Inflation, Growth and Exchange Rate Regimes in Small Open Economies

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Inflation, Growth and Exchange Rate Regimes in Small Open Economies*

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Summary. This paper compares the merits of alternative exchange rate regimes in small open economies where financial intermediaries perform a real allocative function, there are multiple reserve requirements, and credit market frictions may or may not cause credit rationing.

Under floating exchange rates, raising domestic inflation can increase production if credit is rationed. However, there exist inflation thresholds: increasing inflation beyond the threshold level will reduce domestic output.

Instability, indeterminacy of dynamic equilibria and economic fluctuations may arise independently of the exchange rate regime. Private information –with high rates of domestic inflation- increases the scope for indeterminacy and economic fluctuations.

Keywords: Currency Board, Endogenously Arising Volatility, Fixed exchange rates, Floating exchange rates, Growth, Indeterminacy, Inflation, Multiple Reserve Requirements, Private Information, Stabilization

JEL classification: E31, E32, E42, E44, F31, F33, G14, G18, O16

1 Introduction

One of the most basic issues in monetary economics concerns the relative merits of different methods for achieving stability of the price level. In an open economy context, a consideration of this issue necessarily involves a comparison of fixed versus flexible exchange rate regimes.

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Standard quantity theoretic policy prescriptions imply that domestic price level stability can be achieved with a floating exchange rate simply by fixing a low and constant rate of growth for the money supply. However, in countries confronted with high rates of inflation, this is rarely the proposal made for stabilizing the price level. Instead, it is often argued that such economies should fix their rate of exchange against the currency of a country with relatively stable price level -for instance the U.S.[1]

Concerns about the stability of the price level loom particularly large in view of two empirical results. First, it is well-established that there is a strong link between the health of an economy's financial system and its long-run real performance[2]. Second, the level of financial development in an economy is very adversely affected by inflation[3]. These results together suggest that excessively high rates of inflation can have very negative implications for real performance, both in the short and long-run. And, indeed, Bullard and Keating [5] or Khan and Senhadji [7] find that, at low initial rates of inflation, modest increases in inflation can be associated with higher (long-run) levels of real activity. However, above some threshold, further increases in the rate of inflation seem to have adverse effects on short and long-run activity.

This paper investigates the relative merits of different exchange rate regimes along several dimensions, especially with respect to achieving low and stable rates of inflation, promoting financial deepening, and fostering relatively high levels of long-run real activity.

Issues about alternative exchange rate regimes have taken on particular prominence in a Latin American context, where there are long histories of high rates of inflation. A particular motivation for examining different exchange rate regimes is to think about alternative methods for stabilizing high rates of inflation in a small open

[1] Vegh [14], p.42, for example, argues that "the evidence clearly suggests that, in hyperinflationary situations, price stability can be the immediate result of using the exchange rate as a nominal anchor."
[2] See, for instance, King and Levine [9, 10], Levine and Zervos [11], and Levine, Loayza and Beck [12].
[3] See Boyd, Levine and Smith [3] or Khan, Senhadji, and Smith [8].

economy. Here, I focus my attention on the relative merits of two different policies that have been implemented as part of inflation stabilizations in Latin America and, particularly, in Argentina and Perú.

Perú and Argentina are small open economies that experienced episodes of severe hyperinflation in the late 1980s and early 1990s. Both stabilization programs were successful in reducing inflation rates. In addition, these programs had some common aspects with respect to fiscal policies. However, the main difference, and the one I focus on in this paper, is the choice of exchange rate regime. On the one hand, Argentina implemented a currency board, more consistent with a traditional view of what a stabilization program should be: an exchange rate is fixed to an "anchor currency" and automatic convertibility is ensured. In Perú, on the other hand, the exchange rate was left to float freely, under the supervision of the Central Bank. The success of the Peruvian stabilization is extremely interesting in view of the commonly accepted point of view that Latin American countries cannot or will not pursue successful stabilizations based on floating exchange rates.

With these facts in mind, I model a small open economy that reproduces several aspects of the Peruvian and Argentinean economies subsequent to their stabilizations. In each economy, financial intermediaries perform a real allocative function in the presence of obvious credit market frictions that may or may not cause credit to be rationed[4]. As shown by Azariadis and Smith [1] or Boyd and Smith [4] in a closed economy context, when credit is rationed changes in the rate of inflation can have strong effects on the extent to which credit is rationed, and on financial depth. Here I extend the Azariadis-Smith [1] framework to the case of a small open economy. In addition, I add several features to the model that are particularly relevant to Latin American experiences. In particular, a domestic and a foreign currency circulate in the domestic economy, and

[4] Credit rationing is often argued to be a very important aspect of funds allocation in developing economies.

domestic lending is subject to multiple reserve requirements (that are, in general, binding). Finally, there are no legal restrictions either on the use of foreign currency or on investing abroad.

I then consider two such economies that are similar in every respect, except for their choice of exchange rate regime. In the first economy, a floating exchange rate regime will be in place. On the other hand, the second economy will operate under a fixed exchange rate regime, and this economy will be constructed so that a currency board emerges as a special case.

I find that in economies with floating exchange rates, changes in domestic inflation and world (U.S.) inflation affect the domestic capital stock differently according to whether or not credit is rationed. Interestingly -and, in marked contrast to the literature on closed economies[5]- either credit rationing tends to be observed when domestic rates of inflation are low, or else the scope for credit to be rationed depends in a relatively complicated way on the rate of money creation (inflation). The first situation will emerge when the probability of loan default is relatively low while the second will arise when the probability of default is sufficiently high.

In situations where the probability of repaying loans is high and there is a floating exchange rate, moderate increases in the rate of money growth (inflation) stimulate output and lead to financial deepening when credit is rationed (inflation is initially low), but reduce output and financial depth when there is no credit rationing (inflation is initially high). Thus there will be inflation thresholds as are observed empirically: inflation and output are positively (negatively) correlated below (above) the threshold. As a consequence, there is a strict limit to the extent to which domestic inflation can be used to stimulate output. Furthermore, when equilibrium dynamics are considered, I find that -when credit is rationed- endogenously arising volatility can easily be observed. This volatility will be manifested in all endogenous variables, including the rate of inflation.

[5] See, for an example, Azariadis and Smith [1].

Thus, in the short-run, a low and fixed rate of money creation need not imply an absence of price level fluctuations, even in the absence of any exogenous shocks.

On the other hand, in situations where the probability of repaying loans is low and there is a floating exchange rate, increases in the domestic inflation rate always have adverse consequences for real activity. Moreover, private information (together with high rates of inflation) seems always to increase the scope for indeterminacy of dynamic equilibria and for economic fluctuations.

In a small open economy with a fixed rate of exchange, the domestic and foreign inflation rates will be equal. Interestingly again -and, yet in marked contrast to the literature on closed economies- either the scope for credit to be rationed depends in a relatively complicated way on the rate of foreign (and domestic) inflation, or credit rationing tends to be observed when foreign (and domestic) rates of inflation are low. Under a fixed exchange rate regime, the first situation will be associated with a low probability of loan default, while the second situation will be observed when the probability of default is high.

In situations where the probability of repaying loans is high and there is a fixed exchange rate, increases in the foreign rate of inflation always have adverse consequences for real activity. In situations where the probability of repaying loans is low, however, there will be inflation thresholds: foreign (and domestic) inflation and output are positively (negatively) correlated below (above) the threshold.

Of course when the rate of exchange is fixed, the domestic country inherits the inflationary experience of the rest of the world (the U.S.). This is obviously not the case under a flexible exchange rate regime. As the results just described indicate, when credit is rationed the ability to raise the domestic inflation rate above the foreign inflation rate can have positive consequences for financial depth and for real activity, so long as the domestic rate of inflation is not excessively high. In this sense, there can be a real cost to the implementation of a fixed exchange rate regime.

Finally, in economies with fixed exchange rates, a currency board seems to increase the scope for endogenously arising economic fluctuations. Such potential for fluctuations disappears as the backing of the domestic money supply and deposits is reduced. Moreover, indeterminacy of dynamic equilibria may be observed independently of the backing of the domestic money supply. And, in economies with fixed exchange rates, the potential for indeterminacy and fluctuations seems to be positively related to the (world) rate of inflation.

The remainder of the paper is organized as follows. In Section 2, I present a model of a small open economy with floating exchange rates. This economy shares the main stylized characteristics of the Peruvian economy after its stabilization. I then discuss when credit rationing may arise in such an environment as well as the main properties displayed by dynamic equilibria. Next, in Section 3, I consider a model of a small open economy that operates under a fixed exchange rate regime. I again describe when credit may be rationed and equilibrium dynamics. Finally, in Section 4, I present the main conclusions of the analysis.

2 A Flexible Exchange Rate Regime: the Peruvian Economy after the Stabilization

In this section, I build a model of a small open economy that captures the main stylized characteristics of the post-stabilization Peruvian economy. The model is in the spirit of Azariadis and Smith [1], who consider a closed economy in which capital investment requires external finance, and in which credit markets operate subject to various informational asymmetries. I extend this framework to the case of a small open economy where both foreign and domestic currencies circulate and where individual agents can invest both at home and abroad. In addition, domestic lending is subject to multiple reserve requirements (that are, in general, binding) and a flexible exchange rate regime is in place, with no legal restrictions on either the use of foreign currency or on foreign investment.

2.1 The Environment

I consider a small open economy consisting of an infinite sequence of two-period lived, overlapping generations. Time is discrete, and indexed by t=0, 1, 2,... .

Each generation consists of a continuum of agents with unit mass, divided into two types. Type 1 agents comprise a fraction $\lambda \in (0,1)$ of the population, while the remaining fraction $(1-\lambda)$ consists of Type 2 agents.

Every period, both physical capital and labor are used to produce a single tradable final good. K units of physical capital and N units of labor produce $F(K,N)$ units of the final good, where $F(\cdot)$ is a constant returns to scale production function. Let $f(k) \equiv F(k,1)$ denote the intensive production function, with k being the capital-labor ratio, $k \equiv K/N$. I assume that $f(\cdot)$ is a smooth, increasing, concave function such that $f(0)=0$. Finally, we also assume, without real loss of generality, that physical capital depreciates completely in the production process.

All agents are risk neutral and, for simplicity, care about consumption only in the final period of life.

2.1.1 Endowments

Young Type 1 agents are endowed with one unit of labor, which is supplied inelastically. These agents have no labor endowment when old. In addition, Type 1 agents are endowed with access to two investment technologies. One of these is a pure storage technology whereby one unit of the good stored at t returns $x>0$ units of consumption at $t+1$. x should be thought of as relatively small, so that the storage technology is not efficient. The second investment technology available to Type 1 agents transforms one unit of the final good at t into one unit of capital at $t+1$ with probability $\pi \in (0,1)$. With probability (1-π), investments in this technology produce nothing. If capital is received when old, a Type 1 agent making an investment can then hire young labor, and produce final goods using the commonly available final goods production technology. For simplicity I assume that this technology can be utilized only by agents who receive capital from previous investments; there are no rental markets in physical capital.

Type 2 agents have no labor endowment when young, but supply one unit of labor inelastically when old. When young, a Type 2 agent is endowed with an investment technology that allows him to transform one unit of the final good at t into one unit of capital at $t+1$ with certainty. Once this capital is obtained, old Type 2 agents can combine their own labor with labor they hire from young Type 1 agents, and they can then produce the final good. Again, purely for simplicity, Type 2 agents are assumed to work only for themselves.

In addition to young agents, there is an initial old generation at t=0. These agents are all endowed with one unit of labor and K_0>0 units of physical capital. No other agents have an initial endowment of capital, nor are any agents endowed with the final good.

2.1.2 Informational Structure

At the beginning of each period, each agent knows his own type. However, the agent's type is private information. Since Type 2 agents are natural borrowers, having access to a productive investment technology but no young period income, this private information gives rise to a conventional adverse selection problem in credit markets.

In addition, if they obtain credit, at some point each young Type 1 agent learns whether or not he can productively invest in physical capital. This information is also private to the agent. However, age and all market transactions (like working, making deposits in or borrowing from the financial system) are observable. The activity of storing goods does not require market transactions, and, therefore, the storage activity is unobservable.

Given the information structure, young Type 2 agents cannot credibly claim to be of Type 1 and supply labor when young. However, young Type 1 agents can credibly claim to be of Type 2. In order to do so, young Type 1 agents must borrow the same amount that young Type 2 agents do and they cannot supply labor. However, only a fraction π of Type 1 agents have the ability to create physical capital. The remaining fraction cannot operate the production process when old and they would then be discovered as having misrepresented their type. I assume that they can be punished prohibitively. Consequently, the fraction $(1-\pi)$ of young Type 1 agents who obtain credit will avoid punishment only if they "abscond" with their loan. They can do so by taking any credit received when young, investing in the storage technology, and "going underground" when old[6]. The agents both escape punishment, and avoid repaying their loan. Finally, notice that Type 2 agents have no access to the storage technology and, consequently, they choose never to abscond.

2.2 Trading and Financial Intermediation

There are several types of trade that can take place in this economy. First, old producers can hire labor from young Type 1 agents at the competitive real wage, w_t. Second, Type 1 agents who work when young save all their labor income, and part of their savings can be lent to domestic agents claiming to be of Type 2. I will think of domestic lending as being intermediated.

There is free entry into the domestic activity of intermediation. I let R_t denote the gross real interest rate offered on deposits by domestic financial intermediaries between t and $t+1$, and ρ_t the gross interest rate charged on loans made at t and maturing at $t+1$.

[6] Alternatively, x can be regarded as representing the punishment incurred after misrepresenting one's type and taking an unproductive loan.

Third, young Type 1 agents can also invest their savings abroad[7]. One unit of goods invested abroad at t returns r>1 goods at $t+1$, where r is the gross international real interest rate. Of course the assumption that the domestic economy is small implies that no events in the domestic economy influence r. Also notice that the storage technology being inefficient implies that $R_t>x$ and $r>x$.

In addition, two types of currency circulate in the domestic economy. One is issued by the domestic government. Let M_t be the outstanding stock of domestic currency at t and p_t denote the domestic price level. In addition, foreign currency may circulate in the domestic economy. I let Q_t denote the outstanding stock of foreign currency in the domestic country, while p_t^* denotes the price level in the rest of the world. I also let e_t denote the market determined nominal exchange rate at t, defined as units of domestic currency required to purchase a unit of foreign currency at t. The law of one price implies that $e_t p_t^* = p_t$, for all t.

Each initial old agent in the domestic economy is endowed with $M_{-1}>0$ units of domestic currency. From then on, the supply of domestic currency evolves according to

$$M_{t+1} = (1+\sigma)M_t \ , \quad \sigma > -1 \quad (1)$$

with σ, the net rate of money creation, exogenously determined by the domestic monetary authority. Any injection or withdrawal of domestic currency is done by lump-sum transfers to young agents claiming to be of Type 2. Since capital investment is intended to be done by young Type 2 agents, the transfer scheme can be thought of as a program run by the domestic government intended to subsidize capital investment. This program is financed by printing money. If we let τ_t denote the real value of the transfer received by a young agent claiming to be of Type 2 at t, and μ_t be the fraction of young Type 1 agents claiming to be of Type 2 at t, the government budget constraint for that period will be

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[7] One could also think of such investments as cross-border deposits.

$$\left[(1-\lambda)+\mu_t\lambda\right]\tau_t = \frac{(M_t - M_{t-1})}{p_t}, \quad t \geq 0 \qquad (2)$$

All domestic lending is subject to the financial regulations of the domestic country. It is assumed that all agents lending domestically must hold currency reserves. Some of these reserves may be held in domestic, and some in foreign currency. Let $\phi_d \in (0,1)$ denote the fraction of deposits that must be held in the form of domestic currency. Domestic currency reserves held from *t* to *t+1* earn the gross real return $\left(\frac{p_t}{p_{t+1}}\right)$. Similarly, let $\phi_f \in (0,1)$ denote the fraction of deposits that must be held in the form of foreign currency reserves by lenders. Foreign currency reserves held between *t* and *t+1* earn the gross real return $\left(\frac{e_{t+1}p_t}{e_t p_{t+1}}\right)$. Obviously it will be assumed that $\phi_d + \phi_f < 1$. Finally, I will focus on the situation where both reserves requirements are binding. This will transpire if $\left(\frac{p_t}{p_{t+1}}\right) < r$ and $\left(\frac{p_t^*}{p_{t+1}^*}\right) < r$ both hold, so that (net) nominal rates of interest are positive both domestically, and in the rest of the world. Clearly, in contexts like Latin America, the assumption of binding reserve requirements is a highly relevant one.

2.2.1 Credit Markets

In keeping with standard practice in the literature on adverse selection (Rothschild and Stiglitz [12]; Azariadis and Smith, [1]), I seek a separating equilibrium in credit markets. In particular, I seek an equilibrium where only Type 2 agents obtain credit.

Let b_t denote the real value of borrowing by young agents claiming to be of Type 2 at *t*. I assume free entry into intermediation, which implies that domestic intermediaries earn zero profits in equilibrium. This requires that the gross real loan rate, ρ_t, satisfy

$$\rho_t = \frac{\left[R_t - \phi_d \left(\frac{p_t}{p_{t+1}} \right) - \phi_f \left(\frac{e_{t+1} p_t}{e_t p_{t+1}} \right) \right]}{\left(1 - \phi_d - \phi_f \right)}. \qquad (3)$$

2.3 Agents' Behavior and Factor Markets

Type 2 agents cannot store goods and they do not wish to consume when young. Therefore, they invest in physical capital all the resources they obtain in youth, and each old Type 2 agent at $t+1$ will have a capital stock equal to

$$K_{t+1} = b_t + \tau_t \quad (4)$$

reflecting both credit received and the government investment subsidies. In addition, at $t+1$ Type 2 agents combine their inherited capital stock with their own unit of labor, plus L_{t+1} units of young Type 1 labor. Finally, these agents repay their loans. Therefore, the consumption of an old Type 2 agent born at t, $c_{2,t+1}$ is given by

$$c_{2,t+1} = F\left(K_{t+1}, 1 + L_{t+1}\right) - w_{t+1} L_{t+1} - \rho_t b_t . \qquad (5)$$

Type 2 agents choose L_{t+1} to maximize this expression, implying that

$$w_{t+1} = F_2\left(K_{t+1}, 1 + L_{t+1}\right) \qquad (6)$$

Combining (5) with (4) and (6), and using Euler's Law, I get that the lifetime utility of a Type 2 agent born at t is

$$c_{2,t+1} = \left[F_1\left(K_{t+1}, 1 + L_{t+1}\right) - \rho_t\right] b_t + w_{t+1} + F_1\left(K_{t+1}, 1 + L_{t+1}\right) \tau_t \ (7)$$

The first term on the right-hand side of (7) reflects profits (if any) derived from borrowing and investing in physical capital. The second term reflects the value of a Type 2 agent's old labor endowment, and the third term reflects the value of the investment subsidy received from the government.

In a nontrivial separating equilibrium, the total demand for labor at $t+1$ is $(1-\lambda)L_{t+1}$ while the total supply is λ. Therefore, labor market clearing at $t+1$ requires

$$L_{t+1} = \frac{\lambda}{1-\lambda} \quad (8)$$

Hence, the capital-labor ratio in such an equilibrium is given by

$$k_{t+1} \equiv \frac{K_{t+1}}{1+L_{t+1}} = (1-\lambda)K_{t+1} \quad (9)$$

and (6) can be rewritten as

$$w_{t+1} = f(k_{t+1}) - k_{t+1}f'(k_{t+1}) \equiv w(k_{t+1}) \; (10)$$

where $w(k_{t+1})$ is an increasing function of k_{t+1}. Notice that equation (10) implies that

$$k_{t+1} = w^{-1}(w_{t+1}) \equiv \Psi(w_{t+1}) \quad (11)$$

so that the maximized consumption of an old Type 2 agent can be written as

$$\begin{aligned} c_{2,t+1} &= \{F_1[\Psi(w_{t+1}),1] - \rho_t\}b_t + w_{t+1} + F_1[\Psi(w_{t+1}),1]\tau_t \\ &= \{f'[\Psi(w_{t+1})] - \rho_t\}b_t + w_{t+1} + f'[\Psi(w_{t+1})]\tau_t \end{aligned} \quad (12)$$

Notice that Type 2 agents will be willing to take loans only if

$$f'(\Psi(w_{t+1})) = f'(k_{t+1}) \geq \rho_t \quad (13)$$

2.4 Loan Contracts

In equilibrium, lenders must design loan contracts that channel funds to natural borrowers. Therefore the loan contracts offered, in equilibrium, must prevent Type 1 agents from misrepresenting their type (since it is unprofitable to lend to these agents).

Thus loan contracts must induce self-selection[8]. I now describe the determination of equilibrium contracts.

I begin by describing the incentive constraint that must obtain in order to induce self-selection. A Type 1 agent who misrepresents his type at t borrows b_t, as Type 2 agents do, and receives the investment subsidy τ_t. Subsequent to receiving these resources, the agent learns whether he can produce capital when old. This occurs with probability π. If capital can be produced, the agent will operate the final goods production process when old[9]. To do so, the agent will hire $\tilde{L}_{t+1}$ units of young labor. In addition, the agent will repay his loan. Thus, with probability π, a dissembling Type 1 agent has the old-age consumption $F(K_{t+1}, \tilde{L}_{t+1}) - \rho_t b_t - w_{t+1}\tilde{L}_{t+1}$. Alternatively, with probability (1-π) a dissembling Type 1 agent cannot produce capital. In this event, a Type 1 agent who borrows when young stores the good, and has old-age consumption equal to $x(b_t + \tau_t) = xK_{t+1} = xk_{t+1}/(1-\lambda)$. It follows that the expected old-age consumption of a young Type 1 agent who misrepresents his type is

$$\tilde{c}_{1,t+1} = \pi\left[F\left(K_{t+1}, \tilde{L}_{t+1}\right) - \rho_t b_t - w_{t+1}\tilde{L}_{t+1}\right] + (1-\pi)x(b_t + \tau_t) \quad (14)$$

The agent in question chooses $\tilde{L}_{t+1}$ to maximize this expression[10]. Hence

$$F_2\left(K_{t+1}, \tilde{L}_{t+1}\right) = w_{t+1} \quad (15)$$

This equation implies that a dissembling Type 1 agent who operates the production process will utilize the same capital-labor ratio as a Type 2 agent. It then follows that

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[8] In equilibrium, lenders must design loan contracts that channel funds to natural borrowers. Therefore the loan contracts offered, in equilibrium, must prevent Type 1 agents from misrepresenting their type (since it is unprofitable to lend to these agents). Thus loan contracts must induce self-selection.

