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Abstract
This paper develops economic definitions of energy quality for individual fuels and energy aggregates. There are both use- and exchange-value concepts as well as marginal and total measures of energy quality. A factor augmentation or quality coefficients approach corresponds to the use-value definition while indicators based on distance functions and relative prices are exchange-value based definitions. These indicators are identical when the elasticity of substitution between fuels is infinity but diverge or cannot be computed for other interfuel elasticities of substitution. Under zero substitutability only the quality coefficients approach is defined. I also find that the ratio of an energy volume index to aggregate joules cannot be considered a complete indicator of aggregate energy quality as it does not account for quality changes in the component fuels.

JEL Codes: Q40, D24, O47
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1. Introduction

Not all energy sources and fuels are of equal economic productivity. These differences in productivity are termed energy quality. Some fuels can be used for a larger number of activities and/or for more valuable activities. For example, coal cannot be used to directly power a computer while electricity can. The productivity of a fuel is determined in part by a complex set of attributes unique to each fuel: physical scarcity, capacity to do useful work, energy density, cleanliness, amenability to storage, safety, flexibility of use, cost of conversion, and so on. Fuel and energy quality is not necessarily fixed over time as changes in technology in terms of both new techniques of production and new products and activities change the opportunities for using fuels. However, it is generally believed that electricity is the highest quality energy vector followed by natural gas, oil, coal, and wood and other biomass in descending order of quality. This is supported by the typical prices of these fuels per unit of energy, which is one way of measuring relative energy quality.

There are both biophysical and economic approaches to measuring energy quality. The leading physical approach to energy quality is the ratio of exergy – energy that is available to perform work – to total energy. The higher the ratio, the higher energy quality is (Cleveland *et al*., 2000). But as discussed by Cleveland *et al*. (2000), exergy is only one property of energy sources that affects their economic usefulness. And the exergy/energy ratio is essentially the same for all chemical fuels and electricity. Cleveland *et al.* proposed fuel prices or marginal products as the sole economic indicators of energy quality. But price is not the most fundamental definition of energy quality and other indicators can be considered. This paper discusses alternative definitions of economic energy quality and proposes a comprehensive set of economic definitions and indicators.

I show that energy quality is only uniquely defined when the elasticity of substitution between fuels is infinity – in which case all the proposed measures are equal – or zero, in which case only one of the approaches is defined. For intermediate values of the elasticity of substitution energy quality is a more ambiguous concept. The concept that is most relevant depends on the elasticity of substitution and the application.

We can consider both the quality of individual fuels and the quality of an energy aggregate. Most statistical agencies and economists, however, tend to linearly aggregate energy sources together according to their heat content, which implicitly assumes that the different energy sources are infinitely substitutable and of equal quality. If this is not the case, then estimates of productivity and production relations based on these aggregates are biased. Therefore, appropriate aggregation methods are important. The economic energy quality literature has focused on these and conflated them with measuring quality itself (e.g. Cleveland *et al*., 2000; Ho and Jorgenson, 1999). This paper examines whether this interpretation is accurate.

Throughout the paper, I discuss the production case rather than the consumption case. Production seems to be simpler to understand without the problems of wealth constraints and nonmeasurable utility to contend with. I believe that arguments similar to those in this paper would easily transfer into the consumption realm.

The second section of the paper reviews the various concepts in the literature that are relevant to defining fuel quality. The third and fourth sections provide in depth definitions of the quality coefficients and substitution approaches to measuring fuel quality. The fifth section synthesizes these ideas and discusses when each is most relevant. The sixth section examines what a quality adjusted aggregate energy index actually measures and the seventh section concludes.

2. Concepts of Fuel Quality

> **“**Quality -- you know what it is, yet you don't know what it is…. But some things are better than others, that is, they have more quality. But when you try to say what the quality is, apart from the things that have it, it all goes poof! …... But for all practical purposes it really does exist. What else are … grades based on? Why else would people pay fortunes for some things and throw others in the trash pile? Obviously some things are better than others -- but what's the "betterness"? -- So round and round you go, spinning mental wheels and nowhere finding anyplace to get traction. What the hell is Quality? What is it?**”** (Pirsig, 1974, Chapter 15).

Energy quality is easier to pin down than the concept of quality in general but still elusive. From an economic perspective one fuel is better than another if it is more productive in producing economic outputs or utility. The difficulty is only then in defining that productivity.

Various definitions of energy quality have been proposed and are in some cases fairly widely used. Cleveland *et al*. (2000) define energy quality as "the relative economic usefulness per heat equivalent of different fuels". In that paper, I go on to imply that the quality of individual fuels is proportional to their marginal products and, therefore, in competitive input markets the ratio of their prices. The rationale is that if when I substitute one fuel for another, output increases the fuel whose quantity increases is of higher quality. Berndt (1978) just asserts that it is reasonable to use prices as weights in constructing an index of energy, generalizing the approach suggested by Turvey and Nobay (1965).

