Equilibrium price dispersion with heterogeneous searchers

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Equilibrium Price Dispersion with Heterogeneous Searchers*

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Abstract. Firms simultaneously set prices in a homogeneous-product market where uninformed consumers search for price information. Some uninformed consumers are local searchers who visit only one seller, possibly due to high search costs or bounded rationality; whereas others search sequentially with an optimal reservation price. Equilibrium prices may follow a mixture distribution, with clusters of high and low prices separated by a zero-density gap. The presence of local searchers raises prices for high-value products but can lower prices for low-value products. A reduction in search cost sometimes leads to higher equilibrium prices.

Key words: price dispersion, search, search cost, bounded rationality

JEL Classification: D43, D83

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1. INTRODUCTION

One of the most striking observations in consumer markets is the existence of substantial price dispersion for seemingly identical products. In his classic model of sales, Varian (1980) shows how price dispersion in a homogeneous-product market arises due to the presence of consumers who are uninformed about market prices. Price-setting firms balance their incentives to compete for the informed consumers who will pay the lowest price and to exploit the uninformed consumers who each purchase from a randomly selected seller, producing an equilibrium price distribution.[1] The behavior of the uninformed consumers can be justified as their having prohibitively high search cost to find the price of a second seller. Alternatively, as Ellison (2006) notes, the uniformed consumers in Varian's model can be viewed as being boundedly rational–their reservation price is set at their product valuation, which may not be optimal.

An alternative approach to understanding price dispersion is through models of optimal search. Stahl (1989) considers a model that is similar to Varian (1980) except that all the uninformed consumers search optimally given their (not too high) search cost. In equilibrium, firms price according to a probability distribution function (much as in Varian); the informed consumers (whom Stahl calls shoppers who have zero search cost) will pay the lowest price, whereas the uninformed consumers will engage in optimal price search, sampling sellers sequentially with an optimal reservation price. An important contribution of the paper is to bridge the Bertrand equilibrium of marginal cost pricing and the Diamond equilibrium of monopoly pricing (Diamond, 1971) in a unified model of optimal consumer search: equilibrium prices monotonically decrease in consumer search cost and converge to marginal cost when search cost goes to zero, whereas the Diamond outcome obtains in the limit when the portion of consumers with strictly positive search costs approaches 1.

While these two influential papers and the related literature have offered significant insights on equilibrium price competition and dispersion,[2] the startling different behaviors of

[1] See also Rosenthal (1981) for a related original contribution.

[2] The literature on price dispersion that originates from Varian (1980) and Rosenthal (1980) has come to be known as the clearinghouse approach, after the more general model of Baye and Morgan (2001), as

the uninformed consumers in the two models are perhaps best viewed as capturing different aspects of the reality: although some consumers may conduct optimal sequential search, who anticipate correctly the equilibrium price distribution and set an optimal reservation price, there may also be other consumers who behave as the uninformed consumers in Varian (1980), searching with a limited scope. It would be desirable to incorporate these heterogeneous searchers in a unified and yet tractable model.

In this paper, we develop an equilibrium model of oligopolistic pricing in homogeneous-product markets with heterogeneous searchers. Each consumer demands at most one unit of the product. As in Varian (1980) and Stahl (1989), some consumers are informed and will purchase from the seller with the lowest price, while others are uninformed about market prices. The uninformed consumes will search for price information: some of whom are what we shall call *global searchers*, who search sequentially with recall (as in Stahl, 1989) and who follow an optimal reservation price that is generally below their product valuation; whereas the others are what we shall call *local searchers*, who search only once and purchase if the price does not exceed their product valuation. As in Varian, these local searchers can be viewed either as having prohibitively high cost for searching beyond once,[3] or as boundedly rational (henceforth BR) searchers whose reservation price is not derived from the equilibrium price distribution (but is rationalizable).[4] When all uninformed consumers are local searchers, our model reduces to that of Varian (1980); when all uninformed consumers are global searchers, the model is the same as Stahl (1989).[5] By varying the uninformed consumers between these two types and by allowing both interpretations of the local searchers,

opposed to the approach of optimal (sequential) search with strategic firms that Stahl (1989) examplifies (see Reinganum, 1979 for another original contribution. Search models with fixed sample sizes include Burdett and Judd, 1983; and Janssen and Moraga-Gonzalez, 2004). Baye, Morgan, and Scholten (2006) provides an excellent review of the literature.

[3] With this interpretation, our model is complementary to Stahl (1996), which considers search costs that follow an absolutely continuous distribution.

[4] The local searchers can also be boundedly rationality due to their loyalty to a particular seller, as in Rosenthal (1980). Our analysis remains valid with this alternative interpretation of the local searchers.

[5] Stahl (1989) considers a downward-sloping demand curve for each consumer. For convenience, we assume unit demand, but our analysis can be extended to admit elastic demand.

our model combines the two approaches and provides a convenient framework to study how heterogeneity among searchers affects equilibrium outcomes.

In equilibrium, there is price dispersion due to firms adopting mixed strategies, and the nature of the equilibrium depends on how the value of the product differs from a benchmark—the optimal reservation price by global searchers if there were no local searchers, which in turn is an increasing function of the global searchers' search cost. For high-value products, for which local searchers' reservation price exceeds the benchmark by a large amount, the equilibrium is a mixture distribution, where firms randomize between a high-price distribution and a low-price distribution, placing zero probability on an interval of intermediate prices. By adopting this clustered pricing strategy, firms swing between targeting the local searchers and global searchers, and equilibrium prices are higher in this case than if there were no local searchers. Another interesting feature of the equilibrium in this region of parameter values is that, unlike in Stahl (1989), global searchers may indeed search more than once and hence there is true equilibrium sequential search.

For low-value products, for which local searchers' reservation price departs from (exceeding or falls short of) the benchmark by a relatively small amount, the equilibrium has the feature familiar in the literature, where firms adopt unclustered pricing, namely that the equilibrium price distribution has positive density (i.e., has no gap) on the entire support. Remarkably, in this case equilibrium prices are either not affected by or are lower due to the presence of local searchers. Our results thus provide testable predictions of how the nature of equilibrium price dispersion depends on the value of the product relative to search costs.

Our analysis also reveals some intriguing comparative statics. Unlike Stahl (1989), which predicts that prices monotonically decrease as search costs become lower, in our model prices can increase as the search cost for the global searchers decreases, holding other parameters of the model constant. When the equilibrium involves a mixture distribution, a lower search cost would lower the global searchers' optimal reservation price, making it less profitable for firms to target them. Consequently, firms increase the probability of choosing high prices that target the local searchers, resulting in higher expected prices in the market. Furthermore, as this search cost goes to zero, global searchers' reservation price will approach zero and they will effectively search all sellers; but equilibrium price dispersion persists. In fact, in this case the equilibrium price distribution converges to that in Varian (1980) with the mass of uninformed consumers equal to that of local searchers. Our findings offer a simple explanation of the puzzling observations that prices for many products do not seem to be lower on the Internet than in conventional markets and that substantial price dispersion remains in the Internet market, although Internet appears to have substantially lowered search cost.[6] We may consider the local searchers in our model as those who have high search cost, because they lack the access to the low-cost search technology made possible by the Internet (e.g., they may not have a computer or may not have Internet access). Then, the low search cost to find prices through the Internet need not reduce prices and reduce price dispersion.[7] In this sense, the "Digital Divide" not only raises an equity issue, it also has important implications for market efficiency. Our model thus complements other recent studies that offer alternative explanations (e.g., Baye and Morgan, 2001; Baye and Morgan, 2004; and Ellison and Ellison, 2008).

If we interpret the local searchers as BR consumers, our paper is related to a small but growing literature that models bounded rationality in the study of industrial organization.[8] As in the literature, the BR consumers in our model can have either positive or negative externalities on the rational consumers. However, in our model a relatively small departure by some consumers from optimizing behavior either does not affect the equilibrium or benefits all consumers, including the BR consumers themselves.

The rest of the paper is organized as follows. Section 2 presents our model. Sections 3 characterizes equilibrium price distribution. Section 4 analyzes comparative statics. Section 5 discusses the implications of considering local searchers as BR consumers. Section 6 concludes. All formal proofs are relegated to an appendix.

[6] See, for example, Baye and Morgan (2004), Baye, Morgan, and Scholten (2006), and Ellison and Ellison (2005) for discussions of evidence.

[7] Similarly, if local searchers are boundedly rational, the low search cost on the Internet need not reduce prices and price dispersion.

[8] See Armstrong and Chen (2009), Baye and Morgan (2004), Ellison (2005), Gabaix and Laibson (2006), and Spiegler (2006) for examples of recent work, and Ellison (2006) for an insightful review of the literature.

2. THE MODEL

There are $N \geq 2$ firms, producing a homogeneous product with a constant marginal cost that is normalized to zero. Firms simultaneously and independently set their prices. As in the literature, we will consider only symmetric equilibrium and assume that each firm randomly chooses a price from a probability distribution function: if the distribution function reduces to a single point, then each firm chooses a pure strategy; otherwise the firm adopts a mixed strategy.

