The Dynamic Effects of Subsidizing the Tourism Sector

Stefan Franz Schubert and Juan Gabriel Brida

Free University of Bozen-Bolzano

2008

Online at http://mpra.ub.uni-muenchen.de/16755/
MPRA Paper No. 16755, posted 11. August 2009 23:37 UTC
The Dynamic Effects of Subsidizing the Tourism Sector

Stefan F. Schubert\textsuperscript{1}  
and  
Juan G. Brida\textsuperscript{2}  
Free University of Bozen-Bolzano, Italy

This version: 24. June 2007

\textsuperscript{1}Mail: Free University of Bozen-Bolzano, School of Economics and Management, Via Sernesi 1, I-39100 Bolzano, E-mail: StefanFranz.Schubert@unibz.it  
\textsuperscript{2}Mail: Free University of Bozen-Bolzano, School of Economics and Management, Via Sernesi 1, I-39100 Bolzano, E-Mail: JuanGabriel.Brida@unibz.it
Abstract

The paper studies the short run and long run effects of a production subsidy to the tourism sector of a small open economy, which can also be thought as a region within a country. We introduce a two-sector dynamic general equilibrium model where the tourism sector is considered to be labor-intensive and produces traded services. The other sector is capital-intensive and produces a nontraded good, which is also used for capital accumulation. Labor and capital can freely move between sectors. Economic decisions are made by forward-looking representative agents, which optimize their intertemporal welfare by choosing consumption of both the nontraded good and tourism services, the sectoral allocation of labor, and the rate of wealth accumulation. We discuss the short run, dynamic and long run effects of a production subsidy to the tourism sector. In the short run, the introduction of a subsidy to tourism production leads to a boom in that sector. As time passes, the economy-wide capital stock is decumulated, and production of tourism is falling. In the long run, compared to the situation before the subsidy was implemented, tourism production remains on a higher level, whereas output of the nontraded good drops.
1 Introduction

Subsidies have a long tradition in economic policy, and textbooks are full of examples. Various definitions of subsidies can be found. In general, a subsidy is a public financial contribution that confers a benefit to the recipient, i.e., a transfer from the government to a private entity without any payment in return. This transfer can have the form of a public provision of goods or services at less than market prices, it may involve direct provision of funds or regulatory interventions with no direct financial implications for the government’s budget.

As it is well understood, subsidies may have various economic effects. The receipt of a subsidy induces a change in the behavior of the recipient, which may translate into a change on market prices, influencing in turn economic decisions of other agents than the recipient (incidence). For example, a producer who receives an output subsidy which depends on his production will increase production, other things equal. If the subsidy is granted to a group of producers, forming a sector in the economy, their increase in output will lead to a fall in the market price of the good being subsidized, and consumers benefit, too. Moreover, the effects of the subsidy may spill over to other sectors, as the demand for scarce inputs changes. It is therefore important to recognize these indirect effects of a subsidy, which may run counter to the government’s objectives. However, the usual textbook analysis of the economic effects of a subsidy uses a partial equilibrium framework by focusing on one market solely. Depending on the magnitude of spill over effects, such an analysis may be misleading. To take care of the indirect effects of a subsidy, the analysis should be based on a general equilibrium framework.\footnote{See, e.g., Blake, Sinclair, and Campos (2006), who argue that general equilibrium models can take account of the interrelationships among tourism and other sectors in the domestic economy.}

Also, the change in behavior of economic agents and the induced effects on economic key variables affects the evolution of the economy over time. E.g., a production stimulus may result in higher investment rates and thus in a larger capital stock and increased output in the future. It seems therefore appropriate to conduct a full analysis of the economic effects of a subsidy in an intertemporal general equilibrium framework.\footnote{For the need of dynamic general equilibrium modeling, applied to tourism, see, e.g., Dwyer, Forsyth, and Spurr (2004).}

In practice, there are many possible forms of a subsidy, including direct payments, tax concessions, contingent liabilities and provision of goods and services. Subsidies can be distinguished between those that are horizontal and those that are industry-specific. Industry specific subsidies are targeted
to one particular industry, whereas horizontal subsidies are generally catego-
ized by functions or objectives and would typically include environmental
and energy-saving subsidies, research and development subsidies, support to
regional development, and so on. A majority of countries uses more horizon-
tal than industry-specific subsidies. Also, subsidies can be direct or indirect.
Direct subsidies are part of government’s expenditures, whereas indirect sub-
sidies tend not to be recognized as subsidies at all. Examples of the latter
are tax benefits, price regulation, export credit facilities, or preferential treat-
ment.

It is evident that subsidies are also granted to the tourism sector of an
economy. The most used indirect subsidies to the tourism sector are:

- exemption from VAT (value added tax) and excise duty on sales on
  airports, in planes and outside territorial waters,
- low rate of VAT on entry to amusement parks, sports events, etc.,
- incomplete cost coverage from tourist tax, entertainment charges, etc.,
- designation of land for recreational purposes as part of spatial planning,
- granting of land below cost price by municipalities to promote tourism
  activity,
- discounts from public airports to airline companies.

The issue of granting subsidies is receiving growing attention in tourism,
because this sector is one of the most frequently targeted sectors by ser-
vices subsidies. The existence of externalities that are not taken into ac-
count in private production and consumption decisions, the fact that cer-
tain tourism resources have a public good character, and the existence of
information asymmetries between producers and consumers are the main
reasons that justify government intervention in the tourism market. Early
work on these issues was done by Hardin (1968). As reported in World-
Trade-Report (2006), subsidy programs for tourism were mentioned in 62
of their 97 members between 1995 and February 2004. In many developing
countries of Africa, Asia, and Latin America the annual growth rate in inter-
national tourist arrivals was always positive over the past decade, including
the difficult years 2001 and 2002, resulting in growth rates of the tourism
sector higher than the growth rate for the world economy as a whole, see
World-Tourism-Organization (2005). On average then, tourism-specialized
countries grow more than others, and as a consequence, tourism has been
promoted in many countries as part of the solution to their economic problems, see, e.g., Sinclair (1998) and Durbary (2004).

Tourism is seen as an important source of foreign exchange earnings, employment of domestic labor and a contributor to economic growth, and it is considered to be a sector with strong growth potential. Thus, policy makers often consider subsidies as a very useful tool to improve the performance of the tourism sector and to stimulate the development of the country. Tourism subsidies are also used in many industrialized countries to promote regional development strategy, to support the agricultural sector and to protect the environment, as van Beers and de Moor (2001) emphasize.

In general, the motivation for subsidies to the tourism sector differs significantly across countries and regions, and so does the incidence of subsidies. It has been argued that government intervention is essential to promote tourism in early stages of its development. Very often, governments use subsidies for investments in infrastructure that is relevant for the tourism sector.