A similar approach is sometimes seen in the literature on labor quality (e.g. Abowd *et al*., 1996; Ho and Jorgenson, 1999; Jorgenson *et al*., 2003). On the other hand, Kazamaki-Ottersten *et al*. (1999) and Mellander (2000) define the quality of an input as a factor, which multiplies that input, wherever it appears in the production function, while Giannis (1998) assumes that quality adjusted labor supply is an affine function of quality. This quality factor or coefficient approach is much more common in the literature on human capital in economic development and growth. For example, Padilla and Mayer (2003) state that a labor quality index that multiplies the quantity of labor is equivalent to an index of human capital. Tallman and Wang (1994) equate labor quality with human capital per worker. Hanushek and Kimko (2000) measure labor force quality as the average of results on international mathematics and science tests.[1]

Formalizing this quality coefficients approach for the special case of a production function, *f*, with a single output, *y*, distinguishing between general factor neutral technological change, *A,* and the quality factors:

$$y = f(A, \theta_1 E_1, \ldots, \theta_n E_n, \mu_1 X_1, \ldots, \mu_m X_m) \quad (1)$$

[1] But Mankiw *et al*. (1992), among others, introduce labor and human capital as two different inputs to production with different output elasticities.

where the θ_i are the quality factors of the n energy inputs E_i, and the μ_i are the quality factors of the m non-energy inputs X_i. The quality coefficients may be related to explanatory variables as in the labor quality literature or could be treated as latent state variables. The restriction to a single output can be relaxed but the restriction on the form of technological change could not be relaxed in the absence of explanatory variables that can be used to identify quality change as something distinct from technological change.

Kander (2002) suggests that energy quality should be measured based on the use-value – in the classical sense of the term [2] – contributed by each fuel rather than on marginal productivities or exchange-values. She uses the usual neoclassical interpretation of the classical concept of use-value as the integral of the demand curve (e.g. Hirschleifer and Hirschleifer, 1997). Figure 1 illustrates this for the producer case. mp_E is the marginal product of energy. The exchange-value is equal to P*E*. For a single input production function, the producer surplus is equal to the profit generated. In this case the use-value generated is equal to the sum of exchange-value and producer surplus, which is equal to total output as shown by:

$$\int_0^{E^*} \frac{\partial y}{\partial E} dE = y \qquad (2)$$

The idea has merit in that it tries to capture the notion that the inframarginal units of energy contribute more to production than the marginal unit. By contrast, the marginal productivity approach deems that any energy source that is very abundant must be of low relative quality as the marginal units of the energy source will be used in low marginal value activities. But there are inherent problems with integrating the area under the demand curve to derive use-value in the multi-input case. For constant elasticity of substitution production functions, except in the case

[2] in the environmental economics literature the term "use value" is confusingly used in a different way to indicate utility derived from actually using the resource as opposed to existence value.

of where the elasticity of substitution is infinity,[3] the sum of the use-values of the individual inputs is greater than total output and when the elasticity of substitution is less than one the use-value of each individual input is equal to total output as each output is essential to production.[4] in the latter case there is, therefore, no way to compare the qualities of the different inputs on this basis.

Though the usual neoclassical way of expressing the idea of use-value is to integrate the area under the demand curve, this is not necessarily what the classical economists were thinking of when contrasting use- and exchange-value. Based on Commons (1934), Stern (1999) explains that for the classical economists:

> "...use value was utility - the happiness or satisfaction derived from using a commodity. The classical economists did not conceive of this utility as declining with increasing consumption. Therefore, there was no relation between use value per unit and the abundance or consumption of the commodity. Use value did change with what neoclassical economists would now call changes in preferences … The use value of a particular material object would also decline through wear and tear over time. Commons suggested that I measure use value in physical units…" (p473)

The quality coefficients approach achieves exactly what is stated in this quotation. Quality coefficients are defined in terms of physical units as they multiply the quantity of each input. Unlike the demand curve integral, they do not decline with the level of consumption. They may increase with technological improvements and decline with depletion or depreciation. Therefore, quality coefficients appear to correspond better to the classical concept of use-values than do demand integrals.

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[3] I am referring to the traditional definition of the direct or Hicks elasticity of substitution, which takes the value of zero for the Leontief production function and infinity for the linear production function (Stern, 2009).

[4] If output is zero when a particular input is zero, irrespective of the quantities of other inputs, then that input is essential.

We can, therefore, approach energy quality from an exchange value approach – using relative prices or other substitution based measures as discussed below – or a use value approach using quality coefficients. The next two sections of the paper deal in depth with the two approaches and following that a synthesis is laid out.

