# Financial Development and Amplification

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**Abstract**

This paper investigates theoretically how financial development affects the magnitude of financial amplification. Financial development yields two competing effects, balance sheet effects and shock cushioning effects. Depending on which of these forces dominates, we find that financial amplification initially increases with financial development and later falls down. Moreover, we examine the role of monetary policy to reduce financial amplification. We find that in the case of unexpected productivity shocks, money growth targeting dampens financial amplification by producing shock cushioning effects. On the other hand, inflation targeting exacerbates the shocks because under the policy, shock cushioning effects are not generated.

Key Words: Financial development, Financial amplification, Balance sheet effects, Shock cushioning effects
JEL Classification: E44, E32, E52

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# 1 Introduction

What are the effects of the development of financial markets on amplification over the business cycle? Traditional wisdom suggests that financial development stabilizes the economy by providing various channels for risk diversification. According to this view, financial innovation not only promotes long-run economic growth by enhancing efficiency in resource allocation, but also it helps to cushion consumers and producers from the effects of economic shocks.[1] This classical view seems to have been widely accepted. Indeed, several empirical and quantitative studies support the positive role of financial development in reducing volatility (See Cecchetti et al, 2006; Dynan et al, 2006; Jerman and Quadrini, 2008).

However, the situation has begun to change dramatically since the outbreak of the credit crisis of 2007-08. A new perspective has emerged: financial development destabilizes the economy by accelerating financial amplification. Before the crisis, it was often pointed out that thanks to financial innovation, the leverage of borrowers increased, and this high leverage generated economic booms. However, once the credit crisis occurred, people began to state that such a high leverage could lead to significant damages in borrowers' balance sheets, and eventually in the financial system as a whole. Financial development is suddenly blamed for increasing volatility. Indeed, IMF (2006, 2008) supports this new view by presenting empirical evidences that in more-advanced financial systems, the shock propagation effects become stronger.[2]

Motivated by these conflicting views, this paper theoretically investigates the following questions: What is the relationship between financial development and financial amplification (macroeconomic volatility)? Does financial development accelerate or decelerate financial amplification? In order to answer these questions, we develop a model of financial development with endogenous growth. The two key elements of this framework are the borrowing constraint and the heterogeneous investment projects—high profit investment projects with agency problems and low profit investment projects with less agency problems. The former captures *balance sheet effects* that magnify

[1] Levine(1997), Beck et al. (2000) show empirically that financial development causes long run economic growth.

[2] IMF reports argue that the sensitivity of real GDP growth rate, corporate investment, household consumption, and residential investment response to equity busts, or business cycles, is increasing in more market-based financial systems.

shocks.[3] The latter plays an important role in describing *shock cushioning effects*. By changing the *degree* of the borrowing constraint, which is defined as financial development, this paper shows that financial development not only strengthens *balance sheet effects* through changing leverage, but also it produces *shock cushioning effects* through an adjustment of the real interest rate. The balance between these two competing forces determines whether financial development magnifies or dampens financial amplification. Moreover, the balance by itself changes according to the degree of financial development.

Our main result shows that in a low development region, while shock cushioning effects do not work well, *balance sheet effects* get strengthened with financial development, thereby accelerating financial amplification. However, once the level of development passes a certain degree, *shock cushioning effects* start working, which in turn weakens *balance sheet effects*, thereby dampening financial amplification. Hence, the relation between financial development and financial amplification is non-monotonic: financial amplification initially increases with financial development and later falls down.

Moreover, we examine government policy to reduce financial amplification. Under the low development level, once negative productivity shocks hit an economy, downward amplification occurs, which impairs agents' welfare such as workers. Thus, there is a potential role for macro policies. In this paper, we analyze the role of monetary policy. We find that in the case of unexpected productivity shocks, money growth targeting policy dampens financial amplification by producing *shock cushioning effects*, thereby stabilizing economies. On the other hand, inflation targeting policy exacerbates the shocks because under the policy, *shock cushioning effects* are not generated, thereby destabilizing economies.

This paper is in line with business cycle theory which emphasizes the role of credit market imperfections. Following the seminal work by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), some researchers put financial factors a central role in accounting for business fluctuations (See Holmstrom and Tirole, 1997; Kiyotaki, 1998; Bernanke et al., 1999; Kocherlakota, 2000; Cordoba and Ripoll, 2004). These studies demonstrate how shocks are amplified through *balance sheet effects*, assuming a fixed degree of the borrowing constraint.[4] Our study relaxes this assumption. By so doing, we show that

[3] See Bernanke et al. (1996) for balance sheet effects.

[4] A recent study by Brunnermeier and Pedersen (2008) shows that amplification in-

there exists not only a region in which *balance sheet effects* dominate *shock cushioning effects*, but also a region in which *shock cushioning effects* negate *balance sheet effects*.

In the point that this paper examines the relation between financial development and financial amplification, our paper is related to Rajan (2006) and Shin (2009). Rajan argues that financial development has made the world better off, however it can accentuate real fluctuations, and economies may be more exposed to financial-sector-induced turmoil than in the past. However, Rajan does not necessarily propose a formal model of how financial development accelerates financial amplification. Shin presents a theoretical model where securitization by itself may not enhance financial stability. Our study shows the mechanisms within one framework that financial development not only accelerates financial amplification, but also decelerates it.

Concerning this non-monotonic relation between financial development and financial amplification (macroeconomic volatility), Aghion et al. (1999) and Matsuyama (2007, 2008) are close to ours. Aghion et al. derive non-monotonicity be developing an endogenous growth model with borrowing constraints and heterogeneous investment projects. They show that volatility is low when the development level is low or high. High volatility (cycles in their paper) occurs when the level has an intermediated value. Our paper also shows that volatility is high when financial development is in an intermediated development region. However, the source of high volatility is different from their paper. In their model, a change in the interest rate has a role in increasing volatility while in our model, it has a role in reducing volatility.[5] In our model, high volatility is caused by *balance sheet effects* togehter with high leverage.

Matsuyama (2007, 2008) develops a model of the borrowing constraint with various types of heterogeneities in an overlapping generations framework, and shows how it leads to a wide range of non-monotonic phenomena. In Matsuyama's model, the source of non-monotonicity lies in the investment projects which do not produce capital goods. He shows that a better credit market might be more prone to financing those investment projects,

creases by the interaction between funding liquidity and market liquidity, which refer to the borrowing constraint and resaleability constraint, respectively.

[5] In Aghion et al.'s model, a rise (decline) in the interest rate during booms (recessions) increases (reduces) debts repayment, which in turn produces recessions (booms). In this way, endogenous cycles with high volatility occur.

and such a change in credit allocation generates non-monotonicity. On the other hand, in our paper, the source of non-monotonicity lies in the change in the interest rate.

The remainder of the paper is organized as follows. Section 2 presents the model. We analyze the dynamics and derive the relation between financial development and financial amplification. In section 3, we examine the role of monetary policy to reduce financial amplification, and discuss its welfare implications. Section 4 presents conclusion.

# 2 The Model

Consider a discrete-time economy with two types of goods, consumption goods and capital goods and two types of agents, entrepreneurs and workers. Let us start with the entrepreneurs, who are the central actors in the paper. At date $t$, a typical entrepreneur has expected discounted utility:

$$E_0\left[\sum_{t=0}^{\infty}\beta^t \log c_t\right], \tag{1}$$

where $c_t$ is the consumption at date $t$, and $\beta \in (0,1)$ is the subjective discount factor, and $E_0[x]$ is the expected value of $x$ conditional on information at date 0.

Each entrepreneur can access investment projects to produce capital. Every entrepreneur can access low profit investment projects, but only some of the entrepreneurs, called H-entrepreneurs can access high profit investment prejects. The rest of the entrepreneurs we call L-entrepreneurs. The investment technology follows

$$k_{t+1} = \alpha^i z_t, \tag{2}$$

where $z_t$ is investment of goods at date $t$. $\alpha$ is the marginal productivity of investment, and $i \in \{H, L\}$ is the index for the marginal productivity of high and low profit investment, respectively. $k_{t+1}$ is capital produced at date $t+1$. We assume $\alpha^H > \alpha^L$.