The market has a unit mass of consumers, each demanding one unit of the product. They make choices after firms set prices. Portion $\mu \in (0, 1)$ of the consumers are informed about all firms' prices in the market,[9] whereas portion $1 - \mu$ of the consumers are (initially) uninformed about prices in the market. The informed consumers will always buy from a seller with the lowest price, while the uninformed consumers will engage in price search. As is commonly assumed in the literature, the first search has zero cost but each additional search incurs a positive search cost. Portion $\lambda \in [0, 1]$ of the uninformed consumers are *global searchers,* who conduct optimal search sequentially with recall and with search cost $s > 0$; portion $1 - \lambda$ are *local searchers* who will only search one seller and purchase if the price does not exceed their valuation for the product. Thus, the local searchers are the same as the uninformed consumers in Varian (1980), whereas the global searchers are the sequential searchers in Stahl (1989). As in Varian, there are two possible interpretations for the local searchers. They can be viewed as optimal searchers, same as the global searchers, but have a sufficiently high search cost, perhaps because they have no access to some low-cost search technology (e.g., a computer or access to the Internet). Alternatively, these may be BR searchers, who have search cost s but have chosen their product valuation as the reservation price, which may or may not be optimal given the equilibrium price distribution. We shall see that an attractive feature of our model is that the equilibrium price distribution will be the same under either interpretation of the local searchers. Hence we shall allow

[9] As Stahl (1989) suggests, these can be consumers who have zero search cost and even enjoy shopping around.

both interpretations and discuss their potentially different implications.

We assume that local searchers' product valuation is b and all other consumers' product valuation is V, where $\tilde{b} < b < \frac{1}{1-\lambda}V$ and $\hat{b}$ $(< V)$) is the lower bound on b to be defined later. Hence, a special case is $b = V$, but we also allow local searchers to have somewhat different product valuation from other consumers.[10] The key parameters of our model are b, V, μ, λ, and s. A (symmetric) equilibrium is a price distribution function $\Phi(p)$ and a reservation price r by the global searchers such that, given r, b, and other firms adopting $\Phi(p)$, it is optimal for each firm to choosing $\Phi(p)$; and given $\Phi(p)$, it is optimal for global searchers to search sequentially with reservation price r.

By familiar arguments (e.g., Varian, 1980; and Stahl, 1989), the game has no pure-strategy equilibrium, and the equilibrium price distribution $\Phi(p)$ is atomless on its entire support.

Denote the upper limit of the support for $\Phi(p)$ by $\hat{p}$. Then, $\hat{p} \leq \max\{b, r\}$, since a firm will earn zero profit by pricing above $\max\{b, r\}$. Also, $\hat{p} \geq \min\{b, r\}$, since if $\hat{p} < \min\{b, r\}$, a firm would sell to the same number of consumers pricing higher at $\min\{b, r\}$ as pricing at $\hat{p}$. Furthermore, it cannot be true that $\min\{b, r\} < \hat{p} < \max\{b, r\}$, because if it were the case, a firm would sell to the same number of consumers pricing higher at $\max\{b, r\}$ as pricing at $\hat{p}$. We thus have:

Lemma 1 *In equilibrium, the upper limit of the support for $\Phi(p)$ is either the local searchers' reservation price b or the global searchers' reservation price r.*

Before proceeding to the analysis of our model, we discuss two of its special cases. If $\lambda = 0$, then our model reduces to that of Varian (1980), where all the uninformed consumers purchase from a randomly selected seller if the price does not exceed their valuation b. The

[10] As it will become clear later, for $b \geq V$, our analysis will be entirely the same as $b = V$; but $b < V$ allows us to consider an additional case of interest. The local searchers may have lower product valuation than the global searchers, for instance, if the local searchers are low-income consumers who lack access to a new search technology, such as a computer or Internet access.

equilibrium price distribution in this case is

$$F^{v}(p)=1-\left(\frac{1-\mu}{N\mu}(\frac{b}{p}-1)\right)^{\frac{1}{N-1}} \quad \text{with } \frac{1-\mu}{1+(N-1)\mu}b\leq p\leq b. \tag{1}$$

If $\lambda=1$, then our model reduces to that of Stahl (1989); the equilibrium price distribution and reservation price by the global searchers are uniquely given by, respectively:

$$G(p;r_g)=1-\left(\frac{1-\mu}{N\mu}(\frac{r_g}{p}-1)\right)^{\frac{1}{N-1}} \quad \text{with } \frac{1-\mu}{1+(N-1)\mu}r_g\leq p\leq r_g, \tag{2}$$

$$\int_{\frac{1-\mu}{1+(N-1)\mu}r_g}^{r_g}(r_g-p)dG(p;r_g)=s, \tag{3}$$

provided that $r_g\leq V$. All firms simultaneously choose prices according to c.d.f. $G(p;r_g)$, and all uninformed consumers search sequentially with recall under the optimally chosen reservation price r_g, stopping search only when she has found a price $p\leq r_g$ or when she has searched all sellers (in which case she purchases from the seller with the lowest price). Since

$$\int_{\frac{1-\mu}{1+(N-1)\mu}r_g}^{r_g}(r_g-p)dG(p;r_g)=\int_{\frac{1-\mu}{1+(N-1)\mu}r_g}^{r_g}G(p;r_g)dp\rightarrow 0 \text{ as } r_g\rightarrow 0,$$

and the partial derivative of the left-hand side of (3) with respect to r_g is

$$1+\int_{\frac{1-\mu}{1+(N-1)\mu}r_g}^{r_g}\frac{dG(p;r_g)}{dp}\left(-\frac{p}{r_g}\right)dp>1-\int_{\frac{1-\mu}{1+(N-1)\mu}r_g}^{r_g}\frac{dG(p;r_g)}{dp}dp=0, \tag{4}$$

$r_g\leq V$ is satisfied when s is small relative to V, which is assumed to hold throughout our analysis.

Applying the implicit function theorem to (3), we have:

$$\frac{\partial r_g}{\partial\mu}<0 \text{ and } \frac{\partial r_g}{\partial s}>0. \tag{5}$$

That is, if all uninformed consumers are global searchers, their optimal reservation price decreases in the portion of informed consumers but increase in search cost.

Our analysis will depend importantly on how b differs from r_g. We shall divide the possible values of b into three connected and mutually exclusive regions: (i) $b>\frac{r_g}{1-\lambda}$; (ii) $\frac{r_g}{1-\lambda}\geq b\geq r_g$; and (iii) $r_g>b\geq\hat{b}$, where $\hat{b}$ is the lower bound on b that we will define later in Lemma 2.

3. EQUILIBRIUM PRICE DISTRIBUTION

We first consider the case when b exceeds r_g by a large amount, or product value is high: $b > \frac{1}{1-\lambda} r_g$.

Proposition 1 *When $b > \frac{1}{1-\lambda} r_g$, there exists a unique symmetric equilibrium, in which each firm prices according to mixed strategy*

$$F(p;\alpha,r_f) = \begin{cases} (1-\alpha)F_1(p;\alpha) & \text{if } t_1 \le p < r_f \\ (1-\alpha) & \text{if } r_f \le p < t_2 \\ 1-\alpha+\alpha F_2(p;\alpha) & \text{if } t_2 \le p \le b \end{cases}, \tag{6}$$

and global searchers adopt reservation price r_f, where $t_1 < r_f < t_2 < b$,

$$F_1(p;\alpha) = \frac{1}{1-\alpha}\left\{1 - \left[\frac{(1-\lambda)(1-\mu)}{N\mu}\left(\frac{b}{p}-1\right) - \frac{\lambda(1-\mu)\left(1-\alpha^N\right)}{N\mu(1-\alpha)}\right]^{\frac{1}{N-1}}\right\}, \tag{7}$$

$$F_2(p;\alpha) = 1 - \frac{1}{\alpha}\left[\frac{(1-\lambda)(1-\mu)}{N(\mu+\lambda(1-\mu))}\left(\frac{b}{p}-1\right)\right]^{\frac{1}{N-1}}, \tag{8}$$

$$t_1 = b\frac{(1-\lambda)(1-\mu)(1-\alpha)}{(1-\lambda)(1-\mu)(1-\alpha)+\lambda(1-\mu)(1-\alpha^N)+N\mu(1-\alpha)}, \tag{9}$$

$$t_2 = b\frac{(1-\lambda)(1-\mu)}{(1-\lambda)(1-\mu)+N\alpha^{N-1}(\lambda+\mu-\lambda\mu)}, \tag{10}$$

and $\alpha \in (0,1)$ and r_f satisfy

$$r_f = b\frac{(1-\lambda)(1-\mu)(1-\alpha)}{(1-\lambda)(1-\mu)(1-\alpha)+\lambda(1-\mu)(1-\alpha^N)+N\mu(1-\alpha)\alpha^{N-1}}, \tag{11}$$

$$(1-\alpha)\int_{t_1}^{r_f}(r_f-p)dF_1(p;\alpha) = s. \tag{12}$$

Each firm's equilibrium profit is $b\frac{(1-\lambda)(1-\mu)}{N}$. Furthermore, $r_f > r_g$, and $F(p;\cdot) < G(p;\cdot)$ so that both the expected price and the expected minimum price in the market are higher under $F(p;\cdot)$ than under $G(p;\cdot)$. Local searchers have lower expected surplus than global searchers, and welfare of all consumers is lower when $\lambda < 1$ than when $\lambda = 1$.

