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A Long-Wave Pattern for Output and Employment in Pasinetti’s Model of Structural Change

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This paper introduces long waves into Pasinetti’s model of structural change on the assumption that productivity growth is fundamentally driven by technological revolutions (radical process and product innovations). The argument is developed at the logical stage of the “natural” system, focussing the investigation at the sectoral level.

Three general results should be mentioned:

i) the overwhelming importance of the pattern of diffusion of the technological revolution, which shapes the productivity curve of the sector;

* This is a revised and substantially reduced version of Reati (1995).

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ii) the pattern of demand which, for process innovations, results from an endogenous price and income mechanism set up by the technological revolution;

iii) the importance of price and income elasticities of demand, which can amplify or reduce the basic impetus coming from productivity.

More specifically, the sectoral analysis for process innovations shows that physical output in the final sectors follows a long-wave (S-shaped) profile while, in the capital goods sectors, it shows a cyclical pattern around the long-wave path displayed by the corresponding final sector.

The inter-sectoral diffusion of such innovations sets in motion a cumulative process of growth bringing the system out of the long stagnation.

The employment outcome is complex. The clearest case is that of product innovations, which show a growing employment trend both at sectoral and global level. For process innovations the results are more uncertain; however, in the realistic case of pervasive radical technical change, the most likely outcome at the macroeconomic level is a stagnating or even declining long term trend for employment.

Dans cet article je développe le modèle de changement structurel de Pasinetti en y introduisant les ondes longues, ce qui implique que l'évolution de la productivité est déterminée par des révolutions technologiques (innovations radicales de procédés et de produit). L'analyse est située au stade logique du « système naturel », concentrant l'attention au niveau sectoriel.

Trois résultats généraux sont à signaler:

1) l'importance cruciale de la forme prise par le processus de diffusion des innovations, qui modèlise la productivité du secteur ;

2) l'évolution de la demande qui, dans le cas des innovations de procédés, est le résultat d'un mécanisme endogène mis en marche par la révolution technologique ;

3) le rôle joué par les elasticités de la demande par rapport au prix et au revenu, qui amplifient ou réduisent l'impulsion venant du changement technique.

De façon plus spécifique, l'analyse sectorielle des innovations de procédés montre que, dans le secteur des biens finaux, la production suit un profil en S – le mouvement typique de l'onde longue – tandis que le secteur des biens d'équipement fait apparaître un cycle qui se
déroule autour de la tendance créée par le secteur des biens finaux.

La diffusion inter-sectorielle des innovations radicales déclenche un processus cumulatif de croissance qui sort le système de la stagnation longue.

Les perspectives d’emploi sont incertaines. Le cas le plus clair est celui des innovations de produits, pour lesquelles il faut s’attendre à une évolution positive de l’emploi aussi bien au niveau sectoriel que global. Par contre, les innovations de procédés donnent lieu à des résultats non-univoques ; toutefois, dans le cas réaliste d’un changement technique diffusant, l’emploi fera très probablement apparaître une tendance à long terme plate ou en déclin, et cela tant au niveau sectoriel que macroéconomique.

**LIST OF SYMBOLS**

**a) Basic variables**

- $X_i$ physical output of sector $i$ ($i = 1, 2, ..., n - 1$)
- $X_{ki}$ physical output of sector $k_i$ (in terms of the number of productive capacities).
- $X_n$ total population, which is supposed to be equal to total labour supply,
- $E_i$ level of employment in sector $i$
- $E_{ki}$ level of employment in sector $k_i$
- $p_i$ price of final commodity $i$
- $p_{ki}$ price of a unit of productive capacity for sector $i$
- $\pi$ rate of profit
- $w$ wage rate
- $\alpha_i$ productivity level of sector $i$
- $\alpha_{ki}$ productivity level of sector $k_i$

**b) Technical coefficients**

- $a_{ni}$ quantity of labour per unit of physical output in sector $i$;
- $a_{nk_i}$ quantity of labour per unit of physical output in sector $k_i$;
- $a_{in}$ per capita demand for final commodity $i$;
- $a_{k_i,n}$ net investment per capita;
- $1/T_i$ coefficient of physical depreciation for sector $i$;
- $1/T_{ki}$ coefficient of physical depreciation for sector $k_i$;
- $\gamma_i$ ratio of the number of units of consumption goods to the number of units of capital goods which can be produced by the same machine;
c) Constants and initial conditions

1. Constants

\[ C1_i = T_{ki} / (T_{ki} - \gamma_i) \]
\[ C2_i = T_{ki} / (T_{ki} - \gamma_i - \pi \gamma_i T_{ki}) \]
\[ C3_i = \pi + (1 / T_i) \]

2. Initial conditions

\[ IC1_i = a_{ni}(0)a_{ii}(0)X_n(0) \]
\[ IC2_i = a_{nk_i}(0)a_{ii}(0)X_n(0) \]
\[ IC3_i = a_{ii}(0)X_n(0) \]
\[ IC4_i = a_{nk_i}(0)\bar{w} \]
\[ IC5_i = a_{ni}(0)\bar{w} \]
\[ \bar{w} = w(0) \]

d) Parameters which depend on time

\[ D3_i = \frac{T_{ki}}{T_{ki} - \gamma_i - (\dot{r}_i + g)\gamma_i T_{ki}} \]
\[ D4_i = \frac{T_{ki}}{T_{ki} - \gamma_i - \dot{r}_i \gamma_i T_{ki}} \]

e) Percentage rates of change

- A variable with a dot refers to the (instantaneous) percentage rate of change with respect to the previous period (the logarithmic derivative).
- A variable without dot concerns the percentage rate of change for longer time spans, from the beginning to period \( t \) (\( t = 2, 3, ..., T \)).
- Superscript \( (iW) \) (innovation waves) denotes a variable which is subject to radical technical change.
  \[ \rho_i \] percentage rate of growth of productivity of sector \( i \)
  \[ \rho_{ki} \] percentage rate of growth of productivity of sector \( k_i \)
  \[ \rho_{i,k_i} \] weighted percentage rate of growth of productivity of sectors \( i \) and \( k_i \)
  \[ \rho_{ir} \] trend increase in productivity (incremental innovations)
  \[ \rho^* \] "standard" rate of growth of productivity
  \[ r_i \] percentage rate of change of demand for final commodity \( i \)
  \[ g \] percentage rate of change of population
f) Others

$\Delta_i$ percentage leap of productivity for the individual innovators (radical process innovations)

$D(t)$ diffusion function of radical innovations (logistic or Gompertz function)

$\varepsilon_i$ price elasticity of demand for commodity $i$

$\eta_i$ income elasticity of demand for commodity $i$

I. INTRODUCTION

In this paper an attempt is made to develop Pasinetti’s (1981) model of structural change by introducing into it technological revolutions in order to derive the changes over time in physical output and employment. In Pasinetti’s model technical change, although taking place at a different pace in the various sectors, remains exogenous, in the sense that it is not the result of the normal functioning of the system. Here I go a step further and postulate that, in line with the long-wave theory, the most salient feature of technical change is the periodic appearance of technological revolutions. These radical innovations entail a substantial leap in labour productivity for the innovator, then spread thorough the sector according to some diffusion function and eventually generate a complex dynamic in the system involving prices, quantities and employment.

My analysis assumes that the long-wave theory holds, and particularly the explanation in terms of technological revolutions (see Van Duijn, 1983; Kleinknecht 1990). In spite of the never ending controversy on the statistical proof of the existence of long-waves in output \(^1\), recent theoretical advances and historical analyses (Freeman and Perez 1988; Tylecote, 1992) have made this approach sound enough to justify my purpose.

To introduce the argument, let us recall the relevant features of Pasinetti’s model.

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\(^1\) The list of references is very long. Let me just quote the case of R. Metz, which provides an interesting example of the slippery nature of such a subject. In 1992, using new filtering techniques, this author presented robust evidence on the existence of long waves (Metz 1992). In 1996, relying on different econometric techniques, he was unable to detect a long wave movement in the same data he used for his 1992 research (output series for Germany) (Metz 1998).
II. THE STARTING-POINT: 
PASINETTI’S MODEL OF STRUCTURAL CHANGE

1. General features of the model

Pasinetti’s model represents a return to the classical tradition of the “production” approach. The emphasis is thus on the reproducibility of commodities instead of scarcity, on production instead of exchange and on labour as the source of value instead of individual preferences (Pasinetti, 1986b).

The classical tradition is also the inspiration for the methodological device of the “natural” system: Pasinetti works out his model by studying “the ‘primary and natural’ determinants of the variables characterizing an economic system” ², which are prior to, and independent of, any institutional set-up (Pasinetti, 1981, p. 149). “The [theoretical] problems... that emerge at this stage are either in terms of necessary relations, if certain goals are to be achieved (e.g. full employment, price stability, etc.), or in terms of logically consistent relations, or in terms of normative rules, or in terms of those problems which are generated by the basic forces at work in a dynamic context” (Pasinetti, 1994, p. 41). All these relations can be developed without referring to specific behavioural and organizational assumptions; they reflect the basic characteristics of any modern industrialized economy ³. Of course, this gives only an initial picture which has to be fleshed out with an analysis of the institutions.

Economic rationality is defined not in the usual narrow sense of profit maximization but rather as the “intelligent” process of learning: the discovery of new and better methods on the production side, and the awareness of alternative patterns of consumption and the formation of new preferences on the demand side.

The model accounts for structural change in a twofold manner. First, technical change takes place at a different pace in the various sectors of the economy. Second, the model is “open” in the sense that technical change implies the creation of new industries and the disappea-

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² For a discussion of Pasinetti’s claim that the natural system is independent from institutions see Bortis (1993).
³ For Pasinetti, the “natural system” is not only a purely methodological device but has also a normative dimension because it characterizes a system with efficiency and fairness. This point is discussed in Reati (1994).
rance of others. Demand is explicitly introduced and plays a crucial role in determining the development of output. When real income per capita grows, the increase in demand for any final commodity \( i \) follows an Engel curve.

Extensively using the analytical tool of vertical integration (Pasinetti, 1973; 1986a), Pasinetti determines the sectoral trends of productivity, output, prices as well the sectoral and macroeconomic conditions for full employment.

2. Basic structure

2.1. Pasinetti (1981) considers three cases of a closed economy with no joint production: i) a "pure labour" economy in which production is carried out by labour alone; ii) an economy in which the final commodities are produced by means of labour and capital goods; iii) a more complex model involving capital goods for the production of capital goods. In Pasinetti (1981) the focus is on the second model. I shall here consider the third model because, if long waves are to be introduced into Pasinetti's model, it is not possible to assume that capital goods are produced by labour alone.

2.2. The economy comprises three sets of sectors: sectors \( i \) \((i = 1, 2, \ldots n - 1)\), concerning the production of final commodities;
sectors \( k_i \), producing the capital goods required by final sectors \( i \) as well as by the capital goods sector itself to replace the capital goods which wear out, and to increase productive capacity;
sector \( n \), which is the households sector: it provides the labour force for sectors \( i \) and \( k_i \) and receives final commodities as well as capital goods for new investments.