While developed and developing countries’ governments may have different expectations about their tourism sector, most of them share the perception that market forces alone cannot achieve their policy objectives. Different forms of government intervention, including the use of subsidies, play a key role in meeting these goals. Critics to this approach argue, however, that some of the most impressive development records in tourist destinations are those where government intervention was kept to the minimum level.

We already pointed out that governments use subsidies in tourism for many reasons and in many different forms. Recently, natural disasters as the Indian Ocean tsunami that occurred December 2004 and affected India, Indonesia, Maldives, Sri Lanka and Thailand, and hurricane Katrina that hit the Gulf of Mexico and the United States in August 2005 and terrorist acts (New York, Madrid, London, and Bali) led to an increase in tourism subsidies.

Because of the relevance of subsidies in the tourism sector, it is important that their economic effects are to be well understood. Our aim is to provide a simple model for such an analysis. Our approach employs a dynamic general equilibrium two sector model, based on intertemporally optimizing representative agents and perfect competition, which parallels the one of Turnovsky and Sen (1995).³ Our model can be viewed as a minimalist model,⁴ as any

³Hazari and Sgro (2004) analyze the consequences of tourism using dynamic models of trade. Their chapter 11 contains a Ramsey-type growth model, based in intertemporal optimization, but in contrast to our analysis, they abstract from current account adjustments.

⁴Our model is analytically tractable. In the literature on the economic effects of tourism, input-output (IO) models and computational general equilibrium (CGE) mod-
meaningful macroeconomic model of tourism should comprise at least two sectors, one of them producing internationally traded tourism services, the other producing a nontraded good, which can be consumed and invested. Because tourism has much to do with consuming services abroad (or with the consumption of domestically provided tourism services by foreigners), we use an open economy version of the two sector model. In particular, we assume that the economy is small in the sense that it faces a given world interest rate, at which it can lend or borrow at the international financial market.

We would like to stress that the small open economy framework refers also to a region within a country and may fit particularly well a region’s economic environment. For the sake of simplicity, we abstract from various different forms of tourism subsidies and assume that the country’s government grants a production subsidy to firms in the tourism sector. The tourism sector is viewed to be labor intensive, an assumption which is confirmed in practice. The industrial sector is capital intensive. The two factors of production, labor and capital, can freely move between sectors.

In our analysis, we will highlight the dynamic and the general equilibrium (spill over) effects of the subsidy. In the short run, the introduction of a subsidy to tourism production leads to a boom in that sector, as it attracts both labor and capital from the nontraded sector. Production in both sectors becomes more capital-intensive. The relative price of the nontraded good in terms of tourism may fall or rise on impact. As time passes, the economy-wide capital stock is decumulated, and production of tourism is falling. The relative price of the nontraded good increases gradually, and production in both sectors gets less capital-intensive.

In the long run, compared to the situation before the subsidy was implemented, tourism production remains on a higher level, whereas output of the nontraded good drops, and its relative price rises. In this sense, there is...
a deindustrialization of the economy, as more resources are devoted to the service sector.\(^8\) The reason is to be found in the economy’s reduced capital stock, but unchanged long-run sectoral capital intensities. The result resembles therefore the Rybczynski theorem. There is an overshooting in tourism services, i.e., its short run boost is larger than its long run increase. Home residents will consume less of the nontraded good but more tourism both in the short and long run. The subsidy may be welfare increasing. Thus, subsidizing the tourism sector may turn out to be an appropriate policy to create a larger service sector and to increase residents’ welfare.

The rest of the paper is structured as follows. Section 2 describes the economic framework. In section 3 the macroeconomic perfect foresight equilibrium is discussed. Section 4 turns to dynamics and the economy’s steady-state. The effects of a subsidy to the tourism sector are analyzed in detail in section 5. Finally, section 6 summarizes our main findings. Some technical details are delegated to an appendix.\(^9\)

2 Small open economy with a tourism and a nontraded sector

The small open economy comprises two sectors, in which a nontraded good, used for consumption and investment, and tourism services are produced. Domestic households consume both the nontraded good and tourism services and supply labor and capital to firms. Both sectors are perfectly competitive, and all economic agents take market prices as given. In contrast to the nontraded good, tourism services can be also exported to/imported from the rest of the world. The small-country assumption refers to a given world interest rate and to the ability to export/import as much tourism services as agents want without changing world market prices.

Without any loss of generality we can consolidate the production and consumption side of the economy to a representative consumer-producer, who produces and consumes a nontraded commodity and traded tourism services. The representative agent is endowed with a fixed supply of labor, \(L\), normalized to unity, and accumulates capital, \(K\). Both labor and capital can freely move between sectors. Tourism services \(T\) are produced using

\(^8\)Copeland (1991) discusses the conditions under which an increase in tourism can lead to deindustrialization. See also Nowak, Sahli, and Sgro (2004). Recently, Blake (2007), using a CGE model comprising 26 sectors, showed that some other sectors of the economy shrink as tourism expands.

\(^9\)A detailed appendix, containing all calculations, is available from the authors upon request.
a quantity of capital $K_T$ and labor $L_T$, by means of a linear homogenous neoclassical production function $F(K_T, L_T)$, with $F_{K_T} \equiv F_K > 0$, $F_{L_T} \equiv F_L > 0, F_{K_T K_T} \equiv F_{KK} < 0, F_{L_T L_T} \equiv F_{LL} < 0, F_{K_T L_T} \equiv F_{KL} > 0$.$^{10}$ The nontraded good $N$ is produced too by combining capital $K_N$ and labor $L_N$, by means of a second neoclassical production function $H(K_N, L_N)$, having the same properties as $F$. The tourism sector is assumed to be labor intensive, meaning that given wage and interest rate, the tourism sector always produces with a lower capital-labor-ratio than the nontraded sector, i.e., $k_T \equiv K_T/L_T < k_N \equiv K_N/L_N$. Finally, only the nontraded good may be used for investment.