[9] It is possible to show that an agent who can operate the production process will prefer to do so, rather than store goods, if the condition $\left[1 - \frac{(1+\sigma)(1-\phi_d-\phi_f)}{(1-\phi_d-\phi_f)+\sigma(1-\phi_f)}\right]\rho_t \geq x$ is satisfied.

[10] The implicit assumption is that total employment is observable, but the composition of labor inputs between own labor supply and hired youthful labor is not.

$$\tilde{c}_{1,t+1} = \pi\{[f'[\Psi(w_{t+1})] - \rho_t]b_t + f'[\Psi(w_{t+1})]\tau_t\} + (1-\pi)x(b_t + \tau_t) \quad (16)$$

Alternatively, a young Type 1 agent who works when young and saves his labor income obtains the lifetime utility $R_t d_t + r i_t^*$, where d_t and i_t^* denote, respectively, deposits in the domestic financial system and investment abroad. Notice that it must be that $d_t + i_t^* = w_t$. It follows that self-selection occurs in the credit market if

$$R_t d_t + r i_t^* \geq \pi\{[f'[\Psi(w_{t+1})] - \rho_t]b_t + f'[\Psi(w_{t+1})]\tau_t\} + (1-\pi)x(b_t + \tau_t) \quad (17)$$

It is now easy to verify that competition among lenders implies that contractual loan terms, ρ_t and b_t, must be chosen to maximize the expected utility (consumption) of Type 2 agents, subject to the zero profit condition (3) and the self-selection constraint (17). That is, (b_t, ρ_t) maximizes $c_{2,t+1}$ subject to (3) and (17), taking $\tau_t, w_{t+1}, R_t, p_t, p_{t+1}, p_t^*$ and p_{t+1}^* as given.

As I have already noted, this problem has a nontrivial solution if and only if (13) is satisfied. If (13) is an equality, then Type 2 agents are indifferent about the loan quantity they receive. In equilibrium, loan quantities must simply be such that the marginal product of capital equals the loan rate. This outcome is what would be expected in the absence of private information. In effect, the adverse selection problem is non-binding. I refer to this as a Walrasian outcome. Alternatively, if (13) holds as a strict inequality, then Type 2 agents would like to borrow arbitrarily large amounts. Of course, excessive lending would violate the self-selection constraint. Hence Type 2 agents experience credit rationing, and the loan quantity b_t is determined by the self-selection constraint (17) at equality. Below I describe when both Walrasian and Credit Rationed equilibria emerge as outcomes.

2.5 A General Equilibrium

There are several conditions that must be satisfied in a full general equilibrium. First, in the absence of any restrictions on international goods trade, the purchasing power parity condition

$$p_t = e_t p_t^* \qquad (18)$$

must hold. In addition, with no restrictions on international capital flows, rates of return on investments must be equated both internationally and domestically. Hence,

$$R_t = r \quad (19)$$

I assume throughout that $r > x$, so that goods storage is inefficient. In addition, I focus throughout on situations where the reserve requirements bind. Thus

$$r > \frac{p_t}{p_{t+1}} \qquad (20)$$

$$r > \frac{p_t^*}{p_{t+1}^*} \qquad (21)$$

both hold. Recall that d_t denotes the per capita quantity of deposits held by banks lending domestically. Then, since both reserve requirements bind, it follows that

$$\frac{M_t}{p_t} = \lambda \phi_d d_t \quad (22)$$

and

$$\frac{e_t Q_t}{p_t} = \lambda \phi_f d_t \quad (23)$$

(since a fraction λ of domestic agents are of Type 1, and hence are savers).

Additionally, the market for loans clears if the supply of deposits less bank reserves equals the demand for loans. This condition obtains if

$$(1-\phi_d-\phi_f)\lambda d_t=(1-\lambda)b_t \quad (24)$$

Finally, recall that I defined i_t^* to be the real value of net investment abroad, per young Type 1 agent. Then

$$i_t^*=w(k_t)-d_t \quad (25)$$

(that is, net investment abroad equals domestic savings less domestic deposits).

In credit markets, four conditions must be satisfied in equilibrium. First, banks earn zero profits so that (3) holds. Second, (13) must hold. Third, the self-selection constraint (17) must be satisfied. Fourth, an absence of arbitrage opportunities requires that $R_t=r$.

Finally, the government budget constraint -along with the condition that self-selection occurs in the credit market- implies that

$$(1-\lambda)\tau_t=\left(\frac{\sigma}{1+\sigma}\right)\frac{M_t}{p_t} \quad (26)$$

2.5.1 Equilibrium Conditions

It is straightforward to show that

$$\frac{p_t}{p_{t+1}}=\frac{k_{t+2}}{(1+\sigma)k_{t+1}} \quad (27)$$

Next, I define $\varepsilon\equiv\frac{b_t}{K_{t+1}}=1-\frac{\tau_t}{K_{t+1}}$. That is, ε denotes the fraction of the capital stock per producer that is financed with loans from the domestic financial system, as opposed to the fraction financed with the subsidy received from the government. It can be easily shown that

$$\varepsilon = \frac{(1+\sigma)(1-\phi_d-\phi_f)}{(1-\phi_d-\phi_f)+\sigma(1-\phi_f)} \quad (28)$$

Then, it is possible to write the main equilibrium conditions compactly as

$$f'(k_{t+1}) \geq \rho_t = \frac{\left[r - \left(\frac{\phi_d}{1+\sigma}\right)\left(\frac{k_{t+2}}{k_{t+1}}\right) - \phi_f\left(\frac{p_t^*}{p_{t+1}^*}\right)\right]}{(1-\phi_d-\phi_f)} \qquad (29)$$

$$r(1-\lambda)w(k_t) \geq \left\{\pi\left[f'(k_{t+1}) - \varepsilon\rho_t\right] + (1-\pi)x\right\}k_{t+1} \qquad (30)$$

$$i_t^* = w(k_t) - \left(\frac{\varepsilon}{1-\phi_d-\phi_f}\right)\frac{k_{t+1}}{\lambda} \qquad (31)$$

Equation (29) asserts that the marginal product of capital must weakly exceed the rate of interest on loans. Equation (30) is the self-selection condition in loan markets, and equation (31) describes net foreign investment. Note that one of the conditions (29) or (30) must hold as an equality. If (29) is an equality, credit is rationed. Note finally that

$$\sigma > \sigma_\varepsilon \equiv -\left(\frac{1-\phi_d-\phi_f}{1-\phi_f}\right) \qquad (32)$$

must hold in order for lending to be positive[11]. (32) is henceforth assumed to hold.

In order to obtain sharp characterizations of equilibria with and without credit rationing, it will henceforth be convenient to assume that the production function has the Cobb-Douglas form $f(k_t) = Ak_t^\alpha; \alpha \in (0,1)$. In addition, I assume that the rest of the

[11] The condition $\sigma > \sigma_\varepsilon$ is needed for $\varepsilon \equiv \frac{b_t}{K_{t+1}} > 0$.

world has a constant rate of inflation equal to its constant (net) rate of money creation, σ^*. Thus

$$\frac{p_t^*}{p_{t+1}^*} = \frac{1}{(1+\sigma^*)} < r \qquad (33)$$

Under these assumptions, I next turn to the analysis of steady-states. Dynamic equilibria are taken up in section 2.7.

2.6 Steady-State Equilibria

Steady-state equilibria will be characterized by allocations in which the pair $\{k, i^*\}$ is constant. In addition to (33), the following will be true in any steady state:[12]

$$\frac{p_t}{p_{t+1}} = \left(\frac{1}{1+\sigma}\right) < r \qquad (34)$$

$$\rho = \frac{\left[r - \left(\frac{\phi_d}{1+\sigma}\right) - \left(\frac{\phi_f}{1+\sigma^*}\right)\right]}{(1-\phi_d - \phi_f)} \qquad (35)$$

I now analyze Walrasian and Credit Rationing regimes separately.

2.6.1 Steady-State Equilibria in a Walrasian Regime

A steady-state Walrasian equilibrium has $f'(k) = \rho$, and the self-selection constraint (30) does not bind. Let $\hat{k}$ and $\hat{i}^*$ denote, respectively, the steady-state capital-labor ratio and net investment abroad in a Walrasian regime. (29) allows us to determine $\hat{k}$

[12] It is easy to show that, if r>1 and $r > \left(\frac{1}{1+\sigma}\right), \sigma > Max\left\{\left(\frac{1}{r}-1\right), \sigma_\varepsilon\right\}$ implies that ρ>0 holds. Hence, this is the only condition that need be imposed thus far on the rate of domestic money creation.

$$\hat{k} = \left[\frac{\alpha A}{\rho}\right]^{\frac{1}{1-\alpha}} = \left\{\frac{\left[r - \left(\frac{\phi_d}{1+\sigma}\right) - \left(\frac{\phi_f}{1+\sigma^*}\right)\right]}{\left[\alpha A\left(1 - \phi_d - \phi_f\right)\right]}\right\}^{\frac{1}{1-\alpha}} \qquad (36)$$

while (31) determines $\hat{i}^*$:

$$\hat{i}^* = (1-\alpha)A\hat{k}^{\alpha} - \left(\frac{\varepsilon}{1-\phi_d-\phi_f}\right)\frac{\hat{k}}{\lambda} \qquad (37)$$

An additional variable of interest is the total fraction of savings invested abroad, denoted by $\hat{\chi}$ in a Walrasian steady-state. Clearly, $\hat{\chi}$ is given by the expression

$$\hat{\chi} \equiv \frac{\hat{i}^*}{w(\hat{k})} = 1 - \left[\frac{\varepsilon\hat{k}^{(1-\alpha)}}{\lambda(1-\alpha)A(1-\phi_d-\phi_f)}\right] \qquad (38)$$

The remainder of this section analyzes the effects of increases in the steady-state rate of domestic inflation (σ), the steady-state rate of inflation in the rest of the world (σ^*), the international interest rate on deposits (r), and the domestic reserve requirements (ϕ_d and ϕ_f). Formal proofs of the propositions stated below can be found in Hernández-Verme [6].

Proposition 1 *In a Walrasian steady-state, an increase in the rate of domestic inflation (σ) reduces the capital-labor ratio ($\hat{k}$), reversing the Mundell-Tobin effect. In addition, the ratio of investment abroad to total savings in a stationary Walrasian allocation ($\hat{\chi}$) is increasing in the steady-state domestic inflation rate.*

Proposition 2 *An increase in either the steady-state rate of inflation in the rest of the world (σ^*) or the international interest rate on deposits (r) reduces the capital-labor* *ratio ($\hat{k}$) and increases the ratio of investment abroad to total savings ($\hat{\chi}$) in a Walrasian steady-state equilibrium.*

Proposition 3 *An increase in either the required reserves held in domestic currency (ϕ_d) or the required reserves held in foreign currency (ϕ_f) reduces the capital-labor ratio ($\hat{k}$) in a Walrasian steady-state.*

Intuitively, an increase in either the domestic or the foreign rate of inflation lowers the return a bank receives on its reserves. As a result, the rate of interest on loans must increase in order for domestic banks to compete for deposits in world markets. The higher rate of interest on loans then leads to a reduction in domestic capital investment. Notice that the strength of the effect of higher foreign inflation depends on the magnitude of foreign reserve holdings by domestic lenders. As these reserves become larger, ceteris paribus, the consequences of higher foreign inflation become more severe.

Interestingly, higher rates of domestic inflation lead to higher levels of capital outflows. While this is perhaps intuitive, it is also true that higher *foreign* rates of inflation lead to higher levels of capital outflows. This transpires because higher foreign inflation erodes the value of foreign currency reserves as a domestic asset. Domestic investors react by shifting assets abroad in forms whose return is not affected by inflation.

It bears emphasis that some evidence (for instance, Barnes, Boyd and Smith [2]) strongly suggests that changes in the rate of inflation in the U.S., for example, have had strong consequences for countries like Perú. The analysis of this section indicates how such consequences could arise.

2.6.2 Steady-State Equilibria in a Credit Rationing Regime

A steady-state equilibrium with Credit Rationing has $f'(k) > \rho$. In addition, the self-selection constraint (30) binds. Let $\tilde{k}$ be the steady-state capital-labor ratio in a Credit

Rationing regime, and let $\tilde{\chi}$ be the steady-state ratio of investment abroad to savings under the same regime. In this regime, (30) will determine the capital-labor ratio:

$$\tilde{k} = \left\{ \frac{\left[(1-\pi)x - \pi\varepsilon\rho\right]}{A\left[r(1-\lambda)(1-\alpha) - \alpha\pi\right]} \right\}^{\frac{1}{(\alpha-1)}} \qquad (39)$$

while (31) will determine $\tilde{\chi}$:

$$\tilde{\chi} \equiv \frac{\tilde{i}^*}{w(\tilde{k})} = 1 - \left[\frac{\varepsilon}{\lambda(1-\alpha)A(1-\phi_d - \phi_f)} \right] \tilde{k}^{(1-\alpha)}. \qquad (40)$$

As before, it is possible to analyze the effects of increases in the domestic rate of inflation, the foreign rate of inflation, and the world real interest rate when credit rationing prevails. The following propositions state some formal results. Once again, proofs of the propositions can be found in Hernández-Verme [6].

Proposition 4 *Suppose that*

$$r(1-\lambda)(1-\alpha) < (>) \alpha\pi \qquad (41)$$

holds[13]. Then an increase in the domestic rate of inflation increases (reduces) the steady-state capital-labor ratio $\tilde{k}$. If an increase in the domestic inflation rate reduces $\tilde{k}$, then the same increase necessarily increases the fraction of savings invested abroad ($\tilde{\chi}$).

Proposition 5 *Suppose that $r(1-\lambda)(1-\alpha) < (>) \alpha\pi$ holds. Then an increase in the foreign inflation rate (σ^*) or the world real interest rate (r) reduces (increases) the domestic capital-labor ratio $\tilde{k}$. These same changes increase (reduce) the ratio of savings done abroad ($\tilde{\chi}$).*

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[13] Of course if $r(1-\lambda)(1-\alpha) > (<) \alpha\pi, (1-\pi)x > (<) \varepsilon\rho$ must hold in order for $f'(\tilde{k})$ to be well-defined.

Proposition 6 *Suppose that* $r(1-\lambda)(1-\alpha)<(>)\alpha\pi$ *holds. Then an increase in the required reserves held in domestic currency* (ϕ_d) *reduces (increases) the capital-labor ratio* ($\tilde{k}$). *On the other hand, an increase in the required reserves held in foreign currency* (ϕ_f) *reduces (increases) the capital-labor ratio* ($\tilde{k}$) *if* $\sigma>\left\{\frac{\sigma^*\phi_d}{\left[r(1+\sigma^*)-1\right]}-1\right\}$ *but increases (reduces)* $\tilde{k}$ *if* $\sigma<\left\{\frac{\sigma^*\phi_d}{\left[r(1+\sigma^*)-1\right]}-1\right\}$.

Propositions 4 and 5 illustrate two important points. First, in a Walrasian equilibrium, changes in the domestic rate of inflation and changes in the world rate of inflation have qualitatively similar effects. When credit is rationed, on the other hand, changes in the domestic rate of inflation and the world rate of inflation always affect the domestic capital stock differently. Intuitively, this occurs because credit rationing breaks the link between the marginal product of capital and the rate of interest on loans. What matters when credit is rationed is how the domestic and foreign rate of inflation affect the self-selection constraint (30), and they affect this differently.

Second, changes in the domestic rates of inflation can have very different effects under credit rationing than they do in a Walrasian equilibrium. Again, this happens because what matters is how the domestic rate of inflation affects the self-selection constraint. Higher domestic inflation can actually relax this constraint by increasing the rate of interest on loans, and hence attenuating the incentives of Type 1 agents to misrepresent their type. Whether or not higher rates of domestic inflation have this effect depends on the probability of a Type 1 agent actually repaying a loan if it is taken (that is, it depends on the magnitude of π).

2.6.3 When Does Credit Rationing Occur?

I now describe when credit rationing does and does not arise, in a steady-state equilibrium. As will be clear, whether or not credit rationing is observed depends on things like the domestic and foreign rates of inflation, and on the world real rate of interest.

As has been previously noted, steady-state equilibria do (not) display credit rationing if $f'(k) > (=) \rho$. I therefore turn attention to a description of when $f'(k) > \rho$ holds.

To do so, the following result will prove useful. The proofs of lemmas and propositions can be found in Hernández-Verme [6].

Lemma 7 *The steady-state interest rate on loans is a monotonically increasing and concave function of the steady-state domestic inflation rate, for any* $\sigma > Max\left\{\left(\frac{1}{r} - 1\right), \sigma_\varepsilon\right\}$, *and it is bounded above.*

If $\rho(\sigma)$ denotes the loan rate as a function of σ, then lemma 7 implies that $\rho(\sigma)$ has the configuration depicted in Figures 1 and 2.

When credit rationing can emerge now depends on assumptions on parameter values. I describe two cases.

Figure 1

Case 1: $\alpha\pi > r(1-\lambda)(1-\alpha)$ When Case 1 obtains, $f'(\tilde{k})$ is a monotonically decreasing function of σ. Moreover, as $\varepsilon\rho$ grows smaller and closer to $\left(\frac{1-\pi}{\pi}\right)x$, $f'(\tilde{k})$ decreases and, typically, converges to a small and positive number. Thus we have the situation depicted in Figure 1. Credit is rationed iff $\sigma < \sigma_c$ holds.

When Case 1 obtains, if the initial domestic rate of inflation is fairly low, increases in the domestic rate of inflation (that is, increases in σ) can be used to stimulate capital formation and long-run output[14]. However, there is a strict limit to the extent to which domestic inflation can be used for this purpose. Once $\sigma > \sigma_c$, the equilibrium will be Walrasian, and further increases in the domestic money growth rate will have adverse consequences for long-run real activity[15]. Thus there will be inflation thresholds, as is observed empirically.

Figure 2a

Case 2: $\alpha\pi < r(1-\lambda)(1-\alpha)$ In a Case 2 economy, $f'(\tilde{k})$ can be shown to be an increasing, concave function of σ. As a result, several possibilities arise regarding the existence of steady states where credit is rationed. The possibilities are illustrated by Figures 2a, 2b and 2c.

Figure 2a For high (low) values of the domestic inflation rate, credit is (is not) rationed. This situation tends to transpire when x is relative large.

Figure 2b

Figure 2b For rates of money creation below σ_L or for rates of money creation above σ_H, credit is not rationed. Credit is rationed only if $\sigma \in (\sigma_L, \sigma_H)$.

Figure 2c

Figure 2c $\rho(\sigma)$ lies everywhere above $f'(\tilde{k})$. This situation tends to arise when x is relatively small.

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[14] This is consistent with evidence reported by Bullard and Keating [5] and Khan and Senhadji [7] that, at low rates of inflation, moderate increases in the rate of inflation can increase the long-run level of real activity.

[15] This is consistent with evidence that, at high enough rates of inflation, further increases in inflation have detrimental effects on the level of long-run activity. Again, see Bullard and Keating [5] or Khan and Senhadji [7].

Notice that, in a Case 2 economy, the scope for credit to be rationed may depend in a relatively complicated way on the rate of money creation (inflation). In particular, the "bindingness" of informational asymmetries need not depend monotonically on the rate of inflation.

2.7 Dynamic Equilibria

2.7.1 Dynamic System in a Walrasian Regime

The dynamic system in a Walrasian regime is given by:

$$(1-\phi_d-\phi_f)\alpha A k_{t+1}^{\alpha-1}=\left[r-\left(\frac{\phi_f}{1+\sigma^*}\right)\right]-\left(\frac{\phi_d}{1+\sigma}\right)\left(\frac{k_{t+2}}{k_{t+1}}\right) \qquad (42)$$

$$i_t^*=(1-\alpha)Ak_t^{\alpha}-\left[\frac{\varepsilon}{\lambda(1-\phi_d-\phi_f)}\right]k_{t+1} \qquad (43)$$

$$r(1-\lambda)(1-\alpha)Ak_t^{\alpha}>\{\pi(1-\varepsilon)\alpha A k_{t+1}^{\alpha-1}+(1-\pi)x\}k_{t+1} \qquad (44)$$

Notice that (42) and (43) constitute a recursive dynamic system: equation (42) completely governs the dynamics of the per capita capital stock. Equation (43) then indicates how cross-border capital flows inherit their dynamics from the dynamics of the capital-labor ratio.

Lagging (42) one period and rearranging terms for the per capita stock yields

$$k_{t+1}=\left(\frac{1+\sigma}{\phi_d}\right)\left\{\left[r-\left(\frac{\phi_f}{1+\sigma^*}\right)\right]k_t-(1-\phi_d-\phi_f)\alpha A k_t^{\alpha}\right\} \qquad (45)$$

Figure 3

Figure 3 depicts equation (45). Evidently, the only non-trivial equilibrium is the steady state. The economy can attain the steady-state after one period by borrowing or lending abroad.

2.7.2 Dynamic System in a Credit Rationing Regime

When credit is rationed, equation (30) governs the dynamics of the capital stock, and equation (29) holds as a strict inequality. Given the assumption of Cobb-Douglas production, equation (30) can be represented by the following dynamic system:

$$r(1-\lambda)(1-\alpha)Ay_t^{\alpha} = (1-\pi)xk_t + \pi\alpha Ak_t^{\alpha} - \left(\frac{\pi\varepsilon}{1-\phi_d-\phi_f}\right)\left[r-\left(\frac{\phi_d}{1+\sigma}\right)\frac{k_{t+1}}{k_t}-\left(\frac{\phi_f}{1+\sigma^*}\right)\right]k_t \quad (46)$$

$$y_{t+1} = k_t \quad (47)$$

Figure 4

The phase diagram for the dynamic system is depicted in Figure 4.

