Splitting Up Value: A Critical Review of Residual Income Theories

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Abstract

This paper deals with the notion of residual income, which may be defined as the surplus profit that residues after a capital charge (opportunity cost) has been covered. While the origins of the notion trace back to the 19th century, in-depth theoretical investigations and widespread real-life applications are relatively recent and concern an interdisciplinary field connecting management accounting, corporate finance and financial mathematics (Peasnell, 1981, 1982; Peccati, 1987, 1989, 1991; Stewart, 1991; Ohlson, 1995; Arnold and Davies, 2000; Young and O'Byrne, 2001; Martin, Petty and Rich, 2003). This paper presents both a historical outline of its birth and development and an overview of the main recent contributions regarding capital budgeting decisions, production and sales decisions, implementation of optimal portfolios, forecasts of asset prices and calculation of intrinsic values. A most recent theory, the *systemic-value-added* approach (also named *lost-capital* paradigm), provides a different definition of residual income, consistent with arbitrage theory. Enfolded in Keynes's (1936) notion of user cost and forerun by Pressacco and Stucchi (1997), the theory has been formally introduced in Magni (2000a,b,c; 2001a,b; 2003), where its properties are thoroughly investigated as well as its relations with the standard theory; two different lost-capital metrics have been considered, for value-based management purposes, by Drukarczyk and Schueler (2000) and Young and O'Byrne (2001). This work illustrates the main properties of the two theories and their relations, and provides a minimal guide to construction of performance metrics in the two approaches.

Keywords and phrases. Accounting, finance, economics, investment analysis, residual income, excess profit, firm valuation, net present value, opportunity cost, counterfactual, performance measurement.

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1 Introduction

Consider an economic agent and consider the profit originated by her business; then consider the profit that would be (or have been) generated if she pursued (had pursued) an alternative business. Take the difference between the former and the latter: the result is what is usually called *residual income* or *excess profit*. In essence, the actual income is contrasted with a hypothetical, fictitious income foregone by the investor, whose nature is that of an *opportunity* cost. This concept is thus originated by one single question:

What would the profit be (have been) if the investor (had) selected a different course of action?

The idea of excess profit dates back to the eighteenth century, but only in the last twenty-five years the literature on residual income has flourished in various fields such as management accounting, corporate finance, financial mathematics. This notion is highly significant because of its theoretical and applicative implications for project and firm valuation, capital budgeting decisions, performance measurement, management compensation, tax policies.[1] This paper offers a critical review of the notion of residual income. In section 2 the basic constituents (income and opportunity cost) are presented and the counterfactual features of residual income are underlined. Section 3 focusses on the standard theory of residual income: some early contributions are mentioned which connect excess profit and a project/firm's present (market) value; the formal relations among return rates, discount functions, accounting values, market values are summarized stressing the roles of Peasnell's (1981, 1982a) and Peccati's (1987, 1989) analyses; in section 4 an overview is presented of the use of this notion for valuation and for managerial purposes and the most prominent issues are underlined. Section 5 examines a most recent theory of residual income, originally labelled *Systemic Value Added* (Magni, 2000a,b,c; 2001a,b, 2003, 2004), later renamed *lost-capital paradigm* (2007a,b); relations with the standard residual income theory and with arbitrage theory are also illustrated. In section 6 some models are constructed on the basis of the two paradigms: they are classified according to the perspective employed (entity, claimholders', equity) and to the implied notions of income and capital (accounting-based, internal-rate-of-return-based, market-based). Section 7 presents a numerical illustration and section 8 ends the paper. To avoid pedantry in definitions, main notational conventions and acronyms are collected in Tables 0a-0b.

2 Residual income and its basic constituents

Income. Income, profit, earnings, interest, return: these terms are massively used in such fields as economic theory, finance, accounting, actuarial and financial mathematics. Income from the point of view of economists is referred to an individual consumer and is typically defined as the maximum which can be consumed by an individual in a determined period without impairing her wealth or capital (Hicks, 1946; see also Fetter, 1937). From the point of view of accountants income is also called profit or earnings, and is referred to the increase in a firm's assets after distributions of dividends to shareholders (Canning,

[1] Under the *Allowance-for-Corporate-Equity* system (also known as the *imputed income method*), only excess profits are taxed, whereas normal returns to capital are exempt from corporate income taxes (Boadway and Bruce, 1984; Rose and Wiswesser, 1998; Andersson et al., 1998. See also Sørensen, 1994, 1998 on the Dual Income Tax).

1929; Penman, 2007). In the theory of financial contracts (and in actuarial sciences) the notion of interest is used since ancient times to represent the remuneration of the lender (Van de Mieroop, 2005) and is computed as the difference between the installment paid by the borrower and the principal repayment (Francis, 2004; Fabozzi, 2006; Promislow, 2006; Werner and Sotskov, 2006). The notion of return in capital budgeting is referred to a project: in a one-period project return is the difference between the end-of-period payoff and the initial outlay. In security analysis, return denotes dividends plus capital gain. All these concepts are conceptually and formally equivalent and may be conjoined in a unified formal framework:

$$\overbrace{\pi_t}^{\text{economic agent's remuneration}} = \overbrace{a_t}^{\text{payment in cash}} + \overbrace{(w_t - w_{t-1})}^{\text{change in capital}}. \tag{1}$$

The fundamental equation (1) is a most general theoretical umbrella covering such terms as income, profit, earnings, return, interest, which may be viewed, conceptually and formally, as synonyms (see Table 1).[2] Two variants of eq. (1) are particularly important: an interest-rate form is

$$r_t = \frac{a_t + (w_t - w_{t-1})}{w_{t-1}} \tag{2}$$

with $r_t := \pi_t / w_{t-1}$. From the point of view of a lender, r_t is the interest rate on the debt; from the point of view of an accountant, r_t is the accounting rate of return; from the point of view of an investor, r_t is just an internal rate of return of a one-period project (because $-w_{t-1} + (a_t + w_t)/(1 + r_t) = 0$). A second variant of the fundamental equation (1) describes the evolution of the capital through time:

$$w_t = w_{t-1}(1 + r_t) - a_t. \tag{3}$$

This form stresses the role of the return rate (interest rate) as a driver of capital increase: it is usual in the construction of amortization tables, in the computation of project balances and in financial and insurance applications (Levi, 1964; Robichek and Myers, 1965; Teichroew, Robichek, Montalbano, 1965a,b; Hansen, 1972; Peccati, 1991; Promislow, 2006). The fundamental equation (1) alongside its equivalents eqs. (2) and (3) represent a general schema that links income, cash flow, capital, rate of return (see also Archer and D'Ambrosio, 1972; Hansen, 1972; Lee, 1985. See also the fundamental eq. (1') in Samuelson, 1964, p. 604). This formal framework is suited for describing any conceivable situation where a stream of cash flows is involved, be it a project, a personal saving account, a financial contract, a security, a business unit or a firm. Simple as it is formally, this schema represents a major converging force of economic theory, finance and accounting.

Opportunity cost. "You face a choice. You must now decide whether to read this [article], to read something else, to think silent thoughts, or perhaps to write a bit for yourself. The value that

[2] It is worth noting that the term "capital" derives from the medieval latin expression *capitalis pars*, which was referred to the principal sum of a money loan (Fetter, 1937, p. 5). The term *capital* thus originated in a financial context and was only later extended to include the worth of any kind of business asset or investment, referred to corporations as well as individuals (Fetter, 1937). This justifies the practice among financial mathematicians (e.g. Peccati, 1987, 1989, 1991; Pressacco and Stucchi, 1997; Magni, 2000a,b, 2001a,b, 2003) of interpreting a project (or a firm) as a loan (see also Vélez-Pareja, 2001, pp. 6-7). The loan is ideally represented by the investors' (shareholders') legal rights. In particular, capital is viewed as a residual debt: "The corporation *owes* the capital, it does not *own* it. The shareholders own it" (Fetter, 1937, p. 9); and income is viewed as interest: "the profit is equal to interest on the capital value existing at the beginning of the period" (Hansen, 1972, p. 15). The same idea is at the core of Anthony's (1975) notion of profit.

you place on the most attractive of these several alternatives is the cost you must pay if you choose to read this [article] now" (Buchanan, 1969, p.vii). When one calculates the benefit from undertaking a course of action one must take other available opportunities into account. The most valuable of these alternatives represents the cost of undertaking that action. If one says 'it is not worth the cost' one means that alternatives are available which one prefers to undertaking the action. The idea of cost as an *opportunity cost* has been developed by Austrian economists (in particular Ludwig von Mises) as well as by economists of the London School of Economics such as Hayek, Coase, Thirlby, Shackle. Conceptually, it is the result of a *counterfactual conditional*: the cost of receiving income π_t is given by the income that would have accrued to the investor if the capital had been invested in a different economic activity. Opportunity cost is an outcome that might occur (*ex ante* analysis) or that might have occurred (*ex post* analysis) if the decision maker selected or had selected a different course of action: "The cost of doing anything consists of the receipts *which could have been obtained if that particular decision had not been taken.*" (Coase, 1938, 1968, p. 118, italics added). Counterfactual conditionals are ubiquitous in daily life (Kahneman and Tversky, 1982; Wells, Taylor and Turtle, 1987; Roese and Olson, 1995), in philosophy of science (Goodman, 1947; Kneale, 1950), and are pervasive in economic thinking as well: they are tools economists often adopt to explore the world and construct their concepts and models (Sugden, 2000; Hülsmann, 2003).[3] Opportunity cost is income of a foregone opportunity; thus, it is a *counterfactual* income as opposed to the *factual* income received (or to be received) in actual facts (see Magni, 2008a, for a counterfactual analysis of RI and empirical testing).

Residual income. Combining income and opportunity cost means contrasting the factual course of action with the counterfactual course of action:

Factual course of action *versus* Counterfactual course of action	$\Longrightarrow$ Residual Income $\Longleftarrow$	Income *versus* Opportunity cost

Mathematically, residual income is a measure of how factual income exceed counterfactual income, that is, how income exceeds opportunity cost. We have then the following:

Definition 1. Residual income is income in excess of opportunity cost:

$$\text{Residual Income} = \text{Income} - \text{Opportunity cost}. \tag{4}$$

Unanimously in the literature, the foregone profit (opportunity cost) is calculated as the product of the alternative return rate and the capital at the beginning of the period ($=i \cdot w_{t-1}$), so that eq. (4) is formalized as

$$\pi_t^{\mathrm{e}} = \pi_t - i \cdot w_{t-1}. \tag{5}$$

The rate i is often called the *opportunity cost of capital*[4] and may be found as a subjectively determined hurdle rate or, if a perfect capital market is assumed, as the return rate of an alternative comparable in

[3]See also Lundberg and Frost (1992) for the use of counterfactuals by individuals in financial decision-making.

[4]The terminology is unfortunate, given that 'opportunity cost' means counterfactual income whereas 'opportunity cost of capital' means counterfactual rate of return.

risk to the asset under consideration. Residual income is therefore income that residues after covering the interest charge on capital, which has the nature of a foregone profit. Such a foregone profit acts as a benchmark, a *norm* in the sense of Kahneman and Miller (1986). The counterfactual profit is a *normal* profit (e.g. Edey, 1957; Bodenhorn, 1964; Carsberg, 1966; Archer and D'Ambrosio, 1972; Begg, Fisher and Dornbusch, 1984). Across the years, a plethora of terms have been attached to the idea of a profit in excess of some *normal* profit (see Table 2); this paper limits the synonyms to the expressions "residual income" and "excess profit".[5]

3 The standard theory

3.1 The early years

The concept of excess profit may be traced back to Marshall (1890), presumably inspired by Hamilton (1777), who clearly underlines the counterfactual feature of opportunity cost: "excess of gross profits above the interest of his stock ... if the profit of his trade be less than his stock *would have yielded at common interest*, he may properly account it a losing one" (Hamilton, 1777, vol. II, p. 246, as quoted in Arnold, 2000, p. 14; italics added. Also quoted in Mepham, 1980, p. 183). Since the last years of the nineteenth century, this concept was used for valuation purposes: Carsberg (1966) testifies of discounting procedures involving excess profits rather than cash flows; among others (e.g. Dicksee, 1897), the author emphasizes Leake's (1921) contribution to valuation of *Goodwill* (NPV), obtained by discounting the surplus of profit over a *normal* return on capital. The idea of a reasonable, fair return was well accepted in those years in professional practice: Sloan's (1929) "fair and equitable" return is just a *normal* profit (see Goetzmann and Garstka, 1999). In later years, Preinreich (1936) hints at the equivalence between the DCF method and the use of excess earnings to find the NPV: "Goodwill is commonly obtained by discounting 'excess earnings'. If the original investment (C) is added to the goodwill the same capital value results as from the discounting of 'services' [cash flows]" (p. 131). The link between value and excess profits is renewed in Preinreich (1937), where the author writes that "the discounted excess profits plus the recorded value will always give the true fair market value . . . This statement is a simple theorem of arithmetic" (p. 220). In Preinreich (1938) the author rephrases Hotelling's formula of the capital value of a single machine to claim again that "capital value equals the book value, plus the discounted excess profits" (p. 240). The formal link between DCF valuation and residual income is made more explicit by Edey (1957). The author assumes a constant perpetual cash flow $a_t{=}a$, which implies zero change in capital ($w_t{=}w_{t-1}{=}w$) so that $a_t{=}\pi_t{=}\pi$ (see eq. (1)); after reminding that, for a perpetuity, the present value is $v_0{=}\sum_{t=1}^{\infty}\pi(1+i)^{-t}{=}\frac{\pi}{i}$, he shows that the same value may be obtained by capitalizing the super-profits $\pi^{\mathrm{e}}_t{=}\pi^{\mathrm{e}}$ and then adding the value of the firm's net tangible assets:

$$w+\sum_{t=1}^{\infty}\pi^{\mathrm{e}}(1+i)^{-t}=w+\frac{\pi^{\mathrm{e}}}{i}=w+\frac{\pi-iw}{i}=\frac{\pi}{i}=v_0.$$

[5]The expression "residual income" is first used in Solomons (1965), who credits General Electric with coining the term (see also Anthony, 1975, p. 63).

The analogous result in a finite-time setting is found by Edwards and Bell (1961): in Appendix B of their book on business income, the authors use the fundamental equation eq. (1) and define the *excess realizable profit* as $\pi_t^{\rm e}{=}w_t + a_t - (1+i)w_{t-1}$, where the capital w_t is valued on the basis of replacement cost. The authors compute the present value of the stream of excess realizable profits $\sum_{t=1}^{n} \frac{w_t+a_t-(1+i)w_{t-1}}{(1+i)^t}$, and after simple algebraic manipulations they show that such a present value equals the NPV of the expected stream of receipts ('subjective goodwill' in the authors' words): $N_0 = v_0 - a_0 = \sum_{t=1}^{n} a_t(1+i)^{-t} - a_0 = \sum_{t=1}^{n} \pi_t^{\rm e}(1+i)^{-t}$ (see also Lücke, 1955). Analogously, Bodenhorn (1964) defines residual income as *pure earnings* and shows that the sum of their present values is equal to the sum of the present values of the net cash flows (p. 27, footnote 19). In addition, he acknowledges that the equivalence "is independent of the depreciation pattern" (p. 29); that is, the equivalence is independent of the sequence $\{w_t\}$ of outstanding capitals.

Notwithstanding these various scattered contributions, only in recent years a full disclosure of the relations among income, present value, accounting value, rate of return and excess profit has been accomplished and extensive use of residual income has been made in both academic fields and real-life applications. In particular, in accounting, Peasnell (1981, 1982a) thoroughly investigates the relations between accounting numbers and market values; in financial mathematics, Peccati (1987, 1989) decomposes the NPV of a project in period margins and provides an inner decomposition of $\pi_t^{\rm e}$ in terms of sources of funds raised to finance the project.

3.2 Peasnell and Peccati

Suppose a firm is incorporated to undertake an n-period project, which costs $a_0{>}0$ and pays off periodic cash flows $a_t{\in}\mathbb{R}$, $t = 1, 2, \ldots, n$. The cash-flow stream for the capital providers may be written in vectorial form as $\vec{a} = (-a_0, a_1, a_2, \ldots, a_n)$. The project's (firm's) net present value is $N_0{=}\sum_{t=1}^{n} \varphi_t(\vec{\imath})a_t - a_0$, where $\vec{\imath}{:=}(i_1, i_2, \ldots, i_t){\in}\ \mathbb{R}^t$, $t = 1, 2, \ldots, n$, is the vector of period costs of capital, and $\varphi_t(\vec{\imath}){:=}[\prod_{k=1}^{t}(1+i_k)]^{-1}$ is the corresponding discount factor; by definition, $\varphi_0(\vec{\imath}){:=}1$.

Peasnell (1981, 1982a) assumes that the capital w_t is the accounting book value of the firm's assets b_t, and that the fundamental eq. (1) (known in accounting as *clean surplus relation*) holds for all periods. As for time n, the author distinguishes cash flow from operations from project's scrap value. Let R_n be the scrap value and a_{n^*} be cash flow from operations, with n^* denoting time n *after* distribution of a_{n^*} but *before* distribution of R_n. The comprehensive last cash flow is such that $a_n{=}a_{n^*}{+}R_n$. The clean surplus relation is assumed to hold for all $t{=}1, 2, \ldots, n^*$ so that, in particular, $a_{n^*}{=}\pi_n + b_{n-1} - b_{n^*}$. From these assumptions, Peasnell shows that the firm's NPV is equal to the discounted sum of accounting-based excess profits plus the difference of discounted accounting error in capital valuation:

$$
\begin{aligned}
N_0 &= \sum_{t=1}^{n} \varphi_t(\vec{\imath})a_t - a_0 = \sum_{t=1}^{n-1} \varphi_t(\vec{\imath})a_t + \varphi_n(\vec{\imath})(a_{n^*} + R_n) - a_0 \\
&= \sum_{t=1}^{n} \varphi_t(\vec{\imath})\pi_t + \sum_{t=1}^{n} \varphi_t(\vec{\imath})b_{t-1} - \sum_{t=1}^{n-1} \varphi_t(\vec{\imath})b_t + \varphi_n(\vec{\imath})R_n - a_0 - \varphi_n(\vec{\imath})b_{n^*} \\
&= \sum_{t=1}^{n} \varphi_t(\vec{\imath})\pi_t + \sum_{t=1}^{n} \varphi_t(\vec{\imath})b_{t-1} - \sum_{t=0}^{n-1} \varphi_t(\vec{\imath})b_t + \varphi_n(\vec{\imath})(R_n - b_{n^*}) - (a_0 - b_0) \\
&= \sum_{t=1}^{n} \varphi_t(\vec{\imath})\pi_t + \sum_{t=1}^{n} (\varphi_t(\vec{\imath}) - \varphi_{t-1}(\vec{\imath}))b_{t-1} + \varphi_n(\vec{\imath})(R_n - b_{n^*}) - (a_0 - b_0).
\end{aligned}
$$

Reminding that $\varphi_t(\vec{\imath}){-}\varphi_{t-1}(\vec{\imath}){=}i_t\varphi_{t-1}(\vec{\imath})$,

$$
\begin{aligned}
N_0 &= \sum_{t=1}^{n} \varphi_t(\vec{\imath})\pi_t - \sum_{t=1}^{n} i_t\varphi_t(\vec{\imath})b_{t-1} + [\varphi_n(\vec{\imath})(R_n - b_{n^*}) - (a_0 - b_0)] \\
&= \sum_{t=1}^{n} \varphi_t(\vec{\imath})(\pi_t - i_t b_{t-1}) + [\varphi_n(\vec{\imath})(R_n - b_{n^*}) - (a_0 - b_0)]
\end{aligned}
$$

(Peasnell, 1982a, p. 364). If, in addition, one assumes that the opening book capital is valued at outlay (i.e. $b_0 = a_0$) and the closing book capital is written down to scrap value (i.e. $b_n^* = R_n$), as it is usual in capital budgeting, accounting valuation errors disappear[6] and net present value is equal to the discounted value of accounting-based excess profits:

$$
N_0 = \sum_{t=1}^{n} \varphi_t(\vec{\imath})a_t - a_0 = \sum_{t=1}^{n} \varphi_t(\vec{\imath})(\pi_t - i_t b_{t-1}) \tag{6}
$$

(Peasnell, 1981, pp. 53-54). As already noted by Bodenhorn (1964), Peasnell himself notes that this NPV-consistency (aka *conservation property*) is independent of the accounting system used for valuing b_t.