The agent also accumulates net foreign bonds (assets), $b$, that pay the exogenously given world interest rate, $r$. The agent’s flow budget constraint is thus

$$\dot{b} = zF(K_T, L_T) + pH(K_N, L_N) - C_T - pC_N - pI - S + rb,$$  \hspace{1cm} (1a)

where $z \equiv 1 + \sigma$ denotes the value of tourism production inclusive the subsidy, $\sigma$; $C_T$ and $C_N$ denote the agent’s consumption of tourism services and the traded good; $p$ is the relative price of the nontraded good in terms of tourism services; $I$ denotes investment; and $S$ lump-sum taxes. Assuming for simplicity that the capital stock does not depreciate, capital accumulation is given by

$$\dot{K} = I. \hspace{1cm} (1b)$$

The constraints for the allocation of labor and capital between the two sectors are

$$L_T + L_N = 1 \hspace{1cm} (1c)$$

$$K_T + K_N = K. \hspace{1cm} (1d)$$

The representative consumer-producer chooses his consumption levels, $C_N$ and $C_T$; the allocations of labor and capital, $L_N$ and $L_T$, and $K_N$ and $K_T$, respectively; the rate of investment, $I$, and the rate of bond accumulation, to maximize the intertemporal utility function

$$W \equiv \int_{0}^{\infty} U(C_T, C_N) e^{-\beta t} dt,$$  \hspace{1cm} (2)

---

$^{10}$Where no ambiguity can arise we shall adopt the convention of letting primes denote total derivatives and appropriate subscripts partial derivatives. Thus, we shall let $f'(x) \equiv \frac{df}{dx}$; $f(x_1, \ldots, x_n) \equiv \frac{df}{dx}$; $f_{ij} \equiv \frac{\partial^2 f}{\partial x_i \partial x_j}$. Time derivatives will be denoted by dots above the variable concerned, $\dot{x} \equiv \frac{dx}{dt}$. 

6
subject to the constraints (1a) – (1d) and the historically given initial stocks of capital $K(0) = K_0$ and traded bonds $b(0) = b_0$. $\beta$ is the rate of consumer time preference, taken to be constant. The instantaneous utility function $U(C_T, C_N)$ is assumed to be concave with the following properties: $U_{C_T} \equiv U_T > 0, U_{C_N} \equiv U_N > 0, U_{C_T}C_T \equiv U_{TT} < 0, U_{C_N}C_N \equiv U_{NN} < 0, U_{C_T}C_N \equiv U_{TN} > 0$, indicating that tourism services and nontraded goods are Edgeworth complements, implying that the representative agent prefers to change consumption of the nontraded good and tourism services in the same direction. Inserting the allocation constraints (1c) and (1d) into the production functions, the Hamiltonian of the optimization problem can be written as

$$H = U(C_T, C_N) + \lambda [zF(K_T, L_T) + pH(K - K_T, 1 - L_T) - C_T - pC_N - pI - S + rb] + \gamma I,$$

where $\lambda$ is the shadow value of wealth in the form of traded bonds and can be interpreted as the marginal utility of wealth in terms of tourism services; and $\gamma$ measures the shadow value of capital. The optimality conditions are given by

$$U_T(C_T, C_N) = \lambda$$  \hspace{1cm} (3a)

$$U_N(C_T, C_N) = p\lambda$$  \hspace{1cm} (3b)

$$zF_L(K_T, L_T) = pH_L(K_N, L_N)$$  \hspace{1cm} (3c)

$$zF_K(K_T, L_T) = pH_K(K_N, L_N)$$  \hspace{1cm} (3d)

$$p = \gamma / \lambda$$  \hspace{1cm} (3e)

$$\beta - \frac{\lambda}{\lambda} = r$$  \hspace{1cm} (3f)

$$\frac{zF_K(K_T, L_T)}{p} + \frac{\dot{p}}{p} = r = H_K(K_N, L_N) + \frac{\dot{p}}{p}$$  \hspace{1cm} (3g)

together with the transversality conditions

$$\lim_{t \to \infty} \lambda be^{-\beta t} = \lim_{t \to \infty} \lambda pKe^{-\beta t} = 0.$$  \hspace{1cm} (3h)

Equations (3a) and (3b) equate the marginal utilities of consumption of tourism services and the nontraded good to the marginal utility of wealth in terms of tourism services and in terms of the nontraded good, $\lambda p$, respectively. Their ratio gives the familiar condition that the marginal rate of substitution between the nontraded good and tourism services must be equal to the relative price of the nontraded good. Equations (3c) and (3d) determine the allocation of labor and capital to the two sectors by equating marginal
products. Their ratio gives rise to the standard optimality condition that the marginal rate of technical substitution between labor and capital must be equalized across sectors, implying an efficient use of inputs. Equation (3e) is the first order condition for investment and relates the shadow values of traded bonds and nontraded capital. Equations (3f) and (3g) are no-arbitrage conditions. The former equates the rate of return on consumption to the rate of return on bonds, i.e., the interest rate. To obtain an interior solution, we require $\beta = r$, which leads to the zero-root property (see Sen (1994)) $\lambda = \bar{\lambda}$, implying a time-constant marginal utility of wealth, which has important consequences for the dynamics (see Schubert and Turnovsky (2002)), as the long-run equilibrium becomes dependent on initial conditions. The latter equates the rates of return on capital invested in the tourism sector and in the nontraded sector to the interest rate.

The government is the other agent in the small open economy, playing a simple role. It collects lump-sum taxes $S$ to finance its production subsidy to the tourism sector, $\sigma F(K_T, L_T)$. For the sake of simplicity and without changing results$^{11}$, we assume that the government maintains a balanced budget. Its budget constraint is therefore

$$\sigma F(K_T, L_T) = S.$$  \hspace{1cm} (4)

3 Macroeconomic equilibrium

The macroeconomic equilibrium of this intertemporal general equilibrium model is defined to be a situation in which all the planned supply and demand functions are derived from optimization behavior, the economy is continually in equilibrium, and all anticipated variables are correctly forecasted. We will call this concept a “perfect foresight equilibrium”.$^{12}$

From the consumption optimality conditions (3a), (3b), we get$^{13}$

$$C_T = C_T(\bar{\lambda}, p); \quad \frac{\partial C_T}{\partial \lambda} < 0, \quad \frac{\partial C_T}{\partial p} < 0$$ \hspace{1cm} (5a) $$C_N = C_N(\bar{\lambda}, p); \quad \frac{\partial C_N}{\partial \lambda} < 0, \quad \frac{\partial C_N}{\partial p} < 0.$$ \hspace{1cm} (5b)

$^{11}$If we introduced government bonds, results would not change, because according to the Ricardian equivalence proposition, government bonds in a model like ours do not constitute part of agents’ net wealth. See the seminal work of Barro (1974).


