Modeling spatial variations in household disposable income with Geographically Weighted Regression

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MODELING SPATIAL VARIATIONS IN HOUSEHOLD DISPOSABLE INCOME WITH GEOGRAPHICALLY WEIGHTED REGRESSION[1]

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ABSTRACT

The purpose of this paper is to analyze the spatially varying impacts of some classical regressors on per capita household income in Spanish provinces. The authors model this distribution following both a traditional global regression and a local analysis with Geographically Weighted Regression (GWR). Several specifications are compared, being the adaptive bisquare weighting function the more efficient in terms of goodness-of-fit. We test for global and local spatial instability using some F-tests and other statistical measures. We find some evidence of spatial instability in the distribution of this variable in relation to some explanatory variables, which cannot be totally solved by spatial dependence specifications. GWR has revealed as a better specification to model per capita household income. It highlights some facets of the relationship completely hidden in the global results and forces us to ask about questions we would otherwise not have asked. Moreover, the application of GWR can also be of help to further exercises of micro-data spatial prediction.

Key words: Geographically Weighted Regression (GWR), spatial non-stationarity, spatial prediction, income, Spanish provinces.

JEL Classification: **C21, C51, C53, R12**

INTRODUCTION

In regional science, the use of linear regression as an analytical technique has been long widely generalized. However, the explicit incorporation of "space" or "location" has not been that commonly considered. In this context, there has been recently a surge in econometric work focusing on the inclusion of spatial effects in econometric models. One strand of this literature has developed several approaches to incorporate the spatial non-stability. For a specific model, the assumption of stationarity or structural stability over space has been recognized highly unrealistic, accepting the possibility that parameters may vary over the study area. Several methods deal with this issue. Spatial analysis of variance (Griffith 1978, 1992) and models with structural change (Anselin 1988, 1990) are good examples of methods accounting for discrete spatial regimes, whilst the spatial expansion model (Casetti 1972, 1986) and the spatial adaptive filtering (Foster and Gorr 1986, Gorr and Olligschlaeger 1994) concern mostly about continuous variation over space. Getis and Ord (1992) and Openshaw (1993) study local patterns of association (hot spots) and local instabilities. Locally weighted regression method and kernel regression method (Cleveland 1979, Casetti 1982) focus mainly on the fit of a regression surface to data, using a weighting system that depends on the location of the independent variables.

Along this line of thinking, in the spatial econometrics literature, McMillen (1996) and McMillen and McDonald (1997) introduced nonparametric locally linear regression in models where the cases are geographical location and Brunsdon *et al.* (1996) labelled them as "Geographically Weighted Regression" (GWR). Hence, GWR produces locally linear regression estimates for every point in space. For this purpose, weighted least squares methodology is used, with weights based on the distances between observations i and all the others in the sample. GWR allows the exploration of the variation of the parameters as well as the testing of the significance of this variation.

This methodology has recently received intensive attention (Brunsdon *et al.* 1996, 1999; Fotheringham and Brunsdon (1999); Fotheringham *et al.* 1997, 1998, 2002; Leung *et al.* 2000a; Huang and Leung 2002, Paez *et al.* 2002(a,b); Yu and Wu 2004, to name a few). Furthermore, LeSage (1999) introduces a Bayesian approach (BGWR) that subsumes GWR as a special case of a much broader class of spatial econometric models. This methodology overcomes outliers and weak data problems using robust estimates and permitting the introduction of subjective prior information, improving at the same time the inference procedure. Some example of the application of GWR methodology can be found in Fotheringham *et al.* (2000), Huang and Leung (2002), Yu and Wu (2004), Mennis and Jordan (2005), Kentor and Miller (2004), Eckey *et al.* (2005), Bivand and Brunstad (2005), Yu (2006), etc.

The purpose of this paper is to analyze the spatially varying impacts of some classical regressors on per capita household disposable income of Spanish provinces. This variable, which is considered a good indicator of regional welfare, has frequently been estimated for microterritorial units (e.g. municipalities) from aggregate data (e.g. provinces, regions). In some countries with scarce availability of economic macromagnitudes for micro-level units, such as Spain or Portugal, there is an interesting literature in what are called "indirect methods of income estimation" (see Chasco 2003, pp. 178 for a broad revision). These are particular cases of the ecological inference topic, which consists in making inferences about individual behavior drawn from data about aggregates. Clearly, observations at an aggregated level of analysis do not necessarily provide useful information about lower levels of analysis,

particularly when spatial heterogeneity (non-stationarity) is present (Peeters and Chasco 2006[2]). For that reason, we propose to test spatial variation of the determinants of household disposable income across Spanish provinces (nut3). If this effect was present, it should be considered, particularly in future ecological inferences of municipal income.

Therefore, we model the provincial household income distribution following a basic global regression, which was also re-specified to take into account spatial effects. We found clear evidence of spatial instability in the distribution of the residuals that could not be totally solved by global specifications. Since spatial autocorrelation in the residuals may be implied by some kind of spatial heterogeneity not always correctly modeled by spatial dependence specifications, we propose the re-specification of the basic model using GWR.

The outline of the paper is as follows. In section 2, we present a theoretical framework for GWR. Section 3 offers the empirical framework: data, econometric results and spatial non-stationarity tests are displayed. Finally, section 4 concludes.

1. GEOGRAPHICALLY WEIGHTED REGRESSIONS (GWR)

2.1. Specification

The technique of linear regression estimates a parameter β that links the independent variables to the dependent variable. However, when this technique is applied to spatial data, some issues concerning the stability of these parameters over the space come out/arise. There have been several approaches to incorporate this spatial non-stability in the model. One of them, developed by Brundston *et al.* (1996), has been labelled Geographically Weighted Regressions (GWR). It is a non-parametric model of spatial drift that relies on a sequence of **locally linear regressions** to produce estimates for every point in space by using a sub-sample of data information from nearby observations. That is to say, this technique allows the modelling of relationships that vary over space by introducing distance-based weights to provide estimates of β_{ki} for each variable k and each geographical location i.

An ordinary linear regression model can be expressed by:

$$y_i = \beta_0 + \sum_{k=1}^{M} \beta_k x_{ki} + u_i \tag{1}$$

where $y_{i,}$ $i = 1, 2, ..., N$ are the observation of the dependent variable y, β_k ($k = 1, 2, ..., M$) represents the regression coefficients, x_{ki} is the ith value of x_k and u_i are the independent normally distributed error terms with zero mean and constant variance.

In matrix notation:

$$y = \beta_0 + \sum_{k=1}^{m} \beta_k x_k + u \tag{2}$$

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[2] In this paper, the authors demonstrated the existence of spatial instability in per capita GDP across Spanish regions (nut2), as well of varying relations in the explanatory variables of a production-function Cobb-Douglas model.

with y as vector of the dependent variable, x_k as vector of the kth independent variable and u as vector of the error term.

In geographically weighted regression the global regression coefficients are replace by local parameters:

$$y_i = \beta_{0i} + \sum_{k=1}^{m} \beta_{ki} \cdot x_{ki} + u_i \tag{3}$$

where β_{ki} ($k = 1, 2, ..., M$) are the regression coefficients for each location i and each variable k. It is worth notice that the ordinary linear regression model is actually a special case of the geographically weighted regression function where the β_{ki} are constant for all $i = 1, 2, ..., N$.

2.2. Estimation

In this GWR-function, however, there are more unknown parameters than degrees of freedom. Thus, the local estimates are made using weighted regression, with weights assigned to observations being function of the distance from point i. Algebraically, the GWR estimator is then:

$$\hat{\beta}_i = \left(X'W_i X \right)^{-1} X'W_i y \tag{4}$$

with $\hat{\beta}_i = \begin{pmatrix} \hat{\beta}_{i0} & \hat{\beta}_{i1} & ... & \hat{\beta}_{iM} \end{pmatrix}'$

and $W_i = \begin{bmatrix} w_{i1} & ... & 0 & 0 \\ 0 & w_{i2} & ... & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & ... & w_{iN} \end{bmatrix}$

where w_{ij} denotes the weight of the data at point j on the calibration of the model around point i. These weights will vary with i, which distinguishes GWR from traditional Weighted Least Squares where the weighting matrix is fixed.

2.3. Weight matrix specification

The role of the **weighting matrix** in GWR is to represent the importance of individual observations among locations. For that, it is necessary to choose a weighting scheme. It is commonly assumed that areas close to a location i will exert more influence on the parameter estimates at location i than those farther away from it.

- **Inverse distance function**

Many options meet this criterion. One possibility is to choose the inverse distance $1/d_{ij}$ to represent the weight between location i and location j, where d is the Euclidean distance $d^2(i,j) = \sum_k \left(x_{ik} - x_{jk} \right)^2$ between the location i and j. However, one drawback of using this weight scheme is that the self-weight at location i is unlimited. Another possibility is to

exclude observations that are further from *i* than a specified radius distance *r*, which is equivalent to setting a zero weight on observations j whose distance from *i* is greater than *r* (Brunsdon et al 1998a, Yildirim and Öcalb 2006):

$$w_{ij} = \begin{cases} 1 \ if \ d_{ij} \leq r \\ 0 \ if \ d_{ij} > r \end{cases} \tag{5}$$

The above weighting function suffers from the problem of discontinuity over the study area: it seems unnatural that the spatial association between the variables ends so abruptly.