We notice several interesting features of the equilibrium:

First, the equilibrium price distribution is a mixture distribution consisting of two separate cumulative distribution functions, $F_1(p;\alpha)$ and $F_2(p;\alpha)$, playing them randomly with respective probabilities $1-\alpha$ and α, and a gap exists between the upper limit of the support for F_1 and the lower limit of the support for F_2. Both α and r_f are functions of b and are determined endogenously. In equilibrium, with probability α, each firm will price above r_f according to c.d.f. F_2, and in doing so it targets the local searchers (and can sell to the other consumers only when the other firm has also priced above r_f). With probability $1-\alpha$, each firm will price below r_f according to c.d.f. F_1. Given the reservation price r_f of the global searchers, a firm is guaranteed to sell to at least $\frac{1}{N}$ of them if pricing at r_f, the upper limit of the support for F_1, whereas with a slight increase of the price above r_f it will lose sales to all the global searchers if another firm prices at or below r_f, which occurs with probability $1-\alpha^{N-1} > 0$. Thus, the lower limit of support for F_2, which achieves the same expected profit as b (the upper limit of the support for both F_2 and F), must be discretely higher than r_f: when raising its price above r_f, a firm's demand jumps down, which must be exactly offset by a jump-up of the price so that the firm's expected profit remains the same. Consequently, an interval of prices ($r_f < p < t_2$) on the support of the equilibrium distribution F will be played with zero probability. This clustered equilibrium price distribution is in sharp contrast to the usual unclustered price distribution found in the literature.

Second, in equilibrium, since both firms will price above r_f with a positive probability, global searchers may search more than once before purchasing; so there is true equilibrium sequential search. This is in contrast to Stahl (1989), where in equilibrium all searchers only search once before purchase.

Third, the confinement to searching only one seller is costly to the local searchers, making their expected surplus lower than that of the global searchers. This is true because of both a direct effect and a strategic effect. Directly, they pay a higher expected price given the equilibrium price distribution, compared to the global searchers who have a lower reservation price. Indirectly, the presence of the local searchers encourages firms to raise prices strategically, resulting in an equilibrium price distribution with a higher expected price. As a result, the local searchers will have a lower expected surplus than the global searchers, even though they may incur a search cost in equilibrium.

Finally, the presence of local searchers exerts a negative externality on the global searchers and the informed consumers by raising the expected prices they will pay.

We illustrate the equilibrium distribution of Proposition 1 in the example below.

Example 1 *Suppose that* $N = 3$, $\mu = 0.5$, $\lambda = 0.5$, $s = 0.5$. *Then* $r_g = 1.26$, *and* $\frac{1}{1-\lambda} r_g = 2.53$. *Let* $b = 5$. *We find* $\alpha = 0.27$ *and* $r_f = 1.79$. *The equilibrium price distribution is*

$$F(p) = \begin{cases} 1 - 1.38\sqrt{\frac{0.83}{p} - 0.39} & if \quad 0.6 \le p < 1.79 \\ 0.27 & if \quad 1.79 \le p < 3 \\ 1 - 3.65\sqrt{\frac{0.56}{p} - 0.11} & if \quad 3 \le p \le 5 \end{cases}.$$

Its density function, shown in Figure 1, is

$$f(p) = \begin{cases} \frac{0.42}{p^2\sqrt{\frac{0.83}{p} - 0.39}} & if \quad 0.6 \le p < 1.79 \\ 0 & if \quad 1.79 \le p < 3 \\ \frac{0.28}{p^2\sqrt{\frac{0.56}{p} - 0.11}} & if \quad 3 \le p \le 5 \end{cases}.$$

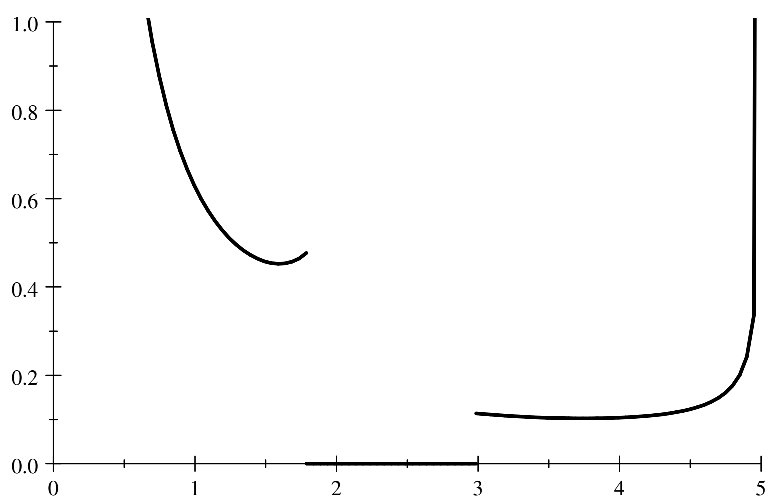

Figure 1

We next consider the case where b exceeds r_g by a relatively small amount.

Proposition 2 *When $r_g \leq b \leq \frac{1}{1-\lambda} r_g$, there exists a unique symmetric equilibrium, in which firms set prices according to $G(p, r_g)$, and global searchers adopt reservation price r_g. Each firm's equilibrium profit is $\frac{1-\mu}{N} r_g$. In equilibrium, local searchers have the same expected surplus as global searchers, and the presence of local searchers has no effect on the equilibrium outcome.*

Remarkably, when the local searchers' reservation price exceeds r_g by a relatively small amount (which can be large if λ is large), their presence has no effect on the equilibrium price distribution, which remains to be $G(p, r_g)$, same as if all searchers were global searchers. In such situations, Stahl (1989)'s analysis is entirely valid. Intuitively, even though the local searchers' reservation price exceeds that of the low-cost searchers, since the difference is relatively small–either because b is close to r_g or because λ is large, firms' price strategy is driven by the consideration of global searchers' reservation price. Consequently, the local searchers have the same expected search outcomes as global searchers. Firm conduct and market performance are not affected by their presence. The global searchers exert a positive externality on the local searchers: firms do not want to lose sales to the global searchers by pricing above r_g, which, given that $r_g \leq b$, means that having the reservation price at b is equivalent to setting it at r_g in equilibrium. Example 2 below illustrates the equilibrium price distribution in Proposition 2.

Example 2 *Everything is the same as in Example 1 except* $1.26 \leq b \leq 2.53$, *where* $1.26 = r_g$ *and* $2.53 = \frac{1}{1-\lambda} r_g$. *The equilibrium price distribution is*

$$G(p) = 1 - \sqrt{\frac{0.42}{p} - 0.33} \qquad \textit{with} \qquad 0.32 \leq p \leq 1.26.$$

Its density function, shown in Figure 2, is

$$g(p) = \frac{0.21}{p^2 \sqrt{\frac{1.26}{p} - 0.33}} \qquad \textit{with} \qquad 0.32 \leq p \leq 1.26.$$

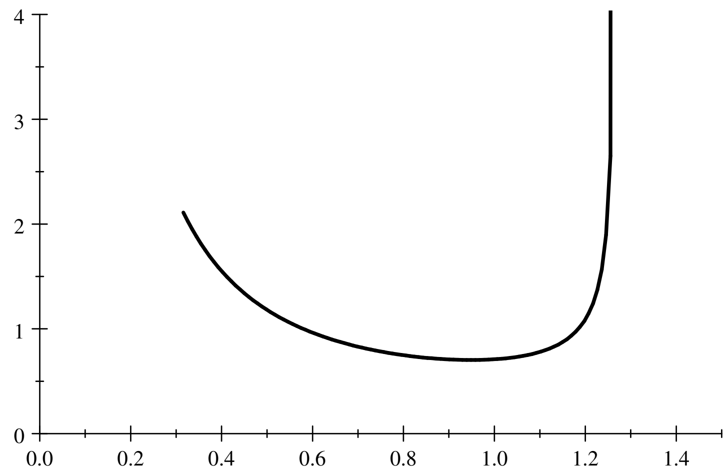

Figure 2

Finally, we consider the case where b is lower than r_g. For $b < r_g$, let $H(p;b)$ and $r_h \equiv r_h(b)$ satisfy

$$H(p;b) = 1 - \left(\frac{1-\mu}{N\mu} (\frac{b}{p} - 1) \right)^{\frac{1}{N-1}} \quad \text{with } \frac{1-\mu}{1-\mu+N\mu} b \leq p \leq b, \tag{13}$$

$$\int_{\frac{1-\mu}{1-\mu+N\mu}b}^{b} (r_h - p) dH(p,b) = s; \tag{14}$$

Given that r_g exists uniquely, it is straightforward to verify that r_h exists uniquely for any given $b > 0$. The result below establishes the unique existence of some $\hat{b} \in (0, r_g)$, which is by assumption the lowest possible value for b.

Lemma 2 *For any given $\lambda < 1$, if $b < r_g$, then $b < r_h < r_g$, $0 < r'_h(b) < 1$, and there exists a unique $\hat{b} \in (0, r_g)$ such that $\hat{b} = \lambda r_h \left(\hat{b}\right)$, with $b > \lambda r_h(b)$ if $b > \hat{b}$ and $b < \lambda r_h(b)$ if $b < \hat{b}$.*

The next result shows that if $b \in \left(\hat{b}, r_g\right)$, then the equilibrium price distribution and global searchers reservation price are given by (13) and (14).

Proposition 3 *Suppose that $r_g > b \geq \hat{b} \equiv \lambda r_h \left(\hat{b}\right)$. Then, there exists a unique symmetric equilibrium, in which firms set prices according to $H(p;b)$ and global searchers adopt* *reservation price r_h, where $H(p;b)$ and $r_h \equiv r_h(b)$ satisfy (13) and (14). Each firm's equilibrium profit is $b\frac{1-\mu}{N}$, lower than when $\lambda = 1$. In equilibrium, local searchers have the same expected surplus as global searchers, and the presence of local searchers increases all consumers' welfare.*

Interestingly, when local searchers' reservation price is below r_g, global searchers choose their optimal reservation price above the upper limit of the support for the equilibrium price distribution. In other words, firms always price strictly below the global searchers' reservation price. While the firms' pricing strategy may seem counter-intuitive, it is easier to understand once the presence of local searchers is taken into account. Since $b < r_h$ but the difference is relatively small, a firm would want to lower its price to b or below in order to sell to the local searchers—it would not be profitable for the firm to raise its price to r_h.

