Portfolio Analysis of Financial Market Risks by Random Set Tools

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Abstract

A new approach to portfolio analysis of financial market risks by random set tools is considered. Despite many attempts, the consistent and global modeling of financial markets remains an open problem. In particular it remains a challenge to find a simple and tractable economic and probabilistic approach to market modeling. This paper attempts to highlight fundamental properties that a market model should possess. The paper suggests a random set approach as a probabilistic base of this model. Using this approach it is possible to establish a corresponding interactive market dynamics that involves a minimal number of sets. These sets include the set of capital surpluses, the set of capital within assets and the set of capital deficits. Several interesting properties related to random volatility of assets quality, probabilities of quality categories and defaults and matrices of transition probabilities of switching among categories can be derived. In addition the random set approach allows to derive the so called transition set-matrices, random set invariants of capital redistribution processes. Empirical evidence will be given that confirm these random set findings. The approach is also illustrated by collapses in U.S. financial markets in 90's and can be used to explain Russian default'98.

Key words and phrases:

risk, financial market, portfolio analysis, random set, Set-at-Risk

1. Introduction

Financial market is a miracle and a puzzle [28]. Periods of smooth flow are followed by sudden going up or unexpected falling down. Who could predict its 1987 crash or a substantial collapse of junk bond market in 1990? What is the way to foresee future cardinal changes in the market or at least to feel that something of the sort is possible? How can one recognize that the certain small market movement is that very "butterfly wings blow"?

The satisfactory mathematical model of a financial market is still to be developed [27]. In the present paper we describe a new set -- theoretic approach and point to some of its applications.

Financial market may be considered as a very large amount of relatively small assets (participants) with ignorant influence of each asset on the whole market, but with recognizable influence of each asset on the behavior of some other assets which we will treat here as belonging to the former's "financial" neighborhood. The total market is divided into independently managed portfolios so that each asset may move between portfolios.

The world economy in the whole as well as any it's part may be represented in the form of a portfolio of assets, which behavior in time resembles capital redistribution among existing assets, new assets birth or existing assets disappearance. Birth and disappearance of assets may be treated as a partial case of capital redistribution if an *empty asset* (asset with zero capital) notion is involved.

Thus, a simple and clear model of capital redistribution may be constructed by the aid of a mathematical language of random set theory. This theory allows to clarify *setlike* properties of capital redistribution in a portfolio of assets, which were not seen before because of a *numberlike* tools usage. Presentation of a financial market as a global portfolio of assets, consisting of a collection of subportfolios, each controlled by independent financial manager in view of global portfolio evolution, is best suited for its description in the set - theoretic process form. The level of detailing may be adopted to researcher needs, so cents may be considered as elementary assets and bonds, stocks, etc. as well.

In the paper we suggest new *random setlike* tools, by which one can describe some features of financial processes, that cannot be taken into account by *numberlike* tools.

This approach allows to involve much more details of the underlying market events than usual quantity - valued models. The latter may be thought of as a result of roughening the set - valued models by using only their power (a number of elements) characteristics. On the other hand our approach directly shows the market's elements moving among portfolios: that cannot be achieved through other set -- valued approaches.

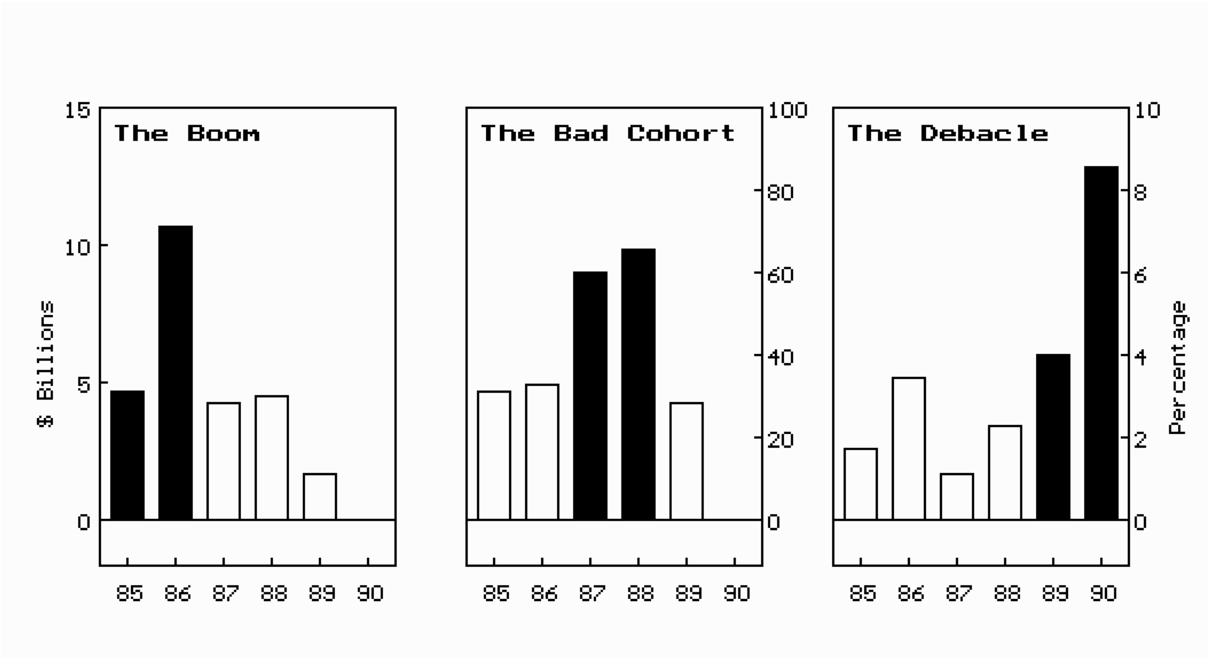
One of our explicit goals is to stimulate broad discussion and further research towards a better understanding of financial market random sets representation and further risk estimation within a full portfolio context. We have sought to make random set approach as competent as possible within an objective and workable framework in natural science and economics. However, we are certain that it is to be improved by comments from the broad community of researchers in the financial and actuarial fields.

Extensive previous work has been done towards developing random set methodologies for estimating various aspects of financial risk. In this paper, we give a brief survey of the academic research so that our efforts with random set approach can be put in context and researches can easily compare our approach to others. We group the previous academic research on financial risk random set estimation within three broad categories:

- modeling *random set processes* (time set-series) such as alphabetical categories for bonds or net capital inflows into mutual funds or new issues of bonds or events of default etc.;
- measuring volatility of random set (termed *random set-losses*) by new measure --- *Set-at-Risk* (generalized Value-at-Risk [25], [20], [18]) with the assumption of bond market level diversification or within the context of a specific portfolio that is not perfectly diversified;
- forecasting financial time set-series by the aid of random set tools.

Also, there have been several results on probabilities of alphabetical categories of bonds and transition matrices, which treat set of bonds as a result of migration set-processes in a set-theoretic framework. There is more recent work in this area which has focused on incorporating high yield bond spread in random set models. For random sets approach in forecasting, we have chosen to focus on Set-at-Risk rather than Value-at-Risk.

On macro-economic level researchers have found that aggregate default likelihood is set-correlated with set-measures of capital inflow and high yield bond issue cycles. Martin S. Fridson [11], [12], [13] and [14] correlates aggregate defaults with new high yield bond issues, and the latter with capital inflows into mutual funds in junk bond markets of the late 1980's.



Source: Investment Dealers Digest, Edward I. Altman,
Moody's Investor Service, Investment Company Institute,
and also: M.S. Fridson, 1991 [12]

Figure 1.1. Fridson's hypothesis.

Adds Fridson [12]: "Pending the descent of the financial archeologists onto the ruins of the 1980's bull market in high yield bonds, accompanied by hordes of graduate students equipped for a massive data-handling project, we offer one tantalizing set of graphs by way of support for our hypothesis (see Fig. \ref{fig1}). The top graph shows that net capital inflows into the high yield mutual funds peaked in 1986 (the "boom" of 1985--1986). During the next two years, a sharp decline was observed in the bond ratings of high yield new issues (the "bad cohort" of 1987--1988). Then in 1989, just as the percentage of debt rated *B*- or lower receded to its more customary range, the default rate began a sharp escalation (the "debacle" of 1989--1990)."