2.3. To these sectors correspond three sets of flows and one set of stocks of capital:
\( X_i \) is the physical output of sector \( i \),
\( X_{k_i} \) is the physical output of sector \( k_i \) (in terms of the number of productive capacities)\(^4\). It is formed by three distinct flows: i) the

\(^4\) The vertically integrated productive capacity is a composite commodity whose elements are derived from a column of the Leontief inverse matrix. The productive capacity is thus a set of different types of physical goods taken in strictly defined pro-
units of productive capacity for sector $i$ to replace what is used up during the current year; ii) the units of productive capacity for sector $k_i$ itself to replace the capital goods used up during the current year; iii) the units of productive capacity for net investment for sector $i$ and for sector $k_i$ (to provide the capital goods required by the expansion of sector $i$),

$X_n$ is total population which, to simplify is supposed to be equal to total labour supply $^5$,

$K_j$ is the stock of capital goods for sector $j$ ($j = 1, 2, ..., n - 1$) at the beginning of the period.

According to the definition of vertically integrated sector, the number of units of productive capacity in each sector must be equal to the number of units of the commodity produced: final commodity $i$ and the capital goods for $i$. To express $X_k_i$ in a homogeneous way with respect to the final commodity to which it refers, Pasinetti (1981, p. 43) introduces the technical coefficient $\gamma_i$ for the capital goods sector. So as not to go on ad infinitum, Pasinetti assumes that each sector $k_i$ produces capital goods for itself and for the corresponding final sector $i$ according to a fixed proportion $\gamma_i$. More precisely, $\gamma_i$ is the ratio of the number of units of consumption goods to the number of units of capital goods which can be produced by the same unit of productive capacity. In other words, the machinery is the same, and it can produce $x_i$ physical units of the consumption goods $i$ or $[(1/\gamma_i)x_i]$ units of capital goods for $i$ (i.e. the productive capacity itself). This makes it possible to fix the number of units of productive capacities required by sector $k_i$ ($K_{ki}$) in terms of $i$ equivalents as: $\gamma_i X_{ki}$. In sum, we have:

$$K_i = X_i \text{ and } K_{ki} = \gamma_i X_{ki} \quad (II.1)$$

2.4. To complete the description of the system, let us define the following coefficients:

$a_{ni}, a_{nk}$, are respectively the quantity of labour per unit of physical output in sectors $i$ and $k_i$;

portions. The vertical integration approach implies, however, that the composition of the productive capacity is not stated in the model: productive capacities are considered only in terms of the units required to perform an activity. For each final commodity $i$ there is a specific unit of productive capacity.

$^5$ When studying the conditions for reaching full employment, Pasinetti (1981, p. 55-56) abandons this simplifying hypothesis by introducing two parameters: the proportion of active to total population ($\mu$) and the ratio of working hours to total hours for the unit of time considered ($v$).
A LONG-WAVE PATTERN FOR OUTPUT AND EMPLOYMENT

\( a_{in} \) is the *per capita* demand for final commodity \( i \); 
\( a_{ki,n} \) is the net investment *per capita*;

\( 1/T_i \) and \( 1/T_{ki} \) are respectively the coefficients of physical deprecation for sectors \( i \) and \( k_i \). They are based on the assumption that, in each sector, a constant proportion of the productive capacity is used up as a result of normal wear and tear.\(^6\)

The formulae for physical outputs and total labour supply are:

\[ X_i = a_{in}X_n \quad \text{(II.2)} \]

\[ X_{ki} = (1/T_i)X_i + \gamma_i(1/T_{ki})X_{ki} + a_{ki,n}X_n; \]

\[ X_{ki} = C_{1i} \left[ a_{ki,n} + \frac{1}{T_i}a_{in} \right] X_n \quad \text{(II.3)} \]

\[ X_n = \sum_{i} a_{ni}X_i + \sum_{i} a_{nk_i}X_{ki} \quad (i = 1,2,\ldots,n-1) \quad \text{(II.4)} \]

where constant \( C_{1i} \) is: \( C_{1i} = T_{ki}/(T_{ki} - \gamma_i) \).

The price system is "dual" with respect to the system of physical quantities.

For this purpose, let:

\( p_i \) and \( p_{ki} \) be, respectively, the price of final commodity \( i \) and the price of a unit of productive capacity for sector \( i \)\(^7\),

\( \pi \) and \( w \) be, respectively, the rate of profit and the wage rate, taken as uniform to simplify notations\(^8\)

the price equations are:

\[ p_i = [\pi + 1/T_i]C_{2i}a_{nk_i} + a_{ni}w \quad \text{(II.5)} \]

\[ p_{ki} = C_{2i}a_{nk_i}w \quad \text{(II.6)} \]

where \( C_{2i} = T_{ki}/(T_{ki} - \gamma_i - \pi \gamma_i T_{ki}) \).

\(^6\) Thus \( T_i \) and \( T_{ki} \) correspond respectively to the average physical life of capital goods in sectors \( i \) and \( k_i \).

\(^7\) Being the price of a composite commodity, \( p_{ki} \) is thus the weighted average of the prices of the capital goods constituting the productive capacity.

\(^8\) In fact, there is nothing to prevent the introduction of a set of differentiated profit rates reflecting non-competitive market structures. This would complicate the results, which would though be of the same kind.
3. Developments over time

3.1. In Pasinetti’s model technical change takes the usual form of product and process innovations.

Product innovations are dealt with by changing the number of sectors in the system. Thus \( n \) is variable: it increases when new final commodities are manufactured and it decreases when some products disappear\(^9\).

Process innovations are dealt with on the (realistic) hypothesis that the most important effect of technical progress is to increase labour productivity. When a new process is adopted, some inputs diminish and others increase. However, vertical integration shows that technical progress exists only when labour coefficients decrease: “technical progress always is ultimately labour-saving” (Pasinetti, 1981, p. 212). Process innovations can either be embodied in capital goods or disembodied; their relative importance changes according to the phases of the long wave but, overall, embodied technical change is certainly the most important.

Of course, technical change does not spread uniformly over the sectors, but each sector has its own rate of increase of productivity: \( \rho_i \) for final sectors and \( \rho_{ki} \) for capital goods sectors \((i = 1, 2, ..., n - 1)\), where \( \rho_{ki} \) is the weighted average of the productivity increases of the individual industries producing the (vertically integrated) productive capacity for sector \( i \). Moreover, these sectoral rates of growth of productivity are not constant over time: \( \rho_i = f(t) \); \( \rho_{ki} = g(t) \).

To avoid misunderstandings, it is useful to state explicitly the links with the input-output analysis. The vertically integrated labour coefficient \( a_{nk_i}(0) \) is calculated from the input-output matrix at the initial time \( A(0) \) on the basis of a rather complex formula, in which appear the vector of direct labour coefficients and the Leontief-inverse matrix (see Pasinetti, 1980, p. 26); \( a_{nk_i}(t) \) is obtained in the same way from matrix \( A(t) \), which differs profoundly from \( A(0) \) because of technical change: new rows and columns are added, some others disappear and the coefficients of those industries which remain are changed. Coefficient \( a_{nk_i}(t) \) thus incorporates the complex movement induced by all types of technical change (embodied and disembodied) which materialized in the time span considered. This movement find a syn-

\(^9\) New intermediate commodities do not appear explicitly because they are subsumed by vertical integration.
thetic expression in $\rho_{ki}$, which is a derived magnitude resulting from
the comparison of $a_{nk_i}(t)$ and $a_{nk_i}(0)$.

If, to simplify notations, we assume that productivity changes are
continuous, although different from one sector to another\(^{10}\), the de-
velopment over time of labour coefficients is:

$$a_{ni}(t) = a_{ni}(0)e^{-\rho_i t} \quad (II.7)$$

$$a_{nk_i}(t) = a_{nk_i}(0)e^{-\rho_{ki} t} \quad (II.8)$$

Obviously, the fact that the sectoral productivity increases change
over time implies that $\rho_i$ and $\rho_{ki}$ in formulae (II.7) and (II.8) refer to
the average rate of change for the period (e.g. from the beginning to
the fifth period, if $t = 5$), and not to the rate of change in each period
with respect to the previous one. In what follows, this latter rate of
change is indicated by putting a dot on the variable concerned.

3.2. As a result of the increase over time in productivity, real per
capita incomes grow, and this has a twofold effect on per capita
consumption: it adds to the size of the basket of consumer goods
through the availability of new commodities and modifies the struc-
ture of consumption with respect to income, according to Engel’s law.

The demand for final commodities is:

$$a_{in}(t) = a_{in}(0) e^{\rho_i t} \quad i = 1,2,\ldots,n - 1 \quad (II.9)$$

Relying on the Keynesian theory, the demand for new investment fol-

$$a_{kn}(t) = a_{kn}(0) e^{\rho_{kn} t} \quad (II.10)$$

Since the rate of change of demand is not constant over time
($r_i = f(t)$), $r_i$ is an average for the period (as in the case for $\rho$), and
not simply the annual rate of change.

3.3. Population is assumed to grow at a constant rate $g$, given exo-
genously. Then,

$$X_n(t) = X_n(0) e^{gt} \quad (II.11)$$

\(^{10}\) Pasinetti (1981, p. 83 and 92) uses, instead, an ingenious notation to represent
productivity increases as segments of straight lines.
4. Some results

4.1. Taking the wage rate as *numéraire*, assuming that it stays at a fixed level \((\bar{w})\) over the entire period: \((w(t) = \bar{w})\), the dynamic expression for *prices* is:

\[
p_i(t) = IC5_i \ e^{-\rho_i t} + \left( \pi + \frac{1}{T_i} \right) C2_i IC4_i \ e^{-\rho_i t} \quad (\text{II.12})
\]

\[
p_{ki}(t) = C2_i IC4_i \ e^{-\rho_i t} \quad (\text{II.13})
\]

where the initial conditions \(IC4_i\) and \(IC5_i\) are respectively:

\[
IC4_i = a_{nk_i}(0)\bar{w}
\]

\[
IC5_i = a_{ni}(0)\bar{w} \quad 11
\]

We thus see that the relative price of any commodity falls at a varying rate equal to that at which labour productivity increases in the corresponding vertically integrated sector. This rate of change is the weighted average of two productivity growth rates: that relating to the production of the commodity concerned (sector \(i\)) and that relating to the production of the corresponding capital goods (sector \(k_i\)).

A convenient way of studying the price movements is to measure them with respect to a general level of prices that, by construction, remains stable over time. This is obtained taking as reference Pasinetti’s *dynamic standard commodity*, a composite commodity for which productivity is growing at the weighted average rate of the entire economic system (Pasinetti, 1981, p. 101 *et seq.*; Pasinetti, 1993, p. 70-74)\(^{12}\). Let us indicate by \(\rho^*\) this “standard” rate of growth of pro-

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11 Constant \(IC4_i\) represents the wage incorporated into one unit of commodity \(k_i\) at the beginning of the period \((t = 0)\). Constant \(IC5_i\) has the same meaning but refers to the direct labour needed to produce commodity \(i\).