In financial mathematics, Peccati (1987, 1989, 1991) proposes a method of decomposing the NPV of a project. To this end, he splits up the project in n one-period subprojects. Each of the subprojects starts at time $t{-}1$ with capital invested w_{t-1} and terminates with end-of-period cash flow a_t plus terminal value w_t (see also Gronchi, 1984, and Manca, 1989, on the splitting up of cash-flow streams). Formally, the cash-flow vector of each subproject is $\vec{a}_t = (\vec{0}_{t-2}, -w_{t-1}, w_t + a_t, \vec{0}_{n-t})$, $t = 1, 2, \ldots, n$, where $\vec{0}_k$ is the null vector in $\mathbb{R}^k$. Note that $\sum_{t=1}^{n} \vec{a}_t = \vec{a}$, that is, the project is equivalent to a portfolio of n one-period assets, where the opening capital of each asset equals the closing capital of the preceding one. Peccati sets the following boundary conditions: $w_0{=}a_0$ (the capital invested in the first period is equal to project A's outlay) and $w_n{=}0$ (the terminal capital, after the liquidating cash flow a_n has been paid to the investor, is zero). The author rests on the fundamental equations (2)-(3) and highlights the univocal correspondence between the outstanding capitals w_t and the internal rates of return r_t: once the values for w_t (respectively, r_t) are *arbitrarily* chosen, the internal return rates r_t (respectively, the outstanding

[6] A less stringent condition is that valuation errors offset each other: $\varphi_n(\vec{\imath})[R_n - b_{n^*}]{=}(a_0 - b_0)$.

capitals w_t) are univocally determined. The net value of each asset is

$$N_0(\vec{a}_t) = -\varphi_{t-1}(\vec{\imath})w_{t-1} + \varphi_t(\vec{\imath})(w_t + a_t).$$

The net value of the portfolio is given by the sum of the values of the constituents assets, which coincides with the project's NPV:

$$\sum_{t=1}^{n} N_0(\vec{a}_t) = \sum_{t=1}^{n} -\varphi_{t-1}(\vec{\imath})w_{t-1} + \sum_{t=1}^{n} \varphi_t(\vec{\imath})(w_t + a_t) = \sum_{t=1}^{n} \varphi_t(\vec{\imath})a_t - a_0 = N_0(\vec{a}). \tag{7}$$

Each asset's net value $N_0(\vec{a}_t)$ is interpretable as the portion of project A's NPV generated in the t-th period. Using eq. (3), Peccati reshapes the periodic quota in a different form:

$$-\varphi_{t-1}(\vec{\imath})\, w_{t-1} + \varphi_{t-1}(\vec{\imath})\, (w_t + a_t) = \varphi_{t-1}(\vec{\imath})\, w_{t-1}(r_t - i_t)$$

which expresses the spread between internal rate of return and cost of capital multiplied by the capital invested in the t-th period. The final form of the decomposition becomes

$$N_0 = \sum_{t=1}^{n} \varphi_{t-1}(\vec{\imath})\, w_{t-1}(r_t - i_t). \tag{8}$$

Owing to the fundamental schema (1)-(3), the above expression becomes $N_0 = \sum_{t=1}^{n} \varphi_{t-1}(\vec{\imath})(\pi_t - i \cdot w_{t-1})$, which resembles Peasnell's eq. (6).

Assumptions. Peasnell's and Peccati's analyses are equivalent but rooted in different traditions. The former author is concerned with accounting values and incomes, the latter is interested in finding a general mathematical framework for decomposing a net present value in periodic quotas. Peasnell makes use of assumptions on the accounting of the project to reach a perfect decomposition of NPV with residual incomes: (i) the clean surplus relation holds and (ii) no accounting valuation errors arise; Peccati does not rest on any particular assumption: he only rests on the standard notion of internal rate of return (of which eq. (2) is a particular case) and the two boundary conditions $w_0 = a_0$ and $w_n = 0$ for the resulting dynamic system represented by eq. (3). These conditions are financially *natural*: as a financial mathematician, Peccati (1991, p. 25) exploits the metaphor "project=loan", so that w_t may be interpreted as the residual debt the firm owes the investors (see footnote 2). The residual debt follows the recursive eq. (3) and the boundary conditions are then obvious: $w_0 = a_0$ says that the residual debt at time 0 is equal to the amount borrowed by the firm from the investors, and $w_n = 0$ is just the usual closing condition of an amortization plan: after the last "installment" a_n has been paid, borrowing and lending sides have no pending amounts left (this assumption is taken by Samuelson, 1964, as well, in his eq. (1'), p. 604). Peasnell's approach does comply with the terminal condition $w_n = 0$ as well, though in an implicit way: as seen, the author does apply the fundamental equation (1) at time n, but *prior* to the distribution of R_n; however, *after* distribution of R_n, the terminal capital invested is necessarily zero: $b_n = b_{n^*} - R_n = 0$.

Internal Financial Law. The role of one-period IRRs in Peccati's analysis is of paramount importance and economically significant. In particular, Peccati uses the notion of *Internal Financial Law* (IFL), which was previously introduced by Weingartner (1966) as a generalization of the IRR (with the label *internal return vector*).[7] Letting $\vec{r}$:=$(r_1, r_2, \ldots r_t) \in \mathbb{R}^t$, $t = 1, 2, \ldots, n$, the IFL determines a discount function $\varphi_t(\vec{r})$:=$[\prod_{k=1}^{t}(1+r_k)]^{-1}$ which is solution to the following equation:

$$-a_0 + \sum_{t=1}^{n} \varphi_t(\vec{r}) a_t = 0. \tag{9}$$

It is worth noting that the above relation is mathematically deduced from iteration of eq. (3) alongside the equalities w_0=a_0 and w_n=0. This implies that the notion of IFL is just a logical consequence of the definition of income and the *natural* boundary conditions; this makes the notion of IFL economically meaningful. If accounting income is assumed (so that w_t=b_t), then the resulting IFL turns out to be the sequence of accounting rates. This sequence has been extensively studied in accounting. For example, Kay (1976), focussing on a continuous setting, shows that "Every sequence of accounting rates of return defines a valuation function under which the present value of the cash flows of the project is zero" (p. 90). This result is found again in Peasnell (1982a, p. 367) for discrete-time projects (see also Peasnell, 1982b; Franks and Hodges, 1984; Brief and Lawson, 1992; Brief, 1999; Feenstra and Wang, 2000). If r_t=r for all t, then the IFL collapses into the IRR and the latter may be written as a weighted average: replacing each i_t with the internal rate r in eq. (8) one gets N_0=$\sum_{t=1}^{n} w_{t-1}(r_t - r)(1+r)^{-t} = 0$ by definition of IRR, whence

$$r = \frac{\sum_{t=1}^{n} r_t \cdot w_{t-1}(1+r)^{-t}}{\sum_{t=1}^{n} w_{t-1}(1+r)^{-t}} \tag{10}$$

(Peasnell, 1982a, Theorem 3; Franks and Hodges, 1984, p. 131; Brief, 1999, p. 3). However, the result suffers from circularity. Peccati (1989, 1991) does not assume existence of IRR and uses the definition of mean given by Chisini[8] to find the project's average yield: he replaces each r_t with a constant r^* and imposes equal NPVs: $\sum_{t=1}^{n} w_{t-1}(r_t - i_t)\varphi_t(\vec{\imath})$=$\sum_{t=1}^{n} w_{t-1}(r^* - i_t)\varphi_t(\vec{\imath})$, whence

$$r^* = \frac{\sum_{t=1}^{n} r_t \cdot \varphi_t(\vec{\imath}) w_{t-1}}{\sum_{t=1}^{n} \varphi_t(\vec{\imath}) w_{t-1}} \tag{11}$$

which differs, in general, from the IRR (if it exists). As a particular case, picking i_t=r_t one finds

$$r^* = \frac{\sum_{t=1}^{n} r_t \cdot \varphi_t(\vec{r}) w_{t-1}}{\sum_{t=1}^{n} \varphi_t(\vec{r}) w_{t-1}} \tag{12}$$

which, contrary to eq. (10), is not circular. It is worth noting that the sequence $\{w_t\}$ is univocally determined by the sequence $\{r_t\}$, not by the internal rate r, and that eq. (12) does not even depend on costs of capital, but only on one-period rates. Generalizing, Peccati finds the average yield of a portfolio of N projects, so that the average return rate of the portfolio is $r^* = \frac{\sum_{j=1}^{N}\sum_{t=1}^{n} r_{tj} \cdot \varphi_t(\vec{\imath}) w_{t-1,j}}{\sum_{j=1}^{N}\sum_{t=1}^{n} \varphi_t(\vec{\imath}) w_{t-1,j}}$ (Peccati, 1989, p. 164; 1991, p. 53), where r_{tj} is the one-period rate of the j-th project and $w_{t-1,j}$ is the corresponding capital invested. With a similar argument, considering a portfolio of projects undertaken in different countries, Peccati (1998) shows that the spreads between the IRR of each investment and the opportunity cost of capital (adjusted to take account of the currencies) may be replaced by an average spread which is the harmonic mean of the various spreads with weights the projects' NPVs.

[7] See Gronchi (1984) and References therein, for an exhaustive historical survey and a thorough theoretical analysis of the notion of internal rate of return.

[8] A function f of n variables x_i leads to a Chisini mean if and only if there exists a unique M such that $f(M, M, ..., M)$=$f(x_1, x_2, ..., x_n)$ (Chisini, 1929. See also de Finetti, 2008).

Given that the IFL is derived from recurrence equation (3), any sequence of IFL is such that the resulting one-period rate r_t has a genuine financial meaning: it represents the rate of capital increase or, using the project=loan metaphor, it is the interest rate on the residual debt. Surprisingly, accounting scholars often grant the IRR a privileged status as opposed to the sequence of accounting rates of return: "it is difficult to assign economic significance to accounting yield except ... as surrogate measures of IRR" (Peasnell, 1982a, p. 380. But see Brief and Lawson, 1992, about the prominent role of accounting rates for valuation). However, to use the IRR boils down to assuming a constant rate of capital increase (constant interest rate on the "loan"). This may be the case when the investment is indeed a loan contract with constant interest rate, or when it is a fixed-income security. In general, the profitability of most projects is not uniformly distributed in time and "the classical troublesome problem of non existence or of multiplicity of IRR arises from the basic and historical error consisting in the aim to describe through a *unique* parameter what happens in *quite different* time periods" (Peccati, 1989, p. 158). Thus, to introduce the IFL as a generalization of the notion of IRR "is not a deficiency of the approach. It simply gives some degrees of freedom in the choice of parameters" (Peccati, 1989, p. 159). And if an aggregate measure of profitability is required, the rates in eqs. (11)-(12) may be employed.

The standard RI approach is generalized by Peccati (1989, 1991), who decomposes the RI itself into equity component and debt component. The line of reasoning is similar to the unlevered case: the author ideally splits up the financing in n one-period sub-financings, so that each sub-project is financed by a sub-financing. Denoting with d_t the installment at time t, the author uses again the fundamental equations (2)-(3) for the financing, so that $D_t + d_t{=}D_{t-1}(1+\delta)$ is the total payment at time t which extinguishes the t-th sub-financing and $\delta_t = (D_t + d_t - D_{t-1})/D_{t-1}$ is the corresponding contractual rate. With respect to the unlevered case, the one-period project's NPV is affected by the amount $D_{t-1}(\delta_t - i_t)$, which is the opportunity cost of financing with debt rather than with equity: $\delta_t D_{t-1}$ is the *factual* interest charge to be paid to debtholders, whereas $i_t D_{t-1}$ is the *counterfactual* interest charge that would be required if the same amount were borrowed from equityholders. The additional element may be either positive or negative, depending on the sign of $(\delta_t - i_t)$. The residual income thus becomes $\pi^{\rm e}_t = w_{t-1}(r_t - i_t) - D_{t-1}(\delta_t - i_t)$. Upon manipulating,

$$\pi^{\rm e}_t = \overbrace{(w_{t-1} - D_{t-1})(r_t - i_t)}^{\text{equity component}} + \overbrace{D_{t-1}(r_t - \delta_t)}^{\text{debt component}}. \tag{13}$$

The first addend is the excess profit generated by equity, the second addend is the excess profit contributed by debt. By using the usual fundamental equation for both the project and the debt, it is easy to show that $\sum_{t=1}^{n} \varphi_t(\vec{\imath})\, \pi^{\rm e}_t = N_0$. Peccati's twofold decomposition lends itself to useful analyses and applications in business, industry, insurance and financial markets (Luciano, 1989; Peccati, 1991; Marena, 1991a,b; Uberti, 1993; Gallo and Peccati, 1993; Magni, 1993; Camillo and Marena, 1994).[9]

[9]It is worth noting that Peccati's analysis does not assume the existence of a Modigliani-Miller world, so that the opportunity cost of capital is subjectively determined and no arbitrage theory is invoked to determine the relation between levered and unlevered project. For this reason, the opportunity cost of capital is invariant under changes in the leverage ratio. In Peccati's analysis, uncertainty is managed either by simulation analysis (Gallo and Peccati, 1993) or by rigorous application of probability theory (Marena, 1991b; Beccacece and Li Calzi, 1991; Luciano and Peccati, 1993).

3.3 The set and its elements

The above described theoretical framework may be actually seen as a basket where many infinite metrics may be fleshed out depending on a particular notion of income and capital. The set of all possible metrics is

$$\Psi = \left\{ \pi_t^{\mathrm{e}} \in \mathbb{R} \quad \text{such that} \quad \pi_t^{\mathrm{e}}(\vec{r}, i_t) = \pi_t(\vec{r}) - \pi_t(\vec{r}, i_t) \right\} \tag{14}$$

where $\pi_t(\vec{r}) = r_t w_{t-1}(\vec{r})$ is the factual profit, $\pi_t(\vec{r}, i_t) = i_t w_{t-1}(\vec{r})$ is the counterfactual profit, with $w_t(\vec{r}){:=}w_t(r_1, \ldots, r_t)$. Since the early 1990s, this set is increasingly exploited by professionals and consulting groups to devise appropriate measures of value creation. The *Economic Value Added* (Stewart, 1991), an accounting-based RI, is popularized by Stern Stewart & Co. and its proponents underline that this measure is helpful for asset valuation, financial analysis, periodic performance assessment and executive compensation. As for the latter, a common compensation plan is based on a bonus bank system which makes the bonus earned by the manager equal to the sum of a target bonus plus a fixed percentage of excess EVA improvement. Such a bonus is credited to a bonus "bank" and the balance of the bonus bank determines the bonus paid (Martin and Petty, 2000; Young and O'Byrne, 2001; Martin, Petty and Rich, 2003). Beside EVA, a multitude of metrics have been put forward in the last twenty years (Stewart, 1991), among which the Oil&Gas Adjusted EVA purported by McCormack and Vytheeswaran (1998); the so-called Edwards-Bell-Ohlson model, (Edwards and Bell, 1961; Ohlson, 1995); the residual income based on the cash flow return on investment (Madden, 1999); Fernández's (2002) Created Shareholder Value (see also Fabozzi and Grant, 2000).

4 Valuation, decision, and management

Valuation. The theoretical equivalence of the RI-based metrics and the NPV (the above mentioned *conservation property*), is well-established and often reproposed in the literature (Martin and Petty, 2000; Lundholm and O'Keefe, 2001; Fernández, 2002; Martin, Petty, and Rich, 2003; Vélez-Pareja and Tham, 2003), and it has been shown to be valid for portfolio of projects as well (Peccati, 1991; see also Reichelstein, 1997). However, the implementation of the RI-based metrics in real-life applications often result in valuations not consistent with the cash-flow-based approach. In their paper, Lundholm and O'Keefe (2001) show that deceptively simple errors may be committed in the practical implementation of the RI models, which result in different value estimates from the cash-flow based models. Lundholm and O'Keefe unmask some subtle internal inconsistencies that often bias real-life applications. Their paper may be considered as an informed guide to avoid three relevant mistakes: (a) inconsistent forecast errors: this error occurs when the starting value from the terminal value perpetuity is incorrect, (b) inconsistent discount rate error: this error occurs when the cost of equity as derived from the equity-cash-flow model differs from the cost of equity implied in the weighted average cost of capital, for example because book values or target values are used instead of market values (see also Fernández, 2002; Cigola and Peccati, 2005), (c) missing cash flows error: this may arise, for example, when the income schema in eq. (1) is not complied with by the financial statement forecasts. Important theoretical advances have been made by O'Hanlon and Peasnell (2002), who provide splitting identities to distinguish realization of value and generation of value through the notion of *unrecovered capital*.[10] Ohlson (1989, 1995) shows that, under assumption of a determined stochastic process for excess profit, total incomes multiplied by an appropriate discount factor approach market value in the long run, which reflects what Penman calls the "aggregation property of accounting" (Penman, 1992, p. 237). The use of *variation* of RI and its relations to value is the focus of O'Byrne's (1996, 1997) investigations, who introduces the notion of excess of EVA improvement (change in EVA minus expected change in EVA), based on Miller and Modigliani's (1961) investment opportunities approach. This very notion, relabelled *Abnormal Earnings Growth*, is later reproposed and studied by Ohlson (2005) and Ohlson and Juettner-Nauroth (2005) (an early anticipation of the concept can be found in Bodenhorn, 1959).

RI maximization and NPV maximization. Given the conservation property of residual income, NPV maximization is equivalent to residual income maximization over the entire life of the project. Pfeiffer (2000) shows that investment decisions with cash-flow-based performance measures are not consistent with NPV maximization (see also Baldenius and Reichelstein, 2005) and Pfeiffer (2004) shows that the class of all NPV-consistent criteria generated by accounting measures and independent of the choice of capital w_t coincide with the set Ψ (up to a linear transformation).[11] Anctil (1996) and Anctil, Jordan and Mukherji (1998a) deal with the case where investment decisions are delegated from the principal (equity's owner) to the agent (manager). While in general RI maximization in a period is not equivalent to NPV maximization, the authors find appropriate assumptions under which even if the manager myopically maximizes residual income ignoring both future residual incomes and future cash flows, the resulting policy will lead, asymptotically, to NPV maximization: the sequence of investment decisions made by manager has the same limit as the NPV-maximizing sequence. This result is particularly important in those cases where the decentralization of cost and benefit information renders the NPV maximization problem unsolvable and evidences that the NPV maximization may be replaced by RI maximization as a useful simplification of the decision process (see previous related results by Tomkins, 1975; Emmanuel and Otley, 1976; Scapens, 1978, 1979). Anctil, Jordan and Mukherji (1998b) also show that an activity-based cost system support RI maximization. Most recently, a discussion in the literature concerns capacity investments: capacity investments maximizing residual income are not optimal according to the NPV rule; however, it may be shown that an appropriate choice can be found for the allocation rule which leads to a situation in which the average historical cost is equal to the long run marginal cost under the assumption of overlapping investments;[12] therefore, capacity investments which maximize residual income of a specific period are exactly those that maximize net present value. If this allocation rule is used, the joined costs of these investments are linearly separable over time, which leads to the above mentioned result (see Rajan and Reichelstein, 2008; Rogerson, 2008).

Investment decisions. Since Solomons's (1965) classical book, the notion of residual income has

[10]Schueler (2000) and Drukarczyk and Schueler (2000) label it "invested capital", given that it is equal to the difference between market value and NPV (see Schueler, 2001, eq.(1); Magni, 2007a, Proposition 3). Vélez-Pareja (2001) use the label "Initial investment not recovered" for the same notion. Young and O'Byrne's (2001) *Adjusted Invested Capital* turns out to be an equivalent notion, if income=cash flows is assumed (see Magni, 2007a).

[11]Focussing on one-period investments, Magni (2007d, 2008b) shows that the use of the CAPM for computing the cost of capital makes RI and NPV nonequivalent (see also Magni, 2009, on the use of CAPM and NPV for capital budgeting).

