Dynamic Effects of Oil Price Shocks and their Impact on the Current Account

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February 2009

Online at http://mpra.ub.uni-muenchen.de/16738/
MPRA Paper No. 16738, posted 11. August 2009 05:44 UTC
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Abstract

Our objective is to study the dynamic effects of an oil price shock on economic key variables and on the current account of a small open economy. To do this, we introduce time non-separable preferences in a standard model of a small open economy, where labor supply is endogenous and imported oil is used both as an intermediate input in production and as a consumption good. Using a plausible calibration of the model, we show that the changes in output and employment are quite small, and that the current account exhibits the J-curve property, both being in line with recent empirical evidence. After an oil price increase, the current account first deteriorates, and after some time it turns into surplus. We explain this non-monotonic behavior with agents’ reluctance to change their consumption expenditures, resulting in an initial trade balance deficit which causes the current account to deteriorate. Over time, with gradually falling expenditures, the trade balance improves sufficiently to turn the current account into surplus. The model thus provides a plausible explanation of recent empirical findings.

Keywords: oil price shocks, time non-separable preferences, current account dynamics

JEL classification: F32, F41,Q43

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† This research was funded by a grant from the Free University of Bozen-Bolzano. The paper was presented on the Latin American Meeting of the Econometric Society in Rio de Janeiro, Brazil, November 20 - 23, 2008, and on the PAESS’08 workshop held in Porto Alegre, Brazil, November 25 - 29, 2008, and has benefited from useful comments of participants. Of course, the usual disclaimer applies.
1 Introduction

Oil prices increased by 140 percent between 2003 and 2007, and prior to the summer of 2008, they reached peaks up to nearly 150 $/barrel. At that time some analysts where predicting that in the near future the oil price could reach 200 $/barrel, although the subsequent turmoil in world financial markets and the accompanying drop in the price of oil has lead to a scaling down of these predictions. But despite their recent dramatic reversal, oil prices are still substantially higher than they where a few years ago. With the rapid development of the BRIC economies and their growing claim on world resources, most economists expect higher oil prices to be a permanent reality and that they will continue to rise over the long term. There is thus a lot of concern of how oil price hikes affect the economy: How do output, investment, and consumption respond, and how do the balance of trade and the current account change? How much does a country suffer under high oil prices? On a personal level, people are concerned about how much they are affected by an oil price shock.

Of course, the price of imported raw materials such as oil has been a concern to economists since the 1970s, with the occurrence of supply shocks associated with the “oil crisis” of that period. This experience spawned substantial research, much of it motivated by the concern of stagflation, a situation in which the economy suffers inflation in conjunction with a decline in output, see, e. g., Corden (1975), Findlay and Rodriguez (1977), Obstfeld (1980), Sachs, Cooper, and Fischer (1981), Bruno and Sachs (1982), Sachs (1983), Golub (1983), Krugman (1983), Marion (1984), Marion and Svensson (1984), Svensson (1984), and Bhandari and Turnovsky (1984). This literature was almost entirely short run in nature, as particular attention was spent on the short run consequences of oil shocks and the appropriate policy reactions to deal with them. Virtually all of the models employed in these papers ignored the role of capital accumulation. One of the conclusions of this literature was that the macroeconomic impacts of oil price shocks depend crucially upon their specific nature.

Ongoing instability in the Middle East and the associated periodic dramatic movements in oil prices made sure that economists still pay attention to the macroeconomic consequences of oil price shocks, see, e. g., Barsky and Kilian (2004) for a recent review. The recent oil price hike dramatically confirms the lasting importance of this issue. But contrary to what one may think at a first glance, a lot of the recent research shows that the oil price shocks occurred in the recent past years have had relatively small effects on real economic activity compared to the experience in the 1970s and 1980s. For example, the loss in output ranges between 1 and 5 percent, depending on the country and on the specific nature of the shock, see Schmidt and Zimmermann (2005, 2007), OECD (2004), Parry and Darmstadter (2004), Dhawan and
Jeske (2006), Nordhaus (2007), and Blanchard and Galí (2007). One reason for this is that the energy intensity of production in developed economies has declined about 50 percent, making an economy less vulnerable to oil price shocks.

Despite the fact that there is now a large literature investigating the macroeconomic impacts of oil price shocks, focusing on output, employment, inflation, and interest rates, surprisingly a much smaller theoretical and empirical literature has studied the impact of oil price shocks on an economy’s external accounts (trade balance, current account, and net foreign asset position). Early work of Agmon and Laffer (1978) based on the monetary approach to the balance of payments found that the trade balance of industrialized countries deteriorated markedly immediately following an oil price increase, but after that initial deterioration these trade balances improved again. Moreover, the trade balance adjustments where almost exclusively in non-oil trade. The current account thus deteriorated sharply following the shock and after some time reverted back to more normal deficits and surpluses. However, the source of the reversal of trade balance and current account deficits was far from being clear. More formal work done by Marion (1984), Marion and Svensson (1984) and Svensson (1984) did not lead to clear-cut results. Instead, ambiguous reactions of the trade balance and the current account to an oil price shock where derived.

Recently, Rebucci and Spatafora (2006) found that oil price shocks have a marked but relatively short-lived impact on current accounts and a noticeable effect on the net foreign asset position of countries. Kilian, Rebucci, and Spatafora (2007) estimate that the net foreign asset position of advanced oil importing countries (with the exception of the US) tends to decline after an oil market specific demand increase, although the decline is not always statistically significant. For middle income countries as well as for Latin America and emerging Asia, they discovered that the current account deteriorates significantly in response to oil supply shocks. Current account deteriorations relative to base-line levels are also reported by the OECD (2004). Gruber and Kamin (2007) point out that changes in the oil trade balance will not have a one-for-one impact on the current account if the non-oil trade balance also responds to oil price shocks.

From an open economy perspective it is therefore of importance to identify how and the channels through which oil price shocks affect not only output and employment, but also trade and thus the balance of payments. Hence, in this paper we address the effects of oil price shocks on internal and external economic performance of a small open economy. The model we shall employ is a variant of the class of model discussed in detail by Turnovsky (2002). We augment that model in several important and new directions:
First, we include a imported good, oil,¹ which is used (i) as a consumption good (e.g. fuel), and (ii) as an intermediate input in production of traded output in the tradition of Sen (1991) who studied the effects of an oil price increase in a small open economy populated with individually optimizing agents, however, without international capital movements.

Second, instead of restricting the production side of the economy to a Cobb-Douglas production structure, we use the more general constant elasticity of substitution (CES) production function approach. The reason for doing this is twofold: (i) there is a lot of empirical evidence that the elasticity of substitution between productive inputs is less than unity, in particular if oil (or energy) is included in the production function, see, e.g., Kemfert (1998) and Van der Werf (2007). (ii) a Cobb-Douglas production function would not be appropriate for the analysis of macroeconomic effects of an oil price shock, as it allows oil to be asymptotically replaced by the capital stock, see Edenhofer, Bauer, and Kriegler (2005). We will take account of the relatively small share of oil in GDP reported by, e.g., Parry and Darmstadter (2004), OECD (2004), Nordhaus (2007), by assigning oil a very low weight in the production function.

Third and most important, we include a reference consumption stock into the representative agent’s utility function, which reflects time non-separable preferences. The addition of habits is a significant augmentation of the standard model and leads to much more plausible results. A lot of empirical evidence has confirmed the importance of time non-separable preferences, see, e.g., Fuhrer (2000), Di Bartolomeo, Rossi, and Tancioni (2005), and Sommer (2007). Gruber (2004) shows that the inclusion of habits significantly improves the empirical performance of the intertemporal current account model, as current account forecasts derived from that model better match the volatility of actual current accounts. Willman (2003) proves that the habit formation hypothesis is strongly supported by the data, and Carroll, Slacalek, and Sommer (2008) find strong evidence of excess smoothness in consumption, supporting therefore the inclusion of habits into the model. As we will show, the small open economy model without habit formation (i.e. with time separable preferences) predicts an improving current account after an unfavorable oil price shock, which is clearly at odds with empirical evidence, whereas the introduction of consumption habits allows the model to match the empirical response of the current account to oil price shocks. We will restrict our attention on the “outward-looking” agent, whose reference stock is based on the average level of consumption in the economy, see Carroll, Overland, and Weil (1997). This restriction keeps the model more tractable, and moreover, the difference between assuming that the reference stock is formed by looking outwards or inwards (i.e. by basing the reference stock on the agent’s own past consumption) is relatively small, although

¹One can also think in terms of imported energy, as oil price movements and price movements of other fossil sources of energy are strongly correlated, see Asche, Gjalberg, and Völker (2003).
it does depend upon the specific shock that hits the economy, as Alvarez-Cuadrado, Monteiro, and Turnovsky (2004) show in a closed economy growth framework.

Of course, the introduction of a reference stock and time non-separable preferences comes at a price. The model becomes too intractable to be fully studied analytically. We therefore will apply numerical simulations to trace the time paths of economic key variables, using a plausible calibration. We also will conduct some sensitivity analysis with respect to the weight of the reference consumption stock in the agent’s utility function, the speed of adjustment of the reference consumption stock, and the oil share in GDP.

In spirit of a large empirical and theoretical literature, we shall focus on a permanent increase in the oil price. Of course, the exact nature of the recent oil price hike is unknown or at least highly uncertain, but both market expectations and an assessment of medium-term oil market fundamentals suggest that a considerable proportion of the shock will be permanent in nature.

There are several key results of our analysis that we want to stress at the outset. The most important finding is that the introduction of time non-separable preferences gives rise to plausible current account dynamics upon an unfavorable oil price shock. The current account dynamics we derive are almost entirely driven by the goods (non oil) trade balance, whose response reflects agents’ reluctance to change their consumption expenditures. After an oil price increase, the current account shows the J-curve property by first deteriorating for a while and then improving. In line with recent empirical evidence, the reactions of other economic key variables like output and employment are moderate. The model thus is able to explain both empirical current account dynamics and empirically small economic effects of oil price shocks.

Price movements of imported goods change the countries terms of trade, i.e. the relative price of its exports in terms of its inputs. Our analysis is therefore related to the broader literature on the effects of terms of trade fluctuations on economic performance, dating back to the seminal contributions of Laursen and Metzler (1950), and Harberger (1950), who predicted that a deterioration in the terms of trade would reduce real income, inducing a reduction in savings and thus a worsening of the current account. The original Laursen-Metzler-Harberger effect was purely static and gave rise to an extensive literature that re-examined the effects of terms of trade shocks in an intertemporal framework.2 Without exception, these papers abstract from the presence of imported inputs. The terms of trade shocks are due to fluctuations in the relative price of goods, and therefore represent pure demand effects. From this standpoint, the paper can be viewed as extending this literature to the important case where fluctuations, being

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due to import price caused terms of trade changes, originate both on the supply and the demand side.

The paper is organized as follows. Section 2 sets out the basic structure of the model. In section 3 we derive the macroeconomic equilibrium dynamics, whereas the steady state is discussed in section 4. Section 5 conducts a numerical analysis. In section 6 some sensitivity analysis is performed. Section 7 summarizes the main findings. A brief appendix derives the equilibrium dynamics.