- **Kernel functions**

In order to solve this problem, it is possible to specify a w_{ij} as a continuous and monotone decreasing function of d_{ij}. The **kernel functions or kernels**[3] has been suggested for constructing this kind of weights, in which a constant value *h* provides some control of the range of the circle of influence of each observation *i*. They are mostly Gaussian distance-decay-based functions and bisquare/tricube functions.

a) Gaussian distance-decay-based functions[4]

They may be achieved by a host of several specifications, as the following (Brunsdon *et al*. 1998a):

$$w_{ij} = \exp(-d_{ij}^2 / h^2) \tag{6}$$

where *h* is a nonnegative parameter known as the *bandwidth*, which produces a decay of influence with distance. If *i* and *j* coincide, the weighting of data at that point will be the unity. Changing the bandwidth results in a different exponential decay profile, which in turn produces estimates that vary more or less rapidly over space. The weighting of other data will decrease according to a Gaussian curve as the distance between *i* and *j* increases. For data a long way from *i* the weighting will fall to virtually zero, effectively excluding these observations from the estimation of parameters for location *i*.

This function is the most commonly used (Huang and Leung 2002) and has received many other specifications, e.g. $w_{ij} = \sqrt{\exp(-d_{ij}/h)}$ (Brundson *et al.* 1996) or $w_{ij} = \exp(-h \cdot d_{ij}^2)$ (Fotheringham *et al.* (1997, 2001).

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[3] A kernel (*K*) is a weighting function that estimates the weights of the nearby data points to a give one. They can be probability density functions. In practice, a kernel should be monotone decreasing function for positive real numbers. Other desirable features of a kernel function are the following: $K(0) = 1$, $\lim_{d \to \infty} K(d) = 1$. Similarites between the spatial kernel approach and the kernel regression are discussed in Wand and Jones (1995). In the kernel regression, weights are based on similarities between the predictor variables themselves, rather than their geographical location, what provides a nonparametric approach to regression modeling.

[4] Distance-decay-based function is the conceptualization of the influence of distance on potential usage expressing the way increasing distance has an inverse effect on the possible usage, that is, data being further away are less likely to be used.

Still another approach is to rely on the Gaussian function ϕ (Le Sage 2004) can be $W_i = \phi(d_i/h\sigma)$, where ϕ denotes the standard normal density and σ represents the standard deviation of the distance vector d_i, which represents a vector of distances between observation i and all other sample data observations.

b) Bisquare and tricube weighting schemes.

Specifically, the formal expression of the bisquare weighting function is the following (Brunsdon et al 1998a, Bivand and Brunstad 2005):

$$w_{ij} = \begin{cases} \left[1 - \left(d_{ij}/h \right)^2 \right]^2 & \text{, if } d_{ij} < h \\ 0 & \text{, otherwise} \end{cases} \tag{7}$$

where h is the maximum -constant- distance for non-zero weights. As h tends to infinity, weights tend to the total sample (N) for all pairs of points so that the estimated parameters become uniform and GWR becomes equivalent to Ordinary Least Squares (OLS)[5]. Conversely, as the bandwidth gets smaller, the estimates of the parameters will increasingly depend on observations close to i and hence will have increased variance. In each case, the constant value h provides some control of the range of the circle of influence of each observation i.

On its side, the tricube kernel function, proposed by McMillen (1996), is specified in a similar way: $w_{ij} = \left[1 - \left(d_{ij}/h \right)^3 \right]^3$, if $I\left(d_i < h \right)$, and 0 otherwise.

c) Adaptive weighting schemes.

In general, all the spatial kernel functions fall within two categories, i.e., fixed or adaptive spatial kernel functions. In a fixed spatial kernel function, one optimum spatial kernel (represented by the spatial bandwidth) is determined and applied uniformly across the study area. Such approach, however, suffer from the potential problem that in some parts of the region, where data are sparse, the local regressions might be based on relatively few data points. As pointed out by Paez *et al.* (2002a, b) and Fotheringham *et al.* (2002), fixed spatial kernels may produce large local estimation variance in areas where data are sparse, which might exaggerate the degree of non-stationarity present. They also might mask subtle spatial non-stationarity at where data are dense.

To offset this problem, spatially adaptive weighting functions can be incorporated into GWR. These functions would have different bandwidths (distances), expressing the number or proportion of observations to retain within the weighting kernel "window", irrespective of distance: on the one hand, relatively small bandwidths in areas where the data points are densely distributed and on the other hand, relatively large bandwidths where the data points are sparsely distributed. In other words, they are able to adapt themselves in size to variations

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[5] One of the advantages of these functions is that they allow saving time in computation processes. In effect, if the number of observations for which weighting is non-zero is significantly smaller than N, then the computation overhead of (4) can be reduced by treating the problem as though it were only applied to the non-zero weighted observation (Brunsdon *et al.* 1998a).

in the density of the data so that the kernels have larger bandwidths where the data are sparse and have smaller ones where the data are plentiful. In these cases, the bandwidth is adaptive in size and acts to ensure that the same number of non-zero weights is used for each regression point in the analysis.

For example, the adaptive bisquare weighting function is the following:

$$w_{ij} = \begin{cases} \left[1 - \left(d_{ij}/h_i\right)^2\right]^2 & \text{, if } d_{ij} < h_i \\ 0 & \text{, otherwise} \end{cases} \tag{8}$$

where h_i represents the different bandwidths (distances), which express the number or proportion of observations to consider in the estimation of the regression at location i.

2.4. Bandwidth selection

The problem is now therefore how to select the optimal bandwidth. If we have strong theoretically based prior beliefs about the value of h in a given situation, then it is reasonable to make use of them. However, that information is usually missing. Anyway, since different h will result in different weight matrices W_i, the estimated parameters of GWR are not unique. The best h can be chosen using the **least squares cross-validation method** (Cleveland 1979; Fotheringham *et al.* 2002). Suppose that the predicted value of y_i from GWR is denoted as a functions of h by $\hat{y}_i^*(h)$. The sum of squared errors may then be written as:

$$\text{RSS}(h) = \sum_i \left[y_i - \hat{y}_i^*(h)\right]^2 \tag{9}$$

A logical choice would be to find h minimizing this equation. However, when h is very large the weighting of all locations except for i itself become negligible ($w_{ij} \rightarrow 0$) and the fitted values at location i will tend to the actual values ($\hat{y}_i^*(h) \rightarrow y_i$). This suggests that an unmodified least squares automatic choice of h would always lead to $h \rightarrow \infty$, or possibly result in computational errors. This problem can be avoided if, for each i, a GWR estimate of y_i is obtained by omitting the ith observation from the model. If the modified GWR estimate of y_i is denoted by $\hat{y}_{\neq i}^*(h)$ then the cross-validates sum of squared errors is denoted by:

$$\text{CVRSS}(h) = \sum_i \left[y_i - \hat{y}_{\neq i}^*(h)\right]^2 \tag{10}$$

Choosing h to minimize this equation provides a method for choosing h automatically that does not suffer from the problems encountered by working with RSS(h). Thus, when h becomes very large, the model is calibrated only on samples near to i and not at i itself (Brunsdon *et al.* 1998b). For the specific case of the adaptive spatial kernels and the tri-cube weighting function, we would compute a value for h (or q) denoting the number of nearest neighbors beyond which we impose zero weights. The score function would be evaluated using alternative values of q to find a value that minimizes the function[6].

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[6] Another estimation method for h consists in minimizing the goodness-of-fit statistics, the corrected Akaike Information Criterion or AICc (Hurvich *et al.* 1998; Fotheringham *et al.* 2002).

2. SPATIAL INSTABILITY IN THE PER CAPITA DISPOSABLE INCOME DISTRIBUTION OF SPANISH PROVINCES

This section examines the spatial distribution of the per capita disposable income of Spanish provinces. This case is well known and has been tackled by many authors (Cuadrado *et al.* 1998, Vicéns and Chasco 1998, Garrido 2002, Alcaide and Alcaide 2005 between others). Thus, our objective is to use this case for applying the methodology that we develop in the previous section. If some kind of continuous spatial instability is found across Spanish provinces, this should be taken into account in some studies, e.g. spatial prediction of municipal income (Alañón 2001, Chasco 2003, Chasco and López 2004, Cravo *et al.* 2004).

In the first subsection, we present the data that we have used. In the second, we specify a basic global model, which is estimated by ordinary least squares; this model is tested for spatial autocorrelation and respecified using different spatial dependence models. In the third subsection, we go deeper into the spatial structure of the data in search of symptoms of spatial instability and estimate a GWR model using different weighting schemes. Next, we test for spatial non-stationarity in the relationship between income and the covariates. Finally, we propose a final spatial regimes model, with different regimes for each non-constant parameter, which fits better than the other models and leads to white noise errors.