On the other hand, the term \(IC4_i \ e^{-\rho_i t}\) in formula (II.13) refers to the wage incorporated in one unit of commodity \(k_i\) at period \(t\). The same applies to the term \(IC5_i \ e^{-\rho_i t}\) in formula (II.12), with reference to the direct labour for commodity \(i\).

12 In other words, the “dynamic standard commodity” is “that particular commodity whose components are in such proportions to one another as to make the weighted average of the corresponding rates of growth of productivity” equal to the average of the system (Pasinetti, 1993, p. 71). This means that such particular commodity is not defined in terms of its composition – which is changing all the time – but in terms of its properties (the decrease of labour requirements of its components).
ductivity and close the price system with the function:

\[ w(t) = \bar{w} e^{\rho^* t} \]  \hspace{1cm} (II.14)

where \( \bar{w} = w(0) \).

This link between wage and productivity has a double dimension. It is, first of all, an equilibrium condition, because it allows price stability. Secondly, it introduces into the system a principle of equity, because all workers benefit from technical change.

Taking into consideration formula (II.14), the price equations (II.12) and (II.13) then become:

\[ p_i(t) = IC5_i e^{(\rho^* - \rho_i)t} + \left( \pi + \frac{1}{T_i} \right) C2_i IC4_i e^{(\rho^* - \rho_{ki})t} \]  \hspace{1cm} (II.15)

\[ p_{ki}(t) = C2_i IC4_i e^{(\rho^* - \rho_{ki})t} \]  \hspace{1cm} (II.16)

It follows that, in terms of the dynamic standard commodity, the price of the commodities produced with above-average productivity growth will decrease and the opposite for the commodities with below-average productivity growth (Pasinetti, 1981, p. 105).

4.2. Assuming that the system tends to follow a dynamic equilibrium path, (Pasinetti, 1981, Chapter III. 2), the change over time in physical output is:

**Final commodities**

\[ X_i(t) = IC3_i e^{(g + r_i)t} \]  \hspace{1cm} (i = 1,2,...,n - 1)  \hspace{1cm} (II.17)

where

\( IC3_i \) stands for the initial conditions (i.e. demand at \( t = 0 \)):

\[ IC3_i = a_{in}(0) X_n(0) \]

**Capital goods**

\[ X_{ki}(t) = \left( g + \dot{r}_i + \frac{1}{T_i} \right) D3_i IC3_i e^{(g + r_i)t} \]  \hspace{1cm} (II.18)\textsuperscript{13}

\textsuperscript{13} The equation for the evolution of output of capital goods is derived by Pasinetti (1981, p. 52-53) in three steps. First he writes the equation for the output of new investment, which grows for two reasons: because of the additions to the productive
where \( D3_i = \frac{T_{ki}}{T_{ki} - \gamma_i - (\dot{r}_i + g) \gamma_i T_{ki}} \)

\( \dot{r}_i \) is the (instantaneous) percentage rate of change of demand relative to the previous period;
\( r_i \) is the average rate of growth of demand from the beginning to period \( t \).

The first two terms on the right in formula (II.18) \([(g + \dot{r}_i + 1/T_i)D3_i]\) are in the nature of an accelerator, in that they establish a proportionality link between the output of final commodities and the output of the corresponding capital goods sector. We shall see later, when dealing with long-waves, that this generates a cyclical movement in \( X_{ki} \).

4.3. Employment can be studied at two levels (sectoral and aggregate) in order to work out the macroeconomic condition for full employment. I shall consider here the first aspect only.

Multiplying the output of the sector (formulae (II.17) and (II.18)) by the quantity of labour per unit of output (the technical coefficients \( a_{ni} \) or \( a_{nk_i} \)) we obtain:

\[
E_i(t) = IC1_i \ e^{(g+r_i-\rho_i)t}
\]  
(II.19)

\[
E_{ki}(t) = D3_i \left( g + \dot{r}_i + \frac{1}{T_i} \right) IC2_i \ e^{(g+r_i-\rho_{ki})t}
\]  
(II.20)

where:

\( E_i(t) \) and \( E_{ki}(t) \) are respectively the level of employment in sectors \( i \) and \( k_i \).

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capacity for final sector \( i \) and because of new investments in sector \( k_i \) to provide the capital goods required by the expansion of sector \( i \). From this he obtains the equilibrium value for \( a_{kn} \) in formula (II.3), which shows that, in each sector, new investments per capita should bear a precise relationship to the consumption coefficient for that sector. This relationship is determined by the growth rate of demand for final commodity \( i \) and by the technical coefficients \( T_{ki}, T_i \) and \( \gamma_i \) (second step: the capital accumulation conditions for keeping full employment over time). Finally, such equilibrium values for \( a_{kn} \) are substituted into formula (II.3).

Let us note that my formula (II.18) is slightly different from Pasinetti's one (1981, p. 92) because here demand evolves in a continuous way \( (r_i = f(t)) \) while, in Pasinetti, it changes by steps. I skip the mathematical derivation of formula (II.18) to save space, but I would be pleased to provide details to any interested person.
IC1i and IC2i stand for the initial employment conditions:

\[ IC1_i = a_n i(0) \ a_i(0) \ X_n(0) \]

\[ IC2_i = a_n k_i(0) \ a_i(0) \ X_n(0) \]

We recognize in formula (II.20) the accelerator term.

III. TECHNOLOGICAL REVOLUTIONS AND OTHER HYPOTHESES

1. To incorporate technological revolutions into Pasinetti’s model, let us first recall the features of the general cycle. As noted by Schumpeter, each long wave develops in four phases: a) prosperity, when growth is high; b) recession, when growth decelerates; c) depression, when growth is near zero or even negative; d) recovery, when growth is modest. Prosperity and recession represent the long expansion while depression and recovery represent the long stagnation (Van Dujn, 1983). In this paper I conventionally assume that a long-wave lasts for 50 years, long expansion and stagnation for 25 years each, and the individual phases for: 20 years (prosperity), 5 years (recession), 15 years (depression) and 10 years (recovery).

In this and in Sections IV and V, I consider only process innovations, not because product innovations are not important but simply because there is no a priori indication of the pattern of productivity changes in these new industries (or new activities). This second type of innovations is dealt with in Section VI.

2. I assume that the sectoral and aggregate changes in labour productivity result from two developments, the second being clearly the more important:
   a) an underlying slow increase (e.g. 0.5-1% per annum) common to all sectors and due to incremental innovations (embodied technical change\(^{15}\), organizational improvements and learning by doing/using (disembodied technical change);
   b) the progressive adoption of radical process innovations (technological revolution) which materialize in a large and sudden increase in

\(^{14}\) I follow here, with minor changes, Reati and Raganelli (1993, p. 9-14).
\(^{15}\) E.g. new “generations” of machinery.
the level of productivity (e.g. a jump of 30-50% with respect to the previous level). This kind of technical change is always embodied in capital goods.

This process is illustrated in figure 1, with reference to individual innovators; the y axis represents the productivity index (volume of output per unit of labour) and the x axis time. The line $\alpha_{tr}$ depicts the general trend and lines $AA'T$, $ABB'T$ and $ACC'T$ the productivity level of the first, second and third innovator. Consider the third innovator: figure 1 shows that in periods 1 and 2 he operates with the old technology, obtaining a modest increase in productivity (1% per annum in this example); in period 3 he adopts the new technology and achieves a massive increase in productivity (30%); from period 4 onwards, there are only minor changes in the new technical bases (incremental innovations), with a small improvement in productivity (again 1% per annum)\textsuperscript{16}.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Productivity level of the innovators (indices)}
\end{figure}

A $\alpha_{tr}$ and $A'T$ seem linear simply because the exponential growth rate is small (1% per annum)

\textsuperscript{16} The mathematical expression of the productivity function of fig. 1 can be found in Reati and Raganeli (1993, p. 12).
3. Turning now to the process of diffusion within a sector, I make two assumptions regarding the duration of the diffusion process and the shape it takes.

a) On the first point, I assume that the diffusion is complete by the end of the phase of the long-wave in which the technological revolution started. If, for instance, a radical innovation is introduced in sector i at the beginning of the depression, on the basis of my conventional schedule it will be only at the end of the 15th year that all the output is obtained with the new technology; if the first innovation appears at the beginning of the recovery, it will take 10 years to become generalized, and so on. Note, however, that this hypothesis is not essential. It is adopted here for the sake of convenience in order to identify the mechanism governing prices and quantities when technological change takes the form of technological revolution; for this purpose, the length of the diffusion period is instrumental and can thus be taken arbitrarily.

b) As regards the pattern of diffusion, my basic assumption is that the introduction of radical process innovations in a sector follows an asymptotic growth path (S-shaped), which mathematically can be expressed by a sigmoid function, e.g. a logistic or a Gompertz curve. It should also be noted that the diffusion process can refer to the adoption of successive “generations” of the same innovation.

The economic justification for a sigmoid pattern of diffusion is empirical. For instance, the enterprises in the sector do not have the necessary information to perceive immediately the advantage of imitating the first innovator or, if they are fully aware of the new opportunities, they prefer to wait so as to avoid the cost of an accelerated scrapping, or they are unable to adopt the new technology for organizational or institutional reasons (they do not know how to master the new technology or do not have the necessary skills; managers are reluctant to change radically the organization of the company).

4. Two methodological problems should be addressed. The first refers to the conceptual level of the analysis, the second to the equilibrium conditions on which Pasinetti’s model is based.

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17 For the analytical expressions of these functions see Van Duijn (1983, p. 42-43). The logistic function is symmetrical, which means that at the mid-point in the diffusion period half of the output of the sector is generated with the new technology. Its derivative function is “bell-shaped” and its instantaneous rate of growth is constantly decreasing. The Gompertz function is not symmetrical, but it too displays an instantaneous rate of growth that is constantly decreasing.

18 See Van Duijn (1983, Chapter II) for a survey of the literature.
a) On the first point, my analysis will be carried out within the logical framework of the natural system, conceived as a *methodological device* to avoid unnecessary complications at this stage of the enquiry. This means that the changes in prices, quantities and employment that will emerge are the movements which are technically possible when there is a technological revolution. What will actually appear on the market depends on the strategies of enterprises and on the influence of institutions, which are, however, constrained by what happens at the level of the natural system.

b) The economic relations in Pasinetti's model are based on the hypothesis that the system is in equilibrium. This implies that there is no idle or insufficient productive capacity at sectoral level 19 and that there is full employment of the labour force at macroeconomic level 20.

In a long-wave context such conditions are not satisfied: the long stagnation is, for instance, characterized by massive unemployment and considerable spare capacity in many sectors. However, the analysis that follows does not necessarily require such stringent conditions for equilibrium. In fact, for the sectoral part of my enquiry, a disequilibrium situation might have two implications: i) the sector concerned by the technological revolution could not find the appropriate productive capacity; ii) the output level which is technically possible is not attained because of insufficient demand. Nevertheless, these possibilities do not necessarily have to be considered in the sense that, at least for the sectors examined, it can be reasonably posited that such obstacles are not operating. This is what is done in the present paper.