Local Stability I now linearize the dynamic system (46) and (47) in a neighborhood of the nontrivial steady state. The Jacobian of the linearized system is

$$J(\tilde{k},\tilde{y}) = \begin{bmatrix} J_{11} & J_{12} \\ 1 & 0 \end{bmatrix},$$

where

$$J_{11} \equiv \left.\frac{\partial k_{t+1}}{\partial k_t}\right|_{(\tilde{k},\tilde{y})} = -E\alpha f'(\tilde{k}) + \left(\frac{1+\sigma}{\phi_d}\right)\left(r-\frac{\phi_f}{1+\sigma^*}\right) - E\left(\frac{1-\pi}{\pi}\right)x \quad (48)$$

$$J_{12} \equiv \left.\frac{\partial k_{t+1}}{\partial y_t}\right|_{(\tilde{k},\tilde{y})} = E\left[\frac{r(1-\lambda)(1-\alpha)}{\pi}\right]f'(\tilde{k}) \quad (49)$$

and where

$$E \equiv \left[\frac{(1-\phi_d - \phi_f) + \sigma(1-\phi_f)}{\phi_d} \right] \qquad (50)$$

Let T and D denote, respectively, the trace and the determinant of the Jacobian. Notice that D<0 for all values of σ associated with a positive marginal product of capital. As a result, both eigenvalues will be real and distinct, and they will have opposite signs. Moreover, it can be shown that

$$1 - T + D = \frac{(1-\alpha)Ef'(\tilde{k})}{\alpha\pi}\left[r(1-\lambda)(1-\alpha) - \alpha\pi\right] \qquad (51)$$

and,

$$1 + T + D = 2 + Ef'(\tilde{k})(1-\alpha)\left[\frac{\alpha\pi - r(1-\lambda)(1+\alpha)}{\alpha\pi}\right] \qquad (52)$$

Therefore, the dynamics in a neighborhood of the nontrivial steady-state will change dramatically depending on whether Case 1 or Case 2 applies. In what follows I describe some formal results. It will be useful to notice that E>0, $\forall\sigma > Max\left\{\left(\frac{1}{r}-1\right), \sigma_\varepsilon\right\}$, and that $Ef'(\tilde{k})$ is always increasing in σ.

Case 1: $\alpha\pi > r(1-\lambda)(1-\alpha)$ In a Case 1 Economy, *1-T+D*<0, $\forall\sigma > Max\left\{\left(\frac{1}{r}-1\right), \sigma_\varepsilon\right\}$. However, it is possible for $\left[\alpha\pi - r(1-\lambda)(1+\alpha)\right]$ to be either positive or negative, and therefore, the properties of *1+T+D* will change accordingly.

Case 1.1 When $\left[\alpha\pi - r(1-\lambda)(1+\alpha)\right] > 0$ obtains, *1+T+D>0>1-T+D,* $\forall\sigma$. The steady state is a saddle with a negative stable eigenvalue. Therefore, dynamic equilibria are determinate and damped oscillations will be observed along the stable manifold.

Case 1.2 When $\left[\alpha\pi - r(1-\lambda)(1+\alpha)\right] < 0$ obtains, *1+T+D* is monotonically decreasing in σ. Hence, it is possible that *1+T+D>0* holds for low values of σ whereas

1+T+D<0 holds as σ increases. Or it is possible that *1+T+D<0* holds for all the values of σ within the range considered. In what follows, I consider both possibilities.

a) *1+T+D>(<)0* for low (high) values of σ. This situation transpires when *x* is relatively large.

a.1) For low values of σ, *1+T+D>0>1-T+D*. The steady state is a saddle with a negative stable eigenvalue, and dynamic equilibria are determinate. Again, damped oscillations will be observed along the stable manifold.

a.2) For high values of σ, *1+T+D<0*. The steady state is a source.

b) *1+T+D<0*, $\forall\sigma > Max\left\{\left(\frac{1}{r}-1\right),\sigma_\varepsilon\right\}$. This situation transpires when *x* is relatively small, and the steady state is always a source.

Case 2: $\alpha\pi < r(1-\lambda)(1-\alpha)$ In a Case 2 Economy, *1-T+D*>0, $\forall\sigma > Max\left\{\left(\frac{1}{r}-1\right),\sigma_\varepsilon\right\}$. However, as observed before, it is possible that either *1+T+D>(<)0* for low (high) values of σ. Alternatively, *1+T+D<0* could hold for all the values of σ within the range considered (when *x* is relatively large).

Whenever *1+T+D* changes sign from positive to negative as σ increases, it is possible to observe the following:

a) For low values of σ, *1+T+D>1-T+D>0* and $|D|<1$. The steady state is a sink and dynamic paths approach the steady state monotonically. Dynamic equilibria are indeterminate.

b) As the domestic rate of money creation increases, *0<1+T+D<1-T+D* and $|D|<1$. The steady state is again a sink, but dynamic paths approaching the steady-state exhibit damped oscillations. Hence, dynamic equilibria are indeterminate and display cyclical fluctuations.

c) For high values of σ, $1\text{-}T\text{+}D>0$ and $1\text{+}T\text{+}D<0$. The steady state becomes a saddle with a positive stable eigenvalue. Equilibria are, then, determinate, and no fluctuations will be observed along the stable manifold.

On the other hand, whenever $1+T+D<0<1-T+D$, $\forall \sigma > Max\left\{ \left(\frac{1}{r} - 1 \right), \sigma_\varepsilon \right\}$, the steady state is always a saddle with a positive stable eigenvalue.

Some Consequences of Equilibrium Dynamics When perfect foresight dynamics allow for oscillations -as can be the case when credit is rationed- then there will be endogenously arising volatility in output, the price level, and net investment abroad. In particular, along dynamic equilibrium paths, all of these variables will fluctuate, even in the absence of exogenous shocks. Thus, when credit is rationed, a policy of floating exchange rates coupled with a constant rate of money creation need not imply the short-run existence of a relatively stable rate of inflation. In addition, the presence of credit rationing and the possibility of associated endogenous volatility can help to explain why net foreign investment tends to be very volatile relative to observed changes in obvious exogenous variables.

3 A Fixed Exchange Rate Regime: Argentina After the Stabilization

In this section, I consider an economy that operates under a fixed rather than a flexible exchange rate regime. This exchange rate regime will be constructed so that a currency board emerges as a special case. Obviously, the intention is to model an economy with a credit market friction that -in certain respects- resembles Argentina subsequent to its stabilization. Of course, in all respects except for the exchange rate regime that is in place, the economy remains as described in the previous sections.

3.1 Government Policy

I consider a regime where, in the initial period, the government sets once and for all the nominal exchange rate *e*. In addition, the domestic monetary authority may hold foreign currency reserves. These reserves may constitute some fraction of the domestic monetary base, or they may even include some fraction of domestic deposits. There is, of course, some issue as to how these reserves are held: they may be held either in the form of (non-interest bearing) foreign currency, or in the form of interest-bearing foreign assets. To fix ideas, I assume that all reserves are held in the form of safe, interest-bearing foreign assets (bonds). But the analysis would be altered in only minor ways if reserves were held in the form of foreign currency.

Let B_t^* denote the foreign bonds held as reserves by the domestic monetary authority. Then,

$$B_t^* = \left(\frac{\theta}{e}\right) M_t + \xi p_t^* \lambda d_t \qquad (53)$$

Here $\theta \in (0,1)$ gives the foreign "currency" reserves that are held against the domestic money supply, measured in units of foreign currency $\left(\frac{M_t}{e}\right)$, and $\xi \in (0,1)$ gives the reserves that the domestic monetary authority holds against domestic bank deposits. In what follows, to fix ideas and without real loss of generality, I use the normalization $e=1$[16]. A situation where $\theta=\xi=0$ defines what I call a *pure exchange rate regime*, in the sense that the government fixes the exchange rate without any backing of either the domestic money supply or domestic deposits[17]. On the other hand, situations where $\theta=1$

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[16] In the absence of the normalization, θ would simply be replaced by $\left(\frac{\theta}{e}\right)$ whenever it appears.

[17] The exchange rate is maintained by injecting or withdrawing money, as required, through the investment subsidy program.

and $\xi \geq 0$ define a *currency board*: the domestic money supply is backed 100% and there may be some backing of domestic deposits too[18].

When the domestic monetary authority holds reserves, it is necessary to modify the government budget constraint. In particular, that constraint now takes the form

$$(1-\lambda)\tau_t = \frac{(M_t - M_{t-1})}{p_t} - \frac{\left[B_t^* - r\left(\frac{p_t^*}{p_{t-1}^*}\right)B_{t-1}^*\right]}{p_t^*} \qquad (54)$$

This constraint asserts that the value of seigniorage revenue less the change in the central bank's reserve position is used to finance an investment subsidy to agents claiming to be of Type 2. Obviously, (54) incorporates the fact that self-selection does occur in a non-trivial equilibrium.

Next observe that, since the nominal rate of exchange is constant,

$$\frac{p_t}{p_{t+1}} = \frac{p_t^*}{p_{t+1}^*}, \forall t \qquad (55)$$

Therefore, the interest rate on loans maturing at *t+1* must equal

$$\rho_t = \frac{\left[r - (\phi_d + \phi_f)\left(\frac{p_t^*}{p_{t+1}^*}\right)\right]}{(1-\phi_d - \phi_f)} \qquad (56)$$

The remaining conditions of equilibrium are not altered by the change in exchange rate regime. Thus, the conditions of non-trivial separating equilibria are as follows:

$$rw(k_t) \geq \{\pi[f'(k_{t+1}) - \rho_t] + (1-\pi)x\}\frac{k_{t+1}}{(1-\lambda)} + \pi\rho_t\tau_t \qquad (57)$$

[18] For technical reasons that will become clearer I assume that $\xi < (1-\theta\phi_d - \phi_f)$.

$$f'(k_{t+1}) \geq \rho_t = \frac{\left[r - (\phi_d + \phi_f)\left(\frac{p_t^*}{p_{t+1}^*} \right) \right]}{(1 - \phi_d - \phi_f)} \tag{58}$$

$$\tau_t = \left(\frac{G_1}{1-\lambda} \right) k_{t+1} + \left(\frac{G_2}{1-\lambda} \right) k_t - G_2 \tau_{t-1} \tag{59}$$

$$i_t^* = w(k_t) + \frac{\left[(1-\lambda)\tau_t - k_{t+1} \right]}{\lambda(1 - \phi_d - \phi_f)} \tag{60}$$

where:

$$G_1 \equiv \left[\frac{(1-\theta)\phi_d - \xi}{1 - \theta\phi_d - \phi_f - \xi} \right] \text{and } G_2 \equiv \frac{(1+\sigma^*) r(\theta\phi_d + \xi) - \phi_d}{(1 - \theta\phi_d - \phi_f - \xi)(1+\sigma^*)} \tag{61}$$

Equation (57) is the self-selection constraint: it holds with equality if credit is rationed. Equation (58) holds with equality in a Walrasian equilibrium. Equation (60), which describes net investment abroad, and equation (59), which is the government budget constraint, hold both in credit rationing and Walrasian equilibria.

I begin by discussing stationary equilibria in which credit is and is not rationed.

3.2 Stationary Equilibria

In a steady state, k_t, i_t^* and τ_t are constant. I begin with a description of Walrasian equilibria.

3.2.1 Steady-State Equilibria in a Walrasian Regime

A Walrasian steady-state equilibrium is such that $f'(k) = \rho$ and the self-selection constraint (57) does not bind. Let $\hat{k}, \hat{i}^*$ and $\hat{\tau}$ denote, respectively, the steady-state capital-labor ratio, net investment abroad and the transfer to producers in a Walrasian regime. We also define $\hat{\chi}$ to be the ratio of investment abroad to total savings in this regime. From (58) we are able to determine that[19, 20]

$$\hat{k} = \left[\frac{\alpha A}{\rho}\right]^{\frac{1}{1-\alpha}} = \left\{ \frac{\left[r - \left(\frac{\phi_d + \phi_f}{1+\sigma^*} \right) \right]}{\alpha A(1-\phi_d - \phi_f)} \right\}^{\frac{1}{\alpha-1}} \qquad (62)$$

(59) and (60), respectively, then give $\hat{\tau}$ and $\hat{\chi}$ as a function of $\hat{k}$:

$$\hat{\tau} = \delta \frac{\hat{k}}{(1-\lambda)} \qquad (63)$$

$$\hat{\chi} = 1 + \frac{(\delta - 1)\hat{k}^{1-\alpha}}{\lambda(1-\phi_d - \phi_f)(1-\alpha)A} \qquad (64)$$

where δ, the subsidy rate on the capital stock per old producer, is defined as

$$\delta \equiv \frac{(1+\sigma^*)(r-1)(\theta\phi_d + \xi) + \sigma^*\phi_d}{(1+\sigma^*)(r-1)(\theta\phi_d + \xi) + (1-\phi_d - \phi_f) + \sigma^*(1-\phi_f)} \qquad (65)$$

Note that

$$\sigma^* > Max\left\{\sigma_\delta^*, \left(\frac{1}{r} - 1\right)\right\}, \quad \sigma_\delta^* \equiv -\frac{\left[(r-1)(\theta\phi_d + \xi) + (1-\phi_d - \phi_f)\right]}{\left[(r-1)(\theta\phi_d + \xi) + (1-\phi_f)\right]} \qquad (66)$$

must hold for lending to be positive and for the domestic reserve requirements to bind. (66) is henceforth assume to hold.

Notice that the steady-state level of the capital-labor ratio in a Walrasian regime is unaffected by the choice of θ and/or ξ, i.e., by how the domestic money supply is

[19] Note that, henceforth, I still use a Cobb- Douglas characterization of the production technology.

[20] Note that, in order for $\hat{k}$ to be well-defined, $\sigma^* > \sigma_\rho^* \equiv \frac{\phi_d + \phi_f}{r} - 1$ must hold.

"backed" in a fixed exchange rate regime. However, increases in either θ or ξ increase δ and therefore, affect the capital subsidy ($\hat{\tau}$) and the share of net investment abroad in total savings ($\hat{\chi}$).

It is also worth noticing that increases in either σ^* (which happens to be both the foreign and domestic steady-state inflation rate), r, or the domestic reserve requirements (ϕ_d or ϕ_f) reduce the steady-state capital-labor ratio, independently of whether a currency board regime or a pure fixed exchange rate regime is in place. Of course, this result transpires from the direct link between the marginal product of capital and the rate of interest on loans that exists when credit is not rationed. This link does not depend on the "backing" of the domestic money supply in a fixed exchange rate regime (i.e., the choice of θ or ξ). In addition, higher inflation abroad always increases both the subsidy rate δ[21] and the share of net investment abroad in total savings ($\hat{\chi}$).

Comparative Statics under a Currency Board The main component of a currency board regime, in an environment like the one described in this section, is the choice to set the policy parameter θ=1. ξ could be chosen by the monetary authority such that $0 \leq \xi < (1-\phi_d-\phi_f)$. Any choice of ξ within this range makes no difference, qualitatively, in terms of the results that I discuss in this section of the paper.

In a currency board regime, an increase in the world real interest rate r obviously increases the real return on the central bank's reserve position, and therefore, it increases the subsidy rate on capital (δ). On the other hand, the same increase in r also increases $\hat{\chi}$, through the resulting increase in δ and reduction in $\hat{k}$. Obviously, this effect operates until the rate of return on domestic deposits equals the new level of r.

Finally, increases in either of the domestic reserve requirements (i.e., either ϕ_d or ϕ_f) always increase δ and $\hat{\chi}$ when a currency board is in place.

[21] This is a result of the increase in domestic seigniorage income associated with a higher value of the money growth rate.

Comparative Statics under a Pure Fixed Exchange Rate In a pure fixed exchange rate regime, both θ and ξ are set equal to zero. This regime is qualitatively different than a currency board in the following sense: increases in r under this regime have no direct effect on the government's finances, leaving δ unaffected. However, for the same reasons as before, increases in r lead to a higher share of investment abroad in total savings, and this effect operates until the rate of return on domestic deposits equals the new level of r.

Finally, increases in either of the domestic reserve requirements (ϕ_d or ϕ_f) will increase (decrease) δ when the foreign rate of inflation is positive (negative). As a result, $\hat{\chi}$ will be increasing in either ϕ_d or ϕ_f for all values but very small values of σ^*.

3.2.2 Steady-State Equilibria in a Credit Rationing Regime

Steady-state equilibria under a Credit Rationing regime have $f'(k)>\rho$, and the self-selection constraint (57) binds. Let $\tilde{k}$, $\tilde{i}^*$ and $\tilde{\tau}$ denote, respectively, the steady-state capital-labor ratio, investment abroad and transfer to producers in a Credit Rationing regime. As we did before, we also define $\tilde{\chi}$ to be the ratio of investment abroad to total savings in this regime. From (59), we can express $\tilde{\tau}$ as a function of the capital-labor ratio

$$\tilde{\tau} = \delta \frac{\tilde{k}}{(1-\lambda)}, \quad (67)$$

where δ has been defined in (65). Using this information, we can determine the steady-state capital stock from (57),

$$\tilde{k} = \left\{ \frac{\left[(1-\pi)x - \pi(1-\delta)\rho\right]}{A\left[r(1-\lambda)(1-\alpha) - \alpha\pi\right]} \right\}^{\frac{1}{\alpha-1}} \qquad (68)$$

and $\tilde{\chi}$ from (60)

$$\tilde{\chi} = 1 + \frac{(\delta - 1)\tilde{k}^{1-\alpha}}{A\lambda(1-\phi_d - \phi_f)(1-\alpha)}. \qquad (69)$$

In contrast with what we observed in stationary equilibria under a Walrasian regime, the steady-state capital-labor ratio in a Credit Rationing equilibrium is affected by the choice of θ and/or ξ made by the monetary authority. Increasing either θ or ξ has a direct positive effect on the government's finances, increasing the subsidy rate on capital, δ. As this subsidy rate changes, it alters the incentives of Type 1 agents to misrepresent their type. In order to induce self-selection, there must be a corresponding change in the degree of credit rationing. Of course, as was true previously, the effects of changes in δ may vary depending on different assumptions on parameter values. I now investigate the effects of increases in the world inflation rate (σ^*), the world interest rate on deposits (r), the domestic reserve requirements (ϕ_d and ϕ_f), and the parameters θ and ξ.

Case 1: $\alpha\pi > r(1-\lambda)(1-\alpha)$ When Case 1 obtains, an increment in δ due to an increase in either θ or ξ, ceteris paribus, increases the subsidy received by agents claiming to be of Type 2 and, in this way, affects the self-selection constraint. Given the high probability of repaying loans (π), $\tilde{k}$ has to adjust upward to maintain the incentives of agents to self-select. As a consequence, the transfer to capital producers ($\tilde{\tau}$) increases and the share of net investment abroad in total savings ($\tilde{\chi}$) falls.

Comparative Statics under a Currency Board When Case 1 obtains, and when a currency board is in place, an increase in the (domestic and) foreign inflation rate σ^* has some potentially complicated consequences. These are described in the following proposition.

Proposition 8 *Let* $\xi_c \equiv \left(\dfrac{\phi_d}{\phi_d+\phi_f}\right)(1-\phi_d-\phi_f)$*, and let Case 1 obtain.*

a) When $\xi \in [0,\xi_c)$*, an increase in* σ^* *causes* $\tilde{k}$ *to fall and* $\tilde{\chi}$ *to increase if*

$$r < r_c \equiv \frac{(\phi_d + \phi_f)(1 - \theta\phi_d - \phi_f - \xi)}{\left[\phi_d - (\phi_d + \phi_f)(\theta\phi_d + \xi)\right]} \quad (70)$$

holds[22].

b) When $\xi \in [\xi_c, (1 - \phi_d - \phi_f))$, *then an increase in* σ^* *causes* $\tilde{k}$ *to fall, whether* $r<r_c$ *holds or not. On the other hand, when* $r<r_c$, $\tilde{\chi}$ *will be increasing (decreasing) in* σ^* *if* $x < (>) \left(\frac{\pi}{1-\pi}\right)\left(\frac{\phi_d + \phi_f}{\phi_d}\right)$ *holds*[23].

Intuitively, under the conditions described, an increase in σ^* increases the net of subsidy effective real interest rate on loans, $(1-\delta)\rho$. This, in turn, affects the incentives of Type 1 agents to misrepresent their type. Given that $\alpha\pi > r(1-\lambda)(1-\alpha)$, the capital stock must fall in order to maintain the incentives of agents to self-select.

With respect to changes in the world interest rate, increments in r in a currency board regime increase the real return on the central bank's reserves, thereby increasing the effective capital subsidy, δ. However, changes in r also affect the self-selection constraint and, under the present configuration of parameters, $\tilde{k}$ must fall. Finally, the share of net investment abroad increases as a result of the higher r.

Under a fixed exchange rate regime, the only instruments of domestic monetary policy are the reserve requirements ϕ_d and ϕ_f. An increase in ϕ_f reduces the capital stock ($\tilde{k}$) when Case 1 obtains and a currency board regime is in operation, but an increase in

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[22] Note that $r_c < \left(\frac{\phi_d + \phi_f}{\phi_d}\right)$ holds. However, for parameter values that seem to obtain in a Latin American context, r_c is fairly large. Thus, this condition is likely to be satisfied in practice.

[23] Note that when $\xi \in [\xi_c, (1 - \phi_d - \phi_f))$ holds, $r_c \geq \left(\frac{\phi_d + \phi_f}{\phi_d}\right)$ holds.

ϕ_d seems to have an ambiguous effect on $\tilde{k}$. Changes in both ϕ_f and ϕ_d have ambiguous effects on the fraction of wealth allocated to foreign assets, $\tilde{\chi}$.

Comparative Statics under a Pure Fixed Exchange Rate Given that $\alpha\pi > r(1-\lambda)(1-\alpha)$, and if $r<r_c$ holds, in a pure fixed exchange rate regime an increase in σ^* must cause $\tilde{k}$ to fall in order to maintain the incentives of agents to self-select. The share of net investment abroad in total savings is always increasing in the rate of foreign (and domestic) inflation.

Increases in the world interest rate r do not affect the subsidy rate δ when the domestic money supply is not backed. However, they do affect the interest rate on loans, and deposits, thereby affecting the self-selection constraint. As a result, $\tilde{k}$ must fall to maintain the incentives of Type 1 agents to self-select. At the same time, when the world rate of interest rises, so does the share of net investment abroad in total savings ($\tilde{\chi}$).

Finally, increases in either of the domestic reserve requirements (ϕ_d or ϕ_f) in general reduce the creation of physical capital ($\tilde{k}$).

Case 2: $\alpha\pi < r(1-\lambda)(1-\alpha)$ When Case 2 obtains, increases in σ^*, r, or the domestic reserve requirements (ϕ_d or ϕ_f) will have the opposite effects on physical capital ($\tilde{k}$) relative to what would be observed when Case 1 obtains.

3.2.3 When Does Credit Rationing Occur?

In this section, I describe when credit rationing does and does not arise, in a steady-state equilibrium. As noted in previous sections, steady-state equilibria do (not) display credit rationing if $f'(\tilde{k}) > (=)\rho$. This condition is equivalent to

$$\alpha\left[(1-\pi)x - \pi(1-\delta)\rho\right] < (>)\left[r(1-\lambda)(1-\alpha) - \alpha\pi\right]\rho \quad (71)$$

whenever $\alpha\pi > (<) r(1-\lambda)(1-\alpha)$. In order to state when (71) holds, the following results will prove useful.