[12]An allocation rule is a one-one correspondence with the outstanding capital w_t and with the IFL. Letting β denote an allocation rule, $\beta_t{=}(w_{t-1}(\vec{r}) - w_t(\vec{r}) + i_t w_{t-1}(\vec{r}))/a_0$, so that RI in Ψ is written as $\pi^e_t = a_t - \beta_t a_0$.

been often advocated; in the 1970s a lively debate took place in management accounting in favour or against the use of residual income for divisional and managerial performance measurement (Flower, 1971; Bromwich 1973; Tomkins, 1975; Amey, 1975; Emmanuel and Otley, 1976). The notion of value-based management (VBM) gradually arose to refer to a managerial approach based on the assumption that the primary purpose is the long-term shareholders' wealth maximization (Arnold and Davies, 2000; Young and O'Byrne, 2001). In a certain sense, VBM is just "net present value analysis or internal rate return analysis ... writ large and applied to strategies, business units, product lines, and so on" (Arnold, 2000, p. 21). Despite conservation property, which holds irrespective of the choice of the sequence $\{w_t\}$, the sign of residual incomes differs, in general, from the the sign of the NPV (Flower, 1971; Bromwich, 1973; Bromwich and Walker, 1998; Drukarczyk and Schueler, 2000; Martin, Petty and Rich, 2003). That is, *strong goal congruence* is not preserved. In this context, a proliferation of recent contributions deal with construction of strong goal congruent measures. Particularly significant is Rogerson's (1997) paper, which copes with investment decisions in decentralized organizations: the principal delegates decisions on investment level to the agent who is better informed about the investment opportunities. The agent is assumed to be "impatient": he has a shorter time horizon and/or uses a higher discount rate than the principal. The principal aims at maximizing the expected NPV and the agent aims at maximizing a utility function which depends on RI via a reward contract that linearly links RI to wages. Assuming positive operating cash flows governed by a specified stochastic path, of which only the distributional parameters are known to the principal, the author shows that there is a unique allocation rule (and thus a unique sequence of $\{w_t\}$), called the "Relative Marginal Benefit" rule, which is optimal in the sense that it maximizes both the principal's expected NPV and the manager's utility function. Thus, the author finds the only RI metric that, under convenient assumptions, guarantees strong goal congruence and constitutes an effective incentive for manager's optimal behavior. It is worth noting that Rogerson's metric is exactly equal to Grinyer's (1985, 1987, 1995) *Earned Economic Income*. Reichelstein (1997) shows that the RI in combination with Relative Marginal Benefit allocation rule is the unique *linear* performance metric that achieves strong goal congruence in this context (see also Bromwich and Walker, 1998). Under the same information structure of Rogerson (1997) and Reichelstein (1997), Mohnen (2003) and Mohnen and Bareket (2007) show that the Relative Marginal Benefit allocation rule is not optimal if exogenous capital constraints (or mutually exclusive projects) are introduced in the decision problem. Without capital constraints, the equity owners' aim is to undertake all projects with an expected positive NPV; if capital constrains are present, the goal is to undertake the highest-NPV portfolio of projects satisfying the constraint, a property which is named *perfect* goal congruence (Mohnen, 2003) or *robust* goal congruence (Dutta and Reichelstein, 2005) and is achieved if the residual-income measure is a (positive) multiple of the NPV, where the proportionality constant is independent of the project (e.g. Mohnen, 2003, Lemma 1; Mohnen and Bareket, 2007, Lemma 1; Pfeiffer and Velthius, 2008, Corollary 17).[13] If the manager is impatient he will tend to undertake, among positive NPV projects, the one which has the quickest (expected) return. Mohnen and Bareket (2007) consider a performance measure of the form $\alpha_t a_t - \beta_t a_0$ and show how α_t and β_t must be chosen so as to induce the agent to optimally select a portfolio of

[13]The problem of finding a goal congruent measure may be interpreted in Peccati's terms as the search for a decomposition of the project's NPV such that the period margin is a multiple of the NPV itself.

projects, whereby the NPV is maximized and robust goal congruence is achieved.[14] Baldenius, Dutta, and Reichelstein (2006) deal with the case of optimal project selection in presence of several divisional managers; Grinyer and Walker (1990) and Stark (2000) focus on real-option frameworks and find that a residual income-type performance measure can be designed which supports optimal investment and disinvestment decisions (see also Friedl, 2005). Schultze and Weiler (2008) deal with a context where the manager communicates the principal the future value of the project. The authors introduce the notion of *Residual Economic Income*,[15] based on O'Hanlon and Peasnell's (2002) *Excess Value Created*, to design a bonus bank system according to which the manager is rewarded on the basis of both past realized value and value generated by future residual incomes. Their system induces optimal investment even if the impatient manager leaves the firm before completion of the project, provided an internal market is created where the quitting manager sells the bonus bank to the entering manager: they show that if the purchase price for the bonus bank is computed with the Nash (1950) bargaining solution, the quitting manager will choose the optimal investment level and will have no incentive to overstate value creation in his reporting.

Operations management. While the focus on investment decisions is predominant in the literature, recent contributions have dealt with several different kinds of decisions. As regards operations management, a significant contribution is Baldenius and Reichelstein (2005), where the authors examine efficient inventory management from an incentive and control perspective: the firm delegates decision-making to a manager who has superior information and affects sales revenues with his productive efforts. They propose to value inventory with a compounded historical cost valuation rule that capitalizes production costs and periodic holding costs and, in addition, treats inventory as an interest-accruing asset (i.e. the value of each unit remaining in ending inventory in a given period increases at the cost of capital i). The authors assume: (i) the manager's objective is to maximize the (expected) NPV of bonus payments, which are proportional to RI, (ii) the optimal sales exceed the available production capacity in each period of the inventory cycle, (iii) the LIFO (last-in-first-out) inventory flow valuation rule is employed. This implies $w_t = \big[\mathfrak{c}(1+i)^{t-t^*} + \sum_{k=0}^{t-t^*}(1+i)^k\big]x_t$, with $\mathfrak{c}$=unit production cost, x_t=ending inventory, t^*= beginning of inventory buildup. The authors show that the optimal production and sales plan that maximizes the firm's NPV is also the one that maximizes the NPV of manager's bonus payments. In case the manager receives updated information about future revenues after the initial production decision the residual income based on the lower-of-cost-or-market rule becomes the optimal incentive mechanism (see also Dutta and Zhang, 2002, on production incentives). A goal congruence approach is also followed by Dutta and Reichelstein (2005) which analyze several different transactions: multi-year construction contracts, long-term leases, asset disposals, research and development (see also Pfeiffer and Schneider, 2007). Stoughton and Zechner (2007) consider capital allocation based on RI in financial institutions (e.g. banks) assuming frictions in the markets and focussing on an institution composed of a risky and a riskless division.[16]

[14]The importance of this strand of literature for practical applications is indirectly evidenced by Balachandran (2006), which provides support that "RI affects real management actions, a necessary condition for assessing the optimality of those actions" (p. 393).

[15]See the analogous notion of *Net Value Created* in Schueler and Krotter (2008).

[16]The reader may also benefit from the overview in Schultze and Weiler (2008) on these topics.

Portfolio management. Residual income structure may also be used for portfolio optimization. In a portfolio, it is essential to rest on reliable estimates of return parameters. The use of RI for extracting implied expected returns from analysts' forecasts is recent: Frankel and Lee (1998) use a three-period version of the RI model based on analysts' forecasts to estimate an intrinsic value measure for firms (see also Lee, 1999). Claus and Thomas (2001) use the approach for forecasting the equity premium, a fundamental variable in portfolio management because it is a component of the cost of capital. The authors argue that the use of RI is superior compared to the dividend growth model and estimate the equity premium for six countries, whose robustness is corroborated by sensitivity analyses. Hagemeister and Kempf (2007) use expected returns (rather than the usual realized returns) implicit in the RI model to test different versions of the Capital Asset Pricing Model. In another context, Hagemeister and Kempf (2006) use the expected returns implied by the RI model for Markowitz-optimization. They optimally combine the RI-based estimator with the time series estimator using the Bayesian approach and find that such a combination results in a better performance when compared to traditional estimation and investment strategies (see also Daske, Gebhardt, and Klein, 2006). Barniv and Myring (2006) contrast two empirical models for assessing the explanatory power for security prices in seventeen countries. The historical model makes price depend on historical book value and earnings, the forecast model makes price depend on ex ante analysts' forecasts of book value and residual income. The authors find that the explanatory power of the forecast model is greater in the Anglo-Saxon and North American countries, as well as in Germany, Japan and three Nordic countries, whereas it is equivalent in Latin countries and in Switzerland. Desroisiers, Lemaire, and L'Her (2007) use RI to deduce the implicit expected rates of return of nineteen countries, claiming that the RI model is "the more reliable and consistent measure of implicit expected rates of returns among countries" (p. 78). They consider zero-investment portfolios and implement a ranking strategy and a mean-variance optimization strategy, finding that the strategies posted positive performances.

5 The lost-capital paradigm

A new alternative concept of residual income, consistent with the fundamental eqs. (1)-(3) has been proposed in recent years. Originally introduced with the name *Systemic Value Added* (Magni, 2000a,b,c) it has been developed, generalized and thoroughly investigated from several points of view: mathematical, theoretical, cognitive, empirical (Magni, 2001a,b; 2003, 2004, 2005, 2006, 2008a). The paradigm has been used to conjoin into a unified perspective disparate models and notions in economic theory and corporate finance (Magni, 2007a,b,c). The theory is essentially based on the idea that the *undoing* of the factual scenario should be accomplished in a genuinely counterfactual way: if the investors had invested in the alternative course of action, the capital that the investor would have owned is different from $w_t(\vec{r})$, so the counterfactual income is not equal to $i_t w_{t-1}(\vec{r})$. In the counterfactual scenario, capital would have increased periodically at the rate i, so that the acceptance of the project implies that a capital equal to $w_t(\vec{\imath}):=w_t(i_1, i_2, \dots i_t)$ is *lost* by the investors: for this reason, the systemic-value-added theory may also be named the *lost capital* theory (Magni, 2007a,b). In value-based management, Drukarczyk and Schueler (2000) and Schueler and Krotter (2004) endorse the use of *Net Economic Income*, which is a market-based lost-capital residual income, while Young and O'Byrne (2001)'s notion of *Adjusted EVA* turns out to be an accounting-based lost-capital residual income in the case where earnings=dividends (see Magni, 2007a).

5.1 The Systemic Value Added

Magni originally introduces the Systemic Value Added by using the following argument. Let $W_0 \in \mathbb{R}$ be the investor's wealth at time 0 and assume it is currently invested in a financial asset F whose periodic interest rate is i_t. Suppose the investor has the opportunity of investing a_0 in project A. The investor may choose to (i) withdraw a_0 from asset F and invest it in the project or, alternatively, (ii) leave wealth invested in asset F. The two alternatives unfold two different financial scenarios for the investor's wealth:
(i) *Factual scenario.* The project is accepted. Then, the investor's wealth is a portfolio of project A and asset F. Let F_t be the value of asset F and w_t be the balance of project A; assuming each cash flow a_t released by A is reinvested in F, one has $F_t = F_{t-1}(1+i_t)+a_t$ and $w_t(\vec{r}) = w_{t-1}(\vec{r})(1+r_t)-a_t$, $t = 1, 2, \ldots, n$, where the sequence $\{w_t\}$ is arbitrary except for the boundary conditions $w_0(\vec{r}) = a_0$ and $w_n(\vec{r}) = 0$. The investor's wealth at time t is a simple dynamic system recursively computed as $W_t = F_{t-1}(1 + i_t) + w_{t-1}(\vec{r})(1 + r_t) = W_{t-1} + i_t F_{t-1} + r_t w_{t-1}(\vec{r})$. The (factual) profit is $W_t - W_{t-1} = i_t F_{t-1} + r_t w_{t-1}(\vec{r})$.[17]
(ii) *Counterfactual scenario.* The project is rejected. Then, the investor's wealth equals the value of asset F, which grows at a rate i_t. Let F^t and W^t be, respectively, the values of the asset and the investor's wealth, which is now governed by a dynamic system expressed by $W^t = F^t = F^{t-1}(1 + i_t)$. Hence, the (counterfactual) profit is $W^t - W^{t-1} = i_t W^{t-1}$.

Contrasting the two profits in the two scenarios a new definition of residual income is generated:

$$\Pi_t^{\mathrm{e}} = \big(i_t F_{t-1} + r_t w_{t-1}(\vec{r})\big) - i_t F^{t-1} \tag{15}$$

(Magni, 2000a, p. 164; 2000b, p. 54; 2001a, eq. (11a); 2004, p. 601) where the foregone return on wealth $i_t F^{t-1}$ has the meaning of opportunity cost. Such an excess profit is labelled *Systemic Value Added* (SVA) because the evolution of wealth in the two scenarios is represented by two different dynamic *systems*.

The assumption of reinvestment of a_t at the cost of capital i_t may be relaxed by dismissing investor's wealth and reframing the two scenarios: in the factual scenario the investor invests the amount a_0 in project A so that the project balance is $w_t(\vec{r}) = w_{t-1}(\vec{r})(1+r_t) - a_t$ (with the usual boundary conditions). In the counterfactual scenario the investor invests a_0 in a financial asset whose interest rate is i_t *and* periodically withdraws the amount a_t from the asset. The asset balance is $w_t(\vec{\imath}) = w_{t-1}(\vec{\imath})(1 + i_t) - a_t$, with obvious initial condition $w_0(\vec{\imath}) = a_0$. At the beginning of each period, the investor invests capital $w_{t-1}(\vec{r})$ at the rate r_t but so doing she loses the opportunity of investing $w_{t-1}(\vec{\imath})$ at the rate i_t. The capital $w_{t-1}(\vec{\imath})$ is thus the capital lost by the investor, the sum that would have been invested if the counterfactual scenario had been chosen. The investor receives a return of $r_t w_{t-1}(\vec{r})$ from A, so losing the opportunity of earning $i_t w_{t-1}(\vec{\imath})$. The latter is the profit foregone, *lost* by the investor (Magni, 2005,

[17] Obviously, such a profit is consistent with the fundamental equations (1)-(2) once the meaning of the variables is made clear: the capital is the investor's entire wealth W_t (inclusive of the project and the financial asset), the rate of return is $\frac{i_t F_{t-1} + r_t w_{t-1}}{F_{t-1} + w_{t-1}}$, a weighted average of r_t and i_t, and net cash flow is zero (cash flows are withdrawn from the project and reinvested in the financial asset).

p. 67). The difference between the two alternative profits may be named the *lost-capital* residual income, which actually is nothing but the SVA, because $F^t - F_t{=}w_t(\vec{\imath})$, with $w_0(\vec{\imath}){=}a_0{=}F^0 - F_0$. Therefore,

$$\Pi^{\mathrm{e}}_t = r_t w_{t-1}(\vec{r}) - i_t w_{t-1}(\vec{\imath}). \tag{16}$$

Reminding that the standard residual income is such that $\pi^{\mathrm{e}}_t(\vec{r}, i_t){=}\pi_t(\vec{r}){-}\pi_t(\vec{r}, i_t)$, the lost-capital paradigm is obtained by replacing $\pi_t(\vec{r}, i_t)$ with $\pi_t(\vec{\imath})$. That is, $\Pi^{\mathrm{e}}_t{=}\pi_t(\vec{r}){-}\pi_t(\vec{\imath})$.[18] The sequence $\{\Pi^{\mathrm{e}}_t\}$ decomposes the Net Final Value N_n. To see it, just consider that the fundamental equations for the two scenarios may be rewritten as $r_t w_{t-1}(\vec{r}) = w_t(\vec{r}) - w_{t-1}(\vec{r}) + a_t$ and $i_t w_{t-1}(\vec{\imath}) = w_t(\vec{\imath}) - w_{t-1}(\vec{\imath}) + a_t$, respectively. Hence,

$$\Pi^{\mathrm{e}}_t = \big(w_{t-1}(\vec{\imath}) - w_t(\vec{\imath})\big) - \big(w_{t-1}(\vec{r}) - w_t(\vec{r})\big). \tag{17}$$

Given that $w_0(\vec{r}){=}w_0(\vec{\imath})$ and $w_n(\vec{r}){=}0$, and solving $w_t(\vec{\imath}){=}w_{t-1}(\vec{\imath})(1 + i_t) - a_t$ for $t{=}n$,

$$\sum_{t=1}^{n} \Pi^{\mathrm{e}}_t = \sum_{t=1}^{n} \Big[\big(w_{t-1}(\vec{\imath}) - w_t(\vec{\imath})\big) - \big(w_{t-1}(\vec{r}) - w_t(\vec{r})\big)\Big] \tag{18}$$

$$= w_n(\vec{r}) - w_0(\vec{r}) - \big(w_n(\vec{\imath}) - w_0(\vec{\imath})\big)$$

$$= -w_n(\vec{\imath}) = N_0 \prod_{t=1}^{n} (1 + i_t) = N_n. \tag{19}$$

Equation (19) shows that the sequence $\{\Pi^{\mathrm{e}}_t\}$ decomposes the Net Final Value, and that such a decomposition is independent of the sequence $\{w_t(\vec{r})\}$ selected: it only depends on the boundary conditions $w_0(\cdot){=}0$ and $w_n(\vec{r}){=}0$. As a result, the sequence $\{\varphi_n(\vec{\imath}) \cdot \Pi^{\mathrm{e}}_t\}$ decomposes the NPV:

$$N_0 = \varphi_n(\vec{\imath})\big(\Pi^{\mathrm{e}}_1 + \Pi^{\mathrm{e}}_2 + \ldots + \Pi^{\mathrm{e}}_n\big) = \varphi_n(\vec{\imath})\Pi^{\mathrm{e}}_1 + \varphi_n(\vec{\imath})\Pi^{\mathrm{e}}_2 + \ldots + \varphi_n(\vec{\imath})\Pi^{\mathrm{e}}_n \tag{20}$$

as opposed to the sequence $\{\varphi_t(\vec{\imath}) \cdot \pi^{\mathrm{e}}_t\}$ in the standard paradigm:

$$N_0 = \varphi_1(\vec{\imath})\pi^{\mathrm{e}}_1 + \varphi_2(\vec{\imath})\pi^{\mathrm{e}}_2 + \ldots + \varphi_n(\vec{\imath})\pi^{\mathrm{e}}_n. \tag{21}$$

Hence, $v_0{=}a_0{+}\varphi_n(\vec{\imath}) \sum_{t=1}^{n} \Pi^{\mathrm{e}}_t$ as opposed to $v_0{=}a_0{+}\sum_{t=1}^{n} \varphi_t(\vec{\imath})\pi^{\mathrm{e}}_t$ (e.g. Magni, 2000a; 2001a,b; 2003; 2005). The lost-capital decomposition then induces a Sum&Discount method as opposed to the standard Discount&Sum method. Penman's (1992) words, referred to earnings, perfectly fit here, as referred to lost-capital *abnormal* earnings: "Unlike dividends (or cash flows), [abnormal] earnings aggregate in a value sense. One does not have to worry about timing. The task does not involve predicting [abnormal] earnings next year, the following year, and so on, but the total dollar [abnormal] earnings that a firm will deliver to the horizon" (p. 237). This brings about relevant implications regarding errors in forecast for investments under uncertainty. Suppose the vectors of theoretically correct residual incomes are $(\hat{\pi}^{\mathrm{e}}_1, \hat{\pi}^{\mathrm{e}}_2 \ldots, \hat{\pi}^{\mathrm{e}}_n)$ and $(\hat{\Pi}^{\mathrm{e}}_1, \hat{\Pi}^{\mathrm{e}}_2 \ldots, \hat{\Pi}^{\mathrm{e}}_n)$ in the standard and lost-capital paradigm respectively, and let $(\pi^{\mathrm{e}}_{x1}, \pi^{\mathrm{e}}_{x2} \ldots, \pi^{\mathrm{e}}_{xn})$ and $(\Pi^{\mathrm{e}}_{x1}, \Pi^{\mathrm{e}}_{x2} \ldots, \Pi^{\mathrm{e}}_{xn})$ be permutations of the correct vectors. Obviously, $\hat{v}_0{=}a_0 + \varphi_n(\vec{\imath}) \cdot \sum_{t=1}^{n} \hat{\Pi}^{\mathrm{e}}_t = a_0 +$

[18] The latter enables one to link residual income to the derivative of the income function (rates assumed constant for notational convenience):

$$\Pi^{\mathrm{e}} = \pi(r) - \pi(i) = \pi'(i)(r - i) + o(|r - i|), \qquad r \to i.$$

$\varphi_n(\vec{\imath}) \cdot \sum_{t=1}^n \Pi^{\rm e}_{xt} = v_0$, where $\hat{v}_0$ and v_0 are, respectively, the theoretically correct and the forecasted project value. Thus, errors in timing are neutralized. In contrast, in the standard paradigm one finds, in general, that the forecasted value differs from the correct value if residual incomes are incorrectly attributed to periods: $\hat{v}_0 = a_0 + \sum_{t=1}^n \varphi_t(\vec{\imath})\hat{\pi}^{\rm e}_t \neq a_0 + \sum_{t=1}^n \varphi_t(\vec{\imath})\pi^{\rm e}_{xt}{=}v_0$ (see also Magni, 2007c). This implies that the heuristic of the arithmetic mean $(\sum_{t=1}^n \Pi^{\rm e}_t)/n$ of expected residual incomes for forecasting purposes is highly relevant in this context (see also Remark 1 below).