This difficulty reappears at the end of my investigation, when I outline the overall dynamic of the system. Since there is now a steadily growing number of sectors concerned by the technological revolution, it becomes difficult to avoid bottlenecks or insufficient demand. However, if we remember that my analysis is situated at the logical level of the "natural" system, the difficulty disappears here too.

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19 Actually, this condition is embodied in the equation for the output of the capital goods sector (Pasinetti, 1981, p. 52-53)

20 This should not be interpreted in the sense that full employment results automatically from the market forces. It is just the opposite. In fact, following Keynes, Pasinetti emphasizes that in a capitalist economy with no regulatory authority for the labour market full employment will be never reached spontaneously because the learning processes in production and consumption operate independently. Thus, when technical and structural changes are deep-seated, the most likely outcome is unemployment, and full employment can be attained only through an active economic policy.
because my purpose is not to reconstruct the whole long-wave movement but merely to show how the inter-sectoral diffusion of the technological revolution produces a long-wave pattern for output. Sectoral bottlenecks or insufficient demand could give a bias to or delay the impetus coming from the technological revolution but, as we shall see later, there is no inconsistency between my results for the basic forces underlying any industrial system and the conditions for the appearance of a long upswing.\footnote{Bortis (1993, p. 367) suggests an easy way to introduce involuntary unemployment in Keynes’s sense into Pasinetti’s model. If past accumulation was not sufficient to bring about full employment of the labor force, this would mean that the quantity levels of $X_i$ in formula (II.2) will have to be multiplied by some scalar smaller than unity (e.g. 0.9) — which means that 10% of the labour force remain permanently unemployed. “This would leave all the other properties of the natural system unchanged. A fully adjusted situation would thus coexist with permanent unemployment due to a lack of effective demand” (id.).}

\section*{IV. \ PROCESS INNOVATIONS: LONG-WAVES IN PRODUCTIVITY AND PRICES}

This section and the following are devoted to the study of changes in prices, output and employment at \textit{sectoral level}, the aim being to provide a basis for an understanding of the overall dynamic of the system. Since my purpose is to study the basic mechanisms of the model, I focus, for the sake of convenience, on the depression phase of the long-wave, \textit{i.e.} the period during which, according to long-wave theory, innovation is more intense (Van Duijn 1983, p. 137; Table 1 below).

\subsection*{1. The productivity function of the sector}

The productivity level of any sector $i$ ($\alpha_i$) or $k_i$ is obtained by combining the technological revolution function depicted in fig. 1 with a diffusion function $(D(t))$.\footnote{\textit{D(t)} — which is a logistic or a Gompertz function — shows the cumulative share of total production of the sector affected by the technological revolution at period $t$. It varies from 0 to 1.}

At time $t$ we have:

$$\alpha_i(t) = \alpha_{ji}(t) \cdot D(t) + \alpha_{zi}(t) \cdot [1 - D(t)] \quad \text{(IV.1)}$$
where \( j \) refers to the last innovators and \( z \) represents the other enterprises.

After some manipulations formula (IV.1) becomes:

\[
\alpha_i(t) = \alpha_i(0)[1 + \Delta_i D(t)] e^{\rho_{tr} t}
\]  \hspace{1cm} (IV.2)

where

\( \Delta_i \) is the percentage leap in productivity due to the radical innovation.

The result of formula (IV.2) is worth noting: it shows, in fact, that changes in the productivity of any sector \( i \) are strongly influenced by the pattern of diffusion of the technological revolution. Since the leap in productivity (\( \Delta_i \)) is multiplied by the diffusion function, the greater the intensity of the technological revolution (quantified by the magnitude of \( \Delta_i \)), the more sectoral changes in productivity will reflect the shape of the diffusion function. In figure 2, I present an example of technological revolution in sector \( k_i \) operating with a logistic diffusion pattern. The productivity function of the innovators is that of figure 1 with three shocks: 70% (line \( \Delta_i = 0.7 \)), 30% (line \( \Delta_i = 0.3 \)) and 10% (line \( \Delta_i = 0.1 \)).

If the technological revolution follows a diffusion pattern which differs from \( k_i \) to \( i \), the diffusion function of the vertically integrated sector will be the weighted average of the diffusion curves of the individual sectors \( i \) and \( k_i \), and the resulting productivity curve will reproduce the shape of the diffusion function. If, contrary to what I have assumed so far, the diffusion process is shorter than the phase of the long-wave in which the technological revolution started, the productivity of the sector follows the diffusion pattern for the time required for the diffusion to be completed and then tracks the trend.

The (instantaneous) percentage rate of growth of the productivity function is obtained simply by calculating the derivative of formula (IV.2) with respect to time and then dividing it by the function itself (the logarithmic derivative). If we denote the first derivative by a prime (') \(^{23}\) we have:

\[
\dot{\rho}_{iw}^{(i)}(t) = \rho_{tr} + \frac{\Delta_i D'(t)}{1 + \Delta_i D(t)}
\]  \hspace{1cm} (IV.3) \(^{24}^{25}\)

\(^{23}\) In this paper, a prime always refer to the first derivative, while a variable with a dot (\( \cdot \)) designates the instantaneous percentage rate of change.

\(^{24}\) In what follows the time index is omitted to simplify the notation.

\(^{25}\) The superscript (\( iw \)) ("innovation waves") characterizes a variable which is influenced by radical technical change.
Figure 2
Diffusion function and labour productivity in sector $k_i$ according to different strength of the technological revolution

\[ D(t) = \text{diffusion function (logistic)} \]
\[ \alpha_{k_i} = \text{productivity level of sector } k_i \text{ (index)} \]
\[ \Delta = \text{productivity shock} \]

For formula (IV.3) we can make the same observations as for formula (IV.2): apart from the small influence of the trend rate of growth ($\rho_{tr}$, which empirically would amount to 0.5-1% per annum), the main determinants of the instantaneous rate of increase in productivity are the intensity of the technological revolution ($\Delta_i$) and the shape of the diffusion function ($D(t)$). When $D(t)$ is a logistic the rate of change fluctuates in a “bell-shaped” manner, as in figure 3; when the diffusion function is a Gompertz curve, the “bell” is skewed. Below, this pattern of change will be associated with the notion of long-waves (simply
because a "bell-shaped" rate of change of a function generates an S-shaped curve, which is the typical long-wave movement).

Changes in productivity have a twofold effect: a direct influence on the price of the commodity produced with the new technology and an indirect effect on the rate of growth of the demand for that commodity.

\[
\Delta = 0.7
\]

\[
\Delta = 0.3
\]

\[
\Delta = 0.1
\]

**Figure 3**
Instantaneous rate of change of the productivity function (logistic diffusion pattern)

2. Long Waves in Prices

Relying on the strong assumption that coefficients \( T_i, T_k, \) and \( \gamma \) as well as the rate of profit remain constant\(^{26}\) (in order to isolate the

\(^{26}\) This is, obviously, an unrealistic hypothesis: as a matter of fact, the rate of profit shows a clear long-wave pattern, with fluctuations ranging sometimes from 1 to 10. For a theoretical discussion and empirical evidence see Reati (1990).
effects of technological revolution), Reati and Raganelli (1993) show that Pasinetti's general finding (formulae (II.12) and (II.13)) also holds when there are long waves: the long-term price movements depict an inverted S-shaped curve, which strongly depends on the features of the diffusion function.

3. The Percentage Rate of Change of Prices

3.1. I consider the case in which the technological revolution occurs in only one branch of the final commodities sector (which is generically designed by $i$) as well as in its corresponding capital goods sector ($k_i$), while in all the other branches (i.e. the remaining $n - 2$ final goods and capital goods branches, indicated by $j$ and $k_j$) there are only incremental innovations, and their rate of increase in productivity is thus $\rho_{tr}$.

This assumption on technical change in the vertically integrated sector $i$ is rooted in the Schumpeterian theory, which has shown that a technological revolution brings with it many clusters of radical (and incremental) innovations in related fields; this creates a new "technological paradigm" which progressively replace the previous one.

Of course, this must not be interpreted in the sense that, in all industries forming the vertically integrated sector, there is the same increase in productivity or an identical pattern of diffusion. Such extreme hypotheses are not at all necessary for my purpose, which is to detect a mechanism. Indeed, it is quite possible that some industries belonging to $k_i$ have a technical change or a diffusion function which are different from the rest of the sector (or even no technical change at all). This will be reflected in the shape of the productivity function of $k_i$ and on its rate of change - which, as I precised above, is a shorthand expression for the whole set of changes in the individual industries forming the vertically integrated sector - but it does not alter the basic mechanism in question.

My hypothesis of a technological revolution in sectors $i$ and $k_i$ implies that changes in the "standard" rate of productivity growth are not a direct function of time: in fact, $\rho^*$ varies only because the individual branches of the consumer and capital goods sectors under consi-

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27 For a more complete treatment of this topic see Reati (1998).
28 Kleinknecht (1990) provides robust empirical evidence of the clustering of radical innovations from 1860 until now.
deration have a productivity growth higher than all the other branches; without this influence, \( \rho^* \) would have been constant (and equal to \( \rho_{tr} \)): 

\[
\rho^*(t) = f[\rho_i(t); \rho_{ki}(t)]
\]

\[
\rho_i = g(t) \quad \text{and} \quad \rho_{ki} = h(t); \quad \frac{\partial \rho^*}{\partial \rho_j} = 0; \quad \frac{\partial \rho^*}{\partial \rho_{ki}} = 0
\]

The "standard" rate of productivity growth varies as follows:

\[
\frac{d \rho^*}{dt} = \frac{\partial \rho^*}{\partial \rho_i} \frac{d \rho_i}{dt} + \frac{\partial \rho^*}{\partial \rho_{ki}} \frac{d \rho_{ki}}{dt} = \rho^{*'} \quad \text{(IV.4)}
\]

3.2. Taking account of formula (IV.4), the percentage rate of change of price of final commodity \( i \) is obtained by calculating the logarithmic derivative of formula (II.15) with respect to time. At this purpose, let us work out in detail the first derivative (numéraire "dynamic standard commodity"). Recalling the meaning of constants \( IC4_i \) and \( IC5_i \) given in paragraph II.4.1 above (see formula (II.12)), and defining the following final conditions:

\[
FC4_i = IC4_i e^{(\rho^* - \rho_{ki})t}
\]

\[
FC5_i = IC5_i e^{(\rho^* - \rho_i) t}
\]

we have:

\[
p_i' = FC5_i [\rho^* + \rho'^* t - (\rho_i + \rho_i') t]
\]

\[
+ C3_i C2_i FC4_i [\rho^* + \rho'^* t - (\rho_{ki} + \rho_{ki}') t]
\]

where \( C3_i = \pi + (1/T_i) \)