4.1. Data

The data employed in the analysis come from the Spanish Office for Statistics (INE) databank. The sample includes the 50 Spanish provinces (the Autonomous Cities of Ceuta and Melilla have been excluded). The INE provides provincial data of household disposable income in the Regional Accounts for the period 2000-2004. The series for 2004 shows (Figure 1) that the highest per capita GDP is observed mainly in the Northeast provinces, which forms a big quadrant determined by the Bask Country, Navarre, *La Rioja* and Catalonia, including Madrid and the Balearics (above 13,800 euros per capita). On the other end of the distribution, the lowest per capita income is registered in Southern provinces, mainly Extremadura, Andalusia and Castile-La Mancha (below 10,700 euros per capita).

In this paper, we want to test the existence of continuous spatial instability of the parameters in a model of per capita household income of Spanish provinces for the year 2004. In spatial prediction, the main aim is to find explanatory variables that exert a good correlation with the endogenous, though it is not "caused" by them in a Granger sense. The explanatory variables must be available not only for the provincial level but also for the municipal one. This imposes an important restriction since some "a priori" good explanatory variables cannot be used or must be proxied. For that purpose, it is preferable to select as many regressors as possible and avoid multicollinearity with principal components analysis.

We have specified the basic model shown in "la Caixa" (2005). In this book, the authors present a spatial prediction exercise of municipal income data from the provincial series. This kind of estimation can be criticized because it estimates only one –national average- coefficient for each explanatory variable, which is applied afterwards to estimate the whole set of Spanish municipalities. If spatial instability is proved in the provincial (aggregate) model of household income, a GWR approach could improve these spatial prediction exercises as it allows for estimating one different coefficient for each province.

Figure 1. Per capita household disposable income of Spanish provinces for 2004
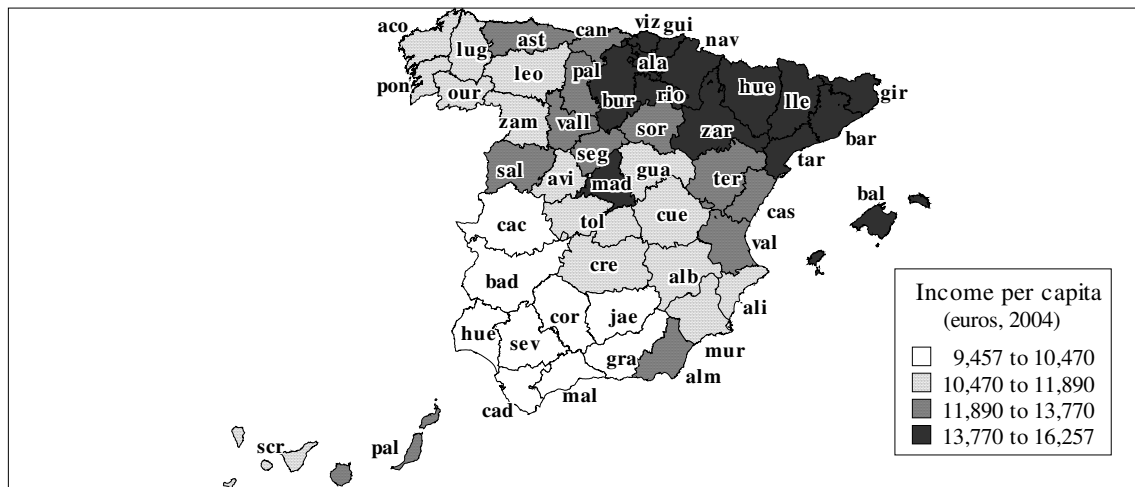
Descriptive statistics:

Label	Minimum	Lwr quartile	Median	Upr quartile	Maximum
Household income per capita	9,457	10,652	11,940	13,936	16,257
Provinces	Badajoz	Albacete	Valencia	Huesca	Álava
	(bad)	(alb)	(val)	(hue)	(ala)

Source: INE 2007. The variable *income per capita* has been classified in the map with a method called "natural breaks", which identifies breakpoints between classes using Jenks optimization (Jenks and Caspall 1971). This method is rather complex, but basically it minimizes the sum of the variance within each of the classes, finding groupings and patterns inherent in the data.

First, we have grouped the original set of 7 explanatory variables into two factors using principal components analysis and Varimax rotation. These variables are the following (all rated by population): domestic telephone lines, broadband telephone lines, population with secondary and university degrees, population with qualified jobs, unemployment, average distance to the capital and commercial heads and average housing price. They are fully specified in Table 2.

The use of principal components is clearly necessary since some of the explanatory variables are highly correlated each other. This is the case of average distance to the capital and commercial heads, which gets a Pearson correlation coefficient above 0.7 with three variables: population with secondary and university degrees, population with qualified jobs and average housing price. We find also very high correlation coefficients (above 0.8) between average housing price and two variables: population with secondary and university degrees and population with qualified jobs. Moreover, the latter are also correlated (Pearson above 0.7) with broadband telephone lines.

The principal components analysis produces 2 factors with 77% of cumulative variance and communalities over 0,7 in all cases, except broadband lines and unemployment rate. The rotated factors can be interpreted as follows: factor 1 (F1) contains high scores of variables as secondary and university degrees, qualified jobs, housing price and average

distance to capitals (negative), which are related to educated-qualified people that live in well-communicated provinces with higher housing price. Regarding factor 2 (F2), it is mainly based on telephone lines (domestic and broadband) and unemployment (negative), which are related to provinces with higher economic activity and lower unemployment rates. In Table 3, we present the communalities and the rotated factors matrix.

Table 1   Definition of the variables

Variable		Source
	Dependent variable:	
INC	Per capita household disposable income (in euros), year 2004	*Contabilidad Regional de España* (INE)
	Explanatory variables:	
❖	*Original variables:*	
T	Per capita domestic telephone lines, year 2004	*Telefónica de España, S.A.*
B	Per capita broadband telephone lines, year 2004	*Telefónica de España, S.A.*
D	Percentage of population with secondary and university degrees, year 2004	2001 Census, *Encuesta de Población Activa* (INE)
J	Percentage of population with qualified jobs, year 2001	2001 Census (INE)
U	Unemployment rate, year 2004	*Instituto Nacional de Empleo* (INEM)
D	Average distance to capitals and commercial heads, year 2004	Estimation and *Nomenclator* (INE)
H	Average housing price, year 2004	*Ministerio de la Vivienda*
❖	*Rotated factors:*	
F1	Factor 1: "education-qualification"	Self-elaboration
F2	Factor 2: "employment-activity"	Self-elaboration
WF2	Employment-activity in neighboring provinces	Self-elaboration

We have called the first factor as "education-qualification" and second one as "employment-activity". It must be highlighted that these two factors are orthogonal, which leads to a first interesting conclusion: in Spain, there is no –or at least, very small- correlation between variables like education-qualification and employment-activity. That is to say, there are provinces in which higher levels of education coexists with lower employment rates (*Córdoba*, *Huelva*, *Biscay*), and vice versa (*Alicante*, *Burgos*, *Tarragona*).

With respect to "la Caixa" (2006), in this paper we have incorporated a new variable to consider the cases in which households obtain their income not only in the same province but also in neighboring ones. In effect, in some Spanish provinces, commuting to work to proximal provinces is a common practice, particularly when they are close to a big urban area (e.g. *Guadalajara* and *Segovia*, near *Madrid*). For that reason, we have built the spatial lag of factor 2 (employment-activity) –using a connectivity weight matrix- as a proxy of this phenomenon.

Table 2   Principal components analysis results

Comunalities

	Initial	Extraction
T	1.000	.708
B	1.000	.694
U	1.000	.638
D	1.000	.817
E	1.000	.862
J	1.000	.842
H	1.000	.837

Note: Extraction method: principal components analysis.

Rotated components matrix

	Component	
	F1	F2
T	.170	.824
B	.603	.575
U	-.149	-.785
D	-.904	.021
E	.855	.363
J	.878	.266
H	.846	.348

Note: Extraction method: principal components analysis. Rotation method: Varimax normalization with Kaiser. The rotation has converged in 3 iterations.

4.2. Estimation of household disposable income: specification of a basic global model

Therefore, our initial specification is a basic regression model, which relates per capita household income with three explanatory variables: education-qualification and employment activity levels existent in a province, as well as the employment-activity present in proximal ones.

$$INC_i = \beta_0 + \beta_1 F1_i + \beta_2 F2_i + \beta_3 WF2_i + u_i \tag{11}$$

where β_0, β_1, β_2, β_2 are parameters to be estimated and u_i is a stochastic error term with the usual properties.

We have estimated this model by *OLS*. We have also carried out various tests to detect the presence of spatial dependence using the spatial weight matrices previously defined in equation (24). As is shown in Table 4, all the coefficients are quite significant and the goodness-of-fit can be considered as acceptable (due to the transformation of the variables), though improvable[7] ($\bar{R}^2 = 0.72$). In this case, the multicollinearity figure is 2.23 (low level of multicollinearity), because of the perfect orthogonality of the factors (F1, F2). The Jarque-Bera non-normality statistic on the residuals takes on a very low value (0.20), which can be considered as non-significant at all ($p = 0.90$). Consequently, we can rely on the results of the misspecification tests which depend on the normality assumption, such as the various Lagrange Multiplier (LM) tests. This is the case for the LM tests for spatial autocorrelation in the residuals, the LM test for spatial error dependence, and the LM test for spatial lag

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[7] The $\bar{R}^2$ value of 0.72 of the above model means that the equation explains 72% of the variance of the level of household per capita income in 2004.

dependence, which are highly significant in this model. These results indicate that there is evidence of the presence of spatial dependence in the OLS error terms[8].