Remark 1. As previously seen, one must average the one-period rates with *discounted* capitals (see eqs. (10)-(12)) to find the average yield (or the IRR) of the project; it is widely accepted in the literature that the *plain vanilla* average of one-period rates weighted by the (undiscounted) outstanding capitals w_t is not meaningful (e.g. Kay, 1976, p. 91). In fact, the $\mathcal{L}$ paradigm enables one to give that plain vanilla average a genuine meaning of profitability index: by imposing $\varphi_n(\vec{\imath})\sum_{t=1}^n(r_t w_{t-1}(\vec{r}) - i_t w_{t-1}(\vec{\imath}))=$ $\varphi_n(\vec{\imath})\sum_{t=1}^n(r^*_{\mathcal{L}} w_{t-1}(\vec{r}) - i_t w_{t-1}(\vec{\imath}))$, one finds the Chisini mean

$$r^*_{\mathcal{L}} = \frac{\sum_{t=1}^n r_t \cdot w_{t-1}(\vec{r})}{\sum_{t=1}^n w_{t-1}(\vec{r})}. \tag{22}$$

Therefore, the weighted average of one-period rates is unsatisfactory only if one looks at return rates with the standard-theory eyes. Wearing the $\mathcal{L}$ "glasses" one is able to recognize $r^*_{\mathcal{L}}$ as being a Chisini mean return. While the existence of the IRR is not required for eq. (22) to hold, the IRR itself may be found by replacing each i_t with the internal rate r in eq. (16) so that $\sum_{t=1}^n(r_t w_{t-1}(\vec{r}) - r w_{t-1}(r)) = 0$, whence

$$r = \frac{\sum_{t=1}^n r_t w_{t-1}(\vec{r})}{\sum_{t=1}^n w_{t-1}(r)} \tag{23}$$

where $w_{t-1}(r){:=}w_{t-1}(r, r, \ldots, r)$ is the IRR-based outstanding capital (not to be confused with the one-period-rate-based capital $w_{t-1}(\vec{r}){=}w_{t-1}(r_1, r_2, \ldots, r_{t-1})$. Note also that $r \neq r^*_{\mathcal{L}}$. Furthermore, with the same Chisini-based argument one gets, from the equation $\sum_{t=1}^n(\pi_t - i_t w_{t-1}(\vec{\imath})){=}\sum_{t=1}^n(\pi - i_t w_{t-1}(\vec{\imath}))$, the average profit $\pi{=}(\sum_{t=1}^n \pi_t)/n$; and, from the equation $\sum_{t=1}^n \Pi^{\rm e}_t{=}\sum_{t=1}^n \Pi^{\rm e}$, the average residual income $\Pi^{\rm e}{=}(\sum_{t=1}^n \Pi^{\rm e}_t)/n$, which confirms the above mentioned implications for forecast purposes.

5.2 The arbitrage connection

An important feature of the lost-capital paradigm is that it may be naturally derived from an arbitrage theory perspective. Suppose p is a portfolio traded in the market which replicates project A's cash flows. Let the yield term structure be represented by $\varphi_t(\vec{\imath})$, which is an IFL representing the unit price of a t-period zero-coupon bond issued at time 0. The value of p is $p_0 = \sum_{t=1}^n \varphi_t(\vec{\imath})a_t = a_0 + N_0$. If $p_0 \neq a_0$ (i.e. $N_0{\neq}0$) the investor may exploit arbitrage opportunities. For example, assuming (with no loss of generality) $p_0 > a_0$, the investor may take a long position in A and a short position in p and reinvest the arbitrage gain $(p_0 - a_0)$ in portfolio p. The resulting net cash flow will be zero at each date, and the investor will receive a net final cash flow Γ, such that $\Gamma = (p_0 - a_0)\prod_{t=1}^n(1 + i_t){=}N_0\prod_{t=1}^n(1 + i_t)$ (see Table 3). We aim at decomposing the terminal arbitrage gain Γ in period margins. To this end, note that the long and short positions in p may be netted out to result in a net short position (see Table 4). A short position in an asset is financially equivalent to a financing, so asset p may be viewed as a "debt" whose (variable) rate of interest is i_t. The arbitrage strategy may thus be interpreted as an investment in

A wholly financed by asset p. The project's cash flows are used to repay the debt, so the project balance is $w_t(\vec{r}) = w_{t-1}(\vec{r})(1+r_t) - a_t$, and the residual debt is $w_t(\vec{\imath}) = w_{t-1}(\vec{\imath})(1+i_t) - a_t$. At time n, the balance of the debt is $w_n(\vec{\imath}){=}{-}N_0\prod_{t=1}^{n}(1+i_t){=}{-}\Gamma{<}0$. The final $w_n(\vec{\imath})$ is negative because the total final payment to close off the position on p ($=a_n - \Gamma$) is smaller than the amount received from the project ($=a_n$).[19] Therefore, $r_t w_{t-1}(r_t)$ is the return from the long position, while $i_t w_{t-1}$ is interest on the short position. The latter represents the cost of the arbitrage strategy. As a result, the periodic gain from the arbitrage strategy is $r_t w_{t-1}(\vec{r}){-}i_t w_{t-1}(\vec{\imath})$, but the latter is just the lost-capital residual income $\Pi_t^{\rm e}$ above introduced, so that

$$\Gamma = -w_n(\vec{\imath}) = N_0\prod_{t=1}^{n}(1+i_t) = \Pi_1^{\rm e} + \Pi_2^{\rm e} + \ldots + \Pi_n^{\rm e}.$$

The arbitrage gain sequence $\{\Pi_t^{\rm e}\}$ decomposes the grand total arbitrage gain Γ in period margins (see also Magni, 2007b, section 3.1).

5.3 Relations between the two theories

The two theories lead, in general, to residual incomes which differ in terms of value and, possibly, in terms of sign (see Ghiselli Ricci and Magni, 2006). However, they are formally related. Let $\mathcal{L}: \pi^{\rm e} \to \Pi_t^{\rm e}$ be the operator transforming standard RIs in lost-capital RIs: we have

$$\pi_t^{\rm e} \to \mathcal{L}(\pi_t^{\rm e}) = \pi_t^{\rm e} - i_t(w_{t-1}(\vec{\imath}) - w_{t-1}(\vec{r})). \tag{24}$$

$\mathcal{L}$ is a bijection, so the inverse function $\mathcal{S} = \mathcal{L}^{-1}$ exists, transforming lost-capital excess profit into standard excess profit:

$$\Pi_t^{\rm e} \to \mathcal{S}(\Pi_t^{\rm e}) = \Pi_t^{\rm e} + i_t(w_{t-1}(\vec{\imath}) - w_{t-1}(\vec{r})). \tag{25}$$

($\mathcal{L}$=lost-capital, $\mathcal{S}$=standard). The constant in the linear affine functions is a *converting factor* representing the lost (or earned if negative) interest on the surplus of capital the investor would own if she rejected the project and invested his funds a_0 in the alternative asset. As we have seen, performance in the $\mathcal{L}$ paradigm is measured not only in terms of which interest rate could have been exploited by the investor, but also in terms of which capital could have been exploited. Thus, while $r_t > i_t$ signals positive performance in the classical $\mathcal{S}$ paradigm (because it implies $\pi_t^{\rm e} > 0$), the capital lost by the investor may be greater than the actual capital invested (i.e $w_{t-1}(\vec{\imath}) > w_{t-1}(\vec{r})$), so that the $\mathcal{L}$ excess profit may signal a smaller performance with respect to the $\mathcal{S}$-paradigm: the interest that could have been yielded by the surplus of capital may be so great as to offset the positive effect of the rate of return: whenever $0 < \pi_t^{\rm e} < i_t[w_{t-1}(\vec{\imath}) - w_{t-1}(\vec{r})]$, one gets $\Pi_t^{\rm e} < 0 < \pi_t^{\rm e}$, which informs that a negative performance is measured by the $\mathcal{L}$ paradigm. The additional component may symmetrically act as a sort of insurance bonus: if $r_t < i_t$, performance may still be regarded as positive if $w_{t-1}(\vec{\imath}) < w_{t-1}(\vec{r})$, which means that past performance has been so positive that the factual capital is greater than the capital lost, so that the smaller rate of return in the period is more than compensated by the greater basis $w_{t-1}(\vec{r})$ to which it is

[19] Rigorously speaking, in analogy with the description of Peasnell's approach, one should write $w_{n^*}{=}{-}\Gamma$, and $w_n(\vec{\imath}){=}w_{n^*} - (-\Gamma) = 0$, because, *after* distribution of the arbitrage gain Γ, no pending positions are left.

applied. This may be rephrased in a rate-of-return comparison:

$$\Pi_t^{\rm e} > 0 \quad \text{if and only if} \quad r_t > i_t^* = i_t + i_t \frac{w_{t-1}(\vec{\imath}) - w_{t-1}(\vec{r})}{w_{t-1}(\vec{r})}. \tag{26}$$

The second addend in the right-hand side is the product of the opportunity cost of capital and the relative increase (decrease) in capital due to acceptance of the project. For example, suppose i_t=0.1, $w_{t-1}(\vec{r})$=80, $w_{t-1}(\vec{\imath})$=100; then, if project had been rejected, the counterfactual capital would be higher than the the factual capital employed; in particular, it would be higher by a 25%=(100−80)/80. This means that the investor could have invested a 25% more capital than she actually invests, and she could have earned a 10% on that 25%, so that an additional 2.5% would accrue to her. Therefore, for a positive performance to occur, the period internal rate must be greater than 10%; in particular, the threshold level is i^*=12.5%=10%+2.5%. In general, the required cutoff rate i_t^* may be greater, equal or smaller than the cost of capital i_t. The latter case occurs whenever the additional-interest component is negative, which means that the factual capital exceeds the counterfactual capital and therefore the investor foregoes (not a return but) a cost. Under this perspective the $\mathcal{L}$ excess profit lends itself to a reinterpretation: it may be seen as a $\mathcal{S}$ residual income where the opportunity cost of capital includes the (positive or negative) additional interest on the surplus $w_{t-1}(\vec{\imath}) - w_{t-1}(\vec{r})$. Formally, $\Pi_t^{\rm e} = \pi_t(\vec{r}) - \pi_t(\vec{r}, i^*)$ so that $\Pi_t^{\rm e} = w_{t-1}(\vec{r})(r_t - i^*)$, where i^* is a comprehensive (i.e. all-inclusive) opportunity cost of capital. Any $\mathcal{L}$ residual income may therefore be dressed as a $\mathcal{S}$ residual income by using a comprehensive cost of capital:[20]

$$\begin{array}{llll} \pi_t^{\rm e} & = \pi_t(\vec{r}) - \pi_t(\vec{r}, i) & = w_{t-1}(\vec{r})(r_t - i_t) & \text{with conservation property} \quad N_0 = \sum \varphi_t(\vec{\imath})\pi_t^{\rm e} \\ \Pi_t^{\rm e} & = \pi_t(\vec{r}) - \pi_t(\vec{r}, i^*) & = w_{t-1}(\vec{r})(r_t - i_t^*) & \text{with conservation property} \quad N_0 = \varphi_n(\vec{\imath}) \sum \Pi_t^{\rm e}. \end{array} \tag{27}$$

Remark 2. Pfeiffer (2004) shows that the class of accounting-based performance measures in Ψ is the only one that satisfies conservation property for *any* sequence of $\{w_t(\vec{r})\}$. Eqs. (20) and (27) induce an enlargement of that class if the following notion of residual income is introduced:

Definition 2. A real number ψ_t is said to be a residual income if, for any sequence $\{w_t(\vec{r})\} \in \mathbb{R}^{n-1}$, there exists a sequence $\{j_t\} \in \mathbb{R}^n$ and a $\xi > 0$ such that $\psi_t = w_{t-1}(\vec{r}) \cdot (r_t - j_t)$ and $N_0 = \sum_{t=1}^n \varphi_\xi(\vec{\imath}) \cdot \psi_t$.

Note that ψ_t=$\pi_t(\vec{r}) - \pi_t(\vec{r}, j_t)$, and the set Ψ is enlarged to encompass both the $\mathcal{S}$ paradigm (if $j_t = i_t$ and $\xi = t$) and the $\mathcal{L}$ paradigm (if $j_t = i_t^*$ and $\xi = n$).

Remark 3. Manipulating the fundamental eq. (1) for both the capital $w_{t-1}(\vec{r})$ and the lost capital $w_{t-1}(\vec{\imath})$, one finds two relations connecting the $\mathcal{S}$ paradigm to the $\mathcal{L}$ paradigm:

- in terms of standard RIs: we have $w_{t-1}(\vec{r}) - w_{t-1}(\vec{\imath})$=$\sum_{k=1}^{t-1} \pi_k^{\rm e} \prod_{k=1}^{t-1}(1 + i_k)$, which implies

$$\Pi_t^{\rm e} - \pi_t^{\rm e} = i_t \sum_{k=1}^{t-1} \pi_k^{\rm e} \prod_{k=1}^{t-1}(1 + i_k) \tag{28}$$

[20] A further formal dressing is $\Pi_t^{\rm e}$=$w_{t-1}(\vec{\imath})(\overline{r}_t - i_t)$ with $\overline{r}_t = r_t w_{t-1}(\vec{r})/w_{t-1}(\vec{\imath})$ where $w_{t-1}(\vec{\imath})$ may be interpreted as the balance of a shadow project (see Magni, 2000a, 2003, 2004, 2005).

- in terms of lost-capital RIs: we have $w_{t-1}(\vec{r}) - w_{t-1}(\vec{\imath}) = \sum_{k=1}^{t-1} \Pi^{\mathrm{e}}_k$ which implies

$$\Pi^{\mathrm{e}}_t - \pi^{\mathrm{e}}_t = i_t \sum_{k=1}^{t-1} \Pi^{\mathrm{e}}_k \tag{29}$$

(see Magni, 2000a, eqs. (6)-(7); 2001b, eqs. (12) and (14); 2004, eqs. (13)-(14)).[21] Owing to eqs. (28) and (29), the $\mathcal{L}$ paradigm may be said to take account of the past performances. As seen above, if management had invested the initial sum a_0 in the counterfactual alternative, the capital invested in each period would have been different. Positive performances in the past, resulting in positive residual incomes, positively reverberate on the current residual income, whereas negative performances will have a negative effect on future excess profit. Therefore, excess profits are chained one another: managers' performance one year reverberates on the following year, so that positive (negative) past performances magnify (shrink) current performances.

5.4 The forerunners: (a) Keynes

The notion of lost-capital residual income may be drawn from Keynes's (1936) notion of *user cost*. Referring to an entrepreneur, the author defines user cost counterfactually as the difference between "the value of his capital equipment at the end of the period ... and ... the value it might have had at the end of the period if he had refrained from using it" (Keynes, 1967, p. 66). Coase (1968) renames it "depreciation through use", stressing the fact that it represents a depreciation charge not with respect to *time*, but to the *use* of the capital equipment. It reflects a depreciation due to "the choice between ... using a machine for a purpose and using it for another" (Coase, 1968, p. 123). Such a depreciation represents the "opportunity cost of putting goods and resources to a certain use" (Scott, 1953, p. 369); it is an economic measure of "the opportunity *lost* when another decision is carried through" (Scott, 1953, p. 375, italics added). The capital equipment is used to undertake a project whose value is equal to "the present value of the net receipts ... by discounting them at a rate of interest" (Coase, 1968, p. 123) and this "rate of discount coincides with that in the market" (Scott, 1953, p. 378). Keynes himself claims that the user cost at a given time t is "the discounted value of the additional prospective yield which would be obtained at some later date" (Keynes, 1967, p. 70). Magni (2007a, section 9) shows that user cost may be formalized as $w_t(\vec{\imath}) - v_t$. It is possible to consider its own depreciation through time by focussing on the periodic variation of user cost:

$$\text{depreciation through time of user cost} = \big(w_{t-1}(\vec{\imath}) - v_{t-1}\big) - \big(w_t(\vec{\imath}) - v_t\big).$$

The above expression is actually a particular case of eq. (17), where $w_{t-1}(\vec{r}) := v_{t-1}$, which implies that the one-period rate r_t is the market rate of return: $r_t := i_t$ for $t>1$ (for $t=1$ the initial boundary condition $w_0(\vec{r}) = a_0$ implies $r_1 = (v_1 - a_0 + a_1)/a_0$). As a result, Keynes's analysis of user cost unfolds an implicit market-based lost-capital residual income, which we here name Keynesian Excess Profit (KEP) and rewrite in terms of differential profits:

$$\mathrm{KEP}_1 = r_1 a_0 - i_1 a_0, \qquad \mathrm{KEP}_t = i_t v_{t-1} - i_t w_{t-1}(\vec{\imath}) \quad \text{for } t > 1 \tag{30}$$

[21] Obviously, one also finds, from eq. (27), the relation $\Pi^{\mathrm{e}}_t - \pi^{\mathrm{e}}_t = w_{t-1}(\vec{r})(i_t - i^*_t)$, which represents the difference between two opportunity costs.

where, obviously, $N_n{=}N_0/\varphi_n(\vec{\imath}){=}\sum_{t=1}^n \text{KEP}_t$.[22] The user cost is thus a building brick of the $\mathcal{L}$ theory, which discloses two important properties. In first place, it is usually thought that "economists cannot afford to lump together, as 'depreciation', changes in present value caused by the passage of time, and by use" (Scott, 1953, p. 371): on the contrary, a lost-capital approach on RI does enable one to lump together *depreciation through time* and *depreciation through use*. In second place, the KEP satisfies (strong and) robust goal congruence: we have $\text{KEP}_1 = v_1 + a_1 - a_0 = N_0/\varphi_1(\vec{\imath})$ and $\text{KEP}_t = i_t(v_{t-1} - w_{t-1}(\vec{\imath})){=}N_t - N_{t-1}{=}i_tN_{t-1}{=}i_tN_0/\varphi_t(\vec{\imath})$ for $t > 1$ (see Magni, 2007a, sections 8-10).

5.5 The forerunners: (b) Pressacco and Stucchi

Pressacco and Stucchi (PS) (1997) focus on Peccati's decomposition model and aim at generalizing the standard theory by considering return rates depending on the sign of the project balance (see Teichroew, Robichek and Montalbano, 1965a,b); however, in the final section of the paper, PS introduce the notion of *external financing* (p. 181) as opposed to *self-financing*; as may be easily checked, the former is equal to the lost capital $w_t(\vec{\imath})$ and the latter is equal to the difference $w_t(\vec{r}) - w_t(\vec{\imath})$. In the final remarks of that section they hint at a period margin obtained as the sum of two components: the *operating margin*, which is equal to the factual income $r_tw_{t-1}(\vec{r})$, and the interest on self-financing, which is equal to the converting factor $i_t[w_{t-1}(\vec{r}) - w_{t-1}(\vec{\imath})]$. In such a way, the authors change their very approach to residual income and construct a period margin by making use of the lost-capital approach.

Remark 4. O'Hanlon and Peasnell (2002) use the notion of *unrecovered capital*, which is equivalent to the notion of PS's *external financing*, and therefore equivalent to the *lost capital* (see also Table 5). In their Proposition 1, they relate unrecovered capital, book value, and past residual incomes: using our symbols, $b_t{=}w_t(\vec{\imath}){+}\sum_{k=1}^t \pi_k^{\text{e}}(1+i)^{t-k}$. It is worth noting that this result is formally anticipated in a general form by PS's (1997) Theorem 7.1 (Schueler, 2001, hints at the same result in his eqs. (2)-(3). See also Magni, 2007a; Schueler and Krotter, 2004, 2008, for relations between lost capital, capital invested, and NPV).

