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Changing the Research Patenting Regime: Schumpeterian Explanation

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Abstract
Starting in the early 1980s, the U.S. patent regime experienced major changes that allowed the patenting of numerous scientific findings lacking in current commercial applications. We assess the rationality of these changes in the legal and institutional environment for science and technology policy. In order to model these changes in the incentives for the commercialization of new ideas, we extend the standard multisector Schumpeterian growth theory by decomposing the product innovation into a two-stage uncertain research activity. This analytical structure, beside suggesting new sources of market and non-market failures, allows us to compare the general equilibrium innovative performance of an economy where early-stage scientific results are patentable with the general equilibrium innovative performance of an economic system where these early-stage results are unpatentable and freely disseminated by public research institutions such as the universities. If researchers are unguided by the invisible hand they risk to invent redundant half-ideas, but public universities are better at internalizing research externalities. When scientists can patent their research, monopolistic research firms restrict entry in the applied R&D. This makes a regime choice a priori controversial and dependent on the exogenous data on technologies. We calibrate the model to the US data and show that in the 70s, a relatively higher applied R&D complexity magnified the public basic R&D inefficiencies and justified the patentability of basic scientific findings.

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1. Introduction

Innovation economics recognizes sequentiality as a distinctive characteristic of the innovative process. Among them Susanne Scotchmer (1991) argued that "Most innovators stand on the shoulders of giants, and never more so than in the current evolution of high technologies, where almost all technical progress builds on a foundation provided by earlier innovators. For example, most molecular biologists use the basic technique for inserting genes into bacteria that was pioneered by Herbert Boyer and Stanley Cohen at the early 1970s, (...). In pharmaceuticals, many drugs like insulin, antibiotics, and anti-clotting drugs have been progressively improved as later innovators bettered previous technologies." Scotchmer’s words do not miss to highlight that sequentiality plays a central role in advanced biotechnologies. According to Roger Brent, the head of Molecular Sciences Institute of Berkeley, in this field, the recent expiring of the U.S. patent on polymerase chain reaction (a genetic procedure patented by F. Hoffman-La Roche and used in almost all research fields of life sciences) will have terrific consequences on research by raising the expectation$^3$ of discovering new drugs for tropical diseases$^4$.

According to Hecht (1999), at the end of 1926 Clarence W. Hansell, researcher at the RCA Rocky Point Laboratory in Long Island, had already outlined the principles of optical fibres bundle functioning; in 1927 RCA was awarded the U.S. patent. However, until 1970, optical fiber had very little practical applicability for commercial use. The second fundamental step to innovation came only with the development of laser technology and the increasing demand for high frequency telecommunication tools in the late 1960’s.

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$^3$Heller and Eisenberg (1998) have pointed out that the patenting of gene sequences produce a tragedy of the anticommons, i.e. a crumbling of rights which greatly amplify transaction cost, thus hampering downstream research for biomedical advance.

$^4$See "Patent Ending", The Economist, April 9th 2005. For further examples of patented research inputs in the process of developing new marketable applications and therapies in biotechnology see National Research Council (2004, pp.74, 75).
when a group of researchers at Corning Glass Works (today Corning Inc.), following a proposal of Charles Kao and Charles Hockam of the Standard Telecommunication Laboratory in England, began to work on purifying glass. In 1970 they refined an optical fiber bundle using pure SiO2 (it was the purest glass ever made) and were awarded the patent for the Optical Waveguide Fibers capable of transmitting 65,000 times more information than metallic wire.

Several studies in the law and economics of intellectual property documented how, over the last 25 years, U.S. Court decisions switched from the traditional jurisprudential limitation of the patentability of early-stage scientific findings lacking in current commercial value to the conception that also fundamental basic scientific discoveries with no current tradeable application (such as scientific theories, algorithms and genetic engineering procedures) fall in the general applicability of the patent system design. For example, in 1980, in the Diamonds v. Chakrabarty case, the Supreme Court of United States ruled that microorganism produced by genetic engineering could be patented. The Supreme Court’s decision arrived two years before the introduction of the first commercial product, human insulin, obtained with recombinant DNA techniques. Jensen and Thursby (2001) study the licensing practices of 62 US universities. They find that "Over 75 percent of the inventions licensed were no more than a proof of concept (48 percent with no prototype available) or lab scale prototype (29 percent) at the time of license!". Moreover, most of the inventions licensed were in such an embryonic state of development, that no one could estimate their commercial potential and the inventor’s cooperation was required to get a successful commercial development.

Universities and public laboratories have always been the main performers of basic R&D in the United States and in Europe. Though an important reason for the relatively low private contribution to basic R&D is often found in the high degree of uncertainty that this activity involves in terms of future commercial application and success, the legal permission to appropriate the fruits of years of investigations is making a big difference between post-1980 US and current European innovation systems. If basic findings are
not patentable, as was in the pre-Eighties US and in the current EU regime, the publicly funded researchers or the university professors will likely investigate undirected by the profitability concerns. Therefore they may end up with discovering economically useless ideas, as a response to the random nature of their personal intellectual interests. Allowing them to patent scientific findings would introduce an interface between intellectual speculation and consumer needs, thereby channelling scientific investigation in the direction more demanded by the economy. Hence the post-Eighties US patenting regime allows its national system of innovation to count on a maximum flow of basic research findings per researcher. Instead, in the pre-Eighties US or in a European patenting regime, a smaller fraction of the scientists’ discoveries would be useful for further commercial applications. On the other side, the patentability of early-stage ideas, by granting monopolistic rents on basic discoveries charges a heavier burden on the applied R&D industry, which restricts the licensing decision of the research tool patent holders. This shifts the debate on the merits of public basic research over private basic research from appropriability concerns to other issues, as, for instance, the organization and the objectives of research activity and the monetary and non monetary incentives guiding research activity in the two systems (Lach and Shankerman, 2003; Aghion, Dewatripont and Stein, 2005).

Despite this controversial aspect of the innovation and growth policy, in the standard Schumpeterian growth theory (Grossman and Helpman, 1991; Aghion and Howitt, 1992 and 1998; Segerstrom, 1998; Howitt 1999) each R&D firm is assumed to undertake an independent innovation process in order to produce a probability of discovering an idea whose value immediately transfers into a tradeable application in the form of higher quality products. In this literature, the innovation process is governed by a Poisson process, by which an idea is instantaneously conceived and implemented into a new product to be sold to the marketplace. Of course, the Schumpeterian growth theory acknowledges the intertemporal spillover and the sequentiality for marketable products. However, it cannot handle the many real world cases in which ideas have to undergo a gradual development process before becoming embodied in a saleable technology, and
R&D firms gain an experience in multiple-stage research activities before developing tradeable applications of basic scientific findings. Thus, the conception that only the concrete embodiment of an idea is provided of economic value prove too restrictive in an age in which a large part of the international academic community, jointly with a large part of the R&D business community, expresses the need for an appropriate intellectual property design to take into account both the sequential and cumulative nature of ideas and the change in the technological paradigm determined by the biotechnology industry\(^5\).

This paper, by taking the R&D sequentiality into the Schumpeterian paradigm investigates the relation between the cumulative uncertainty involved in the two-stages innovation process and the inefficiency in the public university system. We will consider the lack of research direction of large scale publicly funded R&D, but also the ability of public laboratories to coordinate their activities better than the freely entrant private laboratories of the scenario with patentable research tools. Similarly, the positive incentives to basic research set by the possibility to patent research tools will be contrasted by the monopolistic underinvestment in the applied R&D. Our model therefore tries to give an impartial, albeit stylized, representation of the debated issue of the desirability of the patentability of research tools. Whether the positive or negative effects of the post-Eighties US patent system prevail depends on the data. We have performed a numerical attempt to evaluate the desirability of the US regime change in the early Eighties, by plugging the available data on technology, employment and skill premium into our model. After estimating the crucial R&D productivity parameters for the Seventies, we run the model for the two opposite institutional scenarios. We find that during the Seventies the old system was losing grounds on the perspective new patenting system, and the US authorities took institutional decisions which allowed a more efficient innovative activity. In our view, a crucial role was played by the steadily decreasing productivity of applied R&D, which we estimated in a novel way. Other studies documented an increasing com-

\(^5\)According to Gambardella (1995) and Henderson, Orsenigo e Pisano (1999), the pharmaceutical research industry is now more influenced by prior scientific discoveries than in the past.
plexity in the applied R&D activity (Kortum 1993 and 1997; Segerstom, 1998) as well. If applied R&D becomes increasingly more complicated, it is important to have a large flow of half-ideas from basic research. This implies that the inefficiency of public R&D has to be removed more urgently. We claim that this justified the reform undertaken in the US around 1980 and might recommend similar modifications in the European patent law.