2 Analytical Framework

We build upon the one-sector open economy model described in Turnovsky (2002), which we modify and extend in several ways. We abstract from endogenous growth, but include a foreign import good, oil. The economy produces a traded good, \( Y \), that can be consumed, invested, or exported. The imported input, oil, the relative price of which in terms of traded output is \( p \), is used as an intermediate input in production, \( Z \), and as a consumption good, \( M \). The economy is small in the sense that the relative price of oil is determined in the world market. We shall assume that \( p \) and thus the terms of trade, \( 1/p \), remain constant over time and analyze the dynamic effects of a one-time unanticipated permanent increase in \( p \). Furthermore, we assume that the economy is populated with a large number of identical agents,\(^3\) and that each individual \( i \) is endowed with one unit of time, a fraction, \( l_i \), can be allocated to leisure, and the reminder, \( 1 - l_i \), to labor supply. The population grows at the exogenously given constant rate \( \dot{N}/N \equiv n \), where \( N \) denotes the size of population.

Each individual produces traded output, \( Y_i \), using labor, \( 1 - l_i \), imported oil, \( Z_i \), and capital, \( K_i \), according to the constant elasticity of substitution (CES) production function\(^4\)

\[
Y_i = A \left[ \alpha_1 (1 - l_i)^{-\rho} + \alpha_2 Z_i^{-\rho} + \alpha_3 K_i^{-\rho} \right]^{-1/\rho} \tag{1a}
\]

where \( A \) is a scale parameter, and

\[
\alpha_1 + \alpha_2 + \alpha_3 = 1, \quad -1 \leq \rho < \infty
\]

The constant elasticity of substitution is \( \sigma \equiv 1/(1 + \rho) \). The representative agent derives utility from leisure, \( l_i \), and consumption of both the domestically produced good, \( C_i \), and of imported oil, \( M_i \). Moreover, at any point in time, he derives utility from the comparison of the current

\(^3\)Thus, \( X_i \) denote per capita magnitudes, whereas \( X = NX \).  
\(^4\)There is a lot of empirical evidence that the elasticity of substitution between the production factors labor, energy (oil), and capital is positive, but less than unity, see recently Van der Werf (2007).
consumption bundle relative to a reference consumption bundle, denoted by \( H_i \). As in Carroll, Overland, and Weil (1997, 2000), the representative household’s objective is to maximize the intertemporal iso-elastic utility function

\[
\int_0^\infty \frac{1}{\epsilon} \left[ (C_i^{\nu} M_i^{1-\nu})^{\theta} H_i^{-\gamma} \right]^{\epsilon} e^{-\beta t} dt, -\infty < \epsilon < 1, 0 \leq \gamma < 1, \theta \geq 0, 0 \leq \nu \leq 1
\] (1b)

where \( (C_i^{\nu} M_i^{1-\nu}) \) is a linearly homogenous subutility function, which aggregates the domestic good and oil, the share of which is \( 1 - \nu \), into a consumption bundle. The elasticity of leisure (labor) is denoted by \( \theta \). Following Ryder and Heal (1973), the imposed restriction on \( \gamma \) guarantees non-satiation in utility. The long-run intertemporal elasticity of substitution (IES) w. r. t. the aggregator function (i.e. the consumption bundle \( C_i^{\nu} M_i^{1-\nu} \)) is equal to \( 1/(1 - (1 - \gamma)\epsilon) \). In the conventional case of time separable preferences (\( \gamma = 0 \)), the ISE is \( 1/(1 - \epsilon) \). Empirical evidence overwhelmingly suggests that the ISE is smaller than unity, hence we restrict our attention on \( \epsilon < 0 \). In this case, the long-run ISE under time non-separable preferences exceeds the conventional ISE.

The representative agent is outward-looking, as the reference stock \( H_i \) depends on the economy-wide average consumptions of all agents, \( \bar{C} = (1/N) \sum_{i=1}^{N} C_i \), and \( \bar{M} = (1/N) \sum_{i=1}^{N} M_i \), see Carroll, Overland, and Weil (1997, 2000). Since agents are atomistic, they ignore the effect of their individual consumption decisions on the time path of the reference stock, taking it as exogenous. Hence, the reference stock \( H_i \) is an externality. It evolves according to

\[
\dot{H}_i = \zeta \left( \bar{C}^{\nu} \bar{M}^{1-\nu} - H_i \right)
\] (1c)

The speed of adjustment, \( \zeta \), parameterizes the relative importance of recent consumption levels in determining the reference stock. The weight of the consumption bundle over the last ten years in determining the reference stock is given by \( 1 - \exp(-10\zeta) \).\(^5\) The higher \( \zeta \), the more weight is given to recent consumption, the faster the reference stock adjusts, and the lower is the level of persistence in habits. The half-time of the reference stock’s adjustment to a change in the average consumption bundle is \( \frac{1}{\zeta} \ln 0.5 \).\(^5\) See Carroll, Overland, and Weil (2000).

The representative agent accumulates physical capital, \( K_i \). Investment, \( I_i \), is associated with installation costs. We therefore assume a Hayashi (1982) type investment adjustment cost

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\(^5\)This can be seen by noting that integration of equation (1c) gives \( H_i(t) = \zeta \int_{t}^{\infty} \bar{C}^{\nu} \bar{M}^{1-\nu} \exp[\zeta(\tau - t)] d\tau \). Performing the same integration for the weighting function over the last ten years, \( \zeta \int_{t-10}^{t} \exp[\zeta(\tau - t)] d\tau \), yields the result.
function, resulting in a total investment cost function

$$\Phi(I_i, K_i) = I_i + h \frac{I_i^2}{2K_i} = I_i \left(1 + h \frac{I_i}{2K_i}\right)$$

(1d)

where adjustment costs are convex in $I_i$ and proportional to the rate of investment per unit of installed capital, $I_i/K_i$. Letting $\delta$ denote the rate of depreciation of the capital stock, the net rate of capital accumulation per agent, taking population growth into account, is given by

$$\dot{K}_i = I_i - (n + \delta)K_i$$

(1e)

In addition, domestic agents have access to a perfect world capital market, allowing them to accumulate world bonds, denominated in terms of the traded good and paying a fixed given world interest rate, $r$, yielding a net return to individual agents of $(r - n)$. The representative agent’s flow budget constraint, expressed in terms of the traded good, is

$$\dot{B}_i = (r - n)B_i + Y_i - C_i - pM_i - pZ_i - \Phi(I_i, K_i)$$

(1f)

where $B_i > 0$ denotes his (net) holdings of foreign traded bonds. According to (1f), to the extent that the agent’s income from production, $Y_i$, plus net interest, $(r - n)B_i$, exceeds his expenditures on consumption, $C_i + pM_i$, on the imported input, $pZ_i$, and on investment, $\Phi(I_i, K_i)$, he accumulates bonds. For simplicity, we abstract from taxes and from a government.

The agent maximizes intertemporal utility (1b) by choosing the rates of consumptions $C_i$, $M_i$, investment $I_i$, the share of time devoted to leisure $l_i$, oil input $Z_i$, and the rates of bonds and capital accumulation, subject to (1c) and (1f), and the given initial stocks of capital and traded bonds, $K_i(0) = K_{i0}$ and $B_i(0) = B_{i0}$, respectively, leading to the following optimality conditions:

$$\nu C_i^{\varepsilon\mu-1}M_i^{\varepsilon(1-\nu)}H_i^{-\varepsilon\gamma}l_i^\theta = \lambda_i$$

(2a)

$$\dot{}(1-\nu)C_i^{\varepsilon\mu}M_i^{\varepsilon(1-\nu)-1}H_i^{-\varepsilon\gamma}l_i^\theta = p\lambda_i$$

(2b)

$$\theta C_i^{\varepsilon\mu}M_i^{\varepsilon(1-\nu)}H_i^{-\varepsilon\gamma}l_i^\theta-1 = \lambda_i \frac{\partial Y_i}{\partial l_i}$$

(2c)

$$\frac{\partial Y_i}{\partial Z_i} = p$$

(2d)

$$1 + h \frac{I_i}{K_i} = q_i$$

(2e)

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6 In case of $B_i < 0$, the agent is a net debtor.

7 Note that (1c) does not appear in the maximization problem of the outward-looking agent, because the reference stock is treated as given and represents thus an externality.
\[ \frac{\lambda_i}{\lambda_i} = \beta + n - r \quad (2f) \]
\[ \frac{\partial Y_i}{\partial K_i} + \frac{\dot{q}_i}{q_i} + \frac{(q_i - 1)^2}{2h q_i} - \delta = r \quad (2g) \]
\[ \lim_{t \to \infty} \lambda_i B_i e^{-\beta t} = \lim_{t \to \infty} q_i \lambda_i K_i e^{-\beta t} = 0 \quad (2h) \]

where \( \lambda_i \) is the shadow value of wealth in the form of internationally traded bonds, and \( q_i \) is the value of capital in terms of the (unitary) price of foreign bonds, and can be interpreted as Tobin’s \( q \). Conditions (2a) and (2b) are the usual static optimality conditions. They equate the marginal utility of consumption \((C_i \text{ respectively } M_i)\) to the marginal utility of wealth in terms of the traded good and the imported good, respectively. Equation (2c) equates the marginal utility of leisure to the shadow value of its opportunity cost, the real wage (i.e. the marginal product of labor. Equation (2d) states that the marginal product of oil in production has to be equal to the oil price in terms of the domestically produced good, \( p \), and (2e) equates the marginal cost of an additional unit of (new) capital to the market price of capital. Marginal productivities are given by
\[
\frac{\partial Y_i}{\partial (1 - l_i)} = A^{-\rho} a_1 \left( \frac{Y_i}{1 - l_i} \right)^{1+\rho}, \quad \frac{\partial Y_i}{\partial Z_i} = A^{-\rho} a_2 \left( \frac{Y_i}{Z_i} \right)^{1+\rho}, \quad \frac{\partial Y_i}{\partial K_i} = A^{-\rho} a_3 \left( \frac{Y_i}{K_i} \right)^{1+\rho}
\]

The dynamic optimality conditions with respect to \( B_i \), equation (2f), and \( K_i \), (2g), lead to the usual no-arbitrage conditions, equating the rates of return on consumption \( \beta - \dot{\lambda}_i / \lambda_i \) to the net interest rate \((r - n)\), and the rate of return on domestic capital to the world interest rate. The rate of return on domestic capital comprises four terms. The first is the “dividend yield”, the second the capital gain, the third reflects the fact that a benefit of a higher capital stock is to reduce the installation costs (which depend on \( I_i/K_i \) associated with new investment, whereas the fourth element represents a loss due to the depreciating capital stock. Finally, in order to ensure that the agent’s intertemporal budget constraint is met, the transversality conditions (2h) must hold.