Lemma 9 *The steady-state interest rate on loans is a monotonically increasing and concave function of the steady-state inflation rate in the rest of the world, for any* $\sigma^* > Max\left\{\sigma^*_\delta, \left(\frac{1}{r}-1\right)\right\}$, *and it is bounded above. These properties do not depend upon how the domestic money supply is backed in a fixed exchange rate regime.*

Lemma 10 *If* $r < r_c$, *then* $(1-\delta)\rho$ *is an increasing function of the steady-state inflation rate in the rest of the world, for any* $\sigma^* > Max\left\{\sigma^*_\delta, \left(\frac{1}{r}-1\right)\right\}$. *This property does not depend upon how the domestic money supply is backed in a fixed exchange rate regime.*

If $\rho(\sigma^*)$ denotes the interest rate on loans as a function of the inflation rate in the rest of the world, then Lemma 9 implies that $\rho(\sigma^*)$ has the configuration depicted in Figures 5 and 6. When credit rationing can emerge now depends, for given levels of foreign steady-state inflation, on assumptions on parameter values and on the nature of the fixed exchange rate regime in place.

Case 1: $\alpha\pi > r(1-\lambda)(1-\alpha)$ In situations where Case 1 obtains, both the left and the right hand side of (71) are not only negative but also decreasing in the foreign rate of inflation, σ^*. As stated previously, these properties do not depend upon how the domestic money supply is backed in a fixed exchange rate regime. As a result, the scope for credit to be rationed may depend in a relatively complicated way on the rate of foreign inflation. Two of these possibilities are illustrated in Figures 5a, and 5b.

Figure 5a

Figure 5a For low (high) levels of the foreign inflation rate, credit is (is not) rationed. This situation tends to transpire in a Case 1 economy when *x* is relatively large.

Figure 5b

Figure 5b $f^{'}(\tilde{k})$ lies everywhere above $\rho(\sigma^{*})$. This situation tends to transpire in a Case 1 economy when x is relatively small.

Thus, in economies where Case 1 obtains and a fixed exchange rate regime is in place, the scope for credit to be rationed depends in a relatively complicated way on the rate of foreign inflation. In such a situation, increases in the level of steady-state foreign inflation are always detrimental to long-run output. There is no range of inflation rates over which increases in inflation promote real activity.

Case 2: $\alpha\pi < r(1-\lambda)(1-\alpha)$ In situations where Case 2 obtains (71) can be rewritten as

$$\alpha\left[(1-\pi)x - \pi(1-\delta)\rho\right] > \left[r(1-\lambda)(1-\alpha) - \alpha\pi\right]\rho. \qquad (72)$$

Notice that the left-hand side of (72) is positive and decreasing in σ^{*}, while the right-hand side is also positive but increasing in σ^{*}. Again, these properties do not depend upon how the domestic money supply is backed in a fixed exchange rate regime. As a result, two possibilities arise regarding the existence of steady states where credit is rationed. These possibilities are illustrated in figures 6a and 6b.

Figure 6a

Figure 6a For low (high) levels of the foreign inflation rate, credit is (is not) rationed. This situation tends to transpire in a Case 2 economy when x is relatively small.

Figure 6b

Figure 6b $f^{'}(\tilde{k})$ lies everywhere above $\rho(\sigma^{*})$ and credit is always rationed. This situation tends to transpire in a Case 2 economy when x is relatively large.

As a result, in economies where Case 2 obtains and a fixed exchange rate regime is in place, low levels of steady-state inflation will in general be associated with credit being rationed. Moreover, there will be inflation thresholds as are observed empirically: foreign inflation and output are positively (negatively) correlated below (above) the threshold.

3.3 Dynamic Equilibria

This section takes up the analysis of dynamic equilibria under fixed exchange rates. It begins with an analysis of dynamics when credit is not rationed.

3.3.1 Dynamic System in a Walrasian Regime

The dynamic system in a Walrasian regime is given by (58) at equality, (59) and (60). Equations (58) and (60) can be rewritten, respectively, as

$$\alpha A k_{t+1}^{\alpha-1} = \rho = \frac{\left[r - \frac{(\phi_d + \phi_f)}{(1+\sigma^*)} \right]}{(1-\phi_d-\phi_f)} \qquad (73)$$

$$i_t^* = (1-\alpha)Ak_t^{\alpha} + \frac{\left[(1-\lambda)\tau_t - k_{t+1}\right]}{\lambda(1-\phi_d-\phi_f)} \qquad (74)$$

Notice that (73) and (59) constitute a recursive dynamic system. Equation (73) implies that the capital-labor ratio is constant. Then (59) governs the dynamic behavior of the investment subsidy τ_t:

$$\tau_t = \left(\frac{G_1 + G_2}{1-\lambda}\right)\hat{k} - G_2\tau_{t-1}, \qquad (75)$$

where G_1 and G_2 are as defined in (61). Finally, equation (74) can be rewritten as

$$i_t^* = (1-\alpha)A\hat{k}^{\alpha} - \frac{\hat{k}}{\lambda(1-\phi_d-\phi_f)} + \left[\frac{(1-\lambda)}{\lambda(1-\phi_d-\phi_f)}\right]\tau_t \qquad (76)$$

Notice that the dynamic properties of τ_t (and, thus, of i_t^* too) are determined by $\frac{\partial \tau_t}{\partial \tau_{t-1}} = -G_2$. Also notice that G_2>(<)0 under a currency board (pure fixed exchange rate regime). Under a currency board, either $\left|\frac{\partial \tau_t}{\partial \tau_{t-1}}\right|$>1 or $\left|\frac{\partial \tau_t}{\partial \tau_{t-1}}\right|$<1 can hold, while in a pure fixed exchange rate regime typically $\frac{\partial \tau_t}{\partial \tau_{t-1}} \in (0,1)$. When -1<$\frac{\partial \tau_t}{\partial \tau_{t-1}}$<0, as can occur with a currency board, then fluctuations in the value of the investment subsidy can be observed. These fluctuations will then be translated into fluctuations in the magnitude of capital flows (net investment abroad). Notice that fluctuations in the government's fiscal position, and in net foreign investment can only occur if a currency board is in place. Such fluctuations are not possible under a pure fixed exchange rate regime. Thus, backing domestic currency with foreign assets does not prevent fluctuations in net foreign investment; rather, it can promote the occurrence of such fluctuations.

3.3.2 Dynamic System in a Credit Rationing Regime

The dynamic system under credit rationing is given by

$$r(1-\lambda)(1-\alpha)Ak_t^{\alpha} = \pi\alpha Ak_{t+1}^{\alpha} + \left[(1-\pi)x - \pi\rho\right]k_{t+1} + \pi\rho(1-\lambda)\tau_t, \qquad (77)$$

(59), (74) and

$$\alpha Ak_{t+1}^{\alpha-1} > \rho \qquad (78)$$

Equations (77), and (59) jointly govern the dynamics of the capital-labor ratio and the capital investment subsidy. Equation (74) then describes the dynamics of net foreign investment.

Rearranging terms in equations (77) and (59), and defining $q_t = \tau_{t-1}$, I obtain the following dynamic system:

$$k_{t+1} = -\left(\frac{G_2}{G_1}\right)k_t + \left(\frac{1-\lambda}{G_1}\right)\tau_t + \left[\frac{G_2(1-\lambda)}{G_1}\right]q_t = g(k_t, \tau_t, q_t) \quad (79)$$

$$\begin{aligned}\tau_{t+1} = &\left[\frac{r(1-\alpha)A}{\pi\rho}\right]k_{t+1}^{\alpha} - \left[\frac{\alpha A}{(1-\lambda)\rho}\right]\left[g(k_{t+1}, \tau_{t+1}, q_{t+1})\right]^{\alpha} \\ &-\left[\frac{(1-\pi)x - \pi\rho}{(1-\lambda)\pi\rho}\right]g(k_{t+1}, \tau_{t+1}, q_{t+1})\end{aligned} \quad (80)$$

$$q_{t+1} = \tau_t \quad (81)$$

Local Stability I now linearize the dynamic system (79), (80) and (81) in a neighborhood of the nontrivial steady state. The Jacobian of the linearized system is

$$J\left(\tilde{k}, \tilde{\tau}, \tilde{q}\right) = \begin{bmatrix} g_1 & g_2 & g_3 \\ g_1\left(\dfrac{P}{R}\right) & \left(\dfrac{S}{R}\right) & g_3\left(\dfrac{P}{R}\right) \\ 0 & 1 & 0 \end{bmatrix}, \text{ where}$$

$$g_1 \equiv -\left(\frac{G_2}{G_1}\right), \quad g_2 \equiv \left(\frac{1-\lambda}{G_1}\right), \quad g_3 \equiv \left[\frac{G_2(1-\lambda)}{G_1}\right] \quad (82)$$

$$P \equiv r(1-\lambda)(1-\alpha)f'(\tilde{k}) - g_1\left[\alpha\pi f'(\tilde{k}) + (1-\pi)x - \pi\rho\right] \quad (83)$$

$$R \equiv (1-\lambda)\pi\rho + g_2\left[\alpha\pi f'(\tilde{k}) + (1-\pi)x - \pi\rho\right] \quad (84)$$

$$S \equiv r(1-\lambda)(1-\alpha)g_2 f'(\tilde{k}) - (g_1 g_2 + g_3)\left[\alpha\pi f'(\tilde{k}) + (1-\pi)x - \pi\rho\right] \quad (85)$$

It can be easily shown that the determinant of $J\left(\tilde{k}, \tilde{\tau}, \tilde{q}\right)$ is equal to zero. Then, one of the eigenvalues of J will be equal to zero, while the remaining two eigenvalues are given by the roots η of the following quadratic equation:

$$\eta^2 - H_1\eta - H_2 = 0 \quad (86)$$

It can be shown that

$$H_1 \equiv g_1 + \left(\frac{S}{R}\right) = \frac{\left[r(1-\lambda)(1-\alpha) - \alpha\pi G_2\right] f'(\tilde{k}) - G_2(1-\pi)x}{\pi\rho(G_1 - 1) + \alpha\pi f'(\tilde{k}) + (1-\pi)x} \tag{87}$$

and

$$H_2 \equiv (g_1 g_2 + g_3)\left(\frac{P}{R}\right) - g_1\left(\frac{S}{R}\right) = \frac{G_2 r(1-\lambda)(1-\alpha) f'(\tilde{k})}{\pi\rho(G_1 - 1) + \alpha\pi f'(\tilde{k}) + (1-\pi)x} \tag{88}$$

As described above, G_1 and G_2 vary according to the nature of the fixed exchange rate regime. This, in turn, implies that the properties of dynamic equilibria near a nontrivial steady state when credit is rationed differ according to whether or not a currency board is in place. In the remainder of this section I present numerical examples[24].

Case 1: $\alpha\pi > r(1-\lambda)(1-\alpha)$ When a Case 1 economy obtains, it is possible to observe the following:

A Currency Board regime

When Case 1 obtains and a currency board regime is in place, either both eigenvalues are real and negative or they are complex conjugates.

Typically, it is possible to observe the following:

a) For low levels of foreign inflation, the steady state is a saddle. Then, dynamic equilibria are determinate and damped oscillations will be observed along the stable manifold.

b) As σ^* increases, the nontrivial steady state becomes a sink. Therefore, the steady state is indeterminate and dynamic paths approaching it will display damped oscillations.

[24] The following parameter values were kept constant across scenarios in the numerical examples: $\phi_d=\phi_f=0.085$, $r=1.1$, $x=1.05$, $\alpha=0.35$, $A=1$, and $\lambda=0.7$. Notice that the values used for the domestic reserve requirements correspond to the actual values observed in Argentina. In order to obtain the conditions under which Case 1 obtains, I used $\pi=0.95$, while for Case 2 I used $\pi=0.05$. Obviously, $\theta=\xi=0$ when there is a pure fixed exchange rate regime, while the values $\theta=1$ and $\xi\in\{0,0.1,0.2,0.5\}$ defined the different scenarios simulated for a currency board.

c) For high rates of foreign inflation, the eigenvalues become complex conjugates of the form $\left(\frac{H_1}{H_2}\right) \pm \left(\frac{\sqrt{-H_1^2 - 4H_2}}{2}\right) \cdot i$, where $i \equiv \sqrt{-1}$. Moreover, the modulus of the complex eigenvalues, given by $\sqrt{-H_2}$, is an increasing function of σ^*, but it seems that it is never greater than 1. Thus, the nontrivial steady state is a sink with complex roots.

Interestingly, complex eigenvalues are more likely to be observed whenever the policy parameter ξ is relatively large. It is possible that the eigenvalues are complex conjugates for all levels of σ^* when ξ is large enough. On the other hand, when ξ=0, no complex roots seem to be observed. Thus backing domestic deposits with government-held foreign currency reserves promotes endogenously generated volatility.

A Pure Fixed Exchange Rate regime

When Case 1 obtains and a pure fixed exchange rate regime is in place, we typically observe that both eigenvalues are real, distinct and positive. Moreover, $\forall\sigma^* > \left(\frac{1}{r} - 1\right)$, the steady state is a saddle and dynamic paths approach the steady state monotonically. Dynamic equilibria are then determinate.

Case 2: $\alpha\pi < r(1-\lambda)(1-\alpha)$ When a Case 2 economy obtains, it is possible to observe the following:

A Currency Board regime

In a currency board regime, both eigenvalues will be real and distinct, with opposite signs. The positive eigenvalue will typically be less than one and decreasing in σ^*. On the other hand, it is possible for the negative eigenvalue to be greater or less than -1, depending on the magnitude of ξ and σ^*. Moreover, the negative eigenvalue is decreasing in these parameters. Therefore, it will be possible to observe the following:

a) If ξ is relatively large:

a.1) For low rates of foreign inflation, the nontrivial steady state is a sink with dynamic paths that display damped oscillations. Therefore, dynamic equilibria are indeterminate.

a.2) As σ^* increases, the steady state becomes a saddle. Then, dynamic equilibria will be determinate and no oscillations will be observed along the stable manifold.

b) If ξ is relatively low:

b.1) For low values of σ^*, the nontrivial steady state is a sink, and dynamic paths will display monotonic convergence. Dynamic equilibria are, thus, indeterminate.

b.2) For high levels of foreign inflation, the steady state is still a sink, but dynamic paths will display damped oscillations. Dynamic equilibria are still indeterminate.

It is worth noticing that as $\xi \to 0$, the scope for economic fluctuations is reduced for given levels of foreign inflation, and the steady state becomes a sink with dynamic paths that display monotonic convergence.

A Pure Fixed Exchange Rate regime

In a pure fixed exchange rate regime, the steady state is always a sink with real and positive eigenvalues. Thus, there is again an indeterminacy of dynamic equilibria. However, endogenous volatility cannot be observed near the steady state.

4 Conclusions

This paper presents a model of a small open economy where financial intermediaries perform a real allocative function in the presence of multiple reserve requirements and obvious credit market frictions that may or may not cause credit to be rationed. I then consider the relative merits of different exchange regimes along several dimensions including the attainment of low and stable rates of inflation, the promotion of financial deepening, and the avoidance of stagnation in output. I focus my attention on policies that have been implemented in Latin America and, particularly, in Argentina and Perú.

Concerning economies with floating exchange rates, I find that changes in domestic inflation and world (U.S.) inflation affect the domestic capital stock differently according to whether or not credit is rationed. What matters when credit is rationed is how the domestic and foreign rates of inflation affect the self-selection constraint, and they affect this differently. In marked contrast to the literature on closed economies, either credit rationing tends to be observed when domestic rates of inflation are low, or else the scope for credit to be rationed depends in a relatively complicated way on the rate of money creation (inflation). In the first situation, moderate increases in the rate of money growth (inflation) stimulate output when credit is rationed (inflation is initially low), but reduce output when there is no credit rationing (inflation is initially high). Thus there will be inflation thresholds as are observed empirically: inflation and output are positively (negatively) correlated below (above) the threshold. As a consequence, there is a strict limit to the extent to which domestic inflation can be used to stimulate output. Furthermore, the presence of credit rationing seems to increase the scope for both instability and economic fluctuations. When the second situation obtains, however, increases in the domestic rate of inflation always have adverse consequences for real activity and private information (together with high rates of inflation) seems to increase the scope for indeterminacy of dynamic equilibria and for economic fluctuations.

In a small open economy with a fixed rate of exchange, the domestic and foreign inflation rates will be equal. Interestingly again -and, yet in marked contrast to the literature on closed economies- either the scope for credit to be rationed depends in a relatively complicated way on the rate of foreign inflation, or credit rationing tends to be observed when foreign rates of inflation are low. In the first situation, increases in the foreign (and domestic) rate of inflation always have adverse consequences for real activity. In the second situation, however, there will be inflation thresholds: foreign (and domestic) inflation and output are positively (negatively) correlated below (above) the threshold.

Finally, in economies with fixed exchange rates, a currency board seems to increase the scope for economic fluctuations. Such potential for fluctuations disappears as the backing of the domestic money supply and deposits is reduced. Moreover, indeterminacy of dynamic equilibria may be observed independently of the backing of the domestic money supply. And, in economies with fixed exchange rates, the potential for indeterminacy and fluctuations seems to be positively related to the (world) rate of inflation.

5 Appendix

5.1 Proofs of Propositions and Lemmas in Section 2

5.1.1 Proof of Proposition 1

Using equation (36), and differentiating both sides with respect to σ, one obtains

$$\frac{\partial \hat{k}}{\partial \sigma} = -\frac{\phi_d \left[r - \left(\frac{\phi_d}{1+\sigma} \right) - \left(\frac{\phi_f}{1+\sigma^*} \right) \right]^{\frac{2-\alpha}{\alpha-1}}}{(1-\alpha)(1+\sigma)^2 \left[\alpha A (1-\phi_d - \phi_f) \right]^{\frac{1}{\alpha-1}}} < 0. \qquad \text{(A.1)}$$

Equation (A.1) proves the first part of the proposition. For the second part, differentiate the definition of ε in (28). One can easily determine that

$$\frac{\partial \varepsilon}{\partial \sigma} = -\frac{\phi_d (1-\phi_d - \phi_f)}{\left[(1-\phi_d - \phi_f) + \sigma(1-\phi_f) \right]^2} < 0. \qquad \text{(A.2)}$$

Then, given that $\varepsilon > 0, \forall \sigma > \sigma_\varepsilon$, we can easily determine from equation (38) that

$$\frac{\partial \hat{\chi}}{\partial \sigma} = -\left[\frac{1}{\lambda(1-\alpha)A(1-\phi_d - \phi_f)} \right] \left[\hat{k}^{1-\alpha} \frac{\partial \varepsilon}{\partial \sigma} + \varepsilon(1-\alpha)\hat{k}^{-\alpha} \frac{\partial \hat{k}}{\partial \sigma} \right] > 0. \quad \text{(A.3)}$$

Equation (A.3) proves the second part of the proposition. *Q.E.D.*

5.1.2 Proof of Proposition 2

From equation (36), one obtains the following:

$$\frac{\partial \hat{k}}{\partial \sigma^*} = -\frac{\phi_f \hat{k}}{(1-\alpha)(1+\sigma^*)^2 \left[r - \left(\frac{\phi_d}{1+\sigma} \right) - \left(\frac{\phi_f}{1+\sigma^*} \right) \right]} < 0, \qquad \text{(A.4)}$$

and

$$\frac{\partial \hat{k}}{\partial r} = -\frac{\hat{k}}{(1-\alpha) \left[r - \left(\frac{\phi_d}{1+\sigma} \right) - \left(\frac{\phi_f}{1+\sigma^*} \right) \right]} < 0. \quad \text{(A.5)}$$

(A.4) and (A.5) prove the first part of the proposition. In addition, differentiating (38) with respect to σ^* and r, respectively, one obtains

$$\frac{\partial \hat{\chi}}{\partial \sigma^*} = \frac{\alpha \varepsilon \phi_f}{(1-\alpha)\lambda(1+\sigma^*)^2 \left[r - \left(\frac{\phi_d}{1+\sigma} \right) - \left(\frac{\phi_f}{1+\sigma^*} \right) \right]^2} > 0, \qquad \text{(A.6)}$$

and

$$\frac{\partial \hat{\chi}}{\partial r} = \frac{\alpha \varepsilon}{(1-\alpha)\lambda \left[r - \left(\frac{\phi_d}{1+\sigma} \right) - \left(\frac{\phi_f}{1+\sigma^*} \right) \right]^2} > 0. \qquad \text{(A.7)}$$

(A.6) and (A.7) prove the second part of the proposition. *Q.E.D.*

5.1.3 Proof of Proposition 3

Differentiating equation (36) with respect to ϕ_d we obtain

$$\frac{\partial \hat{k}}{\partial \phi_d} = -\left[\frac{\alpha A}{(1-\alpha)\rho^2}\right]\hat{k}^{\alpha}\left[\frac{r - \left(\frac{1}{1+\sigma}\right) + \phi_f\left(\frac{1}{1+\sigma} - \frac{1}{1+\sigma^*}\right)}{(1-\phi_d - \phi_f)^2}\right], \qquad \text{(A.8)}$$

which implies that

$$\frac{\partial \hat{k}}{\partial \phi_d} < (>) \text{ if } \sigma > (<)\sigma_1 \equiv \frac{(1-r) + \phi_f\left(\frac{1}{1+\sigma^*} - 1\right)}{\left(r - \frac{\phi_f}{1+\sigma^*}\right)}. \text{ (A.9)}$$

However, it can be easily shown that $r > \left(\frac{1}{1+\sigma^*}\right)$ implies

$$\sigma_1 < \left(\frac{1}{r} - 1\right). \quad \text{(A.10)}$$

Therefore, since the reserve requirements bind, $\sigma > \sigma_1$ holds and, thus $\frac{\partial \hat{k}}{\partial \phi_d} < 0$, $\forall \sigma > Max\left\{\sigma_{\varepsilon}, \left(\frac{1}{r} - 1\right)\right\}$. This proves the first part of this proposition. To prove the second part of this proposition, differentiate (36) with respect to ϕ_f to obtain

$$\frac{\partial \hat{k}}{\partial \phi_f} = -\left[\frac{\alpha A}{(1-\alpha)\rho^2}\right]\hat{k}^{\alpha}\left[\frac{r - \left(\frac{\phi_d}{1+\sigma}\right) - \left(\frac{1-\phi_d}{1+\sigma^*}\right)}{(1-\phi_d - \phi_f)^2}\right], \quad \text{(A.11)}$$

which in turn implies that

$$\frac{\partial \hat{k}}{\partial \phi_f} < (>) 0 \text{ if } \sigma > (<) \sigma_2 \equiv \frac{\phi_d + \left(\frac{1-\phi_d}{1+\sigma^*}\right) - r}{\left[r - \left(\frac{1-\phi_d}{1+\sigma^*}\right)\right]}. \qquad \text{(A.12)}$$