As already noted, \( \rho_i \) and \( \rho_{ki} \) refer to the average percentage rate of change for the time span considered, and it is the same for \( \rho^* \). The terms \( (\rho_i + \rho_i') \) and \( (\rho_{ki} + \rho_{ki}') \) are a particular way of denoting the (instantaneous) percentage rate of change of \( \rho_i \) and \( \rho_{ki} \) with respect to the preceding period \(^{29}\); \( \rho^* + \rho'^* t \) has the same meaning but refers to the standard rate of growth of productivity. Taking account of this –

---

\(^{29}\) This is a general result. In fact, for any magnitude \( A \) developing exponentially \( A(t) = A(0)e^{bt} \), (where \( b = f(t) \)) the instantaneous (percentage) rate of change with respect to the previous period is:

\[
\frac{dA(t)/dt}{A(t)} = b + b't
\]
and stressing the fact that in sectors $i$ and $k_i$ there is a technological revolution – $^{30}$ the formula above becomes:

$$p_i' = FC5_i (\dot{\rho}^* - \dot{\rho}_{i}^{(iw)}) + C3_i C2_i FC4_i (\dot{\rho}^* - \dot{\rho}_{k_i}^{(iw)})$$

Then the (instantaneous) rate of change of the price of final commodity $i$ with respect to the preceding period is:

$$\dot{p}_i = -\dot{\rho}_i^{(iw)} \omega_1 + \dot{\rho}_{k_i}^{(iw)} \omega_2 + \dot{\rho}^*$$  \hspace{1cm} (IV.5)

where:

$$\omega_1 = \frac{FC5_i}{FC5_i + C3_i C2_i FC4_i}$$  \hspace{1cm} (IV.6) \hspace{1cm} ^{31}

$$\omega_2 = \frac{C3_i C2_i FC4_i}{FC5_i + C3_i C2_i FC4_i}$$  \hspace{1cm} (IV.7)

and $\omega_1 + \omega_2 = 1$

The weightings $\omega_1$ and $\omega_2$ have a clear economic meaning, which is to provide an indicator of the degree of mechanization of the sector. Let us consider, for this purpose, the denominator of $\omega_1$ and $\omega_2$, which is a synthetic expression for $p_i(t)$ (formula (II.15)): $FC5_i$ is the direct wage incorporated into commodity $i$ at period $t$, while $[C3_i \cdot C2_i \cdot FC4_i]$ is the indirect wage (i.e. the wage incorporated into the capital goods for one unit of $i$) and the profit component of $p_i$, expressed in terms of wages. Thus $\omega_1$ is the share of direct wages for $i$ with respect to price, while $\omega_2$ is the share of indirect wages and profits.

Expressing the weighted rate of change of productivity more synthetically as

$$\dot{\rho}_{i,k_i}^{(iw)} = \dot{\rho}_i^{(iw)} \omega_1 + \dot{\rho}_{k_i}^{(iw)} \omega_2,$$

$^{30}$ Thus the rates of change of productivity takes the subscript "iw"

$^{31}$ Since the magnitude of $\omega_1$ and $\omega_2$ changes over time, they should be written: $\omega_1(t)$ and $\omega_2(t)$. The time index is omitted to simplify the notation.

Formula (IV.6) shows that the time trend of $\omega_1$ depends on the relative strength of the technological revolution affecting sectors $i$ and $k_i$. If the productivity leap in sector $i$ is not too pronounced (e.g. $\Delta = 0.3$) while in the capital goods sector it is very important (e.g. $\Delta = 0.7$), $\omega_1$ will increase, and the opposite when the technological shock is important in sector $i$ and not so strong in the capital goods sector.
formula (IV.5) shows even more clearly the determinants of price changes:

$$\dot{p}_i = -\dot{\rho}_{i,k_i}^{(iw)} + \dot{\rho}^*$$  \hspace{1cm} (IV.8)

The size of $\dot{\rho}^*$ depends on the relative importance of sectors $i$ and $k_i$: if their share of total employment is not large (e.g. less than 10%), $\dot{\rho}^*$ will be small. Thus, the main factors determining the magnitude of the percentage decrease in the price of final commodity are the productivity growth of the sector itself and of the corresponding capital goods sector.

3.3. In practice, formula (IV.8) poses some problems because it requires the computation of $\dot{\rho}^*$. This is avoided taking as the numéraire any commodity $h$ unaffected by the technological revolution, and whose productivity increases at the trend rate ($\rho_{tr} = \rho_h$). We are, of course, deprived of the advantage of the stability of the general price level: in terms of the new numéraire, the decrease in $p_i$ will be stronger and the wage increases weaker ($\rho_h < \rho^*$). However, in so far as the technological revolution affects only a small part of the economy (just one vertically integrated sector $i$ in the present case), the difference between $\rho_h$ and $\rho^*$ is small, and the former gives a good approximation of the latter.

Equation (IV.8) is then rewritten substituting $\dot{\rho}^*$ with $\rho_{tr}$

$$\dot{p}_i = -\dot{\rho}_{i,k_i}^{(iw)} + \rho_{tr}$$  \hspace{1cm} (IV.9) \hspace{1cm} (32)

Let us now consider the second effect of productivity growth, that on demand.

IV. PROCESS INNOVATIONS:
PHYSICAL QUANTITIES AND EMPLOYMENT

1. The second effect of productivity changes is even more interesting because it generates an endogenous mechanism that explains the rate of growth of demand.

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(32) Changing the numéraire, the definition of $\omega_1$ and $\omega_2$ remains the same, but $FC4_i$ and $FC5_i$ are modified accordingly in the sense that $\rho^*$ is now replaced by $\rho_{tr} (= \rho_h)$ in the exponent of $e$. 

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Let us refer, as before, to final commodity $i$, assuming that the technological revolution occurs in this sector as well as in the corresponding capital goods sector—while the rest of the economy achieves only incremental innovations.

Given consumer preferences, the level and rate of change of per capita demand for commodity $i$ ($r_i$) depend on the following additive elements:

- a general factor, or purchasing power effect. As already noted, when income grows, per capita demand for $i$ also grows according to an Engel curve path;

- a specific factor, or price effect, given by changes in the price of commodity $i$. This second effect derives directly from the technological revolution in sector $i$ and, indirectly, from the technical change in sector $k_i$. In fact, the Keynesian character of the investment function implies that a radical innovation in any sector $k_i$ has no direct influence on the demand for the capital goods in question but merely an indirect effect through the increased demand for final commodity $i$. The chain of causation is the following: the increase in productivity in sector $k_i$ produces a decrease in the price of capital goods which, in turns, reduces the price of the final commodity, with a corresponding increase in demand for it (and in the output of sectors $i$ and $k_i$).

To this one should add the changes in the price structure. However, in order to focus on the effects of technological revolutions, this last point is disregarded here.

The technological revolution and the ensuing increase in productivity exert a decisive influence on both factors above. The income effect results from the link between wages and productivity increases: if we take the “dynamic standard commodity” as the numéraire of the system, the technological revolution in sectors $i$ and $k_i$ exerts an upward pressure on the “standard” rate of productivity growth which will push wages and consumption upwards. The price effect has just been examined, and does not require further comments.

The precise impact on $r_i$ of the two factors in question depends on two elasticities of demand for commodity $i$: the income elasticity $\eta_i$ and the (own) price elasticity $\varepsilon_i$. These elasticities vary over time as income changes; however, for the sake of simplicity (and in order not to divert attention from the other aspects that I would like to highlight), in the simulations below $\eta_i$ and $\varepsilon_i$ are kept constant for the whole diffusion period.

The above relationships are summarized in figure 4, in which the numéraire of the system is the “dynamic standard commodity”. Since
productivity growth follows a long-wave pattern, $r_i$ will also reflect the same movement, which in turn will generate an S-shaped curve for physical output.

Figure 4
From the technological revolution to demand and output

To show it analytically, let us write the percentage growth rate of demand for commodity $i$ with respect to the previous period, applying formula (IV.5) for the price decrease (*numéraire* the "dynamic standard commodity"):

$$
\dot{r}^{(i,w)}_i = \varepsilon_i \left( \dot{\rho}^{(i,w)}_i \omega_1 + \dot{\rho}^{(i,w)}_{ki} \omega_2 - \dot{\rho}^* \right) + \eta_i \dot{\rho}^* \tag{33}
$$

In the first term on the right we recognize the price effect, while the second term is the income effect. By rearranging, we obtain:

$$
\dot{r}^{(i,w)}_i = \varepsilon_i \left( \dot{\rho}^{(i,w)}_i \omega_1 + \dot{\rho}^{(i,w)}_{ki} \omega_2 \right) + (\eta_i - \varepsilon_i) \dot{\rho}^* \\
= \varepsilon_i \dot{\rho}^{(i,w)}_{i,ki} + (\eta_i - \varepsilon_i) \dot{\rho}^* \quad \text{(see form. IV.8)} \tag{V.1}
$$

An examination of this formula shows that the main factor determining the evolution of demand is the productivity growth in sectors $i$ and $k_i$ and, since in both sectors productivity follows a long-wave pat-

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33 The first term on the right of this formula is positive because it is the product of the price elasticity of demand, which is negative, and the percentage rate of change of price, which is also negative. However, I adopt here the usual convention of considering the absolute value of $\varepsilon_i$. 

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tern, this pattern of change will also appear in the growth rate of demand.

When the technological revolution concerns just one vertically integrated sector, one can posit that the "standard" rate of growth of productivity ($\rho^*$) is quite small and near to the trend growth of productivity. The term $(\eta_i - \varepsilon_i)$ is also small. In fact, Bosworth's (1987) survey of empirical studies shows that income elasticity of demand is higher than 1.5 only in few instances (durable goods and services) and that price elasticity is usually less than one. We can thus realistically expect that $(\eta_i - \varepsilon_i)$ is also less than one in most of the relevant cases.

To facilitate numerical simulations, let us rewrite formula (V.1) adopting as numéraire for prices any commodity $h$ unaffected by the technological revolution. Relying on formula (IV.9) for price changes we have:

$$\dot{r}_{i}^{(i w)} = \varepsilon_i \dot{r}_{1,k_i}^{(i w)} + (\eta_i - \varepsilon_i) \rho_{tr}$$

(V.2)

2. The physical output of final commodity $i$ displays a long-wave (S-shaped) profile. To demonstrate this, let us assume that the population is constant ($g = 0$) and modify formula (II.17) to take into consideration the new expression for the rate of increase of demand. Indicating by $r_{i}^{(i w)}$ the average rate of change from the beginning to period $t$, we have:

$$X_i(t) = IC3_i e^{r_{i}^{(i w)} t}$$

(V.3)

where

$IC3_i$ is the level of demand at $t = 0$: $IC3_i = a_{in}(0) \cdot X_n(0)$

For capital goods, the result is:

$$X_{ki}(t) = \left(\dot{r}_{i}^{(i w)} + \frac{1}{T_i}\right) D4_i IC3_i e^{r_{i}^{(i w)} t}$$

(V.4)

where: $\dot{r}_{i}^{(i w)}$ is given by formula (V.1), and

$$D4_i = \frac{T_{ki}}{T_{ki} - \gamma_i - \dot{r}_{i}^{(i w)} \gamma_i T_{ki}}$$

Figure 5 provides an illustration of the changes in physical output in sectors $i$ and $k_i$, based on the following assumptions (which will be

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34 This hypothesis of a constant population will be maintained for the rest of this paper.
maintained in the rest of the paper, except for price and income elasticities of demand):

- to simplify computations, the intensity of the technological revolution in sectors $i$ and $k_i$ is the same;
- the productivity function upon which it is derived the percentage rate of change of demand is that of figure 2, with a 30% productivity shock for each innovator ($\Delta_i = 0.3$, as in figure 1) \(^{35}\);
- the levels of the technical coefficients and the rate of profit are:

$$T_i = 12; \quad T_{k_i} = 10; \quad \gamma_i = 2; \quad \pi = 0.2.$$  

- the price and income elasticities of demand are the same: $\varepsilon_i = \eta_i = 0.5$.