6 Constructing performance metrics

In this section we consider some RI models presented in the literature and in the practice, as well as some natural extensions of them. The various models may be categorized on the basis of the notion of (income and) capital employed and of the relevant cash flows considered for valuation. As for the notion of capital, we consider three main categories: book value, market value, IRR-based capital. As for the cash flows, we consider three perspectives: the entity approach, the claimholders' approach, the equity (or proprietary) approach. In an entity approach the relevant cash flows are the *free cash flows* (i.e. the cash flow that equityholders would receive if the project were unlevered); in a claimholders' approach the *capital cash flows* (i.e. the cash flows to all claimholders) are used; in an equity approach the relevant cash flows are the *equity cash flows* (i.e. dividends+share repurchases−new shares).[23]

[22]When, after more than sixty years, Drukarczyk and Schueler (2000) advocate the use of their Net Economic Income, they unawarely compute the depreciation through time of Keynes's user cost (see section 6 below).

[23]One should not confuse capital cash flows with free cash flows: the relation among the four types of cash flows may be summarized by the equalities $c_t{=}e_t + d_t{=}f_t + \tau I_t$ (see Ruback, 2002; Fernández, 2002; Tham and Vélez-Pareja, 2004).

Henceforth, it is assumed that the project is uncertain and that a MM market exist (Modigliani and Miller, 1958, 1963). This implies that the cash flows a_t, the RIs and the costs of capital should be interpreted as expected values.

Accounting-based RI models (Table 6). These models are characterized by the adoption of an accounting notion of (income and) capital. Thus, the book value of capital is used, and the corresponding one-period IRR is computed from the fundamental equation eq. (2). In an entity perspective, the Economic Value Added (EVA) model is originated (Stewart, 1991), where the net operating profit after taxes (*nopat*) is the relevant income and the free cash flows are the relevant cash flows, which implies that the one-period IRR is the return on investment (*roi*). Correspondingly, the opportunity cost of capital is the well-known *weighted average cost of capital*: $\rho_t = \frac{\rho_t^{e} v_{t-1}^{e} + \rho_t^{D} D_{t-1}(1-\tau)}{v_{t-1}^{e} + D_{t-1}}$. A modified version of the EVA model is proposed by Vélez-Pareja and Tham (2004), who adopt a claimholders' perspective and suggest the use of the claimholders' profit (net profit to shareholders+interest on debt) as the relevant income. Such a profit is equal to the *nopat* plus the tax shield, and the ratio of this adjusted *nopat* to the book value of assets is here named return on liabilities (*rol*). The cash flows considered are the capital cash flows and the opportunity cost of capital is the pre-tax weighted average cost of capital: $\rho_t^{\tau} = \frac{\rho_t^{e} b_{t-1}^{E} + \rho_t^{D} D_{t-1}}{v_{t-1}^{e} + D_{t-1}}$. Their metric is here labelled Claimholders' Residual Income (CRI).[24,25] The so-called Edwards-Bell-Ohlson (EBO) model (Edwards and Bell, 1961; Ohlson, 1989, 1995; Lee, 1999) is constructed from the point of view of the equity owners, so that the relevant income is the profit after taxes (*pat*) resulting from the income statement, and the one-period rate is the return on equity (*roe*); the cash flows are the equity cash flows and the opportunity cost of capital is the required return to equity (aka cost of equity) $\rho_t^{e} = (v_t^{e} - v_{t-1}^{e} + e_t)/v_{t-1}^{e}$.

Remark 5. It is worth stressing that each of the three accounting rates originates an IFL for the corresponding cash-flow stream: $\sum_{t=1}^{n} f_t \varphi_t(\overrightarrow{roi}) = \sum_{t=1}^{n} c_t \varphi_t(\overrightarrow{rol}) = \sum_{t=1}^{n} e_t \varphi_t(\overrightarrow{roe}) = 0$.

IRR-based RI models (Table 7). These metrics start from the computation of an IRR for the cash-flow stream. A widespread metric in this class is derived from Madden (1999)'s cash flow return on investment (*cfroi*), which is but the IRR of the (inflation-adjusted) free-cash-flow stream, so that $\sum_{t=1}^{n} f_t (1+cfroi)^{-t} = 0$. Comparing the *cfroi* with the weighted average cost of capital ρ_t and multiplying the spread by the outstanding capital computed from the fundamental recursive eq. (3), the *cfroi*-based residual income (RI^{cfroi}) is constructed. Such a metric pertains to an entity approach and a natural extension of this metric is possible if one considers the claimholders' point of view. Using the capital-cash-flow stream to compute the corresponding IRR, one finds what might be named the cash flow return

[24] Vélez-Pareja and Tham (2004) emphasize that if the discount rate for the tax shield is the unlevered cost of assets ρ^U, then the latter coincides with the pre-tax weighted average cost of capital: $\rho^{\tau} = \rho^U$, so that circularity issues are avoided.

[25] The claimholders' profit is hinted at in Grant (1998), where it is called *levered net operating profit after taxes*. However, the author develops a pre-tax version of EVA different from Vélez-Pareja and Tham's, based as it is on earnings before interest and taxes (*ebit*) (which is, by definition, equal to $nopat_t/(1-\tau)$); using our symbols, the author defines

$$\text{Pre-tax EVA} = b_{t-1}\left[\frac{ebit_t}{b_{t-1}} - \frac{\rho_t^{e}/(1-\tau) v_{t-1}^{e} + \rho_t^{D} D_{t-1}}{v_{t-1}^{e} + D_{t-1}}\right]$$

so that EVA=Pre-tax EVA$(1-\tau)$. While the discounted sum of the EVAs gives the project's NPV, the discounted sum of these Pre-tax EVAs only leads to a pre-tax NPV: one has to multiply by $(1-\tau)$ to reach the NPV (see also Abate and Grant, 2002).

on liabilities (*cfrol*): $\sum_{t=1}^{n} c_t(1+\mathit{cfrol})^{-t}=0$. Comparing *cfrol* with ρ_t^{τ}, the *cfrol*-based claimholders' residual income ($\mathrm{RI}^{\mathit{cfrol}}$) is carried out. Analogously, an equity approach leads to the computation of the cash-flow return on equity (*cfroe*) as the IRR of the equity-cash-flow stream: $\sum_{t=1}^{n} e_t(1+\mathit{cfroe})^{-t}=0$. This equity IRR, compared to the cost of equity ρ_t^{e}, gives rise to the *cfroe*-based residual income ($\mathrm{RI}^{\mathit{cfroe}}$).[26]

Market-based models (Table 8). This class is based on market values. Except at time 0, where capital invested is equal to the initial outlay, the market value of capital is chosen for $t<n$, where market value is defined as the sum of the discounted values of the project's cash flows at the appropriate cost of capital. This choice boils down to choosing the cost of capital as the one-period IRR in each period (except the first one). In particular, Fernández (2002) adopts a proprietary perspective and endorses the use of Created Shareholder Value (CSV), which is a reproposal of Bodenhorn's (1964) (cash-flow based) *pure profit*:[27] the capital considered is the market value of equity, the relevant cash flows are the equity cash flows, and the opportunity cost of capital is the required return to equity ρ_t^{e}. This means that the first-period IRR is $r_1=(v_1^{\mathrm{e}}-e_0+e_1)/e_0$ (see Fernández, 2002, p. 281), while in the other periods $r_t=\rho_t^{\mathrm{e}}$. Given that Bodenhorn-Fernández's metric belongs to the class of standard RI models, the excess profit in the first period is $w_0(\vec{r})(r_1-i_1)=e_0(\frac{v_1^{\mathrm{e}}-e_0+e_1}{e_0}-\rho_1^{\mathrm{e}})$, whereas in the other period is zero. Following an entity approach Drukarczyk and Schueler (2000) propose the Net Economic Income (NEI), which belongs to the class of $\mathcal{L}$ metrics. For their metric, the authors choose the market value of the firm's assets, and the capital charge is computed by taking $w_t(\vec{\rho})$ as the lost capital. If one assumes a claimholders' perspective, the NEI turns into a new metric, which is here labelled Claimholders' Economic Income (CEI): the relevant cash flows are the capital cash flows and the opportunity cost of capital is the pre-tax weighted average cost of capital ρ^{τ}.

Remark 6. It is worth noting that NEI, CEI and $\mathcal{L}$(CSV) (the lost-capital companion of CSV) belong to the class of KEP models defined in eq. (30): all of the three metrics coincide with the periodic variation of user cost and satisfy (strong and) robust goal congruence.

A twofold classification of the nine metrics (according to the notion of capital and according to the perspective taken) is summarized in Table 9.

7 A numerical example

The $\mathcal{L}$ theory is a further addition to the toolkit of the financial engineer: valuation theory may now be said to consist of ten basic methodologies, four of which are based on cash flows, three are based on $\mathcal{S}$ residual income, and the remaining three are based on $\mathcal{L}$ residual income (see Table 10). All ten methods are logically consistent and give the same result (in the RI models the initial outlay must be obviously added to the NPV to reach the value). Consider a firm that is incorporated to undertake a 5-year project. Tables 11 and 12 collect the input data (in boldface), the firm's accounting statements, the resulting expected cash flows and rates of return, while Table 13 employs the APV method and the three DCF techniques to find the market values. To this end, it is assumed that ρ^U is exogenously given

[26] It is evident that *cfroi*, *cfrol*, and *cfroe*, may be viewed, respectively, as constant *roi*, *rol*, and *roe*.

[27] See also Dutta and Reichelstein (2005, section 3).

and that nominal debt is equal to market value of debt, which means $\delta_t = \rho_t^D$ and $I_t = \rho_t^D D_{t-1}$. The market value of equity is 22,134 and the NPV turns out to be $N_0 = v_0 - f_0 = v_0^{\rm e} - e_0 = 1{,}134$. Logically, the four methods give the same result (see Fernández, 2002, 2007, for a review of these valuation methods). Table 14 focusses on the 3+3 groups of metrics, and the RIs are calculated for each period. Logically, each of them represents a decomposition of $N_0 = 1{,}134$, which is obtained by the Discount&Sum procedure for the $\mathcal{S}$ metrics and by the Sum&Discount method for the $\mathcal{L}$ metrics. Inspecting Table 14, the reader may appreciate the considerable differences across paradigms and across metrics. It is worth noting that the KEP metrics are perfectly aligned in sign with N_t (robust goal congruence). The fact that most of the IRR-based metrics in the example have the same sign as N_t in each year does not hold in general. As a counterexample, consider the standard IRR-based metrics and suppose that the first-year sales are equal to 13,410, other things equal: one finds NPV=3.6 and the vectors of the three metrics turn out to be, respectively, (-54.7, -30.3, -14.1, 51.6, 89.1), (5.6, 2.0, -0.2, 0.2, -1.9), and (144.2, 74.3, 25, -138.2, -248.8). Contrasting $\mathcal{S}$ metrics with $\mathcal{L}$ metrics, the discrepancies may be enormous; for example, considering ρ^U=23%, other things equal, the NPV is $N_0 = -5{,}977$ and it is easily checked that in the third period the accounting-based standard metrics signal positive performances: EVA=417, CRI=399, EBO=218, whereas their $\mathcal{L}$ companions are deeply negative $\mathcal{L}$(EVA)=-1136 $\mathcal{L}$(CRI)=-1276, $\mathcal{L}$(EBO)=-2874. As a result, the use of either theory for executive compensation may well impact on managers' compensation in profoundly opposite ways.

8 Conclusion

This papers presents a review of the notion of residual income, also known as excess profit. This concept has gone a long road since its origins in the last years of the 19th century (see the synopsis in Table 15). The relation residual income bears to value had been recognized early, but only in relatively recent years it has been thoroughly investigated in various fields: accounting, corporate finance, financial mathematics. This relation has proved useful for valuation purposes and performance analysis, and the use of residual income as a governance tool is at the core of the well-entrenched value-based management literature: linking management compensation to periodic performance is a way for aligning managers' and shareholders' objectives and reduce agency costs.

This paper stresses the role of income and opportunity cost as the two basic ingredients of residual income. The role of counterfactual conditionals in the definition of opportunity cost, highly neglected in the literature, is here emphasized, so that opportunity cost is defined as the counterfactual income the investor would (have) earn(ed) if she (had) rejected the project. The *undoing* of the factual scenario in the standard residual income theory implies that only the return rate is undone. In the systemic-value-added approach, aka lost-capital paradigm, the capital is undone as well so that the lost income is given by the product of the opportunity cost of capital times the capital that the investors would have owned if they had invested in the counterfactual course of action. The new paradigm is naturally embraced in an arbitrage theory perspective and set interesting relations to the notion of *depreciation*: both depreciation through time and depreciation through use are lumped together. This link is provided by Keynes's (1936) user cost (depreciation through use), which is a basic ingredient of the lost-capital theory. The procedure for computing the project value in the two theories is, so to say, specular: a Discount&Sum procedure in the standard paradigm, a Sum&Discount procedure in the lost-capital paradigm. The latter procedure unmasks an aggregation property which enables to neutralize the role of timing in forecasting: it is not necessary to know *when* residual incomes are generated. This induces a simple heuristic for valuation: rather than forecasting each and every residual income one may rest on an average residual income. The new paradigm is also capable of retrieving the average of one-period (e.g accounting) rates weighted by the undiscounted capitals (e.g. book value): dismissed in the accounting literature as nonsignificant, this plain vanilla average represents the project's average yield as seen from the lost-capital theory perspective; the IRR itself may be seen as a generalized weighted average of outstanding capitals. Moreover, it is possible to interpret the lost-capital RI as a standard RI with a comprehensive cost of capital and a Sum&Discount conservation property, which allows for a generalized definition of residual income encompassing both paradigms (see Definition 2).

The standard notion of residual income has triggered a flourishing literature on various problems and perspectives: decisions on investments, production, sales; implementation of optimal portfolio strategies; short-run policies as proxies for long-run policies; conflicts between principal and agent; forecasts of asset prices and valuation of intrinsic values. Hopefully, the theoretical enlargement of the set of performance measures with the lost-capital residual income will attract interest of academics and stimulate research on its possible use for managerial and financial applications. However, using a single theory or "using only a single measure cannot necessarily achieve all of the objectives usually required of such a measure" (Bromwich and Walker, 1998, p. 404). No single theory or metric can be said to be the "best" index. Both theories, along with their corresponding metrics, could fruitfully be conjoined in a multicriteria approach to provide appropriate models satisfying the needs of the evaluator to solve specific decision problems. To this end, the use of vague theories such as fuzzy logic (Zadeh, 1965; Zimmermann, 2001) or supervaluationist logic (Keefe, 2000, 2008; Qizilbash, 2003) might perhaps prove helpful.

References

Abate, J.A., Grant, J.L. 2002. Security analysis using value-based metrics. In Frank J. Fabozzi & Harry M. Markowitz (Eds.), *The Theory and Practice of Investment Management.* Hoboken, NJ: John Wiley & Sons.

Amey, L. 1975. Tomkins on 'Residual Income'. *Journal of Business Finance & Accounting*, 2(1), 55–68

Anctil, R. 1996. Capital budgeting using residual income maximization. *Review of Accounting Studies*, 1(1), 297–229.

Anctil, R., Jordan, J.S., Mukherji, A. 1998a. The asymptotic optimality of residual income maximization. *Review of Accounting Studies*, 2, 207–229.

Anctil, R., Jordan, J.S., Mukherji, A. 1998b. Activity-based costing for Economic Value Added. *Review of Accounting Studies*, 2(3), 231–264.

Andersson K., Kanniainen, V., Södersten, J., Sørensen, P.B. 1998. Corporate tax policy in the Nordic countries. In Sørensen, P.B. (Ed.), *Tax Policy in the Nordic Countries*, MacMillan.

Antle, R., Eppen, G.D. 1985. Capital rationing and organizational slack in capital budgeting. *Management Science*, 31(2) (February), 163–174.

Anthony, R.N. 1975. *Accounting for the Cost of Interest.* Lexington, MA: D. C. Heath and Company.

Archer, S.H., D'Ambrosio, A.A. 1972. *Business Finance. Theory and Management*, second edition. New York: MacMillan Company

Arnold, G. 2000. Tracing the development of value-based management. In G. Arnold & M. Davies (Eds.), *Value-based Management: Context and Application.* Chichester, UK: John Wiley & Sons.

Arnold, G. 2005. *Corporate Financial Management*, third edition. Harlow, UK: Prentice Hall.

Arnold, G. 2007. *Essentials of Corporate Financial Management.* Harlow, UK: Prentice Hall.

Arnold, G., Davies, M. (Eds.) 2000. *Value-based Management: Context and Application.* Chichester, UK: John Wiley & Sons.

Arya, A., Baldenius, T., Glover, J. 1999. Residual income, depreciation, and empire building. Available at <https://littlehurt.tepper.cmu.edu/gsiadoc/wp/2000-E2.pdf>.

Arzac, E.R., Glosten, L.R. 2005. A reconsideration of tax shield valuation, *European Financial Management*, 11(4), 453–461.

Balachandran, S.V. 2006. The role of prior performance measures. *Management Science*, 52(3), 383–393.

Baldenius, T. 2003. Delegated investment decisions and private benefits of control. *The Accounting Review*, 78, 909–930.

Baldenius, T., Dutta, S., Reichelstein, S. 2006. Cost allocation for capital budgeting decisions. Available at SSRN: <http://ssrn.com/abstract=938509>.

Baldenius, T., Reichelstein, S. 2005. Incentives for efficient inventory management: The role of historical cost. *Management Science*, 51(7), 1032–1045, July.

Barniv, R., Myring, M. 2006. An international analysis of historical and forecast earnings in accounting-based valuation models. *Journal of Business Finance & Accounting*, 33(7/8) (September/October), 1087–1109.

Beccacece, F., Li Calzi, M. 1991. On the decomposition of stochastic discounted cash flows. *Decisions in Economics and Finance*, 14(2) (September), 59–73.

Begg, D., Fischer, S., Dornbusch, R. 1984. *Economics*, Mc-Graw Hill, British Edition.

Biddle, G.C., Bowen, R.M., Wallace, J.S. 1999. Evidence on EVA. *Journal of Applied Corporate Finance*, 12(2), 69–79.

Boadway, R., Bruce, N. 1984. A general proposition on the design of a neutral business tax. *Journal of Public Economics*, 24(2) (July), 231–239.

Bodenhorn, D. 1959. On the problem of capital budgeting. *Journal of Finance*, 14(4) (December), 473–492.

Bodenhorn, D. 1964. A Cash-flow concept of profit. *Journal of Finance* (March), 19(1), 16–31.

Brief, R.P. 1999. The accounting rate of return as a framework for analysis Available at SSRN: <http://ssrn.com/abstract=184228>, September.

Brief, R.P. 2007. Accounting valuation models: A short primer. *Abacus*, 43(4), 429–437.

Brief, R.P., Lawson, R.A. 1992. The role of the accounting rate of return in financial statement analysis. *The Accounting Review*, 67(2), 411-426. Reprinted in R. Brief & K.V. Peasnell (Eds.), *A Link Between Accounting and Finance*. New York: Garland, 1996.

Brief, R.P., Peasnell, K.V. (Eds.) 1996. *Clean Surplus: A Link Between Accounting and Finance*. New York and London: Garland Publishing.

Bromwich, M. 1973. Measurement of divisional performance: A comment and extension. *Accounting and Business Research*, 3(2) (Spring), 123–132.

Bromwich, M., Walker, M. 1998. Residual income past and future, *Management Accounting Research*, 9(4), 391-419.

Buchanan, J. 1969. *Cost and Choice. An Inquiry in Economic Theory*. Chicago: Markham. Republished as Midway reprint, Chicago: University of Chicago Press, 1977.

Camillo R., Marena, M. 1994. Un metodo di valutazione di un portafoglio assicurativo vita [An evaluation method of a life insurance portfolio]. *Decisions in Economics and Finance*, 17(2), 61–77.

Canning, J.B. 1929. *The Economics of Accountancy: A Critical Analysis of Accounting Theory*. New York: The Ronald Press

Carsberg, B.V. 1966. The contribution of P.D. Leake to the theory of Goodwill valuation. *Journal of Accounting Research*, 4(1), 1–15, Spring.