Here Fridson forms the working hypothesis about an inevitable connection between the "boom", the "bad cohort" and the "debacle" which follows in the inexorable succession. One of our goals is to confirm Fridson's hypothesis by results of random set financial market modeling.

2. Random set models

Rating alphabetical categories for bonds or net capital inflows into mutual funds, or new issues of bonds or events of default are driven by the probability distributions of specific random sets. The problem of estimating the *set-chance* of these events turned out so difficult that now many researchers devote all their efforts to it. We will discuss three approaches used in the theory:

- the accounting set-analytic approach which is used only by specific theorists now;
- new set-statistical methods which encompass many possibilities;
- the set-theoretic approach which is a common academic paradigm for risk events in financial markets.

It is worth emphasizing that random set approach is not another modeling, measuring and forecasting method. We think that implicit attempts of using random set approach has already been labeled by discrete rating alphabetical categories of bonds. Random set models can be fit to any categorical rating system having historical data. Actually we would argue that each rating system should be fit with its own random set model for financial risk processes. For some users with their own internal rating systems, this will be a necessary first step before applying random set approach to their portfolios.

Next we will give a survey of some known approaches to financial risk evaluation and management.

2.1 Accounting set-analytic approach

The two major U.S. agencies, S & P's and Moody's have published historical default likelihoods for their letter rating categories.

There have been many studies of the historical default frequency of corporate publicly rated bonds [3], [7], [29]. Random set studies [34] are indispensable, and it is important to highlight some relevant topics:

- the set-evolution and set-change in the original issue high yield bond market is unique in its set-history;
- future high yield bond issuance will be different in numerical sense but will be identical in set-sense and will be described by the same set-distribution;

Thus, usage of historical data must be accompanied with set-knowledge of how they were set-generated and what they set-represent.

2.2 Statistical forecasting of financial risks

There is a large body of usual statistically focused work devoted to building risk estimation models, which seek to predict future default. One can identify three basic approaches to estimating default likelihood [26]:

- quantitative dependent variable models;
- discriminant analysis;
- neural networks and genetic algorithms.

All of these approaches are strictly quantitative and will at least yield a ranking of anticipated default likelihoods and often can be tend to provide an estimate of default likelihood.

Value at Risk became quite popular during last decade; it is a stochastic quantity -- valued model for portfolio management [25], [18] and may be formulated as follows: determine the probability distribution for the profit/loss of a portfolio over a given horizon and then summarize the distribution with a single statistic. The quantile of a distribution is often used as the latter statistic. VaR is often treated as (almost sure) maximal possible loss. This model summarizes all assets behavior into a single distribution (or even a single number) on the construction stage, so it definitely looses the detailed behavior of the underlying market.

Linear *discriminant analysis* applies a classification model to categorize defaulted versus survived funds. In this quantitative approach a historical sample is assembled of defaulted funds and a matched collection of similar funds that had not default. Then, the statistical quantitative estimation approach is applied to identify which variables (and which combination !) can best classify funds into either group. This quantitative approach yields a continuous numerical score based on a linear function of relevant fund variables, which - with additional processing - can be mapped to default likelihoods.

The academic literature is full of alternative techniques ranging from principal component analysis, *self-organizing feature maps*, logistic / logit analysis to hierarchical classification models. All of these models can be shown to have some ability to distinguish high from low default likelihood funds. In [2], [4], [9] authors compare the predictive strength of these diverse techniques. All of these models are based on quantitative analysis and neither use the random set analysis nor even speak about it. The application of *neural network* techniques to risk estimating was considered intensively [8], [19], [30], [17]. Neural networks are represented by a set of "neurons" having the possibility to establish links with their neighbors. These networks may be taught by passing them an input data and telling them if their output data (answer) is proper. After teaching the network on a sequence of test problems it establishes interior links in such a way that may solve analogous problems. Anyway neural networks do have the disadvantage of being unable to explain their outcome; besides it is not obvious how to determine the degree of "goodness" of the network's output in the frames of our present problem.

There are applications of neural networks to some large volume risk decisions, but the application of these neural network techniques to large corporate risks seems to have not appeared yet.

Genetic algorithms [6], [15] represent rather new heuristic approach to solution of complex problems. They are set - valued and work as follows: given an initial (e.g. random) population, it is then stochastically developed in time in such a way that "good" members with large probability stay alive in future generations and even produce "children", residing not far from them, and "bad" members used to die (disappear from the population) with large probability. This approach is very effective but needs a measure of "goodness" defining all process behavior. Such a measure is naturally present in optimization problems - it is their goal function. Unfortunately, there is no similar natural measure for the processes of market development.

2.3 Random set approach to financial risk estimating

A literature on financial market models considered may be arranged into two groups: methods of quantity - valued and set - valued analysis. However, methods of random set approach to modeling, measuring and forecasting of financial risks are essentially distinguished from any one mentioned above. These methods use a notion of random set to describe financial events and to distinguish between high and low default likelihood funds. The random set approach was proposed in the context of forest fire spread forecasting [32], and subsequently developed in [34] to financial and actuarial applications. From this point of view a fund, as a random set, has a market covering probability evolving randomly through time along with evolution of the random set distribution as new set-information about future prospects of the fund and of other funds, capital flows and default events become known. Default occurs when the market covering probability of the fund falls low enough to its assets worth less than obligations. This approach has served as an academic set-paradigm for default set-risk, but it is also used as a basis for default set-risk estimation. There is no commercial exemplar of this approach. In general, this method yield a continuous numeric value such as the probability of default, which - without additional processing - can be mapped to default likelihoods.

Only externally the random set approach is similar to set - valued methods, for example, to self-organizing feature maps, neural network techniques and genetic algorithms. Essentially the random set approach is based on random set theoretic operations only in contrast to methods mentioned above which use exclusively quantitative analysis or stochastic numerics.

2.4 A portfolio view as a set view

Capital used to move from assets with less interest rate to assets with greater interest rate as well as from assets with large risk to assets with less risk. This contradictory but evident empirical observation represents two moving forces of capital redistribution.

Subportfolios include less sorts of assets with different interest rate and risk than the global portfolio. The latter has the global interest rate and risk, which are unknown but estimable with such integral characteristics as Dow - Jones index. Subportfolios have well defined interest rate and risk.

If interest rate and risk for each subportfolio coincide with the global interest rate and risk, then financial system is in equilibrium state and it cannot leave this state without external enforcement. We will consider preferably non-equilibrium systems here, meaning the global portfolio development through capital redistribution among subportfolio due to differences in interest rates and risks. Such models are not completely new and were considered before, but only *numberlike* tools were used in their investigation.

We consider a portfolio of assets as a set of assets. Any analysis of a set of exposures may be called a portfolio analysis. We use the term here in the sence of set-type analysis where the total set-risk of a portfolio is measured by explicit consideration of the relationships between individual set-risks and exposure amounts in set-variance - set-covariance framework. This portfolio analysis is a set-analog of a Markowitz-type analysis within variance -- covariance framework.

The classic type of analysis was originated by Harry Markowitz [23], and has subsequently gone through considerable development, primarily in application to equity portfolios. The set-type of portfolio analysis was originated in [34] and now it is making first steps.

A growing number of major institutions estimate the portfolio effect of financial risk only in a Markowitz-type framework now. Most institution still rely on an intuitive and quantitative assessment as to what level of cover concentration to any single area may cause trouble. The random set approach on the contrary offers to rely on a set assessment as to what set of cover concentration to any single area may cause trouble.

2.5 Migration analysis and set-migration analysis

Migration analysis is one of fundamental techniques of CreditMetrics [26]. Knowing today's rating category and transition matrix allows to estimate the possible mode of behavior in future. There exists a vast literature applying migration analysis to risk evaluation, e.g. papers on transition matrices by Professor Edward Altman of New York University [5] and by Lucas and Lonski of Moody's Investor Service [22] may be mentioned.

However migration analysis does not allow to take into account the behavior of other funds in financial neighborhood. Obviously, the behavior of surrounding items has significant influence on the fund's future.

In our random set approach the analogous method calculates the volatility of value due to risk quality *set-changes* rather than just quantitative changes. The great advantage of random sets approach is its ability to operate with *local transition random sets* while standard migration analysis operates only with global transition matrix. This prevents the latter from possibility to look "inside" the group (cohort, static pool) of assets, thus hiding the reasons of migration.