Each type of investment projects is associated with agency problems (Hart and Moore (1994), Tirole (2006)). The entrepreneurs who undertake high (low) profitable investment projects can pledge only a fraction $\theta^H$ ($\theta^L$) of future returns from the investment. This fraction $\theta^H$ or $\theta^L$ can be collateral in borrowing. We assume that $\theta^H$ is less than $\theta^L$. That is, the degree of agency frictions is less severe in low profit investment.

In addition, each entrepreneur knows his/her own type at date $t$ of whether or not he/she has high profit investment projects, but only knows it with probability after date $t+1$. That is, each entrepreneur shifts stochastically between two states according to a Markov process: the state with high profit investment or the state without it. Specifically, an entrepreneur who has high (low) profitable investment at date $t$ may have high profit investment at date $t+1$ with probability $p$ $(X(1-p))$. This probability is exogenous, and independent across entrepreneurs and over time. Assuming that the initial ratio of the entrepreneurs who have high and only low profit investment is $X:1$, the population ratio is constant over time. We assume that the probability is not too large:

$$\text{Assumption} : p > X(1-p). \tag{3}$$

This assumption implies that there is a positive correlation between the present period and the next period. That is, the entrepreneur who has high profit investment in the current period continues to have it next period with higher probability than the one who has only low profit investment in the current period.

The entrepreneur's flow of funds constraint is given by

$$c_t + z_t = q_t k_t - r_{t-1} b_{t-1} + b_t, \tag{4}$$

where $r_{t-1}$ and $b_t$ are the gross real interest rate, and the amount of borrowing at date $t-1$ and $t$, respectively. $q_t$ is the relative price of capital to consumption goods. The left hand side of (4) is expenditure: consumption and investment. The right hand side is financing: the returns from investment in the previous period minus debts repayment, which we call net worth in this paper, and the amount of borrowing.

Because of the agency problems concerning the investment projects, the entrepreneur faces the borrowing constraint.[6] In such a situation, in order for debt contracts to be credible, debts repayment does not exceed the value of collateral. That is, the borrowing constraint becomes

[6] As Matsuyama (2007, 2008) points out, there are several causes to justify the borrowing constraints from microeconomic literature (see Tirole 2006). Here, we do not get into the details about which ones are more appropriate.

$$r_t b_t \le \theta^i q_{t+1} \alpha^i z_t. \tag{5}$$

Here, without loss of generality, we assume that $\theta^L$ is equal to one, which implies that there is no agency friction on low profit investment. We also define $\theta^H$ to be $\theta$. The parameter $\theta$ partly reflects the legal structure and the transaction costs in the liquidation of investment. In this sense, $\theta$ provides a simple measure of financial development. In this paper, we define an increase in $\theta$ as a financial development.

Each entrepreneur chooses consumption, investment, capital, and borrowing $\{c_t, z_t, k_{t+1}, b_t\}$ to maximize the expected discounted utility (1) subject to (2), (4), and (5).

Now, let's turn to the workers. There is only one type of workers. Each worker is endowed with one unit of labor each period, and supplies it inelastically in the labor market. Workers do not have investment project to produce capital, and therefore, do not have any collateral asset in order to borrow. At date $t$, a typical worker has expected discounted utility:

$$E_0 \left[ \sum_{t=0}^{\infty} \beta'^t \log c'_t \right], \tag{6}$$

where $c'_t$ is consumption of the workers at date $t$, and $\beta'$ is the subjective discount factor of the workers. We assume $\beta' < \beta$. This assumption implies that the workers are impatient relative to the entrepreneurs, and ensures that in equilibrium workers will not choose to lend.

Each worker chooses consumption, and the amount of borrowing to maximize (6) subject to the flow of funds constraint and the borrowing constraint.

$$c'_t = w_t - r_{t-1} b'_{t-1} + b'_t, \tag{7}$$

$$r_t b'_t \le 0, \tag{8}$$

where $w_t$ and $b'_t$ are the wage rate and the borrowing of the worker at date $t$.

There is a competitive final goods market. Production function of a representative firm is

$$Y_t = A K_t^{\prime,\sigma} N_t^{1-\sigma} \bar{k}_t^{1-\sigma}, \tag{9}$$

where $A$ is productivity, and $Y_t$ is output of the representative firm at date $t$.[7] $K'_t$ and $N_t$ are capital and labor inputs of the firm at date $t$. $\bar{k}_t$ is per-labor capital of this economy at date $t$, capturing the positive externality in the sense of Romer (1986).[8]

Each firm chooses capital and labor inputs to maximize its profit, given the relative price of capital to consumption goods, $q_t$, the wage rate, $w_t$, and the externality, $\bar{k}_t$. Considering the equilibrium of $k'_t = \bar{k}_t$, we obtain $y_t = Ak'_t$, where $k'_t$, and $y_t$ are per-labor capital and output of the firm. Because the worker's population is one, the aggregate capital input and output equal per-labor capital and output. Competitive factor prices produce

$$q_t = \sigma A, \qquad w_t = A(1-\sigma)k'_t. \tag{10}$$

Let us denote aggregate consumption of H-entrepreneurs, L-entrepreneurs, and workers at date $t$ as $C_t^H, C_t^L$, and $C'_t$. Similarly, let $Z_t^H, Z_t^L, B_t^H, B_t^L$, and $B'_t$ be aggregate investment, and the amount of borrowing of each type. Then, the market clearing for goods, credit, and capital are

$$C_t^H + C_t^L + C'_t + Z_t^H + Z_t^L = Y_t, \tag{11}$$

$$B_t^H + B_t^L + B'_t = 0, \tag{12}$$

$$k'_t = K_t, \tag{13}$$

where $K_t$ is the aggregate capital stock produced by the entrepreneurs at date $t$.

## 2.1 Equilibrium

The competitive equilibrium is defined as a set of prices $\{r_t, q_t, w_t\}_{t=0}^{\infty}$ and quantities $\left\{c_t, c'_t, b_t, b'_t, z_t, C_t^H, C_t^L, C'_t, B_t^H, B_t^L, B'_t, Z_t^H, Z_t^L, K'_t, K_t, Y_t\right\}_{t=0}^{\infty}$ which satisfies the conditions that (i) each entrepreneur and worker maximizes util-

[7] Here, we suppose that each firm is operated by workers. Since the net profit of each firm is zero in equilibrium, the flow of funds constraint of the workers does not change, and is the same as (7).

[8] The reason we use an endogenous growth model is that we want to analyze not only how financial development affects long-run growth, but also growth volatility through financial amplification. See Aghion et al (1999, 2007) for similar analyses.

ity, and each firm maximizes its profit, and (ii) the market for goods, labor, credit, and capital all clear. Because there is no shock except for the idiosyncratic shocks to the state of the entrepreneurs, there is no aggregate uncertainty, and the agents have perfect foresight about future prices and aggregate quantities in the equilibrium.