Although local searchers' reservation price is below that of the global searchers', all searchers have the same expected search outcomes and expected payoffs. This is similar to the case when b exceeds r_g by a small amount. The difference is that here firms change their pricing strategy in response to the reservation price of local searchers, and all consumers are better off compared to the equilibrium where $\lambda = 1$. So this is a case where (low-valuation) local searchers exert a positive externality on global searchers.

Example 3 below illustrates the equilibrium price distribution in Proposition 3.

Example 3 *Everything is the same as in Example 1 except $b = 0.6$. We first compute $\hat{b} = 0.36$. Recall that $r_g = 1.26$. The equilibrium price distribution is*

$$H(p;b) = 1 - \sqrt{\frac{0.17}{p} - 0.33} \qquad \text{with} \qquad 0.15 \leq p \leq 0.6.$$

Its density function, shown in Figure 3, is

$$h(p) = \frac{0.1}{p^2\sqrt{\frac{0.2}{p} - 0.33}} \qquad \text{with} \qquad 0.15 \leq p \leq 0.6.$$

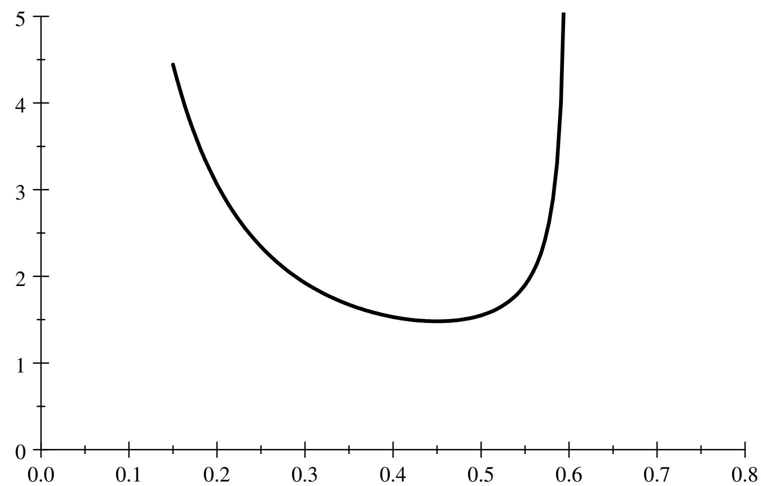

Figure 3

4. COMPARATIVE STATICS

Our analysis has shown how the nature of equilibrium changes with b: When b exceeds r_g by a relatively large amount, global searchers set reservation price $r_f > r_g$ and firms adopt clustered pricing. When b exceeds r_g by a small amount, global searchers set reservation price r_g and firms price as if there were no local searchers. When b is lower than r_g (but by a limited amount), global searchers' reservation price will be $r_h < r_g$, and firms will price lower than if local searchers were not present.

The comparative statics on b suggests an empirically testable prediction. Since $V \geq r_g$, we can consider situations where $b > \frac{r_g}{1-\lambda}$ as having a high-value product, and situations where $b \leq \frac{r_g}{1-\lambda}$ as having a low-value product, relative to the search cost s. From Propositions 1, 2, and 3, we immediately have:

Corollary 1 *Given* $\lambda \in (0,1)$, *for high-value products, firms tend to adopt clustered pricing, randomizing between clusters of high and low prices while avoiding intermediate prices; and for low-value products, firms tend to adopt unclustered pricing, with a price distribution that has positive density on its entire support.*

We next discuss how other parameters of the model, s, μ, and λ, affect market outcomes. Let r be global searchers' equilibrium reservation price, $\bar{p}$ the expected market price, π^* the equilibrium profit of each firm, w^* each consumer's welfare, and W^* aggregate consumer welfare. We first consider the effects of search cost s.

Corollary 2 *Holding all else constant:*
(i) $\frac{d\bar{p}}{ds} > 0$ *if* $b \in [r_g, \frac{1}{1-\lambda} r_g)$.
(ii) It is possible that $\frac{d\bar{p}}{ds} < 0$ *if* $b > \frac{r_g}{1-\lambda}$*; in particular,* $\frac{d\bar{p}}{ds} < 0$ *if* $b > \frac{r_g}{1-\lambda}$ *and* $N = 2$.
(iii) $\frac{dw^*}{ds} < 0$ *for all consumers if* $b \in [r_g, \frac{r_g}{1-\lambda})$ *but it is possible that* $\frac{dW^*}{ds} > 0$ *if* $b > \frac{r_g}{1-\lambda}$*, and a lower* s *need not reduce* π^*.
(iv) As $s \to 0$, π^* *does not converge to* 0, *and price dispersion persists.*

When b exceeds r_g by a relatively small amount, the equilibrium price distribution is the same as in Stahl (1989), and a decrease in search cost has the familiar effect on equilibrium price and consumer welfare, lowering $\bar{p}$ and raising w^*. It is surprising, however, that a reduction in search cost can raise the expected price in the market and reduce aggregate consumer welfare when b exceeds r_g by a large amount. To see the intuition behind this result, recall that when $b > \frac{r_g}{1-\lambda}$, in equilibrium firms randomize between a set of high prices targeting the local searchers and a set of low prices targeting the global searchers. As s becomes lower, the global searchers lower their reservation price, reducing the expected payoff from trying to sell to them. Firms thus find more profitable to target the local searchers, placing a higher probability on the interval of high prices. Consequently, the expected price in the market is higher. This can lead to higher expected price for all consumers, reducing aggregate consumer welfare.[11]

Price dispersion persists in our model even as s vanishes, because there are local searchers whose reservation price is $b > 0$, and $b > \frac{r_g}{1-\lambda}$ as $s \to 0$ ($r_g \to 0$). In fact, as $s \to 0$, the equilibrium price distribution in our model converges to that in Varian (1980) with the

[11] Since the global searchers benefit from the lower search cost, our numerical examples show that they are better off from the lower search cost, despite the higher expected price. The local searchers and the informed consumers are worse off due to the higher prices.

number of uninformed consumers becoming $(1-\lambda)(1-\mu)$. The presence of local searchers also means that a reduction in s need not lower equilibrium firm profit; and for given $b > 0$, equilibrium profit remains positive as s vanishes.

Corollary 2 offers an explanation of the puzzling observation that the Internet, which has substantially reduced search cost, has not significantly reduced prices and price dispersions for many products (e.g., Baye and Morgan, 2004; Baye, Morgan, and Scholten, 2006; and Ellison and Ellison, 2005). Our theory suggests that this can happen if there are local searchers whose reservation price is above r_g. These may be consumers who lack the access to the new search technology made available by the Internet and hence their search cost remains high. Thus even as other consumers who search on the Internet have reduced their search cost dramatically, equilibrium price dispersion remains. In this sense, the so-called "Digital Divide" is not only an equity issue but also has important implications for market efficiency.

Next, we consider the effects of changes in the portion of global searchers (λ) among uninformed consumers. Recall that the equilibrium price distribution is denoted by $\Phi(\cdot)$, and the equilibrium price distributions in Varian (1980) and in Stahl (1989) are denoted by $F^v(\cdot)$ and $G(\cdot)$, respectively.

Corollary 3 *(i) An increase in λ lowers $\bar{p}$ and π^* while benefits all consumers when the higher λ moves the parameter region from $b > \frac{r_g}{1-\lambda}$ to $\frac{r_g}{1-\lambda} > b > r_g$; and an increase in λ has no effect on market outcomes if $\hat{b} < b \leq \frac{r_g}{1-\lambda}$. (ii) $\Phi(p) \to F^v(\cdot)$ if $\lambda \to 0$; and $\Phi(p) \to G(\cdot)$ if $\lambda \to 1$.*

As one would expect, an increase in the number of global searchers tends to reduce market prices and benefit consumers. This happens when $b \geq \frac{r_g}{1-\lambda}$ and an increase in λ changes the nature of the equilibrium price distribution. Numerical analysis suggests that this is also the case for a marginal increase in λ when $b > \frac{r_g}{1-\lambda}$, although we have not been able to show this analytically. However, it is also possible that a higher λ does not lower prices, as in the case when $b \leq \frac{r_g}{1-\lambda}$.

To the extent that the portion of local searchers (e.g., consumers who have no access to computers or the Internet) may decrease over time, or λ may increase over time, our result suggests that as time passes, prices on the Internet might become less dispersed and become closer to marginal cost.

Corollary 3 implies that the equilibrium in Varian (1980) is a limiting case of our model when $\lambda \to 0$, while the equilibrium in Stahl (1989) with unit demand is a special case of our model either when $\lambda \to 1$.

Next, changes in the portion of informed consumers (μ) have expected effects, as in the following:

Corollary 4 *(i)* $\frac{d\pi^*}{d\mu} < 0$, *and (ii)* $\bar{p} \to 0$ *and* $\pi^* \to 0$ *as* $\mu \to 1$.

Thus, more informed consumers result in lower equilibrium profits. As all consumers approach to being fully informed, prices approach marginal cost and firm profits approach zero.

Finally, we can find out the limiting distribution when N goes to infinite. Let $\delta(p)$ denote the degenerate probability distribution with unit mass at p. We have:

Corollary 5 *As* $N \to \infty$, $\Phi(p) \to \delta(V)$ *if* $b \geq V$ *and* $\Phi(p) \to \delta(b)$ *if* $V > b$.