3. Quality Coefficients Approach

Quality coefficients, as introduced in equation (1), multiply each input wherever they appear in the production function and other functions derived from it. However, there are significant problems in actually identifying and measuring such quality changes empirically. A first problem is to differentiate between a technological change and a change in energy quality. Take, for example, a single input model:

$$y = f(A, \theta E) \tag{3}$$

where A is technology, θ is energy quality, y is output, E is joules of energy, and f is an arbitrary production function. How can I distinguish between A and θ? We cannot, unless I define θ in some specific way using additional *a priori* information or data. The simple restriction $y = Af(\theta E)$ will work if there are variable returns to scale and I use both the original production function and the first order condition to estimate the two variables A and θ.[5] But this decomposition is fairly arbitrary. There are four alternative approaches: assuming that the energy quality of each fuel is an intrinsic factor that is constant for all time; assuming that all energy augmenting technical change is a change in energy quality; treating energy used in different uses as being of different qualities but assuming that that quality is fixed over time; or using additional data to model energy quality as is done in much of the labor quality literature referenced.

[5] For example if $f()$ is a quadratic in logarithms then the first order condition is $\partial \ln y / \partial \ln E = \beta_E + 2\beta_{EE}(\ln\theta + \ln E)$, which does not involve A. The mean of quality is still not identifiable (and is meaningless for a single energy input) but I can now measure the change in quality over time.

Assuming that energy quality is an intrinsic factor that is constant for all time, is rather unreasonable. It would mean that inventions that create new productive uses for fuels have no influence on their perceived quality. So in this case, the relative quality of particular grades of coal and gasoline would be considered to be the same today as it was in 1870. At the opposite extreme, assuming that all factor-augmenting technological change associated with an input represented changes in its quality is also seems unreasonable. Much of total TFP growth may consist of labor augmenting technical change as is assumed in most mainstream growth models (Acemoglu, 2000). This large gain could not surely be seen as purely representing an increase in labor quality? A large part of the gain might for example be seen as improved management techniques that use labor more effectively.

One variable that might be used to proxy energy quality is exergy conversion efficiency - the percentage of useful work performed per unit of exergy in the fuel (Ayres and Warr, 2005). This conversion efficiency depends on the use to which the fuel is put and the state of technology. Ayres and Warr compiled estimates of these conversion efficiencies for the United States from 1900 to 1998 for five uses (in order of efficiency in 1998): electric power, high temperature industrial heat, medium temperature heat, other mechanical work, and low temperature space heat. Before the First World War, electricity conversion efficiencies were worse than those of medium or high temperature heat. All conversion efficiencies have improved over time (Figure 2). Qualities of fuels could be determined based on their allocation to these different uses. But there are two issues with this approach as a measure of fuel quality.[6] First it attributes all the technological change that results in efficiency improvements to improvements in energy quality. As shown by Ayres and Warr (2005) most of the "Solow Residual" might be explained by these efficiency improvements. Second, it treats all uses of electricity or process heat as being equally productive. The latter could be partly dealt with by determining whether electricity is used downstream for motive power, heating etc. but that is not a full solution. It is likely that computing generates more waste heat than electric motors do. But that does not necessarily mean electricity is used less productively in computing.

[6] Ayres and Warr (2005) do not claim that this is a measure of energy quality.

Treating each fuel used in each application as a different input is a similar idea to Jorgenson and Griliches's (1967, 269) suggestion that: "in principle it would be desirable to distinguish among categories of labour services classified by age, sex, occupation, number of years schooling completed, industry of employment, and so on." Ho and Jorgenson (1999) actually do classify workers by gender, age, employee/self-employed status, education, and industry. Each unique combination is a separate input. If we differentiate between the uses of fuels, aggregate energy efficiency might then increase over time either because of a shift from lower quality to higher quality fuels or because of a shift in the mix of applications of fuels and most importantly the invention of new more valuable uses. [7]The invention of computers would add a new energy use category in the same way that it introduced a new occupation: computer programmers. Productivity improving technical change within a use – the invention of faster and faster computers say would still be assigned to TFP growth. The more disaggregated data that is available on the uses of energy the more TFP would likely be assigned to improvements in energy quality rather than technological change. This is the logical conclusion of the Jorgenson and Griliches (1967) approach to measuring productivity change – product innovations are assigned to changes in input while process innovations are assigned to TFP growth. Fuel qualities might be estimated using panel data. That panel data would need a measure of output in order to measure productivity. Data on how much of each fuel is used in each use alone is insufficient.[8]

We need to take care in order to estimate quality coefficients or trends that are meaningful indicators of fuel qualities in the multi-input case. For example, for the translog cost function, for the general case of time-varying quality trends, where I do not have additional variables to use in identifying the trends, the cost share equation is given by:

$$S_{it} = \beta_i + \sum_j \beta_{ij} \ln P_{jt} - \sum_j \beta_{ij} \ln A_{jt} + \varepsilon_{it} \quad (4)$$

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[7] Assuming competitive pricing, the price weighted index of energy volume in (17) would not be affected by the use of energy in different end-uses because the price of one kilowatt hour of electricity will be the same whether I use it to run a space heater or a computer.