However, it can be easily shown that $r > \left(\frac{1}{1+\sigma^*}\right)$ implies

$$\sigma_2 < \left(\frac{1}{r} - 1\right). \quad \text{(A.13)}$$

Then, again $\sigma > \sigma_2$ holds and, thus $\frac{\partial \hat{k}}{\partial \phi_f} < 0$, $\forall \sigma > Max\left\{\sigma_\varepsilon, \left(\frac{1}{r} - 1\right)\right\}$, proving the second part of this proposition. *Q.E.D.*

5.1.4 Proof of Proposition 4

Differentiating equation (39) with respect to σ, one obtains

$$\frac{\partial \tilde{k}}{\partial \sigma} = \left\{\frac{\pi}{(1-\alpha)A[r(1-\lambda)(1-\alpha) - \alpha\pi]}\right\} \tilde{k}^{2-\alpha} \left\{\varepsilon \frac{\partial \rho}{\partial \sigma} + \rho \frac{\partial \varepsilon}{\partial \sigma}\right\}, \qquad \text{(A.14)}$$

where

$$\varepsilon \frac{\partial \rho}{\partial \sigma} + \rho \frac{\partial \varepsilon}{\partial \sigma} = \frac{\phi_d \left[(1-r) + \phi_f \left(\frac{1}{1+\sigma^*} - 1\right)\right]}{\left[(1-\phi_d - \phi_f) + \sigma(1-\phi_f)\right]^2} < 0, \text{ given } r > 1 \text{ and } r > \left(\frac{1}{1+\sigma^*}\right). \qquad \text{(A.15)}$$

Thus, $\frac{\partial \tilde{k}}{\partial \sigma} > (<) 0$ if $r(1-\lambda)(1-\alpha) < (>) \alpha\pi$, which proves the first part of the proposition. Differentiating equation (40) with respect to σ, yields

$$\frac{\partial \tilde{\chi}}{\partial \sigma} = -\left[\frac{\tilde{k}^{-\alpha}}{\lambda(1-\alpha)A(1-\phi_d - \phi_f)}\right] \left\{\tilde{k} \frac{\partial \varepsilon}{\partial \sigma} + \varepsilon(1-\alpha) \frac{\partial \tilde{k}}{\partial \sigma}\right\}. \qquad \text{(A.16)}$$

Using (A.2), and the fact that $\varepsilon > 0, \forall \sigma > \sigma_{\varepsilon}$, it is clear that if $\frac{\partial \tilde{k}}{\partial \sigma} < 0$, then it must be the case that $\frac{\partial \tilde{\chi}}{\partial \sigma} > 0$. *Q.E.D.*

5.1.5 Proof of Proposition 5

Differentiating equation (39) with respect to σ^*, yields

$$\frac{\partial \tilde{k}}{\partial \sigma^*} = \left\{ \frac{\pi \tilde{k}^{2-\alpha}}{(1-\alpha)A\left[r(1-\lambda)(1-\alpha)-\alpha\pi\right]} \right\} \left[\frac{\varepsilon \phi_d}{(1-\phi_d-\phi_f)(1+\sigma^*)^2} \right], \quad \text{(A.17)}$$

and, thus $\frac{\partial \tilde{k}}{\partial \sigma^*} < (>) 0$ if $r(1-\lambda)(1-\alpha) < (>) \alpha\pi$. Differentiating equation (39) with respect to *r*, yields

$$\frac{\partial \tilde{k}}{\partial r} = \tilde{k} \left\{ \frac{(1-\lambda)}{\left[r(1-\lambda)(1-\alpha)-\alpha\pi\right]} + \frac{\pi\varepsilon\left[(1-\pi)x-\pi\varepsilon\rho\right]^{-1}}{(1-\alpha)(1-\phi_d-\phi_f)} \right\}, \quad \text{(A.18)}$$

and thus, $\frac{\partial \tilde{k}}{\partial r} < (>) 0$ if $r(1-\lambda)(1-\alpha) < (>) \alpha\pi$. Notice that (A.17) and (A.18) prove the first part of the proposition. Differentiating (40) with respect to σ^*, gives the following:

$$\frac{\partial \tilde{\chi}}{\partial \sigma^*} = -\left[\frac{\varepsilon(1-\alpha)\tilde{k}^{-\alpha}}{\lambda(1-\alpha)A(1-\phi_d-\phi_f)} \right] \frac{\partial \tilde{k}}{\partial \sigma^*}, \quad \text{(A.19)}$$

and thus, $\frac{\partial \tilde{\chi}}{\partial \sigma^*} > (<) 0$ if $r(1-\lambda)(1-\alpha) < (>) \alpha\pi$. Differentiating (40) with respect to *r* one gets

$$\frac{\partial \tilde{\chi}}{\partial r} = -\left[\frac{\varepsilon(1-\alpha)\tilde{k}^{-\alpha}}{\lambda(1-\alpha)A(1-\phi_d-\phi_f)} \right] \frac{\partial \tilde{k}}{\partial r}, \quad \text{(A.20)}$$

and thus, $\frac{\partial \tilde{\chi}}{\partial r} > (<) 0$ if $r(1-\lambda)(1-\alpha) < (>) \alpha\pi$. Notice that (A.19) and (A.20) prove the second part of the proposition. *Q.E.D.*

5.1.6 Proof of Proposition 6

Differentiating equation (39) with respect to ϕ_d allows one to show that

$$\frac{\partial \tilde{k}}{\partial \phi_d} = \left\{ \frac{\pi \tilde{k}^{2-\alpha}}{A(1-\alpha)[r(1-\lambda)(1-\alpha)-\alpha\pi]} \right\} \left\{ \varepsilon \frac{\partial \rho}{\partial \phi_d} + \rho \frac{\partial \varepsilon}{\partial \phi_d} \right\} \quad \text{(A.21)}$$

where

$$\left\{ \varepsilon \frac{\partial \rho}{\partial \phi_d} + \rho \frac{\partial \varepsilon}{\partial \phi_d} \right\} = \frac{(1+\sigma)\left[(r-1) + \phi_f \left(1 - \frac{1}{1+\sigma^*} \right) \right]}{[(1-\phi_d-\phi_f)+\sigma(1-\phi_f)]^2} > 0, \quad \text{(A.22)}$$

given $r > 1$ and $r > \frac{1}{1+\sigma^*}$.

Thus, from (A.21) and (A.22), $\frac{\partial \tilde{k}}{\partial \phi_d} < (>) 0$ if $r(1-\lambda)(1-\alpha) < (>) \alpha\pi$. This proves the first part of the proposition.

Similarly, differentiating (39) with respect to ϕ_f, one gets

$$\frac{\partial \tilde{k}}{\partial \phi_f} = \left\{ \frac{\pi \tilde{k}^{2-\alpha}}{A(1-\alpha)[r(1-\lambda)(1-\alpha)-\alpha\pi]} \right\} \left\{ \varepsilon \frac{\partial \rho}{\partial \phi_f} + \rho \frac{\partial \varepsilon}{\partial \phi_f} \right\} \quad \text{(A.23)}$$

where

$$\left\{ \varepsilon \frac{\partial \rho}{\partial \phi_f} + \rho \frac{\partial \varepsilon}{\partial \phi_f} \right\} = \frac{(1+\sigma)\left\{ (1+\sigma)\left[r - \left(\frac{1}{1+\sigma^*} \right) \right] - \phi_d \left(\frac{\sigma^*}{1+\sigma^*} \right) \right\}}{[(1-\phi_d-\phi_f)+\sigma(1-\phi_f)]^2} \quad \text{(A.24)}$$

Then, it is possible to show that

$$\left\{\varepsilon\frac{\partial\rho}{\partial\phi_f}+\rho\frac{\partial\varepsilon}{\partial\phi_f}\right\}>(<)0 \text{ if } \sigma>(<)\left\{\frac{\sigma^*\phi_d}{\left[r(1+\sigma^*)-1\right]}-1\right\}. \quad \text{(A.25)}$$

Therefore, $\frac{\partial\tilde{k}}{\partial\phi_f}<(>)0$ when $r(1-\lambda)(1-\alpha)<(>)\alpha\pi$ and $\sigma>\left\{\frac{\sigma^*\phi_d}{\left[r(1+\sigma^*)-1\right]}-1\right\}$, but $\frac{\partial\tilde{k}}{\partial\phi_f}>(<)0$ when $r(1-\lambda)(1-\alpha)<(>)\alpha\pi$ and $\sigma<\left\{\frac{\sigma^*\phi_d}{\left[r(1+\sigma^*)-1\right]}-1\right\}$. The latter proves the second part of the proposition. *Q.E.D.*

5.1.7 Proof of Lemma 7

Using equation (35), it is easy to show that

$$\frac{\partial\rho}{\partial\sigma}=\frac{\phi_d}{(1-\phi_d-\phi_f)(1+\sigma)^2}>0,\ \forall\sigma>Max\left\{\left(\frac{1}{r}-1\right),\sigma_\varepsilon\right\} \quad \text{(A.26)}$$

and that

$$\frac{\partial^2\rho}{\partial\sigma^2}=-\frac{2\phi_d}{(1-\phi_d-\phi_f)(1+\sigma)^3}<0,\ \forall\sigma>Max\left\{\left(\frac{1}{r}-1\right),\sigma_\varepsilon\right\} \quad \text{(A.27)}$$

Therefore, ρ is monotonically increasing and strictly concave in σ, for any $\sigma>Max\left\{\left(\frac{1}{r}-1\right),\sigma_\varepsilon\right\}$. Moreover,

$$\lim_{\sigma\to\infty}\rho=\frac{\left[r-\left(\frac{\phi_f}{1+\sigma^*}\right)\right]}{(1-\phi_d-\phi_f)}>0 \quad \text{(A.28)}$$

proving that ρ is bounded above. *Q.E.D.*

5.2 Technical Notes on Local Stability Analysis in Section 2

Equation (46) can be arranged to obtain the following expression:

$$\left(\frac{1+\sigma}{\phi_d}\right)\left[r-\left(\frac{\phi_f}{1+\sigma^*}\right)\right]-E\left(\frac{1-\pi}{\pi}\right)x=1+Ef'\left(\tilde{k}\right) \\ -Ef'(\tilde{k})\left[\frac{r(1-\lambda)(1-\alpha)}{\alpha\pi}\right]. \tag{A.29}$$

Using (A.29), it is possible to obtain the following expression for the trace of the Jacobian:

$$T=J_{11}=1+(1-\alpha)Ef'(\tilde{k})-Ef'(\tilde{k})\left[\frac{r(1-\lambda)(1-\alpha)}{\alpha\pi}\right]. \tag{A.30}$$

Then, using (A.30) equations (51) and (52) are easily obtained.

In addition, recall from equation (32) that it is necessary to impose $\sigma>\sigma_\varepsilon$ for lending to be positive. Then, *E*>0 follows directly from this condition.

Finally, in the remainder of this section, I show that $Ef'(\tilde{k})$ is always increasing in σ. Using equations (39) and (50), yields

$$Ef'(\tilde{k})=\Gamma(1-\pi)x\left[(1-\phi_d-\phi_f)+\sigma(1-\phi_f)\right] \\ -\Gamma\pi\left[r(1+\sigma)-\phi_d-(1+\sigma)\left(\frac{\phi_f}{1+\sigma^*}\right)\right] \tag{A.31}$$

where

$$\Gamma\equiv\left\{\frac{\alpha}{\phi_d\left[r(1-\lambda)(1-\alpha)-\alpha\pi\right]}\right\} \tag{A.32}$$

Notice that $\Gamma<(>)0$ if $r(1-\lambda)(1-\alpha)<(>)\alpha\pi$. Differentiating (A.31) with respect to σ, we obtain

$$\frac{\partial\left[Ef^{'}(\tilde{k})\right]}{\partial\sigma}=\Gamma\left\{(1-\pi)x(1-\phi_f)-\pi\left[r-\left(\frac{\phi_f}{1+\sigma^*}\right)\right]\right\}. \quad \text{(A.33)}$$

It is also useful to notice that

$$\lim_{\sigma\to\infty}\wp=\frac{\left[r-\left(\frac{\phi_f}{1+\sigma^*}\right)\right]}{(1-\phi_f)}. \quad \text{(A.34)}$$

Thus, the following must be true:

a) If $r(1-\lambda)(1-\alpha)<\alpha\pi$, then $(1-\pi)x<\pi\frac{\left[r-\left(\frac{\phi_f}{1+\sigma^*}\right)\right]}{(1-\phi_f)}$ for $\tilde{k}$ to be well defined. In addition, as I mentioned before, Γ<0.

b) If $r(1-\lambda)(1-\alpha)>\alpha\pi$, then $(1-\pi)x>\pi\frac{\left[r-\left(\frac{\phi_f}{1+\sigma^*}\right)\right]}{(1-\phi_f)}$ for $\tilde{k}$ to be well defined. Also, notice that, as I mentioned before, Γ>0.

Thus, from (A.33), it follows that $\frac{\partial\left[Ef^{'}(\tilde{k})\right]}{\partial\sigma}>0$ always holds.

5.3 Technical Notes on (Steady-State) Comparative Statics in Section 3

5.3.1 Steady-State Equilibria in a Walrasian Regime

Comparative Statics with respect to the Policy Parameters θ and ξ

Differentiating equation (65) with respect to θ and ξ, respectively, I obtain

$$\frac{\partial \delta}{\partial \theta} = \frac{\phi_d (1+\sigma^*)^2 (r-1)(1-\phi_d-\phi_f)}{\left[(1+\sigma^*)(r-1)(\theta\phi_d+\xi)+(1-\phi_d-\phi_f)+\sigma^*(1-\phi_f)\right]^2} > 0, \text{given } r > 1, \qquad \text{(A.35)}$$

and

$$\frac{\partial \delta}{\partial \xi} = \frac{(1+\sigma^*)^2 (r-1)(1-\phi_d-\phi_f)}{\left[(1+\sigma^*)(r-1)(\theta\phi_d+\xi)+(1-\phi_d-\phi_f)+\sigma^*(1-\phi_f)\right]^2} \text{ given } r > 1. \quad \text{(A.36)}$$

Thus, δ is increasing in both θ and ξ.

In addition, differentiating equation (63) also with respect to θ and ξ, respectively, one gets

$$\frac{\partial \hat{\tau}}{\partial \theta} = \left(\frac{\hat{k}}{1-\lambda}\right)\frac{\partial \delta}{\partial \theta} > 0, \text{given } \frac{\partial \hat{k}}{\partial \theta} = 0, \quad \text{(A.37)}$$

and

$$\frac{\partial \hat{\tau}}{\partial \xi} = \left(\frac{\hat{k}}{1-\lambda}\right)\frac{\partial \delta}{\partial \xi} > 0, \text{ given } \frac{\partial \hat{k}}{\partial \xi} = 0. \quad \text{(A.38)}$$

Finally, differentiating equation (64) with respect to θ and ξ, respectively, yields

$$\frac{\partial \hat{\chi}}{\partial \theta} = \left[\frac{\hat{k}^{1-\alpha}}{\lambda A(1-\alpha)(1-\phi_d-\phi_f)}\right]\frac{\partial \delta}{\partial \theta} > 0, \text{ given } \frac{\partial \hat{k}}{\partial \theta} = 0, \qquad \text{(A.39)}$$

and

$$\frac{\partial \hat{\chi}}{\partial \xi} = \left[\frac{\hat{k}^{1-\alpha}}{\lambda A(1-\alpha)(1-\phi_d-\phi_f)}\right]\frac{\partial \delta}{\partial \xi} > 0, \text{ given } \frac{\partial \hat{k}}{\partial \xi} = 0. \qquad \text{(A.40)}$$

Effects of Increases in σ^*, r, ϕ_d or ϕ_f on the capital-labor ratio ($\hat{k}$)

Given equation (62), it is obvious that there is a negative relationship between $\hat{k}$ and ρ. Then, it will be sufficient to show the effects of σ^*, r, ϕ_d and ϕ_f on ρ. Differentiating (56) with respect to σ^*, r, ϕ_d and ϕ_f respectively, gives

$$\frac{\partial \rho}{\partial \sigma^*} = \frac{(\phi_d + \phi_f)}{(1 - \phi_d - \phi_f)(1 + \sigma^*)} > 0, \qquad \text{(A.41)}$$

$$\frac{\partial \rho}{\partial r} = \frac{1}{(1 - \phi_d - \phi_f)} > 0, \qquad \text{(A.42)}$$

$$\frac{\partial \rho}{\partial \phi_d} = \frac{\partial \rho}{\partial \phi_f} = \frac{\left[r - \left(\frac{1}{1 + \sigma^*} \right) \right]}{(1 - \phi_d - \phi_f)^2} > 0. \qquad \text{(A.43)}$$

Thus, it follows from (A.41), (A.42) and (A.43) that increases in σ^*, r, ϕ_d and ϕ_f reduce $\hat{k}$.

Effects of Increases in σ^* on both δ and $\hat{\chi}$

Differentiating equation (65) with respect to σ^* yields

$$\frac{\partial \delta}{\partial \sigma^*} = \frac{\phi_d (1 - \phi_d - \phi_f)}{\left[(1 + \sigma^*)(r - 1)(\theta \phi_d + \xi) + (1 - \phi_d - \phi_f) + \sigma^*(1 - \phi_f) \right]^2} > 0. \text{ (A.44)}$$

Thus, δ is always increasing in σ^*.

Differentiating equation (64) with respect to σ^* we obtain the following expression

$$\frac{\partial \hat{\chi}}{\partial \sigma^*} = \left[\frac{1}{\lambda A (1 - \alpha)(1 - \phi_d - \phi_f)} \right] \left\{ (\delta - 1)(1 - \alpha) \hat{k}^{-\alpha} \frac{\partial \hat{k}}{\partial \sigma^*} + \hat{k}^{1 - \alpha} \frac{\partial \delta}{\partial \sigma^*} \right\} > 0. \quad \text{(A.45)}$$

Notice that $(\delta - 1) < 0$ for lending to be positive.

Comparative Statics under a Currency Board

In a currency board, the subsidy rate on capital can be written as

$$\delta = \frac{(1+\sigma^*)(r-1)(\phi_d + \xi) + \sigma^* \phi_d}{\left[(1+\sigma^*)(r-1)(\phi_d + \xi) + (1-\phi_d - \phi_f) + \sigma^*(1-\phi_f)\right]}. \quad \text{(A.46)}$$

Differentiating (A.46) with respect to *r*, one obtains

$$\frac{\partial \delta}{\partial r} = \frac{(1+\sigma^*)^2(\phi_d + \xi)(1-\phi_d - \phi_f)}{\left[(1+\sigma^*)(r-1)(\phi_d + \xi) + (1-\phi_d - \phi_f) + \sigma^*(1-\phi_f)\right]^2}. \quad \text{(A.47)}$$

Thus, the subsidy rate on capital δ is increasing in *r*.

On the other hand, differentiating equation (64) with respect to *r*, we obtain the following expression

$$\frac{\partial \hat{\chi}}{\partial r} = \left[\frac{1}{\lambda A(1-\alpha)(1-\phi_d - \phi_f)}\right]\left\{(\delta - 1)(1-\alpha)\hat{k}^{-\alpha}\frac{\partial \hat{k}}{\partial r} + \hat{k}^{1-\alpha}\frac{\partial \delta}{\partial r}\right\} > 0. \quad \text{(A.48)}$$

Then, the share of net investment abroad in total savings is increasing in the world interest rate *r*.

Regarding the effects of changes in domestic reserve requirements, I start with the analysis of increases in ϕ_d. Differentiating (A.46) with respect to ϕ_d, one gets

$$\frac{\partial \delta}{\partial \phi_d} = \frac{(1+\sigma^*)\left\{(1+\sigma^*)(r-1)(1-\phi_f + \xi) + \sigma^*(1-\phi_f)\right\}}{\left[(1+\sigma^*)(r-1)(\phi_d + \xi) + (1-\phi_d - \phi_f) + \sigma^*(1-\phi_f)\right]^2}. \quad \text{(A.49)}$$

Notice that the expression $\left\{(1+\sigma^*)(r-1)(1-\phi_f + \xi) + (1-\phi_f)\right\}$ in the numerator of (A.49) is increasing in σ^*. Thus, it follows that

$$\frac{\partial \delta}{\partial \phi_d} > 0 \Leftrightarrow \sigma^* > -\left[\frac{(r-1)(1-\phi_f+\xi)}{(r-1)(1-\phi_f+\xi)+(1-\phi_f)}\right] > -1. \qquad \text{(A.50)}$$

This condition always holds if

$$\left(\frac{1}{r}-1\right) \geq -\left[\frac{(r-1)(1-\phi_f+\xi)}{(r-1)(1-\phi_d+\xi)+(1-\phi_f)}\right] \Leftrightarrow \xi \geq 0. \quad \text{(A.51)}$$

Obviously, (A.51) always holds under a currency board and, therefore, $\frac{\partial \delta}{\partial \phi_d} > 0$.