We are now in a position to characterize equilibrium behavior of entrepreneurs. Let us consider the case where $\theta$ is lower than $\theta_1$ ($\theta_1$ is defined later in Proposition 1. We use a method of guess-and verify here.). If $\theta$ is lower than $\theta_1$, in the neighborhood of the steady state, the real interest rate equals the rate of return on low profit investment (This can be verified in Proposition 1.). That is, we have

$$r_t = q\alpha^L. \tag{14}$$

And so, H-entrepreneurs prefer high profit investment with maximum leverage. The borrowing constraint of H-entrepreneurs binds because the rate of return on their investment is greater than the real interest rate. Since the utility function is log, they consume a fraction $(1-\beta)$ of the net worth, $c_t = (1-\beta)(qk_t - r_{t-1}b_{t-1})$. Then, by using (4), and (5), the investment function of H-entrepreneurs becomes

$$z_t = \frac{\beta(qk_t - r_{t-1}b_{t-1})}{1 - \dfrac{q\theta\alpha^H}{r_t}}. \tag{15}$$

The numerator of (15) is the required down payment for unit investment. From (15), we see that the investment equals the leverage, $1/\left[1 - (q\theta\alpha^H/r_t)\right]$ times savings, $\beta(qk_t - r_{t-1}b_{t-1})$. The leverage is greater than one, and increases with $\theta$. This implies that when $\theta$ is large, H-entrepreneurs can finance more investment with smaller net worth. We also see that the sensitivity of investment response to a change in the net worth becomes higher with $\theta$. This implies that even a small decline (increase) in the net worth can have a large negative (positive) effect on the investment.

Concerning workers, in the neighborhood of the steady state, the borrowing constraint binds (This can be verified later in footnote 9.). Thus, they consume all the income at every date, $c'_t = w_t$. From this behavior of workers, credit market equilibrium, (12) becomes

$$B_t^H + B_t^L = 0. \tag{16}$$

To L-entrepreneurs, they are indifferent between lending and investing by themselves because the real interest rate is the same as the return on their investment. Their saving rate is also a fraction $\beta$ of their net worth. Then, the aggregate lending and investment of them are determined by goods market clearing condition, (11).

Since consumption, debt and investment are linear functions of the net worth, we can aggregate across agents to find the law of motion of the aggregate capital:

$$\begin{aligned} K_{t+1} &= K_{t+1}^H + K_{t+1}^L = \alpha^H \frac{\beta E_t^H}{1 - \dfrac{q\theta\alpha^H}{r_t}} + \alpha^L \left( \beta\sigma Y_t - \frac{\beta E_t^H}{1 - \dfrac{q\theta\alpha^H}{r_t}} \right) \\ &= \left[ 1 + \left( \frac{\alpha^H - \alpha^L}{\alpha^L - \theta\alpha^H} \right) s_t \right] A\beta\sigma\alpha^L K_t, \end{aligned} \tag{17}$$

where $K_{t+1}^H$ and $K_{t+1}^L$ are the aggregate capital stock produced by H-entrepreneurs and L-entrepreneurs at date $t+1$, respectively. $E_t^H$ is the aggregate net worth of H-entrepreneurs, and $s_t \equiv E_t^H / \sigma Y_t$ is their net worth share against the aggregate net worth of all entrepreneurs. Since $Y_t = AK_t$ holds in equilibrium, and from (17), economic growth rate becomes

$$g_{t+1} \equiv \frac{Y_{t+1}}{Y_t} = \left[ 1 + \left( \frac{\alpha^H - \alpha^L}{\alpha^L - \theta\alpha^H} \right) s_t \right] A\beta\sigma\alpha^L. \tag{18}$$

From (18), once $s_t$ is determined, economic growth rate is also determined. (18) implies that economic growth rate increases with financial development. Intuitively, when financial development improves, the borrowing constraint of H-entrepreneurs becomes relaxed. In the credit market, more credit can be allocated to high profit investment projects, which promotes capital accumulation, and eventually economic growth. As in a traditional endogenous growth setting, capital accumulation is the engine of economic growth.

The movement of the aggregate net worth of H-entrepreneurs evolves according to

$$E_t^H = p(q_t K_t^H - r_{t-1} B_{t-1}^H) + X(1-p)(q_t K_t^L - r_{t-1} B_{t-1}^L). \tag{19}$$

The first term of (19) represents the aggregate net worth of the entrepreneurs who continue to have high profit investment from the previous period. The second term represents the aggregate net worth of the entrepreneurs who switch from the state of having only low profit investment to the state of having high profit investment. By using (18) and (19), we can derive the law of motion of the net worth share of H-entrepreneurs:

$$s_{t+1} = \frac{p\dfrac{\alpha^H(1-\theta)}{\alpha^L-\theta\alpha^H}s_t + X(1-p)(1-s_t)}{1+\dfrac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}s_t} \equiv \Phi(s_t,\theta). \tag{20}$$

The dynamic evolution of the economy is characterized by the recursive equilibrium: $(w_t, K_{t+1}, Y_{t+1}, g_{t+1}, s_{t+1},)$ that satisfies (10), (13), (17), (18), and (20) as functions of the state variables $(K_t, Y_t, s_t)$.

## 2.2 Steady State Equilibrium

The stationary equilibrium of this economy depends upon the degree of financial development. That is, we have the following proposition (See Figure 1.1 and 1.2. Proof is in Appendix 1).

**Proposition 1** *There are three stages of financial development, corresponding to three different values of $\theta$. The characteristics of each region are as follows:*

*(a) Region 1: $0 \leq \theta < \theta_1 \equiv (1-p)/\left[\alpha^H/\alpha^L - p + X(1-p)\right]$. Since the real interest rate equals the rate of return on low profit investment, the borrowing constraint of H-entrepreneurs binds. Both H-and L-entrepreneurs produce capital. The steady state values of $g^*$, $s^*$, and $r^*$ satisfy*

$$g^* = \left[1+\left(\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}\right)s^*\right]A\beta\sigma\alpha^L,\ s^* = \Phi(s^*,\theta),\ r^* = \sigma A\alpha^L. \tag{21}$$

*(b) Region 2: $\theta_1 \leq \theta < \theta_2 \equiv 1/(1+X)$. Since the real interest rate takes the value of $r^* \in \left[\sigma A\alpha^L, \sigma A\alpha^H\right)$, the borrowing constraint of H-entrepreneurs binds, and they produce capital. However, L-entrepreneurs do not produce capital because the real interest rate is greater than the rate of return on their*

*investment. The steady state values satisfy*

$$g^* = A\beta\sigma\alpha^H,\ s^* = p(1-\theta) + X(1-p)\theta,\ r^* = \frac{\sigma A\alpha^H}{(1-p)/\theta + p - X(1-p)}. \tag{22}$$

*(c) Region 3: $\theta_2 \leq \theta \leq 1$. Since the real interest equals the rate of return on high profit investment, the borrowing constraint of H-entrepreneurs does not bind. Only H-entrepreneurs produce capital. The steady state values satisfy*

$$g^* = A\beta\sigma\alpha^H,\ s^* = \frac{X}{1+X},\ r^* = \sigma A\alpha^H. \tag{23}$$

In region 1 where financial development is relatively low, the financial system can not transfer enough savings to high profit investment because of agency problems. In the credit markets, some of the savings flow to low profit investment because they are not subject to agency frictions. In this region, as financial development improves, more credit is allocated to high profit investment. This improvement of credit allocation promotes capital accumulation, the wage rate, and economic growth (See Figure 1.1). However, in this region the real interest rate is unchanged. This property is similar to Stiglitz and Weiss (1981) model. In their model, when information asymmetry is large, the real interest rate is insensitive, and becomes constant where the bank's profit is maximized. Similarly, in our model, when financial development is low, the real interest rate is sticky (See Figure 1.2).

In region 2 where financial development is high, but not so high, the situation changes. As financial markets develop, the real interest rate starts rising because of the tightness in the credit market, and all the savings are allocated to high profit investment, even though the borrowing constraint still binds for H-entrepreneurs. In this region, since only H-entrepreneurs produce capital, the growth rate of the economy becomes constant, and independent of $\theta$. This implies that once the financial system is developed to some degree, it can transfer enough purchasing power to the entrepreneurs who have high profit investment from the entrepreneurs who have only low profit investment. In addition, in region 1 and 2, since the interest rate is lower than the rate of return on H-entrepreneurs' investment, income distribution is different between H-and L-entrepreneurs.

When financial markets grow further, and reaches region 3, the real interest rate becomes equal to the rate of return on high profit investment.