[8] This is an issue because most countries collect energy data using a different system of use categories than that which they use to collect production data.

where S_{it} is the share of input i in total costs in period t, ε_{it} is a random error term, the P_{jt} are the prices of the various inputs and the A_{jt} the quality coefficients or trends. β_i and the β_{ij} are parameters to be estimated. As the cost function is homogenous of degree one in prices, $\sum_i \beta_i = 1$ and $\sum_j \beta_{ij} = 0$. We can think of each quality trend as being composed of two components:

$$\ln A_{it} = \ln \overline{A}_t + \ln \hat{A}_{it} \qquad (5)$$

where $\overline{A}_t$ is a common factor neutral TFP trend and $\hat{A}_{it}$ are the deviations of each quality trend from the common trend. This is the multivariate generalization of the decomposition I proposed in connection with equation (1) above. Because $\sum_j \beta_{ij} = 0$ the common trend is swept out of (4) and only the deviations can be estimated, so that (4) is replaced by:

$$S_{it} = \beta_i + \sum_j \beta_{ij} \ln P_{jt} - \sum_j \beta_{ij} \ln \hat{A}_{jt} + \varepsilon_{it} \qquad (6)$$

Time-varying quality trends can be treated as either linear deterministic trends or stochastic trends estimated using a structural time series approach (e.g. Harvey and Marshall, 1991). In the latter case, the quality trends are then estimated as state variables using the Kalman filter. Harvey and Marshall note that the trends in (6) are only identified if I require that $\sum \ln \hat{A}_{jt} = 0$ for all t and I either imposing a restriction on their initial values or the mean of each individual trend or on the values of the β_i. The first condition is innocuous given (6) but I cannot impose conditions on the means of the trends if I want to recover meaningful quality factors. Therefore, I need an independent estimate of the β_i's. β_i depends on the units in which the price variables are measured (Hunt and Lynk, 1993). For example, if I measure prices of joules of energy I will obtain a different result than if I use prices of BTUs, or if I index all prices to 1 in the initial year or any other simple rebasing of prices.[9] Estimating the production or cost function itself in addition to the share equations identifies the β_i's as these are now attached to observed variables. For constant returns to scale:

$$\ln C_t = \beta_C + \ln y_t + \sum_i \beta_i(\ln P_{it} - \ln A_{it}) + 0.5\sum_i \sum_j \beta_{ij}(\ln P_{it} - \ln A_{it})(\ln P_{jt} - \ln A_{jt}) + \varepsilon_{Ct} \qquad (7)$$

where C is total cost and y output. If (6) and (7) are estimated jointly, the β_i are identified from (7) and, therefore, the initial values of the relative quality factors $\hat{A}_{it}$ are identified in (6). However, the presence of β_C in (7) means that the mean of the neutral technical change trend $\ln \overline{A}_t$ is not identified. The model can be identified by setting $\beta_C = 0$ or $\ln \overline{A}_1 = 0$, thus giving the common trend an arbitrary starting value. This does not affect the relative energy quality factors nor the changes over time in the absolute energy quality factors of each fuel. The major complication is that (7) is nonlinear in the state variables and requires estimation using the extended Kalman filter (see Harvey, 1989 for details).

Treating $\ln \hat{A}_{it} = \ln \theta_{it}$ is again an arbitrary approach to separating changes in energy quality from changes in technology. It makes the assumption that technological change is unbiased and that there is no net trend in the sum of the quality factors even though they may each individually be trending. Some energy qualities will be declining over time.

Other functional forms, such as the generalized Leontief, have the same or similar identification issues. No individual quality trends can be identified at all for the Cobb-Douglas function. The CES production function is given by:

$$y = \left[\sum_i \alpha_i^{\frac{1}{\sigma}} (A_i X_i)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \qquad (8)$$

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[9] in order to measure relative fuel qualities the prices must be in terms of currency units per energy unit and not indexed to an arbitrary base year.

where σ is the elasticity of substitution. Again some restriction is needed either on the quality trends or the parameters α_i. The limit of the CES function as the elasticity of substitution tends to infinity is:

$$\lim_{\sigma \to \infty} \left[\sum_i \alpha_i^{\frac{1}{\sigma}} \left(A_i X_i \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} = \sum_i A_i X_i \tag{9}$$

So that the coefficients of the linear production function are quality indices. This is the only case where the absolute levels of the quality factors can be unambiguously identified. They are also the marginal products of the inputs. But still there is no unique way to separate quality change from technological change.

4. Substitution Approach

In this section of the paper, I instead define energy quality as how much of one fuel is required to replace another while maintaining output or how much output changes when one fuel replaces another. The former is an input-oriented approach to measuring energy quality and the latter an output-oriented approach. We can also generalize to the multiple output case with potentially some outputs being bads – for example pollution.