The rest of this paper is organized as follows. Sections 2 and 3 set up a Schumpeterian model with sequential innovation in two different patent policy scenarios. In the first scenario basic R&D achievements are patented and, afterwards, developed into tradable applications within a completely privatized economy. In the second scenario basic research findings are conceived and put into the public domain, and subsequently embodied into marketable products by a large number of perfectly competitive private R&D firms. In Section 4 we try to assess, on the basis of the analytical structure provided by the model, the relative advantages and disadvantages of granting no intellectual protection to half-ideas. In Section 5 we calibrate the model with the U.S. data. Finally, the main results are summarized in Section 6.

2. The Model

2.1 Overview

Consider an economy made up of a differentiated final good sector and a differentiated research and development (R&D) sector, along the lines of Grossman and Helpman (1991), where product improvements occur in the consumption good industries. Within each industry, firms are distinguished by the quality of the final good they produce. When the state-of-the-art quality product in an industry $\omega \in [0,1]$ is $j_t(\omega)$, research firms compete in order to learn how to produce the $j_t(\omega) + 1$st quality product. This learning process involves a two-stage innovation path, so first a firm catches a glimpse of innovation through the $j_t(\omega) + \frac{1}{2}$th inventive half-idea and then other firms engage in a patent race to implement it in the $j_t(\omega) + 1$st quality product.

In what follows we refer to the term "quality leader" to denote the firm that produces
the current state-of-the-art quality product. We will call "half-idea follower", or simply the follower, any R&D firm that owns the first half-idea invented in order to introduce, in the second stage, the product’s innovation that pushes up the quality ladder. Finally, we let the term "outsider" denote each R&D firm that tries to invent a new first half idea in the basic research sector (i.e. tries to become the new follower). We assume that firms are able to instantaneously patent both the inventive half-idea and the product innovation. Then, patent protection may determine a monopolistic position both in the applied R&D sector and in the final good sector, and the winner of the final patent R&D race becomes the sole producer of a \( j_i(\omega) + 1 \) quality consumption product. Research firms rationally choose at which stage of the innovation process to settle in by simply comparing the expected benefits of their R&D efforts and their corresponding costs.

Time is continuous with an unbounded horizon and there is a continuum of infinitely-lived dynasties of expanding households with identical intertemporally additive preferences. Heterogeneous labour, skilled and unskilled, is the only factor of production. Both labour markets are assumed perfectly competitive. In the final good sectors \( \omega \in [0, 1] \) monopolistically competitive firms produce differentiated consumption goods by combining skilled and unskilled labour, whereas research firms employ only skilled labour.

2.2 Households

Time \( t \geq 0 \) population \( P(t) \) is assumed growing at rate \( g > 0 \) and its initial level is normalized to 1. The representative household’s preferences are represented by the following intertemporal utility function:

\[
U = E_0 \left[ \int_0^\infty e^{gt} e^{-\rho t} u(t) \, dt \right],
\]

where \( \rho > g \) is the subjective discount rate and \( E_0 \) denotes the expectation operator as of time \( t = 0 \). Instantaneous utility \( u(t) \) is defined as:

\[
u(t) = \int_0^1 \ln \left[ \sum_j \gamma^j d_{j\ell}(\omega) \right] d\omega,
\]
where \( d_{jt}(\omega) \) is the quantity consumed of a good of quality \( j \) (that is, a product that underwent \( j \) quality jumps) and produced in industry \( \omega \) at time \( t \). Assume that \( j \) is forced to assume integer values\(^6\). Parameter \( \gamma > 1 \) measures the size of the quality upgrades (i.e., the magnitude of innovations). This formulation, the same as Grossman and Helpman (1991) and Segerstrom (1991), assumes that each consumer prefers higher quality products.

The representative consumer is endowed with \( L > 0 \) units of skilled labor and \( M > 0 \) units of unskilled labor summing to 1. Since labour bears no disutility it will be inelastically supplied for any level of non negative wages. Since initial population is normalized to 1, \( L \) and \( M \) will also equal, in equilibrium, the percapita supply of skilled, respectively, unskilled labour. Unskilled labor can only be employed in the final goods production. Skilled labour is the most versatile, being also able to perform R&D activities.

In the first step of the consumer’s dynamic maximization problem, she selects the set \( J_t(\omega) \) of the existing quality levels with the lowest quality-adjusted prices. Then, at each instant, the households allocate their income to maximize the instantaneous utility (2) taking product prices as given in the following static (instantaneous) constraint equation:

\[
E(t) = \int_0^1 \sum_{j \in J_t(\omega)} p_{jt}(\omega)d_{jt}(\omega) d\omega.
\]  

(3)

Here \( E(t) \) denotes percapita consumption expenditure and \( p_{jt}(\omega) \) is the price of a product of quality \( j \) produced in industry \( \omega \) at time \( t \). Let us define \( j^*_t(\omega) \equiv \max \{ j : j \in J_t(\omega) \} \)

Using the instantaneous optimization results, we can re-write (2) as

\[
u(t) = \int_0^1 \ln \left[ \gamma^{j^*_t(\omega)} E(t)/p_{jt}(\omega) \right] d\omega = \ln[E(t)] + \ln(\gamma) \int_0^1 j^*_t(\omega)d\omega - \int_0^1 \ln[p_{jt}(\omega)]d\omega
\]

(4)

(5)

\(^6\)This assumption is common in the quality-ladder endogenous growth literature; still, in our framework, it has the meaning of explicitly stating that half-ideas discoveries do not affect consumer’s utility.
The solution to this maximization problem yields the static demand function:

\[ d_{jt}(\omega) = \begin{cases} 
E(t)/p_{jt}(\omega) & \text{for } j = j'_t(\omega) \\
0 & \text{otherwise.} 
\end{cases} \tag{6} \]

Only the good with the lowest quality-adjusted price is consumed, since there is no demand for any other good. We also assume, as usual, that if two products have the same quality-adjusted price, consumers will buy the higher quality product - although they are formally indifferent between the two products - because the quality leader can always slightly lower the price of its product and drive the rivals out of the market.

Therefore, given the independent and - in equilibrium and by the law of large number - deterministic evolution of the quality jumps and prices, the consumer will only choose the piecewise continuous expenditure trajectory, \( E(\cdot) \), that maximizes the following functional

\[ U = \int_0^\infty e^{-(\rho-g)t} \ln[E(t)] dt. \tag{7} \]

Assume that all consumers possess equal shares of all firms\(^7\) at time \( t = 0 \). Letting \( A(0) \) denote the present value of human capital plus the present value of asset holdings at \( t = 0 \), each individual’s intertemporal budget constraint is:

\[ \int_0^\infty e^{-R(t)} E(t) dt \leq A(0) \tag{8} \]

where \( R(t) = \int_0^t r(s) ds \) represents the equilibrium cumulative real interest rate up to time \( t \).

Finally, the representative consumer chooses the time pattern of consumption expen-

\(^7\)It may be pointed out the difficulty in reconciling the hypothesis of consumers’ owning equal shares of all firms with this Schumpeterian framework. Since there is a continuum of structurally identical industries, each consumer can diversify completely the industry-specific risk associated with the discovery of higher quality products. Thus, as creative destruction determines a sequence of monopolistic positions by R&D firms, forward looking consumers should be induced to carefully evaluate each research project in order to optimize the return-risk profile of their asset portfolio. Nevertheless, the Cobb-Douglas specification adopted for the instantaneous utility function, by satisfying the Gorman conditions, allows this model to admit a representative consumer representation regardless of the initial wealth distribution.
diture to maximize (7) subject to the intertemporal budget constraint (8). The optimal expenditure trajectory satisfies the Euler equation:

$$\frac{\dot{E}(t)}{E(t)} = r(t) - (\rho - g)$$  \hspace{1cm} (9)$$

where \( r(t) = \dot{R}(t) \) is the instantaneous market interest rate at time \( t \).

Euler equation (9) implies that a constant (steady state) per-capita consumption expenditure is optimal when the instantaneous market interest rate equals the consumer’s subjective discount rate. Since preferences are homothetic, in each industry aggregate demand is proportional to the representative consumer’s one. \( E \) denotes the aggregate consumption spending and \( d \) denotes the aggregate demand.