Therefore, the borrowing constraint for H-entrepreneurs no longer binds. As in region 2, the financial system can allocate all the savings to only high profit investment. Moreover, since H-and L-entrepreneurs earn the same rate of return, there is no difference in income distribution.[9][10]

## 2.3 Dynamics

Now, let us look at how this economy responds to an unexpected shock to productivity. Suppose that at date $\tau-1$ the economy is in region 1, and in the steady state: $g_{\tau-1} = g^*, s_{\tau-1} = s^*$ and $r_{\tau-1} = r^*$. There is then an unexpected shock to productivity at date $\tau$: $A$ declines by $\varepsilon$, and becomes $A_\tau = A(1-\varepsilon)$. However, the shock is known to be temporary. The productivity at date $\tau+1$ and thereafter returns to $A$. Here since we consider a negative shock, we set $\varepsilon$ to be positive.

Following Kocherlakota (2000), we measure financial amplification (volatility) of a downward shock $\varepsilon$ to be how far economic growth rate from $\tau$ to $\tau+1$ jumps down from the steady-state growth rate through the borrowing constraint. Considering $q_\tau = \sigma A(1-\varepsilon)$ and $A_\tau = A(1-\varepsilon)$, from (18) and (19), we obtain

$$\text{Amplification} \equiv \frac{dg_{\tau+1}}{d\varepsilon}|_{\varepsilon=0} = \left(\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}\right)\underbrace{\frac{ds_\tau}{d\varepsilon}|_{\varepsilon=0}}_{\ominus} A\beta\sigma\alpha^L < 0. \tag{24}$$

Since H-entrepreneurs have a net debt in the aggregate, and debts repayment does not change by this shock, the net worth share of H-entrepreneurs decreases at date $\tau$, $\frac{ds_\tau}{d\varepsilon} < 0$ (See Appendix 2). Because the adjustment of the real interest rate does not work well in region 1, their borrowing constraint

[9] In our model, in the neighborhood of the steady state equilibrium, the borrowing constraint of the workers binds in all three regions because $\beta' r_t/g_{t+1} < 1$ holds. This can be verified by embedding (21), (22), and (23) into the inequality. Of course, considering a model where workers also choose to lend may be an interesting extention.

[10] The difference between Kiyotaki(1998)'s paper and ours is that although his paper does not explicitly mention it, Kiyotaki's analysis implicitly assumes a certain low $\theta$, which is within region 1 in this paper, and then, keeping the $\theta$ fixed, he examines how amplification occurs. On the other hand, our paper analyzes whether or not the magnitude of amplification by itself increases or decreases together with $\theta$ not only in low $\theta$ region, but also high $\theta$ region.

becomes tightened. As a result, the investment function of H-entrepreneurs is shifted to the left as in Figure 2, and they are forced to cut back on their investment from $Z_{\tau}^{H\prime,0}$ to $Z_{\tau}^{H\prime,1}$. (In Figure 2, $Z^{H\prime}$ represents the aggregate investment curve of H-entrepreneurs as a share against the aggregate savings, and SV represents the aggregate saving curve as a share against the aggregate savings.) Moreover, these balance sheet effects cause more credit to flow to the investment without agency frictions. What is called "flight to quality" occurs. Through these effects, less capital is produced at date $\tau+1$, so that economic growth rate at date $\tau+1$ jumps down from the steady state growth rate.

## 2.4 Financial Development and the Magnitude of Amplification

Now, we are in a position to examine whether financial development accelerates or dampens these financial amplification effects.

First, let's check region 1. By differentiating (24) with respect to $\theta$, we obtain

$$\frac{\partial^2 g_{\tau+1}}{\partial\theta\partial\varepsilon}|_{\varepsilon=0} = \underbrace{\frac{\partial}{\partial\theta}\left(\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}\right)}_{\oplus}\underbrace{\frac{\partial s_\tau}{\partial\varepsilon}|_{\varepsilon=0}}_{\ominus}A\beta\sigma\alpha^L+\left(\frac{\alpha^H-\alpha^L}{\alpha^L-\theta\alpha^H}\right)\underbrace{\frac{\partial^2 s_\tau}{\partial\theta\partial\varepsilon}|_{\varepsilon=0}}_{\ominus}A\beta\sigma\alpha^L<0. \tag{25}$$

The first term represents the sensitivity of the H-entrepreneurs' investment response to a change in the net worth share. Since it becomes higher with $\theta$, with even a small decline in the net worth share, H-entrepreneurs are forced to reduce their investment substantially. The second term represents the degree of a decline in the net worth share. It says that the decline by itself becomes larger with $\theta$ (See Appendix 2). This implies that when $\theta$ is high, the leverage and debt/asset ratios of H-entrepreneurs also rise. In such a situation, even a small negative productivity shock can cause a large decline in the net worth share. Taken together, H-entrepreneurs have to make deeper cuts in their investment. Moreover, this causes a substantial credit shift from the investment with agency frictions to the one without agency frictions. That is, balance sheet effects and flight to quality are significant. Hence, in region 1, financial development accelerates financial amplification effects, thereby leading to increased macroeconomic volatility.

Once the economy enters region 2, the situation changes dramatically. The adjustment of the real interest rate starts operating. As a result, financial amplification is dampened. In order to clarify this point, let's look at the equilibrium of the credit market in region 2 at date $\tau$:

$$\frac{s_\tau}{1 - \dfrac{q_{\tau+1}\theta\alpha^H}{r_\tau}} = 1. \tag{26}$$

The left hand side and the right hand side of (26) are the investment function and the saving function, respectively. From (26), the real interest rate is determined once $s_\tau$ is given. Remember that since the productivity shock is temporary, the relative price of capital to consumption goods at date $\tau + 1$ becomes $q_{\tau+1} = \sigma A$.

Next, let's look at how the net worth share of H-entrepreneurs changes by this shock. The aggregate net worth of H-entrepreneurs and the aggregate output at date $\tau$ follow

$$E_\tau^H = p\left[\sigma A(1-\varepsilon)K_\tau^H - r_{\tau-1}B_{\tau-1}^H\right]) + X(1-p)r_{\tau-1}B_{\tau-1}^H, \tag{27}$$

$$Y_\tau = A(1-\varepsilon)\alpha^H\beta\sigma Y_{\tau-1}. \tag{28}$$

From (27) and (28), the net worth share of H-entrepreneurs at date $\tau$ follows

$$s_\tau = \frac{p(1-\theta-\varepsilon) + X(1-p)\theta}{1-\varepsilon}. \tag{29}$$

And so, by using (26) and (29), we obtain an expression for the equilibrium interest rate at date $\tau$:

$$r_\tau = \frac{\sigma A\theta\alpha^H(1-\varepsilon)}{(1-p)(1-\varepsilon) + \left[p - X(1-p)\right]\theta}. \tag{30}$$

From (30), we observe that the real interest rate declines at the time of the shock. Intuitively, following the shock, the borrowing constraint becomes tightened as in region 1. And then, the investment function is shifted to the left. However, in region 2, together with this shift, the real interest rate goes down in the credit market as in Figure 3. This decline in the real interest rate in turn relaxes the borrowing constraint, thereby weakening the balance sheet effects and preventing flight to quality. As a result, financial amplification is dampened. This implies that once financial development passes a certain degree, the adjustment of the real interest rate recovers, so that even if the economy is hit by the shock, all the credit flow only to high profit investment. Therefore, the shock does not get amplified. Financial development leads to macroeconomic stability.[11]

When financial development reaches region 3, even the shock hits the economy, the financial system can transfer enough purchasing power to those who have high productive investment from those who have only low profit investment without the adjustment of the real interest rate (See Figure 4). The real interest rate at date $\tau$, $\sigma A\alpha^H$ and the growth rate from $\tau$ to $\tau+1$, $\sigma A\alpha^H\beta$ are unchanged. So, no financial amplification occurs. The following proposition summarizes the results.