$^{13}$The signs of the partial derivatives are reported in appendix A.1.
The analysis of the production side can be simplified by working with production functions in intensive form, i.e.,

\[ f(k_T) \equiv F(K_T, L_T)/L_T; \quad h(k_N) \equiv H(K_N; L_N)/L_N. \]

Thus, the production block (3c) and (3d) can be written as

\[
\begin{align*}
  z[f(k_T) - f'(k_T)k_T] &= p[h(k_N) - h'(k_N)k_N] \quad (3c') \\
  zf'(k_T) &= ph'(k_N) \quad (3d')
\end{align*}
\]

and may be solved together with (1c) and (1d) to yield

\[
\begin{align*}
  k_T &= k_T(p, \sigma); \quad \frac{\partial k_T}{\partial p} < 0, \frac{\partial k_T}{\partial \sigma} > 0 \quad (6a) \\
  k_N &= k_N(p, \sigma); \quad \frac{\partial k_N}{\partial p} < 0, \frac{\partial k_N}{\partial \sigma} > 0 \quad (6b) \\
  L_T &= L_T(K, p, \sigma); \quad \frac{\partial L_T}{\partial K} < 0, \frac{\partial L_T}{\partial p} < 0, \frac{\partial L_T}{\partial \sigma} > 0 \quad (6c) \\
  L_N &= L_N(K, p, \sigma); \quad \frac{\partial L_N}{\partial K} > 0, \frac{\partial L_N}{\partial p} > 0, \frac{\partial L_N}{\partial \sigma} < 0 \quad (6d)
\end{align*}
\]

The outputs of tourism services and nontraded goods are obtained by substituting (6a) – (6d) into the sector-specific production functions:

\[
\begin{align*}
  Y_T &= L_T(K, p, \sigma)f[k_T(p, \sigma)] \equiv Y_T(K, p, \sigma); \quad \frac{\partial Y_T}{\partial K} < 0, \frac{\partial Y_T}{\partial p} < 0, \frac{\partial Y_T}{\partial \sigma} > 0 \quad (7a) \\
  Y_N &= L_N(K, p, \sigma)h[k_N(p, \sigma)] \equiv Y_N(K, p, \sigma); \quad \frac{\partial Y_N}{\partial K} > 0, \frac{\partial Y_N}{\partial p} > 0, \frac{\partial Y_N}{\partial \sigma} < 0 \quad (7b)
\end{align*}
\]

Equations (6) and (7) deserve further explanation. An increase in the economy-wide capital stock \( K \) reduces production of tourism services and increases production of the nontraded good. This result is due to the Rybczynski theorem\(^{14}\): An increase in the capital stock, given relative output price and hence the wage-rental-rate-ratio and sectoral capital intensities, increases output of the good whose production is capital intensive and reduces production of the labor intensive good. Therefore, labor employed in the tourism sector falls, whereas it rises in the nontraded sector. An increase in the relative price of the nontraded good leads to a change in the output mix, as resources are shifted from the tourism sector into the nontraded sector, which can offer higher factor rewards, and this is reflected in opposite output reactions. Production in both sectors becomes less capital intensive,\(^{14}\)See Rybczynski (1955).
and employment in the tourism sector falls along with contracted tourism output, whereas labor used in the nontraded sector is increased. On the other hand, a higher subsidy on tourism production attracts resources to this sector, and given overall labor and capital endowment, the tourism sector expands, whereas the nontraded sector shrinks, as labor and capital move from the nontraded sector into the tourism industry. Both sectors become more capital intensive.

Finally, macroeconomic equilibrium requires that the market for the nontraded good clears, i.e.

\[ Y_N(K, p, \sigma) = C_N(\bar{\lambda}, p) + I, \tag{8} \]

which states that nontraded output must be allocated either to consumption or investment. Inserting (4) and (8) into (1a), we get the country’s current account

\[ \dot{b} = [Y_T - C_T] + rb, \tag{9} \]

where the term in brackets represents the balance of payments on goods and services (which in our model is simply the service balance), and \( rb \) denotes (net) interest income from abroad.\(^{15}\)

### 4 Equilibrium dynamics and steady state

Denoting steady-state values with tildes, the linearized dynamics for the relative price and the overall capital stock follow from (1b), (3g), and (8) and are given by

\[
\begin{pmatrix}
\dot{p} \\
\dot{K}
\end{pmatrix} = 
\begin{pmatrix}
-\frac{zf}{p(k_N-k_T)} & 0 \\
\frac{h}{k_N-k_T} & 0
\end{pmatrix}
\begin{pmatrix}
0 \\
\partial Y_N / \partial p - \partial C_N / \partial p
\end{pmatrix}
\begin{pmatrix}
p - \bar{p} \\
K - \bar{K}
\end{pmatrix} \tag{10}
\]

Because the determinant of the matrix in (10) is negative, the system has one positive and one negative eigenvalue, denoted by \( \mu_1 < 0 \), and is therefore saddle-path stable. The stable solutions for the relative price, \( p \), and the economy’s capital stock, \( K \), are:

\[
p(t) - \bar{p} = X(K_0 - \bar{K})e^{\mu_1 t} \tag{11a}
\]

\[
K(t) - \bar{K} = (K_0 - \bar{K})e^{\mu_1 t}. \tag{11b}
\]

\(^{15}\)In case that the small economy is a region rather than a country, no official balance of payments statistics may exist. Nonetheless, the economic relationships remain exactly the same as for a country.
Equations (11a) and (11b) can be combined to obtain the stable saddle-path 
\[ p(t) - \tilde{p} = X[K(t) - \tilde{K}], \]
which is a negative line in \((K, p)\)-space with slope \(X\):
\[ X \equiv \left( \mu_1 - \frac{h}{k_N - k_T} \right) \left( \frac{\partial Y_N}{\partial p} - \frac{\partial C_N}{\partial p} \right)^{-1} < 0. \]
Using (11a), (5a) and (7a), after some algebra the linearized bonds accumulation equation (9) can be written as
\[ \dot{b} = \Omega(K - \tilde{K}) + r(b - \tilde{b}), \] (12)
where
\[ \Omega \equiv \left[ \frac{\tilde{p}}{z} \left( r - z\mu_1 - (1 - z) \frac{\partial Y_N}{\partial K} \right) - \left( \frac{\tilde{p}}{z} (1 - z) \frac{\partial Y_N}{\partial p} + \frac{\partial C}{\partial p} \right) X \right]. \]
The stable solution for the economy’s net foreign asset position can then be found to be
\[ b(t) - \tilde{b} = \frac{\Omega}{\mu_1 - r} (K(t) - \tilde{K}). \] (13)
Setting \( t = 0 \) in (13) yields the economy’s intertemporal budget constraint which ensures that the economy remains intertemporally solvent. After a shock, e.g., introducing a production subsidy to the tourism sector, proper adjustments of the marginal utility of wealth and the relative price of the nontraded good guarantee the economy’s long-run solvency. The relationship between the evolution of the economy’s capital stock and its net foreign assets (traded bonds) depends on the sign of \( \Omega/(\mu_1 - r) \), which measures a direct and an indirect effect of an increase in the capital stock on the current account and thus on bonds accumulation. The direct effect is negative and reflects the Rybczynski effect that an increase in the capital stock decreases tourism production, which — viewed in isolation — deteriorates the balance of trade and services and therefore the current account. The indirect effect describes the impact of capital accumulation on savings. Along the saddle-path, an increase in the capital stock towards steady-state lowers the relative price \( p \). Because of \( p(t) > \tilde{p} \), this implies lower real consumption \( C \equiv C_T + pC_N \) and higher real income \( Y \equiv Y_T + pY_N \) than in steady-state, and therefore positive savings, which together with declining investment improve the current account. Unless this indirect effect is large, the current account will move inversely with the capital stock, which we therefore shall assume (i.e. \( \Omega > 0 \)).