For marginal changes, relative marginal products or prices are the relevant indicators for the single output case as their ratio is equal to the marginal rate of substitution between inputs. It is easy to show that these do not generally correspond to the quality indicators discussed in the previous section. A simple, single input, Cobb-Douglas production function example can illustrate this:

$$y = \left(\theta E \right)^{\alpha} \tag{10}$$

Energy quality is then defined by:

$$\theta = y^{1/\alpha} E^{-1} \tag{11}$$

but the marginal product is given by:

$$\frac{\partial y}{\partial E} = \alpha y E^{-1} \tag{12}$$

(11) and (12) are not equal unless $\alpha = 1$, which is the single input case of the linear production function or a Cobb-Douglas function with constant returns to scale. Furthermore, in the multi-input case the relative marginal products of a Cobb-Douglas function do not involve the intrinsic energy qualities. For more complex functions the ratios of marginal products may involve the augmentation factors. But they generally involve complicated functions of all the augmentation factors.

Therefore, relative prices do not in general measure relative energy qualities in the sense that they were defined in the previous section. But for the linear production function (9) the two approaches do coincide.

If the elasticity of substitution is greater than unity between two inputs so that neither is essential to production then, as illustrated in figures 2 and 3, total measures of energy quality can also be constructed using the substitution approach. In both examples, I assume that the quantities of all other inputs are held constant.

Figure 3 presents the set $L(y^0)$ that indicates feasible combinations of two energy inputs E_1 and E_2 to produce a given level of output y^0. The boundary of the set is the traditional isoquant for output y^0. The point q^0 on the E_1 axis indicates the minimum feasible quantity of input E_1 required to produce y^0 when E_2 is not used. When instead q^0 units of input E_2 are used instead of E_1 it is no longer feasible to produce y^0. Instead, to fully replace E_1 in production, q^1 units of E_2 are required. q^0/q^1 is a measure of the total energy quality of E_2 relative to E_1.

Figure 4 shows the feasible output set $P(q^0,0)$ for outputs y_1 and y_2 given input of q^0 of E_1 and zero of E_2. $y^0(q^0,0)$ is one of the efficient output combinations that can be produced with these levels of inputs. But for the input combination $(0,q^0)$ only a maximum of y^1 can be produced given the same output mix. The ratio of distances $|y^1|/|y^0|$ is a measure of the total energy quality of E_2 relative to E_1.

How does this measure relate to intrinsic qualities? Using the CES function in (12) the output oriented total energy quality of E_2 relative to E_1 is given by:

$$\frac{y(0,q^0)}{y(q^0,0)} = \left(\frac{\alpha_2}{\alpha_1}\right)^{\frac{1}{\sigma-1}} \frac{A_2}{A_1} \tag{13}$$

which is the ratio of the augmentation factors multiplied by a constant which is a summary statistic of the technology. The translog function cannot handle inputs with values of zero but for a two input homogeneous generalized Leontief function the ratio of total energy quantities is:

$$\frac{y(0,q^0)}{y(q^0,0)} = \frac{\alpha_{22}}{\alpha_{11}} \frac{A_2}{A_1} \tag{14}$$

The two concepts are, therefore, closely related. For more than two inputs or for non-homogeneity such a simple relationship does not hold.

The indicators of energy quality discussed here would most likely be estimated using distance functions estimated on cross-sectional data using either econometrics or more likely data envelopment techniques.

Is the assumption that the interfuel elasticity of substitution is greater than one important? Stern (2009) finds that at the level of the industrial sector the elasticity is only significantly greater than unity for substitution between coal and gas and significantly less than unity for substitution between oil and electricity. The degree of substitutability appeared to be less at the macroeconomic level and more at the sub-industry level. However, these means based on the existing empirical literature are likely to be biased downwards. The total substitution indicator of relative energy quality may, therefore, be relevant, especially at the micro-level.

5. Fuel Quality: A Synthesis

We have seen that energy quality can be defined along the following dimensions:

Total vs. Marginal: Factor augmentation indices and distance-based indicators are measures of total energy quality while relative prices are indicators of marginal energy quality.

Intrinsic vs. Substitution Based: Augmentation indices are measures of intrinsic energy quality that do not depend on the quantities of other inputs or on how much energy is used and relate well to the classical notion of use-value. They do, however, depend on the state of technology unless some *a priori* assumption can be used to distinguish between changes in fuel quality and technological change. In simple two input cases, the distance-based indicators are linear functions of the ratios of the augmentation indices but in more general cases they depend on the quantities of other inputs used and their augmentation indices. Relative marginal products are usually a function of the quantities of inputs.

Absolute vs. Relative: An individual augmentation trend is an absolute indicator of energy quality. All the other indicators are relative, though a single marginal product in real terms could perhaps also be an absolute indicator. Relative augmentation trends net out the contribution of factor neutral technical change and hence achieve a crude decomposition of changes in energy quality and technological change.

But the existence and relevance of these various indicators also depend on the elasticity of substitution between fuels. We can differentiate between the following cases:

Infinite Substitutability ($\sigma = \infty$): For the linear production function, marginal products are equal to augmentation indices. There is, therefore, a single indicator of absolute energy quality. But still a question of separating energy quality and technical change. In the two input case, the substitution-based measure of total energy quality is also equal to the ratio of marginal products and the ratio of augmentation indices. Therefore, there is a single definition of relative energy quality.