Dividing (2a) by (2b), we get the standard optimality condition that the marginal rate of substitution between the domestic consumption good and imported oil (for consumption) has to be equal to the relative price of \( C_i \) in terms of oil, \( 1/p \), i.e. the terms of trade.

\[
\frac{\nu C_i^{-1}}{(1 - \nu) M_i^{-1}} = \frac{1}{p} \iff p M_i = \frac{1 - \nu}{\nu} C_i.
\]

Defining the agent’s consumption expenditure as \( E_i \equiv C_i + p M_i \), we can solve for the two
consumption rates as functions of expenditure

\[
C_i = \nu E_i \quad (3a)
\]

\[
M_i = \frac{1 - \nu}{p} E_i \quad (3b)
\]

Dividing (2c) by (2a) gives the well-known optimality condition that the marginal rate of substitution between leisure and consumption (of the domestically produced good) has to be equal to the real wage. Rearranging this condition yields

\[
\frac{C_i}{Y_i} = \frac{\nu \alpha_1}{\theta A^\rho} \left( \frac{l_i}{1 - l_i} \right) \left( \frac{Y_i}{1 - l_i} \right)^\rho \quad (3c)
\]

or, in terms of expenditure

\[
\frac{E_i}{Y_i} = \frac{\alpha_1}{\theta A^\rho} \left( \frac{l_i}{1 - l_i} \right) \left( \frac{Y_i}{1 - l_i} \right)^\rho \quad (3c')
\]

This equation states that the expenditure-output ratio depends both on the leisure-labor ratio and the output-labor ratio. The conditional factor demand for oil, given production, can be derived from (2d) and reads

\[
Z_i = \left( \frac{pA^\rho}{\alpha_2} \right)^{-\frac{1}{1+\rho}} Y_i \quad (3d)
\]

It follows that the higher the relative price of oil, the lower its usage in the production of a given quantity of output. Using (3d), we can eliminate \( Z_i \) in the production function (1a) to get

\[
Y_i = A \left[ \alpha_1(1 - l_i)^{-\rho} + \alpha_3 K_i^{-\rho} \right]^{-1/\rho} \left[ 1 - \alpha_2 \left( \frac{p}{\alpha_2 A} \right)^{\frac{\rho}{1+\rho}} \right]^{-1/\rho} \quad (1a')
\]

similarly, the marginal product of capital can be expressed as

\[
\frac{\partial Y_i}{\partial K_i} = A \alpha_3 \left[ 1 - \alpha_2 \left( \frac{p}{\alpha_2 A} \right)^{\frac{\rho}{1+\rho}} \right]^{(1+\rho)/\rho} \left( \alpha_1(1 - l_i)^{-\rho} K_i^\rho + \alpha_3 \right)^{-(1+\rho)/\rho} \quad (4)
\]

3 Macroeconomic equilibrium

In macroeconomic equilibrium, all static and dynamic optimality conditions (2) must hold continuously for all agents. Moreover, in steady-state equilibrium of this economy all aggregate quantities grow at the constant rate \( n \), whereas the market price of capital, \( q_i \), and the labor allocation, \( l_i \), remain constant. Since all agents are identical, it is convenient to express the dynamics in per-capita (or average) magnitudes, which are constant in steady-state equilibrium.
Note that because all agents are identical, $\bar{C} = C_i$ and $\bar{M} = M_i$. Since in steady-state the agent’s consumption rates have to remain constant, (2f) requires the marginal utility of wealth to remain constant over time to guarantee an interior equilibrium. Hence, this imposes the knife-edge condition $\beta = r - n$, see Turnovsky (2002), which makes the steady state dependent on the economy’s initial state. The equation of motion for the capital stock follows from (1e), using (2e), as

$$\frac{\dot{K}_i}{K_i} = \frac{q_i - 1}{h} - \delta - n$$  \hspace{5cm} (5)

Using (4), The equation of motion (2g) for $q_i$ can be written as

$$A\alpha_1 \left[ 1 - \alpha_2 \left( \frac{p_i}{\pi^2 A} \right)^{\tau \pi} \right] (1+\rho)^{\rho} \left[ \alpha_1 (1 - l_i)^{-\rho} K_i^\rho + \alpha_3 \right]^{-\rho} q_i - \frac{\dot{H}_i}{H_i} - A_1(l_i, K_i) A_2(l_i, K_i) \frac{\dot{K}_i}{K_i}$$

The dynamic equation for leisure is derived in the appendix\(^8\) as

$$\dot{l}_i = A_1(l_i, K_i) c_\gamma \frac{\dot{H}_i}{H_i} - A_1(l_i, K_i) A_2(l_i, K_i) \frac{\dot{K}_i}{K_i}$$  \hspace{5cm} (7)

where $A_1(l_i, K_i), A_2(l_i, K_i)$ are defined in the appendix. This equation reveals that $l_i$ is a function of $H_i$ and $K_i$. It thus introduces thus a linear dependence into the dynamic system, see Turnovsky (2002). The differential equation for the reference stock is derived in the appendix too, and reads

$$\dot{H}_i = \zeta \left[ \left( \frac{1 - \nu}{\nu} \right) \frac{1 - \nu}{\rho} \frac{\nu A^\nu}{\theta} \left( \frac{l_i}{1 - l_i} \right) \left( \frac{1}{1 - l_i} \right) ^\rho \left[ \frac{\alpha_1 (1 - l_i)^{-\rho} + \alpha_3 K_i^{-\rho}}{1 - \alpha_2 \left( \frac{p_i}{\pi^2 A} \right)^{\tau \pi}} \right]^{1+\rho} - H_i \right]$$  \hspace{5cm} (8)

Equations (5) - (8) describe the economy’s internal dynamics.

Finally, the external dynamics of net foreign assets, $B_i$, are governed by equation (1f), which — noting $C_i + pM_i \equiv E_i$, $\Phi(I_i, K_i) = \frac{q_i^{n-1}}{2n} K_i$, and using equations (3d) and (1a') — can be expressed in terms of $B_i, K_i, l_i,$ and $q_i$ as

$$\dot{B}_i = (r - n) B_i + \left( 1 - p \left( \frac{p A^\rho}{\alpha_2} \right)^{\tau \pi} \right) \frac{A \left[ \alpha_1 (1 - l_i)^{-\rho} + \alpha_3 K_i^{-\rho} \right]^{-1/\rho} - \frac{\theta}{A^\rho} \left( \frac{p}{\pi^2 A} \right)^{\tau \pi}}{1 - \alpha_2 \left( \frac{p_i}{\pi^2 A} \right)^{\tau \pi}}$$

$$\dot{B}_i - \alpha_1 \theta A^\rho \left( \frac{l_i}{1 - l_i} \right) \left( \frac{1}{1 - l_i} \right) ^\rho \left[ \frac{\alpha_1 (1 - l_i)^{-\rho} + \alpha_3 K_i^{-\rho}}{1 - \alpha_2 \left( \frac{p_i}{\pi^2 A} \right)^{\tau \pi}} \right]^{1+\rho} - \frac{\dot{q}_i^2 - 1}{2h} K_i$$

Because the evolution of $K_i, H_i, \text{and} q_i$ is independent from $B_i$, we can solve the dynamics sequentially by first deriving the solution for the internal dynamics and second for the external dynamics.\(^8\)

\(^8\)A detailed appendix, containing also the linearization procedure, is available from the author upon request.
dynamics. To do this, we linearize the dynamic equations (5), (6), and (8), around the steady state, the values of which are denoted by tildes, noting that according to equation (7) up to a linear approximation the distances of \( l_i, K_i, \) and \( H_i \) from their steady states are related by

\[
l_i - \bar{l}_i = F_1(\bar{l}_i, \bar{H}_i, \bar{K}_i) \left( H_i - \bar{H}_i \right) - F_2(\bar{l}_i, \bar{H}_i, \bar{K}_i) \left( K_i - \bar{K}_i \right)
\]

where

\[
F_1(\bar{l}_i, \bar{H}_i, \bar{K}_i) \equiv \frac{c\gamma A_1(\bar{l}_i, \bar{K}_i)}{H_i}, \quad F_2(\bar{l}_i, \bar{H}_i, \bar{K}_i) \equiv \frac{A_1(\bar{l}_i, \bar{K}_i) A_2(\bar{l}_i, \bar{K}_i)}{K_i}
\]

Performing the linearization, we obtain in matrix form

\[
\begin{pmatrix}
\dot{\bar{K}}_i \\
\dot{H}_i \\
\dot{q}_i
\end{pmatrix} = \begin{pmatrix}
a_{21} & a_{22} & 0 \\
a_{31} & a_{32} & (r - n)
\end{pmatrix} \begin{pmatrix}
0 & \bar{K}_i/h \\
0 & H_i - \bar{H}_i \\
q_i - \bar{q}_i
\end{pmatrix}
\]

where

\[
a_{21} \equiv \frac{\epsilon \theta + (\epsilon - 1) \left(1 + \frac{(\epsilon + \rho)\alpha K_{K_i}^{-\rho} \left(\frac{\bar{H}_i}{\bar{K}_i} - 1\right)}{\alpha_1 (1 - l_i)^{-\rho + \alpha_3 K_i^{-\rho}}} \right)}{\alpha_1 (1 - l_i)^{-\rho + \alpha_3 K_i^{-\rho}}}
\]

\[
a_{22} \equiv \frac{\epsilon \theta + (\epsilon - 1) \left(1 + \frac{(\epsilon + \rho)\alpha K_{K_i}^{-\rho} \left(\frac{\bar{H}_i}{\bar{K}_i} - 1\right)}{\alpha_1 (1 - l_i)^{-\rho + \alpha_3 K_i^{-\rho}}} \right)}{\alpha_1 (1 - l_i)^{-\rho + \alpha_3 K_i^{-\rho}}}
\]

\[
a_{31} \equiv \frac{(\epsilon(1 + \theta) - 1) A_{01} \alpha_{13} (1 + \rho) \left(1 - \alpha_2 \left(\frac{\bar{H}_i}{\bar{K}_i} \right) \frac{a_{11} K_{K_i}^{-(\rho + 1)}}{(1 - l_i)^{-\rho + \alpha_3 K_i^{-(\rho + 1)}}} \right)}{\alpha_1 (1 - l_i)^{-\rho + \alpha_3 K_i^{-\rho}}}
\]

\[
a_{32} \equiv \frac{\epsilon(1 + \theta) - 1 + \frac{(\epsilon - 1) (1 + \rho) \alpha K_{K_i}^{-\rho} \left(\frac{\bar{H}_i}{\bar{K}_i} - 1\right)}{\alpha_1 (1 - l_i)^{-\rho + \alpha_3 K_i^{-\rho}}} \left(1 - \frac{l_i}{\bar{l}_i} \right)}{\alpha_1 (1 - l_i)^{-\rho + \alpha_3 K_i^{-\rho}}}
\]

It is straightforward to show that, provided \( \epsilon < 0 \), what we have assumed, the system (11) has two negative eigenvalues and one positive eigenvalue. We cannot rule out the possibility of conjugate complex roots, in this case the real part of them is negative. In the following, we will focus on real roots, but allow for the possibility of conjugate complex roots in our simulations.
In case of real eigenvalues, the stable solution of system (11) is

\[
\begin{pmatrix}
K_i(t) - \bar{K}_i \\
H_i(t) - \bar{H}_i \\
q_i(t) - \bar{q}_i
\end{pmatrix} = c_1 \begin{pmatrix}
\frac{1}{\mu_1 - a_{21}} \\
\mu_1 \beta_i \\
K_i
\end{pmatrix} e^{\mu_1 t} + c_2 \begin{pmatrix}
\frac{1}{\mu_2 - a_{22}} \\
\mu_2 \beta_i \\
K_i
\end{pmatrix} e^{\mu_2 t}
\]

(12)

with

\[
\mu_1 < \mu_2 < 0 < \mu_3
\]

and where the \(c_i\) are constants to be determined by initial conditions on \(K_i(0) = K_{i0}\) and \(H_i(0) = H_{i0}\). Imposing these, we get

\[
c_1 = \frac{(K_i(0) - \bar{K}_i)\omega_{22} - (H_i(0) - \bar{H}_i)}{\omega_{22} - \omega_{21}}
\]

\[
c_2 = \frac{(H_i(0) - \bar{H}_i) - \omega_{21}(K_i(0) - \bar{K}_i)}{\omega_{22} - \omega_{21}}
\]

where \(\omega_{jk}\) denotes the \(j\)-th element in eigenvector \(k\), \(\omega_k = (1, a_{21}/(\mu_k - a_{22}), \mu_k h/\bar{K}_i)\). Unfortunately, the system is too complex to determine the signs of the eigenvectors analytically and to compare them. Therefore, we will utilize numerical simulations.