2.3 Production

In this section we examine the production side of the economy. We assume constant returns to scale technologies in the (differentiated) manufacturing sectors represented by the following production functions:

$$y(\omega) = X^\alpha (\omega) M^{1-\alpha} (\omega), \text{ for all } \omega \in [0, 1],$$ \hspace{1cm} (10)$$

where \( \alpha \in (0, 1) \), \( y(\omega) \) is the output flow per unit time, \( X(\omega) \) and \( M(\omega) \) are, respectively, the skilled and unskilled labour employment flows in industry \( \omega \in [0, 1] \). Letting \( w_s \) and \( w_u \) denote the skilled and unskilled wage rates, in each industry the quality leader seeks to minimize its total cost flow \( C = w_s X(\omega) + w_u M(\omega) \) subject to constraint (10). For \( y(\omega) = 1 \), the solution to this minimization problem yields the conditional unskilled (11) and skilled (12) labour demands (i.e. the per-unit labour requirements):

$$M(\omega) = \left( \frac{1-\alpha}{\alpha} \right)^\alpha \left( \frac{w_s}{w_u} \right)^\alpha,$$ \hspace{1cm} (11)$$

$$X(\omega) = \left( \frac{\alpha}{1-\alpha} \right)^{1-\alpha} \left( \frac{w_u}{w_s} \right)^{1-\alpha}.$$ \hspace{1cm} (12)$$
Thus the (minimum) cost function is:

\[ C(w_s, w_u, y) = c(w_s, w_u)y \]  \hspace{1cm} (13)

where \( c(w_s, w_u) \) is the per-unit cost function:

\[ c(w_s, w_u) = \left[ \left( \frac{1-\alpha}{\alpha} \right)^{-(1-\alpha)} + \left( \frac{\alpha}{1-\alpha} \right)^{-\alpha} \right] w_s^\alpha w_u^{1-\alpha}. \] \hspace{1cm} (14)

Since unskilled labour is uniquely employed in the final good sectors and all price variables (including wages) are assumed to instantaneously adjust to their market clearing values, unskilled labour aggregate demand \( \int_0^1 M(\omega) \, d\omega \) is equal to its aggregate supply, \( MP(t) \), at any date. Since industries are symmetric and their number is normalized to 1, in equilibrium \(^8\) \( M(\omega) = MP(t) \).

Letting \( w_u = 1 \), from equations (11) and (12) we get the firm’s skilled labour demand negatively depending on skilled (/unskilled) wage (ratio):

\[ X(\omega) = \frac{1}{w_s} \left( \frac{\alpha}{1-\alpha} \right) MP(t) \] \hspace{1cm} (15)

In per capita terms,

\[ x(\omega) \equiv \frac{X(\omega)}{P(t)} = \frac{1}{w_s} \left( \frac{\alpha}{1-\alpha} \right) M. \] \hspace{1cm} (16)

In each industry, at each instant, firms compete in prices. Given demand function (6), within each industry product innovation is non-drastic\(^9\), hence the quality leader will fix its (limit) price by charging a mark-up \( \gamma \) over the unit cost (remember that parameter \( \gamma \) measures the size of product quality jumps).

\(^8\)More generally, with mass \( N > 0 \) of final good industries, in equilibrium \( M(\omega) = \frac{MP(t)}{N} \).

\(^9\)We are following Aghion and Howitt’s (1992) and (1998) definition of drastic innovation as generating a sufficiently large quality jump to allow the new monopolist to maximize profits without risking the re-entry of the previous monopoly. Given the unit elastic demand, here the unconstrained profit maximizing price would be infinitely high: that would induce the previous incumbent to re-enter.
\[ p = \gamma c(w_s, 1) \Rightarrow d = \frac{E}{\gamma c(w_s, 1)}. \]  

(17)

Hence each monopolist earns a flow of profit, in per capita terms, equal to

\[ \pi = \frac{\gamma - 1}{\gamma} E = (\gamma - 1) \frac{w_s x}{\alpha} \]

\[ \pi = (\gamma - 1) \frac{1}{1 - \alpha} M. \]  

(18)

From eqs (18) follows:

\[ \frac{\gamma - 1}{\gamma} E = (\gamma - 1) \frac{1}{1 - \alpha} M \Rightarrow E = \frac{\gamma}{1 - \alpha} M. \]  

(19)

Interestingly, eq. (19) implies that in equilibrium total expenditure is always constant. Therefore, eq. (9) implies a constant real interest rate:

\[ r(t) = \rho - g \equiv r. \]  

(20)

2.4 R&D Sectors with Patentable Research Tools.

In each industry, the R&D activity is a step-by-step process by which, first a new idea is invented and then it is used to find the way to introduce a higher quality product. First half-idea are new, non-obvious, non-tradeable, patentable and necessary to get to the product innovation: first half-ideas are research tools.

In this section, stylizing a post-1980 US scenario, we assume that once the half-idea is invented it gets protected by an infinitely-lived patent. With this assumption we impose the publicity of the state-of-the-art in every sector and exclude the possibility for an outsider firm to lie by announcing a false half-idea finding in order to discourage its competitors.

Following Aghion and Howitt (1998, Ch.7), we assume that each R&D firm faces a
\( \cup \)-shaped unit cost function\(^{10} \). Let \( i = F, O \) denote a follower and an outsider R&D firm respectively, \( N_i \), with \( i = F, O \), indicates the mass of follower and, respectively, outsider firms in each R&D sector. The individual firm’s Poisson process probability intensity to succeed in inventing a half-idea or completing one (i.e. introducing the product innovation) is \( \theta_i (z_i - \phi, N_i, P(t)) \), increasing and concave in \( z_i - \phi \geq 0 \), depending on the R&D effort \( z_i \) in excess of the fixed cost, in terms of labour input, \( \phi > 0 \), that each firm has to pay per-unit time in order to engage in the R&D race. In particular, we specify the per-unit time Poisson probability intensity to succeed for an outsider and a follower firm respectively as

\[
\theta_O(z_O - \phi, N_O, P(t)) = \frac{\lambda_0}{P(t)} \sqrt{\max(z_O - \phi, 0)} \left( \frac{N_O}{P(t)} \right)^{-a} \tag{21}
\]

\[
\theta_F(z_F - \phi, N_F, P(t)) = \frac{\lambda_1}{P(t)} \sqrt{\max(z_F - \phi, 0)} \left( \frac{N_F}{P(t)} \right)^{-a} \tag{22}
\]

where \( \lambda_k > 0, k = 0, 1 \), are R&D laboratory productivity constants; \( N_i (i = O, F) \) represent the number of laboratories in each industry and constant \( a > 0 \) is an intra-sectoral congestion parameter, capturing\(^{11} \) the risk of R&D duplications, knowledge theft and other diseconomies of fragmentation in the R&D. Each Poisson process - with arrival rates described by (21)-(22) - governing the assumed two-stage innovative process is supposed to be independent across laboratories and across industries.

Eq.s (21)-(22) state that the probability intensity of the invention of a half-idea decreases with population. This assumption, common to Dinopoulos and Segerstrom (1999), captures the complexity of improving a good in a way that renders a larger population happier. Notice that also the congestion externality is assumed to decrease with population, as we deem it reasonable that the risk of R&D duplications declines with the

\(^{10} \)Assuming \( \cup \)-shaped R&D cost curves introduces some additional analytical complexity, but -beside being more realistic than the usual linear private R&D technologies - deliver more robust equilibria under different institutional scenarios. This renders our framework useful for additional extensions. It is interesting to point out that the square roots are not necessary: any exponent between 0 and 1 would work.

\(^{11} \)As, for example, in Romer’s (1990) specification of the R&D technology.
difficulty of duplications, that the industrial espionage activities are rendered more complicated with the technological complexity of the ideas being targeted, etc. The specific form postulated for our assumption of increasing technological complexity is sufficient to guarantee that the equilibrium long run per capita growth rates do not increase with population, thereby rendering our model immune to the embarrassing strong scale effect (Jones 2003) that plagued the early generation endogenous growth models, without leading to "semi-endogenous" growth (Jones 1995, Segerstrom 1998).