Well known problem [26] of the volatility of transition matrices during time is due to the fact that these matrices are rough approximation of a real financial situation. The random set approach is an underlier for quantitative analysis and it allows to find a set-invariants of real financial situation, *local transition random sets*, which model the volatility of transition matrix to be invariable itself. Thus, a matrix of *local transition random sets* used in the random set approach replaces a transition matrix used in CreditMetrics [26]. In other words the random set approach provides a "migration migration analysis" or a "double migration analysis", so called *set-migration analysis*, because it analyzes both migration of categories among other categories of risk quality and migration of transition matrices due to a drift of the real financial situation.

Default and risk quality migration.

A fundamental source of risk is that the risk quality of a bond may change over the risk horizon. "Risk quality" is commonly used to refer to only the relative chance of default. As we show here, however, random set approach makes use of an extended definition that includes also the volatility of this chance.

Existing rating systems typically assign an alphabetic or numeric label to rating categories. By itself, this only gives an ordinal ranking of the default likelihoods across the categories. A more quantitative framework, such as random set approach, can assign a value to each rating category by linking it with a default probability.

In academic research, even the definition of the default event has evolved over time. For example, default rates may be sufficiently different depending upon the population under study. If rates are tabulated for the first few years of newly issued tools, then default rate will be much lower than if the population broadly includes the full extent debt.

Fitting probabilities of default with a transition matrix.

Morgan utilizes risk ratings as an indication of the chance of default and rating migration likelihood. Based on historical default studies from risk rating systems, Morgan has transition matrices which include historically estimated one year default rates [26] There are two ways of using historical statistics depending upon historical information available. For tabulating a transition matrix one can use both individual rating histories and cumulative default histories by rating category[1].

Bond quality migration.

Risk rating migration can be thought of as Morgan's extension of the model of bond defaults [26]. Morgan states that a bond has some underlying value and changes in this value suggest changes in risk quality. The data driving this model are the default likelihood and risk rating migration likelihoods for each rating.

One can compactly represent these migration probabilities using a transition matrix model. Essentially a transition matrix is nothing more than a square table of probabilities. These probabilities give the likelihood of migrating to another rating category (or perhaps default) during unit period given the current bond rating.

Many practical financial and economical events can be triggered by a rating change. Thus, Morgan find it very convenient to explicitly incorporate awareness of rating migration into risk models [26].

Historical tabulation of transition matrices.

One can tabulate historical risk rating migration probabilities through time series of risk ratings over many bonds. There are several sources of transition matrices, each specific to a particular risk rating service. Morgan list three of these sources: Moody's, S&P's, and KMV. Moody's transition matrix is shown in Table 1a.

[1] {Moody's calls these aggregate groupings "cohorts" and S & P calls them "static pools".

Table 1a

Moody's Investors Service: one - year transition matrix (%)

Initial Rating	Aaa	Aa	A	Baa	Ba	B	Caa	Default
Aaa	93.40	5.94	0.64	0	0.02	0	0	0
Aa	1.61	90.55	7.46	0.26	0.09	0.01	0	0.02
A	0.07	2.28	92.44	4.63	0.45	0.12	0.01	0
Baa	0.05	0.26	5.51	88.48	4.76	0.71	0.08	0.15
Ba	0.02	0.05	0.42	5.16	86.91	5.91	0.24	1.29
B	0	0.04	0.13	0.54	6.35	84.22	1.91	6.81
Caa	0	0	0	0.62	2.05	4.08	69.20	24.06

Source: Lea Carty of Moody's Investor Service [7]

Rating risk profile across the bond market.

Another desirable fitting property of a transition matrix is proper exhibition of a long-term steady state that approximates the observed profile of the overall financial markets. By this Morgan means [26] that - among those bonds which do not default - there exists some distribution of their risk quality across the available risk rating categories. To represent the risk rating profile across the bond market, Morgan have taken the following data (Table 1b) from S&P's [29].

Table 1b

Morgan's probabilities of risk rating categories

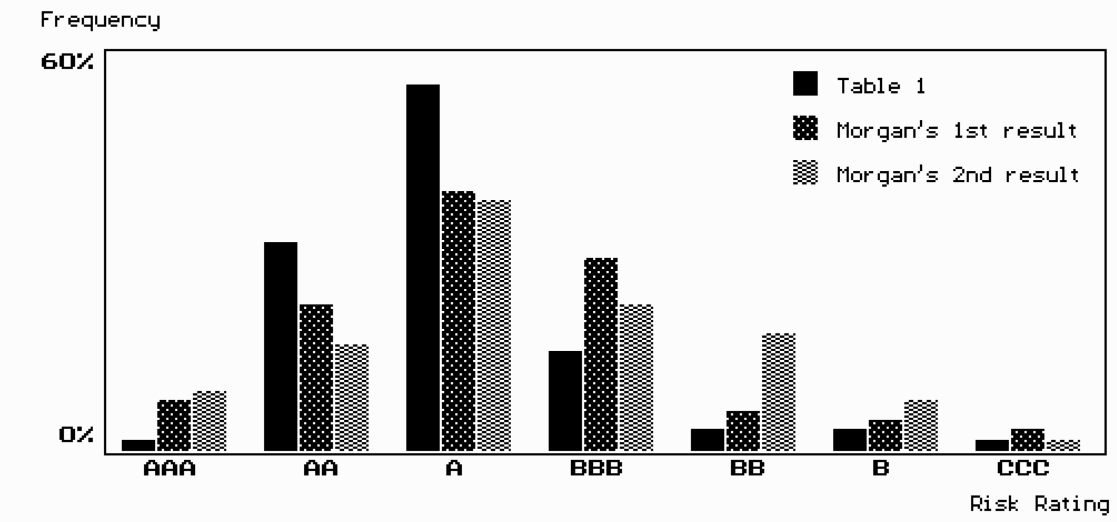
S & P 1996	AAA	AA	A	BBB	BB	B	CCC
Count	85	200	487	275	231	87	13
Proportion	6.2	14.5	35.3	20.0	16.8	6.3	0.9

Source: J.P.Morgan CreditMetrics [26].

Mathematically, Morgan's transition matrix Markov process will have two long-term properties (i.e., more than 100 periods).

- First, since default is an *absorbing* Markov state, eventually all bonds will default.
- Second, since the initial Markov state has geometrically less influence on future states, the profile of non-defaulted bonds will converge to some steady state regardless of the bond's initial rating.

As the chart below shows, Morgan's fitting algorithms can achieve a closer approximation of the anticipated long-term state. Though some Morgan's transition matrices show a tendency to migrate towards some categories more or less strong than one shown in Table 1b.



Source: J.P.Morgan's CreditMetrics, 1997 [26].

Figure 2.1. Morgan's Risk Rating Probabilities.

Morgan promises to move towards a better approximation of the historical transition matrix shown in Table 1a.

For this purpose Morgan assume a monotonicity (smoothly changing) transition matrices. Though it is certainly not a requirement of a transition matrix, Morgan's expectation is that there is certain rank ordering the likelihood of migrations as follows:

- Better ratings should never have a higher chance of default;
- The chance of migration should become less as migration distance (in rating notches) becomes greater; and
- The chance of migrating to a given rating should be greater for more closely adjacent rating categories.

Then Morgan brings all this assumptions together in Table 1c to give an estimate of one-year transition matrix which is rooted in the historical data (Table 1a) and is also sensitive to Morgan's expectation of long-term behavior.

Table 1c

J.P.Morgan: one-year transition matrix (%)

Initial Rating	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	87.74	10.93	0.45	0.63	0.12	0.10	0.02	0.02
AA	0.84	88.23	7.47	2.16	1.11	0.13	0.05	0.02
A	0.27	1.59	89.05	7.40	1.48	0.13	0.06	0.03
BBB	1.84	1.89	5.00	84.21	6.51	0.32	0.16	0.07
BB	0.08	2.91	3.29	5.53	74.68	8.05	4.14	1.32
B	0.21	0.36	9.25	8.29	2.31	63.89	10.13	5.58
CCC	0.06	0.25	1.85	2.06	12.34	24.86	39.97	18.60

Source: J.P.Morgan CreditMetrix [26].

This Morgan's transition matrix is meant to be close to the historically tabulated probabilities while being adjusted somewhat to better approximate the long-term behavior.