Linearizing the external dynamics (9), using (10), gives

\[
\dot{B}_i = (r - n) (B_i - \bar{B}_i) - \frac{\bar{q}_i \bar{K}_i}{K_i} (q_i - \bar{q}_i) + \Gamma_1 \left( K_i - \bar{K}_i \right) + \Gamma_2 \left( H_i - \bar{H}_i \right)
\]

where\(^9\)

\[
\Gamma_1 \equiv \left( \frac{\partial \dot{B}_i}{\partial K_i} \right) F_1(\cdot), \quad \Gamma_2 \equiv \left( \frac{\partial \dot{B}_i}{\partial l_i} \right) F_1(\cdot)
\]

Inserting the stable solutions for \((K_i - \bar{K}_i), (H_i - \bar{H}_i)\), and \((q_i - \bar{q}_i)\) yields after rearranging

\[
\dot{B}_i - (r - n) (B_i - \bar{B}_i) = c_1 \Omega_1 e^{\mu_1 t} + c_2 \Omega_2 e^{\mu_2 t}
\]

(13)

where

\[
\Omega_1 \equiv \left[ \Gamma_1 - \bar{q}_i \mu_1 + \Gamma_2 \frac{a_{21}}{\mu_1 - a_{22}} \right], \quad \Omega_2 \equiv \left[ \Gamma_1 - \bar{q}_i \mu_2 + \Gamma_2 \frac{a_{21}}{\mu_2 - a_{22}} \right]
\]

\(^9\)The partial derivatives are given as

\[
\frac{\partial B_i}{\partial K_i} = -\frac{\bar{q}_i^2}{2h} + \frac{a_3 Y_i}{K_i} \left[ a_1 (1 - \bar{L}_i)^{-\rho} + a_3 \bar{K}_i^{-\rho} \right]^{-1} \bar{K}_i^{-\rho} \left[ \left( 1 - \rho \frac{p \bar{A}^{\rho}}{a_2} \right) \bar{l}_i - (1 + \rho) \frac{a_1}{\bar{A}^{\rho}} \left( \bar{l}_i \left( \frac{1}{1 - \bar{L}_i} \right) \bar{Y}_i^{\rho} \right) \right]
\]

\[
\frac{\partial B_i}{\partial l_i} = a_1 \left( \frac{\bar{Y}_i}{1 - \bar{L}_i} \right)^{1 + \rho} \left[ \frac{p \left( \frac{\bar{A}^{\rho}}{a_2} \right) - 1}{\bar{A}^{\rho}} - 1 \right] + \frac{a_1}{\alpha (1 - \bar{L}_i)^{-\rho} + a_3 \bar{K}_i^{-\rho}} - \frac{1}{\bar{A}^{\rho}} \left[ (1 + \rho) \left( \frac{\bar{l}_i}{1 - \bar{L}_i} \right) \bar{Y}_i^{\rho} \right] \right]
\]
Integrating (13) and applying the transversality conditions (2h), the stable solution for traded bonds is

$$B_i(t) - \tilde{B}_i = \frac{c_1\Omega_1}{\mu_1 + n - r} e^{\mu_1 t} + \frac{c_2\Omega_2}{\mu_2 + n - r} e^{\mu_2 t}$$  \hspace{1cm} (14)$$

whereas the transversality conditions require

$$B_i(0) - \tilde{B}_i = \frac{c_1\Omega_1}{\mu_1 + n - r} + \frac{c_2\Omega_2}{\mu_2 + n - r}$$  \hspace{1cm} (15)

which is the agent’s intertemporal solvency condition. It is exactly this equation which makes the steady state dependent on the initial values of $B_i(0)$, $H_i(0)$, and $K_i(0)$. Equations (12) and (14) completely describe the dynamics of the economy (per capita). We now turn to the steady state.

4 Steady state

The steady-state is defined by $\dot{Y}_i = \dot{K}_i = \dot{H}_i = \dot{q}_i = \dot{l}_i = \dot{B}_i = 0$. We get the following steady-state relationships

$$\left( \frac{1 - \nu}{\nu p}\right)^{1 - \nu} \tilde{C}_i = \tilde{H}_i$$  \hspace{1cm} (16a)

$$\tilde{C}_i = \nu \tilde{E}_i$$  \hspace{1cm} (16b)

$$\tilde{M}_i = \frac{1 - \nu}{p} \tilde{E}_i$$  \hspace{1cm} (16c)

$$\tilde{q}_i = 1 + h(n + \delta)$$  \hspace{1cm} (16d)

$$\tilde{Y}_i = A \left[ \alpha_1 (1 - \tilde{l}_i)^{-\rho} + \alpha_2 \tilde{Z}_i^{-\rho} + \alpha_3 \tilde{K}_i^{-\rho} \right]^{-1/\rho}$$  \hspace{1cm} (16e)

$$\tilde{C}_i = \tilde{Y}_i \left[ \frac{\alpha_1 (1 - \tilde{l}_i)}{\theta A} \right] \left( \frac{\tilde{l}_i}{1 - \tilde{l}_i} \right) \left( \frac{\tilde{Y}_i}{1 - \tilde{q}_i} \right)^{\rho}$$  \hspace{1cm} (16f)

$$\tilde{Z}_i = \left( \frac{p A^p}{\alpha_2} \right)^{-\frac{1}{1 + \rho}} \tilde{Y}_i$$  \hspace{1cm} (16g)

$$r = \frac{A \alpha_3 \left[ 1 - \alpha_2 \left( \frac{p}{\theta A^p} \right)^{-\frac{1}{1 + \rho}} \right]^{(1 + \rho)/\rho} \left[ \alpha_1 (1 - \tilde{l}_i)^{-\rho} \tilde{K}_i^\rho + \alpha_3 \right]^{-(1 + \rho)/\rho} + (\tilde{q}_i - 1)^2}{2 h \tilde{q}_i} - \delta$$  \hspace{1cm} (16h)

$$(r - n) \tilde{B}_i + \left[ 1 - p \left( \frac{p A^p}{\alpha_2} \right)^{-\frac{1}{1 + \rho}} \right] \tilde{Y}_i - \frac{\alpha_1}{\theta A^p} \frac{l_i}{1 - l_i} \left( \frac{1}{1 - l_i} \right)^{\rho} \tilde{Y}_i^{1 + \rho} - \frac{\tilde{q}_i^2}{2 h} \tilde{K}_i = 0$$  \hspace{1cm} (16i)

$$B_i(0) - \tilde{B}_i = \frac{c_1\Omega_1}{\mu_1 + n - r} + \frac{c_2\Omega_2}{\mu_2 + n - r}$$  \hspace{1cm} (16j)
where we note that \( c_1 \) and \( c_2 \) depend on \( H_{i0}, K_{i0} \) and \( \tilde{H}, \tilde{K} \). These ten equations jointly determine the steady-state values \( \tilde{C}_i, \tilde{M}_i, \tilde{E}_i, \tilde{q}_i, \tilde{Y}_i, \tilde{\tilde{Z}}_i, \tilde{\tilde{H}}_i, \tilde{\tilde{K}}_i, \tilde{l}, \tilde{B}_i \).

The following comments can be made: (16a) determines the steady-state level of the reference stock, given \( \tilde{C}_i \). Equations (16b) and (16c) relate steady-state consumption expenditures \( \tilde{E}_i \) and consumption of the domestically produced traded good \( \tilde{C}_i \) and oil \( \tilde{M}_i \). Given the level of consumption expenditures, the higher the oil price, the lower oil consumption. Equation (16d) determines the steady-state market price of installed capital. The higher the adjustment cost parameter \( h \), and the higher population growth \( n \) and the depreciation rate \( \delta \), the higher the steady-state value of Tobin’s \( q \). Equation (16e) gives steady-state production. Equation (16f) is the optimality condition (3c); it relates steady-state consumption of the domestic good and leisure to output. Equation (16g) gives the steady-state level of imported input (oil) as a function of steady-state output. Given output, the higher the price of the imported input, \( p \), the lower its usage in production. Equation (16h) is the no-arbitrage condition for capital, requiring that in the long run, the rate of return on capital (the marginal product of capital, valued at its market price, the gain from reducing adjustment cost via investment, and the loss due to depreciation) has to be equal to the interest rate, i.e. the rate of return on traded bonds. Together with an unchanged \( \tilde{q} \), this condition requires that the steady-state marginal productivity of capital has to remain constant, implying that output and the capital stock change by the same percentage amount. Equation (16i) is the long-run zero current account condition. It states that in steady state the interest income on bonds, corrected by population growth, has to finance the trade balance, which can be split up into the non-oil trade balance

\[
\text{Non-oil TB} = \tilde{Y}_i - \frac{\tilde{q}_i^2 - 1}{2h} \tilde{K}_i - \frac{\alpha_1 \nu}{\theta A^\rho} \frac{\tilde{l}_i}{1 - \tilde{l}_i} \left( \frac{1}{1 - \tilde{l}_i} \right)^\rho \tilde{Y}_i^{1+\rho} \\
\Phi(\tilde{l}_i, \tilde{K}_i)
\]

and the oil trade balance

\[
\text{Oil balance} = - \left[ p \left( \frac{pA^\rho}{\alpha_2} \right)^{-\frac{1}{1+\rho}} \tilde{Y}_i + \frac{\alpha_1 (1 - \nu)}{\theta A^\rho} \frac{\tilde{l}_i}{1 - \tilde{l}_i} \left( \frac{1}{1 - \tilde{l}_i} \right)^\rho \tilde{Y}_i^{1+\rho} \right].
\]

Of course, the oil balance is always negative, as the country does not produce oil. Because the interest rate is exogenous, (16i) requires that the overall trade balance and the stock of bonds have to change by the same percentage amount. Finally, equation (16j) is the agent’s intertemporal solvency condition. It links the initial stocks of bonds, capital and habits to the steady state in a way that the agent remains solvent. The satisfaction of (16j) is achieved by
an appropriate initial adjustment in consumption expenditure \( E_i(0) \) and thus \( C_i(0) \) and \( M_i(0) \).

Note that in steady-state all aggregate magnitudes \( X \equiv NX_i \) grow at the common and constant rate \( n \), whereas the fraction of time allocated to leisure and the market price of capital remain constant.

