>(0.135)</td>
<td>(0.080)</td>
</tr>
<tr>
<td>Green Book</td>
<td>0.916**</td>
<td>0.708**</td>
<td>0.104</td>
</tr>
<tr>
<td></td>
<td>(0.052)</td>
<td>(0.122)</td>
<td>(0.067)</td>
</tr>
</tbody>
</table>

Note: ** indicates statistical significance at the 5% significance level. Estimation results are obtained using equation (3) and (4).

Table 2: Root Mean Square Error

<table>
<thead>
<tr>
<th>Forecast</th>
<th>Without Bias Correction</th>
<th>With Bias Correction</th>
<th>Using Information Available</th>
<th>% Change in Improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPF</td>
<td>1.359</td>
<td>1.069</td>
<td>at t</td>
<td>21.3%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.141</td>
<td>at t-1</td>
<td></td>
</tr>
<tr>
<td>Green Book</td>
<td>1.187</td>
<td>0.997</td>
<td>at t</td>
<td>16.0%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.028</td>
<td>at t-1</td>
<td>13.4%</td>
</tr>
</tbody>
</table>

Note: The root mean square error (RMSE) is $\sqrt{(1/T) \sum_{t=0}^{T} \epsilon_t^2}$ where $\epsilon_t$ is forecast error for period $t$. 
Table 3: **Sources of Bias: SPF and Green Book (GB)**

<table>
<thead>
<tr>
<th>Data</th>
<th>$\delta_1$</th>
<th>$\delta_2$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
<th>$\theta_4$</th>
<th>$\theta_5$</th>
<th>$R^2$</th>
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</thead>
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<td>SPF</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k=1$</td>
<td>0.051**</td>
<td>0.140**</td>
<td>-0.120**</td>
<td>-0.034**</td>
<td>-0.017</td>
<td>-0.035**</td>
<td>-0.037*</td>
<td>0.72</td>
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<tr>
<td></td>
<td>(0.011)</td>
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<td>(0.022)</td>
<td>(0.015)</td>
<td>(0.021)</td>
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<tr>
<td>$k=2$</td>
<td>0.048**</td>
<td>0.150**</td>
<td>-0.099**</td>
<td>-0.022</td>
<td>-0.037**</td>
<td>-0.036**</td>
<td>-0.038**</td>
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<td>(0.010)</td>
<td>(0.031)</td>
<td>(-0.019)</td>
<td>(-0.021)</td>
<td>(-0.018)</td>
<td>(0.015)</td>
<td>(0.019)</td>
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<tr>
<td>$k=3$</td>
<td>0.045**</td>
<td>0.158**</td>
<td>-0.072**</td>
<td>-0.043**</td>
<td>-0.038**</td>
<td>-0.034*</td>
<td>-0.039*</td>
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<tr>
<td></td>
<td>(0.010)</td>
<td>(0.036)</td>
<td>(0.020)</td>
<td>(0.017)</td>
<td>(0.019)</td>
<td>(0.020)</td>
<td>(0.021)</td>
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<tr>
<td>$k=4$</td>
<td>0.039**</td>
<td>0.156**</td>
<td>-0.078**</td>
<td>-0.047**</td>
<td>-0.034</td>
<td>-0.040**</td>
<td>-0.022</td>
<td>0.68</td>
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<tr>
<td></td>
<td>(0.010)</td>
<td>(0.040)</td>
<td>(0.019)</td>
<td>(0.018)</td>
<td>(0.023)</td>
<td>(0.018)</td>
<td>(0.024)</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$k=1$</td>
<td>0.049**</td>
<td>0.119**</td>
<td>-0.090**</td>
<td>-0.031**</td>
<td>-0.029</td>
<td>-0.017</td>
<td>0.019</td>
<td>0.72</td>
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<td></td>
<td>(0.009)</td>
<td>(0.025)</td>
<td>(0.019)</td>
<td>(0.013)</td>
<td>(0.017)</td>
<td>(0.014)</td>
<td>(0.017)</td>
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<tr>
<td>$k=2$</td>
<td>0.047**</td>
<td>0.129**</td>
<td>-0.083**</td>
<td>-0.033*</td>
<td>-0.022</td>
<td>0.000</td>
<td>0.001</td>
<td>0.69</td>
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<td></td>
<td>(0.009)</td>
<td>(0.030)</td>
<td>(0.018)</td>
<td>(0.019)</td>
<td>(0.016)</td>
<td>(0.012)</td>
<td>(0.017)</td>
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</tr>
<tr>
<td>$k=3$</td>
<td>0.044**</td>
<td>0.135**</td>
<td>-0.078**</td>
<td>-0.028*</td>
<td>-0.007</td>
<td>-0.017</td>
<td>0.002</td>
<td>0.66</td>
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<td>(0.009)</td>
<td>(0.034)</td>
<td>(0.022)</td>
<td>(0.017)</td>
<td>(0.016)</td>
<td>(0.018)</td>
<td>(0.019)</td>
<td></td>
</tr>
<tr>
<td>$k=4$</td>
<td>0.041**</td>
<td>0.133**</td>
<td>-0.071**</td>
<td>-0.015</td>
<td>-0.023</td>
<td>-0.018</td>
<td>0.005</td>
<td>0.60</td>
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<tr>
<td></td>
<td>(0.009)</td>
<td>(0.038)</td>
<td>(0.018)</td>
<td>(0.017)</td>
<td>(0.024)</td>
<td>(0.017)</td>
<td>(0.026)</td>
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</tr>
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</table>

Note: ** and * indicate statistical significance at the 5%, 10% significance level. Estimation of equation (6) is conducted by the Ordinary Least Squares considering Newey-West HAC Standard Errors and Covariance (lag truncation=4). Sample ranges from 1968Q4 to 2005Q4 for the SPF and from 1968Q4 to 2001Q4 for the Green Book.
Figure 1: Biases and Forecast Errors \( (\pi_{t+1} - \hat{\pi}_{t+1}) \)

Figure 2: Fitted Value (Equation 6) and Bias from the SPF
Figure 3: Fitted Value (Equation 6) and Bias from the Green Book

Figure 4: Biases and the Real Interest Rate

Note: Each series is normalized around zero and one-standard deviation.