The most striking aspect is that sector $k_i$ evolves in a cyclical-like manner around the long-wave pattern displayed by sector $i$. This interesting path is due to the fact that the accelerator term $[(r_{i}^{(lw)} + 1/T_i) D4_i]$ is "bell-shaped", with a range of variation which could be quite large \(^{36}\). Thus, it is not necessary to have time-lags for $X_{k_i}$ to show cyclical fluctuations.

3. Employment is obtained from formulae (II.19) and (II.20), in which demand and productivity are shaped by the technological revolution.

3.a) For final sector $i$ we have

$$E_i(t) = IC_1_i e^{(r_{i}^{(lw)} - \rho_i^{(lw)}) t}$$  \hspace{2cm} (V.5)

where $IC_1_i$ stands for the initial employment conditions:

$I C_1_i = a_{ni}(0) a_{in}(0) X_n(0)$

Formula (V.5) can be alternatively stated as follows:

$$E_i(t) = E_i(t - 1) e^{(r_{i}^{(lw)} - \rho_i^{(lw)})} \quad (t = 1, 2, ..., T; \quad T = 15)$$  \hspace{2cm} (V.6)

when $t = 0 \rightarrow E_i(0) = IC_1_i$.

Writing in full the exponent (taking formula (V.1) for the rate of increase of demand with respect to the previous period), we see that at period $t$ employment increases if:

\(^{35}\) Clearly, when the scope of the technological revolution is wider (a larger productivity shock), the results that I obtain for prices, quantities and employment are magnified, and vice versa when the productivity shock is less than the 30% assumed here.

\(^{36}\) For instance, in figure 5 the "accelerator" term at $t = 7$ is 32% higher than its level at $t = 0$. 

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Price and income elasticities of demand $= 0.5$;
Technical change in both sectors (pervasive technical change)

\[ \varepsilon_i \dot{\rho}_{i,k_i}^{(i,w)} + (\eta_i - \varepsilon_i) \dot{\rho}^* > \dot{\rho}_i^{(i,w)} \] (V.7)

Considering that, in most practical cases, the magnitude of the weighted average of productivity growth in the two sectors is not very different from the percentage rate of change of productivity of sector $i$, formula (V.7) shows that year-on-year changes in employment depend crucially on price and income elasticities of demand. Employment can grow only when these elasticities are sufficiently high, i.e. larger than one. Since such high values are quite rare in practice (Bosworth, 1987), this means that the most likely outcome of a technological revolution in sectors $i$ and $k_i$ is a decline in employment.

3.b) For the capital goods sector the results are straightforward (formula (II.20)):

\[ E_{k_i}(t) = (r_i^{(i,w)} + 1/T_i) D4_i IC2_i \ e^{(r_i^{(i,w)} - \rho_{k_i})t} \] (V.8)

where

\[ IC2_i = (a_{nk_i} / a_{ni}) IC1_i \]
We notice the accelerator term as in formula (V.4): employment in the capital goods sector thus exhibits the same cyclical profile as output, which implies that employment can grow substantially even when it is stationary in the final commodities sector (which happens for \( \varepsilon_i = \eta_i = 1 \)). For instance, a numerical simulation with the usual values for productivity and the technical coefficients shows that, for \( \varepsilon_i = \eta_i = 1 \), at \( t = 7 \) employment in the capital goods sector is 66% higher than its initial level. Of course, at the end of the period of diffusion of the technological revolution, employment in the capital goods sector has fallen to the level at the outset.

3.c) The overall effect on employment resulting from the technological revolution in sector \( i \) and \( k_i \) is the sum of the levels derived from formulae (V.5) and (V.8). Unfortunately, the simple addition of these two expressions does not allow us to reach general conclusions since everything depends on the relative size of the two sectors. We have thus to rely on numerical simulations based on alternative assumptions regarding the degree of mechanization of the sectors concerned as well as the magnitudes of the price and income elasticities of demand. In the examples which follow, the size of employment in sector \( k_i \) with respect to sector \( i \) is determined on this basis:

\[
IC_2i = \left( a_{nk_i} / a_{ni} \right) IC_1i = \frac{\omega_2(0)}{\omega_1(0)} \frac{C3_i}{C2_i} IC_1i
\]

where \( IC_1i = 100 \) \(^{37}\)

For price and income elasticities higher than one, total employment increases, with a more or less pronounced cyclical component according to the relative importance of the capital goods sector (which depends on the degree of mechanization of sector \( i \)). For \( \varepsilon_i = \eta_i = 1 \), total employment reproduces the cyclical pattern of sector \( k_i \) around a flat long-term trend. For \( \varepsilon_i \) and \( \eta_i \) less than one, it tends to decline. For instance, when \( \varepsilon_i = \eta_i = 0.5 \), a numerical simulation with the usual values (\( \omega_1(0) = 0.4 \) for the size of sector \( k_i \)) shows a continuous decline in total employment which, at the end of the period (\( T = 15 \)), is 18.4% lower than at the beginning (figure 6). Comparing this out-

\(^{37}\) \( C2_i \) and \( C3_i \) have been calculated taking the usual values for the technical coefficients and the rate of profit.

Let us remember that, when \( \omega_1 \) and \( \omega_2 \) are computed at \( t = 0 \), then \( FC4_i = IC4_i = a_{nk_i}(0)\bar{w} \), and \( FC5_i = IC5_i = a_{ni}(0)\bar{w} \).
come with that for output, we see that rapid growth in output is not at all incompatible with a decline in employment.

It could be argued that no general conclusions can be drawn from such numerical simulations because the results depend on the specific values assigned to the parameters. This objection is not really important because I am concerned with the direction of the long-term trends, and not with the precise magnitudes of the variables. Several tests on the sensitivity of the final outcome to the values assigned to \( T_i, T_{ki} \) and \( \gamma_i \) have shown that, when coefficients \( T_i, T_{ki} \) change, this does not modify substantially the final result: different values of the coefficients of depreciation alter the size of the capital goods sector but have almost no effect on its dynamic. In fact, calculating \textit{coeteris paribus} the term \( D4_i \) for \( 5 \leq T_{ki} \leq 15 \), I have found that the level of \( D4_i \) is somewhat different but its evolution over time is practically the same. For the technical coefficient \( \gamma_i \) things are different. In fact, \( \gamma_i \) has a double effect on employment outcome, via the term \( C2_i \) and via the accelerator. Testing alternative values for \( \gamma_i \), from a very low (\( \gamma_i = 0.5 \)) to \( \gamma_i = 3 \) (which is near the maximum level of 3.33 for \( C2_i > 0 \)), it appeared that the size of employment in the capital goods sector can change substantially, but total employment still follows a path which is quite near to what is outlined above. My general conclusions are not, therefore, affected.
VI. PRODUCT INNOVATIONS

1. I consider radical product innovations (as opposed to incremental innovations) to mean here that a completely new final product is launched on the market, "a product that is a radical departure from existing ways of performing a service" (Dean, 1950, p. 46). This new commodity may replace another commodity in satisfying a perceived consumer need or may be in response to an entirely new need.

2. The demand for these new commodities does not spring endogenously from the model as in the case of process innovations, but it results essentially from changes in consumer preferences. In long waves and marketing literature (Van Duijn, 1983; Levitt, 1965; Dean, 1950; Mahajan, Muller and Bass, 1990) the usual reference is to the product life-cycle. According to Levitt (1965, p. 81), demand for and sales of new products pass through four stages:
   i) market development (introduction), when the product is first brought to the market;
   ii) growth, when demand begins to accelerate and the size of the total market expands rapidly;
   iii) maturity, when demand levels off and grows, for the most part, only at the replacement and new family-formation rate;
   iv) decline, when the product begins to lose consumer appeal and sales drift downward.

   The reasons put forward to explain consumer behaviour in the first and second stages are not so different from those identified above for the S-shaped diffusion of process innovations. First of all, there is the gradual spread among consumers of information on the existence of the new commodity, its characteristics and its appropriateness in satisfying a particular need: it is the "epidemic" model (Stoneman, 1983).

   Next we turn to prices. Very often the introduction of a new product requires heavy investment in research and development as well as consi-

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38 Falkinger (1994) develops another approach in which the demand for new commodities is derived from the hierarchical nature of consumer demand: when people have satisfied higher-priority wants, they turn to new wants. Thus, product innovations must be related to income because demand for new products evolves as demand for old products is saturated. Seen in this way, product innovations are no longer exogenous. (I thank J. Falkinger for drawing my attention to this point).

   This approach could be compatible with the findings of the long-wave theory summarized in Table 1 below. In fact, during the long stagnation, income inequalities usually widen and this could foster demand for new products.
derable expenditure on marketing. In such circumstances, the price at the initial stage in the product life-cycle will frequently be set at a high level to allow the innovator to recoup his costs before too many imitators enter the market. However, this price level, which "skims the cream of the demand", will be progressively abandoned during the later stages of the product life-cycle so as to stimulate demand from other segments of the market. Further price reductions of this kind will be engendered by the growing competition from newcomers as well as by process innovations in the sector concerned and in the corresponding capital goods sector.

In the literature, the demand for radically new products during the first three stages of their life-cycle is represented by a Gompertz curve (e.g. Levitt, 1965), which implies that its percentage rate of change with respect to the previous period \( r_i^{(np)} \) decreases over time. Unfortunately, it is not possible to add anything precise regarding the magnitudes of the values taken by \( r_i^{(np)} \) at any period because they depend on the initial level of demand for the new commodity as well as on the slope of the diffusion curve. The only thing that one can say is that \( r_i^{(np)} \) will usually be quite high during the first two stages of the product life-cycle.

The length of the diffusion period, i.e. the number of years taken to move from the introduction stage to the maturity stage, varies a great deal from one product to another. Thus, unlike in the case of process innovations, it can no longer be realistically assumed that the maturity stage is reached at the end of the phase at the long-wave during which the innovation was first introduced. Empirical research by Gort and Klepper (1982) based on a sample of "basic" product innovations first

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39 The strategic choice regarding the initial price level is discussed by Dean (1950, p. 49 et seq.).
40 The Gompertz curve is:

\[
D(t) = Ka^{bt}
\]

where:
(as already noted,) \( D(t) \) is the cumulative share of the total production of the sector affected by the diffusion at period \( t \),

\( K \) is the asymptote of the function (the saturation level)

\( a \) and \( b \) are constants. Constant \( a \) determines the value of the function when \( t = 0 \) and \( b \) determines the slope of the curve.