### 5 Numerical analysis of an oil price shock

#### 5.1 Benchmark calibration

Because the model is too complex to calculate the shape of time paths, we refer to numerical simulations. We calibrate the model to reproduce some key features of a set of OECD countries, e.g., Germany, France or Italy, being hit by an oil price shock.\(^{10}\) Table 1 summarizes the parameters upon which our simulations are based. Empirical evidence on the elasticity of substitution in production \( (\sigma) \) is not unique. While Edenhofer, Bauer, and Kriegler (2005) work with \( \sigma = 0.4 \), Van der Werf (2007) estimated \( \sigma \) for different countries in the range between 0.2 and 0.6, and Kemfert (1998) reports elasticities in the range between zero and one for Germany. Therefore, we chose an intermediate value and set \( \rho = 1/3 \), which gives \( \sigma = 0.75 \). The parameter on the production function for capital \( \alpha_3 = 0.4 \) is noncontroversial, whereas the weight of oil \( \alpha_2 \) is a crucial parameter for the magnitude of adverse supply side effects of an oil price hike. \( \alpha_2 = 0.004 \) is chosen in a way that, together with the oil share in the consumption bundle, \( 1 - \nu = 0.008 \), the ratio of oil imports to output equals 0.0227, and the ratio of oil consumption to oil input equals 0.429.\(^{11}\) The world interest rate, \( r \), and the rate of time preference,
Table 2: Base equilibrium

<table>
<thead>
<tr>
<th>$Y_i$</th>
<th>$K_i$</th>
<th>$l_i$</th>
<th>$Z_i$</th>
<th>$q_i$</th>
<th>$E_i$</th>
<th>$C_i$</th>
<th>$M_i$</th>
<th>Stable eigenvalues</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4614</td>
<td>1.0208</td>
<td>0.6854</td>
<td>0.0073</td>
<td>1.75</td>
<td>0.3889</td>
<td>0.3858</td>
<td>0.0031</td>
<td>-0.1507 -0.0638</td>
</tr>
</tbody>
</table>

$qK_i/Y_i$, $E_i/Y_i$, $B_i/Y_i$, Oil share in $Y_i$, $M_i/Z_i$, $GTB_i$, $OTB_i$, $TB_i$

3.87 0.84 0.27 0.0226 0.4239 0.00545 0.01045 -0.005

$GTB_i$, $OTB_i$, and $TB_i$ denote the goods trade balance (exclusive of oil), the oil trade balance and the overall trade balance, respectively.

$\beta$, are set equal to 0.04, a value which is non-controversial, see, e.g., Karayalcin (2003) and Willman (2003). The rate of depreciation equals 0.05, a value that is usually assumed, see, e.g., Alvarez-Cuadrado, Monteiro, and Turnovsky (2004) and Chatterjee and Turnovsky (2006). The population growth rate $n$ is set equal to zero. On the one hand, this has the advantage that the aggregate equilibrium is stationary, and on the other hand this reflects the fact that in the recent past years population in many countries has not grown at all. The initial stock of bonds is chosen in a way to yield a net foreign asset-GDP ration of approximately 0.25.\(^\text{12}\)

The elasticity of leisure, $\theta$, is the key determinant of the equilibrium labor–leisure allocation and has been set to ensure that this is empirically plausible. A value of 1.75 accords with the standard value in the business cycle literature and yields an equilibrium fraction of time devoted to leisure of around 0.7, consistent with the empirical evidence, see, e.g., Turnovsky (2004). The choice of adjustment costs is less obvious and $h = 15$ lies in the consensus range of 10 to 16.\(^\text{13}\)

The initial relative price of oil is normalized to unity.

The critical parameters pertain to the relative importance of the reference stock, $\gamma$, and the speed of which it is adjusted, $\zeta$. Carroll, Overland, and Weil (2000) set the adjustment speed of habits equal to 0.2, an assumption we shall adopt. This corresponds to a half-time of the reference stock’s adjustment of 3.47 years or, equivalently, to a weight of the consumption bundle over the last ten years of 0.865. Regarding $\gamma$, they work with values of 0.25, 0.5, and 0.75.

\(^{12}\)E.g., in 2006, the ratio was 0.28 for Germany, 0.06 for France, -0.21 for Austria, -0.06 for Italy, -0.80 for Portugal, and -0.83 for Greece. [Calculations from the author, based on data from the IMF.] However, the economic effects of oil price shocks in the calibrated model are quite insensitive to changes in the initial stock of bonds.

\(^{13}\)Ortigueira and Santos (1997) show that the speed of convergence is sensitive to $h$, and choose $h = 16$ on the grounds that it generates a speed of convergence of around 2 percent per annum, consistent with much empirical evidence. Auerbach and Kotlikoff (1987) assume $h = 10$, and recognize that this is at the low value of estimates, while Barro and Sala-i-Martin (2003) propose a somewhat larger value.
Fuhrer (2000) estimates $\gamma$ in the range of 0.8 to 0.9, and McCallum and Nelson (1999) calibrate their model with $\gamma = 0.8$. More recently, Willman (2003) estimated values for Germany up to 0.924. Gruber (2004) analyzes Canada, France, Italy, Japan, the Netherlands, Spain, the UK, and the US, and estimates very high values for $\gamma$, too. Di Bartolomeo, Rossi, and Tancioni (2005) estimate $\gamma$ for G7 countries. They find values in the range of 0.6 to 0.8, where the lowest (0.61) was found for Germany, and the highest (0.818) for Italy. For the US, Sommer (2007) estimates $\gamma$ in the range of 0.7. Carroll, Slacalek, and Sommer (2008) find strong evidence of habit formation with the value of $\gamma$ typically ranging between 0.6 and 0.8. For Germany, they find $\gamma = 0.66$, see their table 1. We therefore choose $\gamma = 0.6$ as the benchmark, and we will conduct some sensitivity analysis with respect to $\gamma$ and $\zeta$.\footnote{There is some difficulty in translating empirical estimates of $\gamma$, which are based on discrete time models, to our continuous time model.}

Calibrating the model with the parameters summarized in table 1 gives the initial base equilibrium shown in table 2. The consumption-output ratio is 0.84, and about 69 (31) percent of time is allocated to leisure (labor). The capital coefficient equals 3.87. Note that the baseline equilibrium is independent of the particular values of the preference parameters $\gamma$ and $\zeta$.

### 5.2 Conventional preferences

Before discussing the adjustments in the model with time non-separable preferences, it is useful to run the model for the case of conventional time separable preferences. This is achieved by setting $\gamma = 0$, so that the dynamics decouple and the evolution of the reference stock becomes irrelevant. The base equilibrium remains the same, only the two stable eigenvalues change and become -0.2 and -0.0593, respectively. The reference stock adjusts with a speed of 20 percent, whereas the capital stock, $q$, and the stock of traded bonds monotonically converge to the steady state with an adjustment speed of 5.9 percent.

Assume now that the oil price $p$ rises unexpectedly and permanently about 100 percent from 1 to 2. The short-run and long-run changes of economic key variables are reported in the first column of table 3.

On impact, the increase in the oil price induces producers to cut back their oil usage by around 41 percent. This in turn lowers the marginal product of labor and hence, given the real wage, labor demand. Consumers reduce their expenditures about 1.37 percent, because of the wealth effect. The reduction in consumption increases the marginal rate of substitution between consumption and leisure. Given the real wage rate, households increase their labor supply. In sum, the increase in labor supply exceeds the reduction in labor demand, and labor increases by 0.175 percent, whereas the real wage falls. Given the initial capital stock, output drops about
Table 3: Increase of $p$ from 1 to 2 — sensitivity analysis

<table>
<thead>
<tr>
<th></th>
<th>Oil share = 0.0226</th>
<th>Oil share = 0.03505</th>
<th>Oil share = 0.05047</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma = 0$</td>
<td>$\gamma = 0.6$</td>
<td>$\gamma = 0.8$</td>
</tr>
<tr>
<td>$\Delta Y(0)%$</td>
<td>-0.796</td>
<td>-1.105</td>
<td>-1.695</td>
</tr>
<tr>
<td>$\Delta \tilde{Y}%$</td>
<td>-1.656</td>
<td>-1.452</td>
<td>-2.236</td>
</tr>
<tr>
<td>$\Delta \tilde{K}%$</td>
<td>-1.656</td>
<td>-1.452</td>
<td>-2.236</td>
</tr>
<tr>
<td>$\Delta(1 - l(0))%$</td>
<td>0.175</td>
<td>-0.279</td>
<td>-0.435</td>
</tr>
<tr>
<td>$\Delta(1 - \tilde{l})%$</td>
<td>-0.333</td>
<td>-0.126</td>
<td>-0.198</td>
</tr>
<tr>
<td>$\Delta Z(0)%$</td>
<td>-41.013</td>
<td>-41.197</td>
<td>-41.548</td>
</tr>
<tr>
<td>$\Delta \tilde{Z}%$</td>
<td>-41.524</td>
<td>-41.403</td>
<td>-41.868</td>
</tr>
<tr>
<td>$\Delta E(0)%$</td>
<td>-1.368</td>
<td>-0.977</td>
<td>-1.489</td>
</tr>
<tr>
<td>$\Delta \tilde{E}%$</td>
<td>-1.616</td>
<td>-1.709</td>
<td>-2.622</td>
</tr>
<tr>
<td>$\Delta GTB(0)%$</td>
<td>61.545</td>
<td>10.952</td>
<td>7.129</td>
</tr>
<tr>
<td>$\Delta \tilde{GTB}%$</td>
<td>-4.976</td>
<td>16.787</td>
<td>11.139</td>
</tr>
<tr>
<td>$\Delta OTB(0)%$</td>
<td>12.216</td>
<td>12.075</td>
<td>12.028</td>
</tr>
<tr>
<td>$\Delta \tilde{OTB}%$</td>
<td>11.424</td>
<td>11.567</td>
<td>11.630</td>
</tr>
<tr>
<td>$\Delta TB(0)%$</td>
<td>-41.548</td>
<td>13.298</td>
<td>22.500</td>
</tr>
<tr>
<td>$\Delta \tilde{TB}%$</td>
<td>28.776</td>
<td>5.877</td>
<td>-4.351</td>
</tr>
<tr>
<td>$\Delta B%$</td>
<td>28.776</td>
<td>5.877</td>
<td>8.859</td>
</tr>
<tr>
<td>$\Delta \tilde{B}%$</td>
<td>-2.051</td>
<td>-3.227</td>
<td>-5.003</td>
</tr>
<tr>
<td>$\Delta W(0)%$</td>
<td>-1.959</td>
<td>-2.346</td>
<td>-3.642</td>
</tr>
<tr>
<td>long-run oil share</td>
<td>0.025660</td>
<td>0.025640</td>
<td>0.025631</td>
</tr>
<tr>
<td>half-time $K$</td>
<td>11.69</td>
<td>9.71</td>
<td>8.71</td>
</tr>
<tr>
<td>time minimum $B$</td>
<td>—</td>
<td>4.75</td>
<td>4.77</td>
</tr>
<tr>
<td>stable eigenvalues</td>
<td>-0.2</td>
<td>-0.1507</td>
<td>-0.1506</td>
</tr>
<tr>
<td></td>
<td>-0.0593</td>
<td>-0.0638</td>
<td>-0.0636</td>
</tr>
</tbody>
</table>

α₂ = 0.004, ν = 0.992
α₂ = 0.007, ν = 0.987
α₂ = 0.011, ν = 0.98
γ = 0.6
0.8 percent. The effects on welfare reported are equivalent variation measures, calculated as the percentage change in the permanent flow of consumptions $C_i$ and $M_i$ necessary to equate the initial level of welfare to what it would be following the shock.\footnote{We apply the method shown in Alvarez-Cuadrado, Monteiro, and Turnovsky (2004). Initial welfare means the welfare at instant time zero and refers to the utility function, whereas overall (long-run) welfare refers to intertemporal welfare as given by the welfare integral (1b).} Initial welfare of the representative agent falls by 2.05 percent.