From the random set approach standpoint we see that there is noticeable and non-removable barrier for quantitative methods to better approximate the long-term behavior. This barrier is a setlike type of real financial space which cannot be properly described by quantitative methods only. There is no transition matrix which is rooted in the historical data and defined the rating migration. Quantitative transition matrices should have changes [1] which are rooted in a dynamics of historical financial space and cannot be estimated from historical data. At the same time transition set-matrices are invariants of financial market set-dynamics and can be estimated from historical data and can be more reliable.

3 Random set approach to financial market modeling

In the paper we use materials from universal guidance [26], where the concept of measuring credit risk exposed on the base of *Value-at-Risk* notion, default probabilities, and transition probabilities matrices for bonds migrating over quality rating categories.

To describe our financial market models we need some new mathematical notions from random set theory. We use the *random set redistribution process* as a model defined by a table of local random sets of redistribution, which form the so called *transition set-matrix*. Our random set model of capital redistribution yields all quantitative characteristics of Morgan's model CreditMetrics [26] as a result. Moreover the random set model observes the course of financial events in more details to fix them in an abstract financial space and to use them for forecasting future behavior of financial markets.

We will omit most mathematical details in what follows; however rigorous and exact mathematical model serves an underlier for our approach; it will be published separately [34].

3.1 Two portfolio descriptions of financial markets

Quantitative portfolio mathematical models and *setlike portfolio models* of financial markets are almost similar at a glance. The only distinction is the substitution of quantities by sets and quantitative vectors by set-vectors, collections of sets.

However, this formal trivial distinction results from essential substitution of the whole quantitative apparatus with fundamentally different set - oriented apparatus. This one cannot be generated by trivial analogy and requires a specific set theory techniques [33].

A novelty and an essence of set theory operations for portfolio analysis yields advantages pointed to in introduction: a simplicity of models and extension of practical problems sphere, which can be solved by them.

An advantage of random set models most clearly may be seen on dependency notion. If in order to describe a dependent quantitative behavior one uses a quantitative model, then almost always we can use more fundamental set model under assumptions of independence. Generally speaking, a random set independence cannot be described by independent quantitative models. The simplest example is a power of union of independent random sets, a random variable, which cannot be described as a sum or another combination of independent random variables (powers of summands), but which can be represented in some way as a sum of dependent random variables. Thus, an independent set model allows to describe such situations, that require assumptions of dependence for it's quantitative description. This simple remark is of great importancesince the power of a set serves as the main bridge from setlike models to quantitative results interested the user: these may be the amount of free capital in the market, the amount of capital invested into certain bond, defaults percentage, etc.

3.2 Random set models of financial processes

Although a mathematical model to be described has no close relation to processes in financial markets, the same or more complicated models can be applied to describe a capital redistribution among financial assets. As well these models may be applied to describe mathematically similar set processes from absolutely different practical fields.

As is well known, the most recognized world financial assets are permanently ranging by different rating agencies (S&P's, Moody's) with respect to their quality. The ratings are based on interest rates and risks of these assets. Among them there are alphabetic *categories, cohorts and pools* from "AAA" to "CCC" and "defaults" in order of increasing the risk of assets. Probabilities of defaults depending on asset category and time as well as matrices of probabilities of asset transition from one category to another at year end are permanently published.

Representing assets by sets of elements (stocks, bonds, other financial instruments) a process of capital redistribution among them may be represented by sets or collection of sets which are developing in time. One can think of a set of assets with "AAA" category and so on.

Our purpose is to construct a model of set redistribution, which can be used to describe the process of capital redistribution among financial assets. As soon as the capital redistribution process is only available as a result of statistical observations and cannot be described by some general law, the model of random set redistribution is involved.

We shall start with some general definitions. A process of random set redistribution is a sequence of random sets

$$\{K_t, t=0,1,...\},$$

forming Markov process, where random sets are related by

$$K_{t+1}=F(K_t).$$

Here the random set-to-set operator F is formed by the set - theoretic operations (union, intersection, complement etc.) with the state K_t, random neighborhoods of its elements and probably other random sets and related processes. Examples may be found in [32], [34], they are: random spread process, symmetric difference process and so on.

While random spread processes are usually described in terms of "fire", random redistribution processes would be simply described in terms of "finance". However, remind that this model can be used in other fields too.

3.2.1. Abstract financial space

First of all we need to find a simple and tractable economic and probabilistic approach to financial market modeling [27]. This section attempts to highlight fundamental properties that a market model should possess. Assuming these properties it is possible to establish a corresponding interactive market dynamics that involves a minimal set of factors.

One can imagine a *real financial space* consisting of primary financial elements (PFEs), for example: capitals, bonds and other financial instruments. Evidently, there is a very complicated structure of financial dependencies between these PFEs. Each of them may depend on neighbor PFEs. Thus, each of PFEs defines it's financial neighborhood of PFEs, a set of PFEs. In practice this system of financial neighborhoods is hardly observed. Obviously one can only observe some small fragments of a global financial dependencies picture. Usually quantitative characteristics of global behavior of sets of PFEs are readily available. For example, one can observe a volume of capital, a volume of new issues of bonds, and a volume of defaults. Now one can try to solve the so called inverse problem: reconstruct an *abstract financial space* corresponding to real quantitative characteristics. Such reconstruction of the abstract financial space can be considered as a set model of real financial space.

3.2.2. A primitive random set model

We shall start with a primitive model with a set of bonds at time t as a random set process state. Thus, the abstract financial space is roughly divided into two parts: "bonds" B_t and "non-bonds", the latter being the set theoretic complement of bonds set.

Capital redistribution between bonds and non-bonds takes the form of symmetric difference process in this model. This process has the property to attract non-bonds from its neighborhood and repulse some bonds in a random way.

In the simplest case random neighborhoods may be assumed to be totally independent and having a point - independent distribution. Then all *probabilities of local redistribution* together with a distribution of initial random set B_0, define a random redistribution process completely since it is Markov process.

It is interesting to note the following features of the primitive model. Each bond adds new elements to B_t from non-bonds part (these may be interpreted as newly issued bonds at time t) and repulses to non-bonds another random set of financial elements (these ones go to category "defaults").

Note two important things.

- A process of new bonds issues can only be the result of existance of a free capital.
- A process of "defaults" is disappearing bonds from financial space. Of course, this process is accompanied by capital outflow from a given financial space.

3.2.3. A model of redistribution

Now we have several set processes, call them a *set process of bonds*, a *set process of capital deficits* a *set process of capital surpluses* and a background *set process of free financial elements*, that models a capital flow through abstract financial space considered.

Everything is ready to construct a model of capital redistribution within some abstract financial space. This model is described by system of four recurrent interconnected set relations:

- first describes a process of capital surpluses generated by capital flow within free financial elements and capital deficits;
- second describes a process of capital deficits generated by capital flow within free financial elements and bonds, the latter being a result of "defaults" of bonds;
- third describes a process of bonds generated by capital flow within capital surpluses and disappearance of bonds into "defaults";
- fourth describes a passive process of free financial elements, a part of financial space free from capital surpluses, capital deficits and bonds.

Consider more details each set process bringing to the model of capital redistribution within the abstract financial space.

First of all we give a general note. All of three processes include as a part the well known [32] set model of random spread, which is defined by random sets, describing a local behavior of processes within corresponding neighborhoods of a point of abstract financial space.

Process of capital surpluses.

This process is represented in the form of a random spread process, describing a *capital surpluses growth* within free financial elements and capital deficits. Note, that by the aid of this process one can describe a process of *capital inflow* as well. In the context random financial neighborhoods describe a local capital redistribution within a neighborhood of an element during unit time.

Process of bonds.

This process is represented in the form of random spread process describing *bonds growth* within capital surpluses. Note, that a *process of new bonds issues* can be described by these processes too. Here random financial neighborhoods describe a local bonds growth per time unit.

Process of capital deficits.

This process is defined by a random spread process describing a spread of capital deficits within free financial elements and bonds. Here random financial neighborhoods describe a local spread of capital deficits. Note, that a process of bonds defaults is also described by this process.

Process of free financial elements.

This process is defined by a background random spread process describing a passive spread of free financial elements, not occupied by other processes.