In these cases, the classic modelling strategy (Florax *et al.* 2003) suggests a re-specification of the basic initial model to take into account the spatial effects. The robust version of LM test is only significant for the spatial lag dependence. We have tested different spatial dependence specifications, such as spatial lag, spatial cross-regressive, mixed spatial autoregressive cross-regressive models (see Anselin 1988 for a review) but in all cases, we found remaining spatial autocorrelation and/or spatial heterogeneity in the residuals.

Table 3   Estimation results for the basic model of per capita income

Model	Basic model	Spatial lag model	Spatial cross-regressive model	Mixed spatial autorregres. cross-regressive model
Estimation	OLS	ML	OLS	ML
Constant	12,212.1***	3,434.65**	12,168.3***	7,333.8***
	(148.69)	(1543.05)	(119.18)	(2333.35)
F1 (education-qualification)	706.407***	608.652***	595.333***	579.626***
	(166.52)	(127.09)	(134.84)	(122.60)
F2 (employment-activity)	721.533***	482.52***	455.466***	415.161***
	(188.01)	(149.09)	(158.80)	(145.73)
WF2 (F2 spatial lag)	1,253.46***	520.471**	1229.53***	832.8***
	(288.79)	(251.33)	(230.93)	(271.21)
WF1 (F1 spatial lag)	-	-	1981.6***	1,298.25***
			(381.59)	(473.29)
Endogenous spatial lag	-	0.7127***	-	0.3938**
		(0.127)		(0.1911)
$\overline{R}^2$	0.722	-	0.822	-
AIC	841.510	823.540	820.033	818.091
Jarque Bera	0.2018	-	1.6033	-
Breush-Pagan	2.7306	7.8296**	2.6207	6.8313
White test	12.8841	-	10.6734	-
LMERR	4.5933**	10.0515***	0.1149	6.4461**
Robust LMERR	2.2749	-	5.2601**	-
LMLAG	25.0736***	-	4.5176**	-
Robust LMLAG	22.7553***	-	9.6629***	-

Notes: * Significant at $p < 0.10$. ** Significant at $p < 0.05$. *** Significant at $p < 0.01$. *In brackets:* estimates standard deviation. $\overline{R}^2$ is the adjusted R^2. *LMERR* is the Lagrange multiplier test for spatial autocorrelation in the error term. *LMLAG* is the Lagrange multiplier test for an additional spatially-lagged endogenous variable in the model. *Robust LMERR* is the Lagrange multiplier test for spatial autocorrelation in the error term robust to the presence of spatial lag dependence. *Robust LMLAG* is the Lagrange multiplier test for an additional spatially lagged endogenous variable robust to the presence of spatial error dependence.

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[8] Here neighbors have been defined such that each element of the spatial weight matrix, w_{ij}, is set equal to the inverse of the squared distance between centroids of provinces i, j. We have obtained similar results for other specifications, e.g. connectivity or nearest neighbor matrices.

What happens? As <u>Brunsdon *et al.* 1999</u> state, sometimes spatial autocorrelation in the residuals may be implied by some kind of spatial heterogeneity that is not correctly modeled by spatial dependence specifications. Consequently, we estimate model (26) with the GWR method and test the goodness-of-fit and the remaining spatial dependence in the residuals.

4.3. Estimation of spatial instability of household disposable income with a GWR model

The first step in computing GWR estimates is to find the weighting matrix W_i ($i = 1, 2, ..., n$), given that $W_i = diag(w_{i1}, w_{i2}, ..., w_{in})$ where w_{ij} is the weight between location *i* and location *j*. In this paper, we have adopted three kernel functions[9]: 1) the fixed gaussian function shown in (6), for which the influence of each neighboring province *j* around province *i* is a continuous decreasing function of the distance that separates them, 2) the fixed bisquare weighting function shown in (7) and 3) the adaptive bisquare weighting function shown in (8). These approaches correspond to the situation that one province will have no influence on the provincial household income of another province if the distance between them is sufficiently long.

Since W_i is a function of the bandwidth we must choose the optimal *h*. For this purpose, we use the least-squares cross-validation (CVRSS) method shown in (9). For each province *i*, this method obtains a GWR estimate of per capita income in this province by omitting the ith observation. The complete process starts with a weighted OLS calibration using different possible values of *h* that leads to different weighting matrices W_i (always omitting the observations at location *i*). Thus, many different values of the estimated independent $\hat{y}^*_{\neq i}(h)$ can be estimated at this stage, and therefore the scores of the residuals sum of squares, $\sum_i \left[y_i - \hat{y}^*_{\neq i}(h) \right]^2$, can also be calculated. Finally, the best value of *h* is selected by minizing the sum of the squared errors.

Applying the above procedure to the analysis of per capita income in Spanish provinces, the method computes 11, 9 and 8 different weighted OLS calibrations (iterations) for the fixed Gaussian, fixed bisquare and adaptive bisquare functions, respectively, and thus, different CVRSS values.

When we apply a fixed Gaussian function, the minimum score of the CVRSS value is obtained when the bandwidth *h* equals approximately 167 km. Thus, the weighting matrix W_i is estimated, where $w_{ij} = \exp(-d_{ij}^2 / 167^2)$. In the case of the fixed bisquare function, the minimum score of the CVRSS value is obtained for h equals 807 km. In this case, the weighting matrix W_i is estimated as $w_{ij} = \left[1 - \left(d_{ij} / 807 \right)^2 \right]^2$, if $d_{ij} < 807$. Finally, for the adaptive bisquare function, the minimum score of CVRSS is obtained when the local sample size includes approximately a 46% of the total units. The weighting matrix W_i is here estimated considering different local bandwidths or distances (h_i), such that each sub-sample always includes the 46% of the Spanish provinces (about 23 of 50): $w_{ij} = \left[1 - \left(d_{ij} / h_i \right)^2 \right]^2$, if $d_{ij} < h_i$. It is particularly highlighting the difference in bandwidths between both fixed Gaussian and bisquare functions (167 and 807 km, respectively). Too large a bandwidth will produce a flat surface with little spatial variation and too small a

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[9] To give a point location value to each observation, we used the centroid of each province.

bandwidth will result in estimation problems with some of the local regressions. On the contrary, the bisquare adaptive function establishes different local bandwiths (such that each sub-sample always includes the 46% of the Spanish provinces, which is about 23 of 50), in a range from 290 to 963 km, depending on the density of neighbors around each province.

Figure 2 shows the spatial range of the two fixed weighting schemes used in this application, Gaussian and bisquare, computed for their corresponding cross-validated bandwidths (167 and 807 km, respectively). In current GWR techniques, bandwidth is first established globally, with fixed weighting schemes (Gaussian or bisquare), meaning that regions in denser parts of the study area have a larger number of weighted neighbors contributing to their estimates than in less dense parts. The bandwidth of the Gaussian fixed weighting function (167 km) is about half the greatest nearest neighbor distance in the data set (264 km, for the Balearic Islands and *Tarragona*), and lower than the first quartile (277 km). The bandwidth of the bisquare fixed function establishes a much wider bandwidth (807 km), which is higher even than the third quartile (600 km). Regarding the bisquare adaptive weighting method, it considers a range of different local bandwidths (h_i). The lowest bandwidth (290 km) corresponds to *Segovia* ("seg" in Figure 1), which is a Castilian province located in the center of Spain, the denser area (e.g. the zone with more proximal neighbors). On the other side, *Santa Cruz de Tenerife* ("scr"), in the Canary Islands, constitutes one of the sparsest zones; for this reason, it has the largest bandwidth, 963 km.

Figure 2. **Values of weighting functions by distance for bandwidths chosen by cross-validation of the basic model**
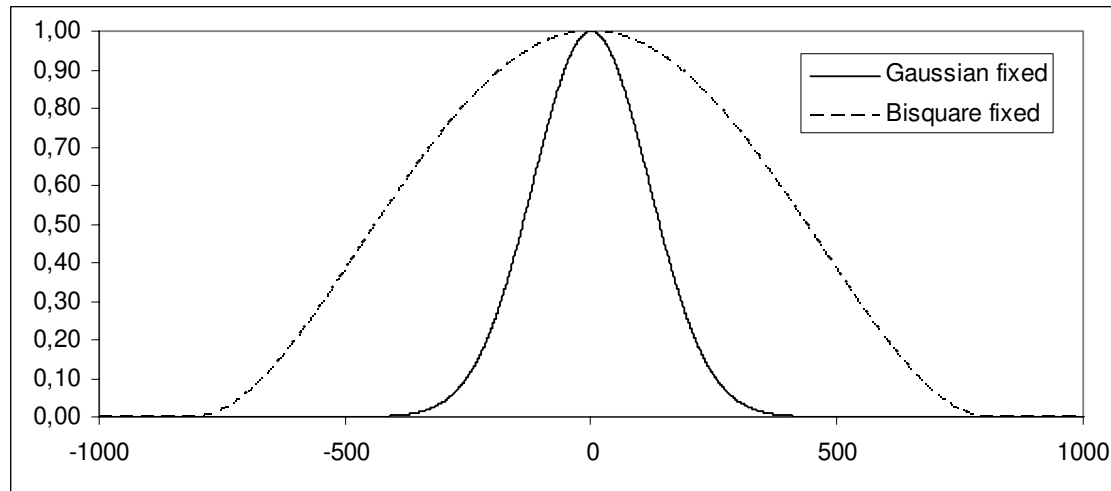
Source: Self-elaboration.