Chew, D.H., Stern, Stewart, G.B. 1995. The EVA financial management system, *Journal of Applied Corporate Finance*, 8(2), 32–46, Summer.

Chisini, O. 1929. Sul concetto di media [On the concept of mean], *Periodico di Matematiche*, 4, 106–116.

Christensen, P.P., Feltham, G.A. and Wu, M.G.H., 2002. "Cost of Capital" in residual income for performance evaluation. *The Accounting Review*, 77(1) (January), 1-23.

Cigola, M., Peccati, L. 2005 On the comparison between the APV and the NPV computed via the WACC. *European Journal of Operational Research*, 161, 377-385.

Claus, J., Thomas, J. 2001. Equity premia as low as three percent? Evidence from analysts' earnings forecasts for domestic and international stock markets. *Journal of Finance*, 56(5) (October), 1629–1666.

Cnossen, S. 1998. Reform and coordination of company taxes in the European Union. In Sørensen, P.B. (Ed.), *Public Finance in a Changing World*, MacMillan.

Coase, R.H. 1938. Business Organization and the Accountant, *The Accountant*, October-December. Reprinted as "The nature of costs" in D. Solomons, (Ed.), *Studies in Cost Analysis*. London: Sweet & Maxwell, 1968, second edition.

Coase, R.H. 1968. The nature of costs. In D. Solomons (Ed.), *Studies in Cost Analysis*, London: Sweet & Maxwell, second edition.

Cooper, I.A., Nyborg, K.G. 2006. The value of tax shields IS equal to the present value of tax shields, *Journal of Financial Economics*, 81, 215–225.

Damodaran A. 2006. *Damodaran on Valuation. Security Analysis for Investment and Corporate Finance.* Hoboken, NJ: John Wiley & Sons.

Daske, H., Gebhardt, G., Klein, S. 2006. Estimating the expected cosst of equity capital using analysts' consensus forecasts. *Schlamenbach Business Review*, 58(1) (January), 2–36.

de Finetti, B. 2008. The concept of mean. In de Finetti, B., *Philosophical Lectures on Probability*, edited by Alberto Mura. Springer Science.

Desrosiers, S. Lemaire, N., L'Her, J.-F. 2007. Residual income approach to equity country selection. *Financial Analysts Journal*, 63(2) (April/March), 76–89.

Dicksee, L.R. 1897. *Goodwill and Its Treatment in Accounts.* London.

Drukarczyk, J., Schueler, A. 2000. *Approaches to value-based performance measurement.* In G. Arnold & M. Davies (Eds.), *Value-based Management: Context and Application.* Chichester, UK: John Wiley & Sons

Dutta, S., Reichelstein, S. 2002. Controlling investment decisions: depreciation- and capital charges. *Review of Accounting Studies*, 7, 253–281.

Dutta, S., Reichelstein, S. 2005. Accrual accounting for performance evaluation. *Review of Accounting Studies* , 10, 527–552.

Dutta, S., Zhang, X. 2002. Revenue recognition in a multiperiod agency setting, *Journal of Accounting Research* 40, 67–83.

Edey, H.C. 1957. Business valuation, goodwill and the super-profit method. *Accountancy*, January/February. Reprinted in W.T. Baxter & S. Davidson (1962) (Eds.), *Studies in Accounting Theory.* London: Sweet & Maxwell, pp. 201–217.

Edwards, E., Bell, P. 1961. *The Theory and Measurement of Business Income.* Berkeley University of California Press.

Ehrbar, A. 1998. *Eva: The Real Key to Creating Value.* New York: John Wiley & Sons.

Emmanuel, C.R., Otley D.T. 1976. The usefulness of residual income. *Journal of Business Finance & Accounting*, 3(4), 43–51.

Fabozzi, F.J. 2006. *Fixed-Income Mathematics*, fourth edition. New York: McGraw-Hill.

Fabozzi, F.J., Grant, J.L. (Eds.) 2000. *Value-Based Metrics: Foundations and Practice.* New Hope, PA: Frank J. Fabozzi Associates

Feenstra, D.W., Wang, H. 2000. Economic and accounting rates of return. Research Report 00E42, *University of Groningen*, *Research Institute SOM (Systems, Organisations and Management).* Available at <http://som.eldoc.ub.rug.nl/FILES/reports/themeE/2000/00E42/00E42.pdf>.

Feltham, G.A., Ohlson J.A. 1995. Valuation and clean surplus accounting for operating and financial activities. *Contemporary Accounting Research*, 11(2) (Spring), 689–731.

Fernández, P. 2002. *Valuation Methods and Shareholder Value Creation.* San Diego, CA: Elsevier Science.

Fernández, P. 2005. Reply to "Comment on The value of Tax Shields is NOT equal to the present value of tax shields". *Quarterly Review of Economics and Finance*, 45(1), 188–192.

Fernández, P. 2007. Valuing companies by cash flow discounting: then methods and nine theories, *Managerial Finance*, 33(11), 853–876.

Fetter, F.A. 1937. Reformulation of the concepts of capital and income in economics and accounting. *The Accounting Review*, 12(1), 3–12, March.

Flower, J. 1971. Measurement of divisional performance, *Accounting and Business Research*, 1(3) (Summer), 205–214.

Forker, J., Powell, R. 2000. A review of accounting and VBM alternatives. In G. Arnold & M. Davies, *Value-based Management: Context and Application*. Chichester, UK: John Wiley & Sons.

Francis, A. 2004. *Business Mathematics and Statistics*, sixth edition. London, UK: Thomson Learning.

Francis, J., Olsson , P., Oswald, D.R. 2000 . Comparing the accuracy and explainability of dividend, free cash flow, and abnormal earnings equity value estimates. *Journal of Accounting Research*, 38(1), 45–70, Spring.

Frankel R., Lee, C.M.C. 1998. Accounting valuation, market expectation, and cross-sectional stock returns, *Journal of Accounting and Economics*, 25, 283–319.

Franks, J.R., Hodges, S.D. 1984. The meaning of accounting numbers in target setting and performance measurement: implications for managers and regulators, presented at the *Annual Meeting of the American Finance Association*, San Francisco, 28-30 December 1983. Reprinted in R. Brief & K.V. Peasnell (Eds.), *A Link Between Accounting and Finance*. New York: Garland, 1996.

Friedl, G. 2005. Incentive properties of residual income when there is an option to wait. *Schmalenbach Business Review* 57(1) (January), 3-21.

Gallo, P. Peccati, L. 1993. The appraisal of industrial investments: a new method and a case study. *International Journal of Production Economics*, 465–476.

Garrod, N., Valentincic, A. 2005. The term structure of implicit discount rates in security valuation. *Journal of Business Finance & Accounting*, 32(7/8) (september/October), 1237–1274.

Ghiselli Ricci, R., Magni, C.A. 2006. Economic Value Added and Systemic Value Added: Symmetry, additive coherence and differences in performance. *Applied Financial Economics Letters*, 2(3), 151–154, May.

Goetzmann, W.N., Garstka, S.J. 1999. The Development of Corporate Performance Measures: Benchmarks Before EVA. Yale ICF Working Paper No. 99-06. Available at SSRN: <http://ssrn.com/abstract=170674>.

Goodman, N. 1947. The problem of counterfactual conditionals. *Journal of Philosophy*, 44, 113-128.

Göx, R.F. 2005. Discussion of "Incentive properties of residual income when there is an option to wait. *Schmalenbach Business Review* 57(1) (January), 22-28.

Grant J. 1996. Foundations of EVA for investment managers. *Journal of Portfolio Management*, Fall, 41–48.

Grant J. 1998. Security analysis using EVA. In F.J. Fabozzi (Ed.), *Handbook of Portfolio Management*. New Hope, PA: Frank J. Fabozzi Associates.

Grinyer, J.R. 1985. Earned Economic Income – A theory of matching. *Abacus*, 21(2), 130–148.

Grinyer, J.R. 1987. A new approach to depreciation. *Abacus*, 23(2), 43–54.

Grinyer, J.R. 1995. Analytical Properties of Earned Economic Income – a response and extension. *British Accounting Review*, 27, 211–228.

Grinyer, J.R., Walker, M. 1990. Deprival value accounting rates of return. *Economic Journal*, 100(3) (September), 918–922.

Gronchi, S. 1984. *Tasso di rendimento e valutazione dei progetti: una analisi teorica* [*Return Rate and Project Evaluation: A Theoretical Analysis*]. Siena: Università degli Studi di Siena, Collana dell'Istituto di Economia. Reprinted 1987 by Franco Angeli, Milano.

Gronchi, S. 1986. On investment criteria based on the internal rate of return. *Oxford Economics Papers*, 38(1), 174–180.

Hagemeister, M., Kempf, A. 2006. Employing the residual income model in portfolio optimization. Available at <http://www.wiso.uni-koeln.de/finanzierung/version2004/mitarbeiter/ehem_ma/Employing_RIM.pdf>

Hagemeister, M., Kempf, A. 2007. CAPM und erwartete Renditen: Eine Untersuchung auf Basis der Erwartung von Marktteilnehmern. CFR-working paper no. 07-01, University of Cologne. Available at <http://www.cfr-cologne.de/download/workingpaper/cfr-07-01.pdf>.

Hamilton, R. 1777. *An Introduction to Merchandize*. Edinburgh.

Hansen, P. 1972 *The Accounting Concept of Profit*, second edition. Amsterdam: North-Holland.

Harris, R.S., Pringle, J.J. 1985. Risk-adjusted discount rates extensions from the average-risk case, *Journal of Financial Research*, 215–225, Fall.

Hicks, J. 1946. *Value and Capital: An Inquiry into Some Fundamental Principles of Economic Theory*, second edition. Oxford: Clarendon Press.

Hülsmann, J. G. 2003. Facts and counterfactuals in economic law. *Journal of Libertarian Studies* 17(1) (Winter), 57-102.

Kahneman, D., Miller, D.T. 1986. Norm theory: Comparing reality to its alternatives. *Psychological Review*, 93, 136-153.

Kahneman, D. Tversky, A. 1982. The simulation heuristic. In D. Kahneman, P. Slovic & A. Tversky (Eds.), *Judgment under uncertainty: Heuristics and biases.* New York: Cambridge University Press.

Kay, J.A. 1976. Accountants, too, could be happy in the golden age: The accountants rate of profit and the internal rate of return. *Oxford Economic Papers*, 28(3) (November), 447–460.

Keefe R. 2000. *Theory of Vagueness.* Cambridge, UK: Cambridge University Press.

Keefe R. 2008 Vagueness: Supervaluationism. *Philosophy Compass*, 3(2), 315-324.

Kellison, S.G. 1991. *The Theory of Interest*, second edition. Homewood, IL: Irwin/McGraw-Hill.

Keynes J.M. 1936. *The General Theory of Employment, Interest and Money.* London: Macmillan.

Keynes, J.M. 1967. *The General Theory of Employment, Interest and Money.* London: MacMillan.

Kneale, W. 1950. Natural Laws and Contrary to Fact Conditionals. *Analysis*, 10, 121–125. Landman,

Leake, P.D. 1921. *Goodwill. Its History, Value and Treatment in Accounts.* London: Pitman and Sons Ltd.

Lee, M.C. 1999. Accounting-based valuation: impact on business practices and research. *Accounting Horizons*, 13(4), 413–425.

Lee, T. 1985. *Income and Value Measurement. Theory and Practice,* third edition. Worcester, UK: Billing & Sons.

Levi E. 1964. *Corso di matematica finanziaria e attuariale* [Handbook of Financial and Actuarial Mathematics]. Milano: Giuffrè.

Lücke, W. 1955. Investitionsrechnungen auf der Grundlage von Ausgaben oder Kosten. *Zeitschrift für betriebswirthschaftliche Forschung*, 7, 310–324.

Luciano, E. 1989. A new perspective on dynamic portfolio policies. *Decisions in Economics and Finance*, 12(1) (First Semester), 91–104.

Luciano, E., Peccati, L. 1993. A decomposition of random net present values in E.J. Stokking & L. Zambruno (Eds.), *Recent Research in Financial Modelling*. Heidelberg: Physica-Verlag.

Lundberg, C.G., Frost, D.E. 1992. Counterfactuals in financial decision making, *Acta Psychologica*, 79, 227-244.

Lundholm, R., O'Keefe, T. 2001. Reconciling value estimates from the discounted cash flow model and the residual income model. *Contemporary Accounting Research*, 18(2) (Summer), 311–335.

Madden, B.J. 1999. *Cash Flow Return On Investment: A Total System Approach to Valuing the Firm*. Oxford, UK: Butterworth-Heinemann.

Magni, C.A. 1993. Valore attuale netto e politiche ottimali d'indebitamento [Net Present value and optimal debt policies], *Proceedings of the XVII AMASES Conference (Association for Mathematics Applied to Economic and Social Sciences)*, Ischia, Italy, September.

Magni, C.A. 2000a. Decomposition of a certain cash flow stream: differential systemic value and net final value, *Proceedings of the XXIV AMASES Conference (Association for Mathematics Applied to Economic and Social Sciences)*, Padenghe, Italy, September. Available at SSRN: <http://ssrn.com/abstract=1096213>.

Magni, C.A. 2000b. Tir, Roe e Van: convergenze formali e concettuali in un approccio sistemico [IRR, ROE and NPV: formal and conceptual convergences in a systemic approach]. *Finanza Marketing e Produzione*, 18(4) (December), 31–59. Available at SSRN: <http://ssrn.com/abstract=1104597>.

Magni, C.A. 2000c. Systemic Value Added, residual income and decomposition of a cash flow stream. *Working paper* n. 318, Department of Economics, University of Modena and Reggio Emilia.
Available at SSRN: <http://ssrn.com/abstract=1032011>

Magni, C.A. 2001a. Valore aggiunto sistemico: un'alternativa all'EVA quale indice di sovraprofitto periodale [Systemic Value Added: an alternative to EVA as a residual income model]. *Budget*, 25(1), 63–71. Available in bilingual version at SSRN: <http://ssrn.com/abstract=1103492>.

Magni, C.A. 2001b. Scomposizione di sovraprofitti: Economic Value Added e Valore Aggiunto Sistemico [Excess-profit decomposition: Economic Value Added and Systemic Value Added]. *Finanza marketing e produzione*, 19(4), 94–119, December. Available at SSRN: <http://ssrn.com/abstract=1140163>.

Magni, C.A. 2003. Decomposition of net final values: Systemic Value Added and residual income, *Bulletin of Economic Research* 55(2), 149–176.

Magni, C.A. 2004. Modelling excess profit, *Economic Modelling*, 21(3), 595–617.

Magni, C.A. 2005. On decomposing net final values: EVA, SVA and shadow project. *Theory and Decision.* 59(1), 51–95.

Magni, C.A. 2006. Zelig and the art of measuring excess profit, *Frontiers in Finance and Economics*, 3(1) (June), 103–129.

Magni, C.A. 2007a. Residual income and value creation: An investigation into the lost-capital paradigm. Available at SSRN: <http://ssrn.com/abstract=1029680>.

Magni, C.A. 2007b. Measuring performance and valuing firms: In search of the lost capital. Available at SSRN: <http://ssrn.com/abstract=1029244>.

Magni, C.A. 2007c. A Sum&Discount method of appraising firms: An illustrative example. Available at SSRN: <http://ssrn.com/abstract=1033522>.

Magni, C.A. 2007d. CAPM and capital budgeting: present versus future, equilibrium versus disequilibrium, decision versus valuation. Available at SSRN: <http://ssrn.com/abstract=1024716>.

Magni, C.A. 2008a. Opportunity cost, excess profit, and counterfactual conditionals, *Frontiers in Finance and Economics*, forthcoming. Available at SSRN: <http://ssrn.com/abstract=1028749>.

Magni, C.A. 2008b. Economic profit, NPV and CAPM: Biases and violations of Modigliani and Miller's Proposition I. *The ICFAI Journal of Applied Finance*, 14(10) (October), 59–72.

Magni, C.A. 2009. Correct or incorrect application of the CAPM? Correct or incorrect decisions with the CAPM?. *European Journal of Operational Research*, 192(2) (January), 549–560.

Manca, P. 1989. The splitting up of a financial project into uniperiodic consecutive projects, *Decisions in Economics and Finance*, 12(1) (March), 107–117.

Marena M. 1991a. The decomposition of NPV in insurance. *Ninth Meeting of the EURO Working Group on Financial Modelling*, Curaçao.

Marena M. 1991b. The decomposition of NPV in non-life Insurance, *Tenth Meeting of the EURO Working Group on Financial Modelling*, London, November 21-23.

Marshall, A. 1890. Principles of Economics. London

Martin, J.D., Petty, J.W. 2000. *Value Based Management. The Corporate Response to Shareholder Revolution.* Boston, MA: Harvard Business School.

Martin, J.D., Petty, J.W., Rich, S.P. 2003. An analysis of EVA and other measures of firm performance based on residual income. Hankamer School of Business Working paper.
Available at SSRN: <http://papers.ssrn.com/sol3/papers.cfm?abstract_id=412122>.

McCormack, J., Vytheeswaran, J. (1998) How to use EVA in the oil and gas industry. *Journal of Applied Corporate Finance* 11(3), 109–131.

Mepham, M.J. 1980. The residual income debate. *Journal of Business Finance & Accounting*, 7(2), 183–199.

Miller, M., Modigliani, F. 1961. Dividend policy, growth, and the valuation of shares. *Journal of Business*, 34, 411–433.

Modigliani, F., Miller, M. 1958. The cost of capital, corporation finance and the theory of investment, *American Economic Review*, 48(3), 261–297, June.

Modigliani, F., Miller, M. 1963. Corporate Income Taxes and the Cost of Capital: A correction. *American Economic Review*,53(3) (June), 433-443.

Mohnen, A. 2003. Managerial performance evaluation with residual income - limited investment b budget and NPV-maximization. Available at SSRN: <http://ssrn.com/abstract=481203>.

Mohnen, A., Bareket, M. 2007. Performance measurement for investment decisions under capital constraints. *Review of Accounting Studies*, 12(1) (March), 1–22.

Mowen, M.M., Hansen, D.R. 2008. *Cornerstones of Managerial Accounting*, second edition. Thomson South-Western.

Myers, S.C. 1974. Interactions of corporate financing and investment decisions - Implications for capital budgeting. *J. of Finance*, **29**(1) 1–25, March.

Nash, J. 1950. The bargaining problem. *Econometrica*, 18: 155–162. 48

O'Byrne. S.F. 1996. EVA and market value. *Journal of Applied Corporate Finance*, 9(1), 116–125, Spring.

O'Byrne. S.F. 1997. EVA and shareholder return. *Financial Practice and Education*, Spring/Summer.

O'Byrne. S.F., Young, D.S. 2006. Incentives and investor expectations. *Journal of Applied Corporate Finance*, 18(2) 98–105, Spring.

O'Hanlon, J., Peasnell, K.V. 2000. Residual income and EVA. *Economic and Financial Computing*, Summer, 53–95.

O'Hanlon J., Peasnell, K.V. 2002. Residual income and value creation: the 'missing link', *Review of Accounting Studies*, **7**(2/3), 229–245.

Ohlson, J.A. 1989. Accounting earnings, book values, and dividends: The theory of the clean surplus equation in equity valuation. Unpublished manuscript. Reprinted in R. Brief & K.V. Peasnell (Eds.), *A Link Between Accounting and Finance*. New York: Garland, 1996.

Ohlson. J.A. 1995. Earnings, book values, and dividends in equity valuation. *Contemporary Accounting Research*, 11(2) (Spring), 661–687.

Ohlson. J.A. 2003. Positive (zero) NPV projects and the behavior of residual earnings. *Journal of Business Finance & Accounting*, 30(1/2) (January/March), 37–15.

Ohlson. J.A. 2005. On accounting-based valuation formulae. *Review of Accounting Studies*, 10, 323–347.

Ohlson. J.A., Juettner-Nauroth, B.E. (2005). . Expected EPS and EPS growth as determinants of value. *Review of Accounting Studies*, 10, 349–365.

Peasnell, K.V. 1981. On capital budgeting and income measurement. *Abacus*, 17(1), 52–67, June.

Peasnell, K.V. 1982a. Some formal connections between economic values and yields and accounting numbers. *Journal of Business Finance & Accounting*, 9(3), 361–381.