From the Poisson process proprieties, the probability of simultaneously inventing two half-ideas in a tiny interval of time of duration $\Delta t$ is a zero of order higher than the first. If instead an outsider R&D firm invents a second half-idea after the inventor of a first half-idea has already been granted a patent, the patent office will not grant a second patent on that half-idea, based on the legal principle of patentability requirement (O’Donoghue 1998, O’Donoghue and Zweimueller, 2003), because the second half-idea does not promise to generate further utility gains to the consumer. Therefore no second R&D firm will ever invest resources in inventing a half-idea on which it will not be able to claim intellectual property rights. As a result, no industry has more than one follower and the whole set of industries $\omega \in [0, 1]$ gets partitioned into two sets of industries: industries $\omega \in A_0$ (temporarily) with no half-ideas and, therefore, with one quality leader (the final product patent holder), no followers and a mass of outsider firms, and the industries $\omega \in A_1 = [0, 1] \setminus A_0$ industries, with one half-idea and, therefore, one half-idea leader (the final product patent holder) and one follower (the half-idea patent holder). Firms engage in basic R&D only in $\omega \in A_0$ industries and engage in applied R&D activity aimed at a direct product innovation only in $A_1$ industries. When a quality improvement occurs in an industry the half-idea follower becomes the new quality leader and the industry switches from $A_1$ to $A_0$. When an inventive half-idea discovery arises in an industry $\omega \in A_0$ this industry switches to $A_1$ and the winner of the first-stage patent race is the new follower. Figure 1 illustrates the flow of industries from a condition to the other:

Insert Figure 1
Notice that the two sets $A_0$ and $A_1$ change over time, even if the economy will eventually admit a steady state. At any instant we can measure the mass of industries without any half-idea as $m(A_0) \in [0, 1]$, and the mass of industries with an uncompleted half-idea as $m(A_1) = 1 - m(A_0)$. Clearly, in a steady state these measures will be constant, as the flows in and out will offset each other. In light of the definitions so far, we can express the skilled labor market equilibrium in per capita terms as:

$$L = x + m(A_0)n_Oz_O + m(A_1)n_Fz_F,$$

where $n_O \equiv \frac{N_O}{N(t)}$ and $n_F \equiv \frac{N_F}{N(t)}$ are the per capita number of laboratories in each basic and applied R&D sector. Eq. (L') states that, at each date, the aggregate supply of skilled labor, $LP(t)$, finds employment in the manufacturing firms of all $[0, 1]$ sectors, $x$, and in the R&D firms of the $A_0$ sectors, $n_Oz_O$, and of the $A_1$ sectors, $n_Fz_F$.

The stock value of all firms is determined by privately arbitraging between risk free consumption loans, firm bonds and equities, viewed as perfect substitutes also due to the ability of financial intermediaries to perfectly diversify portfolios and eliminate risk\textsuperscript{12}. Letting $V_O$, $V_F^1$, $V^0_L$, and $V^1_L$ denote respectively the present expected value of being an outsider firm ($V_O$), a half-idea follower ($V_F^1$), an $A_0$ industry quality leader ($V^0_L$) and an $A_1$ industry quality leader ($V^1_L$), costless arbitrage between risk free activities and firms’ equities imply that at each instant the following Bellman’s equations must hold in

\textsuperscript{12}Hence, despite individuals’ being risk averse, average returns will be deterministic, the risk premia will be zero, and agents will only compare expected returns. As usual in this class of models, we invoke the law of large numbers, which allows individuals who invest in a continuum of sectors with idiosyncratic risk, thereby transforming probabilities into frequencies.
Equilibrium:

\[ rV_O = \max_{z_O} \lambda_0 \sqrt{z_O - \phi} \left( \frac{N_O}{P(t)} \right)^{-a} (V_F - V_O) - w_sz_O + \frac{dV_O}{dt} \]  

(23a)

\[ rV_F = \max_{N_F, z_F} N_F \lambda_1 \sqrt{z_F - \phi} \left( \frac{N_F}{P(t)} \right)^{-a} (V_L^0 - V_{LF}) - w_sN_Fz_F + \frac{dV_F}{dt} \]  

(23b)

\[ rV_{L0}^0 = \pi P(t) - \left( \frac{N_O}{P(t)} \right)^{1-a} \lambda_0 \sqrt{z_O - \phi} (V_L^0 - V_L^1) + \frac{dV_{L0}^0}{dt} \]  

(23c)

\[ rV_{L1} = \pi P(t) - \left( \frac{N_F}{P(t)} \right)^{1-a} \lambda_1 \sqrt{z_F - \phi} V_L^1 + \frac{dV_{L1}}{dt} \]  

(23d)

Equation (23a) states that the risk free income deriving from the liquidation of the expected present value of an outsider R&D firm in an \( A_0 \) industry, \( rV_O \), is equal to the expected gain from becoming a follower, \( \lambda_0 \sqrt{z_O - \phi} \left( \frac{N_O}{P(t)} \right)^{-a} (V_F - V_O) \), i.e. the patent holder of the next half-idea in an \( A_1 \) industry minus the R&D expenditure, \( w_sz_O \), plus the gradual stock market appreciation in the case of the half-idea not arriving, \( \frac{dV_O}{dt} \).

Equation (23b) equals the risk free income deriving from the liquidation of the expected present value of the follower in an \( A_1 \) industry, \( rV_F \), and the expected increase in value from becoming a quality leader (i.e. completing the product innovation process), \( \lambda_1 \sqrt{z_F - \phi} (V_L^0 - V_F) \), minus the relative R&D cost, \( w_sN_Fz_F \), plus the gradual appreciation in the case of R&D success not arriving, \( \frac{dV_F}{dt} \).

Equation (23c) states that the risk free income deriving from the liquidation of the stock market value of a leader in an \( A_0 \) industry, \( rV_{L0}^0 \), equals the flow of profit \( \pi \) minus the capital loss from being challenged by a half-idea on a better product in the case a follower appears, \( \lambda_0 \sqrt{z_O - \phi} (V_{L0}^0 - V_{L1}) \), plus gradual appreciation in the case of such event not occurring, \( \frac{dV_{L0}^0}{dt} \).

Finally, equation (23d) equals the risk free income deriving from the liquidation of

---

13 The reader may note that, by making the assumption that the patent holder of a research tool, i.e. a first half-idea follower, can run any number of R&D laboratories and/or license its patent to any number of R&D firms (the licensee of the patent for the research tool is the only one who can use it to invent a completed new product), the patent holder is able to appropriate the profits potentially generated by its half-idea in its industry without incurring in the strong decreasing returns to laboratory size implied by our assumed inverse-U shaped laboratory unit cost curve.
the stock market value of a leader in an $A_1$ industry, $rV_L^1$, and the relative flow of profit $\pi$ minus the expected loss deriving from the follower’s success, $\lambda_1 \left( \frac{N_e}{P(t)} \right)^{1-a} \sqrt{Z_F - \phi V_L^1}$, plus the gradual appreciation if obsolescence does not occur, $\frac{dV_L^1}{dt}$.

Each perfectly competitive outsider firm determines the amount of labour devoted to basic research $z^*_O$ by trying to maximize its expected profit flow.

**Lemma 1**

a) The equilibrium amount of skilled labour employed by each outsider R&D firm in each $A_0$ industry is $z^*_O = 2\phi$.

b) The positive R&D equilibrium value of the skilled wage ratio is $w_s = \max \left( \frac{\lambda_1 V_L^0 (N_e \phi)^{-a}}{2\sqrt{z_F - \phi}}, 1 \right)$.

**Proof** (in Appendix 1.A).

Interestingly, unlike the usual features of previous quality ladders models, in this model, due to decreasing returns at the industry level, there is no indeterminacy in the intersector allocation of R&D labor$^{14}$.

Let us now turn to the follower’s problem. Problem (23b)’s first order conditions are:

$$\frac{\left( \frac{N_F}{P(t)} \right)^{1-a} \lambda_1 (V_L^0 - V_F)}{2\sqrt{z_F - \phi}} = w_s N_F$$

$$(1 - a) \left( \frac{N_F}{P(t)} \right)^{-a} \frac{\lambda_1}{P(t)} \sqrt{z_F - \phi} (V_L^0 - V_F) = w_s z_F,$$

which imply that the following **Lemma 2** holds:

**Lemma 2** The equilibrium amount of skilled labour employed in each R&D follower laboratory and the equilibrium number of laboratories in each $A_1$ industry are

$$z^*_F = 2\phi \frac{1 - a}{1 - 2a}$$

$$N^*_F = \left[ \frac{\lambda_1 (V_L^0 - V_F)}{P(t)2w_s \sqrt{\frac{\phi}{1 - 2a}}} \right]^\frac{1}{a} P(t).$$

$^{14}$See Cozzi (2006) for a proof of indeterminacy in quality ladders models.
It is interesting to note that the patent holder is using its research tool in a small enough number of R&D laboratories so that its R&D industry congestion is fully internalized.