Therefore, as N grows large, in the limit the equilibrium price distribution collapse to the local searchers' or the global searchers' product valuation, whichever is smaller, consistent with the findings of Varian (1980) and Stahl (1989). Intuitively, as $N \to \infty$, the price distribution $G(\cdot)$ will concentrate at the reservation price r_g because the probability of being the firm with lowest price diminishes to zero. The concentration of prices increases the reservation price as search benefit decreases, which in turn increases the incentive for firms to set higher prices. As a result, r_g converges to V and $G(\cdot)$ converges to $\delta(V)$. Similarly, as $N \to \infty$, the price distribution $H(\cdot)$ converges to $\delta(b)$. Furthermore, since by assumption $b < \frac{V}{1-\lambda}$, as $N \to \infty$ we must have $b < \frac{r_g}{1-\lambda}$ due to $r_g \to V$. Hence, if $b > V$, as $N \to \infty$ we have $\frac{r_g}{1-\lambda} > b > r_g$ and $\Phi(p) = G(p) \to \delta(V)$; if $b < V$, $\Phi(p) = H(p) \to \delta(b)$; and if $b = V$, $\Phi(p) \to \delta(V) = \delta(b)$.

5. DISCUSSIONS

Like the uninformed consumers in Varian (1980), the local searchers in our model can have two alternative interpretations. All of our formal results in Sections 3 and 4 are valid whether the local searchers search only once because of high search cost or of some behavioral search rule.[12] We now discuss the implications of our result, focusing on the interpretation that the local searchers are boundedly rational—they have the same search cost s as the global searchers but their reservation price is not derived from optimal sequential search. This connects our analysis closely to a small but growing literature that considers bounded rationality in the study of industrial organization.[13] From Propositions 2 and 3, we have:

Corollary 6 *Suppose that the local searchers are boundedly rational in choosing their reservation price b. If the degree of departures from optimal search is relatively small in the market (in the sense that $\frac{r_g}{1-\lambda} \geq b \geq \hat{b}$), then the BR searchers will have the same expected payoffs as rational searchers, and the equilibrium outcome is either the same as or is better for all consumers than that when all consumers are fully rational.*

Our finding is in contrast to the result in many other models that BR consumers are always harmed by their non-optimizing behavior in equilibrium (e.g., Armstrong and Chen, 2009; Gabaix and Laibson. 2006; and Spiegler, 2006). Unlike in these models, in our model a small degree of non-optimizing behavior by some consumers is harmless and can benefit all consumers. Thus, BR searchers can have a positive externality on rational searchers and informed consumers.

However, when the departure from optimal search is relatively large, with $b > \frac{r_g}{1-\lambda}$, non-optimizing search behavior is costly to the BR searchers, reducing their welfare below that

[12] As long as s is not too large, we will have $s < \hat{b} \leq b$, and hence the search rule of having the reservation price at b is not irrational, but is boundedly rational, since any $b \in (s, V]$ is rationalizable under some market price distribution. Furthermore, as we have seen from Propostions 2 and 3, for $b \in \left[\hat{b},\ \frac{r_g}{1-\lambda}\right]$, althouth $b \neq r_g$, it is effectively optimal to choose b as the reservation price in equilibrium.

[13] See Armstrong and Chen (2008), Baye and Morgan (2004), Ellison (2005), Gabaix and Laibson (2006), and Spiegler (2006) for examples of recent work, and Ellison (2006) for an insightful review of the literature.

of the rational searchers. Furthermore, the presence of BR searchers now exerts a negative externality on rational searchers and informed consumers by encouraging firms to raise prices. We thus have:

Corollary 7 *If the degree of departures from optimal search is relatively large in the market, with $b > \frac{r_g}{1-\lambda}$, then the BR searchers will have lower expected surplus than rational searchers, and their presence makes all consumers worse off.*

We have confined our analysis to the three cases of $b > \frac{r_g}{1-\lambda}$, $\frac{r_g}{1-\lambda} \geq b \geq r_g$, and $r_g > b \geq \hat{b}$. Since b is local searchers' product valuation and since $V \geq r_g$, it is appropriate to assume that b is not too much smaller than r_g. There is another motivation to confine our analysis to $b \geq \hat{b}$. If $b < \hat{b}$, it can be shown that the equilibrium will depend on whether searchers with the exogenous reservation price b are (i) optimal searchers with (prohibitively) high search cost or (ii) BR searchers whose reservation price is b: under (i), in equilibrium these searchers will still search only once in equilibrium; but under (ii) they may search multiple sellers if b is low. The equilibrium will still be in mixed strategies, but the equilibrium price distribution would depend on which interpretation we adopt. Our purpose in this paper is to develop a theory of equilibrium price dispersion that is robust to alternative (plausible) interpretations of consumer behavior. We thus wish to avoid conclusions that are driven by (or are only valid under) a specific view of consumer behavior. This is accomplished by assuming $b \geq \hat{b}$.

6. CONCLUSION

We have developed a simple search model that unifies two different approaches of studying homogeneous-product markets with imperfect consumer information. Our analysis suggests that including the two types of searchers from Varian (1980) and Stahl (1989) in a single model yields interesting new insights about oligopolistic pricing. Most strikingly, equilibrium prices may follow a mixture distribution, with clusters of high and low prices separated by a zero-density gap in the middle; and a reduction in search cost sometimes leads to higher market prices. The equilibrium price distribution is robust with respect to alternative interpretations of the heterogeneous searchers. Under a boundedly rational interpretation for the local searchers, a small degree of bounded rationality in the market either has no effect on equilibrium outcomes or benefits all consumers while reducing firm profits. Furthermore, our analysis provides the testable empirical implication that firms tend to adopt clustered pricing for high-value products but unclustered pricing for low-value products.

While our mixture-distribution equilibrium is in contrast with the standard result in the literature (e.g., Varian, 1980; Stahl, 1989) that the equilibrium price distribution is gapless, the existence of such an equilibrium requires certain conditions. In our specific model, we have identified plausible conditions under which the equilibrium price distribution does or does not have a gap. More generally, our analysis suggests that the nature of price distribution will be sensitive to the specifications of consumer search costs as well as to other market conditions.[14]

For future research, it would be interesting to extend our model to study markets with horizontal product differentiation. It would also be interesting to study how firms might separate the local searchers from the global searchers in order to engage in price discrimination. Another direction for future research is to extend our model to settings where there is information imperfection in multiple dimensions, such as in both price and product quality. Also, it would be desirable to empirically test our model's prediction concerning how the nature of products affects the nature of equilibrium price dispersions, and more generally to understand empirically price dispersions in different markets.

[14] When the number of firms is large, our mixture-distribution equilibrium can be alternatively interpreted as an asymmetric equilibrium where α portion of the firms price higher according to F_2 whereas $1-\alpha$ portion of the firms price lower according to F_1. This suggests that high-price stores and low-price stores might coexist persistently, with price dispersion among each type of stores.

APPENDIX

Proof of Proposition 1.

Step 1. We verify that $F(p;\cdot)$ is a c.d.f. Since $F_1(r_f;\cdot) = 1$, $F_2(t_2;\cdot) = 0$, $F_1(p;\cdot)$ and $F_2(p;\cdot)$ increase in p, it follows that $F(p;\cdot)$ is continuous and weakly increases in p. Furthermore, $F(t_1;\cdot) = (1-\alpha)F_1(t_1;\cdot) = 0$, and $F(b;\cdot) = 1-\alpha+\alpha F_2(b;\cdot) = 1$. Therefore $F(p;\cdot)$ is a continuous c.d.f.

Step 2. We show that each firm is optimizing following $F(p;\cdot)$, given that other firms choose prices according to F, local searchers' reservation price is b, and global searchers' reservation price is r_f. The expected profit when a firm chooses price p is:

(i) If $t_1 < p \leq r_f$,

$$\begin{aligned}
\pi &= p\left[\frac{(1-\lambda)(1-\mu)}{N} + \lambda(1-\mu)\sum_{i=0}^{N-1}\frac{\binom{N-1}{i}\alpha^{N-1-i}(1-\alpha)^i}{i+1}\right] \\
&\quad + p\mu\sum_{i=0}^{N-1}\binom{N-1}{i}\alpha^{N-1-i}(1-\alpha)^i(1-F_1(p,\cdot))^i \\
&= p\left[\frac{(1-\lambda)(1-\mu)}{N} + \frac{\lambda(1-\mu)\left(1-\alpha^N\right)}{N(1-\alpha)} + \mu(\alpha+(1-\alpha)(1-F_1(p,\cdot)))^{N-1}\right],
\end{aligned}$$

because the firm sells to $\frac{(1-\lambda)(1-\mu)}{N}$ of local searchers with probability 1, to $\frac{\lambda(1-\mu)}{i+1}$ of global searchers when i other firms also price below r_f (which occurs with probability $\binom{N-1}{i}\alpha^{N-1-i}(1-\alpha)^i$), and also to all informed consumers (μ) when its price is lowest (which occurs with probability $[\alpha+(1-\alpha)(1-F_1(p,\cdot))]^{N-1}$). The equality above then follows from operations of combinations.

(ii) If $t_2 \leq p < b$,

$$\pi = p\left[\frac{(1-\lambda)(1-\mu)}{N} + [\lambda(1-\mu)+\mu]\,\alpha^{N-1}(1-F_2(p,\cdot))^{N-1}\right],$$

because the firm sells to $\frac{(1-\lambda)(1-\mu)}{N}$ of local searchers with probability 1 and to all global searchers and informed consumers with probability $\alpha^{N-1}(1-F_2(p,\cdot))^{N-1}$ (when p is the lowest price).

(iii) If $p = r_f$, $\pi = r_f(\frac{(1-\lambda)(1-\mu)}{N} + \frac{\lambda(1-\mu)\left(1-\alpha^N\right)}{N(1-\alpha)} + \mu\alpha^{N-1})$.