**Proposition 2** *The relationship between financial development and financial amplification is non-monotonic: financial amplification initially increases with financial development (in region 1) and later falls down (in region 2 and 3).*

This non-monotonicity is consistent with empirical studies. For example, Easterly et al. (2000) demonstrate that the relationship between financial development and growth volatility is non-monotonic. They show that while developed financial systems offer oppurtunities for stabilization, they may also imply higher leverage of firms and thus more risks and less stability. A recent study by Kunieda (2008) also show empirically that the relationship is hump-shaped, i.e., in early stages of financial development, as the financial sector develops in an economy, it becomes highly volatile. However, as the financial sector matures further, the volatility starts to reduce once again.

Based on the above analysis, we might be able to explain why we observe two conflicting views. The traditional view might discuss region 2 or 3 where financial markets are well developed. Indeed, in Arrow-Debreu economy where there are no agency frictions in credit markets, $\theta$ is equal to one, which is within region 3 in this paper. On the other hand, the new view might discuss region 1 where financial development is not so high, and there are agency frictions to some degree in financial markets (See Figure 5). In

[11] Indeed, the growth rate of the economy from date $\tau$ to $\tau+1$ can be written as $g_\tau = A\alpha^H\beta s_\tau\sigma/(1-q_{\tau+1}\theta\alpha^H/r_\tau)$. By embedding (26) into this, we obtain $g_\tau=\sigma A\alpha^H\beta$, which implies that the growth rate from at date $\tau$ to $\tau+1$ is unchagend.

this sense, the discrepancy between two views might arise from the difference in the degree of financial development.[12]

The implications of Proposition 1 and 2 are that in region 1, financial development produces more capital, promotes economic growth, and leads to higher wage rate. Therefore, it improves welfare of all agents[13]. However, once negative productivity shocks hit the economy, since the economy is highly leveraged, downward amplification is significant. In this sense, there is a trade-off between higher economic growth and macroeconomic stability. But, once financial development reaches region 2 or 3, both go together.

Moreover, from Proposition 2, our model may also have implications for asymmetric movements of business fluctuations. As Kocherlakota (2000) emphasizes, macroeconomics looks for an asymmetric amplification and propagation mechanism that can turn small shocks to the economy into the business cycle fluctuations. Our model might deliver this. For example, if the economy is around $\theta_2$, to positive productivity shocks, even though the borrowing constraint for H-entrepreneurs is binding, the economy will not respond upwardly because the interest rate will go up in the credit market. On the other hand, to negative productivity shocks, it will react downwardly because the interest rate does not adjust.[14] We summarize this result in Proposition 3.

[12] You may wonder why large downward amplification occurs repeatedly in the real economy where financial development keeps increasing over time, even though our model suggests that financial amplification eventually becomes small in high $\theta$ region. Here is one interpretation from this model. In this model, the important factor which affects the size of financial amplification is $\theta^H$, which is put on high profitable investment, not on low profitable investment. Considering this point, think about the case where the existing projects with $\alpha^L$ disappper, and new investment opportunities with higher profitability than the existing $\alpha^H$ come into the economy. In such a situation, the $\theta$ which is put on those new investment projects matters. If the $\theta$ is low, the economy will get into region 1 again even if it was in region 2 or 3 before. In the real economy, this process might repeats itself.

[13] To the entrepreneurs and the workers, since the net returns from high profitable investment, $\alpha^H(1-\theta)/(1-\theta\alpha^H/\alpha^L)$ and the wage rate are higher in more developed financial system, the level of their expected consumption is also higher, and so is welfare.

[14] Here we consider small shocks. However, if we think about relatively large productivity shocks, business fluctuations may become asymmetric, even if the economy is far from $\theta_2$. In the case with relatively large positive shocks, positive propagation occurs, but the degree of it is weakened because the adjustment of the interest rate works. However, to the negative shocks, because the adjustment does not work, the economy experiences large downward propagation.

**Proposition 3** *If the level of financial development is around $\theta_2$, business fluctuations are asymmetric.*

# 3 Policy Analysis

As we analyzed in the previous section, we learn that in region 1, once negative productivity shocks hit the economy, donward amplification occurs, which causes capital accumulation and economic growth rate to drop down. From a welfare point of view, this impairs the workers' welfare because the wage rate declines. A natural question is can a government mitigate the drop in the economic growth rate and the workers' welfare? In this section, as a stabilization policy, we examine the role of monetary policy, and discuss its welfare implications.[15] In order to do so, we extend the model of the previous section, and get money into it.

In the monetary economy, the flow of funds constraints for the entrepreneurs and the workers, (5) and (7) can be rewritten as follows

$$\begin{aligned} \text{for entrepreneurs, } \frac{m_t}{P_t} + c_t &= q_t k_t - \frac{P_{t-1}}{P_t} i_{t-1} b_{t-1} + b_t + \frac{m_{t-1}}{P_t}, &(31)\\ \text{for workers, } \frac{m'_t}{P_t} + c'_t &= w_t - \frac{P_{t-1}}{P_t} i_{t-1} b'_{t-1} + b'_t + \frac{m'_{t-1}}{P_t}, &(32) \end{aligned}$$

where $m_t$ and $m'_t$ are the nominal money demand of the entrepreneurs and the workers, respectively. $P_t$ is the price level at date $t$, and $i_{t-1}$ is gross nominal interest rate at date $t-1$. We assume that debt contracts are nominal.[16] Then, the borrowing constraints become

$$\text{for entrepreneurs, } \frac{P_t}{P^e_{t+1}} i_t b_t \leq \theta q_{t+1} \alpha^i z_t, \quad \text{for workers, } \frac{P_t}{P^e_{t+1}} i_t b'_t \leq 0, \quad (33)$$

where $P^e_{t+1}$ is the price level at date $t+1$ expected at date $t$.

In the monetary economy, all agents face cash-in-advance (CIA) constraint following Lucas and Stocky (1984):

[15] Aghion et al. (1999) analyzes fiscal policies.

[16] Iacoviello (2005) points out that in almost all the low inflation countries, debt contracts are nominal.

$$\text{for entrepreneurs,}\quad m_{t-1} \geq P_t c_t, \quad \text{for workers,}\quad m'_{t-1} \geq P_t c'_t. \tag{34}$$

Each entrepreneur and worker holds money to consume. We consider the equilibria where CIA constraint for both agents binds.

The competitive equilibrium is defined as a set of prices $\{i_t, w_t, q_t, P_t\}_{t=0}^{\infty}$ and quantities $\left\{c_t, c'_t, b_t, b'_t, z_t, m_t, m'_t, C_t^H, C_t^L, C'_t, B_t^H, B_t^L, B'_t, Z_t^H, Z_t^L, K'_t, K_t, Y_t\right\}_{t=0}^{\infty}$ which satisfies the conditions that (i) each entrepreneur maximizes (1) subject to (31), (33), and (34), and each worker maximizes (6) subject to (32), (33), and (34), and each firm maximizes its profit, given the relative price of capital to consumption goods, the wage rate, and the externality. (ii) The markets for goods, labor, capital, credit, and money all clear. Since there is no aggregate uncertainty, all agents have perfect foresight about future prices and quantities in equilibrium. That is, $P_{t+1}^e = P_{t+1}$ hold.

Since we focus on binding CIA constraint, and the utility function is log, then we have $m_t = P_t(1-\beta)(q_t k_t - r_{t-1}b_{t-1})$, and $m'_t = P_t w_t$. That is, each entrepreneur uses a fraction $(1-\beta)$ of the net worth to buy money. Each worker uses all income to buy money. When we aggregate across all agents, we obtain the aggregate money demand at date $t$, $M_t^D$:

$$M_t^D = P_t(1-\sigma\beta)Y_t. \tag{35}$$

(35) implies that when aggregate output declines, the aggregate demand for money also decreases.