Because the reduction in oil input, the marginal product of capital falls, and $q_i$ drops, which in turn reduces investment expenditures. The reductions in consumption of the domestically produced good and investment expenditure outweigh the reduction in output by far, hence the goods trade balance sharply improves. On the other hand, the oil balance deteriorates, because the value of oil imports increases. Because the improvement of the goods trade balance outweighs the deteriorated oil balance by far, the overall trade balance (which was negative in steady state) improves by 41.55 percent. Hence, the economy starts running a current account surplus, accumulating bonds, and decumulates its capital stock,\footnote{Graphically, in $(K_i, B_i)$-space the transition is characterized by a negatively sloped line.} and the transition to the new steady state is monotonic, as it is usual in the standard model of a small open economy, see Turnovsky (1997, ch. 3).

Note that the half-time of capital stock adjustment is roughly 11.75 years. During transition, consumption expenditures on both goods are continually reduced, and labor and oil input fall, as firms cut back employment and oil usage due to the reducing capital stock. Output falls monotonically. Because investment expenditures recover during transition and further reductions in consumption expenditures are moderate compared to output reduction (the expenditure-output ratio increases) the goods trade balance starts deteriorating immediately after its impact improvement, whereas the oil balance slightly improves due to small, but ongoing cut backs in oil imports. The overall trade balance thus starts deteriorating, slowing down the accumulation of traded bonds.

Compared to the base equilibrium, in the new steady state output and the capital stock have fallen by 1.66 percent, and labor drops by 0.33 percent. Oil input is reduced by 41.52 percent, from which it follows that almost all adjustment in oil input happens on impact. Long run consumption expenditures are reduced by 1.62 percent. The goods trade balance (positive) deteriorates by more than 4.5 percent, which, together with a more negative oil balance (11.42 percent) increases the trade balance deficit by 28.55 percent. This higher trade balance deficit is completely financed by increased interest earnings, as the stock of traded bonds rises by 28.55 percent, too.

The overall welfare effect of the oil shock is quite small, as intertemporal welfare falls by 1.96
percent, indicating that agents are indifferent between the oil price shock and its aftermath and a permanent reduction in consumption levels by 1.96 percent.

It is interesting to note that with respect to output, oil input and consumption expenditures, most of the adjustment happens right on impact, whereas the short-run and long-run reactions of the goods trade balance and the trade balance go in opposite directions. Of course, the current account dynamics are at odds with empirical evidence.

5.3 The benchmark economy

We now turn to the effects of an 100 percent oil price shock in the benchmark economy with time non-separable preferences, and parameter values given by the benchmark calibration (table 1). The dynamics of economic key variables and the reference stock are now interdependent. The two stable eigenvalues are -0.1507 and -0.0638, respectively. Thus, the speed of convergence of any variable at any point in time is a weighted average of the two stable eigenvalues. Over time, the weight of the smaller eigenvalue (-0.1507) declines, hence the larger eigenvalue (-0.0638) describes the asymptotic speed of adjustment.\footnote{For further discussion on this point see Eicher and Turnovsky (1999, 2001).}

The flexibility provided by the additional eigenvalue allows the model to match some features of the data, in particular with respect to the current account.

Impact effects

The impact and long-run effects of an unanticipated permanent 100 percent increase in the oil price are reported in bold in the second column of table 3. Starting off from the base equilibrium, the 100 percent rise in the oil price leads producers to reduce oil input instantaneously around 41 percent. The marginal product of labor is reduced, and given the real wage, labor demand falls. The impact reaction of firms is thus quite the same, regardless of the specific form of preferences.

However, the reaction of households differs dramatically. The presence of a reference consumption stock dampens the utility associated with a change in initial consumption relative to the reference stock and makes agents more reluctant to change their consumption pattern. This is the “status effect” described by Alvarez-Cuadrado, Monteiro, and Turnovsky (2004). The negative wealth effect of the oil price shock impacts on consumption expenditures ($E$) with a 0.98 percent reduction only. The marginal rate of substitution between consumption and leisure increases, and at the going real wage, agents increase their labor supply, however at a smaller amount compared to the conventional preferences case, because the marginal rate of substitution
Figure 1: Dynamic effects of an 100 % increase in $p$ – benchmark economy
changes less, due to the “status effect”. On the labor market, the reduction in labor demand outweighs the increase in labor supply, the real wage drops, and hence employment falls by 0.28 percent. Given the capital stock, output drops by 1.11 percent. Compared to the conventional case, output drops by a larger amount. The reason is the reduction in labor and oil input, whereas in the conventional case labor input increases, dampening thus output reduction. In figures 1(a), (b) and (c) the drops in output, labor, and consumption expenditures are illustrated by the difference between the dashed and solid lines.\textsuperscript{18} Because of the “status effect”, the expenditure-output ratio increases, as figure 1(d) illustrates.

Instantaneous welfare of the representative agent falls now by 3.23 percent, meaning that the agent is indifferent between the shock and a 3.23 percent reduction in his instantaneous consumption levels of the traded good and oil.

Oil input and oil consumption both fall, but valued at the higher oil price, the value of oil imports increases, hence the (negative) oil trade balance (\(OTB\)) deteriorates by roughly 12 percent. This is shown in figure 1(e).\textsuperscript{19}

Investment expenditures are reduced because of the drop in the marginal productivity of capital. The reductions in consumption of the domestically produced good and investment expenditure are larger than the output drop, hence the (positive) goods trade balance (\(GTB\)) improves by 10.95 percent, see also figure 1(f). However, the improvement is much smaller as in the conventional preferences case (where it was 61.55 percent), because the “status effect” dampens the consumption response to the oil price shock and increases the output reaction. The improvement in the goods trade balance is not sufficient to outweigh the deteriorated oil balance, and the (negative) overall trade balance deteriorates by about 13.30 percent, as illustrated in 1(g)\textsuperscript{20} The increased overall trade deficit starts a decumulation of net foreign assets, shown in figure 1(h) by an initially downward-sloping time path of bonds.

\textbf{Dynamic adjustments}

The cut in investment expenditure initiates a decumulation of the capital stock. The dynamic evolution of the capital stock is monotonic and illustrated in figure 1(i). The half-time of the capital stock adjustment is roughly 9.7 years. Compared to the conventional case, this means that the adjustment in the capital stock is fastened. Thus, the introduction of a reference consumption stock counter-intuitively speeds up the dynamics.\textsuperscript{21}

\textsuperscript{18}The dashed lines refer to the base equilibrium.
\textsuperscript{19}We have not drawn the time paths for \(C_i, M_i\), and \(Z_i\). Their time paths are similar to that of expenditure \(E_i\) and output \(Y_i\). This is because \(C_i, M_i\) and \(E_i\) move proportionally according to equations (3a) and (3b), and \(Z_i\) is proportional to \(Y_i\), as equation (3d) states.
\textsuperscript{20}In the figure, the dashed line, illustrating the base equilibrium, and the horizontal axis coincide.
\textsuperscript{21}This property is pointed out in Alvarez-Cuadrado, Monteiro, and Turnovsky (2004).
As time proceeds, because of reduced consumption expenditures the reference stock gradually declines. This in turn makes agents less reluctant to further reduce their consumption expenditures over time, as can be seen in figure 1(c). At the same time, together with falling expenditures agents reduce their leisure and thus increase their labor supply, and in the earlier stage of the dynamic transition firms are willing to hire more labor. Hence, employment increases over roughly 15 years following the shock. However, the gradual, but ongoing reduction of the capital stock reduces marginal productivity of labor, and firms become more and more reluctant to hire additional labor. In fact, beyond 15 years, labor employed begins to fall slightly towards its steady-state level. A can be seen from table 3, the difference between the impact and steady-state change of oil input is extremely small (0.206 percentage points), implying that almost all of the oil input adjustment happens instantaneously after the shock hits the economy.

The falling capital stock and reduced labor (which is always below the base level) lead to an ongoing output reduction. As figure 1(a) reveals, in the very early stage of adjustment output falls at an increasing amount, as capital decumulation is highest there, whereas after roughly 9.5 years output decline slows down. The evolution of the expenditure-output ratio is illustrated in figure 1(d). After the initial increase it starts to fall, quickly achieving levels below the base line and partially recovering in the latter stage of the dynamic adjustment.

The most interesting part of the dynamics is the evolution of the trade balance and its components, and the time path for traded bonds. First, the oil trade balance slightly improves during transition, as firms and households further cut back their oil usages. However, the improvement of the oil balance is small. This can also be seen in table 3, which reveals that the impact and steady-state changes of the oil trade balance differ only by roughly 0.5 percentage points. Second, output, consumption and investment dynamics lead to an improving goods trade balance in the first 11.47 years of transition. After that time, the ongoing reduction in output overweighs the cut backs in investment expenditures (because more than half of the capital stock’s adjustment is already done) and consumption expenditures on the domestically produced good, and the goods trade balance deteriorates. Third, putting the time paths of the oil balance and the goods trade balance (figures 1(e),(f)) together gives the evolution of the overall trade balance. Improvements of the goods trade balance and the oil balance raise the trade balance. Since almost all adjustment of the oil trade balance happens on impact, the dynamics of the overall trade balance are almost entirely governed by the goods trade balance, hence the phase of trade balance improvements ends slightly after that of the goods trade balance (i.e. after roughly 11.76 years). From thereon, the trade balance deteriorates towards its new steady-state level.
Taking interest income on traded bonds into account, the accumulation of bonds (and hence the current account) evolves as depicted in figure 1(h). Traded bonds are decumulated, but this decumulation slows down, and after 4.75 years the current account reverts into a surplus. From thereon, bonds are accumulated, and at steady-state the economy has improved its net foreign asset position. The reason for this quite early switch in the current account stems from the fact that the overall trade balance returns quickly (after roughly 4.25 years) to its pre-shock level. Because during this time the stock of bonds declined, interest income has fallen, and hence the current account turns into surplus a little bit later.

The evolution of traded bonds and hence of the current account shows the J-curve property. After the increase in the oil price (the deterioration in the country’s terms of trade), the current account worsens instead of improving, and it takes some time until the current account switches back to a “normal” reaction upon a deterioration in the terms of trade. By comparing the benchmark economy with time non-separable preferences with the conventional one, it is clear that the model’s J-curve phenomenon is due to the presence of consumption habits. The emergence of a J-curve when habit formation is present is described in detail by Cardi (2007), although his model is different in several aspects.