(iv) If $p = b$, $\pi = b\frac{(1-\lambda)(1-\mu)}{N}$.

Equal profits from (i) and (iii) yield

$$\begin{aligned} & F_1(p;\alpha, r_f) \\ = & \frac{1 - \left[\alpha^{N-1} + \left(\frac{(1-\lambda)(1-\mu)}{N\mu} + \frac{\lambda(1-\mu)\left(1-\alpha^N\right)}{N\mu(1-\alpha)} + \alpha^{N-1}\right)\left(\frac{r_f}{p} - 1\right)\right]^{\frac{1}{N-1}}}{1-\alpha}, \end{aligned} \tag{15}$$

which, after substituting r_f from (11), yields equation (7), where t_1 is given by equation (9). Equal profits from (ii) and (iv) yield equation (8), where t_2 is given by equation (10). And equal profits from (iii) and (iv) yield the expression for r_f, equation (11). Note that $t_1 \leq r_f \leq t_2 \leq b$. Therefore, the firm is optimizing choosing prices $p \in [t_1, b]$ according to $F(p; \cdot)$. Moreover, global searchers will search optimally, which gives equation (12), and the assumption that $b < \frac{1}{1-\lambda}V$ ensures that $r_f < V$.

Step 3. We show the existence of $\alpha \in (0, 1)$. Substituting r_f, t_1, and F_1 as functions of α and b into the left-hand side of equation (12), which can then be denoted as

$$\Gamma(\alpha; b) \equiv (1-\alpha)\int_{t_1(\alpha;b)}^{r_f(\alpha;b)} F_1(p;\alpha, r_f(\alpha;b))dp. \tag{16}$$

Γ is a continuous function of α, for any given b (and given all other parameter values). Thus, the proposed pricing strategy F and the search strategy r_f constitute an equilibrium if there exists some $\alpha \in (0, 1)$ that solves $\Gamma(\alpha; b) = s$. Since

$$\frac{\partial r_f(\alpha;b)}{\partial b} = \frac{(1-\lambda)(1-\mu)(1-\alpha)}{(1-\lambda)(1-\mu)(1-\alpha) + \lambda(1-\mu)(1-\alpha^N) + N\mu(1-\alpha)\alpha^{N-1}} > 0$$

and for $p \in [t_1, r_f]$

$$\frac{\partial F_1(p;\alpha)}{\partial r_f} = \frac{\partial F_1(p;\alpha)}{\partial p}\left(-\frac{p}{r_f}\right) > -\frac{\partial F_1(p;\alpha)}{\partial p}, \tag{17}$$

we have

$$\begin{aligned} \frac{\partial \Gamma(\alpha;b)}{\partial b} &= (1-\alpha)\frac{\partial r_f(\alpha;b)}{\partial b} + (1-\alpha)\int_{t_1(\alpha;b)}^{r_f(\alpha;b)} \frac{\partial F_1(p;\alpha)}{\partial r_f}\frac{\partial r_f(\alpha;b)}{\partial b}dp \\ &= (1-\alpha)\frac{\partial r_f(\alpha;b)}{\partial b}\left[1 + \int_{t_1(\alpha;b)}^{r_f(\alpha;b)} \frac{\partial F_1(p;\alpha)}{\partial r_f}dp\right] \\ &> (1-\alpha)\frac{\partial r_f(\alpha;b)}{\partial b}\left[1 - \int_{t_1(\alpha;b)}^{r_f(\alpha;b)} \frac{\partial F_1(p;\alpha)}{\partial p}dp\right] = 0. \end{aligned}$$

If $\alpha = 0$, we would have $b = \frac{1}{1-\lambda} r_g$ from equations (11), (7), and (3). Thus, for $b > \frac{1}{1-\lambda} r_g$, $\Gamma(\alpha; b) > s$ if $\alpha = 0$, and $\Gamma(\alpha; b) = 0 < s$ if $\alpha = 1$. Hence for any $b \in (\frac{1}{1-\lambda} r_g, \left(1 + \frac{\lambda}{1-\lambda}\right) V]$, there exists some $\alpha \in (0,1)$ that solves $\Gamma(\alpha; b) = s$.

Step 4. We establish equilibrium uniqueness. It is straightforward to verify that F is the only possible symmetric equilibrium price strategy of the game given any α. The uniqueness of the equilibrium is then established if, for any given b, $\alpha(b)$ uniquely solves $\Gamma(\alpha; b) = s$, which would be true if $\alpha(b)$ is monotonically increasing in b. Rewriting

$$F_1 = \frac{1}{1-\alpha} \left[1 - A^{\frac{1}{N-1}}\right],$$

where from (15)

$$A \equiv \alpha^{N-1} + \left(\alpha^{N-1} + \frac{(1-\mu)(1-\lambda)}{N\mu} + \frac{(1-\mu)\lambda\left(1-\alpha^N\right)}{N\mu(1-\alpha)}\right)\left(\frac{r_f}{p} - 1\right) > 0,$$

we have $\frac{\partial A}{\partial \alpha} > 0$ and

$$\begin{aligned}
\frac{\partial F_1}{\partial \alpha} &= \frac{1}{(1-\alpha)^2}\left[1 - A^{\frac{1}{N-1}}\right] - \frac{1}{1-\alpha}\frac{1}{N-1} A^{\frac{1}{N-1}-1}\frac{\partial A}{\partial \alpha} \\
&= \frac{F_1}{(1-\alpha)} - \frac{1}{1-\alpha}\frac{1}{N-1}\frac{1-(1-\alpha)F_1}{A}\frac{\partial A}{\partial \alpha}.
\end{aligned}$$

Thus

$$(1-\alpha)\frac{\partial F_1}{\partial \alpha} - F_1 = -\frac{1}{N-1}\frac{1-(1-\alpha)F_1}{A}\frac{\partial A}{\partial \alpha} < 0. \tag{18}$$

Therefore

$$\begin{aligned}
&\frac{\partial \Gamma(\alpha; b)}{\partial \alpha} \\
= & -\int_{t_1(\alpha;b)}^{r_f(\alpha;b)} F_1(p; \alpha, r_f(\alpha; b)) dp + (1-\alpha)\left[\frac{\partial r_f(\alpha; b)}{\partial \alpha} + \int_{t_1(\alpha;b)}^{r_f(\alpha;b)} \left(\frac{\partial F_1}{\partial \alpha} + \frac{\partial F_1}{\partial r_f}\frac{\partial r_f}{\partial \alpha}\right) dp\right] \\
= & \int_{t_1(\alpha;b)}^{r_f(\alpha;b)} \left[(1-\alpha)\frac{\partial F_1}{\partial \alpha} - F_1(p; \alpha, r_f(\alpha; b))\right] dp + (1-\alpha)\frac{\partial r_f(\alpha; b)}{\partial \alpha}\left(1 + \int_{t_1(\alpha;b)}^{r_f(\alpha;b)} \frac{\partial F_1}{\partial r_f} dp\right) \\
< & \ 0,
\end{aligned}$$

because $\frac{\partial r_f(\alpha;b)}{\partial \alpha} < 0$ from (11), $1 + \int_{t_1(\alpha;b)}^{r_f(\alpha;b)} \frac{\partial F_1}{\partial r_f} dp > 0$ from (17), and $(1-\alpha)\frac{\partial F_1}{\partial \alpha} - F_1(p; \alpha, r_f(\alpha; b)) < 0$ from (18). It follows that

$$\frac{d\alpha(b)}{db} = -\frac{\frac{\partial \Gamma(\alpha;b)}{\partial b}}{\frac{\partial \Gamma(\alpha;b)}{\partial \alpha}} > 0. \tag{19}$$

Step 5. We show $r_f > r_g$ by proving the two claims below

Claim 1. r_f must be monotonic in b.

Suppose to the contrary that r_f is non-monotonic in b. By the continuity of r_f in b, there will be some $b \neq \tilde{b}$ associated with some common r_f. Suppose $b > \tilde{b}$. Then $\alpha > \tilde{\alpha}$ from (19) and $t_1 > \tilde{t}_1$ from (9) and (11). Thus, using (15) for F_1,

$$\begin{aligned} s &= (1-\alpha)\int_{t_1}^{r_f} F_1(p;\alpha, r_f(b))dp \\ &= \int_{t_1}^{r_f}\left(1-\left[\alpha^{N-1}+\left(\frac{(1-\lambda)(1-\mu)}{N\mu}+\frac{\lambda(1-\mu)\left(1-\alpha^N\right)}{N\mu(1-\alpha)}+\alpha^{N-1}\right)\left(\frac{r_f}{p}-1\right)\right]^{\frac{1}{N-1}}\right)dp \\ &< \int_{\tilde{t}_1}^{r_f}\left(1-\left[\tilde{\alpha}^{N-1}+\left(\frac{(1-\lambda)(1-\mu)}{N\mu}+\frac{\lambda(1-\mu)\left(1-\tilde{\alpha}^N\right)}{N\mu(1-\tilde{\alpha})}+\tilde{\alpha}^{N-1}\right)\left(\frac{r_f}{p}-1\right)\right]^{\frac{1}{N-1}}\right)dp \\ &= (1-\tilde{\alpha})\int_{\tilde{t}_1}^{r_f} F_1\left(p;\tilde{\alpha}, r_f\left(\tilde{b}\right)\right)dp = s, \end{aligned}$$

a contradiction.

Claim 2. r_f cannot be decreasing in b for all $b \geq \frac{1}{1-\lambda}r_g$.