The economy’s steady-state equilibrium is reached when \( \dot{p} = \dot{K} = \dot{\tilde{b}} = 0 \). Hence, we get the steady-state relationships
\[ zf'(\tilde{k}_T) = \tilde{p}h'(\tilde{k}_N) \] (14a)
Several aspects of this equilibrium merit comment. The steady-state optimality conditions for the allocation of capital (14a), (14b) and labor (14c) jointly determine the two sectors’ capital intensities $\tilde{k}_i$ and the long-run relative price; they are only dependent on the subsidy $\sigma$. Given the values of $\tilde{k}_T, \tilde{k}_N, \tilde{p}$, the capital allocation constraint (14d), the market clearing condition for the nontraded good (14e), the long-run zero current account (14f), and the intertemporal solvency condition (14g) yield then the economy’s steady-state capital stock $\tilde{K}$, the long-run labor allocation between sectors, $\tilde{L}_T$ and $\tilde{L}_N = 1 - \tilde{L}_T$, the long-run stock of net foreign assets, $\tilde{b}$, and the equilibrium value of the marginal utility of wealth, $\bar{\lambda}$. From the equilibrium relative price and the marginal utility of wealth steady-state consumption of the nontraded good and of tourism services of domestic residents are derived. We therefore see that a subsidy to the tourism sector has direct effects on the production side of the economy, summarized by (14a) – (14c), and indirect effects on the demand side.

5 Analysis of a subsidy to the tourism sector

Steady state changes

Since our model assumes perfect foresight, the dynamic evolution of the economy and hence the transitional adjustment is determined in part by agents’ expectations of the ultimate steady-state. It is therefore convenient to start our analysis with the investigation of the long-run steady-state effects of the introduction of a subsidy $\sigma$ (respectively $z = 1 + \sigma$) to the tourism sector. These are given by

\[
\frac{d\tilde{k}_T}{d\sigma} = \frac{d\tilde{k}_N}{d\sigma} = 0 \quad \text{(15a)}
\]

\[
\frac{d\tilde{p}}{d\sigma} = \frac{\tilde{p}}{z} > 0 \quad \text{(15b)}
\]
and merit further comment. First, subsidizing tourism production has no effect on long-run capital intensities, as (15a) states. The reason is that the steady-state capital intensity in the nontraded sector, $\tilde{k}_N$, is determined by the non-traded sector’s production function $h$ and the interest rate $r$ alone, both of them which are not affected by the subsidy. Since an efficient factor allocation requires the equality of both sectors’ marginal rates of technical substitution between capital and labor, it follows that the long-run capital intensity in the tourism sector, $\tilde{k}_T$, remains unchanged, too. Therefore, all that happens on the production side of the economy is that the subsidy raises the relative price of the nontraded good, $\tilde{p}$, as (15b) indicates, to guarantee equality between the rates of return on capital in both sectors. Second, (15c) tells us that the economy’s long-run capital stock drops. This is a result of the dynamic adjustment caused by the subsidy. Together with the change in labor allocation, given in (15d), it follows that long-run production of the nontraded good (15f) falls, whereas steady-state tourism services (15e) increase. In the long-run, there is a deindustrialization of the economy, as the economy’s overall capital stock and thus its capital-labor-ratio falls, the
capital intensive sector shrinks, and resources are shifted to the labor intensive tourism sector.\footnote{This is again a consequence of the Rybczynski theorem. On tourism and deindustrialization, see, e.g., Copeland (1991).} Third, (15g) indicates that the marginal utility of wealth falls, implying a positive wealth effect on the side of domestic consumers. Viewed in isolation, the wealth effect increases consumption of both goods. But as (15h) and (15i) show, only consumption of tourism services is raised, and consumption of the nontraded good falls. The reason is that the negative substitution effect of the relative price increase works against consumption of the nontraded good and outweighs the positive wealth effect. Fourth, looking at (15j), the economy’s long-run net foreign asset position increases, indicating that during transition the country runs a current account surplus. This in turn implies higher net interest earnings on traded bonds, and thus a deterioration in the balance of trade and services. The increase in net interest income allows domestic agents to increase their consumption of tourism services over and above the growth in its production, indicating that domestic residents can afford to spend more on tourism activities abroad.

**Impact effects**

Having described the long-run effects of the subsidy, we turn to the short-run (impact) effects. Initially, the subsidy attracts labor and capital out of the nontraded sector into the tourism industry. Hence, production of tourism services increases and nontraded output falls. Both sectors’ capital intensities increase.\footnote{These results are proven in appendix A.2.}

\[
\begin{align*}
\frac{dk_T(0)}{d\sigma} > 0, & \quad \frac{dk_N(0)}{d\sigma} > 0, & \quad \frac{dK_T(0)}{d\sigma} > 0, & \quad \frac{dK_N(0)}{d\sigma} < 0 \\
\frac{dL_T(0)}{d\sigma} = -\frac{dL_N(0)}{d\sigma} > 0, & \quad \frac{dY_T(0)}{d\sigma} > 0, & \quad \frac{dY_N(0)}{d\sigma} < 0
\end{align*}
\]

The impact effect on the relative price, $p$, is ambiguous. This can be seen by taking the derivative of equation (11a) at time $t = 0$:

\[
\frac{dp(0)}{d\sigma} = \left(\frac{d\tilde{p}}{d\sigma}\right) + \left(\frac{d\tilde{K}}{d\sigma}\right) \tilde{X} \geq 0; \quad \frac{dp(0)}{d\sigma} < \frac{d\tilde{p}}{d\sigma}.
\]

The reason for the ambiguity is that $p$ plays a dual role both as a price for the nontraded good and for an asset, i.e. nontraded capital $K$. The no-arbitrage
condition for capital (3g) requires the equality between the rates of return on capital and on bonds

\[ h'(k_N) + \frac{\dot{p}}{p} = r \]

to hold continuously except at moments where new information arrives. On impact, the marginal product of capital \( h'(k_N) \) falls, and agents know that the price of capital \( p \) increases over time, creating therefore a capital gain. Depending on which effect dominates, the market price of capital has to fall or to rise on impact, but it always will settle below its new steady-state level.