To estimate the parameter vectors $\hat{\beta}_i^{\text{gwr}} \left(i = 1, 2, ..., 50 \right)$ equation (4) is used. These parameter vectors are the expected estimates parameters of the GWR when the value of $h = 167$ (Gaussian fixed), $h = 807$ (bisquare fixed) and an adaptive 46% (23 provinces) of local sample size (bisquare adaptive). The main results of the estimations are in Table 5. For the global model (OLS basic model in Table 4), we show the estimates and their corresponding standard errors, as well as a *2-sigma* value, which is twice the estimate standard errors. For the GWR regressions, we show 4 descriptive statistics that summarize the

distribution of the $\hat{\beta}_{ij}^{\text{gwr}}$ -coefficients: the minimum and maximum values, the inter-quartile range and the standard deviation of each distribution. In addition, we present a some goodness-of-fit measures: adjusted R^2, SSR and standard error of the estimate[10].

Table 4   Estimation results for the global OLS and GWR models of per capita income

	Global model (OLS)		Gaussian fixed model (GWR)	Bi-square fixed model (GWR)	Bi-square adaptive model (GWR)
	Coeff.		Coeff.	Coeff.	Coeff.
$\hat{\beta}_0$ constant	12,212*** (148.69) 297.38	Min Max Int. range St.dev. 2-sigma	11,017 13,328 713.832 552.203	11.115 12.654 190.000 283.461	11,112 13,618 799.099 611.750
$\hat{\beta}_1$ Education-qualification	706.407*** (166.52) 333.04	Min Max Int. range St.dev. 2-sigma	-362.00 1,067.97 576.538 341.933	349,24 1.026,12 203.500 140.847	-241.78 1,083.59 483.323 307.955
$\hat{\beta}_2$ Employment-activity	721.533*** (188.01) 376.02	Min Max Int. range St.dev. 2-sigma	80.41 1,180.40 257.596 225.313	265,69 1.001,14 187.800 144.884	123.89 1,373.72 301.762 271.531
$\hat{\beta}_3$ Employment-activity spatial lag	1,253.46*** (288.79) 577.58	Min Max Int. range St.dev. 2-sigma	455.74 2,125.51 531.828 335.593	598,60 1.428,52 267.000 168.552	147.37 1,884.24 750.428 445.774
Bandwidth	-		167.45 km.	807.23 km.	46% 23 provinces
$\overline{R}^2$	0.72		0.83	-	0.84
AIC	841.51		806.63	827.22	804.69
SSR	50,847,587		21,195,415	37,974,257	19,317,962
$\hat{\sigma}_u$	1,051.37		803.96	871,48	774.04

Notes: * Significant at $p < 0.10$. ** Significant at $p < 0.05$. *** Significant at $p < 0.01$. *In brackets:* standard errors of the OLS coefficient estimates, $SE\left(\hat{\beta}_j^{ols}\right)$. *2-sigma* is twice the OLS standard errors. *St.dev* is the standard deviation of the GWR coefficient estimates, $\sqrt{\upsilon_k}$. *SSR* is sum of squared residuals. Underlined are the coefficient interquartile ranges above *2-sigma* and the GWR standard deviations above OLS standard deviations. $\hat{\sigma}_u$ is the standard error of the estimate.

In terms of goodness of fit, the bi-square adaptive model is better than the bisquare and Gaussian fixed ones, and the three GWR estimations improves significantly the OLS global model. The adjusted R^2 indicates that about 84% of the per capita household income is

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[10] The adjusted R^2 is constructed from a comparison of the predicted values from different models at each regression point and the observed values.

explained by the three elected independent variables in the GWR bi-square adaptive model, what can be considered a good result.

Concerning the individual significance of the parameters, though they are all statistically significant in the global regression, there are several non-significant parameters in the GWR models. A pseudo t-test can be built dividing each local estimate by the corresponding local standard error of the estimate. This measure does not allow refusing the null of zero value, with at least a 90% of confidence, in 17, 9 and 19 cases for F1, F2 and WF2, respectively. In addition to the estimation of model parameters at each location of interest, a local estimate of the coefficient of determination (R^2) may be derived. These regression model parameters and diagnostic statistics are interpreted in the same way as with a conventional global regression, although the local estimates of the R^2 should not be interpreted with the same confidence as that from a global model (Fotheringham et al., 2002). In the three GWR regressions, local R^2 are sufficiently high, above 0.7, except 3 and 4 regressions in the Gauss fixed and bisquare adaptive models, respectively, corresponding to the provinces of *Cádiz* ("cad"), *Girona* ("gir"), *Málaga* ("mal") and *Navarre* ("nav"). The bisquare fixed model registers local R^2 above 0.7. In this case, the relatively better goodness of fit in local regressions is due to a large bandwidth, which produces little spatial variation and local estimates very close to the global one.

Figure 3. Plots of the GWR estimate distributions
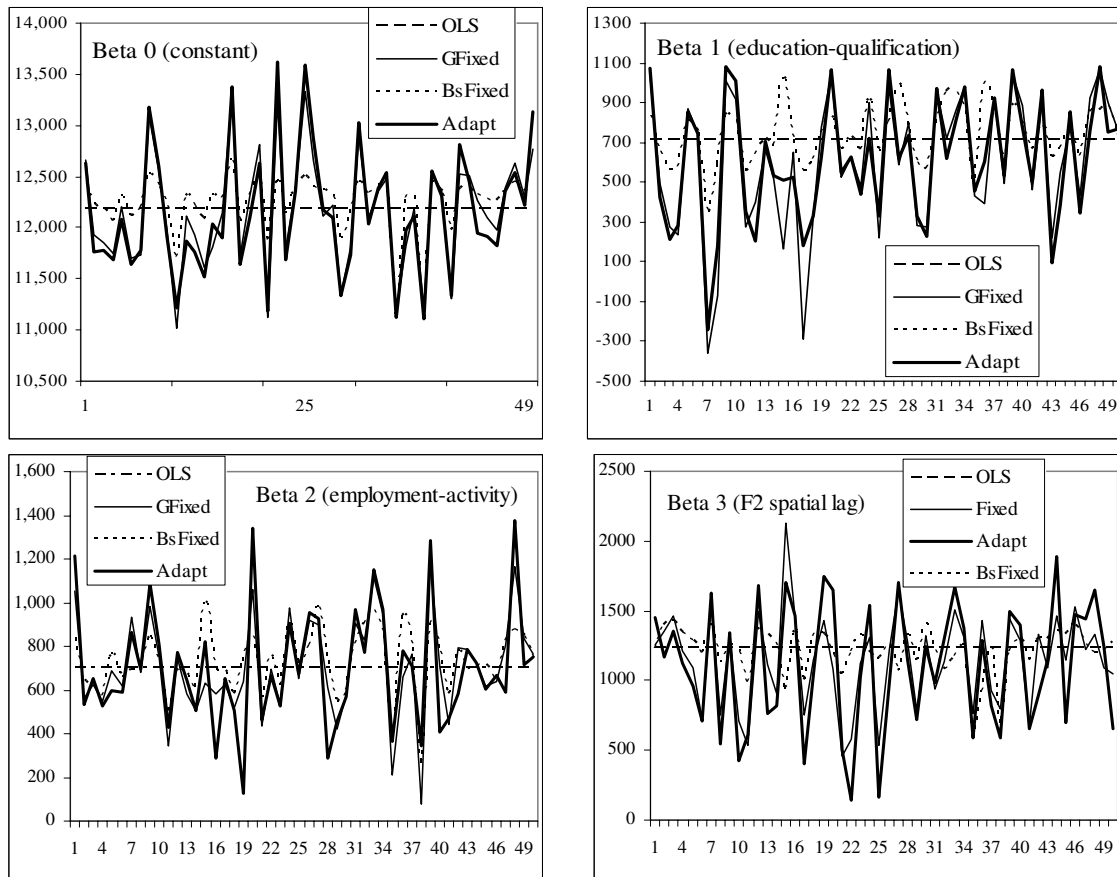
Note: OLS is global model, *GFixed* is Gaussian fixed, *BsFixed* is bisquare fixed, *Adapt* is bisquare adaptive.