High Substitutability ($\infty > \sigma > 1$): All the measures of energy quality can be computed in theory, though in practice strong identifying assumptions are needed to identify augmentation trends as quality factors. However, each indicator has a different value.

Cobb-Douglas ($\sigma = 1$): Quality coefficients or augmentation indices cannot be identified.

Low Substitutability ($1 > \sigma > 0$): The substitution-based indicator of total energy quality cannot be computed. Otherwise the indicators are similar to the high substitutability case.

Zero Substitutability ($\sigma = 0$): We have not considered this case explicitly in the paper so far. The Leontief production function is given by:

$$y = \min\left[E_1/\gamma_1,...,E_n/\gamma_n,X_1/\gamma_{n+1},...,X_m/\gamma_{n+m}\right] \tag{15}$$

where the γ's are the minimum input requirements to produce one unit of output y. I few believe that there is no reason for the technology to require varying amounts of energy from the different fuels apart from differences in their quality, then the energy quality indices for each energy input, E_i, therefore are equal to $1/\gamma_i$. The function could be re-written in factor augmentation form as:

$$y = \min\left[A_1E_1,...,A_nE_n,A_{n+1}X_1,...,A_{n+m}X_m\right] \tag{16}$$

Therefore, only factor augmentation makes sense as an indicator of energy quality in this case and neither of the substitution measures can be defined. On the other hand, if we believe that the differences in the minimum input requirements are not entirely due to differences in quality we cannot measure energy quality in the zero substitutability case.

As discussed in section 2, the quality coefficients can also:

- vary across fuels but do not depend on the use of fuels and do not vary over time.
- vary by fuel and use but do not vary change over time.
- vary by fuel (and possibly by use) and change over time possibly as a function of other variables.

Marginal products might do any of the above, as will the total substitution measure. Neither is defined based on restrictions in the way that quality coefficients are.

6. Aggregate Energy Quality

Using discrete Divisia aggregation:

$$\Delta \ln Q_t = \sum_i 0.5\left(S_{it} + S_{it-1}\right)\Delta \ln E_{it} \qquad (17)$$

the change in the logarithm of the quality adjusted quantity index of energy *Q*, in period t, is the sum of the cost share weighted changes in the logarithms of the quantities of the various fuels E_i. Dividing this index by the simple unweighted aggregate of fuels measured in heat equivalents (*E*) gives an index of aggregate energy quality. This approach is used almost universally in the literature on quality adjustment of inputs.

Ho and Jorgenson (1999) define the quality of labor purely as the difference between the volume index that takes into account substitution between different labor inputs (Equation 4 in their paper) and the simple sum of labor hours. They are very explicit that one should not confuse quality with factor augmentation, which is a particular parameterization of technological change. Jorgenson and Griliches (1967, 257) go so far as to say that use of the term "quality change" for this kind of adjustment for the effects of changes in the mix of inputs is a misnomer as all a supposed increase in quality reflects is more rapid growth in the use of higher quality inputs than lower quality inputs with no account taken of any change in the qualities of the individual inputs themselves. In the context of labor quality if the education of college educated workers is improving over time, this change will be attributed to TFP while a shift from non-college educated to college educated workers is attributed to quality change.

We can also develop an index of aggregate energy quality using the standard growth accounting approach applied to (1). We take the total differential of the production function with respect to time, divide both sides by *y* and multiply and divide each of the RHS variables by itself, yielding:

$$\frac{d\ln y}{dt} = \frac{\partial \ln y}{\partial \ln A}\frac{d\ln A}{dt} + \sum_{i=1}^{n}\frac{\partial \ln y}{\partial \ln \theta_i}\frac{d\ln \theta_i}{dt} + \sum_{i=1}^{n}\frac{\partial \ln y}{\partial \ln \mu_i}\frac{d\ln \mu_i}{dt} + \sum_{i=1}^{n}\frac{\partial \ln y}{\partial \ln E_i}\frac{d\ln E_i}{dt} + \sum_{i=1}^{m}\frac{\partial \ln y}{\partial \ln X_i}\frac{d\ln X_i}{dt} \tag{18}$$

Now as each quality factor multiplies its factor of production the output elasticities with respect to the quality factors are equal to those with respect to their inputs. In competitive equilibrium under constant returns to scale the output elasticities are equal to the relevant cost shares and so each of the latter four summations in (18) can be approximated in discrete form by a Divisia index:

$$\Delta \ln Z_t = \sum_i 0.5\left(S_{it} + S_{it-1}\right)\Delta \ln Z_{it} \tag{19}$$

where the Z's are either energy inputs, other inputs, or quality factors. Of interest is an index of energy quantity (20) and one of energy quality (21):

$$\Delta \ln Q_t = \sum_i 0.5\left(S_{it} + S_{it-1}\right)\Delta \ln E_{it} \tag{20}$$

$$\Delta \ln \theta_t = \sum_i 0.5\left(S_{it} + S_{it-1}\right)\Delta \ln \theta_i \tag{21}$$

Equation (20) is the standard index of aggregate energy input as proposed by Berndt (1978) and already given in (17). The second index is constant if there are no changes in the quality factors of the inputs. If there are no such changes, then (20) is an appropriate method of aggregating the energy inputs so that we can write:

$$y = f(A, \theta_1 E_1, ..., \theta_n E_n, \mu_1 X_1, ..., \mu_m X_m) = g(A, Q, \mu_1 X_1, ..., \mu_m X_m) \quad (22)$$

If there are changes in the qualities, θ_i, but (21) is not actually computed then it will be absorbed into the total factor productivity residual A.