Government budget constraint is

$$P_t G_t = M_t - M_{t-1}. \tag{36}$$

where $G_t$ and $M_t$ are the government (consolidated government) expenditure and the money supply at date $t$, respectively. The government finances expenditure by printing money. We assume that the government expenditure does not affect utility of the agents.

Monetary policy rule is

$$M_t = \mu M_{t-1}, \tag{37}$$

where $\mu$ is gross money growth rate. The monetary authority keeps the money growth rate constant.

Money market clearing condition is

$$M_t = M_t^D. \tag{38}$$

The dynamic evolution of the economy is characterized by the recursive equilibrium: $(w_t, K_{t+1}, Y_{t+1}, g_{t+1}, s_{t+1}, G_t)$ that satisfies (10), (13), (17), (18), (20), (35), (36), and (38) as functions of the state variables, $(K_t, Y_t, s_t)$ and monetary policy rule, (37).

In order to understand the dynamics in the monetary economy, we consider the same experiment as in section 2. At date $\tau$, under a fixed $\theta$, there is an unexpected negative shock to productivity by $\varepsilon$. Following the shock, if other things were kept constant, the net worth share of H-entrepreneurs would decrease. Then, the investment function would be shifted to the left through the balance sheet effects, which would cause flight to quality (See Figure 7). However, in the monetary economy, this does not happen in equilibrium. There is an additional feedback effect to the credit market, which is not generated in the nonmonetary economy.

In order to make this point clear, let's look at the money market equilibrium at date $\tau$:

$$M_\tau = P_\tau(1 - \sigma\beta)(1 - \varepsilon)Y_\tau^e, \tag{39}$$

where $Y_\tau^e$ is the aggregate output at date $\tau$ expected at date $\tau - 1$. Given the negative shock of size $\varepsilon$, the aggregate output declines by $\varepsilon$. Together with this decline, since the net worth of all entrepreneurs and the wage rate of the workers decrease, the aggregate money demand (the right hand side of (39)) also falls down. Then, from (39), if monetary authority keeps the money growth rate constant, for the money market to clear, the price level goes up. This rise in the price level in turn reduces the real burden of debts repayment for borrowers (H-entrepreneurs at date $\tau - 1$) by $\varepsilon$, which produces a shift-back effect as in Figure 6. Consequently, in equilibrium, the net worth share of H-entrepreneurs at date $\tau, s_\tau$ is unchanged, which implies that the aggregate net worth of H-entrepreneurs and the aggregate net worth of all entrepreneurs fall in the same proportion. As a result, no financial amplification occurs as if the economy were in region 2 or 3. We summarize this in Proposition 4.[17]

[17]Nominal contracts also play an important role to produce shock cushioning effects. If the contracts are index, the effects are not generated. This point is different from the existing view that nominal contracts magnify the shocks. Iacoviello (2005) also derives simillar results with simulation by extending the model with price stickiness while our

**Proposition 4** *Suppose that debt contracts are nominal, and the economy is in region 1 of the monetary economy. In the case of an unexpected productivity shock, the money growth targeting policy dampens financial amplification by generating the shock cushioning effects.*

Proof: By using the money market clearing condition, the aggregate real debts repayment at date $\tau$ can be rewritten as follows: $i_{\tau-1}B_{\tau-1}P_{\tau-1}/P_{\tau} = (1-\varepsilon)i_{\tau-1}B_{\tau-1}P_{\tau-1}/P_{\tau}^{e}$, where $P_{\tau}^{e}$ is the price level at date $\tau$ expected at date $\tau - 1$. By putting this into (19), and then solving $s_{\tau}$, we see that the net worth share at date $\tau$ remain unchanged.

On the other hand, if monetary authority adopts inflation targeting policy, the shock is exacerbated through the balance sheet effects and flight to quality. This is because, under inflation targeting policy, since the monetary authority tries to keep the inflation rate of each period the same as the one in the steady state, it decreases the money growth rate accommodatively with the decline in the aggregate money demand. As a result, since the real burden of debts repayment is unchanged, the shock cushioning effects and the shift-back effect are not generated. We summarize this result in Proposition 5.

**Proposition 5** *Suppose that debt contracts are nominal, and the economy is in region 1 of the monetary economy. If monetary authority adopts inflation targeting policy in the case of an unexpected productivity shock, the shock cushioning effects are not generated, so that financial amplification occurs.*

Proof: Since $P_{\tau}$ is equal to $P_{\tau}^{e}$, the real burden of the aggregate debts is unchanged. Considering this point, if we embed $q_{\tau} = \sigma A(1-\varepsilon)$ and $A_{\tau} = A(1-\varepsilon)$ into (18) and (19), we have $\frac{ds_{\tau}}{d\varepsilon} < 0$.

## 3.1 Discussion: Welfare Implications

From the previous section, although we learn that the money growth targeting policy stabilizes the economy by weakening financial amplification, does this policy improve agents' welfare compared to inflation targeting policy? In this section, we discuss this point.

Let's compare welfare of each agent. Let $V_t^{MG}, V_t^{IT}, V_t'^{MG}, V_t'^{IT}$ be welfare of an entrepreneur and a worker under the money growth targeting (MG) and

model is based on flexible price settings, and derives the results analytically.

inflation targeting (IT) policies, respectively. Similarly, let $c_t^{i,MG}$, $c_t^{\prime,MG}$, $c_t^{i,IT}$, $c_t^{\prime,IT}$, $w_t^{MG}$, $w_t^{IT}$, $e_t^{i,MG}$, $e_t^{i,IT}$, and $\pi_t^{MG}, \pi_t^{IT}$ be consumption of the entrepreneurs and workers, the wage rate, the net worth of the entrepreneurs, and the inflation rate at date $t$, where $\pi_t \equiv P_{t-1}/P_t$.

For the worker, the welfare becomes

$$V^{\prime MG} = E_\tau \left[ \sum_{n=0}^{\infty} \beta^n \log c_{\tau+n}^{\prime,MG} \right] = E_\tau \left[ \sum_{n=0}^{\infty} \beta^n \log \left( \frac{w_{\tau+n-1}^{MG}}{\pi_{\tau+n}^{MG}} \right) \right], \tag{40}$$

$$V^{\prime IT} = E_\tau \left[ \sum_{n=0}^{\infty} \beta^n \log c_{\tau+n}^{\prime,IT} \right] = E_\tau \left[ \sum_{n=0}^{\infty} \beta^n \log \left( \frac{w_{\tau+n-1}^{IT}}{\pi_{\tau+n}^{IT}} \right) \right]. \tag{41}$$

The welfare depends upon the inflation rate and the wage rate at date $\tau$ and thereafter. By subtracting (40) from (41), we obtain

$$V^{\prime MG} - V^{\prime IT} = \underbrace{\log \left( \frac{\pi_\tau^{IT}}{\pi_\tau^{MG}} \right)}_{\ominus} + E_\tau \left[ \underbrace{\sum_{n=1}^{\infty} \beta^{n+1} \log \left( \frac{w_{\tau+n}^{MG}}{w_{\tau+n}^{IT}} \right)}_{\oplus} \right]. \tag{42}$$

From (42), we can understand whether or not the MG policy improves welfare of the worker compared to the IT policy. The first term of (42) represents the difference in the inflation rate at date $\tau$ under the two policies. Under the MG policy, following the shock, the higher inflation occurs unexpectedly at date $\tau$. That is, we have $\pi_\tau^{MG} > \pi_\tau^{IT}$. This reduces the purchasing power of money, so that the worker's consumption at date $\tau$ decreases. Thus, the first term is negative. Note that the inflation rate after $\tau + 1$ is the same under the two policies.[18]

The second term represents the difference in the wage rate. Under the MG policy, because of the unexpected higher inflation, the redistribution of wealth occurs at date $\tau$ from L-entrepreneurs at date $\tau - 1$, who are lenders, to H-entrepreneurs at date $\tau - 1$, who are borrowers (note that $p > X(1-p)$). This increases the aggregate net worth of H-entrepreneurs at date $\tau$. Consequently, the borrowing constraint of H-entrepreneurs at date $\tau$

[18]Under the MG policy, since no financial amplification occurs, the inflation rate after date $\tau + 1$ equals to the one in the steady state.

becomes relaxed, so that more capital is going to be produced at date $\tau+1$ and thereafter, which pushes up the wage rate after date $\tau+1$, $w_{\tau+n}^{MG} > w_{\tau+n}^{IT}$ $(n \geq 1)$. Thus, the second term is positive. Note that the wage rate at date $\tau$ is the same, $w_{\tau}^{MG} = w_{\tau}^{IT}$. Hence, whether or not the MG policy improves the worker's welfare compared to the IT policy depends upon the above two effects. If $A$ is high or $\sigma$ is low, there is a large positive spillover effect on the wage rate. Then, the positive effect might become larger than the negative effect.