Using this result, as well as the fact that $\hat{k}$ is decreasing in ϕ_d, it is straightforward to show that $\hat{\chi}$ is increasing in ϕ_d: differentiating (64) gives

$$\frac{\partial \hat{\chi}}{\partial \phi_d} = \left[\frac{1}{\lambda A(1-\alpha)(1-\phi_d-\phi_f)}\right]\left\{(\delta-1)(1-\alpha)\hat{k}^{-\alpha}\frac{\partial \hat{k}}{\partial \phi_d} + \hat{k}^{1-\alpha}\frac{\partial \delta}{\partial \phi_d}\right\} > 0 \qquad \text{(A.52)}$$

Similarly, differentiating (46) with respect to ϕ_f yields

$$\frac{\partial \delta}{\partial \phi_f} = \frac{(1+\sigma^*)\left\{(1+\sigma^*)(r-1)(\phi_d+\xi)+\sigma^*\phi_d\right\}}{\left[(1+\sigma^*)(r-1)(\phi_d+\xi)+(1-\phi_d-\phi_f)+\sigma^*(1-\phi_f)\right]^2}. \qquad \text{(A.53)}$$

Notice that the expression $\left\{(1+\sigma^*)(r-1)(\phi_d+\xi)+\sigma^*\phi_d\right\}$ in the numerator of (A.53) is increasing in σ^*. It follows, then, that

$$\frac{\partial \delta}{\partial \phi_f} > 0 \Leftrightarrow \sigma^* > -\left[\frac{(r-1)(\phi_d+\xi)}{(r-1)(\phi_d+\xi)+\phi_d}\right] > -1. \qquad \text{(A.54)}$$

This condition always holds if

$$\left(\frac{1}{r}-1\right) \geq -\left[\frac{(r-1)(\phi_d+\xi)}{(r-1)(\phi_d+\xi)+\phi_d}\right] \Leftrightarrow \xi \geq 0. \quad \text{(A.55)}$$

Again, it is obvious that (A.55) always holds under a currency board and therefore, $\frac{\partial \delta}{\partial \phi_f} > 0$. Using this result, as well as the fact that $\hat{k}$ is decreasing in ϕ_f, it is straightforward to show that $\hat{\chi}$ is increasing in ϕ_f: Thus, differentiating (64), one obtains

$$\frac{\partial \hat{\chi}}{\partial \phi_f} = \left[\frac{1}{\lambda A(1-\alpha)(1-\phi_d-\phi_f)} \right] \left\{ (\delta-1)(1-\alpha)\hat{k}^{-\alpha} \frac{\partial \hat{k}}{\partial \phi_f} + \hat{k}^{1-\alpha} \frac{\partial \delta}{\partial \phi_f} \right\} > 0 \quad \text{(A.56)}$$

Comparative Statics under a Pure Fixed Exchange Rate

In a pure fixed exchange rate regime, the subsidy rate on capital can be written as

$$\delta = \frac{\sigma^* \phi_d}{\left[(1-\phi_d-\phi_f) + \sigma^*(1-\phi_f) \right]}. \qquad \text{(A.57)}$$

By inspection of (A.57), it is obvious that $\frac{\partial \delta}{\partial r} = 0$. It follows that

$$\frac{\partial \hat{\chi}}{\partial r} = \left[\frac{1}{\lambda A(1-\alpha)(1-\phi_d-\phi_f)} \right] \left\{ (\delta-1)(1-\alpha)\hat{k}^{-\alpha} \frac{\partial \hat{k}}{\partial r} \right\} > 0. \quad \text{(A.58)}$$

Thus, the share of net investment abroad in total savings is always increasing in r.

Next, differentiate (A.57) with respect to ϕ_d to obtain

$$\frac{\partial \delta}{\partial \phi_d} = \frac{\sigma^*(1+\sigma^*)(1-\phi_f)}{\left[(1-\phi_d-\phi_f) + \sigma^*(1-\phi_f) \right]^2}. \text{ (A.59)}$$

In addition, differentiating equation (64) with respect to ϕ_d, yields

$$\frac{\partial \hat{\chi}}{\partial \phi_d} = \left[\frac{\alpha}{\lambda(1-\alpha)(1-\phi_d-\phi_f)} \right] \left[\frac{\rho \frac{\partial \delta}{\partial \phi_d} + (1-\delta)\frac{\partial \rho}{\partial \phi_d}}{\rho^2} \right]. \qquad \text{(A.60)}$$

Using (A.43), (A.59), and (A.60) I am able to determine that the share of net investment abroad is increasing in ϕ_d for all but very small values of σ^*.

In similar way, differentiating (A.57) with respect to ϕ_f I obtain

$$\frac{\partial \delta}{\partial \phi_f} = \frac{\sigma^*\left(1+\sigma^*\right)\phi_d}{\left[\left(1-\phi_d-\phi_f\right)+\sigma^*(1-\phi_f)\right]^2}. \quad \text{(A.61)}$$

Differentiating equation (64) with respect to ϕ_f, I obtain

$$\frac{\partial \hat{\chi}}{\partial \phi_f} = \left[\frac{\alpha}{\lambda(1-\alpha)(1-\phi_d-\phi_f)}\right]\left[\frac{\rho\dfrac{\partial \delta}{\partial \phi_f}+(1-\delta)\dfrac{\partial \rho}{\partial \phi_f}}{\rho^2}\right]. \qquad \text{(A.62)}$$

Using (A.43), (A.61) and (A.62) I am able to determine that the share of net investment abroad is increasing in ϕ_f for all but very small values of σ^*.

5.3.2 Steady-State Equilibria in a Credit Rationing Regime

Comparative Statics with respect to the Policy Parameters θ and ξ

Differentiating equation (68) with respect to θ and with respect to ξ, and using (A.35) as well as (A.36) I am able to determine, respectively, that

$$\begin{aligned}\frac{\partial \tilde{k}}{\partial \theta} &= -\left\{\frac{\pi\rho\tilde{k}^{2-\alpha}}{A(1-\alpha)\left[r(1-\lambda)(1-\alpha)-\alpha\pi\right]}\right\}\frac{\partial \delta}{\partial \theta} \\ &= \begin{cases} >0, \text{ if Case 1 obtains,} \\ <0, \text{ if Case 2 obtains} \end{cases}\end{aligned} \quad \text{(A.63)}$$

and

$$\frac{\partial \tilde{k}}{\partial \xi} = -\left\{ \frac{\pi \rho \tilde{k}^{2-\alpha}}{A(1-\alpha)\left[r(1-\lambda)(1-\alpha)-\alpha\pi\right]} \right\} \frac{\partial \delta}{\partial \xi}$$
$$= \begin{cases} >0, \text{ if Case 1 obtains,} \\ <0, \text{ if Case 2 obtains} \end{cases}. \qquad \text{(A.64)}$$

Then, differentiating equation (67) also with respect to θ and with respect to ξ, and using (A.35), (A.36), (A.63), and (A.64), I am able to determine that

$$\frac{\partial \tilde{\tau}}{\partial \theta} = \frac{\left[\tilde{k}\frac{\partial \delta}{\partial \theta} + \delta \frac{\partial \tilde{k}}{\partial \theta} \right]}{(1-\lambda)} > 0 \text{ if Case 1 obtains,} \qquad \text{(A.65)}$$

and

$$\frac{\partial \tilde{\tau}}{\partial \xi} = \frac{\left[\tilde{k}\frac{\partial \delta}{\partial \xi} + \delta \frac{\partial \tilde{k}}{\partial \xi} \right]}{(1-\lambda)} > 0 \text{ if Case 1 obtains.} \qquad \text{(A.66)}$$

Finally, differentiating equation (69) with respect to θ and with respect to ξ, and using (A.35), (A.36), (A.63) and (A.64), I am able to determine, respectively, that

$$\frac{\partial \tilde{\chi}}{\partial \theta} = \left\{ \frac{\left[r(1-\lambda)(1-\alpha)-\alpha\pi\right]}{\lambda(1-\alpha)(1-\phi_d-\phi_f)} \right\} \left\{ \frac{(1-\pi)x\frac{\partial \delta}{\partial \theta}}{\left[(1-\pi)x-\pi(1-\delta)\rho\right]^2} \right\}$$
$$= \begin{cases} <0, \text{ if Case 1 obtains} \\ >0, \text{ if Case 2 obtains} \end{cases}, \qquad \text{(A.67)}$$

and

$$\frac{\partial \tilde{\chi}}{\partial \xi} = \left\{ \frac{\left[r(1-\lambda)(1-\alpha) - \alpha\pi \right]}{\lambda(1-\alpha)(1-\phi_d - \phi_f)} \right\} \left\{ \frac{(1-\pi)x \dfrac{\partial \delta}{\partial \xi}}{\left[(1-\pi)x - \pi(1-\delta)\rho \right]^2} \right\} \qquad \text{(A.68)}$$

$$= \begin{cases} < 0, \text{ if Case 1 obtains} \\ > 0, \text{ if Case 2 obtains} \end{cases}.$$

Comparative Statics under a Currency Board

Proof of Proposition 8 In this section I proceed to prove Proposition 8 as well as the corresponding results for a Case 2 economy arises for the case when a currency board is in place.

I start with the analysis of the effects of increases in σ^* on the capital-labor ratio $\tilde{k}$. Differentiating equation (68) with respect to σ^* one obtains

$$\frac{\partial \tilde{k}}{\partial \sigma^*} = \left\{ \frac{\pi \tilde{k}^{2-\alpha}}{A(1-\alpha)\left[r(1-\lambda)(1-\alpha) - \alpha\pi \right]} \right\} \left\{ \frac{\partial \left[(1-\delta)\rho \right]}{\partial \sigma^*} \right\}. \qquad \text{(A.69)}$$

In addition, using equations (A.58) and (A.65), gives

$$\frac{\partial \left[(1-\delta)\rho \right]}{\partial \sigma^*} = \frac{\left\{ r\left[(\phi_d + \phi_f)(\theta\phi_d + \xi) - \phi_d \right] + (\phi_d + \phi_f)(1 - \theta\phi_d - \phi_f - \xi) \right\}}{\left\{ (1+\sigma^*)(r-1)(\theta\phi_d + \xi) + (1-\phi_d - \phi_f) + \sigma^*(1-\phi_f) \right\}^2}. \quad \text{(A.70)}$$

Obviously, under a currency board θ=1 and $\xi \in [0,(1-\phi_d-\phi_f))$, and therefore (A.70) becomes

$$\frac{\partial \left[(1-\delta)\rho \right]}{\partial \sigma^*} = \frac{\left\{ r\left[(\phi_d + \phi_f)(\phi_d + \xi) - \phi_d \right] + (\phi_d + \phi_f)(1 - \phi_d - \phi_f - \xi) \right\}}{\left\{ (1+\sigma^*)(r-1)(\phi_d + \xi) + (1-\phi_d - \phi_f) + \sigma^*(1-\phi_f) \right\}^2}. \qquad \text{(A.71)}$$

Notice that the first term in the numerator of (A.71) is positive if

$$\xi \geq \xi_c \equiv \left(\frac{\phi_d}{\phi_d + \phi_f} \right)(1 - \phi_d - \phi_f). \qquad \text{(A.72)}$$

In addition, recall that 0≤ξ≤(1-ϕd-ϕf) by assumption. Thus, $(1-\delta)\rho$ is unambiguously increasing in σ^* $\forall r$ in situations where $\xi \in [\xi_c, (1-\phi_d-\phi_f))$ holds and a currency board is in place. As a consequence, $\tilde{k}$ is decreasing (increasing) in σ^* if Case 1 (Case 2) obtains or, equivalently if $r(1-\lambda)(1-\alpha) < (>) \alpha\pi$.

On the other hand, notice that if $\xi \in (0, \xi_c)$, the first term in the numerator of (A.71) is negative and, obviously, $\frac{\partial[(1-\delta)\rho]}{\partial\sigma^*}$ is decreasing in r. Also, while the denominator in (A.71) is always positive, we can rewrite the numerator as

$$(\phi_d+\phi_f)(1-\phi_d-\phi_f-\xi) - r\lfloor\phi_d-(\phi_d+\phi_f)(\phi_d+\xi)\rfloor, \qquad \text{(A.73)}$$

and, therefore

$$\frac{\partial[(1-\delta)\rho]}{\partial\sigma^*} > 0 \Leftrightarrow r < r_c \equiv \frac{(\phi_d+\phi_f)(1-\phi_d-\phi_f-\xi)}{[\phi_d-(\phi_d+\phi_f)(\phi_d+\xi)]} \qquad \text{(A.74)}$$

under a currency board when $\xi \in (0, \xi_c)$. However, r_c seems to be arbitrarily high for parameter values similar to the ones found in Latin American countries and, therefore, it seems that $r<r_c$ always holds. Therefore, under plausible conditions, $(1-\delta)\rho$ is also an increasing function of σ^* when $\xi \in (0, \xi_c)$ and, as a consequence, $\tilde{k}$ is decreasing (increasing) in σ^* if Case 1 (Case2) obtains or, equivalently if $r(1-\lambda)(1-\alpha) < (>) \alpha\pi$.

Next, I proceed to analyze the effects of increases in σ^* on the share of net investment abroad in total savings, $\tilde{\chi}$. Differentiating equation (69), I am able to determine that

$$\begin{aligned}\frac{\partial\tilde{\chi}}{\partial\sigma^*} &= \left\{\frac{[r(1-\lambda)(1-\alpha)-\alpha\pi]}{\lambda(1-\alpha)(1-\phi_d-\phi_f)[(1-\pi)x-\pi(1-\delta)\rho]^2}\right\} \\ &\times\left\{(1-\pi)x\frac{\partial\delta}{\partial\sigma^*} - \pi(1-\delta)^2\frac{\partial\rho}{\partial\sigma^*}\right\}.\end{aligned} \qquad \text{(A.75)}$$

Using (A.41) and (A.44), (A.75) can be rewritten as

$$\frac{\partial \tilde{\chi}}{\partial \sigma^*} = \left\{ \frac{\left[r(1-\lambda)(1-\alpha) - \alpha\pi \right]}{\lambda(1-\alpha)\left[(1-\pi)x - \pi(1-\delta)\rho \right]^2} \right\} \times \left\{ \frac{(1-\pi)x\phi_d - \pi(\phi_d + \phi_f)}{\left[(1+\sigma^*)(r-1)(\theta\phi_d + \xi) + (1-\phi_d - \phi_f) + \sigma^*(1-\phi_f) \right]^2} \right\}. \quad \text{(A.76)}$$

Obviously, under a currency board θ=1 and $\xi \in [0,(1-\phi_d-\phi_f))$, and therefore (A.76) can be rewritten as

$$\frac{\partial \tilde{\chi}}{\partial \sigma^*} = \left\{ \frac{\left[r(1-\lambda)(1-\alpha) - \alpha\pi \right]}{\lambda(1-\alpha)\left[(1-\pi)x - \pi(1-\delta)\rho \right]^2} \right\} \times \left\{ \frac{(1-\pi)x\phi_d - \pi(\phi_d + \phi_f)}{\left[(1+\sigma^*)(r-1)(\phi_d + \xi) + (1-\phi_d - \phi_f) + \sigma^*(1-\phi_f) \right]^2} \right\}. \quad \text{(A.77)}$$

Define $x_c \equiv \left(\frac{\pi}{1-\pi} \right)\left(\frac{\phi_d + \phi_f}{\phi_d} \right)$. Then $\left. \frac{\partial \tilde{\chi}}{\partial \sigma^*} \right|_{x=x_c} = 0$. Moreover, when Case 1 obtains

$$\frac{\partial \tilde{\chi}}{\partial \sigma^*} = \begin{cases} > 0, \text{if } x < x_c \\ < 0, \text{if } x > x_c \end{cases}, \quad \text{(A.78)}$$

while when Case 2 obtains

$$\frac{\partial \tilde{\chi}}{\partial \sigma^*} = \begin{cases} < 0, \text{if } x < x_c \\ > 0, \text{if } x > x_c \end{cases}. \quad \text{(A.79)}$$

I now analyze when each of the conditions in (A.78) and (A.79) holds, respectively.

Notice that, under a currency board

$$\lim_{\sigma^* \to \infty} (1-\delta)\rho = \frac{r}{(r-1)(\phi_d + \xi) + (1-\phi_f)}. \quad \text{(A.80)}$$

Also notice that if Case 1 (Case 2) obtains

$$\left(\frac{1-\pi}{\pi}\right)x < (>) \frac{r}{(r-1)(\phi_d + \xi) + (1-\phi_f)} \quad \text{(A.81)}$$

for $\lim_{\sigma^* \to \infty} f^{'}(\tilde{k})$ to be well-defined. Next, I start by analyzing situations where $\xi \in [0, \xi_c)$. Recall that, under these circumstances, $\lfloor \phi_d - (\phi_d + \phi_f)(\phi_d + \xi) \rfloor > 0$ holds. If this is the case, it is straightforward to show that

$$\lim_{\sigma^* \to \infty} (1-\delta)\rho = \frac{r}{(r-1)(\phi_d + \xi) + (1-\phi_f)} < \frac{(\phi_d + \phi_f)}{\phi_d} \Leftrightarrow r < r_c. \quad \text{(A.82)}$$

Notice that when Case 1 obtains

$$\left(\frac{1-\pi}{\pi}\right)x > \frac{r}{(r-1)(\phi_d + \xi) + (1-\phi_f)} < \left(\frac{\phi_d + \phi_f}{\phi_d}\right) \Rightarrow x < x_c. \quad \text{(A.83)}$$

Thus, when Case 1 obtains and when both $\xi \in [0, \xi_c)$ and $r < r_c$ hold, $\tilde{\chi}$ is always an increasing function of σ^*. On the other hand, notice that when Case 2 obtains and when both $\xi \in [0, \xi_c)$ and $r < r_c$ hold,

$$\left(\frac{1-\pi}{\pi}\right)x > \frac{r}{(r-1)(\phi_d + \xi) + (1-\phi_f)}, \quad \text{(A.84)}$$

but (A.82) holds too. Therefore, it is possible to observe either $x < x_c$ or $x > x_c$ and, then, from (A.79), $\tilde{\chi}$ will be decreasing or increasing in σ^*, accordingly.

Regarding situations where $\xi \in [\xi_c, (1-\phi_d - \phi_f))$, I focus again, for consistency, on the case where $r < r_c$. It is straightforward to show that

$$\lim_{\sigma^*\to\infty}(1-\delta)\rho=\frac{r}{(r-1)(\phi_d+\xi)+(1-\phi_f)}\Leftrightarrow r<r_c,\ \text{(A.85)}$$

since $\left[\phi_d-(\phi_d+\phi_f)(\phi_d+\xi)\right]<0$ when $\xi\in[\xi_c,(1-\phi_d-\phi_f))$. Using (A.81) and (A.85), it is possible to determine that either $x<x_c$ or $x>x_c$ when Case 1 obtains and both $\xi\in[\xi_c,(1-\phi_d-\phi_f))$ and $r<r_c$ hold. Thus, $\tilde{\chi}$ will be increasing or decreasing in σ^* accordingly. On the other hand, using also (A.81) and (A.85), it is possible to determine that $x>x_c$ when Case 2 obtains and both $\xi\in[\xi_c,(1-\phi_d-\phi_f))$ and $r<r_c$ hold. Thus, under these circumstances, $\tilde{\chi}$ is always increasing in σ^*. *Q.E.D.*

Effects of Increases in *r* on $\tilde{k}$ and $\tilde{\chi}$ I start with the analysis of the effects of increases in r on $\tilde{k}$. Differentiating equation (68) with respect to r, I obtain

$$\frac{\partial\tilde{k}}{\partial r}=\frac{\tilde{k}^{2-\alpha}\left\{\pi\left[r(1-\lambda)(1-\alpha)-\alpha\pi\right]\left[(1-\delta)\frac{\partial\rho}{\partial r}-\rho\frac{\partial\delta}{\partial r}\right]+\left[(1-\pi)x-\pi(1-\delta)\rho\right]\right\}}{\left\{(1-\alpha)A\left[r(1-\lambda)(1-\alpha)-\alpha\pi\right]^2\right\}}.\ \text{(A.86)}$$

Using (A.42) and (A.47) I obtain

$$(1-\delta)\frac{\partial\rho}{\partial r}-\rho\frac{\partial\delta}{\partial r}=\frac{(1+\sigma^*)\left[(1-\phi_d-\phi_f)(1-\phi_d-\xi)+\sigma^*(1-\phi_d-\phi_f-\xi)\right]}{\left[(1+\sigma^*)(r-1)(\phi_d+\xi)+(1-\phi_d-\phi_f)+\sigma^*(1-\phi_f)\right]^2}.\qquad\text{(A.87)}$$

Notice that (A.87) is unambiguously positive $\forall\sigma^*>-\dfrac{(1-\phi_d-\phi_f)(1-\phi_d-\xi)}{(1-\phi_d-\phi_f-\xi)}$.

Also, recall that both domestic and foreign currency being dominated in rates of return implies that $\sigma^*>\left(\frac{1}{r}-1\right)$. Also, under a currency board, $\left(\frac{1}{r}-1\right)>\sigma^*_\delta$, unambiguously. If $\left(\frac{1}{r}-1\right)>-\dfrac{(1-\phi_d-\phi_f)(1-\phi_d-\xi)}{(1-\phi_d-\phi_f-\xi)}$, then $(1-\delta)\rho$ is always increasing in r. I next proceed to analyze when the last condition holds.

$$\left(\frac{1}{r}-1\right) > -\frac{(1-\phi_d-\phi_f)(1-\phi_d-\xi)}{(1-\phi_d-\phi_f-\xi)}$$
$$\Leftrightarrow \left(\frac{1}{\mathrm{r}}\right) > \frac{\left[\phi_d(1-\phi_d-\phi_f)-(\phi_d+\phi_f)\xi\right]}{(1-\phi_d-\phi_f-\xi)}. \qquad \text{(A.88)}$$

Obviously, if $\xi \in [\xi_c, (1-\phi_d-\phi_f))$, the right-hand-side of the second inequality in (A.88) is negative, and the inequality always holds, resulting in $(1\text{-}\delta)\rho$ being always increasing in *r*. On the other hand, if $\xi \in [0, \xi_c)$, the right-hand-side of the second inequality is positive. In this case, it is helpful to rewrite the second inequality in (A.88) as

$$r < \frac{(1-\phi_d-\phi_f-\xi)}{\left[\phi_d(1-\phi_d-\phi_f)-(\phi_d+\phi_f)\xi\right]}. \qquad \text{(A.89)}$$

Notice that, given that both $r>1$ and $\frac{(1-\phi_d-\phi_f-\xi)}{\left[\phi_d(1-\phi_d-\phi_f)-(\phi_d+\phi_f)\xi\right]} > 1$, (A.89) might hold (or not), especially if ξ is very close to ξ_c. Thus, when $\xi \in [0, \xi_c)$, $(1\text{-}\delta)\rho$ is increasing in *r* for all values of σ^* but maybe those arbitrarily close to $\left(\frac{1}{r}-1\right)$. I can then say that $(1\text{-}\delta)\rho$ is typically increasing in *r*, $\forall \xi \in [0, (1-\phi_d-\phi_f))$, $\forall \sigma^* > Max\left\{\left(\frac{1}{r}-1\right), \sigma^*_\delta\right\}$.

Now, I return to the analysis of equation (A.86). When Case 1 (Case 2) obtains, both $r(1-\lambda)(1-\alpha) < (>) \alpha\pi$ and $\left[(1-\pi)x - \pi(1-\delta)\rho\right] < (>) 0$ hold, and, therefore, $\tilde{k}$ is decreasing (increasing) in *r*, given that, as we have previously proved, $(1\text{-}\delta)\rho$ is typically increasing in *r*.