It is clear, however, that the standard approach to computing a "quality-weighted" index in equation (20) does not account for the effects of change in augmentation index style qualities. While it might be weighted for differences in marginal products *Q/E* is not really an index of energy quality from this perspective.

7. Conclusions

The arguments in this paper show that an index of energy volume/divided by aggregate heat equivalent is not a complete indicator of aggregate energy quality. It may reflect the shift of the energy mix towards higher or lower quality fuels but it will register no change when the relative prices of any of the fuels in the mix changes but their quantity is unchanged. Also, I have presented arguments to show why traditional demand curve use-value calculations cannot be aggregated and are not informative regarding energy quality when an input is essential.

Several alternative indicators of energy quality are introduced. They are all equivalent only in the case of linear production or infinite substitutability. I distinguished between measures of total and marginal energy quality and between definitions of quality based on intrinsic properties of each fuel (use-value) and based on substitution of one fuel for another (exchange-value).

Factor augmentation indices are an intrinsic total energy quality indicator. This indicator has already been assumed to be the indicator of labor quality by Kazamaki-Ottersten *et al*. (1999) and Mellander (2000) and is a development of the use-value indicators discussed in Stern (1999). However, except in the cases of zero and infinite substitutability, measurement depends on estimating complex nonlinear time series models that are likely to require long time series in order to achieve any precision in estimation. There is also no obvious way to distinguish between quality change and more general technological change.

I also introduced a new approach to measuring energy quality using distance functions. This is a substitution based total energy quality indicator. The method can only be used if the elasticity of substitution between energy inputs is greater than one. Relative prices are a straightforward approach to measuring marginal quality from the substitution approach. They are useable for any degree of substitutability between the energy inputs greater than zero. However, they only collapse to the intrinsic quality indicator in the case of linear production.

Of course, there is nothing energy-specific about any of the arguments in this paper and the proposed indicators can be applied to any other inputs too.

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Figure 1. Exchange- and Use-Value

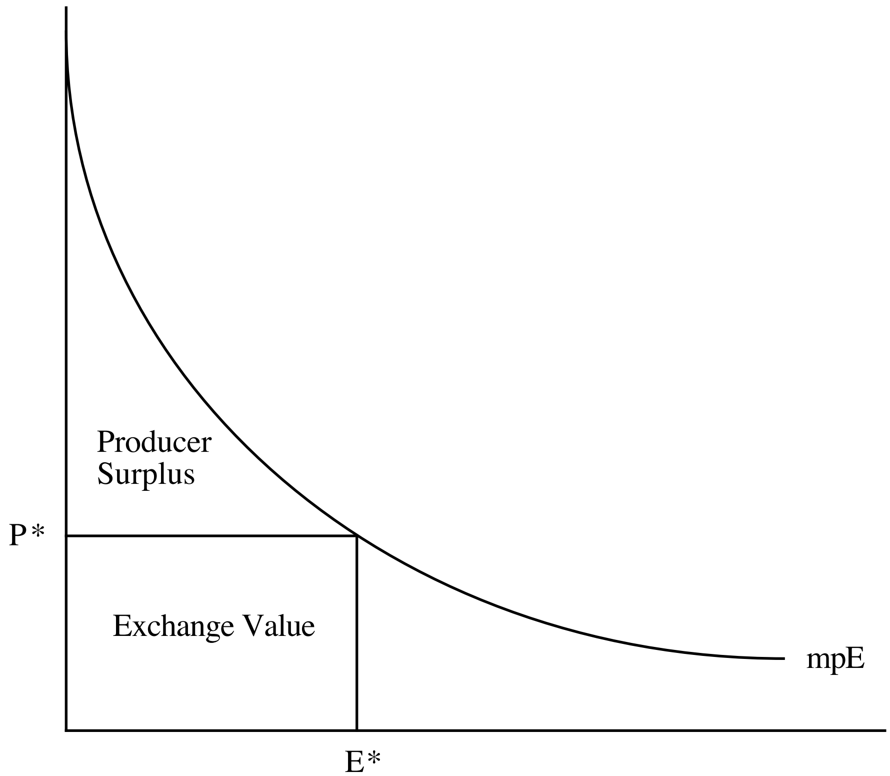

Figure 2. Exergy Conversion Efficiencies, USA, 1900-1998

Source: Ayres and Warr (2005)