Similarly, for the entrepreneur, we obtain

$$V^{MG} - V^{IT} = \underbrace{\log\left(\frac{\pi_{\tau}^{IT}}{\pi_{\tau}^{MG}}\right)}_{\ominus} + E_0 \left[ \begin{array}{c} \underbrace{\sum_{n=0}^{\infty} \beta^{n+1} \log\left(\frac{e_{\tau+n}^{i,MG}}{e_{\tau+n}^{i,IT}}\right)} \\ \oplus \text{ for H-entrepreneurs at date } \tau-1. \\ \ominus \text{ for L-entrepreneurs at date } \tau-1. \end{array} \right]. \tag{43}$$

The first term is the same as the worker. The second term represents the difference in the net worth under the two policies. Under the MG policy, for the entrepreneurs who had high productive investment at date $\tau-1$, who are borrowers, they gain at date $\tau$, $e_{\tau}^{H,MG} > e_{\tau}^{H,IT}$ because the real burden of debts repayment is reduced. Therefore, their net worth after $\tau+1$ will also increase, $e_{\tau+n}^{H,MG} > e_{\tau+n}^{H,IT}$ $(n \geq 1)$. For them, if the positive effect becomes larger than the negative effect (the first term), their welfare improves under the MG policy. On the other hand, for the entrepreneurs who had low productive investment at date $\tau-1$, who are lenders, they lose at date $\tau$, $e_{\tau}^{L,MG} < e_{\tau}^{L,IT}$. Therefore, their net worth after $\tau+1$ will also decrease, $e_{\tau+n}^{L,MG} < e_{\tau+n}^{L,IT}$ $(n \geq 1)$. For them, the MG policy impairs their welfare. Hence, since our model has heterogeneity among agents, the welfare impacts of a particular monetary policy rule are also heterogeneous between the agents.[19][20]

[19] Woodford (2003) discusses optimal monetary policy with a single agent model.

[20] In stead of monetary policy, we can think of a tax cut policy. For example, suppose that the government imposes tax on the entrepreneur's net worth. Imagine that the

# 4 Concluding Remarks

In this paper, we investigate theoretically how financial development affects the magnitude of financial amplification. Extending a model with borrowing constraints and heterogeneous investment projects, we show that the effect of financial development on financial amplification is non-monotonic: financial amplification initially increases with financial development and later falls down.

Moreover, we study the role of monetary policy to reduce financial amplification. We find that in the case of unexpected productivity shocks, money growth targeting policy dampens financial amplification by generating shock cushioning effects, thereby stabilizing economies. On the other hand, inflation targeting exacerbates the shocks because under the policy, shock cushioning effects are not generated, thereby destabilizing economies.

As future research, the next step would be that we want to develop quantitative assessment into the relationship between the development of financial markets and volatility of the economy. Another step would be to consider the welfare cost of volatility in a heterogeneous agents model with aggregate uncertainty. These directions will be promising.

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economy experiences an unexpected negative productivity shock at date $\tau$ as in section 2. Under laisser-fair economy, since the net worth of all entrepreneurs at date $\tau$ decreases by this shock, downward amplification occurs. However, if the government conducts a tax cut policy at date $\tau$ (at the same time of the shock), then the entrepreneurs' net worth increases at date $\tau$. As a result, downward amplification is dampened. The economy is insulated from the negative shock. Moreover, this policy improves all the entrepreneurs' welfare because their consumption increases at date $\tau$ and thereafter.

## Appendix 1

In order to verify that (14) holds in equilibrium, we only need to check that the entrepreneurs with low profit investment invest positive amounts of goods:

$$Z_t^L = \beta\sigma Y_t \left( 1 - \frac{s_t}{1 - \dfrac{\theta\alpha^H}{\alpha^L}} \right). \tag{44}$$

Using (20), we find that (44) becomes positive in the neighborhood of the steady state if, and only if $\theta$ is lower than $\theta_1$.

Moreover, from (22), if $\theta < 1/(1+X)$, then $r^* < \sigma A\alpha^H$. That is, the real interest rate is lower than the marginal productivity of the entrepreneurs with high profit investment. Thus, the borrowing constraint for H-entrepreneurs binds. For L-entrepreneurs, since the real interest rate is greater than the rate of return on their investment, they would prefer lending to investing by themselves.

We also see that if $\theta = 1/(1+X)$, then $r^* = \sigma A\alpha^H$. Thus, the borrowing constraint for H-entrepreneurs no longer binds. Furthermore, If $\theta$ is greater than $1/(1+X)$, then for the credit market to clear, the real interest rate has to equal $\sigma A\alpha^H$ (If the real interest rate is greater than $\sigma A\alpha^H$, nobody is willing to borrow in the credit markets. This can not be an equilibrium.).

## Appendix 2

By embedding $q_\tau = \sigma A(1-\varepsilon)$ and $A_\tau = A(1-\varepsilon)$ into (18) and (19), and differentiating $s_\tau$ with respect to $\varepsilon$, we obtain

$$\frac{\partial s_\tau}{\partial \epsilon}|_{\varepsilon=0} = [p - X(1-p)] \frac{-\theta\alpha^H s^*}{\alpha^L - \theta\alpha^H + (\alpha^H - \alpha^L)s^*} < 0. \tag{45}$$

And then, by using (45), we have

$$\frac{\partial^2 s_\tau}{\partial\theta\partial\epsilon}|_{\varepsilon=0} = [p - X(1-p)]\, \alpha^H \frac{-\theta\dfrac{\partial s^*}{\partial\theta}(\alpha^L - \theta\alpha^H) - \alpha^L s^* - (\alpha^H - \alpha^L)s^{*2}}{[\alpha^L - \theta\alpha^H + (\alpha^H - \alpha^L)s^*]^2} < 0. \tag{46}$$

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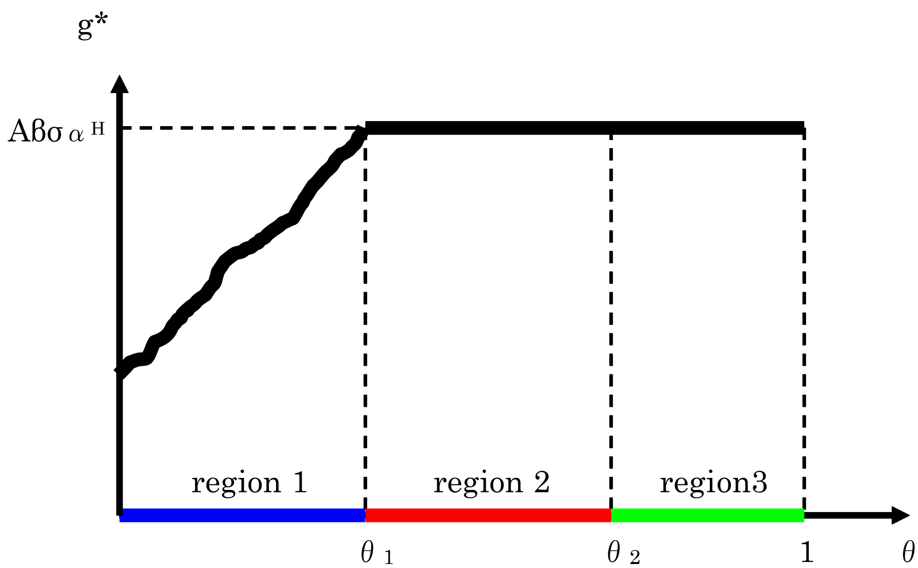