For each variable and each weighting method, all the estimates have a typical marked banded pattern (Figure 3). In general terms, Gaussian fixed and bisquare adaptive functions are more similar, while the bisquare fixed scheme leads to more smoothed results, due to its larger bandwidth. If we compare the GWR estimates with the single OLS one, we realize that the latter is not always an average of the former. Particularly in Factor 2, the OLS estimate is higher than all the GWR ones, with the exception of 9 provinces. Therefore, it seems quite evident that the single OLS estimates could be highly biased in certain provinces.

We should test for the degree of collinearity existent between GWR estimates. As Wheeler and Tiefelsdorf (2005) show, the local regression coefficients can be potentially collinear even if the underlying exogenous variables in the data generating process are uncorrelated. They demonstrate that in many cases, it appears that when one coefficient estimate moves in one direction, it seems to force at least one of the others in the other direction. In the present case, Table 5 indicates that in there is scarce correlation between the explanatory variables, mainly in the bi-square adaptive model. This results raise no doubts about the potential for using geographically weighted regression, mainly the bisquare adaptive model, for explaining per capita income in Spanish provinces.

Table 5   Correlation coefficients of the GWR coefficient estimates

Gaussian fixed weighting method:

	Intercept	F1	F2	WF2
Intercept	1.00			
F1	0.23	1.00		
F2	0.64	0.49	1.00	
WF2	0.06	0.09	0.43	1.00

Bi-square fixed weighting method:

	Intercept	F1	F2	WF2
Intercept	1.00			
F1	0.45	1.00		
F2	0.79	0.82	1.00	
WF2	0.56	-0.08	0.21	1.00

Bi-square adaptive weighting method:

	Intercept	F1	F2	WF2
Intercept	1.00			
F1	0.30	1.00		
F2	0.47	0.45	1.00	
WF2	-0.07	0.09	0.38	1.00

4.4. Testing spatial non-stationarity

The remaining questions are whether the GWR model describes the relationships significantly better than an OLS model and whether each set of parameter estimates exhibit significant spatial variation over the study areas. In effect, although the techniques of GWR seem to be very appealing in analyzing spatial non-stationarity, from the statistical viewpoint, two critical questions need to be addressed. The first one is a goodness-of-fit *vs.* simplicity

question. As pointed out by Leung *et al*. (2000a) and Fotheringham *et al*. (2002), the GWR model will certainly fit a given data set better than a global OLS model. However, in practice, a simpler model is usually preferred over a more complex one if there is no significant improvement from the latter. To justify the employment of GWR instead of OLS in this study, a Monte Carlo F test (Brunsdon *et al*. 1999, Fotheringham *et al*. 2002) and two other F tests (F1 and F2 test in Leung *et al.* 2000a) have been formulated to address this concern.

Table 6   Spatial stability tests of GWR models

		Gaussian fixed model (GWR)		Bi-square fixed model (GWR)		Bi-square adaptive model (GWR)	
		BFC local p-values	LMZ F_3 p-values	BFC local p-values	LMZ F_3 p-values	BFC local p-values	LMZ F_3 p-values
$\hat{\beta}_0$ constant		0.030^{**}	0.605	-	0.232	0.00^{**}	0.520
$\hat{\beta}_1$ Education-qualification		0.320	0.088^*	-	0.230	0.43	0.000^{***}
$\hat{\beta}_2$ Employment-activity		0.760	1.000	-	1.000	0.70	1.000
$\hat{\beta}_3$ F1 spatial lag		0.890	1.000	-	1.000	0.72	1.000
Global variability p-values	BFC	0.017^{**}		0.039^{**}		0.008^{***}	
	LMZ F_1	0.123		0.313		0.092^*	
	LMZ F_2	0.119		0.073^*		0.092^*	
Moran's *I*		0.011		0.162^{**}		-0.059	

Notes: * Significant at $p < 0.10$. ** Significant at $p < 0.05$. *** Significant at $p < 0.01$.

These F-tests are based on analysis of variance (ANOVA) and use generalized degrees of freedom (no longer integer because of the varying weights sums per estimate) to compare the improved sum of squares accounted for by the GWR estimates as compared with the global OLS estimates. These tests are established so that a hat matrix may be constructed from rows from the GWR model, each row using its appropriate weights. The Monte Carlo test derived by Brunsdon *et al.* (1999) compares the difference in the residual sums of squares of the OLS and GWR models with the residual sum of squares of the GWR model. We will call it "global BFC test". On their side, Leung *et al.* (2000a) proposed two other Monte Carlo tests, which are very similar to the global BFC: LMZ F_1 and F_2. The LMZ F_1 compares the residual sums of squares of the GWR model with the residual sums of squares of the OLS one, with the corresponding degrees of freedom. On its side, the LMZ F_2 is very similar to global BFC: it compares the same difference than in global BFC with the residual sum of squares of the OLS model, choosing a different denominator for the F-ratio, as well as slightly different degrees of freedom. Results of these tests are shown in Table 6, from which we can see that the GWR estimates significantly reduce the residual sum of squares over and above the OLS estimates, specially in the case of the bisquare specifications. The global BFC tests

are significant in the three GWR models, though the F_1 and F_2 are not significant in the Gaussian fixed model.

The second question concerns the core concept of employing a GWR model, namely, whether there is significant spatial non-stationarity among the relationships or not. To answer this question, another F test (the F_3 test in Leung *et al.* 2000a), based on the sample variance of the estimated model coefficients have been suggested as well. If any independent variable (including the constant term) shows spatial stationarity by this test, a mixed GWR model may be more appropriate than OLS. In Table 6, we display two local statistics to test spatial variation in each GWR individual coefficients: BFC local test and LMZ F_3 test. The BFC local is a Monte Carlo test to see if the local parameter estimates are stationary or non-stationary. BFC local test shows that only the independent term varies significantly over space, while LMZ F_3 highlights spatial variation in F1 (education-qualification) parameter.

Brunsdon *et al.* 1998a, 1999, Mei 2004 and Fotheringham *et al.* 2002 also suggest the use of intuitive alternative methods that can overcome some of the disadvantages of the significance tests. In this sense, they present two informal proceedings. First, they compare the variability of the β_{ij}^{gwr}-coefficients with the standard errors of the β_j^{ols}-coefficients of the global regression model (1). This can be done by tabulating $\sqrt{\upsilon_k} = \sqrt{\sum_i \left(\beta_{ik} - \beta_{.k} \right)^2 / N}$, for each variable, against $SE\left(b_k \right)$ from model (1). If the null hypothesis of stationarity were rejected for some but not all parameters, a mixed GWR-model could be appropriate. In Table 5, all the values of $\sqrt{\upsilon_k}$ exceeds $SE\left(\hat{\beta}_j^{ols} \right)$, although by less amounts in the cases of the non-significant variables (F2 and WF2). This suggests that there is some justification in considering the patterns of spatial variation in all the coefficients in the model.

Secondly, we can also compare the range of the local parameter estimates ($\hat{\beta}_{ij}^{gwr}$) with a confidence interval around the global estimate of the equivalent parameter ($\hat{\beta}_j^{ols}$). Recall that 50% of the local parameter values will be between the upper and lower quartiles and that approximately 68% of values in a normal distribution will be within ± 1 standard deviations of the mean. This gives us a reasonable, although very informal, means of comparison. We can compare the range of values of the local estimates ($\hat{\beta}_{ij}^{gwr}$) between the lower and upper quartiles with the range of values at ±1 standard deviations of the respective global estimate, which is simply $2 \times SE\left(\hat{\beta}_j^{ols} \right)$. Given that 68% of the values would be expected to lie within this latter interval, compared to 50% in the inter-quartile range, if the range of local estimates between the inter-quartile range is greater than that of 2 standard errors of the global mean, this suggests the relationship might be non-stationary.

As we can see in Table 5, in the bi-square adaptive regression, only the interquartile range of the F2 (employment-activity) estimates is less than 2 standard errors (*2-sigma*) of its corresponding global estimate. These results reinforce the conclusion about the existence of spatial variation in the intercept and F1 (education-qualification) variable, though some spatial instability has also been found in WF2 (employment-activity in neighboring provinces).

To give a further indication of the nature of the variation in the coefficients, we can consider the range of coefficient values estimated for the explanatory variables of income per capita with the help of a density plot[11] ([Bivand and Brunstad 2002](#)). Figure 4 shows for each coefficient the density plot of the GWR coefficient distributions, as well as the OLS global estimate (solid vertical line) with its corresponding confidence bands (+/- twice the standard error) in dashed vertical lines. In all the cases, the GWR estimates group more or less around the OLS global estimates. These plots highlight the skewed shape of the bisquared fixed GWR distributions, due to the wider bandwidth (807 km) reached by this method with respect to the others. As stated before, too large a bandwidth will produce a flat surface with little spatial variation, and vice versa.