The (instantaneous) percentage rate of change with respect to the previous period is:

\[
C \frac{b^t}{a^{bt}}
\]

where \( C \) is a constant: \( C = \ln(b) \ln(a) \)
commercially introduced between 1887 and 1960 shows that, on average, the maturity stage was reached in 37 years.\textsuperscript{41}

3. New products can be manufactured using an existing technique or a completely new technique. In the first case only incremental process innovations take place and the rate of increase of productivity is $\rho_{tr}$. This is referred to below as the “pure” case.

Alternatively, when the technology which is becoming dominant in the economic system is pervasive in nature (as it is at present the case with information technologies), the manufacture of the new product will also be affected by radical process innovations.

a) The “pure” case: product innovations alone

4. Demand is completely exogenous: its evolution is described by the product life-cycle and relies essentially on changes in consumer preferences.

5. The physical output of the new commodity does not present any conceptual problem. All that needs to be done is to insert into formulae (V.3) and (V.4) the rates of change of demand $r^{(np)}_i$ and $r^{(np)}_i$. Since these rates are derived from a Gompertz curve, the output of the final commodity will reflect closely this movement, while the output of the capital goods will exhibit the well-known cycle. The difference compared with process innovations is that the diffusion could now be much longer and could extend, for instance, over the entire stagnation phase of the long wave or even beyond.

6. Employment is also straightforward:

$$E_i(t) = IC_{i1} e^{(r^{(np)}_i - \rho_{tr})t} \quad (VI.1)$$

$$E_{ki}(t) = (r^{(np)}_i + \frac{1}{T_i}) D_{4i} IC_{2i} e^{(r^{(np)}_i - \rho_{tr})t} \quad (VI.2)$$

Since $r^{(np)}_i > \rho_{tr}$, employment increases in both sectors: in this “pure” case, product innovations are thus a major source of employment.

\textsuperscript{41} However, the interval required for successful imitation has systematically declined over time. While the overall average length of the first stage (introduction) is 14.4 years, for products introduced before 1930 this interval was 23.1 years; it is 9.6 years for those introduced in the period 1930-1939 and only 4.9 years for products introduced in 1940 or later (Gort and Klepper, 1982, p. 640).
b) The "mixed" case: product innovations are coupled with radical process innovations

On this point I shall be very brief because the analysis is very similar to chapter IV above.

b.1) Demand and physical output

7. Demand has two additive components because the process of changing consumer preferences (the exogenous part, described by a Gompertz curve) is coupled with the demand stimulus resulting from the technological revolution (the price and income effects, or the endogenous component):

\[
\dot{r}_i^{(np)} = \dot{r}_i^{(ex)} + \dot{r}_i^{(iw)}
\]

(VI.3)

where

\[\dot{r}_i^{(ex)}\]

is the (instantaneous) percentage rate of change in the exogenous component of demand.

Taking into consideration formula (V.1), the preceding formula becomes:

\[
\dot{r}_i^{(np)} = \dot{r}_i^{(ex)} + \varepsilon_i \dot{r}_i^{(iw)} + (\eta_i - \varepsilon_i) \dot{\rho}^*
\]

(VI.4)

During the first few years of the diffusion period \(\dot{r}_i^{(ex)}\) will be the dominant factor shaping \(\dot{r}_i^{(np)}\) because, as already noted, \(\dot{r}_i^{(ex)}\) is quite high while \(\dot{r}_i^{(iw)}\) is low. Moreover, since the diffusion period for the new product could be longer than the time span required by the diffusion of the process innovations, when the technological revolution has come to an end, the demand for the new product could continue to grow substantially under the influence of \(\dot{r}_i^{(ex)}\).

8. As for physical output, I would simply note that, for the final commodity, we find once again the S-shaped movement resulting from the exogenous and the endogenous components of demand. Output in the capital goods sector displays the usual cycle around the long-wave pattern followed by \(X_i\).

b.2) Employment

9. For the final commodities sector we have, as usual:

\[
E_i(t) = E_i(t-1) e^{r_i^{(np)} - \dot{\rho}_i^{(iw)}}
\]

(VI.5)
Writing in full the exponent on the basis of formula (VI.4) and rearranging, we have:

$$\dot{r}_{i}^{(np)} - \dot{r}_{i}^{(iw)} = \dot{r}_{i}^{(ex)} + \epsilon_{i} \dot{\rho}_{i,k_{i}}^{(iw)} - \dot{\rho}_{i}^{(iw)} + (\eta_{i} - \epsilon_{i}) \dot{\rho}^{*} \quad (VI.6)$$

The comments in point V.3. a on the importance of price and income elasticities of demand for determining the changes in employment stemming from process innovations (the endogenous components) still hold. I would just add that, even when such elasticities are less than one (and when, as a result, the endogenous component of employment is zero or negative), employment in sector $i$ will increase under the influence of the exogenous component of demand provided that:

$$\dot{r}_{i}^{(ex)} > |\epsilon_{i} \dot{\rho}_{i,k_{i}}^{(iw)} - \dot{\rho}_{i}^{(iw)} + (\eta_{i} - \epsilon_{i}) \dot{\rho}^{*}| \quad (VI.7)$$

Assuming, so as to simplify matters, that the diffusion of product and process innovations starts at the same time, employment in sector $i$ will certainly rise in the first stage of the product life-cycle. When the product life-cycle is longer than the diffusion period for the process innovations, employment could also increase during the period beyond the end of that period if:

$$\dot{r}_{i}^{(ex)} > |\rho_{tr} (\eta_{i} - 1)|^{42} \quad (VI.8)$$

For the period covering the intermediate stages of the product life-cycle, the case in which the two elasticities are less than one does not permit a clear-cut outcome: depending on the value taken by $\epsilon_{i}$, $\eta_{i}$ and $\dot{r}_{i}^{(ex)}$, employment could increase or stagnate. In the case of capital goods, the only thing to note is that they add a cyclical component (the accelerator term) to the general trends outlined above.

In conclusion, we can say that, in spite of some uncertain cases, product innovations on the whole offer positive prospects for employment, even when the effects of process innovations are job-destroying.

**VII. OUTLINE OF THE OVERALL DYNAMIC OF THE SYSTEM**

The purpose of this section is not to reconstruct the long-wave movement, a task which is beyond the limits of this paper and which

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42 Formula (VI.8) is obtained taking into account that, during the period in question, there are only incremental process innovations. Thus: $\rho^{*} = \rho_{tr}$. 

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includes many institutional aspects, but to address those aspects which are relevant for my attempt to introduce long-waves into Pasinetti’s model. I will, therefore, consider only the effects of the diffusion of the technological revolution (which is nevertheless one of the main factors explaining the appearance of long waves) in order to give some broad indications as to how the sectoral trends already examined generate a long upswing for the whole economy. The transition from long expansion to long stagnation is thus left out of my analysis (for this see Mandel, 1976, 1980).

1. Van Duijn (1983) shows that, during the depression phase of the long-wave, the major innovations tend to appear in existing industries and concern processes as well as products. During the recovery, the number of major process innovations in existing industries falls sharply, while the flow of product innovations continues. However, the dominant feature of this phase is the appearance of radical product innovations leading to the creation of new industries. The propensity to innovate, therefore, seems to change as described in Table 1.

Table 1
Propensity to innovate during the phases of the long-wave
(Van Duijn, 1983, p. 137)

<table>
<thead>
<tr>
<th></th>
<th>Stagnation</th>
<th>Expansion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Depression</td>
<td>Recovery</td>
</tr>
<tr>
<td>1. Process innovations (existing industries)</td>
<td>•••</td>
<td>•</td>
</tr>
<tr>
<td>2. Product innovations (existing industries)</td>
<td>•••</td>
<td>••</td>
</tr>
<tr>
<td>3. Product innovations (new industries)</td>
<td>•</td>
<td>•••</td>
</tr>
<tr>
<td>4. Process innovations (basic sectors)</td>
<td>•</td>
<td>•</td>
</tr>
</tbody>
</table>

The more dots, the greater the propensity to innovate.

43 By disregarding this part of the long-wave theory, we need not introduce the effects of the saturation of demand for many commodities, and this justifies my previous assumption of constant income elasticity of demand for the numerical simulations.

Falkinger's (1994) model, in which income distribution and demand for new commodities play a central role in the long-run growth path of the economy, helps to complete the picture.
2. Starting with *process innovations*, the first element which characterizes the overall dynamic is the growing number of sectors affected by the technological revolution (Table 1). If the new technology is pervasive, a substantial part of the economy (including services) will operate on the new technical base by the end of the long stagnation. The diffusion of the dominant technology forming the new "technological paradigm" (Freeman and Perez, 1988; Dosi, 1984) is further reinforced by the appearance of a cluster of radical innovations in other fields that are not necessarily related to the "core" of the technological revolution (Schumpeter, 1977).

3. The consequence of the inter-sectoral diffusion of the technological revolution is a change in the composition of the "dynamic standard commodity" and a corresponding acceleration in the "standard" growth rate of productivity. The changes in productivity associated with the "dynamic standard commodity" will reflect the S-shaped productivity functions of the individual sectors affected by the technological revolution.

This modifies my previous analysis where, to facilitate the numerical simulations, the *numéraire* was any commodity \( h \) not affected by radical technical change. In fact, since the "standard" growth rate of productivity is now much more important, this implies that:

i) the price effect of the demand functions is reduced. In fact, since \( p^* > \rho_{11} \), in terms of the new *numéraire* the decline in the prices of the commodities affected by the technological revolution is less marked,

ii) the income effect is magnified and becomes increasingly important as the technological revolution extends to other sectors of the economy.

As a matter of fact, process innovations occur throughout the entire long wave (Table 1). This means that the two effects arising from the increase in the "standard" rate of productivity are constantly being fuelled. During the first phase of long stagnation (the depression) the stimulus comes from existing industries; when the number of innovations from these sectors falls, there is a wave of innovations from "basic" sectors, intensifying in the first phase of the long expansion, and so on.

4. The other major effect of the inter-sectoral diffusion of the technological revolution is on physical output. We have seen that, when a sector is affected by such technological change, its output follows a long-wave path. The multiplication of this phenomenon during the long stagnation sets in motion a cumulative process of growth which is then further sustained by process innovations in "basic" sectors.
during the prosperity phase of the long expansion. To this should be added the general growth in demand associated with the fulfilment of the equilibrium condition for wages, i.e. the wage rate follows the "standard" growth rate of productivity. This effect operates throughout the long wave and becomes stronger when radical technical change intensifies.

Being the aggregation of all sectoral outputs, aggregate output will exhibit the familiar S-shaped profile, which now extends over the (conventionally assumed) fifty years of the long-wave. The capital goods sectors add a "rolling" component to the basic trend set by the final commodities sectors.