It is worth reminding here the classic quantitative models "shark - victim" introduced by Lotka [21] and Volterra [31]. These models defined by ordinary differential equations describes behavior of number of elements which has an oscillatory mode. The desire to take into account random fluctuations in the model "shark - victim" had led to birth and death processes, describing behavior of random number of elements that are well known in probability theory [10]. Set model of capital redistribution suggested here are similar to models "shark - victim" and birth and death processes, but it is more primary and describe a behavior of random set of elements by set theoretic operations only. Differences between possibilities of set theoretic tools and a traditional mathematical analysis or random process theory allow to catch random set features of process behavior, this is unavailable under quantitative analysis. At the same time we observe a profound connection between these mathematical descriptions: from Lotka's and Volterra's deterministic model of quantitative oscillations to our stochastic set model of random set redistribution. This connection can be explained by qualitative progress of modern mathematical theory.

3.2.4. The random set model of capital redistribution

In this section a simple class of random set models of real financial space is presented, as setlike, stochastic dynamic models that can be effectively used to describe the dynamics of financial markets. There are many factors forming financial markets. For simplicity in the paper only one factor, capital flows, is considered to show how the random set model may describe three interactive quantitative processes: capital inflows, new issues of bonds and defaults [13], [12]. Thus, here we consider a process of capital flows to be a main financial process defining a dynamics of financial markets.

Consider an abstract financial space X and subsets of this space occupied by capital, and random subset of the capital occupied by bonds at time t.

We have three sorts of capital in the financial space: a capital surpluses, a capital bound within bonds and a capital deficits.

- A developing *capital surpluses* includes an addition of a capital newly generated and a deletion of a capital turned out to be within bonds.
- A developing *capital bound within bonds* is defined by two processes. The first one is a new bonds issues resulting in transformation of some part of capital surpluses to capital bound within bonds. The second one is a capital disappearance from bonds explained by defaults.
- A developing *capital deficits* includes an addition of a capital deficits newly generated within bonds (so called defaults) and free financial elements and a deletion of a capital deficits turned out to be within capital surpluses.

All of these capital properties can be described by the random set model of capital redistribution within abstract financial space *X*, or by the so called random set-vector process. We consider a capital redistribution process between four elements: capital surpluses elements, capital deficits elements, bond elements and free financial elements in the abstract financial space *X*.

Now we are ready to construct a model of capital redistribution within some abstract financial space *X*. This model is described by the system of recurrent interconnected set theoretic relations for only the process of capital redistribution. Set theoretic description of a *process of defaults* and a *process of new bonds issues* are obtained as a set theoretic differences of capital redistribution components.

Note that the process is the vector-generalization of well known set model of random spread, introduced for forest fire spread modeling [32]. The process is defined by corresponding random sets, describing their local behavior within neighborhoods of an element of financial space and also by random sets describing processes of "births" and "deaths" of capital surpluses, capital deficits and bonds within corresponding financial elements.

3.2.5. Set-vector case

In contrast to random spread processes described by a simple recurrent set theoretic relation modeling fire spread [32] one can present some ways to generalize this model. It is possible to introduce a model of independent generation of new process elements. This model is defined as a random set with distribution independent of a set state of capital process. Then the simplest random set process may be defined as this very random set. This is a process of independent random set states. The generating model can be included into a classic scheme of random spread to obtain a random spread process by union operation with spread and independent random set states processes.

PFEs with different properties may exist in financial space. For example, they can be capital surpluses, bonds and capital deficits and each bond can also have different category of quality from say "AAA" to "CCC". Thus, here we get the so called dynamic financial space consisting of several nonintersecting parts. These parts may be active and passive (as a background for a process). For example, active parts may be capital surpluses, capital deficits and bonds in different categories of quality and passive part may be a set of PFEs free from capital surpluses, capital deficits and bonds. It is necessary to use more sophisticated random set model to describe capitals flows within the dynamic multipart medium *X*. To model capital flows within this medium we introduce the system of interactive random spread subprocesses (capital surpluses, capital deficits and bonds), defined by transition matrices of local random sets with elements, describing spread of a subprocess within another subprocess.

Characteristic condition of non-intersecting for set-vector case.

Note that local random sets (financial neighborhoods) have to be nonintersecting in pairs. This condition is very important for a definition of a set-vector random spread process. One can say that this condition characterizes a random set-vector model. Thus, due to characteristic condition we obtain a system of dependent subprocesses. The condition allows to correctly define a probability of transition from one subprocess to another. For the sake of simplicity we usually assume that each subprocess does not develop within itself.

4 Set-at-Risk financial analysis

A set approach can be used for financial risk estimating. A notion of *Set-at-Risk* defined as a quantile, or a set-quantile of a random set and is a new set-measure of financial risk. We give the only simple example of Set-at-Risk measuring. Consider modeling a migration of bonds among categories of quality by random redistribution process. Note that if the distribution of a random set of bond is known then for each financial element a probability p for it to be a bond is defined. A set of financial elements for which the probability p is less than a is a a-quantile of random set "Bonds" and is called *Set-at-Risk* set-measuring a risk quality of "Bonds". This probability p may be treated as an indicator of financial element's quality. In the following table for example:

$$1>a_{AAA}>a_{AA}>a_{A}>a_{BBB}>a_{BB}>a_{B}>a_{CCC}>0$$

are ranging parameters, defined from historical statistics. Hence a change of the probability defines a migration of financial elements among *categories, cohorts and pools* mentioned above.

Table 2a

Risk rating categories as intervals of probabilities

Categories	
AAA	$1 \geq p > a_{AAA}$
AA	$a_{AAA} \geq p > a_{AA}$
A	$a_{AAA} \geq p > a_{A}$
BBB	$a_{A} \geq p > a_{BBB}$
BB	$a_{BBB} \geq p > a_{BB}$
B	$a_{BB} \geq p > a_{B}$
CCC	$a_{B} \geq p > a_{CCC}$
Default	$a_{CCC} \geq p > 0$

5 Conclusions

In the paper a random set technology is catching up with "booms", "bad cohorts" and "debacles". Sophisticated random set modeling approach is helping financiers and researchers better understand how, when and where capital flows transform into defaults. Knowing how capital flows gives them clue as to whether "booms", "bad cohorts" or "debacles" are set. The model is getting more into the set-dynamics of capital flows, how it propagates throughout a financial set-structure, so that we have a better understanding of its properties. Knowing how long a capital redistribution process takes to progress from stage of "boom" to the next stage of "bad cohort" can give researchers a clear understanding of when a "debacle" will started.

The computer code for the random set models of capital redistribution is now available for simulation of financial markets using PC. The random set approach today involves a lot of technology and science. Modeling provides information on how a capital should redistribute through a financial set-structure based on financial neighborhoods set-configuration. When the program is ready for release, probably early next year, users will be challenged by the computer to use "best practices" in investigating a computerized financial space. The computer tracks their progress and tells them where their deficiencies are.

Advantages and novelties of random set approach to financial market modeling:

- introducing two new "risk rating categories" ("capital surpluses" and "free financial elements") for elements of financial space to confirm Fridson's hypothesis about bond markets;
- working out set-migration analysis with it's probabilities of local migration and with it's migration set-matrix combined from local random sets;
- discovering a close connection between set-migration analysis and migration analysis in the sense of J.P.Morgan;
- possibility of solution of inverse problems to find of set-migration observed hardly from migration parameters observed easily;
- possibility of solution of inverse problem to reconstruct an abstract financial space considered as a set-imprint, a set-cast or a set-photo of real financial space observed hardly;
- proposing the set-photo of real financial space now as a base to forecast a behavior of financial markets in future.

Confirmation of Fridson's hypothesis about U.S. collapse'90 by random set approach.

Fridson's hypothesis is verified by the random set model of capital redistribution. The "boom", the "bad cohort" and the "debacle" of the late 1980's are quantitative results of random set processes in abstract financial space. Random set approaching results are shown in Figures \ref{fig3}, \ref{fig4}, \ref{fig5}, \ref{fig7} and \ref{fig6} and Tables 3, 4 and 5.

Of course, we consider Figure \ref{fig7} as a "set-joke" because future and retrospective extrapolations are only possible under an assumption of the same conditions during 126 years from 1930 to 2056 as well as during 1985 - 1990. Though the latter is not likely a general idea of random set forecasting one financial stage ("debacle") starting from properties of others ("boom" and "bad cohorts") is remained to be true.