Figure 4. Density plots of the GWR estimates
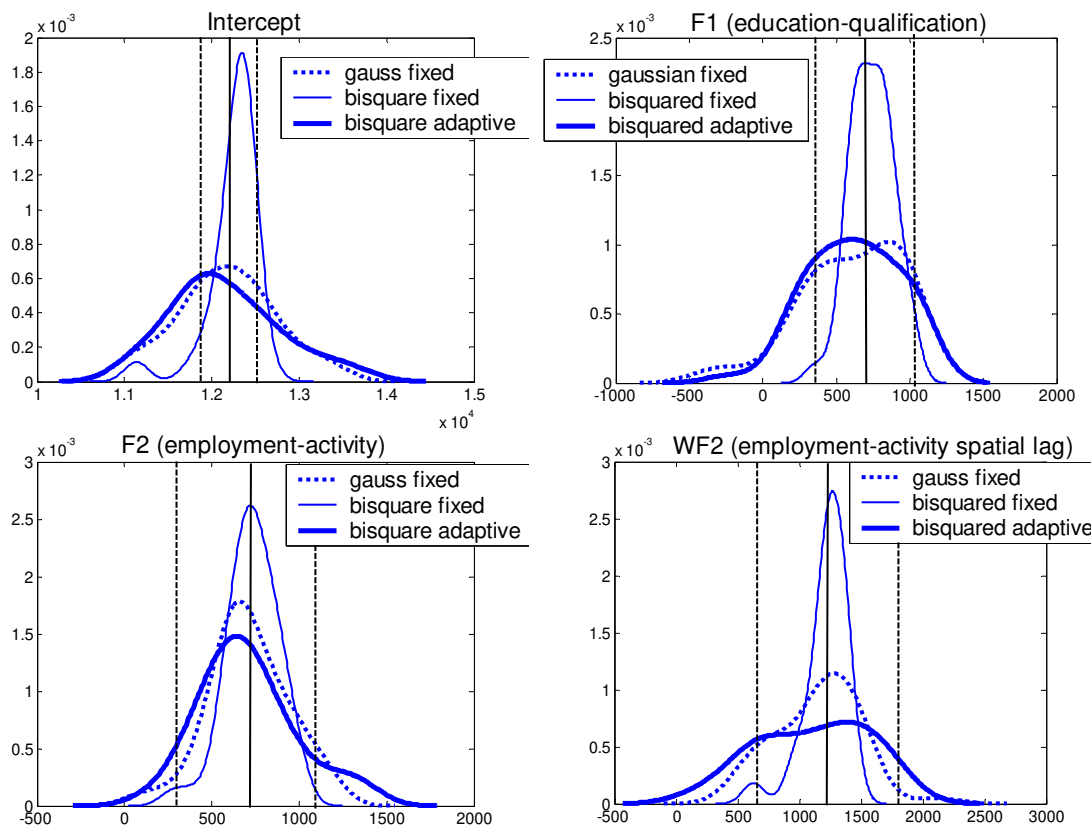
Notes: Vertical solid lines are the OLS estimators of the global basic model; vertical dashed lines are their corresponding *2-sigma* values (+/- twice the estimator standard errors).

It also seems clear that there is a certain degree of structural instability in GWR coefficients with respect to OLS, since some of the former are out the latter *2-sigma* bands, especially in the bisquare adaptive distributions. In the case of the intercept and F1 (education-qualification), spatial variability is more acute because there are more GWR

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[11] The density plot draws the non-parametric kernel density estimates of the coefficient distributions. It may be interpreted as the continuous equivalent of a histogram in which the number of intervals has been set to infinity and then to the continuum.

coefficients significantly exceeding the OLS bands. On the contrary, spatial instability is lower in F2 (employment-activity) and WF2 (employment-activity spatial lag) because they have a higher density mass inside the OLS *2-sigma* bands. This result confirms the output of the local instability tests: though the informal measures are more or less significant for all the coefficients, in the GWR bisquare adaptive model (Table 4) the formal tests (Table 5) are only clearly significant for the intercept (BFC local) and the variable F1 of education-qualification (LMZ F_3).

Finally, we can test for spatial autocorrelation in the residuals with the help of the Moran's *I* test[12] (Cliff and Ord 1981), which also allows us to detect remaining spatial non-stationarity (Fingleton 1999). Except for the bisquare fixed function, GWR models lead to uncorrelated and stationary residuals what proves the superiority of GWR over the OLS global model.

In short, since results demonstrate the existence of spatial instability in the parameters, we can conclude that GWR regression is better than a global model to explain income in Spanish provinces. We have tested this hypothesis using three different weighting functions and all them address to this same conclusion. We choose the bisquare adaptive function since it leads to a better goodness of fit and can adapt to the different spatial configurations of Spanish provinces across territory (e.g. it considers different bandwidths for each province depending on its spatial sparseness). Moreover, this function also overcomes better the multicollinearity problems between the estimates and produces white noise residuals. Regarding local non-stationarity of the bisquare adaptive GWR estimates, the formal tests are only clearly significant for the intercept and F1 (education-qualification), though the informal local tests are also significant (in a lesser extent) for the other two variables. Next, we analyze in depth the distribution of these estimates across the Spanish territory.

4.5. Analysis of spatial instability in bisquare adaptive GWR estimators

First, we map the spatial variation of these two variables, intercept and F1, to have a better knowledge of variation on their fitted parameters (Figure 5). Indeed, especially when working with large data sets, mapping, or some other form of visualization, is the only way to make sense of nature of spatial instability.

In our model, the intercept or constant parameter measures the fundamental level of per capita household income excluding the effects of all factors on regional economic development across Spain. It can be referred to as "the basic level of regional economic development". There is a clear spatial variation with higher constant parameters in the northeastern provinces and lower ones in the southern provinces. It means that, other things being equal, the northeastern Spanish provinces had the highest basic level of economic development; the southern provinces had the lowest, while the level in the central -and northwestern- areas are between the northeastern and the southern areas in 2004. Thus, the basic level of economic development in the Spanish provinces displays a ladder-step distribution, which varies from high in the northeast to low in the south.

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[12] The Moran's *I* test has been computed on errors using the expression commonly applied to univariate distributions and the same spatial weight matrix used previously in the LM tests. Inference is based on the permutation approach (999 permutations), as shown in (Anselin 1995). There is also a test procedure for spatial autocorrelation in GWR framework in Leung *et al.* (2000b), which is a three-moment χ^2 approximation.

In Figure 5, we have also represented Factor 1 (education-qualification) coefficients. This variable has more impact on per capita household income in the north (Cantabria, the Bask Country, north Castile, Navarre, La Rioja) and west (*Salamanca* and *Cáceres*). On the other side, the influence of this factor on per capita income is less important in the Mediterranean provinces (Catalonia, The Balearics, *Comunidad Valenciana*, Murcia, south Andalusia and the Canaries), having this variable a negative effect in the Balearics. This is likely because these areas are growing economies based on the service sector, which is mainly labor-intensive and not so dependent on the education level of the labor force. On the contrary, in the north, economic activity is based on heavy industries, which are not so highly labor demanding, though require qualified workers.

**Figure 5.   Map for the intercept and F1 adaptive GWR estimates**
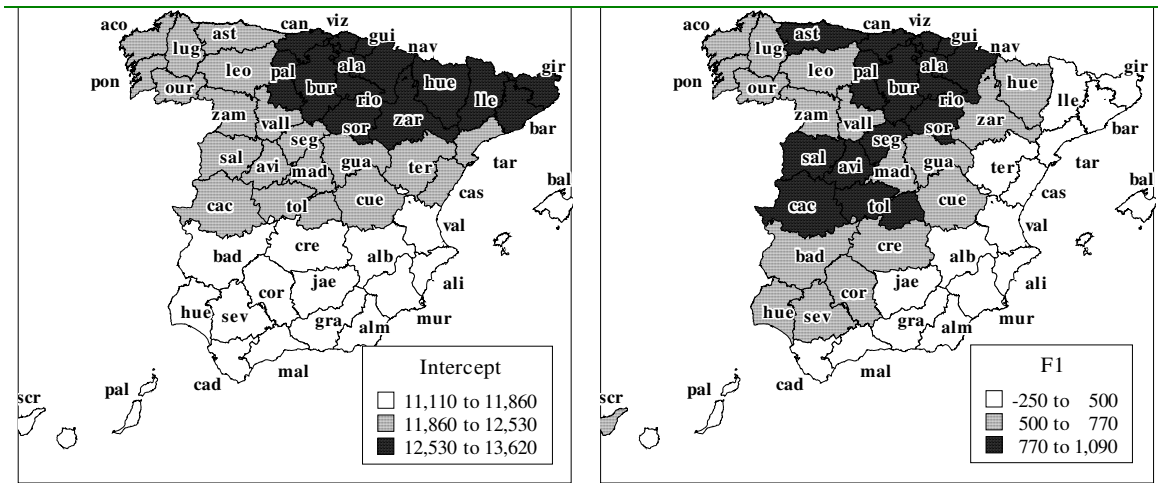
Notes: The classification method is "natural breaks" (Jenks and Caspall 1971)

Mapping the spatial distributions of the GWR raw estimates is interesting. However, as Fotheringham *et al.* 2001 point out, the values represented on these maps do not take into account the standard errors of the estimates. Consequently, it is often more illuminating to view maps of *t*-statistics by dividing each local estimate by the corresponding local standard error of the estimate. These *t*-maps are useful, not in a formal sense, but in a purely exploratory role, to highlight parts of the map where interesting relationships appear to be occurring. In Figure 5, we have shown the *t*-maps for the GWR local parameters of the explanatory variables (we exclude the intercept since it is always significant). In general, a *t*-map of a certain coefficient is very similar to the map of this same raw coefficient, but sometimes reveals interesting spatial variations that would be missed completely if we rely solely on the global analysis of raw parameters.