Since the oil trade balance remains almost constant, it is the reduced change in consumption expenditures on the domestically produced good, due to the “status effect”, which causes the J-curve. This J-curve effect can also be seen in the state-space representation of $K$ and $B$ in figure 1(j). In the conventional preferences case, the trajectory would be a negatively sloped line.

**Steady state effects**

As the second column of table 3 reveals, in the long run output and the capital stock fall by 1.45 percent (w.r.t. the base equilibrium), and labor is reduced by 0.13 percent. Oil input usage is cut back by 41.40 percent, and consumers reduce their expenditures by 1.71 percent. The (positive) goods trade balance improves by 16.79 percent, whereas the oil balance deteriorates by 11.57 percent, resulting in an overall deterioration of the (negative) trade balance by 5.88 percent. Accordingly, the steady-state net foreign asset position (positive) has to rise by 5.88 percent. The long-run oil share ($OTB/Y$) increases by 13.21 percent and is still fairly low at 0.02564.

Perhaps most important from the point of view of the representative agent is the change in his intertemporal welfare $W$. The 100 percent oil price hike lowers the agent’s overall welfare by

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22In Cardi’s model, (i) agents are “inward-looking”, (ii) labor is fixed, (iii) they are no imported inputs, and (iv) there is a valuation effect, as bonds are denoted in terms of the imported good. A terms of trade change exercises thus a direct wealth effect in terms of the domestic good. In our model, however, there is no valuation effect on the agent’s asset position and thus no such wealth effect.
2.35 percent in the sense that he is indifferent between the shock and a 2.35 percent reduction in his permanent consumption levels of the traded good and oil.

Compared to the case of conventional preferences, the model with a reference consumption stock matches data much better, predicting that after an oil price shock, the economy suffers a slump, as output drops, employment falls, and the current account worsens. A large amount of the total steady-state output change occurs right on impact, a feature which is stressed by many empirical studies, see, e.g., Jiménez-Rodríguez and Sánchez (2004), Blanchard and Galí (2007), and Kilian (2007). The dynamic evolution of the current account confirms the early findings of Agmon and Laffer (1978), and more recently of Rebucci and Spatafora (2006) and Kilian, Rebucci, and Spatafora (2007). Moreover, compared to the conventional model, the long-run change in the overall trade balance is strongly reduced. With respect to the output loss, the model fits actual findings that this loss is quite low. The long-run loss (-1.45 percent) is in accordance of what, e.g., Jiménez-Rodríguez and Sánchez (2004) the OECD (2004), Nordhaus (2007), Blanchard and Galí (2007) and Schmidt and Zimmermann (2007) found.

6 Sensitivity analysis

Having discussed the dynamics and the steady-state changes, we now perform some sensitivity analysis with respect to the weight $\gamma$ of the reference consumption stock in preferences, the speed of adjustment $\zeta$ of the reference consumption stock, and the oil share in output.

6.1 Weight of habits in preferences

Starting from the benchmark calibration, we will vary the weight $\gamma$ from zero to 0.8, as recent empirical evidence suggests values of $\gamma$ up to 0.9. We already discussed in detail the case of $\gamma = 0$, when the evolution of the reference stock becomes irrelevant. The results of the sensitivity analysis are summarized in columns one to four of table 3. As $\gamma$ increases, the smaller stable root increases (becomes less negative), whereas the larger stable root falls (becomes more negative). Hence, the asymptotic speed of convergence increases with $\gamma$. The half-time of the capital stock falls with an increasing $\gamma$, whereas the duration of the current account deficit rises.

In the short run, an increasing $\gamma$ raises the initial output loss, increases the reductions in labor and oil input, and lowers agents’ expenditure cuts and therefore the change in the goods trade balance. As agents are increasingly reluctant to cut their consumption expenditures, they are increasingly unwilling to increase their labor supply, resulting in increasingly lower employment. The cut in oil input is almost not affected, as preferences impinge on the production side of the economy only indirectly via labor supply. As the oil trade balance is insensitive to $\gamma$, the
(negative) overall trade balance becomes increasingly negative with rising $\gamma$. The instantaneous welfare loss rises with $\gamma$, too.\textsuperscript{23} In the long run, the higher $\gamma$, the lower the steady-state reductions in output and the capital stock, and the smaller the reduction in labor. Agents’ long-run expenditure change increases with $\gamma$. The improvement of the (positive) goods trade balance becomes larger. Because the long-run oil balance is insensitive to $\gamma$, the (negative) steady-state trade balance deteriorates less as $\gamma$ increases, and for $\gamma = 0.8$ it improves. These changes are mirrored in the net foreign asset position. The long-run welfare loss rises with $\gamma$.

The reason for this sensitivity with respect to $\gamma$ is that in the short run an increase in the weight of habits makes agents more reluctant to reduce their expenditures and to supply more labor, increasing thus the output loss, the trade balance deficit and hence the current account deficit, leading in turn to a longer period of bonds decumulation and to more pronounced changes in the the net foreign asset position. Because agents are forward-looking, an increasing reluctance to reduce consumption expenditures on impact requires a larger long-run expenditure cut to maintain intertemporal solvency. On the other hand, larger long-run consumption cuts increase agents’ willingness to supply labor, thus reducing the steady-state drops in employment, output, and the capital stock.

The case of $\gamma = 0.8$ is of particular interest, as some empirical work suggests this high value for a lot of countries. In that case, the economy ends off with a lower stock of traded bonds. For\textsuperscript{23} The welfare comparison has to be interpreted with care, as different values of $\gamma$ refer to different representative agents having different tastes. Instead, we are suggesting that an analysis based on time-separable preferences or on a too small value of $\gamma$ would understate the short-run (long-run) welfare loss derived by an agent having the “true” $\gamma$. 

\textbf{Figure 2: Time paths of bonds for different weights of habits}
illustrative purposes, figure 2 shows the time path of bonds for different values of $\gamma$, starting off from $B_0 = 0.125$. In general, for $\gamma$ in the range between 0.6 and 0.8, the time paths of economic key variables are similar. A value of $\gamma = 0.8$ leads to the current account dynamics Rebucci and Spatafora (2006) and Kilian, Rebucci, and Spatafora (2007) detected for a lot of countries, resulting in a reduction of the net foreign asset position. Seeing that in practice the steady-state will never be reached because of an ongoing occurrence of shocks, calibrating the model with $\gamma = 0.6$ or 0.7 yields plausible results, too, as the current account is in deficit for several years. On the other hand, we safely can rule out the case of conventional preferences, as the implied current account dynamics are at odds with empirical evidence.

### 6.2 Speed of adjustment of reference stock

Starting from the benchmark calibration, we analyze the effects of changing the speed of adjustment $\zeta$ of the reference stock. Table 4 summarizes the effects of varying $\zeta$ between 0.02 and 1 with respect to the stable eigenvalues and the behavior of the current account. For $\zeta = 0.05$ and 0.1 the stable eigenvalues are conjugate complex numbers. Because of the very small imaginary part, the periodicity of one cycle is extremely long (3491 and 571 years, respectively), so that for practical purposes the adjustment is essentially non-cyclical.

<table>
<thead>
<tr>
<th>$\zeta$</th>
<th>Stable eigenvalues</th>
<th>Time minimum $B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>re = -0.0492</td>
<td>11.14 (max)</td>
</tr>
<tr>
<td>0.05</td>
<td>re = -0.0686</td>
<td>3.87 (max)</td>
</tr>
<tr>
<td>0.1</td>
<td>re = -0.1507</td>
<td>— (monotone)</td>
</tr>
<tr>
<td>0.2</td>
<td>re = -0.3829</td>
<td>4.75</td>
</tr>
<tr>
<td>0.5</td>
<td>re = -0.7673</td>
<td>1.53</td>
</tr>
<tr>
<td>1.0</td>
<td>im = 0.0110</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Sensitivity analysis reveals that both the impact and steady-state output losses, reductions in employment and oil input increase with $\zeta$, whereas the drop in consumption expenditures and the welfare loss become smaller. Most important, for $\zeta = 0.02$ and 0.05 the current account improves on impact before turning into a deficit, whereas for $\zeta = 0.1$ the time path of bonds is monotonically falling. This behavior is against empirical evidence, implying that a reasonable adjustment speed of habits has to be at least 0.2. Adjustment speeds of 0.5 and 1 make the J-curve effect very short-living, as the current account is only for 1.53 respectively 0.75 years in deficit, which is somewhat implausible. Hence, we can conclude that most reasonably the speed of adjustment of the reference stock should be set $\zeta = 0.2$, as we have done in the benchmark

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24However, for $\gamma = 0.8$ after their impact drops, output and oil input slightly increase in the first 3.85 years after the shock and fall then toward steady state.
6.3 Oil share in output

According to empirical evidence, we calibrated the oil share in output to be 0.0226, reflecting the low energy cost share in GDP in the last 20 years. However, as Schmidt and Zimmermann (2007) state, the energy cost share has partially recovered in the very recent past, a trend that cannot be precluded to proceed in the future. In addition, some countries may have higher oil shares. Moreover, from a historical point of view, it is interesting to apply the model to the oil price shocks of the 1970s and early 1980s, a period in which the oil share was much higher than in our days. It is therefore appropriate to conduct some sensitivity analysis with respect to the oil share by varying $\alpha_2$ and $1 - \nu$. The variation in the oil share in the production function is compensated by an equivalent reduction in the share of labor $\alpha_1$. The calibration is done in a way to ensure that the oil input/oil consumption ratio is in line with empirical evidence (0.45 and 0.49, respectively).

The last two columns of table 3 immediately reveal that increasing the oil share has almost no effects on the stable eigenvalues and hence on the speed of adjustment, and thus on the half-time of the capital stock adjustment and the timing of the current account reversal. Also, the percentage changes of oil input (both in the short run and in the long run) are almost the same. What dramatically changes are the reactions of output, expenditure and welfare. Doubling the oil share roughly doubles the short and long run output losses, employment and expenditure reductions, and the welfare losses more than double, whereas the impact and steady-state changes of the oil trade balance differ only by around 1.95 respectively 2.59 percentage points. The changes in the overall trade balance increase with the oil share, implying thus larger current account and net foreign asset changes, whereas the changes in the goods trade balance become significantly smaller. The reason for that is that given the initial stock of bonds, a higher oil share implies a larger oil trade balance deficit, requiring a bigger goods trade balance. Hence a given absolute improvement in the goods trade balance translates into a lower relative change.

The sensitivity analysis with respect to the oil share yields three important results. First, the dynamics but not the magnitude of changes are insensitive with respect to the oil share. Second, the model shows that the observed small effects of oil price shocks in modern economies can be explained by low oil shares. Third, the model calibrated to a relatively large oil share

\[\text{See, e.g., OECD (2004), and Parry and Darmstadter (2004).}\]

\[\text{Alternatively, one could reduce the share of capital, } \alpha_3. \text{ The resulting differences are very small.}\]
adequately describes the effects of the oil crises of the 1970s and 1980s, where oil prices more than doubled.