Defining the per-capita stock market value of firms as \( v_L^0 \equiv \frac{V^0_L}{P(t)} \) and \( v_F \equiv \frac{V_F}{P(t)} \), we can write:

\[
n_F^* = \left[ \frac{\lambda_1(v_L^0 - v_F)}{2w_s \sqrt{\frac{\phi}{1 - 2a}}} \right]^{\frac{1}{2}}.
\]  

(26)

Plugging eq.s (16) and (18) into the expression of the skilled labour wage ratio (Lemma 1b)) and using per capita notation, we obtain

\[
x = \frac{1}{w_s} \left( \frac{\alpha}{1 - \alpha} \right) M = \min \left( \frac{2\sqrt{\phi}}{\lambda_0 v_F n_O^*}, 1 \right) \left( \frac{\alpha}{1 - \alpha} \right) M.
\]  

(27)

Therefore the (per capita) skilled labor employment in the manufacturing sector is inversely related to the market value of half-patented ideas. In fact, higher valued research tools draw more skilled labor from the manufacturing plants into the basic research laboratories, thereby increasing the manufacturing unskilled/skilled labor ratio and consequently raising skilled labor marginal productivity and the relative wage. Since the patent on a half-idea does not derive value from the direct production of a marketable good, \( V_F^1 \) is in turn pinned down by \( V_L^0 \). Therefore, the equilibrium value of the skilled wage is indirectly related to the stream of profits expected from the future commercialization of the product of the completed idea. Unlike the traditional Schumpeterian innovative process, the skilled wage here does not immediately incorporate the discounted expected value of the next commercially fruitful patent, but it does so only one step ahead: the value of the future monopolist is scaled down to current R&D labor wage by the composition of two innovation probabilities.

Let us remind that the skilled labor market clearing condition states:

\[
m(A_0)n_O^* z_O^* + (1 - m(A_0)) n_F^* z_F^* + x = L
\]  

(28)
Hence, since wages are pinned down by the optimal firm size and by the zero profit conditions in the perfectly competitive basic R&D labor markets, the unique equilibrium per-sector mass of entrant basic R&D firms consistent with skilled labor market clearing (28) is determined by solving equation (28) for \( n_O^* \):

\[
n_O^* = \frac{L - x - (1 - m(A_0))}{2\phi m(A_0)} n_F^*,
\]

where we have used Lemma 1 (for \( z_O^* = 2\phi \)) and (24) for \( z_F^* \).

To complete our analysis, let us look more closely at the inter-industry dynamics depicted by Figure 1. In the set of basic research industries a given number of perfectly competitive (freely entered) outsider firms, \( N_O \), employ a flow of skilled labour input \( z_O^* \) to get a flow probability of becoming \( A_1 \) followers, while in the set of innovative industries each of the \( n_F^* \) per-industry follower laboratories employs a flow of skilled labour input \( z_F^* \) to obtain a flow probability to succeed in implementing the state-of-the-art research level. The flows of probability, i.e. the per-unit time probabilities, for the individual firm to pass from \( A_0 \) to \( A_1 \) and from \( A_1 \) to \( A_0 \), at the aggregate level, become deterministic frequencies of industries flowing from \( A_0 \) to \( A_1 \) and from \( A_1 \) to \( A_0 \). Hence the industrial dynamics of this economy is described by the following first order ordinary differential equation:

\[
\frac{dm(A_0)}{dt} = (1 - m(A_0)) N_F \frac{\lambda_1}{P(t)} \left( \frac{N_F}{P(t)} \right)^{-a} \sqrt{z_F^*} - m(A_0) \left( \frac{N_O}{P(t)} \right)^{-a} N_O \frac{\lambda_0 \sqrt{z_O^*} - \phi}{P(t)}
\]

\[
= (1 - m(A_0)) \lambda_1 (n_F^*)^{1-a} \sqrt{\frac{\phi}{1 - 2a}} - m(A_0) (n_O^*)^{1-a} \lambda_0 \sqrt{\phi}
\]

where in the second equality we have used Lemma 1 (for \( z_O^* = 2\phi \)) and (24) for \( z_F^* \). Notice that our R&D technology assumption have allowed us to describe the dynamics of the fraction of sectors performing basic R&D only as a function of the mass of population-adjusted per-industry R&D laboratories. To completely describe the equilibrium dynamics of our economy it is convenient to reduce the dimensionality of the system.
by considering only population-adjusted variables. After using (20) and simplifying, we can rewrite eq.s (23b), (23c) and (23d) as

\[
\begin{align*}
\rho v_F &= (n_F^*)^{1-a} \lambda_1 \sqrt{\frac{\phi}{1-2a}} (v^0_L - v_F) - w_s n_F^* 2\phi \frac{1-a}{1-2a} + \frac{dv_F}{dt} \quad (31a) \\
\rho v^0_L &= \pi - (n_O^*)^{1-a} \lambda_0 \sqrt{\phi} (v^0_L - v^1_L) + \frac{dv^0_L}{dt} \quad (31b) \\
\rho v^1_L &= \pi - (n_F^*)^{1-a} \lambda_1 \sqrt{\frac{\phi}{1-2a}} v^1_L + \frac{dv^1_L}{dt} \quad (31c)
\end{align*}
\]

System (31a), (31b), (31c) and eq. (30) - jointly with cross equation restrictions (27) and (29) - form a system of four first order ordinary differential equations, whose solution describes the dynamics of this economy for any admissible initial value of the unknown functions of time \(v^0_L, v^1_L, v_F\), and \(m(A_0)\). In a steady state, \(\frac{dv^1_L}{dt} = \frac{dv^0_L}{dt} = \frac{dv_F}{dt} = \frac{dm(A_0)}{dt} = 0\).

Given the analytical complexity of such system we resorted to numerical analysis\(^\text{15}\). In all numerical simulations, the steady state exists, it is unique and it is saddle point stable for any set of parameter values. In particular, we obtain a unique economically meaningful equilibrium, and the uniqueness of the equilibrium is (locally) guaranteed because the linear approximation around the steady state has three unstable manifolds - associated with market variables \(v^0_L, v^1_L, v_F\) - and one stable manifold - associated with predetermined variable \(m(A_0)\). In fact, in all our computations the Jacobian of system (31a), (31b), (31c) and eq. (30) computed at the steady state has three eigenvalues with positive real parts and one eigenvalue with negative real part. Therefore, given an initial condition for \(m(A_0)\), there is (locally) only one initial condition for \(v^0_L, v^1_L, v_F\) such that the generated trajectory tends to the steady state vector: the equilibrium is determinate.

\[\text{2.5 R&D Sectors with Unpatentable Research tools.}\]

In this section we drop the assumption of patentable basic scientific results, in order to depict a pre-1980 US normative environment and/or a current European patent

\(^{15}\)The files .mod used to simulate the model in Matlab are available from the authors on request.
regime. Lacking the patent protection of the first half-ideas, the innovative process would need to resort to non-profit motivated R&D organizations to take place: publicly funded universities and laboratories have often been motivated by the induced scientific spillover on potentially marketable future technical applications.

This section also introduces a particular behavioral rule for public researchers: we assume that public researchers are not perfectly mobile across sectors, so that when in a sector $\omega$ that lacked a half-idea, i.e. belonged to $A_0$, a half-idea appears, i.e. it becomes $A_1$, the public R&D workers keep carrying out basic research in that sector. Given our technological assumptions, this behavior will likely lead to the discovery of a second half-idea in sector $\omega$ that is redundant from the economic viewpoint. This may represent the case of university researchers who keep investigating along intellectual trajectories even when they know that no private firm will ever profit from adapting to their market the new knowledge they may create. Unguided by the invisible hand, researchers will keep devoting their efforts proving that they are able to invent a second, third, ..., $n^{th}$ genial - but socially useless - idea to enrich their cv and their academic carrier opportunities. Hence, we will assume from here on that the public researchers are allocated across different industries according to a uniform distribution. This assumption emphasizes the role of markets to give R&D laboratories the right incentives to divert their resources from the unprofitable sectors and to quickly reallocate them towards more profitable aims.