Turning to consumption, domestic demand for tourism services unambiguously increases, because the initial response of \( p \) falls below its steady-state change, and because steady-state \( \tilde{C}_T \) increases, see (15h), whereas the effect on demand for nontraded goods is ambiguous,

\[
\frac{dC_T(0)}{d\sigma} = \frac{\partial C_T}{\partial \lambda} \frac{d\lambda}{d\sigma} + \frac{\partial C_T}{\partial p} \frac{dp(0)}{d\sigma} > 0
\]

\[
\frac{dC_N(0)}{d\sigma} = \frac{\partial C_N}{\partial \lambda} \frac{d\lambda}{d\sigma} + \frac{\partial C_N}{\partial p} \frac{dp(0)}{d\sigma} \geq 0
\]

If \( p \) falls on impact, consumption of the nontraded good rises, because of both the positive wealth effect, embodied in the reduced \( \lambda \), and the lower market price of nontraded goods. However, if \( p \) increases sufficiently on impact, this will outweigh the wealth effect, and \( C_N(0) \) falls. Viewed in another way, despite the fact that production of \( Y_N \) falls on impact, the decumulation of capital, i.e., disinvestment that sets in may allow higher consumption \( C_N \), depending on the magnitudes of output and investment responses.

**Dynamic transition**

On impact, the economy, given its initial stocks of capital \( K_0 \) and traded bonds \( b_0 \), starting off from an “old” steady-state, denoted by \( \tilde{p}_0, \tilde{K}_0, \tilde{b}_0 \), jumps from point A to point B on the stable saddle-path XX depicted in the upper part of figure 1. We have drawn the figure for the case of an initial drop of the relative price. The lower part of the figure portrays the relationship between the economy’s capital stock and its net foreign assets; it is a graphical representation of the intertemporal solvency condition. From thereon, the economy moves along XX and NN from points B and P towards points C and Q, respectively. The dynamic adjustment is characterized by a de-
Figure 1: Increase in subsidy $\sigma$
cumulation of capital, an increasing relative price, and a positive current account, as equations (11) and (13) confirm.

The rising relative price exercises several effects on economic key variables. Taking time derivatives of equations (6a) and (6b),

$$\dot{k}_T = \frac{\partial k_T}{\partial p} \dot{p} < 0, \quad \dot{k}_N = \frac{\partial k_N}{\partial p} \dot{p} < 0$$

reveals that both sectors’ capital intensities are falling over time. This is due to the economy’s deindustrialization, caused by a falling capital stock. The production of tourism services falls over time, and because steady-state production is higher than before granting the subsidy, this indicates that the subsidy causes an overshooting of tourism production on impact, i.e.

$$\frac{dY_T(0)}{d\sigma} > \frac{d\tilde{Y}_T}{d\sigma} > 0, \quad \dot{Y}_T < 0.$$ 

The dynamics of the nontraded good are less clear. The initial reshuffling of the capital stock leads to a drop in $Y_N$. This is a consequence of the Rybczynski theorem. From thereon, as time passes, two opposite forces are at work. The gradually falling economy-wide capital stock lowers production due to the Rybczynski theorem, whereas the rising relative price makes it more attractive. Time differentiating the nontraded goods market equilibrium condition, equation (8), i.e.

$$Y_N = C_N(\lambda, p) + \dot{K},$$

and using the stable adjustment paths for $p(t)$ and $K(t)$, equations (11), we obtain

$$\dot{Y}_N = \left( \frac{\partial C_N(\cdot)}{\partial p} X + \mu_1^{(-)} \right) \left[ K(t) - \tilde{K} \right]^{(+)} + \mu_1^{(-)}.$$

If $X$, the slope of the stable saddle path, is sufficiently negative, that is, if the negative relation between the capital stock and the relative price of the nontraded good is sufficiently strong, the expression within large brackets becomes positive, and we get $\dot{Y}_N < 0$, hence deindustrialization continues after the impact contraction of nontraded output. A sufficient condition for this to happen is that the absolute value of the price elasticity of consumption demand for the nontraded good exceeds the price elasticity of its supply.

To understand the ambiguity of the evolution of nontraded output, note first that the evolution of the demand component investment (in our case actually disinvestment) is given by the time derivative of $I$, i.e.

$$\dot{I} = \mu_1^2 [K(t) -$$
\( \hat{K} > 0 \), and depends – ceteris paribus – not on the slope of the saddle path. Hence, over time investment, although negative, increases. Second, consumption of the nontraded good depends on the marginal utility of wealth, \( \bar{\lambda} \), which changes once and for all at time 0, and on the relative price, \( p \), which rises over time. If \( X \) is large (in absolute value), the relative price \( p \) does not change much on impact, and compared to the new steady-state, consumption \( C_N(0) \) is relatively high, since \( p(0) \) is far below \( \bar{p} \). But this implies that \( C_N \) will have to fall a lot during transition. Together with the given increase in investment demand, aggregate demand for the nontraded good falls, and this calls for production cuts over time. If, on the other hand, \( X \) is small (in absolute value), \( p \) changes quite a lot on impact, and therefore consumption \( C_N(0) \) is already relatively low and will fall only modestly over time. In that case, the increase in investment demand will outweigh the reduction in consumption demand for the nontraded good, hence increasing aggregate demand for the nontraded good stimulates its production over time. In the former case (\( |X| \) sufficiently large), the impact reaction of \( Y_N \) undershoots its steady-state change, and deindustrialization continues over time, that is

\[
0 > \frac{dY_N(0)}{d\sigma} > \frac{d\bar{Y}_N}{d\sigma}, \quad \dot{Y}_N < 0.
\]

In the latter case (\( |X| \) small), \( Y_N \) overshoots it’s steady-state change, that is, deindustrialization is largest on impact, and as time passes, the nontraded sector recovers partially, i.e.

\[
\frac{dY_N(0)}{d\sigma} < \frac{d\bar{Y}_N}{d\sigma} < 0, \quad \dot{Y}_N > 0.
\]

As time passes by, domestic agents’ consumption of both tourism services and nontraded goods falls because of the increasing relative price, as the time derivatives of equations (5a) and (5a) reveal. The reason is that the marginal utility of wealth in terms of the nontraded good, \( \lambda p \), increases, exercising thus a negative wealth effect, which outweighs the substitution effect of the price increase in favor of tourism services. Because long-run consumption of tourism services rises, it follows that \( C_T(0) \) overshoots its steady-state value, too, i.e.

\[
\frac{dC_T(0)}{d\sigma} > \frac{d\bar{C}_T}{d\sigma} > 0, \quad \dot{C}_T < 0.
\]

The initial boom in the tourism sector is so large that it outweighs increased domestic demand by far, the excess being sold (exported) to foreigners.\(^{20}\)

\(^{20}\)To be exact, this holds if the nation is a net debtor (\( b < 0 \)). If the country is a net creditor (\( b > 0 \)), then the boom reduces the trade balance deficit, and less tourism services are imported (net) from abroad.
current account is thus turned into surplus, and the economy accumulates traded bonds/decumulates its debt, as it moves up the $NN$ schedule.