7 Conclusions

Recent empirical evidence showed that the macroeconomic effects of oil price shocks are quite small. Previous research has focused almost entirely on the reaction of output, employment, and inflation, and little attention was spent on an economy’s external dynamics. Exceptions are Rebucci and Spatafora (2006) and Kilian, Rebucci, and Spatafora (2007), who found that the current account deteriorates after an oil price hike. However, this empirical work does not address the reason for the current account adjustment. This paper has examined the effects of oil price shocks in a small open economy framework, paying particular attention to the current account.

Recent empirical evidence strongly suggests the introduction of time non-separable preferences. Because this increases the complexity of the model substantially, most of our work has proceeded numerically by calibrating a plausible open economy model. Our analysis shows the importance of introducing a “status effect” by comparing the results with that of the standard model. Whereas the standard model predicts a current account surplus and a monotonic adjustment of a country’s net foreign asset position, which is at odds with empirical evidence, the presence of the “status effect” enriches the dynamics substantially, predicting that an oil price hike turns the current account into deficit, as consumers are reluctant to sufficiently reduce their consumption expenditures. Moreover, the adjustments in the current account are almost entirely driven by the goods (non-oil) trade balance. Over time, together with falling consumption expenditures the current account deficit is gradually reduced, and after a sufficiently low level of consumption expenditures is achieved, the current account eventually turns into a surplus, showing thus the J-curve property. Depending on the weight of the consumption reference stock and thus on how strong the “status effect” is, the economy ultimately ends up with a higher or lower stock of net foreign assets. The model thus provides a sound theoretical underpinning of empirical evidence described by Agmon and Laffer (1978) and others, who found that current account dynamics upon oil price shocks are non-monotonic. Due to a small oil share in GDP, our model also predicts quite small responses of output and employment upon an oil price shock, as recent empirical research suggests.

Extensive sensitivity analysis showed that a plausible calibration of the agent’s preferences comprises a speed of adjustment of the consumption reference stock of 0.2 and a weight of

\[ 27 \text{See, e.g., Nordhaus (2007).} \]
the reference stock in the utility function of 0.6. A higher weight results in more pronounced and long-lasting current account deficits, and very low or high speeds of adjustments generate implausible current account dynamics. Moreover, contrasting our model with empirical evidence provides strong support for the existence of time non-separable preferences, as they induce a plausible pattern of the current account. In light that the oil share in GDP may rise in the future, we conducted some sensitivity analysis in that direction. Increases in the oil share result in larger output and employment reductions and higher welfare losses, as experienced in the 1970s, where oil shares where roughly twice as high as today, but do not change the dynamics qualitatively.

The model can thus be viewed as an important extension of the standard small open economy model, yielding results in accordance with empirical evidence, and providing a deeper understanding of current account dynamics caused by oil price shocks.

A Appendix: Derivation of equilibrium dynamics

To derive the equation of motion for \( l_t \), we write equations (2a) and (2c) in growth rates, denoted by hats

\[
(\epsilon \nu - 1) \hat{C}_t + \epsilon (1 - \nu) \hat{M}_t - \epsilon \gamma \hat{H}_t + \epsilon \theta \hat{l}_t = \hat{\lambda}_t \quad \text{(A.1a)}
\]

\[
\nu \epsilon \hat{C}_t + \epsilon (1 - \nu) \hat{M}_t - \epsilon \gamma \hat{H}_t + (\epsilon \theta - 1) \hat{l}_t = \hat{\lambda}_t + (1 + \rho) \hat{Y}_t - (1 + \rho) (1 - \hat{l}_t) \quad \text{(A.1b)}
\]

where we used

\[
\frac{\partial Y_t}{\partial (1 - l_t)} = A^{-\rho} \alpha_1 \left( \frac{Y_t}{1 - l_t} \right)^{1+\rho}
\]

From (3), we get \( \hat{C}_t = \hat{E}_t = \hat{M}_t \) because \( \hat{\rho} = 0 \) for all \( t > 0 \). Thus, equations (A.1) become

\[
(\epsilon - 1) \hat{C}_t - \epsilon \gamma \hat{H}_t + \epsilon \theta \hat{l}_t = \hat{\lambda}_t \quad \text{(A.1a')}
\]

\[
\epsilon \hat{C}_t - \epsilon \gamma \hat{H}_t + (\epsilon \theta - 1) \hat{l}_t = \hat{\lambda}_t + (1 + \rho) \hat{Y}_t - (1 + \rho) (1 - \hat{l}_t) \quad \text{(A.1b')}
\]

Solving (A.1b') for \( \hat{C}_t \) gives

\[
\hat{C}_t = -\frac{\epsilon \theta - 1}{\epsilon} \hat{l}_t + \frac{\hat{\lambda}_t}{\epsilon} + \frac{\gamma}{\epsilon} \hat{H}_t + \frac{1 + \rho}{\epsilon} \hat{Y}_t - \frac{1 + \rho}{\epsilon} (1 - \hat{l}_t)
\]

Inserting into (A.1a') yields

\[
-(\epsilon - 1) \frac{\theta - 1}{\epsilon} \hat{l}_t + \frac{\epsilon - 1}{\epsilon} \hat{\lambda}_t + (\epsilon - 1) \gamma \hat{H}_t + \frac{\epsilon - 1}{\epsilon} (1 + \rho) \hat{Y}_t - \frac{\epsilon - 1}{\epsilon} (1 + \rho) (1 - \hat{l}_t) - \epsilon \gamma \hat{H}_t + \epsilon \theta \hat{l}_t = \hat{\lambda}_t
\]
Collecting terms and noting that (i) \( \frac{1}{1-l_i} = -\frac{l_i}{1-l_i} = -\frac{l_i}{1-l_i} \) and (ii) \( \frac{\epsilon - 1}{\epsilon} (\epsilon \theta - 1) - \theta \epsilon = -\epsilon^{(1+\theta)-1} \), we obtain

\[
\left[ (\epsilon(1 + \theta) - 1 + (\epsilon - 1)(1 + \rho)) \frac{l_i}{1-l_i} \right] \frac{\dot{l}_i}{l_i} - \epsilon \gamma \frac{\dot{H}_i}{H_i} + (\epsilon - 1)(1 + \rho) \frac{\dot{Y}_i}{Y_i} = \left( \frac{\dot{Y}_i}{Y_i} \right) \frac{\dot{\lambda}_i}{\lambda_i} \quad (A.2)
\]

Next, we eliminate \( \dot{Y}_i/Y_i \). Equation (2d) implies for constant \( p \): \( \dot{Y}_i = \ddot{Z}_i \). Writing equation (3d) in growth rates, noting (1a'), gives

\[
-\rho \ddot{Z}_i = \left[ \alpha_1 (1 - l_i)^{-\rho} + \alpha_3 K_i^{-\rho} \right]
\]

Hence

\[
\dot{Y}_i \quad \dot{Y}_i = \frac{\dot{Z}_i}{Z_i} = \frac{\left[ \alpha_3 K_i^{-\rho} \frac{K_i}{K_i} - \alpha_1 (1 - l_i)^{-\rho} \left( \frac{l_i}{1-l_i} \right) \frac{l_i}{K_i} \right]}{[\alpha_1 (1 - l_i)^{-\rho} + \alpha_3 K_i^{-\rho}]} \quad (A.3)
\]

Next, we insert (A.3) into (A.2). This yields

\[
\frac{\dot{\lambda}_i}{\lambda_i} = \left\{ \frac{(\epsilon(1 + \theta) - 1 + (\epsilon - 1)(1 + \rho)) \alpha_3 K_i^{-\rho}}{[\alpha_1 (1 - l_i)^{-\rho} + \alpha_3 K_i^{-\rho}]} \left[ \frac{l_i}{1-l_i} \right] \right\} \frac{\dot{l}_i}{l_i}
\]

\[
+ \frac{(\epsilon - 1)(1 + \rho) \alpha_3 K_i^{-\rho}}{[\alpha_1 (1 - l_i)^{-\rho} + \alpha_3 K_i^{-\rho}]} \frac{\dot{K}_i}{K_i} - \epsilon \gamma \frac{\dot{H}_i}{H_i} \quad (A.4)
\]

Noting that \( \dot{\lambda}_i/\lambda_i = \beta + n - r = 0 \), equation (A.4) can be solved for \( \dot{l}_i \):

\[
\dot{l}_i = A_1 \epsilon \gamma \frac{\dot{H}_i}{H_i} - A_1 A_2 \frac{\dot{K}_i}{K_i} \quad (A.5)
\]

where

\[
A_1(l_i, K_i) \equiv \frac{l_i}{\epsilon (1 + \theta) - 1 + \frac{(\epsilon - 1)(1 + \rho) \alpha_3 K_i^{-\rho}}{[\alpha_1 (1 - l_i)^{-\rho} + \alpha_3 K_i^{-\rho}]} \left[ \frac{l_i}{1-l_i} \right]}
\]

\[
A_2(l_i, K_i) \equiv \frac{(\epsilon - 1)(1 + \rho) \alpha_3 K_i^{-\rho}}{[\alpha_1 (1 - l_i)^{-\rho} + \alpha_3 K_i^{-\rho}]}
\]

which is equation (7) in the text.

The equation of motion for the reference stock (1c) can be rewritten as follows: Inserting (3a) and (3b) into the aggregator function \( C_{\nu}^\nu M_i^{1-\nu} \), we get

\[
C_{\nu}^\nu M_i^{1-\nu} = \left( \frac{1 - \mu}{\nu p} \right)^{1-\nu} C_i
\]
Hence, the equation of motion for the reference stock can be written as a function of $C_i$ and $H_i$:

\[
\frac{\dot{H}_i}{H_i} = \zeta \left[ \left( \frac{1 - \nu}{\nu p} \right)^{1-\nu} \frac{C_i}{H_i} - 1 \right].
\] (A.6)

Using (3c) for $C_i$ and substituting (1a') for $Y_i$, we can solve for the $C_i/H_i$ ratio:

\[
\frac{C_i}{H_i} = \frac{\nu \alpha_1 A}{\theta H_i} \frac{l_i}{1 - l_i} \frac{1}{1 - \alpha_2} \frac{\left( \alpha_1 (1 - l_i)^{-\rho} + \alpha_3 K_i^{-\rho} \right)^{-(1+\rho)/\rho}}{\left[ 1 - \alpha_2 \left( \frac{p}{\alpha_2 A} \right)^{1+\rho} \right]^{-\left(1+\rho)/\rho\right)}}
\] (A.7)

Inserting (A.7) into (A.6) and multiplying by $H_i$ gives the differential equation for the reference stock

\[
\dot{H}_i = \zeta \left[ \left( \frac{1 - \nu}{\nu p} \right)^{1-\nu} \frac{\nu \alpha_1 A}{\theta} \frac{l_i}{1 - l_i} \frac{1}{1 - \alpha_2} \frac{\left( \alpha_1 (1 - l_i)^{-\rho} + \alpha_3 K_i^{-\rho} \right)^{-(1+\rho)/\rho}}{\left[ 1 - \alpha_2 \left( \frac{p}{\alpha_2 A} \right)^{1+\rho} \right]^{-\left(1+\rho)/\rho\right)}} - H_i \right]
\] (A.8)

which is equation (8) in the text.

References


Sommer, M., 2007. Habit formation and aggregate consumption dynamics. The B. E. Journal of Macroeconomics 7 (1 (Advances)).