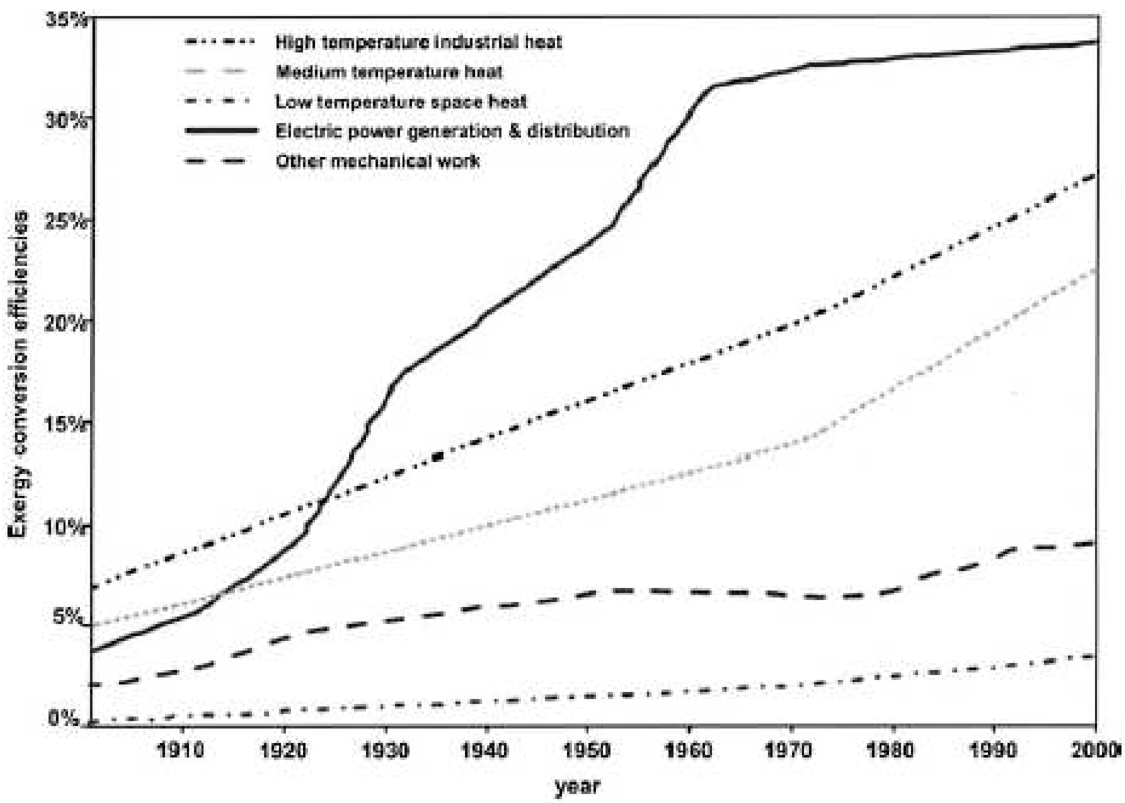

Figure 3. Input Oriented Total Energy Quality

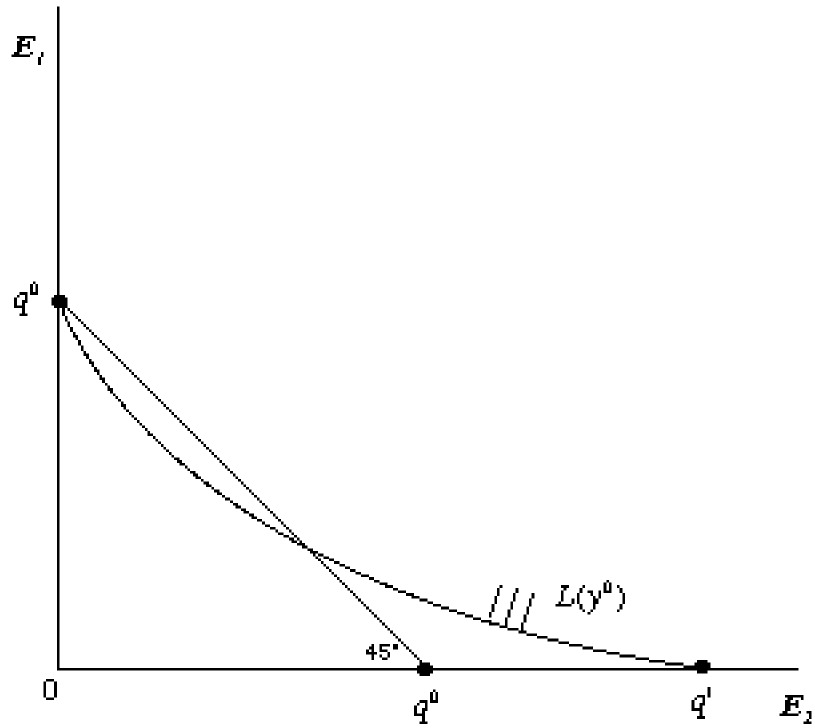

Figure 4. Output Oriented Total Energy Quality