Eventually, the steady-state is reached, and all adjustments are completed. The economy is equipped with a lower capital stock and produces less nontraded goods but more tourism services than before the subsidy was introduced.

**Welfare effects of subsidy**

Let us finally address the question if a production subsidy to the tourism sector is socially desirable. First, it is important to note that the subsidy itself causes a direct wealth effect to agents, despite the fact that all subsidy receipts are taxed away. Second, the subsidy changes the structure of the economy and the consumption pattern, and these changes will have substantial effects on the wellbeing of economic agents. Denoting their instantaneous utility function $Z(t) = U(C_T(t), C_N(t))$, intertemporal utility or welfare

$$W = \int_0^\infty Z(t) e^{-rt} dt$$

(16)

can be approximated by

$$W = \frac{\tilde{Z}}{r} + \frac{Z(0) - \tilde{Z}}{r - \mu_1}.$$  

(17)

The introduction of the subsidy affects both time-zero utility $Z(0)$ and steady-state utility $\tilde{Z}$, and thus welfare

$$\frac{dW}{d\sigma} = \frac{1}{r - \mu_1} \left[ \frac{dZ(0)}{d\sigma} - \frac{\mu_1 d\tilde{Z}}{r d\sigma} \right].$$  

(18)

The effect on utilities is ambiguous, but we have

$$\frac{dZ(0)}{d\sigma} > \frac{d\tilde{Z}}{d\sigma}.$$  

Thus the subsidy doesn’t unambiguously raises welfare $W$ of domestic agents. Initially, the boom in the tourism sector allows higher consumption of tourism services, and consumption of nontraded goods may rise or fall, in sum this may raise instantaneous utility $Z(0)$. If $p(0)$ drops on impact, $Z(0)$ will unambiguously rise. Steady-state utility may increase, too, despite the fact

\[21\] For details, see appendix A.3.
of deindustrialization, because in the long-run the economy’s net foreign asset position, $\tilde{b}$, has improved, yielding higher net interest earnings, which allow domestic agents to permanently raise tourism consumption. If the increase in the long-run relative price $\tilde{p}$ is sufficiently small, the subsidy will be welfare increasing. Loosely speaking, as long as the drops of steady-state nontraded output $\tilde{Y}_N$ and therefore steady-state consumption of the nontraded good $\tilde{C}_N$ are not too large, agents’ steady-state utility $\tilde{Z}$ will increase, thus rising their welfare $W$.

6 Conclusion

In this paper we have modeled a two sector small open economy, producing labor intensive tourism services and a capital intensive nontraded good, intended for consumption and investment. The economy can export/import tourism services and has perfect access to the world financial market, thus facing a given world interest rate.

The model predicts that a production subsidy to the tourism sector leads to a boom in that sector in the short run, as capital and labor move out of the nontraded sector, resulting in a drop in production there. Production in both sectors becomes more capital intensive. On impact, tourism production and consumption both overshoot their new steady-state levels. The dynamic effects of the subsidy comprise an increase in the relative price of the nontraded good, capital decumulation, and a gradual reduction of domestic residents’ consumption of tourism services. The economy runs a current account surplus, thus accumulating net traded bonds. In the new long-run equilibrium, the economy is equipped with a lower stock of capital and an improved net foreign asset position, and produces less nontraded goods but more tourism services that before the subsidy was introduced. A deindustrialization has taken place. Domestic consumers are able to consume more tourism services, either at home or abroad, but cut consumption of the nontraded good. Therefore, the overall effect on domestic residents’ wellbeing is ambiguous. If deindustrialization is not too severe, the introduction of a subsidy will be welfare improving. Before implementing a subsidy to the tourism sector, a careful analysis of the economic structure is therefore necessary.
A Appendix

A.1 Partial derivatives

From (3a) and (3b) we calculate

\[
\frac{\partial C_T}{\partial \lambda} = \frac{U_{NN} - pU_{TN}}{U_{TT}U_{NN} - U_{TN}^2} < 0, \quad \frac{\partial C_T}{\partial p} = \frac{-\lambda U_{TN}}{U_{TT}U_{NN} - U_{TN}^2} < 0 \tag{A.1a}
\]

\[
\frac{\partial C_N}{\partial \lambda} = \frac{pU_{TT} - U_{TN}}{U_{TT}U_{NN} - U_{TN}^2} < 0, \quad \frac{\partial C_N}{\partial p} = \frac{-\tilde{\lambda} U_{TN}}{U_{TT}U_{NN} - U_{TN}^2} < 0 \tag{A.1b}
\]

where \( U_{TT}U_{NN} - U_{TN}^2 > 0 \) because of the concavity of \( U \). From the production block (3c\') and (3d\') together with the capital allocation constraint (1d), noting that \( dz/d\sigma = 1 \), we derive

\[
\frac{\partial k_T}{\partial p} = hzf''(k_N - k_T), \quad \frac{\partial k_T}{\partial \sigma} = -\frac{ph}{z^2f''(k_N - k_T)} \tag{A.2a}
\]

\[
\frac{\partial k_N}{\partial p} = \frac{zf}{p^2h''(k_N - k_T)}, \quad \frac{\partial k_N}{\partial \sigma} = -\frac{f}{ph''(k_N - k_T)} \tag{A.2b}
\]

\[
\frac{\partial L_T}{\partial K} = -\frac{1}{k_T - k_N} = -\frac{\partial L_N}{\partial K} \tag{A.2c}
\]

\[
\frac{\partial L_T}{\partial p} = \left( \frac{L_Nzf}{p^2h''(k_N - k_T)} + \frac{L_Tph}{z^2f''(k_N - k_T)} \right) \frac{1}{(k_N - k_T)^2} = -\frac{\partial L_N}{\partial p} < 0 \tag{A.2d}
\]

\[
\frac{\partial L_T}{\partial \sigma} = -\left( \frac{L_Nzf}{p^2h''(k_N - k_T)} + \frac{L_Tph}{z^2f''(k_N - k_T)} \right) \frac{1}{(k_N - k_T)^2} = -\frac{\partial L_N}{\partial \sigma} > 0, \tag{A.2e}
\]