Peasnell, K.V. 1982b. Estimating the internal rate of return from accounting profit rates. *The Investment Analyst*, April, 26–31.

Peasnell, K.V. 1995. Second thoughts on the analytical properties of earned economic income. *British Accounting Review*, 27, 229–239.

Peccati, L. 1987. DCF e risultati di periodo [DCF and period results], *Proceedings of the XI AMASES Conference (Association for Mathematics Applied to Economic and Social Sciences*, Aosta, Italy, 9-11 September.

Peccati, L. 1989. Multiperiod analysis of a levered portfolio. *Decisions in Economics and Finance*, 12(1) (March), 157–166. Reprinted in J. Spronk & B. Matarazzo (Eds.), 1992, *Modelling for Financial Decisions*. Berlin: Springer-Verlag.

Peccati, L. 1991. *Valutazione analitica e sintetica di attività finanziarie* [Analytic and synthetic evaluation of financial activites]. Supplement to *Rivista milanese di economia*, 39, Lug-Set, serie quaderni n.21. Milano: Laterza.

Peccati, L. 1998. Harmonic means of spread and other. *Proceedings of the XXII AMASES Conference (Association for Mathematics Applied to Economic and Social Sciences*, Genova, Italy, September.

Penman, S.H. 1992. Return to fundamentals. *Journal of Accounting Auditing and Finance*. Reprinted in R. Brief & K.V. Peasnell (Eds.) (1996). *Clean Surplus: A Link Between Accounting and Finance*. New York and London: Garland Publishing, pp. 233–251.

Penman, S.H. 2005. Discussion of "On accounting-based valuation formulae" and "Expected EPS and EPS growth as determinants of value". *Review of Accounting Studies*, 10, 367–378.

Penman S. 2007. *Financial Statement Analysis and Security Valuation*, third edition. Singapore: McGraw-Hill/Irwin.

Pfeiffer, T. 2000. Good and bad news for the implementation of shareholder-value concepts in decentralized organizations. *Schmalenbach Business Review*, 52(1) (January), 68–91.

Pfeiffer, T. 2004. Net present value-consistent investment criteria based on accruals: A generalisation of the residual income-identity. *Journal of Business Finance & Accounting*, 31(7/8) (September), 905–926.

Pfeiffer, T., Schneider, G. 2007. Residual income-based compensation plans for controlling investment decisions under sequential private information. *Management Science*, 53(3), 495–507.

Pfeiffer, T., Velthuis L. 2008. Incentive system design based on accruals. Working paper.

Pope, P. Wang, P. 2003. Positive (zero) NPV projects and the behavior of residual earnings: discussion and extension. *Journal of Business Finance & Accounting*, 30(1-2), 16–22.

Pratt, S.P., Reilly, R.F., Schweihs, R.P. 1996. *Valuing a Business : The Analysis and Appraisal of Closely-Held Companies*, third edition. McGraw-Hill.

Preinreich, G. 1936. The fair value and yield of common stock. *The Accounting Review* Reprinted in R. Brief & K.V. Peasnell (Eds.) (1996). *Clean Surplus: A Link Between Accounting and Finance*. New York and London: Garland Publishing.

Preinreich, G. 1937. Valuation and amortization. *The Accounting Review*, 12(3), 209–226, September.

Preinreich, G. 1938. Annual survey of economic theory: the theory of depreciation. *Econometrica* 6(1), 219–241, January.

Pressacco, F., Stucchi, P. 1997. Su una estensione bidimensionale del teorema di scomposizione di Peccati, [A two-dimensional extension of Peccati's decomposition theorem], *Decisions in Economics and Finance*, 20, 169-185.

Promislow, S.D. 2006. *Fundamentals of Actuarial Mathematics*. Chichester, UK: John Wiley & Sons.

Qizilbash M. 2003. Vague language and precise measurement: the case of poverty. *Journal of Economic Methodology*, 10(1), 41–58.

Rajan, M.V., Reichelstein, S. 2008. The economic relevance of historical cost. Working paper.

Rao, R. 1992. *Financial Management*. MacMillan Publishing Company.

Rappaport, A. 1998. *Creating Shareholder Value*. New York: The Free Press.

Reichelstein, S. 1997. Investment decisions and managerial performance evaluation. *Review of Accounting Studies*, 2(2), 157–180.

Rendleman, R. 1978. Ranking errors in CAPM capital budgeting applications, *Financial Management*, 7(4), 40-44, Winter.

Revsine, L., Collins, D.W., Johnson, W.B. 2005. *Financial Reporting and Analysis*, third edition. Upper Saddle River, NJ: Prentice Hall. Education.

Reynolds, L.G. 1963. *Economics: A General Introduction*. Homewood, IL: Richard Irwin.

Robichek, A.A., Myers, S.C. 1965. *Optimal Financing Decisions*. Englewood Cliffs, NJ: Prentice-Hall.

Roese, N.J., Olson, J.M. (Eds.) 1995. *What Might Have Been. The Social Psychology of Counterfactual Thinking*. Mahwah, NJ: Lawrence Erlbaum Associates.

Rogerson, W.P. 1997. Intertemporal cost allocation and managerial investment incentives: A theory explaining the use of Economic Value Added as a performance measure. *Journal of Political Economy*, 105(4), 770–795.

Rogerson, W.P. 2008 Inter-temporal cost allocation and investment decisions. First version: 2006. Available at <http://www.chicagogsb.edu/research/workshops/theoryoforg/archive/oldwebfilesprefall2006/rogersonapril3spring06paperwto.pdf>.

Rose, M., Wiswesser, R. 1998. Tax reform in transition economies: Experiences from the Croatian tax reform process of the 1990s. In Sørensen, P.B. (Ed.), *Public Finance in a Changing World*, MacMillan.

Ruback, R.S. 2002. Capital Cash Flows: a simple approach to valuing cash flows. *Financial Management*, Summer, 85–103.

Samuels, J.M., F.M. Wilkes and R.E. Brayshaw 1990. *Management of Company Finance*, fifth edition. London: Chapman and Hall.

Samuelson, P. 1964. Tax deductibility of economic depreciation to insure invariant valuations. *Journal of Political Economy*, 72, 604–606.

Scapens, R.W. 1978. A neoclassical measure of profit, *The Accounting Review*, 53(2) (April), 448–469.

Scapens, R.W. 1979. Profit measurement in divisionalised companies, *Journal of Business Finance & Accounting*, 6(3) (Autumn), 281–305.

Schueler A. 2000. Periodische Performance-Messung durch Residualgewinne [Periodical performance via residual income] *Deutsches Steuerrecht*, 49, 2105–2108.

Schueler, A. 2001. Jahresabschlussdaten und Perfomance-Messung [Accounting data and performance measurement]. In G. Seicht (Ed.), *Jahrbuch für Controlling und Rechnungswesen*, pp. 141–158.

Schueler, A., Krotter, S. 2004. Konzeption wertorientierter Steuerungsgrössen: Performance-Messung mit Discounted Cash-flows und Residualgewinnen ex ante und ex post [Designing value based performance measures: Performance measurement with discounted cash flows and residual incomes ex ante and ex post]. *Finanz Retrieb*, 6, 430–437.

Schueler, A., Krotter, S. 2008. The link between residual income and value created for levered firms: A note. *Management Accounting Research*, forthcoming. Available at <http://dx.doi.org/10.1016/j.mar.2008.02.003>.

Schultze, W., Weiler, A. 2008. Performance measurement, value-creation and managerial compensation: The missing link. Available at SSRN: <http://ssrn.com/abstract=1088702>.

Scott, A.D. 1953. Notes on user cost. *Economic Journal*, 63, 368–384, June.

Capital asset prices: a theory of market equilibrium under conditions of risk. *Journal of Finance*, 19(3), 425-442.

Sloan, L.H. 1929. *Corporate Profits: A Study of Their Size, Variation, Use, and Distribution in a Period of Prosperity*. New York: Harper and Brothers Publishers.

Solomons, D. 1965. *Divisional Performance: Measurement and Control*. Homewood, IL: Richard D. Irwin.

Soper, C.S. 1959. The marginal efficiency of capital. *Economic Journal*, 69, 174-177 March.

Sørensen, P.B. 1994. From the global income tax to the dual income tax: Recent tax reforms in the Nordic countries, *International Tax and Public Finance*, 1(1) (February), 57–79.

Sørensen, P.B. (Ed.) 1998. *Tax policy in the Nordic Countries*. McMillan.

Stark, A.W. 1986. More on the discounting of residual income. *Abacus*, 22(1) (March), 20–28.

Stark, A.W. 2000. Real options (dis)investment decision-making and accounting measures of performance. *Journal of Business Finance & Accounting*, 27(3/4), 313-331.

Stewart, G.B. 1991. *The Quest for Value: the EVA^{TM} Management Guide*. HarperCollins Publishers Inc.

Stoughton, N.M., Zechner, J. 2007. Optimal capital allocation using RAROC^{TM} and EVA^{TM}. *Journal of Financial Intermediation*, 16(3), 312–342.

Sugden, R. 2000. Credible worlds: the status of theoretical models in economics. *Journal of Economic Methodology*, 7(1), 1–31.

Teichroew, D., Robichek, A., Montalbano, M. 1965a. Mathematical analysis of rates of return under certainty. *Management Science*, 11, 395-403.

Teichroew, D., Robichek, A., Montalbano, M. 1965b. An analysis of criteria for investment and financing decisions under certainty. *Management Science*, 12, 151-179.

Tham, J., I. Vélez-Pareja 2001. The correct discount rate for the tax shield: the N-period case. Available at SSRN: <http://ssrn.com/abstract=267962>.

Tham, J., Vélez-Pareja, I. 2004. *Principles of Cash Valuation*. Burlington, MA: Elsevier Academic Press.

Tomkins, C. 1975. Residual income–a rebuttal of Professor Amey's Argument. *Journal of Business Finance & Accounting*, 2(2), 161–168.

Uberti M. 1993. Present value decomposition of foreign currency assets. In J. Stokking & G. Zambruno (Eds.), *Recent Research in Financial Modelling*, Heidelberg: Physica-Verlag, pp. 105-116.

Van de Mieroop, M. 2005. *The invention of interest. Sumerian loans*. In In W.N. Goetzmann & K.G. Rouwenhorst (Eds.), *The Origins of Value. The Financial Innovations That Created Modern Capital Markets*, Oxford University Press, pp. 17-30.

Vélez-Pareja, I. 2001. Economic value measurement: Investment recovery and value added — IRVA. Available at SSRN: <http://ssrn.com/abstract=223429>.

Vélez-Pareja, I., Tham, J. 2003. Do the RIM (Residual Income Model), EVA and DCF (Discounted Cash Flow) really match?. *Working Paper* n. 25. <http://ssrn.com/abstract=379740>.

Vélez-Pareja, I., Tham, J. 2004. EVA made simple: Is it possible?. Available at SSRN: <http://ssrn.com/abstract=497545>.

Weaver S.C., Weston, J.F. 2003. A unifying theory of value based management. Paper 4-03, November 27. Available at <http://repositories.cdlib.org/anderson/fin/4-03/>.

Weingartner, H.M. 1966. The generalized rate of return. *The Journal of Financial and Quantitative Analysis*, 1(3), 1–29, September.

Wells, G.L., Taylor, B.R., Turtle, J.W. 1987. The undoing of scenarios. *Journal of Personality and Social Psychology*, 53(3), 421–430.

Werner, F., Sotskov, Y.N. 2006. *Mathematics of Economics and Business*. New York: Routledge.

Yee, K.K. 2005. Aggregation, dividend irrelevancy, and earnings-value relations. *Contemporary Accounting Research*, 22(2), 453–480, Summer.

Young, S.D., O'Byrne, S.F. 2001. *EVA and Value-Based Management*. McGraw-Hill.

Zadeh, L. 1965. Fuzzy sets. *Information and Control*, 8, 338–353.

Zimmermann, H.J. 2001. *Fuzzy Set Theory and its Applications*, fourth edition, Springer.

Table 0a. Main Notational Conventions

Symbol	*Meaning*	*Formalization*
A	project (firm)	
a_t; a_n^*	cash flow; time-n cash flow from operations	
$\vec{a}$; $\vec{a}_t$	project's, sub-project's cash-flow vector	$(-a_0, a_1, \ldots, a_n)$;$(\vec{0}_{t-2}, -w_{t-1}, w_t + a_t, \vec{0}_{n-t})$
b_t	Book value of assets	
c_t	Capital cash flow	$e_t + d_t$ or $rol_t \cdot v_{t-1} + (v_{t-1} - v_t)$
d_t	Installment, Cash flow to debt	$c_t - e_t$ or $\delta_t D_{t-1} + (D_{t-1} - D_t)$
D_t	Debt (nominal value=market value)	$\sum_{k=t+1}^n d_k \frac{\varphi_k(\rho^D)}{\varphi_t(\rho^D)}$
δ_t	nominal debt rate	I_t/D_{t-1}
Δ	Variation	
e_t	equity cash flow	$c_t - d_t$ or $roe_t \cdot v_{t-1}^{\mathrm{e}} + (v_{t-1}^{\mathrm{e}} - v_t^{\mathrm{e}})$
F	Financial asset	
F_t; F^t	Factual, counterfactual value of asset F	$F_{t-1}(1+i_t)+a_t$; $F^{t-1}(1+i_t)$
f_t	Free cash flow	$c_t - \tau I_t$ or $roi_t \cdot v_{t-1} + (v_{t-1} - v_t)$
$\varphi_t(\vec{\imath})$; $\varphi_t(\vec{r})$	Discount factors	$\left[\prod_{k=1}^t (1+i_k)\right]^{-1}$; $\left[\prod_{k=1}^t (1+r_k)\right]^{-1}$
i, i_t, $\vec{\imath}$	Opportunity cost of capital (scalar, vector)	
i^*	Comprehensive cost of capital	$i_t \cdot \left(1 + \frac{w_{t-1}(\vec{\imath}) - w_{t-1}(\vec{r})}{w_{t-1}(\vec{r})}\right)$
I_t	interest on debt	$\delta_t D_{t-1}$
$\mathcal{L}$	Lost capital	
n^*	time n, prior to distribution of R_n	
N_0, $N_0(\vec{a})$	Project's Net Present Value	$-a_0 + \sum_{t=1}^n a_t \varphi_t(\vec{\imath})$
$N_0(\vec{a}_t)$	Sub-project's NPV	$-\varphi_{t-1}(\vec{\imath}) w_{t-1} + \varphi_t(\vec{\imath})(w_t + a_t)$
N_t	Project's time-t Net Present Value	$N_0 \prod_{k=1}^t (1+i_k)$
N_n	Project's Net Final Value	$N_0 \prod_{k=1}^n (1+i_k)$
π_t	Income	$a_t + (w_t - w_{t-1})$
π_t^{e}	Standard residual income	$w_{t-1}(r_t - i_t)$
Π_t^{e}	Lost-capital residual income	$r_t w_{t-1}(\vec{r}) - i_t w_{t-1}(\vec{\imath})$
p	Portfolio traded in the capital market	
p_0	price of p	$\sum_{t=1}^n a_t \varphi_t(\vec{\imath})$
r_t	one-period (internal) rate of return	$\frac{w_t - w_{t-1} + a_t}{w_{t-1}}$
r, $\vec{r}$	internal rate of return/financial law	$\sum_{t=1}^n a_t(1+r)^{-t}$=0; $\sum_{t=1}^n a_t \varphi(\vec{r})$=0
r^*, $r^*_{\mathcal{L}}$	project's average yield	$\frac{\sum_{t=1}^n r_t \varphi_t(\vec{\imath}) w_{t-1}(\vec{r})}{\sum_{t=1}^n \varphi_t(\vec{\imath}) w_{t-1}(\vec{r})}$; $\frac{\sum_{t=1}^n r_t w_{t-1}(\vec{r})}{\sum_{t=1}^n w_{t-1}(\vec{r})}$
R_n	Terminal (scrap) value	$a_n - a_n^*$
ρ	Weighted average cost of capital	$\frac{\rho_t^{\mathrm{e}} v_{t-1}^{\mathrm{e}} + \rho_t^D D_{t-1}(1-\tau)}{v_{t-1}^{\mathrm{e}} + D_{t-1}}$
ρ^D; ρ^{e}	Cost of debt; Cost of equity	$\frac{D_t - D_{t-1} + d_t}{D_{t-1}}$; $\frac{v_t^{\mathrm{e}} - v_{t-1}^{\mathrm{e}} + e_t}{v_{t-1}^{\mathrm{e}}}$
ρ^U	Unlevered cost of equity (cost of assets)	
ρ^τ	Pre-tax weighted average cost of capital	$\frac{\rho_t^{\mathrm{e}} v_{t-1}^{\mathrm{e}} + \rho_t^D D_{t-1}}{v_{t-1}^{\mathrm{e}} + D_{t-1}}$
$\mathcal{S}$	Standard	
τ	tax rate	
v_t, v_t^{e}	Market value, equity value of the project/firm	$\sum_{k=t+1}^n f_k \frac{\varphi_k(\vec{\rho})}{\varphi_t(\vec{\rho})}$; $\sum_{k=t+1}^n e_k \frac{\varphi_k(\vec{\rho}^{\mathrm{e}})}{\varphi_t(\vec{\rho}^{\mathrm{e}})}$
v^U	Market value of the unlevered firm	$\sum_{k=t+1}^n f_k \frac{\varphi_k(\vec{\rho}^U)}{\varphi_t(\vec{\rho}^U)}$
w_t	outstanding capital	$w_{t-1} + \pi_t - a_t$
$w_t(\vec{r})$; $w_t(\vec{\imath})$	Factual, counterfactual capital	$w_{t-1}(\vec{r})(1+r_t) - a_t$; $w_{t-1}(\vec{\imath})(1+i_t) - a_t$
W_t; W^t	Factual, counterfactual investor's wealth	$W_{t-1}\left(1 + \frac{r_t w_{t-1} + i_t F_{t-1}}{w_{t-1} + F_{t-1}}\right)$; $W^{t-1}(1+i_t)$

Table 0b. Acronyms

Acronym	*Meaning*	*Formalization*
APV	Adjusted Present Value	
cfroi	Cash flow return on investment	IRR of $(-f_0, f_1, \dots, f_n)$
cfrol	Cash flow return on liabilities	IRR of $(-c_0, c_1, \dots, c_n)$
cfroe	Cash flow return on equity	IRR of $(-e_0, e_1, \dots, e_n)$
CRI	Claimholders' Residual Income	see Table 6
CSV	Created Shareholder Value	see Table 8
DCF	Discounted-cash-flow	
$ebit_t$	Earnings before interest and taxes	$\frac{nopat_t}{1-\tau}$ or $roi_t \cdot b_{t-1}/(1-\tau)$
EBO	Edwards-Bell-Ohlson's Residual Income	see Table 6
EVA	Economic Value Added	see Table 6
IFL	Internal Financial Law	$\{r_t\}$ such that $\sum_{t=1}^n a_t\varphi(\vec{r}) = 0$
IRR	Internal rate of return	r_t=r for all t such that $\sum_{t=1}^n a_t(1+r)^{-t} = 0$
KEP	Keynesian excess profit	see eq. (30)
CEI	Claimholders' Economic Income	see Table 8
MM	Modigliani and Miller	
NEI	Net Economic Income	see Table 8
$nopat_t$	Net operating profit after taxes	$roi_t \cdot b_{t-1}$
NPV	Net Present Value	$\sum_{t=1}^n a_t\varphi_t(\vec{\imath}) - a_0$
pat_t	Profit After Taxes	$roe_t \cdot b^{\mathrm{e}}_{t-1}$
RI	Residual income (excess profit)	see eq. (4)
RI^{cfroi}	*cfroi*-based Residual Income	see Table 7
RI^{cfrol}	*cfrol*-based Residual Income	see Table 7
RI^{cfroe}	*cfroe*-based Residual Income	see Table 7
roe_t	Return on equity	$\frac{pat_t}{b^{\mathrm{e}}_{t-1}}$
roi_t	Return on investment	$\frac{nopat_t}{b_{t-1}}$
rol_t	Return on liabilities	$\frac{pat_t+I_t}{b_{t-1}}$ or $\frac{nopat_t+\tau I_t}{b_{t-1}}$
VTS	Value of Tax Shield	$\sum_{t=1}^n \tau I_t\varphi_t(\rho^D)$