Suppose to the contrary that r_f monotonically decreases in b. Then $r_f < r_g$ when $b > \frac{1}{1-\lambda}r_g$. But, since $\frac{\left(1-\alpha^N\right)}{1-\alpha} = \sum_{n=0}^{N-1}\alpha^n$,

$$\begin{aligned} r_f &= b\frac{(1-\lambda)(1-\mu)(1-\alpha)}{(1-\lambda)(1-\mu)(1-\alpha)+\lambda(1-\mu)(1-\alpha^N)+N\mu(1-\alpha)\alpha^{N-1}} \\ &= b\frac{(1-\lambda)(1-\mu)}{(1-\lambda)(1-\mu)+\lambda(1-\mu)\sum_{n=0}^{N-1}\alpha^n+N\mu\alpha^{N-1}} \\ &> b\frac{(1-\lambda)(1-\mu)}{(1-\lambda)(1-\mu)+\lambda(1-\mu)N+N\mu} > r_g, \end{aligned}$$

when $b > \tilde{b} = \frac{(1-\lambda)(1-\mu)+\lambda(1-\mu)N+N\mu}{(1-\lambda)(1-\mu)}r_g$, a contradiction.

Together, Claim 1 and Claim 2 imply that r_f monotonically increases in b, and hence $r_f > r_g$ for $b > \frac{1}{1-\lambda}r_g$.

Finally, $F_2 < F_1$, and $F_1 < G$ because

$$\begin{aligned} G - F_1 &= -\frac{\alpha}{1-\alpha}+\frac{\left[\alpha^{N-1}+\left(\frac{(1-\lambda)(1-\mu)}{N\mu}+\frac{\lambda(1-\mu)\left(1-\alpha^N\right)}{N\mu(1-\alpha)}+\alpha^{N-1}\right)\left(\frac{r_f}{p}-1\right)\right]^{\frac{1}{N-1}}}{1-\alpha} \\ &\quad -\left[\frac{1-\mu}{N\mu}\left(\frac{r_g}{p}-1\right)\right]^{\frac{1}{N-1}} \end{aligned}$$

$$> \quad -\frac{\alpha}{1-\alpha}+\frac{1}{1-\alpha}\left[\alpha^{N-1}+\frac{1-\mu}{N\mu}\left(\frac{r_f}{p}-1\right)\right]^{\frac{1}{N-1}}-\left[\frac{1-\mu}{N\mu}(\frac{r_g}{p}-1\ )\right]^{\frac{1}{N-1}}$$

$$> \quad -\frac{\alpha}{1-\alpha}+\frac{1}{1-\alpha}\left(\alpha^{N-1}\right)^{\frac{1}{N-1}}+\frac{\left[\frac{1-\mu}{N\mu}\left(\frac{r_f}{p}-1\right)\right]^{\frac{1}{N-1}}-\left[\frac{1-\mu}{N\mu}(\frac{r_g}{p}-1\ )\right]^{\frac{1}{N-1}}}{1-\alpha}>0.$$

Thus F first-order stochastically dominates G. It follows that the distribution of the minimum prices under $F(\cdot), 1-\ [1-F(\cdot)]^N$, also first-order stochastically dominates the distribution of minimum prices under $G(\cdot)\,,\ 1-[1-G(\cdot)]^N$. Therefore, both the expected price and the expected minimum price are higher under F than under G, and all consumers are worse off compared to the situation where $\lambda = 1$. Local searches have lower surplus than global searchers, since with positive probability they have different search outcomes. *Q.E.D.*

Proof of Proposition 2. Given $G(p; r_g)$, the global searchers are searching optimally with reservation price r_g. To show that the proposed is an equilibrium, we thus only need to show that, given b and r_g, and given other firms choose $G(p; r_g)$, each firm is optimizing choosing any $p \in \left[\frac{1-\mu}{1+(N-1)\mu} r_g,\ r_g\right]$. For any such price, the firm's expected profit is

$$\begin{aligned} & p\frac{1-\mu}{N}+p\mu\left(1-G(p;r_g)\right)^{N-1} \\ = \quad & p\frac{1-\mu}{N}+p\mu\frac{1-\mu}{N\mu}(\frac{r_g}{p}-1\ )=\frac{1-\mu}{N}r_g. \end{aligned}$$

Then, the most profitable deviation is $p = b$, because any $p > b$ would lead to zero profit and any $p \in (r_g, b)$ would result in the same amount of sales as $p = b$ but at a lower price. However, profit at $p = b$ is

$$\frac{(1-\lambda)\,(1-\mu)}{N}b \leq \frac{(1-\lambda)\,(1-\mu)}{N}\frac{1}{1-\lambda}r_g=\frac{1-\mu}{N}r_g.$$

Therefore the firm is maximizing its profit by choosing its price from $G(p; r_g)$, and each firm's equilibrium profit is $\frac{1-\mu}{N} r_g$.

Furthermore, from familiar arguments there can be no other equilibrium price distribution. The uniqueness of r_g then implies that there can be no other symmetric equilibrium. Since $b \geq r_g$, local searchers always have the same search outcomes as global searchers;

and since the equilibrium distribution is identical to that when $\lambda = 1$, the presence of local searchers has no effect on the equilibrium outcome. *Q.E.D.*

Proof of Lemma 2. From (14) and (3), we have

$$r_h - \int_{\frac{1-\mu}{1-\mu+N\mu}b}^{b} pdH(p,b) = r_g - \int_{\frac{1-\mu}{1-\mu+N\mu}r_g}^{r_g} pdG(p,r_g) = s,$$

and thus $r_h < r_g$ since $G(p;r_g) < H(p;b)$ for $b < r_g$. Also, for $b < r_g$, if $r_h \leq b$, we would have

$$\begin{aligned} s &= \int_{\frac{1-\mu}{1-\mu+N\mu}b}^{b} (r_h - p)dH(p,b) \leq \int_{\frac{1-\mu}{1-\mu+N\mu}b}^{b} (b-p)dH(p,b) \\ &< \int_{\frac{1-\mu}{1-\mu+N\mu}r_g}^{r_g} (r_g - p)dH(p,r_g) = s, \end{aligned}$$

where the last inequality follows from (4). This is a contradiction. Therefore, if $b < r_g$, $b < r_h < r_g$.

Rewriting (14) as

$$(r_h - b) + \int_{\frac{1-\mu}{1-\mu+N\mu}b}^{b} H(p,b)dp = s,$$

we have

$$\begin{aligned} 0 &< r_h'(b) = 1 - \frac{d\int_{\frac{1-\mu}{1-\mu+N\mu}b}^{b} H(p,b)dp}{db} \\ &= 1 - 1 - \int_{\frac{1-\mu}{1-\mu+N\mu}b}^{b} \frac{dH(p,b)}{db}dp = -\int_{\frac{1-\mu}{1-\mu+N\mu}b}^{b} \frac{dH(p,b)}{dp}\left(-\frac{p}{b}\right)dp \\ &= \int_{\frac{1-\mu}{1-\mu+N\mu}b}^{b} \frac{dH(p,b)}{dp}\left(\frac{p}{b}\right)dp < \int_{\frac{1-\mu}{1-\mu+N\mu}b}^{b} \frac{dH(p,b)}{dp}dp = 1. \end{aligned}$$

From $0 < r_h'(b) < 1$, together from $b \geq \lambda r_h$ if $b \to r_g$ and $b < \lambda r_h$ if $b \to 0$, there is some $\hat{b} \in (0, r_g)$ that uniquely solves $\hat{b} = \lambda r_h\left(\hat{b}\right)$, with $b > \lambda r_h(b)$ if $b > \hat{b}$ and $b < \lambda r_h(b)$ if $b < \hat{b}$. *Q.E.D.*

Proof of Proposition 3. First, $H(p;b)$ is a continuous c.d.f., with $H(\frac{1-\mu}{1-\mu+N\mu}b;b) = 0$, $H(b;b) = 1$, $G(p;r_g) < H(p;b)$ for $b < r_g$, and $H(p;b) = G(p;r_g)$ if $b = r_g$. Given that all other firms follow the strategy $H(p;b)$ and given r and b, if a firm charges any $p \in$

$\left[\frac{1-\mu}{1-\mu+N\mu}b, b\right]$, its expected profit is

$$p\frac{1-\mu}{N}+p\mu\left[1-H(p;b)\right]^{N-1}=b\frac{1-\mu}{N}.$$

Furthermore, if it prices below $\frac{1-\mu}{1-\mu+N\mu}b$ or above r_h, its expected profit would be lower than $b\frac{1-\mu}{N}$; and if it prices between b and r_h, since $\lambda r_h \leq b$ for $b \geq b^*$, its expected profit would be

$$p\frac{\lambda\left(1-\mu\right)}{N}\leq r_h\frac{\lambda\left(1-\mu\right)}{N}\leq\frac{b}{\lambda}\frac{\lambda\left(1-\mu\right)}{N}=b\frac{1-\mu}{N}.$$

Thus each firm is optimizing by choosing mixed strategy $H(p;b)$.

Next, expecting price distribution $H(p;b)$, it is optimal for global searchers to search with reservation price r_h. Therefore the proposed pricing and search strategies indeed constitute an equilibrium, with $\pi^*=b\frac{1-\mu}{N}$, and there is no other symmetric equilibrium. Since $b<r_g$, $b\frac{1-\mu}{N}<r_g\frac{1-\mu}{N}$, and thus firm profit is lower than when $\lambda=1$. Given firms' equilibrium pricing strategy, local consumers have the same search outcome as the global searchers, and thus have the same expected surplus in equilibrium.