For example, in Figure 5 we see that F1 (education-qualification) has low influence on per capita income in the Mediterranean provinces and the Isles. However, with the help of the *t*-map we can also detect other non-significant provinces, such as the ones located in the northwest (Galicia). Nevertheless, in general, in these places the most significant variable is F2 (employment-activity). That is to say, there is a tradeoff between these 2 factors when explaining per capita household income. This conclusion was also reached before, when computing these factors as orthogonal, demonstrating that in Spain, there are provinces in which higher levels of education coexists with lower employment rates, and vice versa.

Figure 6. *t*-Maps for the adaptive GWR estimates of the explanatory variables
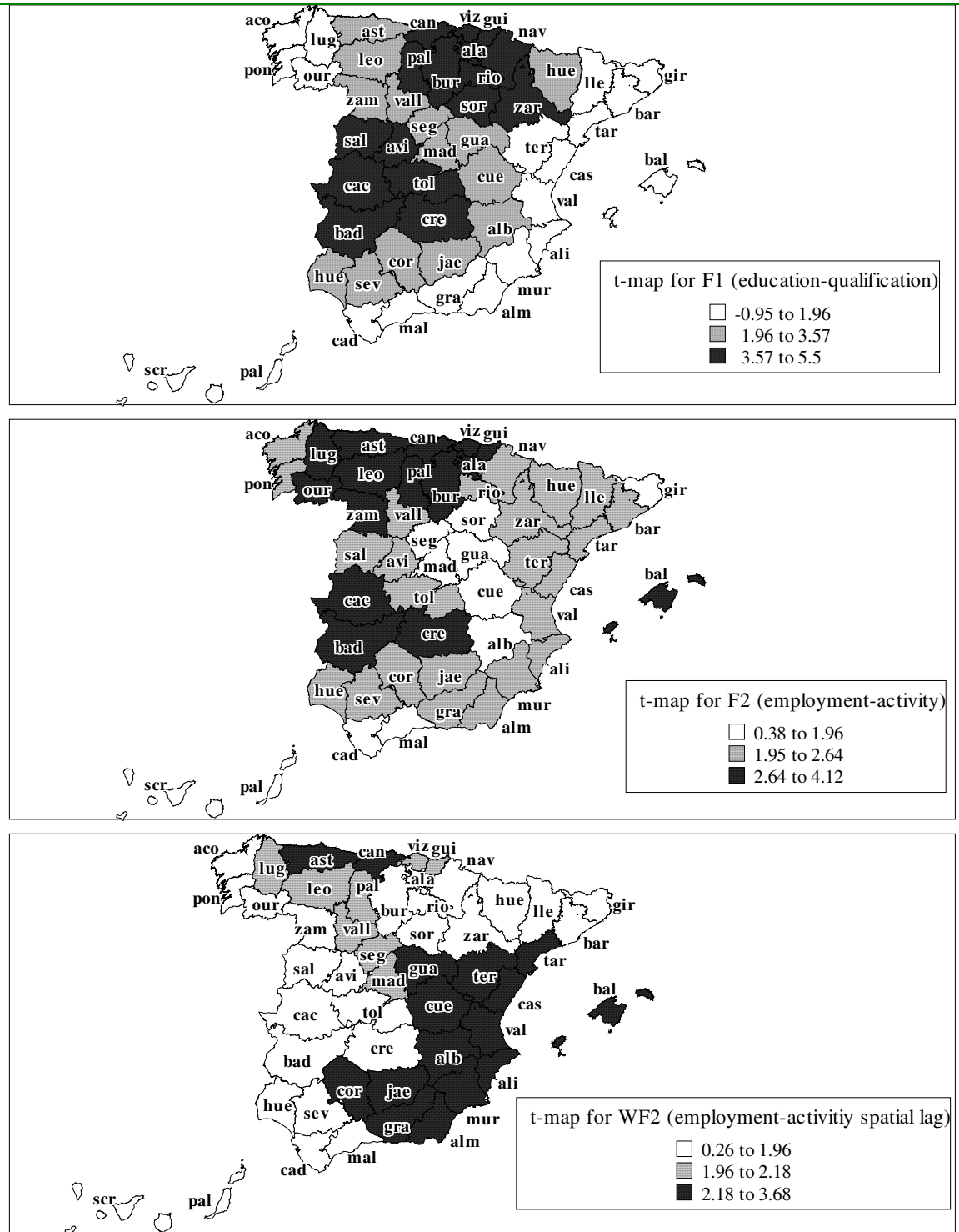
Notes: The classification method is "natural breaks", except the first class that has been established as less than 1.96, which is approximately the critical value for refusing the null hypothesis of non-significant parameter with a 95% of confidence.

Moreover, it is particularly highlighting that F2 coefficients are not significant in some provinces proximal to Madrid (*Guadalajara* and *Segovia*) and Valencia (*Cuenca* and *Albacete*). It is possibly due to the existence of many residents that do not earn their salaries in the provinces they live but in Madrid and Valencia, respectively, where they commute to work. In this cases, WF2 (employment-activity spatial lag), which consider this variable, is highly significant what emphasizes this argument. WF2 measures the role that spatial externalities of employment-activity play in the explanation of per capita household income. This role is very important in the provinces located in the axis Madrid-Valencia-Barcelona. This interesting result is the same to that found in Peeters and Chasco (2006) that detected big agglomeration effects mainly in the provinces of the Valencian Community, as well as in Catalonia, Murcia and Castile-La Mancha.

The t-maps reveal that the GWR model does not explain per capita income the same everywhere. For example, it is more effective for the northern provinces (along the Cantabric coast) than in the southwest (Andalusia and the Canary Islands). It is particularly highlighting the case of *Cádiz*, *Málaga* and the Canaries, which are the only provinces with non-significant *t*-values in all the explanatory variables. In effect, neither education-qualification nor employment-activity (and its spatial lag) seems to be significant enough to explain per capita household income. In the case of the Andalusian provinces, this situation can be explained because in the inland, farmers' income does not fundamentally depend on neither their educational level nor the economic activity, but on the state subsidies. Besides, some provinces are not so well explained by the model, as the case of Galicia (*A Coruña*, *Pontevedra* and *Ourense*), where only F2 coefficient is significant. In the first ones, economic activity is very high, particularly in the coast and highly labor demanding. *Ourense* constitutes a peculiar case, since its residents income depends not only in its own economic activity but also on emigrants' remittances. As seen, the t-maps highlight some facet of the relationship completely hidden in the global results and force us to ask about questions we would otherwise not have asked.

3. MAIN CONCLUSIONS

The purpose of this paper is to analyze the spatially varying impacts of some classical regressors on per capita household disposable income of Spanish provinces. This variable has frequently been modeled as an ecological regression to predict its value for microterritorial units (e.g. municipalities). Nevertheless, observations at an aggregated level of analysis do not necessarily provide useful information about lower levels of analysis, particularly when spatial heterogeneity (non-stationarity) is present. For that reason, we propose to test spatial variation of the determinants of per capita household income across Spanish provinces: if this effect was present, it should be considered in future ecological inferences of municipal income.

First, we modeled the provincial household income distribution following a basic global regression, which was also re-specified to take into account spatial effects. We found clear evidence of spatial instability in the distribution of the residuals that could not be totally solved by global specifications. Since spatial autocorrelation in the residuals may be implied by some kind of spatial heterogeneity not always correctly modeled by spatial dependence specifications, we re-specified the basic model using GWR methods. With this aim, we tested three weighting functions: Gaussian fixed, bisquare fixed and bisquare adaptive. In these three

cases, spatial instability was detected using both formal and informal tests. We finally chose the bisquare adaptive specification since it leads to a better goodness of fit and can adapt to the different spatial configurations of Spanish provinces across territory. Moreover, this function also overcomes better the multicollinearity problems between the estimates and produces white noise residuals.

Regarding local non-stationarity of the bisquare adaptive GWR estimates, the formal tests were only clearly significant for the intercept and F1 (education-qualification), though the informal local tests were also significant (in a lesser extent) for the other two variables. GWR shown that when explaining per capita household income, there is a tradeoff between education-qualification and employment-activity; e.g. in Spain, there are provinces in which higher levels of education coexists with lower employment rates, and vice versa. Regarding WF2, it measures the role of spatial externalities in the explanation of per capita household income. This role revealed as very important in the provinces located in the axis Madrid-Valencia-Barcelona, which coincides with other similar analysis.

GWR is, no doubt, a better specification to model per capita household income than other global models. It highlights some facets of the relationship completely hidden in the global results and forces us to ask about questions we would otherwise not have asked. This analysis demonstrates that GWR should be considered to predict income in Spanish municipalities from an ecological inference estimated with provincial data.

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