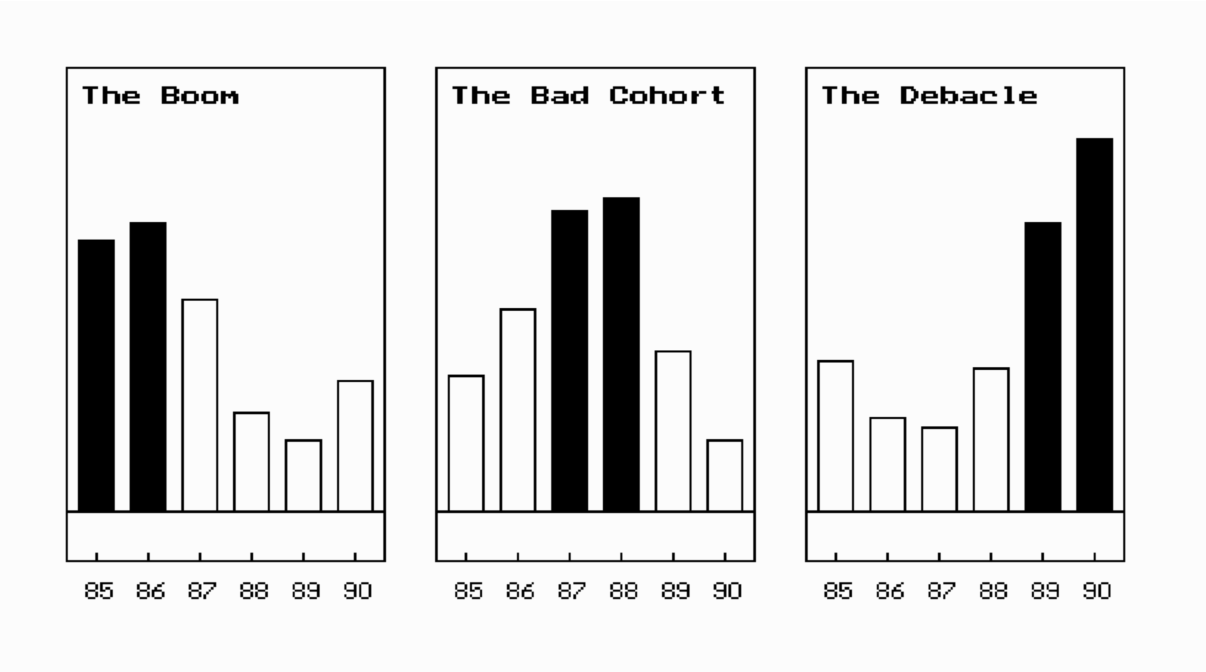


Figure 5.1. Random set approaching confirmation of Fridson's hypothesis.

Left --- The Boom'85--86; Center --- The Bad Cohort'87--88; Right --- The Debacle'89--90.

Random set modeling Russian default'98.

We have attempted to use our random set approach to model Russian default'98. Unfortunately, we haven't had enough reliable statistics about all of three main financial processes of capital redistribution: *Capital Surpluses*, *Capital within Bonds* and *Capital Deficits*. The only statistics about *Capital within GKO (Russia Government Treasure Bonds)* was accessible and reliable during last years. Dealing with the only statistics about the "Russian Bad Cohort" one can accurately model the only duration of the "Russian Boom" and the "Russian Debacle'98". Results of our attempts: estimations of a duration of "Russian Debacle'98" and a duration of preceding "Russian Boom", corresponding transition set-matrices and probabilities of risk rating categories, abstract maps of set-changes in Russian financial space'98 will be published separately.

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Appendix

Table 3a

Random set approach's probabilities
of risk rating categories during the Boom'85-86.

Surpluses	AAA	AA	A	BBB	BB	B	CCC	Deficits
6.88	0.00	1.94	7.96	20.43	26.24	15.05	16.99	4.52

Table 3b

Random set approach's probabilities
of risk rating categories during the Bad Cohort'87-88.

Surpluses	AAA	AA	A	BBB	BB	B	CCC	Deficits
5.44	4.08	17.23	29.01	21.49	12.69	5.53	2.90	1.63

Table 3c

Random set approach's probabilities
of risk rating categories during the Debacle'89-90.

Surpluses	AAA	AA	A	BBB	BB	B	CCC	Deficits
1.25	1.88	10.37	18.25	22.50	19.88	10.00	6.88	9.00

Table 5a.

Local Spread Probabilities
from *i*-th sort PFE x within *j*-th sort PDEs y_k
which define Local Random Set of Redistribution S^{ij}
within neighborhoods of 25 PFEs

$p^{ij}(y_1)$	$p^{ij}(y_2)$	$p^{ij}(y_3)$	$p^{ij}(y_4)$	$p^{ij}(y_5)$
$p^{ij}(y_6)$	$p^{ij}(y_7)$	$p^{ij}(y_8)$	$p^{ij}(y_9)$	$p^{ij}(y_{10})$
$p^{ij}(y_{11})$	$p^{ij}(y_{12})$	$p^{ij}(x)$	$p^{ij}(y_{13})$	$p^{ij}(y_{14})$
$p^{ij}(y_{15})$	$p^{ij}(y_{16})$	$p^{ij}(y_{17})$	$p^{ij}(y_{18})$	$p^{ij}(y_{19})$
$p^{ij}(y_{20})$	$p^{ij}(y_{21})$	$p^{ij}(y_{22})$	$p^{ij}(y_{23})$	$p^{ij}(y_{24})$

Table 4a
Random set approach's transition matrix during the Boom'85-86

	Surpluses	AAA	AA	A	BBB	BB	B	CCC	Deficits
Surpluses	0.00	0.00	0.00	0.00	15.63	31.25	21.88	31.25	0.00
AAA	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
AA	0.00	0.00	100.0	0.00	0.00	0.00	0.00	0.00	0.00
A	0.00	0.00	0.00	88.57	11.43	0.00	0.00	0.00	0.00
BBB	0.00	0.00	0.00	6.45	82.80	8.60	0.00	0.00	2.15
BB	0.00	0.00	0.00	0.00	11.11	81.75	3.97	0.00	3.17
B	0.00	0.00	0.00	0.00	0.00	12.50	69.32	9.09	9.09
CCC	0.00	0.00	0.00	0.00	0.00	0.00	4.88	86.59	8.54
Deficits	21.63	0.00	0.00	0.00	0.00	0.00	0.00	0.00	78.37

Table 4b
Random set approach's transition matrix during the Bad Cohort'87-88

	Surpluses	AAA	AA	A	BBB	BB	B	CCC	Deficits
Surpluses	0.00	0.00	5.00	11.67	15.00	41.67	16.67	10.00	0.00
AAA	0.00	91.11	6.67	0.00	0.00	0.00	0.00	0.00	2.22
AA	0.00	2.14	89.84	8.02	0.00	0.00	0.00	0.00	0.00
A	0.00	0.00	6.51	89.73	2.40	0.00	0.00	0.00	1.37
BBB	0.00	0.00	0.00	18.38	79.49	0.85	0.00	0.00	1.28
BB	0.00	0.00	0.00	0.00	24.72	70.22	2.25	0.00	2.81
B	0.00	0.00	0.00	0.00	0.00	18.46	73.85	4.62	3.08
CCC	0.00	0.00	0.00	0.00	0.00	2.38	21.43	69.05	7.14
Deficits	24.44	0.00	0.00	0.00	0.00	0.00	0.00	0.00	75.56

Table 4c
Random set approach's transition matrix during the Debacle'89-90

	Surpluses	AAA	AA	A	BBB	BB	B	CCC	Deficits
Surpluses	0.00	0.00	0.00	0.00	0.00	30.00	50.00	20.00	0.00
AAA	0.00	46.88	43.75	6.25	0.00	0.00	0.00	0.00	2.22
AA	0.00	0.00	71.13	26.80	0.00	0.00	0.00	0.00	0.00
A	0.00	0.00	0.00	60.82	30.41	0.52	0.00	0.00	1.37
BBB	0.00	0.00	0.00	0.00	67.05	25.57	0.00	0.00	1.28
BB	0.00	0.00	0.00	0.00	1.97	71.77	14.47	0.00	2.81
B	0.00	0.00	0.00	0.00	0.00	4.94	62.96	22.22	3.08
CCC	0.00	0.00	0.00	0.00	0.00	0.00	12.07	63.79	7.14
Deficits	15.37	0.00	0.00	0.00	0.00	0.00	0.00	0.00	75.56