From (7a) and (7b) we get:

\[
\frac{\partial Y_T}{\partial K} = \frac{\partial L_T}{\partial K} f = \frac{f}{k_T - k_N}, \quad \frac{\partial Y_N}{\partial K} = \frac{h}{k_N - k_T} \tag{A.3a}
\]

\[
\frac{\partial Y_T}{\partial p} = \left( \frac{L_Nzf^2}{p^2h''} + \frac{L_Tph^2}{z^2f''} \right) \frac{1}{(k_N - k_T)^2} < 0. \tag{A.3b}
\]

\[
\frac{\partial Y_T}{\partial \sigma} = -\left( \frac{L_Nzf^2}{p^2h''} + \frac{L_Tph^2}{z^2f''} \right) \frac{1}{z^2(k_N - k_T)^2} > 0. \tag{A.3c}
\]

\[
\frac{\partial Y_N}{\partial p} = -\left( \frac{L_Nzf^2}{p^2h''} + \frac{L_Tph^2}{z^2f''} \right) \frac{1}{p(k_N - k_T)^2} > 0 \tag{A.3d}
\]

\[
\frac{\partial Y_N}{\partial \sigma} = \left( \frac{L_Nzf^2}{p^2h''} + \frac{L_Tph^2}{z^2f''} \right) \frac{1}{(k_N - k_T)^2} < 0. \tag{A.3e}
\]

The signs in the text follow from \( k_T < k_N \).
A.2 Impact effects

Proof that \( \frac{dY_T(0)}{d\sigma} > 0 \)

From (13) and (11b), we derive

\[
\dot{b} = \frac{\Omega}{\mu_1 - r} \mu_1 (K - \tilde{K}) > 0 \quad \text{and} \quad \ddot{b} = \frac{\Omega}{\mu_1 - r} \mu_1^2 (K - \tilde{K}) < 0
\]

because \( K(0) > \tilde{K} \). Time-differentiating the current account equation (9), recognizing \( \dot{b} > 0 \) and \( \ddot{b} < 0 \), we get

\[
\ddot{b} = \dot{Y}_T - \dot{C}_T + r \dot{b} < 0
\]

Because \( \dot{C}_T = \frac{\partial C_T}{\partial p} \dot{p} < 0 \), and \( \dot{b} > 0, \ddot{b} < 0 \) if and only if \( \dot{Y}_T \ll \dot{C}_T < 0 \).

Finally, the ultimate steady-state increase of \( Y_T \), following from (15e), and the fact that \( \dot{Y}_T < 0 \) during transition proves

\[
\frac{dY_T(0)}{d\sigma} > \frac{d\dot{Y}_T}{d\sigma} > 0.
\] (A.4)

Proof of initial responses of \( k_i(0), L_T(0), K_T(0) \)

Because steady-state capital intensities do not change, and since \( \dot{k}_i < 0 \), we must have \( k_T(0) > \tilde{k}_T = k_{T0}, \quad k_N(0) > \tilde{k}_N = k_{N0} \), where \( k_{i0} \) denotes the “old” steady-state value before the subsidy was introduced. Thus,

\[
\frac{dk_T(0)}{d\sigma} > 0, \quad \frac{dk_N(0)}{d\sigma} > 0.
\] (A.5)

Rewriting the capital resource constraint (1d) in intensive form, noting that at \( t = 0 \) the overall capital-labor-ratio \( k_0 \) and \( L \) are given, i.e. \( k_T(0)L_T(0) + k_N(0)L_N(0) = k_0L \), and differentiating w. r. t. \( \sigma \), noting \( dL_T/d\sigma = -dL_N/d\sigma \), gives

\[
[k_T(0) - k_N(0)] \frac{dL_T(0)}{d\sigma} + \frac{dk_T(0)}{d\sigma} L_T + \frac{dk_N(0)}{d\sigma} L_N = 0.
\]

Because \( k_T < k_N \), and both sectors’ capital intensities rise on impact, we obtain

\[
\frac{dL_T(0)}{d\sigma} = -\frac{dL_N(0)}{d\sigma} > 0.
\] (A.6)

Finally, (A.5) and (A.6) together with the given overall capital stock \( K_0 = K_T(0) + K_N(0) \) imply

\[
\frac{dK_T(0)}{d\sigma} > 0, \quad \frac{dK_N(0)}{d\sigma} < 0.
\] (A.7)
A.3 Welfare effects

We first note that \( Z(t) \) can be linearly approximated as

\[
Z(t) = \tilde{Z} + \left[ U_T \frac{\partial C_T}{\partial p} + U_N \frac{\partial C_N}{\partial p} \right] (p - \tilde{p}).
\]

(A.8)

Using the stable solution for \( p - \tilde{p} \), this becomes

\[
Z(t) = \tilde{Z} + \left[ Z(0) - \tilde{Z} \right] e^{\mu_1 t}
\]

(A.9)

Using (3a) and (3b), the impact and steady-state effects on instantaneous utility are

\[
\frac{dZ(0)}{d\sigma} = \bar{\lambda} \left[ \frac{\partial C_T}{\partial \lambda} + p \frac{\partial C_N}{\partial \lambda} \right] \frac{d\tilde{\lambda}}{d\sigma} + \bar{\lambda} \left[ \frac{\partial C_T}{\partial p} + p \frac{\partial C_N}{\partial p} \right] \frac{dp(0)}{d\sigma}
\]

(A.10a)

and

\[
\frac{d\tilde{Z}}{d\sigma} = \bar{\lambda} \left[ \frac{\partial C_T}{\partial \lambda} + p \frac{\partial C_N}{\partial \lambda} \right] \frac{d\tilde{\lambda}}{d\sigma} + \bar{\lambda} \left[ \frac{\partial C_T}{\partial p} + p \frac{\partial C_N}{\partial p} \right] \frac{d\tilde{p}}{d\sigma}
\]

(A.10b)

Note that the expressions in brackets have negative sign. Evaluating the welfare integral (16), using (A.9), yields (17). The change in welfare is then

\[
\frac{dW}{d\sigma} = \frac{1}{r - \mu_1} \left[ \frac{dZ(0)}{d\sigma} - \frac{\mu_1 d\tilde{Z}}{r d\sigma} \right].
\]

(A.11)

Substituting (A.10a) and (A.10b) and simplifying gives

\[
\frac{dW}{d\sigma} = \frac{\bar{\lambda}}{r - \mu_1} \left[ \left( \frac{\partial C_T}{\partial \lambda} + p \frac{\partial C_N}{\partial \lambda} \right) \frac{\mu_2 d\tilde{\lambda}}{r d\sigma} + \bar{\lambda} \frac{\partial C_N}{\partial \lambda} \frac{\mu_2 d\tilde{p}}{r d\sigma} - \lambda \frac{\partial C_N}{\partial \lambda} \frac{\tilde{K}}{d\sigma} \right]
\]

The presence of the term

\[
\frac{\bar{\lambda}}{r} \frac{\partial C_N}{\partial \lambda} \frac{\mu_2 d\tilde{p}}{d\sigma} < 0
\]

makes the sign of \( dW/d\sigma \) ambiguous.
References