We also make the assumption that the government chooses the fraction, $\bar{L}_G \in [0, L]$, of population of skilled workers to be allocated to the heterogenous research activities conducted by universities and other scientific institutions. The government basic R&D expenditure, equal to $P(t)\bar{L}_Gw_s$, is funded by lump sum per-capita taxes on consumers. The assumption of lump sum taxation guarantees that government R&D expenditure does not imply additional distortions on private decisions. This allows us to use the previous notation and derivations also for the case of a balanced government budget taxing all households in order to transfer the tax proceeds to the basic R&D workers.
The optimizing behavior of the public sector consists of maximizing the expected flow of half-ideas per sector with respect to the intensity of basic research effort $z_G$, that is the government chooses the optimal scale for the public laboratories.

The fixed per capita amount of skilled workers, $\bar{L}_G$, hired in the basic public R&D is equal to the intensity of basic research effort, $z_G$, multiplied by the number of public laboratories, $N_G$, i.e.:

$$P(t)\bar{L}_G = N_G z_G.$$ \hspace{1cm} (32)

In per capita terms,

$$\bar{L}_G = \frac{N_G}{P(t)} \equiv n_G z_G.$$ \hspace{1cm} (33)

**Lemma 3** The solution of the public sector maximization problem is $z^*_G = 2\phi \frac{1-a}{1-2a}$.

**Proof** (in Appendix 1.B).

Therefore, solving eq. (33) for $n_G$ and substituting the solution of the government maximization problem, we have:

$$n_G = \frac{\bar{L}_G}{2\phi \frac{1-a}{1-2a}}.$$ \hspace{1cm} (34)

The financial arbitrage implies the following market valuations of each sector’s unchallenged\footnote{We here mean "unchallenged" by a follower. However, a monopolist in an $A_0$ industry is indirectly challenged by the basic R&D laboratories trying to invent a new half-idea on which future follower firms will work to render it obsolete.} leader firm, $V^0_L$, directly challenged leader firm, $V^1_L$, and follower R&D firm.
\[ V_F^{17} : \]

\[ rV_L^0 = \pi P(t) - \left( \frac{N_G}{P(t)} \right)^{1-a} \lambda_0 \sqrt{z_G - \phi} \left( V_L^0 - V_L^1 \right) + \frac{dV_L^0}{dt} \quad (35a) \]

\[ rV_L^1 = \pi P(t) - \lambda_1 \left( \frac{N_F}{P(t)} \right)^{1-a} \sqrt{z_F - \phi} V_L^1 + \frac{dV_L^1}{dt} \quad (35b) \]

\[ rV_F = \max_{z_F} \lambda_1 \left( \frac{N_F}{P(t)} \right)^{-a} \sqrt{z_F - \phi} \left( V_L^0 - V_F \right) - w_z z_F + \frac{dV_F}{dt} \quad (35c) \]

Plugging eq. (34) and the optimal size of public laboratories as given in Lemma 3 into (35a) and using percapita variables - as in the last section - allow us to rewrite the equation of leader’s financial arbitrage as:

\[ \rho v_L^0 = \pi - \left( \frac{\bar{L}_G}{2 \phi \sqrt{1-a} \lambda_0} \right)^{1-a} \lambda_0 \sqrt{\frac{\phi}{1-2a}} (v_L^0 - v_L^1) + \frac{dv_L^0}{dt} \quad (36) \]

Solving Bellman eq. (35c) and setting (as a consequence of free entry into the applied R&D sector) \( V_F = 0 \), we get the flow of research labor hired by each follower, \( z_F^* = 2\phi \). Hence, the previous system (35a)-(35c) can be rewritten in percapita terms as:

\[ r v_L^0 = (\gamma - 1) \frac{1}{1-\alpha} M - \left( \frac{\bar{L}_G}{2 \phi \sqrt{1-a} \lambda_0} \right)^{1-a} \lambda_0 \sqrt{\frac{\phi}{1-2a}} (v_L^0 - v_L^1) + \frac{dv_L^0}{dt} \]

\[ r v_L^1 = (\gamma - 1) \frac{1}{1-\alpha} M - \lambda_1 n_F^{1-a} \sqrt{\phi} v_L^1 + \frac{dv_L^1}{dt} \]

\[ 0 = \lambda_1 \sqrt{\phi} v_L^0 n_F^{1-a} - w_z 2\phi \quad (37a) \]

\(^{17}\text{Notice that } \lambda_0 \left( \frac{N_G}{P(t)} \right)^{1-a} \sqrt{z_G - \phi} \text{ captures the expected partial obsolescence of unchallenged leadership in each } A_0 \text{ sector, } \lambda_1 \left( \frac{N_F}{P(t)} \right)^{1-a} \sqrt{z_F - \phi} \text{ the expected final obsolescence of directly challenged leadership in each sector } A_1, \text{ and } \lambda_1 \left( \frac{N_F}{P(t)} \right)^{1-a} \sqrt{z_F - \phi} \text{ the probability per unit time that a single follower succeeds in each sector } A_1.\)
From eq. (37a), we can solve for the skilled/unskilled wage ratio, getting:

\[ w_s = \max \left( \frac{\lambda_1 v_0^F}{2\sqrt{\phi}} n_F^{-a}, 1 \right). \]  

(38)

Let us remember that, from the final production analysis, we have:

\[ x = \frac{1}{w_s} \left( \frac{\alpha}{1 - \alpha} \right) M. \]  

(39)

The dynamics of the industries is now described by the following first order ordinary differential equation:

\[
\frac{dm(A_0)}{dt} = (1 - m(A_0)) N_F \frac{\lambda_1}{P(t)} \left( \frac{N_F}{P(t)} \right)^{-a} \sqrt{z_F - \phi} - m(A_0) N_G \frac{\lambda_0}{P(t)} \left( \frac{N_G}{P(t)} \right)^{-a} \sqrt{z_G - \phi} = \\
= (1 - m(A_0)) n_F^{1-a} \lambda_1 \sqrt{\phi} - m(A_0) \left( \frac{\tilde{L}_G}{2\phi^{1-a/2}} \right)^{1-a} \lambda_0 \sqrt{\frac{\phi}{1 - 2a}}. 
\]  

(40)

From the skilled labor market clearing condition

\[ x + \tilde{L}_G + (1 - m(A_0)) n_F 2\phi = L, \]  

we get to the equilibrium mass of per-sector followers:

\[ n_F = \frac{L - \frac{1}{w_s} \left( \frac{\alpha}{1 - \alpha} \right) M - \tilde{L}_G}{2\phi(1 - m(A_0))}, \quad m(A_0) \in [0, 1]. \]  

(42)

In the stationary distribution \( \frac{dm(A_0)}{dt} = 0 \). Therefore the flow of industries entering the \( A_0 \) group must equal the flow of industries entering the \( A_1 \) group. Given the complexity of our problem, also in this case we performed numerical simulations in Matlab\(^{18}\). In all simulations a unique economically meaningful steady state equilibrium exists and it is determinate.


\(^{18}\)The files .mod used to simulate the model in Matlab are available from the authors on request.
This section draws the implications of the two intellectual property scenarios we have depicted in the previous sections. To compare the two different regimes, we assume that in the scenarios with unpatentable research tools the government chooses the same amount of basic R&D that the private economy would generate in equilibrium. This allows us to compare the different inefficiencies after controlling for the public R&D employment level.

A first analytical result is expressed by the following:

**Proposition 1:** If the congestion externality is very small, i.e. \( a \to 0 \), the equilibrium innovation rate of the private economy with patentable research tools is higher than the equilibrium innovation rate of the economy with unpatentable research tools.

**Proof** (in Appendix 1.C).

As a result of Proposition 1, we can expect that if the information and communication technologies permit a high level of coordination of the investigations in the research community and trade secret laws effectively discourage ideas misappropriation\(^{19}\), the social costs of a public research system that lacks economic incentives to invent new potentially useful ideas are higher than the shortcomings stemming from monopolistic applied R&D.

The following also holds:

**Proposition 2:** If relative applied R&D productivity parameter \( \lambda_1 \) is very large, the equilibrium innovation rate of the private economy with patentable research tools is lower than the equilibrium innovation rate of the economy with unpatentable research tools.