Table 1. The income schema

	π_t	a_t	w_t
Microeconomics	personal income	consumption	consumer's wealth
Accounting	earnings	distribution to claimholder	equity or entity value
Capital budgeting	return	cash flow to investor	project balance
Security analysis	return	cash flow to stockholder	stock's value
Loan contract	interest	installment	principal (residual debt)

Table 2. Synonyms

Abnormal earnings (Ohlson, 1989, 1995; Francis, Olsson and Oswald, 2000; Yee, 2005; Revsine, Collins and Johnson, 2005)—— ***Abnormal gain*** (Grant, 1998)—— ***Abnormal profit*** (Bromwich and Walker, 1998)—— ***Adjusted income*** (Peasnell, 1981, 1982)—— ***Adjusted profit*** (Carsberg, 1966)—— ***Economic income*** (Grant, 1998)—— ***Economic profit*** (Archer and D'Ambrosio, 1972; Boadway and Bruce, 1984; Rao, 1992; Ehrbar, 1998; Grant, 1998; Martin and Petty, 2000; Fabozzi and Grant, 2000; Arnold, 2000, 2005, 2007; Magni, 2008b) —— ***Economic Rent*** (Cnossen, 1998; Ehrbar, 1998)—— ***Economic Value Added*** (Rogerson, 1997; Ehrbar, 1998; Magni, 2000a,b,c; 2001a,b; 2003, 2004, 2005, 2006); Stoughton and Zechner (2007)—— ***Excess earnings*** (Preinreich, 1936; Pratt, Reilly and Schweihs, 1996)—— ***Excess economic income*** (Pratt, Reilly and Schweihs, 1996)—— ***Excess income*** (Peasnell, 1982)—— ***Excess profit*** (Preinreich, 1937, 1938; Edwards and Bell, 1961; Weaver and Weston, 2003; Magni, 2001b, 2004, 2006)—— ***Excess realizable profit*** (Edwards and Bell, 1961)—— ***Excess return*** (Rendleman, 1978; Damodaran, 2006) ***Financial margin*** (Peccati, 1987, 1989, 1991; Marena, 1991a; Camillo and Marena, 1994) ***Period contribution*** (Peccati, 1989)—— ***Period margin*** (Gallo and Peccati, 1993; Pressacco and Stucchi, 1997)—— ***Profit*** (Anthony, 1975; Reynolds, 1963)—— ***Pure earnings, pure profit*** (Bodenhorn, 1964; Boadway and Bruce, 1984; Cnossen, 1998)—— ***Quasi-rent*** (Peasnell, 1981)—— ***Residual earnings*** (Ohlson, 2003; Pope and Wang, 2003; Penman, 2007)—— ***Residual (capital) return*** (Grant, 1998)—— ***Residual income*** (Solomons, 1965; Tomkins, 1975; Emmanuel and Otley, 1976; Kay, 1976; Mepham, 1980; Peasnell, 1981, 1982; Ohlson, 1989, 1995; Grant, 1996; Bromwich and Walker, 1998; Ehrbar, 1998; Biddle, Bowen and Wallace, 1999; Lee, 1999; Arnold, 2000; Martin and Petty, 2000; Magni, 2000a, 2003; Lundholm and O'Keefe, 2001; Martin, Petty and Rich, 2003; Mowen and Hansen, 2008)—— ***Super-profit*** (Leake, 1921; Edey, 1957; Samuels, Wilkes and Brayshaw, 1990; Bromwich and Walker, 1998; Fernández, 2002)—— ***Supernormal profit*** (Begg, Fischer, and Dornbusch, 1984)—— ***Surplus profit*** (Dicksee, 1897; Carsberg, 1966)—— ***Value added*** (Tham and Vélez-Pareja, 2004)

Table 3. Arbitrage strategy

	Cash flows				
Time	0	1	2	…	n
Long position on A	$-a_0$	a_1	a_2	…	a_n
Short position on p	p_0	$-a_1$	$-a_2$	…	$-a_n$
Long position on p	$-(p_0 - a_0)$	0	0	…	Γ
Total	0	0	0	…	Γ

Table 4. Arbitrage strategy: netting out positions on p

	Cash flows				
Time	0	1	2	$\cdots$	n
Long position on A	$-a_0$	a_1	a_2	$\cdots$	a_n
Net short position on p	a_0	$-a_1$	$-a_2$	$\cdots$	$-(a_n-\Gamma)$
Total	0	0	0	$\ldots$	Γ

Table 5. The lost capital $w_t(i)$ in the literature

Label	Authors
External financing	Pressacco and Stucchi (1997)
Balance of the shadow project	Magni (2000-2006. See References)
Invested capital	Schueler (2000), Drukarczyk and Schueler (2000)
Initial investment not recovered	Vélez-Pareja (2001)
Adjusted invested capital	Young and O'Byrne (2001)†
Unrecovered capital	O'Hanlon and Peasnell (2002)
Lost capital	Magni (2007)

†Only under the assumption earnings=cash flows

Table 6. Accounting-based residual income

	IFL	Fundamental Equation	Residual Income
EVA	$roi_t=\frac{nopat_t}{b_{t-1}}$	$b_t=b_{t-1}(1+roi_t)-f_t$	$b_{t-1}(roi_t-\rho_t)$
CRI	$rol_t=\frac{nopat_t+\tau I_t}{b_{t-1}}$	$b_t=b_{t-1}(1+rol_t)-c_t$	$b_{t-1}(rol_t-\rho_t^\tau)$
EBO	$roe_t=\frac{pat_t}{b^{\rm e}_{t-1}}$	$b^{\rm e}_t=b^{\rm e}_{t-1}(1+roe_t)-e_t$	$b^{\rm e}_{t-1}(roe_t-\rho^{\rm e}_t)$

Table 7. IRR-based residual income

	IFL	Fundamental Equation	Residual Income
RI^{cfroi}	$cfroi$	$w_t(cfroi)=w_{t-1}(1+cfroi)-f_t$	$w_{t-1}(cfroi)(cfroi-\rho_t)$
RI^{cfrol}	$cfrol$	$w_t(cfrol)=w_{t-1}(1+cfrol)-c_t$	$w_{t-1}(cfrol)(cfrol-\rho_t^\tau)$
RI^{cfroe}	$cfroe$	$w_t(cfroe)=w_{t-1}(1+cfroe)-e_t$	$w_{t-1}(cfroe)(cfroe-\rho^{\rm e}_t)$

Table 8. Market-based residual income

	IFL	Fundamental Equation	Residual Income
NEI	$r_1{=}\frac{v_1-f_0+f_1}{f_0}$; $r_t{=}\rho_t$	$v_1{=}f_0(1{+}r_1){-}f_1$; $v_t{=}v_{t-1}(1{+}\rho_t){-}f_t$	$f_0(r_1-\rho_1)$; $\rho_t v_{t-1}{-}\rho_t w_{t-1}(\rho_t)$
CEI	$r_1{=}\frac{v_1-c_0+c_1}{c_0}$; $r_t{=}\rho_t^\tau$	$v_1{=}c_0(1{+}r_1){-}c_1$; $v_t{=}v_{t-1}(1{+}\rho_t^\tau){-}c_t$	$c_0(r_1-\rho_1^\tau)$; $\rho_t^\tau v_{t-1}{-}\rho_t^\tau w_{t-1}(\rho_t^\tau)$
CSV	$r_1{=}\frac{v_1^{\mathrm{e}}-e_0+e_1}{e_0}$; $r_t{=}\rho_t^{\mathrm{e}}$	$v_1^{\mathrm{e}}{=}e_0(1{+}r_1){-}e_1$; $v_t^{\mathrm{e}}{=}v_{t-1}^{\mathrm{e}}(1{+}\rho_t^{\mathrm{e}}){-}e_t$	$e_0(r_1-\rho_1^{\mathrm{e}})$; $\rho_t^{\mathrm{e}} v_{t-1}{-}\rho_t^{\mathrm{e}} v_{t-1}=0$

Table 9. Classification of the nine residual income models

	Entity approach (*Free Cash Flow*)	**Claimholders' approach** (*Capital Cash Flow*)	**Equity approach** (*Equity Cash Flow*)
Accounting-based RI	EVA	CRI	EBO
IRR-based RI	RI^{cfroi}	RI^{cfrol}	RI^{cfroe}
Market-based RI	NEI	CEI	CSV

Table 10. Ten models for valuing a project†

Theory	Perspective: *Entity*	*Claimholders*	*Equity*
APV	$\sum_{t=1}^n \varphi_t(\vec{\rho}^u) f_t + \mathrm{VTS}$	–	–
DCF	$\sum_{t=1}^n \varphi_t(\vec{\rho}) f_t$	$\sum_{t=1}^n \varphi_t(\vec{\rho}^\tau) c_t$	$\sum_{t=1}^n \varphi_t(\vec{\rho}^{\mathrm{e}}) e_t$
$\mathcal{S}$-RI	$\begin{cases}\sum_{t=1}^n \varphi_t(\vec{\rho})\ \mathrm{EVA}\\ \sum_{t=1}^n \varphi_t(\vec{\rho})\ \mathrm{RI}^{cfroi}\\ \sum_{t=1}^n \varphi_t(\vec{\rho})\ \mathcal{S}(\mathrm{NEI})\end{cases}$	$\begin{cases}\sum_{t=1}^n \varphi_t(\vec{\rho}^\tau)\ \mathrm{CRI}\\ \sum_{t=1}^n \varphi_t(\vec{\rho}^\tau)\ \mathrm{RI}^{cfrol}\\ \sum_{t=1}^n \varphi_t(\vec{\rho}^\tau)\ \mathcal{S}(\mathrm{CEI})\end{cases}$	$\begin{cases}\sum_{t=1}^n \varphi_t(\vec{\rho}^{\mathrm{e}})\ \mathrm{EBO}\\ \sum_{t=1}^n \varphi_t(\vec{\rho}^{\mathrm{e}})\ \mathrm{RI}^{cfroe}\\ \sum_{t=1}^n \varphi_t(\vec{\rho}^{\mathrm{e}})\ \mathrm{CSV}\end{cases}$
$\mathcal{L}$-RI	$\begin{cases}\varphi_n(\vec{\rho}) \sum_{t=1}^n \mathcal{L}(\mathrm{EVA})\\ \varphi_n(\vec{\rho}) \sum_{t=1}^n \mathcal{L}(\mathrm{RI}^{cfroi})\\ \varphi_n(\vec{\rho}) \sum_{t=1}^n \mathrm{NEI}\end{cases}$	$\begin{cases}\varphi_n(\vec{\rho}^\tau) \sum_{t=1}^n \mathcal{L}(\mathrm{CRI})\\ \varphi_n(\vec{\rho}^\tau) \sum_{t=1}^n \mathcal{L}(\mathrm{RI}^{cfrol})\\ \varphi_n(\vec{\rho}^\tau) \sum_{t=1}^n \mathrm{CEI}\end{cases}$	$\begin{cases}\varphi_n(\vec{\rho}^{\mathrm{e}}) \sum_{t=1}^n \mathcal{L}(\mathrm{EBO})\\ \varphi_n(\vec{\rho}^{\mathrm{e}}) \sum_{t=1}^n \mathcal{L}(\mathrm{RI}^{cfroe})\\ \varphi_n(\vec{\rho}^{\mathrm{e}}) \sum_{t=1}^n \mathcal{L}(\mathrm{CSV})\end{cases}$

†Time subscripts in the metrics are omitted for notational convenience

Table 11. Input data, Balance Sheet, Income Statement

Time	0	1	2	3	4	5
BALANCE SHEET						
Gross fixed assets	**26 000**	**26 000**	**26 000**	**26 000**	**26 000**	**26 000**
−cumulative depreciation	0	−5 200	−10 400	−15 600	−20 800	−26 000
Net fixed assets	26 000	20 800	15 600	10 400	5 200	0
Working capital	**5 000**	**5 000**	**5 000**	**5 000**	**5 000**	**0**
NET ASSETS	31 000	25 800	20 600	15 400	10 200	0
Debt	**10 000**	**9 500**	**8 500**	**8 000**	**7 500**	**0**
Equity (book value)	21 000	16 300	12 100	7 400	2 700	0
TOTAL LIABILITIES	31 000	25 800	20 600	15 400	10 200	0
INCOME STATEMENT						
Sales		**15 300**	**13 400**	**17 400**	**12 400**	**11 500**
Cost of sales		**4 000**	**4 000**	**4 000**	**4 000**	**4 000**
Gen. & Adm. expenses		**1 000**	**1 000**	**1 000**	**1 000**	**1 000**
Depreciation (straight-line)		**5 200**	**5 200**	**5 200**	**5 200**	**5 200**
Earnings before interest and taxes		5 100	3 200	7 200	2 200	1 300
Debt rate (δ)		**8%**	**8%**	**8%**	**8%**	**8%**
Interest		800	760	680	640	600
Profit before taxes		4 300	2 440	6 520	1 560	700
Tax rate		**33%**	**33%**	**33%**	**33%**	**33%**
Taxes		1 419	805	2 152	515	231
Profit after taxes		2 881	1 635	4 368	1 045	469

Table 12. Cash flow and rates

Time	0	1	2	3	4	5
Equity cash flow (e_t)†	−21 000	7 581	5 835	9 068	5 745	3 169
Capital cash flow (c_t)	−31 000	8 881	7 595	10 248	6 885	11 269
Free cash flow (f_t)	−31 000	8 617	7 344	10 024	6 674	11 071
Cash flow to debt (d_t)	−10 000	1 300	1 760	1 180	1 140	8 100
roi		11.02%	8.31%	23.42%	9.57%	8.54%
rol		11.87%	9.28%	24.51%	10.94%	10.48%
roe		13.72%	10.03%	36.10%	14.12%	17.37%
cfroi		12.26%	12.26%	12.26%	12.26%	12.26%
cfrol		13.32%	13.32%	13.32%	13.32%	13.32%
cfroe		16.81%	16.81%	16.81%	16.81%	16.81%

† $e_t = pat_t$ + Depreciation + Δ Debt $-$ Δ Working Capital

Table 13. Valuation with APV and DCF methods

Time	0	1	2	3	4	5
Cost of assets (ρ^U)		**12%**	**12%**	**12%**	**12%**	**12%**
Cost of debt ($\rho^D{=}\delta$)		**8%**	**8%**	**8%**	**8%**	**8%**
v^U	31 207	26 334	22 151	14 785	9 885	0
VTS†	928	738	546	365	183	0
$v{=}v_U{+}\text{VTS}$	32 134	27 072	22 697	15 150	10 068	0
$v^{\mathrm{e}}{=}v_U{+}\text{VTS}{-}D$	22 134	17 572	14 197	7 150	2 568	0
ρ^{e}		13.64%	13.99%	14.24%	16.27%	23.40%
$v^{\mathrm{e}}{=}\sum_{k=t+1}^{n} e_k \frac{\varphi_k(\rho^{\mathrm{e}})}{\varphi_t(\rho^{\mathrm{e}})}$	22 134	17 572	14 197	7 150	2 568	0
ρ		11.06%	10.96%	10.92%	10.51%	9.96%
$v^{\mathrm{e}}{=}\sum_{k=t+1}^{n} f_k \frac{\varphi_k(\rho)}{\varphi_t(\rho)} - D$	22 134	17 572	14 197	7 150	2 568	0
ρ^τ	11.88%	11.89%	11.90%	11.90%	11.93%	
$v^{\mathrm{e}}{=}\sum_{k=t+1}^{n} c_k \frac{\varphi_k(\rho^\tau)}{\varphi_t(\rho^\tau)} - D$	22 134	17 572	14 197	7 150	2 568	0
$N(\vec{e})$	1 134	1 289	1 469	1 679	1 952	2 408
$N(\vec{f})$	1 134	1 260	1 398	1 550	1 713	1 884
$N(\vec{c})$	1 134	1 269	1 420	1 589	1 778	1 990

†The discount rate here used for discounting the tax shield is the cost of debt. While this is irrelevant to the subject of the paper, it is worth noting that there is a lively debate in the literature on the correct discount rate for computing the tax shield. The reader may be willing to turn to the contributions of Myers (1974), Harris and Pringle (1985), Tham and Vélez-Pareja (2001), Arzac and Glosten (2005), Fernández (2005), Cooper and Nyborg (2006) on this topic. The readers may well dismiss the first five rows of the Table, if not in line with their stance, and consider ρ^{e} as exogenously given.

Table 14. Performance metrics and its companions

Period	1	2	3	4	5		1	2	3	4	5
EVA	−13	−686	2 499	45	39	$\mathcal{L}$(EVA)	−13	−685	2 575	−144	−145
CRI	−3	−673	2 596	−148	−148	$\mathcal{L}$(CRI)	−3	−673	2 516	71	80
EBO	17	−646	2 645	−159	−163	$\mathcal{L}$(EBO)	17	−644	2 556	155	325
RI^{cfroi}	371	339	297	258	227	$\mathcal{L}(\text{RI}^{cfroi})$	371	380	379	377	377
RI^{cfrol}	445	375	314	210	138	$\mathcal{L}(\text{RI}^{cfrol})$	445	428	418	364	336
RI^{cfroe}	665	477	358	39	−179	$\mathcal{L}(\text{RI}^{cfroe})$	665	570	534	327	312
$\mathcal{S}$(NEI)	1 260	0	0	0	0	NEI†	1 260	138	153	163	171
$\mathcal{S}$(CEI)	1 269	0	0	0	0	CEI†	1 269	151	169	189	212
CSV	1 289	0	0	0	0	$\mathcal{L}$(CSV)†	1 289	180	209	273	457

†Keynesian Excess Profit

Table 15. Synopsis

1777	**Hamilton** makes reference to a counterfactual income to act as a capital charge
1890	**Marshall** introduces the idea of an excess over a normal profit
1921	**Leake** uses discounted surplus profits in order to compute goodwill
1936	**Keynes** introduces the notion of user cost, which is a basic constituent of the market-based *lost-capital* residual income
1936-1938	**Preinreich** acknowledges that value equals book value plus the discounted excess profits
1957	**Edey** shows, for perpetual streams, that the NPV equals the discounted sum of super-profits
1961	**Edwards and Bell** show, under a finite-horizon assumption, that the NPV is equal to the discounted sum of excess realizable profits
1964	**Bodenhorn** focusses on a two-period horizon and shows that the NPV is equal to the discounted sum of pure earnings
1965	**Solomons** introduces the term "residual income"
1966	**Weingartner** generalizes the notion of internal rate of return by introducing the notion of *internal vector return*
1981, 1982	**Peasnell** investigates in a formal setting relations among residual income, market values, accounting values, accounting rates of return
1987	**Peccati** decomposes a project NPV in terms of spread between internal period rates and opportunity cost of capital and investigates the relations between IRR and internal financial laws.
1989, 1991	**Peccati** splits up residual income into equity component and debt component
1991	**Stewart** propagandizes the use of *Economic Value Added*, an accounting-based residual income in an entity perspective
1995	**Ohlson** finds that, in the long run and under suitable assumptions on the stochastic process of residual incomes, the future market value is approximated by a function of earnings
1997	**Pressacco and Stucchi** (i) generalize Peccati's model, (ii) anticipate O'Hanlon and Peasnell's (2002) Proposition 1, (iii) anticipate the *systemic-value-added* approach.
2000, 2001	**Magni** introduces the *systemic-value-added* (i.e. *lost-capital*) approach as a new theory of RI showing its NPV-consistency via a Sum&Discount procedure.
2000, 2001	**Drukarczyk and Schueler** introduce, in a value-based management context, the *invested capital* (equivalent to lost capital) and the *Net Economic Income*, which is a market-based lost-capital metric (Keynesian Excess Profit)
2001	**Young and O'Byrne** introduce the notion of *Adjusted EVA*, which turns out to be an accounting-based lost-capital metric if earnings equal dividends
2002	**O'Hanlon and Peasnell** use unrecovered capital (equivalent to lost capital) to provide splitting identities that separate realized value from value generated in the past
2004	**Pfeiffer** shows that no accounting-based measure other than the standard RI is NPV-consistent via a Discount&Sum procedure
2007	**Magni** links lost-capital RI with arbitrage theory, Keynes's user cost, abnormal earnings growth, Miller and Modigliani's (1961) investment opportunity approach, Anthony's (1975) profit, O'Hanlon and Peasnell's (2002) splitting identities