Next, I analyze the effects of increases in *r* on the share of net investment abroad in total savings. Differentiating equation (69) with respect to *r*, I obtain

$$\frac{\partial\tilde{\chi}}{\partial r}=\frac{\left[(\delta-1)(1-\alpha)\tilde{k}^{-\alpha}\frac{\partial\tilde{k}}{\partial r}+\tilde{k}^{1-\alpha}\frac{\partial\delta}{\partial r}\right]}{\lambda A(1-\alpha)(1-\phi_d-\phi_f)}. \qquad \text{(A.90)}$$

Using (A.47) and the previous result, it should be obvious that when Case 1 obtains and a currency board is in place, $\frac{\partial\tilde{\chi}}{\partial r}>0$. However, when Case 2 obtains and a currency board is in place, both $\tilde{k}$ and δ are increasing in *r*, resulting in two forces that act in opposite directions. In this case, it is helpful to rewrite (A.90) as

$$\frac{\partial\tilde{\chi}}{\partial r}=\frac{Term1+Term2+Term3}{\lambda A(1-\alpha)(1-\phi_d-\phi_f)\left[(1-\pi)x-\pi(1-\delta)\rho\right]^2}, \qquad \text{(A.91)}$$

where

$$Term1\equiv(\delta-1)\left[(1-\pi)x-\pi(1-\delta)\rho\right]<0 \quad \text{when Case 2 obtains,} \qquad \text{(A.92)}$$

$$Term2\equiv\left[(1-\pi)x-\pi(1-\delta)\rho\right]\left[r(1-\lambda)(1-\alpha)-\alpha\pi\right]\frac{\partial\delta}{\partial r}>0 \qquad \text{(A.93)}$$
$$\text{when either Case 1 or Case 2 obtain,}$$

$$Term3\equiv(\delta-1)\left[r(1-\lambda)(1-\alpha)-\alpha\pi\right]\pi\left\{\frac{\partial\left[(1-\delta)\rho\right]}{\partial r}\right\}<0 \qquad \text{(A.94)}$$
$$\text{when Case 2 obtains}$$

Thus, the effect of increases in *r* on $\tilde{\chi}$ is ambiguous when Case 2 obtains and a currency board is in place.

Effects of Increases in ϕ_d on $\tilde{k}$ and $\tilde{\chi}$ I start with the analysis of the effects of increases in ϕ_d on the capital-labor ratio, $\tilde{k}$. Differentiating equation (68) with respect to ϕ_d, I obtain

$$\frac{\partial \tilde{k}}{\partial \phi_d} = \left\{ \frac{\pi \tilde{k}^{2-\alpha}}{A(1-\alpha)\left[r(1-\lambda)(1-\alpha)-\alpha\pi\right]} \right\} \left\{ \frac{\partial\left[(1-\delta)\rho\right]}{\partial \phi_d} \right\}. \qquad \text{(A.95)}$$

Using (A.43) and (A.49), I am able to determine that

$$\begin{aligned} \frac{\partial\left[(1-\delta)\rho\right]}{\partial \phi_d} &= \frac{\left[r(1+\sigma^*)-1\right]\left[(1+\sigma^*)(r1)(\phi_d+\xi)+(1-\phi_d-\phi_f)+\sigma^*(1-\phi_f)\right]}{(1-\phi_d-\phi_f)\left[(1+\sigma^*)(r1)(\phi_d+\xi)+(1-\phi_d-\phi_f)+\sigma^*(1-\phi_f)\right]^2} \\ &- \frac{\left[r(1+\sigma^*)-(\phi_d+\phi_f)\right]\left[(1+\sigma^*)(r-1)(1-\phi_d-\xi)+\sigma^*(1-\phi_f)\right]}{(1-\phi_d-\phi_f)\left[(1+\sigma^*)(r1)(\phi_d+\xi)+(1-\phi_d-\phi_f)+\sigma^*(1-\phi_f)\right]^2}. \end{aligned} \qquad \text{(A.96)}$$

After some simplifications, (A.96) can be rewritten as

$$\frac{\partial\left[(1-\delta)\rho\right]}{\partial \phi_d} = \frac{\Theta_1 - \Theta_2}{(1-\phi_d-\phi_f)\Theta_3} >< 0, \quad \text{(A.97)}$$

where

$$\Theta_1 \equiv \sigma^*\left[(r-1)+r^2(\phi_f-\xi)+\xi\right], \qquad \text{(A.98)}$$

$$\Theta_2 \equiv r(r-1)(1+\sigma^*)^2 + (r-1)\left[1+(\phi_f)(r+1)\right], \qquad \text{(A.99)}$$

and

$$\Theta_3 \equiv \left[(1+\sigma^*)(r-1)(\phi_d+\xi)+(1-\phi_d-\phi_f)+\sigma^*(1-\phi_f)\right]^2 \quad \text{(A.100)}$$

Therefore, the effect of increases in ϕ_d on the capital-labor ratio $\tilde{k}$ is always (both when Case 1 or Case 2 obtain) ambiguous when a currency board is in place.

Now I turn to the analysis of the effects of increases in ϕ_d on $\tilde{\chi}$ under a currency board. Differentiating equation (69) with respect to ϕ_d, I obtain the following expression

$$\frac{\partial \tilde{\chi}}{\partial \phi_d} = \left\{ \frac{\left[r(1-\lambda)(1-\alpha)-\alpha\pi\right]}{\lambda A(1-\alpha)} \right\} \left\{ \frac{\Lambda_1+\Lambda_2+\Lambda_3}{(1-\phi_d-\phi_f)^2\left[(1-\pi)x-\pi(1-\delta)\rho\right]^2} \right\}, \qquad \text{(A.101)}$$

where

$$\Lambda_1 \equiv (1-\phi_d-\phi_f)\left[(1-\pi)x-\pi(1-\delta)\rho\right]\frac{\partial\delta}{\partial\phi_d} = \begin{cases} <0, \text{ if Case 1 obtains} \\ >0, \text{ if Case 2 obtains} \end{cases}, \qquad \text{(A.102)}$$

$$\Lambda_2 \equiv (\delta-1)\left[(1-\pi)x-\pi(1-\delta)\rho\right] = \begin{cases} >0, \text{ if Case 1 obtains} \\ <0, \text{ if Case 2 obtains} \end{cases}, \quad \text{(A.103)}$$

and,

$$\Lambda_3 \equiv (\delta-1)(1-\phi_d-\phi_f)\pi\frac{\partial\left[(1-\delta)\rho\right]}{\partial\phi_d} < 0 \quad \text{in either Case 1 or Case 2.} \qquad \text{(A.104)}$$

Therefore, the effect of increases in ϕ_d on the share of net investment abroad, $\tilde{\chi}$, are ambiguous for either Case 1 or Case 2 under a currency board.

Effects of Increases in ϕ_f on $\tilde{k}$ and $\tilde{\chi}$ I first analyze the effects of increases in ϕ_f on the capital-labor ratio, $\tilde{k}$. Differentiating equation (68) with respect to ϕ_f, I obtain the following expression

$$\frac{\partial\tilde{k}}{\partial\phi_f} = \left\{\frac{\pi\tilde{k}^{2-\alpha}}{A(1-\alpha)\left[r(1-\lambda)(1-\alpha)-\alpha\pi\right]}\right\}\left\{\frac{\partial\left[(1-\delta)\rho\right]}{\partial\phi_f}\right\}. \qquad \text{(A.105)}$$

Using (A.53) as well as (A.43), I am able to determine that

$$\frac{\partial\left[(1-\delta)\rho\right]}{\partial\phi_f} = \frac{r(1+\sigma^*)^2+\sigma^*\left[1+\phi_f-(r-1)(\phi_d+\xi)\right]+\left[1-(r-1)(\phi_d+\xi)\right]}{(1-\phi_d-\phi_f)\Theta_3} > 0.$$
(A.106)

Thus, when Case 1 (Case 2) obtains, and a currency board is in place, the capital-labor ratio $\tilde{k}$ is decreasing (increasing) in ϕ_f.

Next, I turn to the analysis of the effects of increases in ϕ_f on the share of net investment abroad, $\tilde{\chi}$. Differentiating equation (69) with respect to ϕ_f, I obtain

$$\frac{\partial\tilde{\chi}}{\partial\phi_f}=\left\{\frac{\left[r(1-\lambda)(1-\alpha)-\alpha\pi\right]}{\lambda A(1-\alpha)}\right\}\left\{\frac{\Lambda_1^{'}+\Lambda_2^{'}+\Lambda_3^{'}}{(1-\phi_d-\phi_f)^2\left[(1-\pi)x-\pi(1-\delta)\rho\right]^2}\right\}, \quad \text{(A.107)}$$

where

$$\Lambda_1^{'}\equiv(1-\phi_d-\phi_f)\left[(1-\pi)x-\pi(1-\delta)\rho\right]\frac{\partial\delta}{\partial\phi_f}=\begin{cases}<0, \text{if Case 1 obtains}\\ >0, \text{if Case 2 obtains}\end{cases}, \quad \text{(A.108)}$$

$$\Lambda_2^{'}\equiv(\delta-1)\left[(1-\pi)x-\pi(1-\delta)\rho\right]=\begin{cases}>0, \text{if Case 1 obtains}\\ <0, \text{if Case 2 obtains}\end{cases}, \quad \text{(A.109)}$$

and

$$\Lambda_3^{'}\equiv(\delta-1)(1-\phi_d-\phi_f)\pi\frac{\partial\left[(1-\delta)\rho\right]}{\partial\phi_f}<0 \text{ in either Case Case 2.} \quad \text{(A.110)}$$

Thus, when a currency board is in operation and either Case 1 or Case 2 obtain, increases in ϕ_f have ambiguous effects on the share of net investment abroad, $\tilde{\chi}$.

Comparative Statics under a Pure Fixed Exchange Rate

Effects of increases in σ^* on $\tilde{k}$ and $\tilde{\chi}$ I start with the analysis of the effects of increases in σ^* on $\tilde{k}$. The steps are identical to the ones followed for the case of a currency board. Equation (A.69) still applies. However, using θ=ξ=0 in (A.70), I obtain

$$\frac{\partial\left[(1-\delta)\rho\right]}{\partial\sigma^*}=\frac{-r\phi_d+(\phi_d+\phi_f)(1-\phi_f)}{\left[(1-\phi_d-\phi_f)+\sigma^*(1-\phi_f)\right]^2}, \quad \text{(A.111)}$$

and, therefore

$$\frac{\partial\left[(1-\delta)\rho\right]}{\partial\sigma^*}>0\Leftrightarrow r<r_c\equiv\left(\frac{\phi_d+\phi_f}{\phi_d}\right)(1-\phi_f). \quad \text{(A.112)}$$

It follows from both (A.69) and (A.112) that when $r<r_c$ and Case 1 (Case 2) obtains, the capital-labor ratio $\tilde{k}$ is decreasing (increasing) in σ^*.

I next turn to the analysis of the effects of increases in σ^* on the share of net investment abroad, $\tilde{\chi}$. Equation (A.76) still applies, and using the fact that under a pure fixed exchange rate regime $\theta=\xi=0$, I obtain

$$\frac{\partial \tilde{\chi}}{\partial \sigma^*} = \left\{ \frac{\left[r(1-\lambda)(1-\alpha) - \alpha\pi \right]}{\lambda(1-\alpha)\left[(1-\pi)x - \pi(1-\delta)\rho\right]^2} \right\} \left\{ \frac{(1-\pi)x\phi_d - \pi(\phi_d + \phi_f)}{\left[(1-\phi_d - \phi_f) + \sigma^*(1-\phi_f\right]^2} \right\}. \quad \text{(A.113)}$$

Recall that I have previously defined $x_c \equiv \left(\frac{\pi}{1-\pi} \right) \left(\frac{\phi_d + \phi_f}{\phi_f} \right)$ and that $\frac{\partial \tilde{\chi}}{\partial \sigma^*} = 0$ when $x=x_c$. Then, when Case 1 obtains (A.78) holds, while when Case 2 obtains (A.79) holds. Now, I turn to analyze the conditions under which (A.78) and (A.79), respectively, hold. Again, I focus on situations where $r<r_c$. Notice that under a pure fixed exchange rate regime

$$\lim_{\sigma^* \to \infty} (1-\delta)\rho = \frac{r}{(1-\phi_d)}. \quad \text{(A.114)}$$

In addition, for $\lim_{\sigma^* \to \infty} f'\left(\tilde{k}\right)$ to be well-defined, when Case 1 (Case 2) obtains

$$\left(\frac{1-\pi}{\pi} \right) x < (>) \frac{r}{(1-\phi_d)} \quad \text{(A.115)}$$

must hold. It is straightforward to show that

$$\lim_{\sigma^* \to \infty} (1-\delta)\rho = \frac{r}{(1-\phi_f)} < \left(\frac{\phi_d + \phi_f}{\phi_d} \right) \Leftrightarrow r < r_c. \quad \text{(A.116)}$$

Notice that when Case 1 obtains,

$$\left(\frac{1-\pi}{\pi}\right)x < \frac{r}{(1-\phi_f)} < \frac{(\phi_d + \phi_f)}{\phi_f} \Rightarrow x < x_c. \quad \text{(A.117)}$$

Therefore, when Case 1 obtains and there is a pure fixed exchange rate regime in place, $\tilde{\chi}$ is increasing in σ^*. On the other hand, when Case 2 obtains

$$\left(\frac{1-\pi}{\pi}\right)x > \frac{r}{(1-\phi_d)}, \quad \text{(A.118)}$$

but also (A.116) holds. Then, when Case 2 obtains and a pure fixed exchange rate regime is in place, either $x<x_c$ or $x>x_c$ may be observed and, thus, $\tilde{\chi}$ can be either decreasing or increasing in σ^*, respectively.

Effects of Increases in *r* on $\tilde{k}$ and $\tilde{\chi}$ I start with the analysis of the effects of increases in r on $\tilde{k}$. Recall that, by differentiating equation (68) with respect to r, I obtained equation (A.86), which still holds. Using (A.42) as well as the fact that $\frac{\partial \delta}{\partial r} = 0$ under a pure fixed exchange rate regime, I obtain

$$\frac{\partial \tilde{k}}{\partial r} = \tilde{k}^{2-\alpha} \left\{ \frac{\pi[r(1-\lambda)(1-\alpha) - \alpha\pi](1-\delta)\frac{\partial \rho}{\partial r} + [(1-\pi)x - \pi(1-\delta)\rho]}{(1-\alpha)A[r(1-\lambda)(1-\alpha) - \alpha\pi]^2} \right\} \quad \text{(A.119)}$$
$$= \begin{cases} < 0, \text{if Case 1 obtains} \\ > 0, \text{if Case 2 obtains} \end{cases}.$$

Thus, when Case 1 (Case 2) obtains and a pure fixed exchange rate regime is in place, the capital-labor ratio $\tilde{k}$ is decreasing (increasing) in r.

I next turn to the analysis of the effects of increases in r on the share of net investment abroad $\tilde{\chi}$. Differentiating equation (69), again, using the fact that $\frac{\partial \delta}{\partial r} = 0$ under a pure fixed exchange rate regime, I obtain

$$\frac{\partial\tilde{\chi}}{\partial r}=\frac{\left[(\delta-1)(1-\alpha)\tilde{k}^{-\alpha}\dfrac{\partial\tilde{k}}{\partial r}\right]}{\lambda A(1-\alpha)(1-\phi_d-\phi_f)}=\begin{cases}>0, \text{if Case 1 obtains}\\ <0, \text{if Case 2 obtains}\end{cases}. \quad \text{(A.120)}$$

Thus, when Case 1 (Case 2) obtains, $\tilde{\chi}$ is increasing (decreasing) in r.

Effects of Increases in ϕ_d on $\tilde{k}$ and $\tilde{\chi}$ I start with the analysis of the effects of increases in ϕ_d on $\tilde{k}$, the capital-labor ratio when credit is rationed. Differentiating equation (68) with respect to ϕ_d and using the fact that θ=ξ=0 under a pure fixed exchange rate regime, I obtain

$$\begin{aligned}\frac{\partial\tilde{k}}{\partial\phi_d}&=\left\{\frac{\pi\tilde{k}^{2-\alpha}}{(1-\alpha)A\left[r(1-\lambda)(1-\alpha)-\alpha\pi\right]}\right\}\left\{\frac{\partial\left[(1-\delta)\rho\right]}{\partial\phi_d}\right\}\\&=\left\{\frac{\pi\tilde{k}^{2-\alpha}}{(1-\alpha)A\left[r(1-\lambda)(1-\alpha)-\alpha\pi\right]}\right\}\left\{\frac{(r-1)+\sigma^*(r-1+\phi_d)}{\left[(1-\phi_d-\phi_f)+\sigma^*(1-\phi_f)\right]^2}\right\}.\end{aligned} \quad \text{(A.121)}$$

Notice that $\frac{\partial\left[(1-\delta)\rho\right]}{\partial\phi_d}>0, \forall\sigma^*>\left(\frac{1}{r}-1\right)$. Thus, when Case 1 (Case 2) obtains and a pure fixed exchange rate regime is in place, $\tilde{k}$ is decreasing (increasing) in ϕ_d.

Now, I turn to the analysis of the effects of increases in ϕ_d on the share of net investment abroad $\tilde{\chi}$. Differentiating equation (69) in Chapter I with respect to ϕ_d, and again using the fact that θ=ξ=0, I obtain

$$\frac{\partial\tilde{\chi}}{\partial\phi_d}=\left\{\frac{\left[r(1-\lambda)(1-\alpha)-\alpha\pi\right]}{\lambda A(1-\alpha)}\right\}\left\{\frac{\Delta_1+\Delta_2}{(1-\phi_d-\phi_f)^2\left[(1-\pi)x-\pi(1-\delta)\rho\right]^2}\right\}, \quad \text{(A.122)}$$

where

$$\Delta_1 \equiv -\frac{\left[(1-\pi)x-\pi(1-\delta)\rho\right]\left(1-\phi_d-\phi_f\right)^2(1+\sigma^*)}{\left[(1-\phi_d-\phi_f)+\sigma^*(1-\phi_f)\right]^2} = \begin{cases} >0, \text{if Case 1 obtains} \\ <0, \text{if Case 2 obtains} \end{cases} \quad \text{(A.123)}$$

and

$$\Delta_2 \equiv -\frac{(1+\sigma^*)(1-\phi_d-\phi_f)^2\pi\left[(r-1)+\sigma^*(r-1+\phi_f)\right]}{\left[(1-\phi_f-\phi_f)+\sigma^*(1-\phi_f)\right]^2} < 0, \forall\sigma^* > \left(\frac{1}{r}-1\right). \quad \text{(A.124)}$$

Thus, when Case 1 obtains and a pure fixed exchange rate regime is in place, increases in ϕ_d have ambiguous effects on the share of net investment abroad in total savings, $\tilde{\chi}$. However, when Case 2 obtains, $\tilde{\chi}$ is decreasing in ϕ_d.

Effects of Increases in ϕ_f on $\tilde{k}$ and $\tilde{\chi}$ Again, I start with the analysis of the effects of increases in ϕ_f on the capital-labor ratio $\tilde{k}$. Differentiating equation (68) with respect to ϕ_f and using the fact that $\theta=\xi=0$ under a pure fixed exchange rate regime, I obtain

$$\frac{\partial\tilde{k}}{\partial\phi_f} = \left\{\frac{\pi\tilde{k}^{2-\alpha}}{(1-\alpha)A\left[r(1-\lambda)(1-\alpha)-\alpha\pi\right]}\right\}\left\{\frac{\partial\left[(1-\delta)\rho\right]}{\partial\phi_f}\right\} = \left\{\frac{\pi\tilde{k}^{2-\alpha}}{(1-\alpha)A\left[r(1-\lambda)(1-\alpha)-\alpha\pi\right]}\right\}\left\{\frac{r(1+\sigma^*)^2-\left[1+\sigma^*(1+\phi_d)\right]}{\left[(1-\phi_d-\phi_f)+\sigma^*(1-\phi_f)\right]^2}\right\}. \quad \text{(A.125)}$$

Notice that $\frac{\partial\left[(1-\delta)\rho\right]}{\partial\phi_f} > 0, \forall\sigma^* > \left(\frac{1}{r}-1\right)$. Thus, when Case 1 (Case 2) obtains and a pure fixed exchange rate regime is in place, $\tilde{k}$ is decreasing (increasing) in ϕ_f.

Next, I turn to the analysis of the effects of increases in ϕ_f on the share of net investment abroad $\tilde{\chi}$. Differentiating equation (69) with respect to ϕ_f, and again using the fact that θ=ξ=0, I obtain

$$\frac{\partial\tilde{\chi}}{\partial\phi_f}=\left\{\frac{\left[r(1-\lambda)(1-\alpha)-\alpha\pi\right]}{\lambda A(1-\alpha)}\right\}\left\{\frac{\Delta_1^{'}+\Delta_2^{'}}{(1-\phi_d-\phi_f)^2\left[(1-\pi)x-\pi(1-\delta)\rho\right]^2}\right\}, \quad \text{(A.126)}$$

where

$$\Delta_1^{'}\equiv-\frac{(1-\phi_d-\phi_f)^2(1+\sigma^*)^2\left[(1-\pi)x-\pi(1-\delta)\rho\right]}{\left[(1-\phi_d-\phi_f)+\sigma^*(1-\phi_f)\right]^2} \qquad \text{(A.127)}$$

and

$$\Delta_2^{'}\equiv\frac{(\delta-1)(1-\phi_d-\phi_f)\pi\left\{r(1+\sigma^*)^2-\left[1+\sigma^*(1+\phi_d)\right]\right\}}{\left[(1-\phi_d-\phi_f)+\sigma^*(1-\phi_f)\right]^2}<0,\forall\sigma^*>\left(\frac{1}{r}-1\right).$$

(A.128)

Thus, when Case 1 obtains and a pure fixed exchange rate regime is in place, the effect of increases in ϕ_f on $\tilde{\chi}$ is ambiguous. However, when Case 2 obtains and a pure fixed exchange rate regime is in place, $\tilde{\chi}$ is decreasing in ϕ_f.

5.3.3 When Does Credit Rationing Occur?

I start this section by proving Lemmas 10 and 11.