**Proof** (in Appendix 1.C).

If it is relatively easy to find economic applications of scientific ideas, most of the sectors will tend to lack basic ideas, whose applications rapidly exhaust. Therefore the broadly focussed academic research by pure scientists uninterested in economic applications will turn out to be performing a useful service to the private R&D firms, by

\(^{19}\)See Cozzi (2001) and Cozzi and Spinesi (2006) for models of economic growth with endogenous industrial espionage.
disclosing a large spectrum of potentially profitable innovations. Though blind to the invisible hand, the publicly run universities internalize their academic externalities and - if applied R&D is simple enough or if basic research is difficult enough - cause the economy to reach a stronger innovative performance than if patenting the research tools disclosed by uncoordinated basic R&D institutions were permitted. In fact, relatively high levels of \( \lambda_1 \) imply a large steady state level of \( m(A_0) \), thereby minimizing the inefficiencies of the public researchers (who keep researching as if their activity were needed in the whole product space), compared to the social costs of the monopolistic production (by the research tools patent holders) of the applied R&D success probabilities.

Proposition 2 suggests that a public innovation infrastructure poor of selective economic incentives could have been acceptable in a world in which the industrial applications of basic scientific discoveries were rather straightforward. In the modern industry, in which applications of science are eagerly searched by often highly sophisticated researchers, curing the inefficiencies of basic research may become the top priority for a steadily growing economy. This may have motivated the switch in the US patenting rules in the early Eighties and at the same time may provide an explanation for the growing relative disadvantage of the European system of innovation, in which the patentability of research tools is not allowed.

Though lacking a general analytical result, all our numerical simulations of the two models of this paper show that, even at moderate levels of the congestion externality, the economy where research tools are patentable and basic R&D is privately carried out can lead to more innovation than an economy in which the public basic research is conducted in an inertial way. This happens when the applied R&D productivity parameter, \( \lambda_1 \), becomes very low: in such cases the equilibrium innovative performance of the private economy with patentable research tools becomes better than the equilibrium growth performance of the economy with a public R&D sector. In fact, if \( \lambda_1 \) is very small then \( m(A_0) \) will be small, thereby exalting the wasteful nature of the public R&D activity in \([0, 1] - A_0\): in this case the social cost of a public R&D blind to the social needs signalled
by the invisible hand would overwhelm the social costs of the restricted entry into the applied R&D sector induced by the patentability of research tools.

Table 1 in Appendix 2, built by simulating the model for constant values of all parameters excepted $\lambda_1$, summarizes the economy’s functioning for different values of applied R&D productivity in the two different intellectual property regimes. Table 1 provides an example that as $\lambda_1$ becomes very low the innovation rate in the scenario where research tools are patentable overcomes the equilibrium growth performance of the economy with a public R&D sector.

Figure 2, built on the data provided in Table 1, shows that for sufficiently low levels (below 1.7) of relative applied R&D productivity parameter $\lambda_1$, the innovation rate of the economy where research tools are patentable and basic R&D is privately carried out may overcome the innovation rate of the economy with unpatentable research tools.

**Insert Figure 2**

Among the different forms of intellectual property protection, patents constitute the most common way to allow the inventors to appropriate the economic potential from the inventions\(^{20}\). For this reason, patent data are often used by the economists as indicators for the innovative performance of the economic system\(^ {21}\).

**Insert Figure 3**

Figure 3 represents the total U.S. patent grants divided by the U.S. inflation adjusted applied R&D expenditure. It is easily seen that in the U.S. the ratio of the patents granted each year to US residents on applied R&D expenditure per year (in year 2000

\(^{20}\)Teece (2000) remarks that “Patents are in one sense the strongest form of intellectual property because they grant the ability to exclude, whereas copyrights and trade secrets do not prevent firms that make independent but duplicative discoveries from practicing their innovations and inventions.”

\(^{21}\)For a survey on the literature on patent statistics see Griliches (1990).
dollars) decreased by about four fifths from 1953 to 1982. This confirms the existence of an increasing complexity in the application of the basic scientific results for commercial purposes. Such patents to R&D ratio is an index of applied research productivity. We can view it as an alternative index to our parameter $\lambda_1$.

5. Calibrating the Model

In this section we calibrate our model with U.S. data from 1973 to 1979\textsuperscript{22}. Our exercise, among other things, will allow to obtain an estimation of the complexity of basic R&D, as summarized, inversely, by our parameter $\lambda_0$, whose evolution cannot be inferred by patent statistics, because in the Seventies basic R&D outcomes could not be patented even in the US.

In our calibrations we use the number of patents granted to U.S. residents - expressed in millions - for the years 1973, 1975, 1977, and 1979, as well as the data for the skilled and unskilled full time equivalent workers, mark ups, etc. to compute, for each year, the values of $\lambda_0$ and $\lambda_1$ which generate the observed numbers as the steady state equilibria\textsuperscript{23}. The value of the mark-up has been set equal to 1.68, consistently with estimates of Roeger (1995) and Martins et al. (1996). The values of the skill premium are taken from Krusell et al. (2000). Free parameters $\alpha$, $a$ and $\phi$ have been fixed, respectively, to values\textsuperscript{24} 0.1, 0.2 and 100. Also, we set the subjective rate of time preference to 0.05 throughout this period. Since we have purged our model from the scale effect, in the population-adjusted Bellman’s equation the population growth rate disappeared, thereby waving the need to give a value for it.

By solving for the steady state values of the endogenous variables in a way consistent

\textsuperscript{22}The data for $L_G$ National Science Board (2006) "S&E doctorate holders in research university and other academic institutions". Calibrations also include data for the U.S. skilled (completed four years of college or more) (/unskilled) labour employment available at http://www.census.gov/population/socdemo/education/tabA-2.xls.

\textsuperscript{23}Our decision to use the data only for the odd years is motivated by data availability. However, since we rely on the steady state computations, we think it reasonable to allow for a two year transition to the steady state rather than just one year.

\textsuperscript{24}Though these choices are admittedly arbitrary and motivated by the facility of convergence, we tried several other parameter values without modifying the ordering of results.
with the data we are able to get the values for the applied R&D productivity, $\lambda_1$, listed for each year in the following Figure 4:

**Insert Figure 4**

and for the basic R&D productivity; $\lambda_0$, listed for each year in Figure 5:

**Insert Figure 5**

As the reader can notice, during the Seventies R&D complexity increased in both basic and applied R&D. Hence, in principle, the relative advantage for the patentability of research tools over the public basic R&D system is ambiguous. Therefore, we have used the previously described values of the technological parameters and of the population composition to compute the hypothetical steady state equilibrium for each year for the alternative scenario (with patentable research tools) and compared its growth rate with the growth rate that a public system would deliver after controlling for the different endogenous basic R&D labor. The following Figure 6 lists the comparative growth rates in the two scenarios:

**Insert Figure 6**

We can notice from Figure 6 that the effect of the decrease in the productivity of applied R&D overwhelmed the increasing complexity in the basic R&D. Throughout the decade the unpatentability of the basic scientific findings imposed an inefficiency to the US innovation system. Moreover, the growth rate gap tended to increase over time. If policy makers or the courts were aware of this kind of inefficiency they would have accelerated the patentability of research tools, which anyway prevailed at the beginning of the Eighties. Therefore we can consider the policy change in favour of the research
tools patentability occurred in the United States from the early Eighties as the rational political reaction to a decrease in the applied R&D productivity.

One could wonder why the increase in the applied R&D difficulty did not bring about a decrease in the U.S. applied R&D expenditure. Our model suggests that the expansion of basic research, resulting from non-profit motivated decisions by public authorities and other entities, associated to the increase in $\bar{L}_G$, from 14% of the total U.S. employment in 1973 to 16% of the total U.S. employment in 1979, allowed the generation of enough half ideas to induce the observed increase in the private applied R&D employment.

6. Final Remarks

This paper developed a general equilibrium R&D-driven growth model in which the innovation process is decomposed into two successive innovative stages. The extension of multisector Schumpeterian models to such a more realistic dimension allows us to answer a question about the US policy shift towards the extension of patentability to research tools and basic scientific ideas. These normative innovations have been modifying the industrial and academic lives in the last two decades. Our calibration suggests that one of the reasons why they were efficiently introduced was the increased relative difficulty in the marketable applications of basic scientific discoveries.

As the chances of finding profitable economic applications of each scientific result get more and more remote, a larger number of industries need to be endowed with basic ideas to try to build upon, which means that the scientists should focus their research energies in the sectors where important scientific results are still lagging. This social need is less likely met by academic researchers whose careers proceed when they show themselves able to discover theoretical results regardless of the potential commercial profits that applied R&D firms may make by building on their inventions.