5. **Product innovations** strongly reinforce the tendencies outlined above, particularly during the long stagnation because, as Table 1 shows, it is in this phase of the long wave that the propensity for such innovations to materialize is greater.

To appreciate their overall impact, it is essential to distinguish between product innovations in existing industries and product innovations giving rise to new industries. In fact, in the former case a (completely) new product satisfies a need which was already met by another commodity. Enterprises in this sector thus progressively substitute the old commodity with the new one. The contribution of the sector to aggregate output in the economy is the difference between the expanding output of the new commodity and the declining output of the old commodity.

Product innovations which coincide with the creation of new industries satisfy a new need: their output thus therefore represents a net addition to aggregate output. Since such innovations are more frequent during the recovery phase of the long stagnation, their contribution to the incipient process of growth could be appreciable.

6. **Total employment** is the most puzzling part of the story because it is influenced by countervailing and uncertain factors.

a) Let us start with **process innovations** and consider the realistic case in which the new technology is pervasive (i.e. it concerns both final commodities and capital goods sectors as well as a large part of the economy). This technological revolution has two effects on the level of employment in the economic system: a general effect, with positive repercussions on employment, and a specific effect, with neutral or negative repercussions. Let us first consider the latter.

The sectoral analysis has shown that total employment displays a long-term positive trend only when the price and income elasticities of demand are high (greater than one). Since empirical evidence indicates
that such high values of the elasticities appear only in a few cases, one can expect that, for the whole economic system, the trend will be flat or at best slightly positive, with a more or less pronounced cycle due to the capital goods sector.

The general effect stems from the increase in aggregate demand resulting from the positive influence of the technological revolution on the "standard" growth rate of productivity and on wages. In such circumstances, total employment will be underpinned by the sectors not concerned by radical technical change. In fact, their demand will increase, but this will not be offset by an analogous increase in productivity, which continues to grow at the trend rate. The magnitude of this effect depends on the relative importance of the sectors in question with respect to the total economy.

b) For product innovations the outcome is well defined, in the sense that, even when process innovations also extend to the manufacture of new products, we can expect a positive effect on employment.

c) To sum up, one can tentatively say that, during the long stagnation, the "specific" effect will prevail while, during the long expansion, the main stimulus will come from the demand side. To be more precise:

- in the depression phase of the long stagnation employment will be roughly stationary, for three reasons:
  - process innovations in existing industries will not contribute appreciably to the growth of employment;
  - the same will be true of product innovations in existing industries. The new products, in fact, replace some old ones and, in any case, their relative importance is rather weak because they are at the beginning of the product life-cycle;
  - the demand effect is also rather weak, especially during the first years of the phase;
- during the recovery phase employment will increase under the impact of product innovations in new industries, which will be reinforced by the same type of innovations in existing industries. To this has to be added the demand effect, which has meanwhile gained momentum.

It is perhaps worth repeating that this is only a partial picture of the long-wave, a picture which considers only the effects of the technological revolution. For instance, the fact that in the depression technical change has a rather neutral effect on employment does not prevent employment from actually falling for other reasons (e.g. sectoral restructuring and bankruptcies).
7. To conclude, I would like to stress that my analysis is not inconsistent with the more elaborate theory of long-waves, which considers actual capitalist economies instead of the "natural" system. The case in which this complementarity is the most obvious is the Schumpeterian interpretation of long-waves, focussing on the "techno-economic" paradigms (Dosi, 1988; Freeman, 1982; Freeman and Perez, 1988), but this is also apparent in Mandel's (1976, 1980) contribution, in which social relations and conflicts play a central role. According to this author, three conditions need to be met in order to trigger a new long-term expansion: i) a technological revolution; ii) an exceptional long-term increase in the actual and expected average rate of profit; iii) a long-term expansion of demand. The second and third conditions are the prerequisites for a massive implementation of radical innovations, while the first depends on a number of exogenous factors (Mandel, 1976, Vol. I, p. 224-225).

If we compare these findings with the results of the present paper, we will see that, leaving aside the profit rate condition, the expansion of demand now has an endogenous explanation at the deeper level of the "natural" system. "Institutional" analysis is thus embedded in a classical "high" theory and receives new strength from it.

VIII. CONCLUSIONS

1. Long-waves are introduced into Pasinetti's model of structural change on the assumption that productivity growth is driven essentially by technological revolutions. Radical process innovations result in a leap in productivity for the innovator and progressively extend throughout the sector concerned according to a non-linear path. Demand for completely new products follows a similar profile, which is determined by the product life-cycle.

2. The argument is developed at the logical stage which precedes institutions (the "natural" system) so as to identify the basic forces determining the trend and the boundaries for the actual movements in prices, physical quantities and employment.

The enquiry is conducted mainly at the sectoral level; however, at the end of the paper it is shown that the outcome of the sectoral trends discovered is a long-wave pattern for the whole economic system. This is not, of course, a reconstruction of the complete long-wave movement, but rather an analysis of the effects of the technological revolutions in Pasinetti's "natural" system.
3. Three general results should be mentioned. The first one is the overwhelming importance of the pattern of diffusion of the technological revolution. It is, in fact, this element that shapes the productivity curve of the sector, which, in turn, determines the trend and form of the price movement as well as the scope for the growth of demand.

This last aspect, which constitutes the second general result, deserves particular attention. The technological revolution and the ensuing increase in productivity generate an endogenous mechanism explaining the growth rate of demand via:

- a purchasing power effect which operates when wages are linked to the average productivity growth of the system (the “standard” rate of growth);
- a price effect for the commodity directly concerned by the technological revolution.

The third result is the importance of the price and income elasticity of demand, which can amplify or reduce the basic stimulus coming from productivity.

4. As for process innovations, the sectoral analysis shows that physical output in the final commodities sectors follows a long-wave (S-shaped) profile which is more or less pronounced according to the values taken by the price and income elasticities of demand.

Physical output in the capital goods sectors is characterized by a business-cycle pattern around the long-wave path displayed by the corresponding final commodities sector.

The progressive inter-sectoral diffusion of such innovations sets in motion a cumulative process of growth that helps the system climb out of the long stagnation.

5. The employment outcome is complex.

a) The clearest case involves product innovations, which result in a growing employment trend both at sectoral and aggregate level.

When there are no radical process innovations in the vertically integrated sector producing the new commodity (the “pure” case), product innovations are a major source of employment. When, on the contrary, the output of the new commodity is concerned by radical process innovations (the “mixed” case), the situation is not unambiguous. Nevertheless, even in this case the prospects for employment are on the whole positive because the increase in demand for the new commodity tends to outweigh the job-destroying effect of process innovations.
b) For process innovations the results are more uncertain because employment is exposed to a number of conflicting pressures. I should mention in particular the price and income elasticities of demand.

b.1) At sectoral level, numerical simulations carried out for the realistic case in which the technological revolution affects both final commodities and capital goods sectors lead to the conclusion that, in the most common cases (i.e. when the price and income elasticities of demand are equal to one or less), total employment stagnates or declines, with a cyclical component. Comparing this result with that for output, we notice that, for the vertically integrated sector concerned, substantial growth in output may very well be compatible with stagnating or even declining employment. This is because the rate of change of demand which is (endogenously) generated by the technological revolution is sufficiently high to raise output, but not large enough to compensate for the employment-reducing effect of productivity increases.

b.2) At macroeconomic level, analysis of the inter-sectoral diffusion of the technological revolution in the case of pervasive technological change has shown two conflicting influences on aggregate employment:

- a specific effect reflecting the situation in the sectors directly affected by the technological revolution;

- a general effect resulting from the following sequence: increase in productivity in the sectors affected by the technological revolution; ensuing increase in the "standard" growth rate of productivity; corresponding increase in wages and in aggregate demand.

For the most common values of the price and income elasticities of demand (i.e. for \( \varepsilon_i \) and \( \eta_i \) equal to or less than one), the first effect will be conducive to a rather flat trend for total employment, whereas the second effect will push up employment because the extra demand will also be directed towards sectors untouched by radical technical change and hence not suffering from technological job redundancies. The extent of such job creation depends on the relative importance of these traditional sectors. If their share of the total economy is limited because the technological revolution is pervasive, then the prevailing macroeconomic tendency could be a very slow increase in or a stagnating level of employment, even during the long expansion.\(^{44}\) Thus, one

\(^{44}\) In today's industrial societies the technological revolution could actually encompass manufacturing as a whole and half of the service sector, with the result that the technologically advanced share of the system accounts for about 70% of the private sector. The positive effect on employment stemming from the general increase in demand is thus confined to the remaining 30%.
should not be too impressed if, for some years, total employment rises under the impact of the capital goods sector, because this is only a cyclical movement which does not undermine the basic trend.

6. The theoretical analysis carried out in this paper has at least three implications for economic policy: a) the action to foster the diffusion of the technological revolution; b) the action on the employment front; (c) the guiding role of public authorities in meeting the equilibrium condition for wages.

a) We have seen that the diffusion of radical technical change is conducive to growth. The faster the diffusion, the sooner growth will materialize. However, several obstacles can delay innovations; public authorities can influence the process directly and through R&D policy.

b) Employment policy has two main aspects: i) the measures necessitated in the normal course of events by structural change; ii) the specific measures imposed by the pervasive nature of the present technological revolution.

The diversified impact of technical change entails a permanent shift in the structure of employment which calls for a continuous flow of workers from contracting to expanding sectors. There is considerable scope for government action in this field. The first task is to foster the sectoral shifts in the labour force: besides disseminating appropriate information on labour market opportunities, the public authorities must provide constant retraining and skill development for the population.

If the above measures are not sufficient to achieve full employment, the public authorities can attempt to reduce the overall labour supply by acting on two parameters: the share of the labour force in the total population and the share of working time in total time. As for the first parameter, they could lengthen the period of compulsory education, encourage people to take early retirement, promote part-time work, etc. The second parameter can be influenced by a reduction in annual working time, mainly through a reduction in weekly working hours. This has been happening for a long time: over the last two hundred years, we have moved from the 80-hour week (or more) common in the 19th century to the present 40-hour week.

The technological revolution in computer and information technologies adds a specific problem on account of its pervasive character. While in past long waves the technological revolution affected only some segments of industry – with no influence on services – the present technologies have also spread into this sector, which is no longer a reliable source of employment for those who have lost their jobs in
industry. As my results show, when technological change is pervasive, the usual outcome is growth with a very low increase in or a stagnating level of employment, even for the long expansion phase of the long-wave. Considering that we are now in a situation of high unemployment, the employment prospects for the next decade could thus be very gloomy, and this makes it more necessary to devise ways of reducing total labour supply.

c) Finally, it is important to stress the importance of the equilibrium condition of the model, which links wage dynamics to average productivity growth in the system. This Keynesian component of Pasinetti’s model is particularly important in the recovery phase of the long stagnation and becomes crucial in the long expansion because it is the way to provide the demand for a growing output.

REFERENCES


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