Table 5
Transition Set-Matrix
for Random Set Redistribution Process
with 4 integrated financial areas of PFEs

	Capital Surpluses	Capital within Bonds (AAA,...,CCC)	Capital Deficits	Free Financial Elements
Capital Surpluses	E	E	S^{CD}	S^{CF}
Capital within Bonds (AAA,...,CCC)	S^{BC}	E	E	E
Capital Deficits	E'	S^{DB}	E'	S^{DF}
Free Financial Elements	E	E	E	E

$E=$

0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000
0.000	0.000	1.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000

$E'=$

0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000
0.000	0.000	1.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000

$S^{CD}=$

0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000
0.000	0.000	1.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000

$S^{CF}=$

0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000
0.000	0.000	1.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000

$S^{DB}=$

0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000
0.000	0.000	1.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000

$S^{DF}=$

0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000
0.000	0.000	1.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000

$S^{BC}=$

0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000
0.000	0.000	1.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000

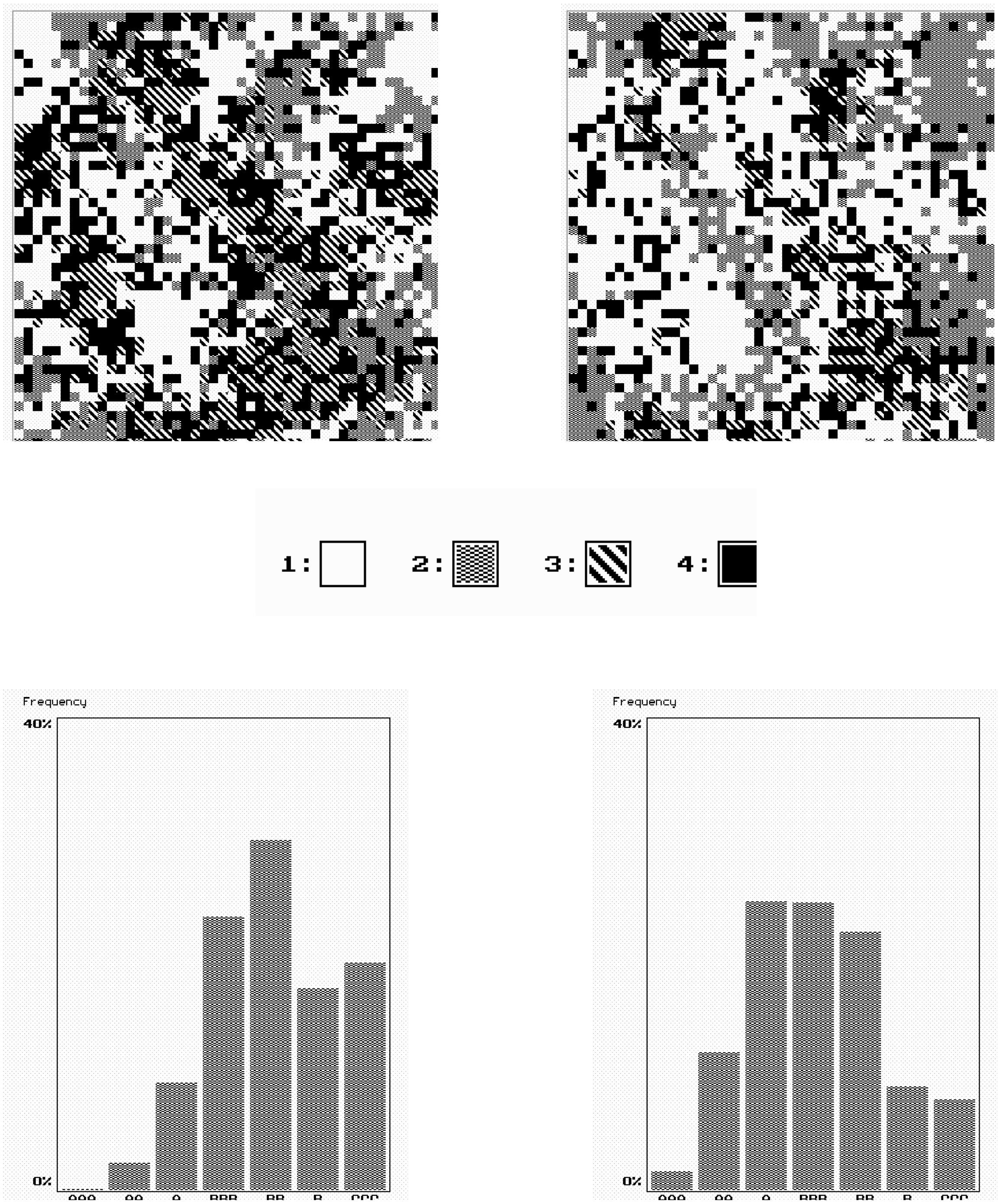

Figure 6.1: Financial Maps of Capital Redistribution Process during the Boom (left, *December'85*) and during the maximum of Capital Surpluses (right, *December'86*). 1 - Capital Surpluses, 2 - Capital within Bonds, 3 - Capital Deficits, 4 - Free Financial Elements. Below Probabilities if Risk Rating Categories of Bonds corresponding *December'85* and *December'86* are shown.

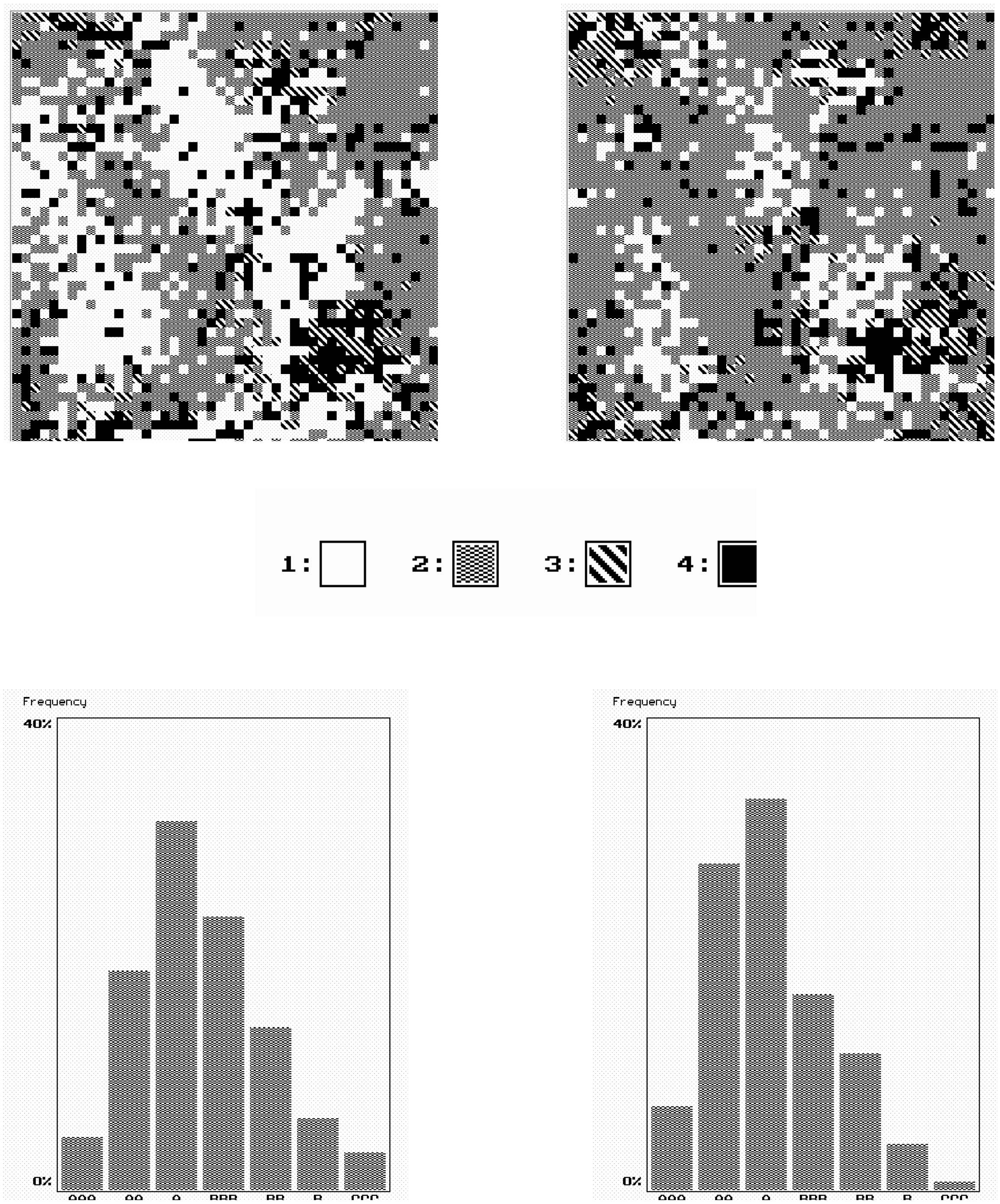

Figure 6.2: Financial Maps of Capital Redistribution Process during the Bad Cohort (left, *October'87*) and during the maximum of Capital within Bonds (right, *October'88*). 1 - Capital Surpluses, 2 - Capital within Bonds, 3 - Capital Deficits, 4 - Free Financial Elements. Below Probabilities if Risk Rating Categories of Bonds corresponding *October'87* and *October'88* are shown.