The analysis carried out in this paper should be extended in order to fully grasp the complexity of the problem on which we tried to cast some light. For example, we can notice that governments often have several policy tools at their disposal: hence their
choice is not merely infinite patents versus public laboratories. There are subsidies, finite patent lives - modeled, for example, by a constant probability of the patent lapsing, as in Grossman and Lai, (2004) - as well as other importation ways to design the rewards to the innovative activities. In particular, the classical Michael Kremer’s (1998) idea of patent buyouts could be incorporated into the general equilibrium framework laid down by this paper. In the case of the polymerase chain reaction or other really valuable basic ideas where the monopoly mark-up is a severe distortion, it seems like buyouts could be especially useful. Therefore in future works it would be potentially very fruitful to try to incorporate the refinements investigated in the more specialized microeconomic literature on rewarding sequential innovation with heterogeneous agents and asymmetric information, such as Hopenhayn, Llobet and Mitchell (2006).

Bibliography


Appendix

34
1.A - PROOF of Lemma 1.

\( a \) Consider the \textit{outsider}'s profit maximization problem in an \( A_0 \) industry (eq. 21a in text):

\[
rV_O = \max_{z_O} \frac{\lambda_0}{P(t)} \sqrt{z_O - \phi} \left( \frac{N_O}{P(t)} \right)^{-a} (V_F^1 - V_O) - w_s z_O + \frac{dV_O}{dt}. \tag{43}
\]

The first order condition for a maximum is:

\[
\frac{\lambda_0 (V_F^1 - V_O)}{2P(t)\sqrt{z_O - \phi}} \left( \frac{N_O}{P(t)} \right)^{-a} = w_s \tag{44}
\]

Since other outsider firms can freely enter each first half-idea patent race, in equilibrium the unskilled labor real wage will adjust so that expected profits will be annihilated at the optimal firm size. Therefore in any equilibrium with positive R&D activity, the stock value of every first half-idea R&D firm, \( V_O \), will be zero, and the average and marginal product of research will be equal, i.e.

\[
V_O \equiv 0 \text{ implies: } \frac{dV_O}{dt} = 0 \text{ and } \lambda_0 \sqrt{z_O - \phi} \frac{V_F^1}{P(t)} \left( \frac{N_O}{P(t)} \right)^{-a} = z_O w_s. \tag{45}
\]

Solving (44) and the last eq. (45) for \( z_O \), we obtain the equilibrium amount of labour, \( z_O^* \), employed by each outsider R&D firm:

\[
z_O^* = 2\phi. \tag{46}
\]

\( b \) Plugging (46) into (45) allows us to find the positive R&D equilibrium value of the skilled wage ratio as:

\[
w_s = \max \left( \frac{\lambda_0 V_F^1 \left( \frac{N_O}{P(t)} \right)^{-a}}{2\sqrt{\phi}}, 1 \right). \tag{47}
\]

In eq. (47) we have constrained the equilibrium skilled wage not to be lower than
the unskilled wage, because otherwise the skilled workers would apply for unskilled jobs. Notice that if the skilled wage were lower than the r.h.s. of (47) there would be excess demand for skilled labor, because the freely entrant basic R&D firms would try to make unboundedly high profits. Hence \( w_s \) would immediately increase. If instead the wage was higher than the r.h.s. of eq. (47) no basic R&D would be carried out, and eventually no R&D at all. Unlike Aghion and Howitt (1992) and Grossman and Helpman (1991), such no growth path would never afflict the economy depicted in our model, because of the assumed form of decreasing sectorial returns in each R&D sector.

Q.E.D.

1.B - PROOF of Lemma 3. From eq.(27) we have:

\[ n_G = \frac{\bar{L}_G}{z_G}. \]  

(48)

The public authorities seek to maximize the per sector- expected flow of half ideas by choosing the optimal scale for public laboratories:

\[
\max_{z_G} P(t)\frac{\bar{L}_G}{z_G}\theta_G(z_G - \phi, N_O, P(t)) = \max_{z_G} \left( \frac{\bar{L}_G}{z_G} \right)^{1-a} \lambda_0 \sqrt{z_G - \phi}. 
\]  

(49)

The solution for the public sector maximization problem (48) is:

\[ z_G^* = 2\phi \frac{1 - a}{1 - 2a}. \]  

(50)

Q.E.D.

1.C - PROOF of Proposition 1. Letting \( m_{0\text{priv}} \) denote the equilibrium mass of \( A_0 \) sectors in the privatized - i.e. with patentable research tools - basic R&D economy, \( m_{0\text{pubineff}} \), the equilibrium mass of \( A_0 \) sectors in the inefficiently run public - i.e. with unpatentable research tools - basic R&D economy, the following relationship holds in all
comparisons we made:

\[ \bar{L}_G = m_{0\text{Priv}} n_O 2 \phi \Rightarrow n_O = \frac{\bar{L}_G}{m_{0\text{Priv}} 2 \phi}. \]  

(51)

In the steady state the innovation rate of the private basic R&D economy is:

\[ \text{Innov}_{\text{Priv}} = m_{0\text{Priv}} \left( \frac{\bar{L}_G}{m_{0\text{Priv}} 2 \phi} \right)^{1-a} \lambda_0 \sqrt{\phi} = m_{0\text{Priv}}^{a} \left( \frac{\bar{L}_G}{2 \phi} \right)^{1-a} \lambda_0 \sqrt{\phi}. \]  

(52)

The innovation rate of the public inefficient economy is:

\[ \text{Innov}_{\text{PuIne}} = m_{0\text{PuIne}} \left( \frac{\bar{L}_G}{2 \phi \frac{1-a}{1-2a}} \right)^{1-a} 2 \lambda_0 \sqrt{\phi} \frac{1}{1-2a}. \]  

(53)

By dividing eq. (52) by (53), and letting \( a \to 0 \), we get:

\[ \frac{\text{Innov}_{\text{Priv}}}{\text{Innov}_{\text{PuIne}}} \to \frac{(m_{0\text{Priv}})^a}{m_{0\text{PuIne}}} \left( \frac{1-a}{1-2a} \right)^{1-a} (1-2a)^{\frac{1}{2}} \to \frac{1}{m_{0\text{PuEff}}} > 1. \]  

(54)

Q.E.D.

**PROOF of Proposition 2.** Consider the following ratio:

\[ \frac{\text{Innov}_{\text{Priv}}}{\text{Innov}_{\text{PuIne}}} = \frac{(m_{0\text{Priv}})^a}{m_{0\text{PuIne}}} \left( \frac{1-a}{1-2a} \right)^{1-a} (1-2a)^{\frac{1}{2}} \]  

(55)

Let us first note that \( \left( \frac{1-a}{1-2a} \right)^{1-a} (1-2a)^{\frac{1}{2}} \) is a positive number lower than 1 for all values of \( a \) between zero and 1/2. To see this, remind that \( \left( \frac{1-a}{1-2a} \right)^{1-a} (1-2a)^{\frac{1}{2}} = 1 \) when \( a = 0 \), and that it is positive when \( 0 < a < 1/2 \). Moreover, it is strictly decreasing for \( 0 \leq a < 1/2 \) which is seen in a simple way by derivating its natural logarithm with respect to \( a \), and getting:

\[ \ln \left( \frac{1-2a}{1-a} \right) < 0. \]
Let us now note that, from the properties of our model, the limit, as $\lambda_1$ tends to infinity, of the measure of $A_0$ tends to 1 in both economies, because in the limit the applied R&D instantaneously completes any new half-idea, immediately reflecting each sector that enters $A_1$ into $A_0^{25}$. Therefore:

$$\frac{\text{Innov}_{Priv}}{\text{Innov}_{Pulnelf}} \xrightarrow{\lambda_1 \to \infty} \frac{(1 - a)^{1-a} \frac{1}{2}}{(1 - 2a)^{1-a}} < 1, \quad 0 < a < 1/2.$$  

Q.E.D.

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25 Notice that in this case our model behaves as the standard quality ladder growth model - in which $A_0 = [0, 1]$ - and no completing half-ideas are required.
Figures to be inserted in the text:

**FIGURE 1:** REPRESENTATION OF THE ECONOMY BY FLOWS OF INDUSTRIES.
FIGURE 2: COMPARISON OF THE INNOVATIVE PERFORMANCES, SIMULATION RESULTS.
Figure 3: U.S. Patent Ratio Over Time

Figure 5: Estimation for the Basic R&D Productivity