Proof of Lemma 10

Equation (A.41) implies that the steady-state interest rate on loans is monotonically increasing in σ^*. In addition, differentiating (A.41) with respect to σ^*, I obtain

$$\frac{\partial^2\rho}{\partial\left(\sigma^*\right)^2}=-\frac{2(\phi_d+\phi_f)}{(1-\phi_f-\phi_f)(1+\sigma^*)^3}<0. \qquad \text{(A.129)}$$

Both (A.41) and (A.129), as well as

$$0 < \lim_{\sigma^* \to \infty} \rho = \frac{r}{(1-\phi_d - \phi_f)} < \infty \qquad \text{(A.130)}$$

imply that the steady-state interest rate on loans is bounded above. In addition, by inspection of equation of equation (62), it is straightforward to deduct that ρ is independent of the policy parameters θ and ξ. Thus, the properties of ρ do not depend upon how the domestic money supply is backed in a fixed exchange rate regime. *Q.E.D.*

Proof of Lemma 11

(A.71), (A.74) and (A.112) prove this lemma. *Q.E.D.*

Next, I turn to the analysis of when does credit rationing occur for the different parameter configurations considered, i.e., for Case 1 and Case 2, respectively.

Case 1

Using condition (71), credit is rationed when Case 1 obtains if

$$\alpha[(1-\pi)x - \pi(1-\delta)\rho] < [r(1-\lambda)(1-\alpha) - \alpha\pi]\rho. \qquad \text{(A.131)}$$

From Lemmas 10 and 11 we know that ρ and $(1-\delta)\rho$ are both increasing in σ^*. In addition, both $[(1-\pi)x - \pi(1-\delta)\rho] < 0$ and $[r(1-\lambda)(1-\alpha) - \alpha\pi] < 0$ when Case 1 obtains. Therefore both the left-hand side and the right-hand side of (A.131) are negative and decreasing in σ^*. Thus, the correspondence between the presence of credit rationing and the foreign (and domestic) rate of inflation may be relatively complicated.

Case 2

Using condition (71), credit is rationed when Case 2 obtains if

$$\alpha[(1-\pi)x - \pi(1-\delta)\rho] > [r(1-\lambda)(1-\alpha) - \alpha\pi]\rho. \qquad \text{(A.132)}$$

Again, from Lemmas 10 and 11 we know that ρ and (1-δ)ρ are both increasing in σ^*. In addition, both $\left[(1-\pi)x-\pi(1-\delta)\rho\right]>0$ and $\left[r(1-\lambda)(1-\alpha)-\alpha\pi\right]>0$ when Case 2 obtains. Then, both the left-hand side and the right-hand side of (A.132) are positive. However, the left-hand side of (A.132) is decreasing in σ^* while the right-hand side is increasing in σ^*. Thus, if (A.132) holds or, equivalently, if credit rationing occurs, it will observed for low foreign (and domestic) inflation rates.

5.4 Technical Notes on the Local Stability Analysis in Section 3

5.4.1 The Dynamic System in a Walrasian Regime

Using equations (61) and (75), I obtain

$$\frac{\partial \tau_t}{\partial \tau_{t-1}} = -G_2 = -\frac{(1+\sigma^*)r(\theta\phi_d+\xi)-\phi_d}{(1-\theta\phi_d-\phi_f-\xi)(1+\sigma^*)}. \quad \text{(A.133)}$$

Next, I analyze local stability under a currency board and a pure fixed exchange rate, respectively.

Local Stability under a Currency Board

Notice that, under a currency board, given that θ=1 and $\xi\in$[0, (1-ϕ_d-ϕ_f)), we can rewrite (A.133) as

$$\frac{\partial \tau_t}{\partial \tau_{t-1}}\Big|_{(\theta=1,\xi)} = -G_2\left(\theta=1,\xi\right) = -\frac{(1+\sigma^*)r(\phi_d+\xi)-\phi_d}{(1-\phi_d-\phi_f-\xi)(1+\sigma^*)}. \quad \text{(A.134)}$$

Notice that $G_2>0, \forall\sigma^*>\left(\frac{1}{r}-1\right)$ and, therefore $\frac{\partial \tau_t}{\partial \tau_{t-1}}<0$ under a currency board. Also notice that $\frac{\partial G_2}{\partial \sigma^*}>0$, and therefore, local stability might depend on the foreign (and domestic) rate of inflation.

If $G_2 > 1$, then the solution to (75) is stable, but fluctuations will be observed. Notice, from (A.134), that

$$\text{If } r > \frac{(1-\phi_d - \phi_f - \xi)}{\xi} \Rightarrow G_2 > 1,\ \forall \sigma^* > \left(\frac{1}{r} - 1\right). \quad \text{(A.135)}$$

On the other hand,

$$\text{If } r < \frac{(1-\phi_d - \phi_f - \xi)}{\xi} \Rightarrow \begin{cases} G_2 > 1, \text{ for } \sigma^* < a \\ G_2 < 1, \text{ for } \sigma^* > a \end{cases}, \quad \text{(A.136)}$$

where

$$a \equiv \frac{\phi_d + (1-\phi_f) - (r+1)(\phi_d + \xi)}{(r+1)(\phi_d + \xi) - (1-\phi_f)}. \quad \text{(A.137)}$$

Thus, when a currency board is in place, it is possible to observe either $\left|\frac{\partial \tau_t}{\partial \tau_{t-1}} > 1\right|$ or $\left|\frac{\partial \tau_t}{\partial \tau_{t-1}} < 1\right|$. Finally, from equations (75) and (76), it can be easily inferred that any fluctuations observed in the government's fiscal position (τ_t) will be transmitted to net investment abroad.

Local Stability under a Pure Fixed Exchange Rate Regime

Under a pure fixed exchange rate, given that θ=ξ=0, we can rewrite (A.133) as

$$\frac{\partial \tau_t}{\partial \tau_{t-1}}\Big|_{(\theta=\xi=0)} = -G_2(\theta = \xi = 0) = \frac{\phi_d}{(1-\phi_f)(1+\sigma^*)} > 0, \forall \sigma^* > \left(\frac{1}{r} - 1\right). \quad \text{(A.138)}$$

Also notice that $\frac{\partial(-G_2)}{\partial \sigma^*} < 0$ and, therefore, as σ^* increases, the solution to (75) becomes more unstable when a pure fixed exchange rate regime is in place. Finally, notice that $|G_2|<1,\ \forall\sigma^* > Max\left\{\sigma^*_\delta, \left(\frac{1}{r}-1\right)\right\}$. Therefore, the solution to (75) is typically unstable under a pure fixed exchange rate regime, but no oscillations are observed.

5.4.2 The Dynamic System in a Credit Rationing Regime

Determinant of the Jacobian evaluated at the nontrivial steady-state

Recall that the Jacobian of the linearized system, evaluated at the nontrivial steady-state, is given by

$$j(\tilde{k},\tilde{\tau},\tilde{q}) = \begin{bmatrix} g_1 & g_2 & g_3 \\ g_1\left(\frac{P}{R}\right) & \left(\frac{S}{R}\right) & g_3\left(\frac{P}{R}\right) \\ 0 & 1 & 0 \end{bmatrix}. \qquad \text{(A.139)}$$

First, I find the matrix $(J-\eta I))$, where η represents an eigenvalue of J:

$$J-\eta I = \begin{bmatrix} g_1-\eta & g_2 & g_3 \\ g_1\left(\frac{P}{R}\right) & \left(\frac{S}{R}\right)-\eta & g_3\left(\frac{P}{R}\right) \\ 0 & 1 & -\eta \end{bmatrix}. \qquad \text{(A.140)}$$

Obviously, the characteristic equation is given by $Det(J-\eta I)=0$. After some steps, I am able to express the characteristic equation as

$$\eta\left\{(g_1-\eta)\left(\eta-\frac{S}{R}\right)+(g_1g_2+g_3)\left(\frac{P}{R}\right)\right\}=0, \quad \text{(A.141)}$$

and thus, one of the roots of this equation is η=0. The remaining roots solve the quadratic equation

$$\eta^2-\left[g_1+\left(\frac{S}{R}\right)\right]\eta-\left[(g_1g_2+g_3)\left(\frac{P}{R}\right)-g_1\left(\frac{S}{R}\right)\right]=0. \qquad \text{(A.142)}$$

Equation (A.142) leads in a very straightforward way to equation (86). Also, equations (87) and (88) follow in a very straightforward but algebraically intensive way from equations (82), (83), (84) and (85).

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Figure 1

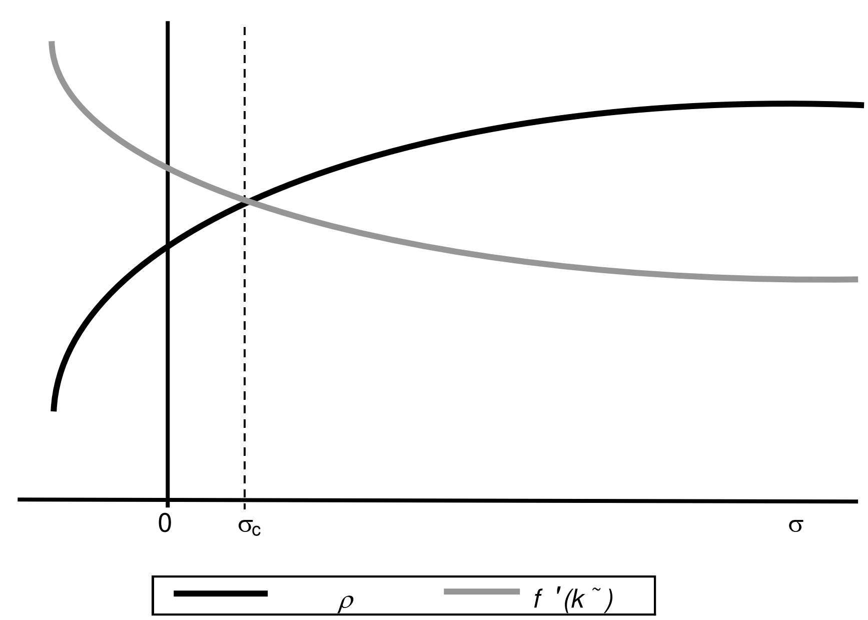

Figure 2a

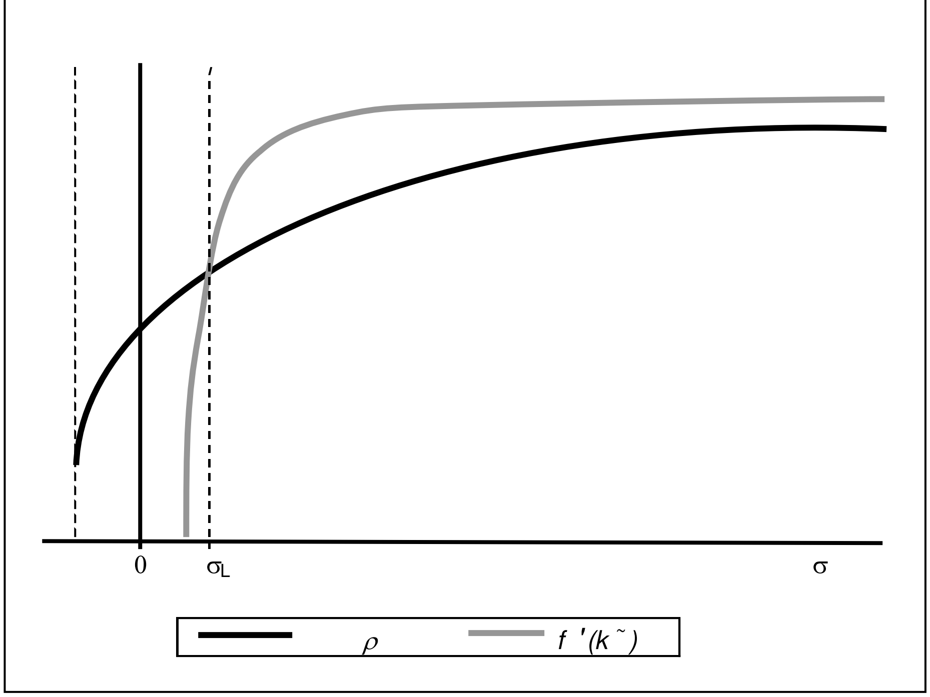

Figure 2b

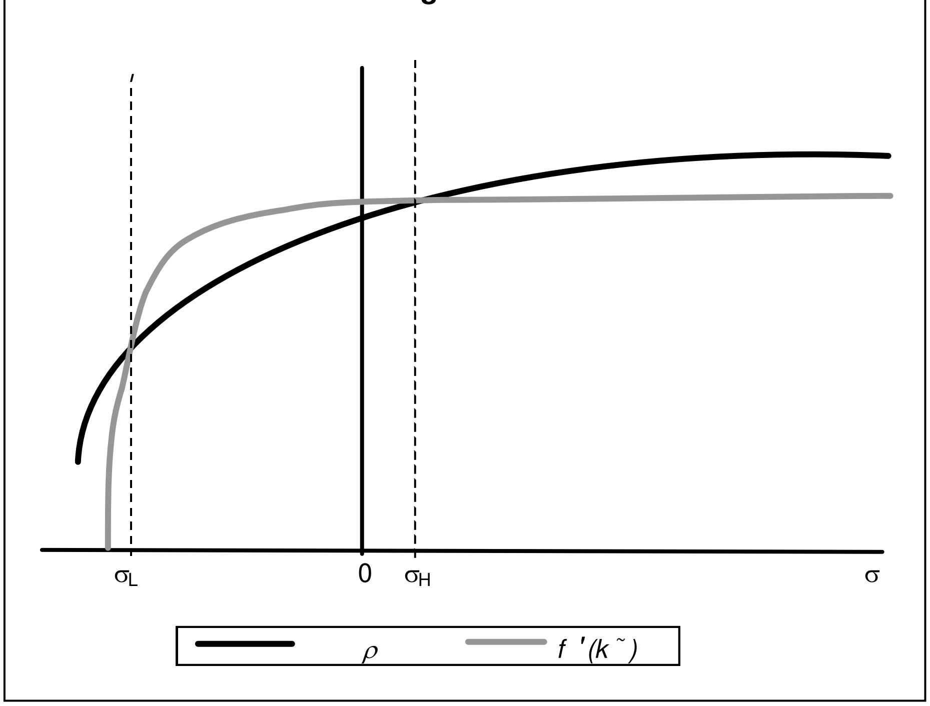

Figure 2c

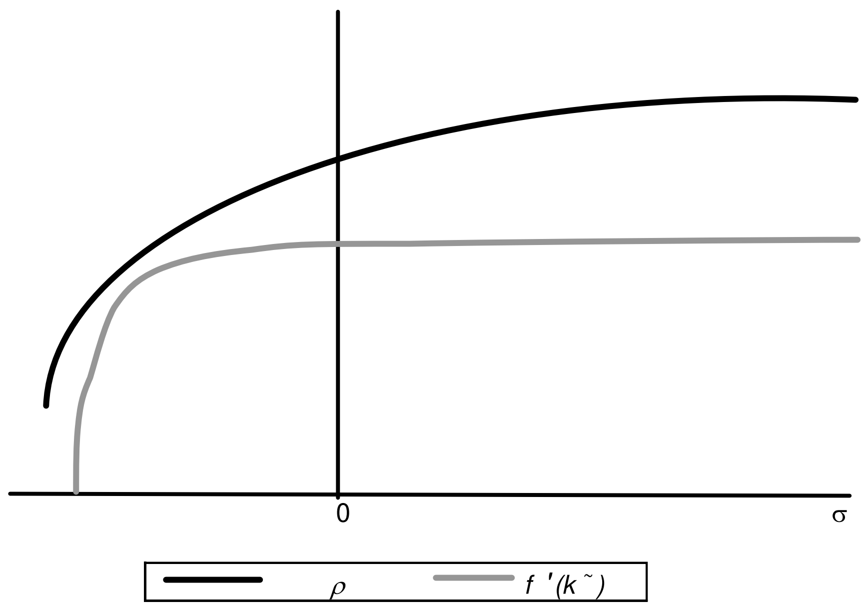

Figure 3
Perú: Equilibrium Law of Motion for the Capital-Labor Ratio in a Walrasian Regime

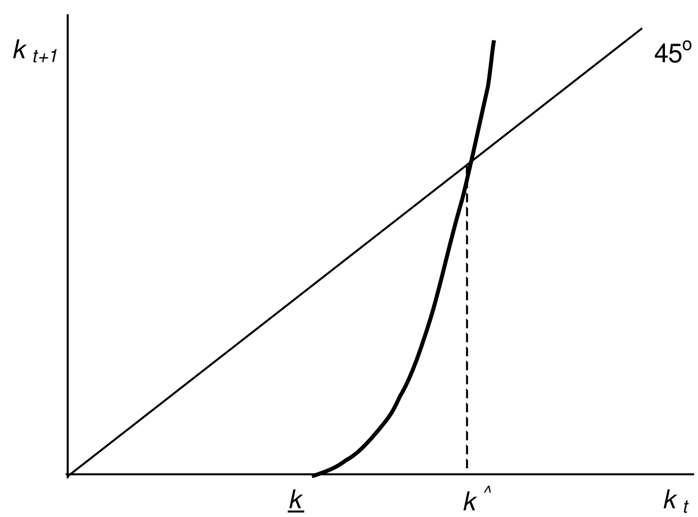

Figure 4
Perú: Phase Diagram
in a Credit Rationing Regime

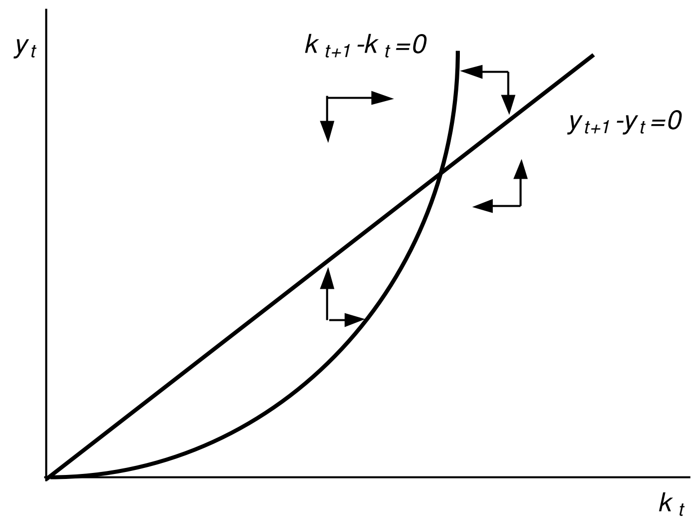

Figure 5a

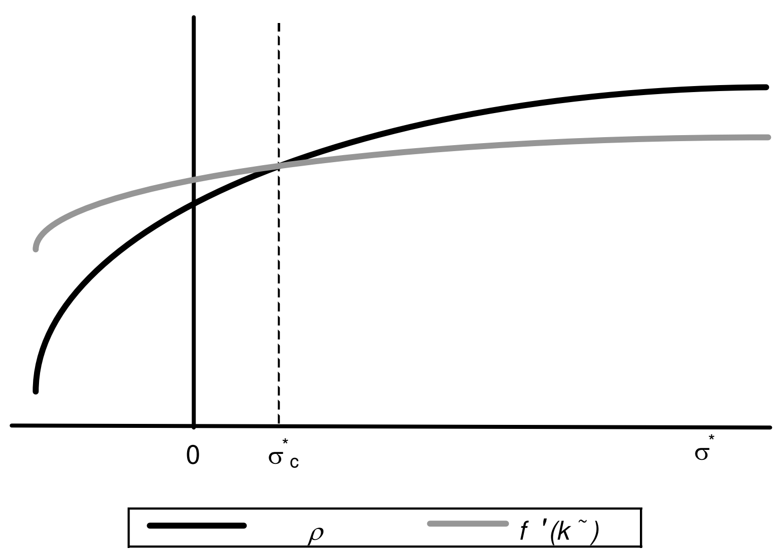

Figure 5b

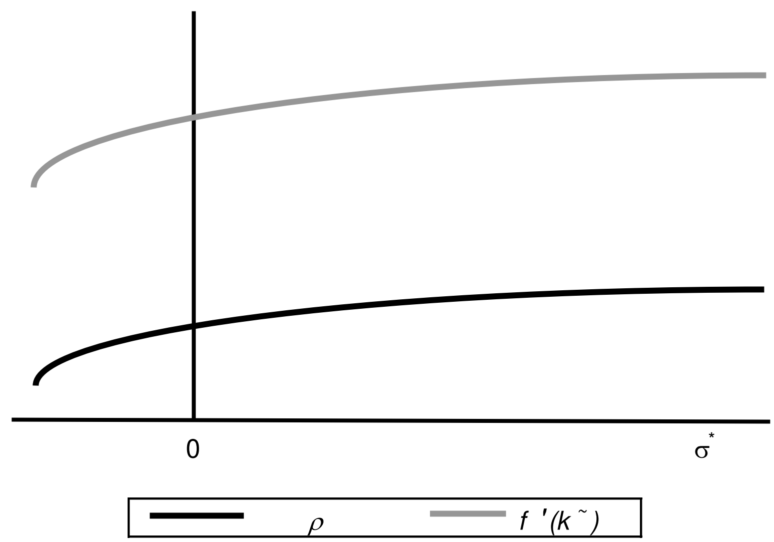

Figure 6a

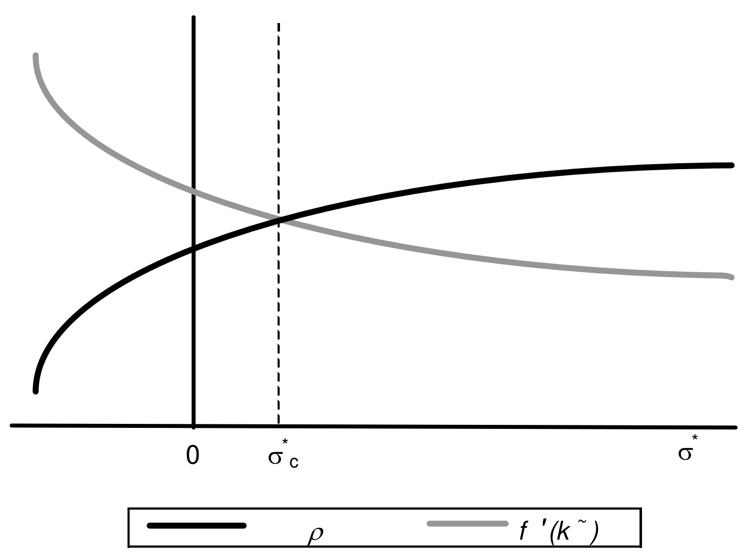

Figure 6b