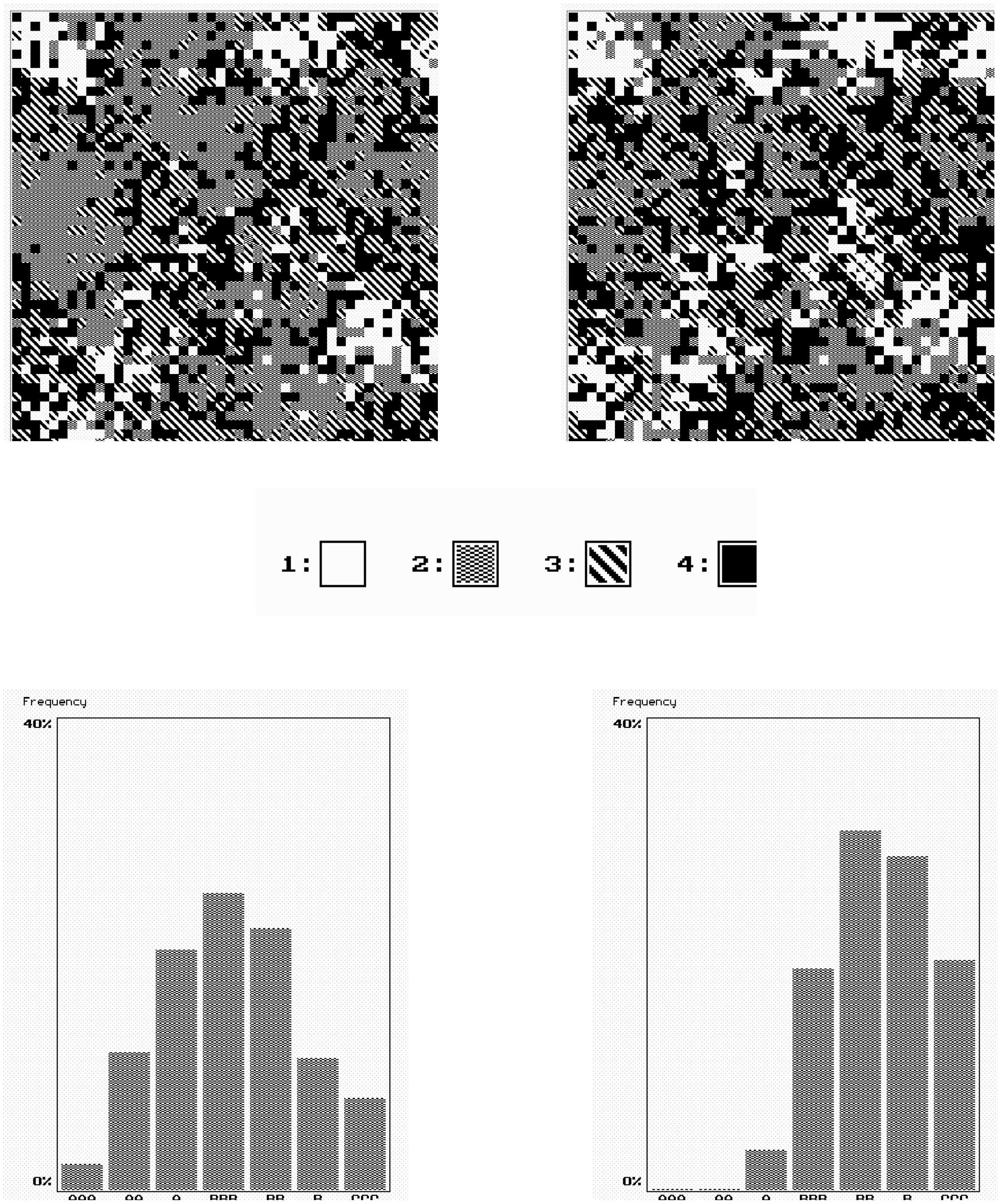

Figure 6.2: Financial Maps of Capital Redistribution Process during the Debacle (left, *August'90*) and during the maximum of Capital within Bonds (right, *February'91*).
1 - Capital Surpluses, 2 - Capital within Bonds, 3 - Capital Deficits, 4 - Free Financial Elements. Below Probabilities if Risk Rating Categories of Bonds corresponding *August'90* and *February'91* are shown.

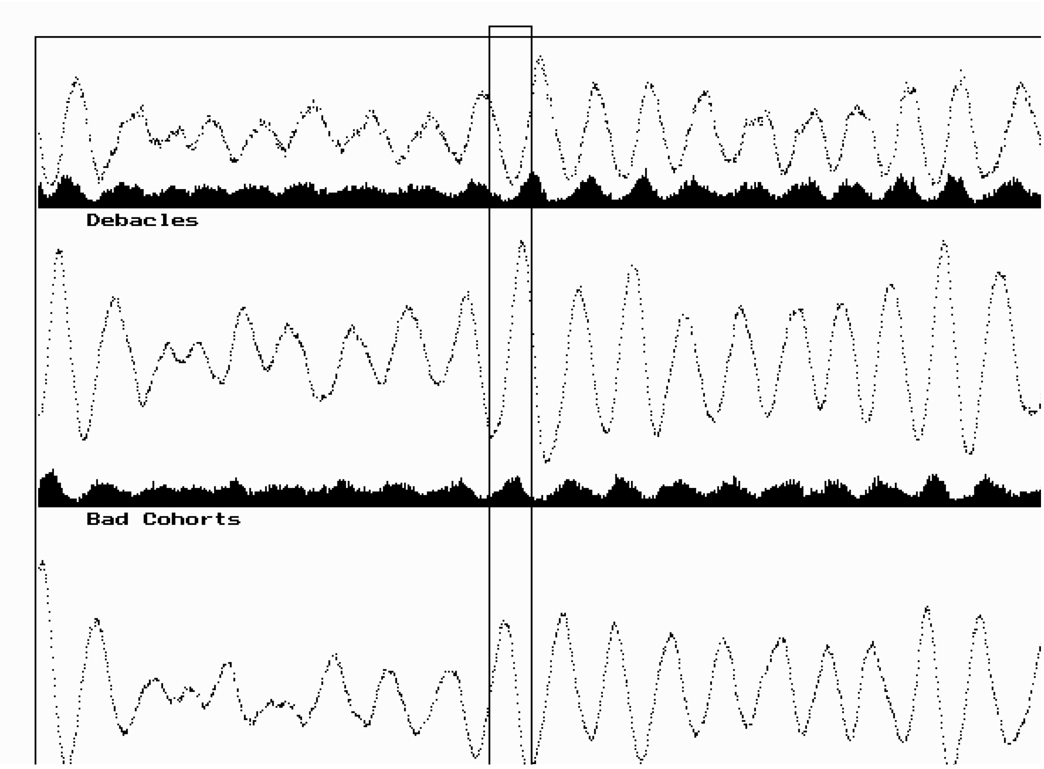

Figure 6.4: A dynamics of three pairs of quantitative characteristics of capital redistribution process during 1930 - 2056. Below - Capital Surpluses and Volume of Free Capital (dashed line); Center - New Issues of Bonds and Volume of Bonds (dashed line); Above - Defaults and Volume of Capital Deficits (dashed line).