Finally, since $G(p;r_g)<H(p;b)$, the expected price is lower under $H(p;b)$. Moreover, since the distributions of the minimum price in the market are $1-\left[1-G(p;r_g)\right]^N$ and $1-\left[1-H(p;b)\right]^N$, respectively, which preserves the stochastic ordering, the expected lowest price is also lower under $H(p;b)$. Thus all consumers are better off due to the presence of the local searchers. *Q.E.D.*

Proof of Corollary 2. (i) If $b\in[r_g,\frac{1}{1-\lambda}r_g)$, a marginal reduction in s reduces r from (5) and hence increases $G\left(p;\cdot\right)$ from (2), lowering $\bar{p}$.

(ii) It suffices to show that $\frac{d\bar{p}}{ds}<0$ if $b>\frac{1}{1-\lambda}r_g$ and $N=2$. From Proposition 1, when $N=2$:

$$\begin{aligned}\bar{p} &= E\left[p|F\right]=\left(1-\alpha\right)E\left[p|F_1\right]+\alpha E\left[p|F_2\right]\\ &= \left(1-\alpha\right)\frac{\left(1+\alpha\lambda\right)\left(1-\mu\right)+2\alpha\mu}{2\left(1-\alpha\right)\mu}r_f\ln\frac{r_f}{t_1}+\alpha\left(\frac{\left(1-\lambda\right)\left(1-\mu\right)}{2\alpha\left(\mu+\lambda\left(1-\mu\right)\right)}b\ln\frac{b}{t_2}\right)\end{aligned}$$

$$\begin{aligned} &= \frac{b(1-\lambda)(1-\mu)}{2\mu}\ln\left(\frac{(1+\alpha\lambda)(1-\mu)+2\mu}{(1+\alpha\lambda)(1-\mu)+2\alpha\mu}\right) \\ &\quad+\frac{(1-\lambda)(1-\mu)}{2(\mu+\lambda(1-\mu))}b\ln\frac{(1+\alpha\lambda-\lambda(1-\alpha))(1-\mu)+2\alpha\mu}{(1-\lambda)(1-\mu)}. \end{aligned}$$

Thus

$$\frac{d\bar{p}}{d\alpha}=\frac{b\lambda(1-\alpha)^2(1-\lambda)(1-\mu)^2(\lambda(1-\mu)+2\mu)}{[(1-\lambda)(1-\mu+2\alpha\mu)+2\alpha\lambda][1+\mu+\alpha\lambda(1-\mu)][(1-\mu)(1+\alpha\lambda)+2\alpha\mu]}>0. \tag{20}$$

From (12) and (9),

$$\begin{aligned} s &= (1-\alpha)(r_f-E[p|F_1])=(1-\alpha)\left(r_f-\frac{(1+\alpha\lambda)(1-\mu)+2\alpha\mu}{2(1-\alpha)\mu}r_f\ln\frac{r_f}{t_1}\right) \\ &= b\frac{(1-\alpha)(1-\lambda)(1-\mu)}{(1+\alpha\lambda)(1-\mu)+2\alpha\mu}-\frac{b(1-\lambda)(1-\mu)}{2\mu}\ln\left(\frac{(1+\alpha\lambda)(1-\mu)+2\mu}{(1+\alpha\lambda)(1-\mu)+2\alpha\mu}\right). \end{aligned}$$

Thus

$$\frac{ds}{d\alpha}=-\frac{2b\mu(1-\alpha)(1-\lambda)(1-\mu)(1-\lambda\mu+\lambda+\mu)}{[1+\mu+\alpha\lambda(1-\mu)](\mu-\alpha\lambda-2\alpha\mu+\alpha\lambda\mu-1)^2}<0. \tag{21}$$

Therefore, from (20), (21), and the fact that there is one-to-one match between s and α from the proof of Proposition 1, we have $\frac{d\alpha}{ds}<0$ and

$$\frac{d\bar{p}}{ds}=\frac{d\bar{p}}{d\alpha}\frac{d\alpha}{ds}<0.$$

(iii) A marginal reduction in s increases welfare for all consumers if $b\in[r_g,\frac{1}{1-\lambda}r_g)$, since it lowers both $\bar{p}$ and the expected minimum price in the market; but it can reduce W^* if $b>\frac{1}{1-\lambda}r_g$. For example, if $N=2$, $\lambda=0.5$, $\mu=0.5$, $V=6$, $b=5$, we have $W^*=4.7364$ if $s=0.8$ while $W^*=4.7286$ if $s=0.4$. From Propositions (1), (2), and (3), corresponding to the three connected and mutually exclusive regions of b values from large to small, equilibrium profits are respectively $\frac{(1-\lambda)(1-\mu)}{N}b$, $\frac{1-\mu}{N}r_g$, and $\frac{\lambda(1-\mu)}{N}b$. Both $\frac{\lambda(1-\mu)}{N}b$ and $\frac{(1-\lambda)(1-\mu)}{N}b$ are not affected by a reduction in s.

(iv) As $s\to 0$, $r_g\to 0$ and hence $b>\frac{1}{1-\lambda}r_g$ for any given $b>0$. Thus, the equilibrium price distribution will be $F(p.\cdot)$ as $s\to 0$. Moreover, from (12), $\frac{\partial\alpha}{\partial s}<0$, and $\alpha\to 1$ as $s\to 0$. Thus, as $s\to 0$, $F(p.\cdot)\to F_2(p.\cdot)$, which is a non-degenerate c.d.f. , and each firm's equilibrium profit is $\frac{(1-\lambda)(1-\mu)}{N}b>0$. *Q.E.D.*

Proof of Corollary 3. (i) First, suppose initially $b > \frac{1}{1-\lambda} r_g$ and the equilibrium price distribution is $F(p;\cdot)$. A higher λ can move the parameter region to $\frac{1}{1-\lambda} r_g > b > r_g$, changing the equilibrium price distribution to $G(p;\cdot) > F(p;\cdot)$, resulting in lower $\bar{p}$ and π^* while benefiting all consumers (who may also save on search costs in equilibrium). Second, if $\hat{b} < b \leq \frac{r_g}{1-\lambda}$, then the equilibrium price distribution is either $G(\cdot)$ or $H(\cdot)$, which is independent of λ.

(ii)As $\lambda \to 0$, $b > \frac{1}{1-\lambda} r_g$ if $b > r_g$, and from Proposition 1

$$\Phi(p) = F(p) \to 1 - \left[\frac{(1-\mu)}{N\mu} (\frac{b}{p} - 1) \right]^{\frac{1}{N-1}} = F^v(p).$$

On the other hand, as $\lambda \to 1$, we have $b < \frac{r_g}{1-\lambda}$, firms will ignore the local searchers and $\Phi(\cdot) \to G(\cdot)$. *Q.E.D.*

Proof of Corollary 4. (i) for π^*, we have three possible cases. If $\pi^* = \frac{\lambda(1-\mu)}{N} b$ or $\pi^* = \frac{(1-\lambda)(1-\mu)}{N} b$, then obviously $\frac{d\pi^*}{d\mu} < 0$. If $\pi^* = \frac{1-\mu}{N} r_g$, then since $\frac{dr_g}{d\mu} < 0$ from (5), we also have $\frac{d\pi^*}{d\mu} < 0$. (ii) as $\mu \to 1$, the equilibrium price distribution converges to 0 (marginal cost), and hence $\bar{p} \to 0$ and $\pi^* \to 0$. *Q.E.D.*

Proof of Corollary 5. First, we show that, as $N \to \infty$, $r_g \to V$. The proof is similar to the proof of Proposition 4 in Stahl (1989). Note that, for $p \leq r_g \leq V$, $\left(\frac{r_g}{p} - 1\right)\left(\frac{1-\mu}{\mu}\right)$ is bounded away from zero. Thus, as $N \to \infty$, from (2), $G(\cdot) \to 0$. That is, as $N \to \infty$, $G(\cdot)$ concentrates at its upper bound. It follows that, as $N \to \infty$, for any $\varepsilon > 0$, we can always find some p' arbitrarily close to r_g such that $G(p') < \epsilon$. Suppose ε is such that $p' = r_g - s + \epsilon V$. Then, as $N \to \infty$,

$$\begin{aligned}
\int_{\frac{1-\mu}{1+(N-1)\mu} r_g}^{r_g} G(p; r_g) dp &= \int_{p'}^{r_g} G(p; r_g) dp + \int_{\frac{1-\mu}{1+(N-1)\mu} r_g}^{p'} G(p; r_g) dp \\
&< \left(r_g - p'\right) + \epsilon \left(p' - \frac{1-\mu}{1+(N-1)\mu} r_g\right) \\
&< \left(r_g - p'\right) + \epsilon V = s,
\end{aligned}$$

which implies that the benefit from search is smaller than s. Therefore, as $N \to \infty$, reservation price r_g must approach to V.

Next, Since $b < \frac{1}{1-\lambda}V$ by assumption, we must have $b < \frac{1}{1-\lambda}r_g$ as $N \to \infty$. If $b > V$, as $N \to \infty$, $\Phi(p) = G(\cdot) \to \delta(V)$. If $b < V$, as $N \to \infty$, $b < r_g$ and $\Phi(p) = H(p)$. But for $p < b$, $\left(\frac{b}{p} - 1\right)\left(\frac{1-\mu}{\mu}\right)$ is bounded away from zero. Thus, as $N \to \infty$, from (13), $H(p) \to 0$. Therefore, if $b < V$, $\Phi(p) = H(\cdot) \to \delta(b)$ as $N \to \infty$.

Finally, if $b = V$, $\Phi(p) \to \delta(V) = \delta(b)$ as $N \to \infty$. *Q.E.D.*

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