Figure 1.1－The Relation between Growth and Financial Development

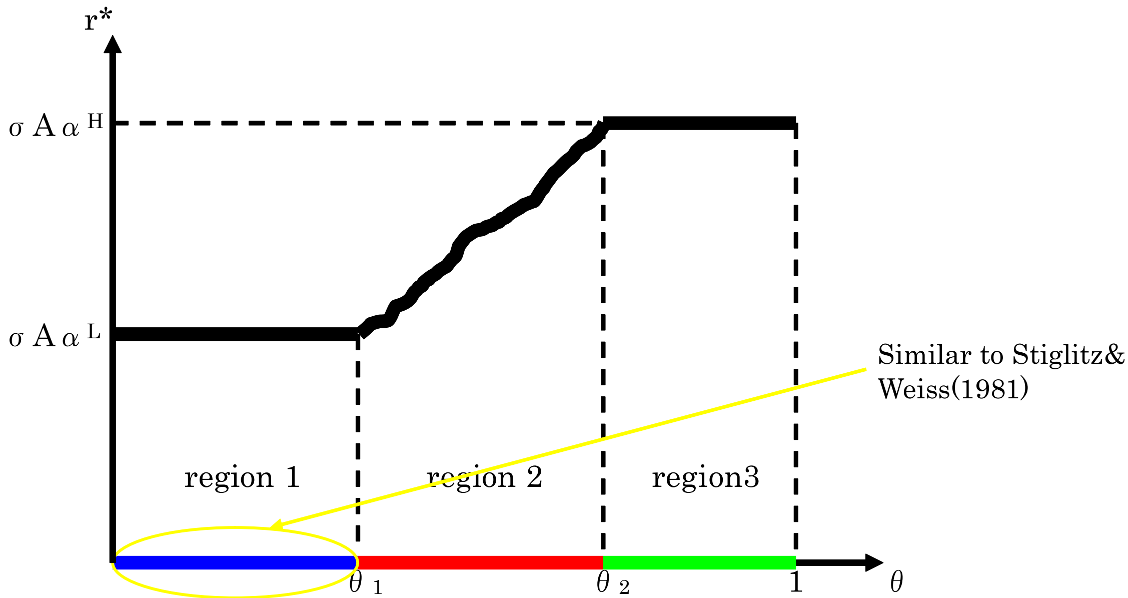


Figure 1.2－The Relation between Interest Rate and Financial Development

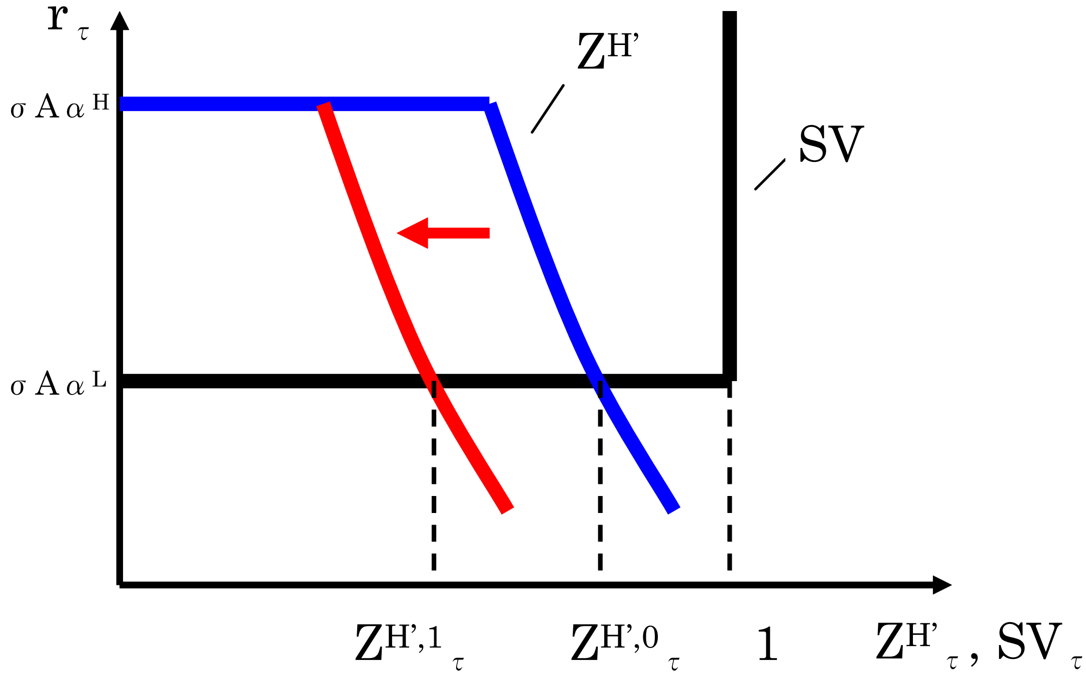


Figure 2－Equilibrium of Credit Market (region 1)

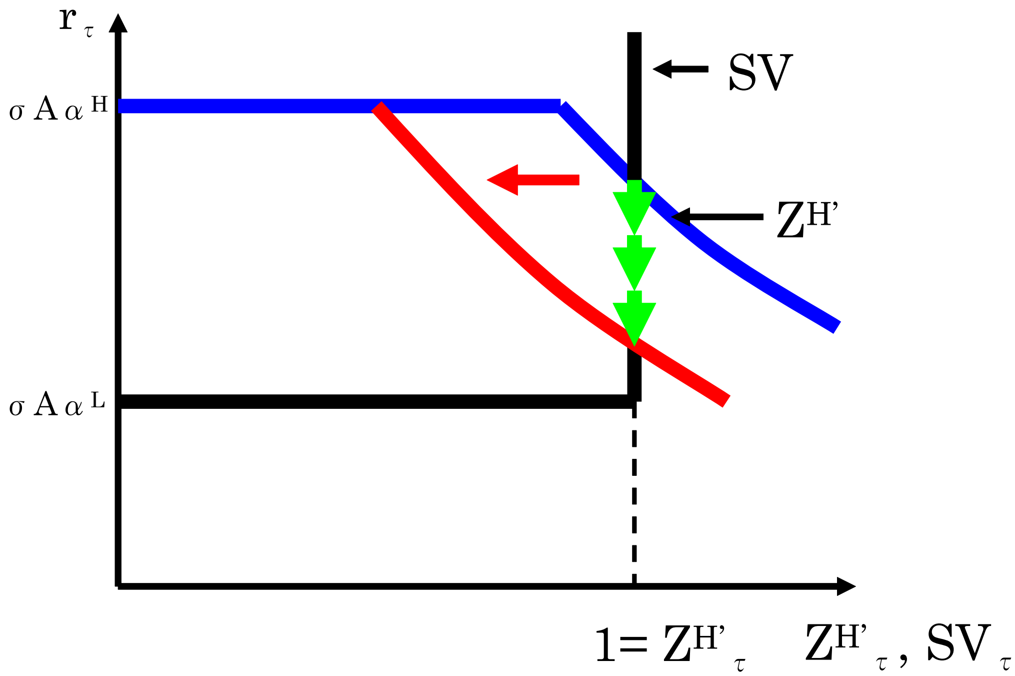


Figure 3－Equilibrium of Credit Market (region 2)

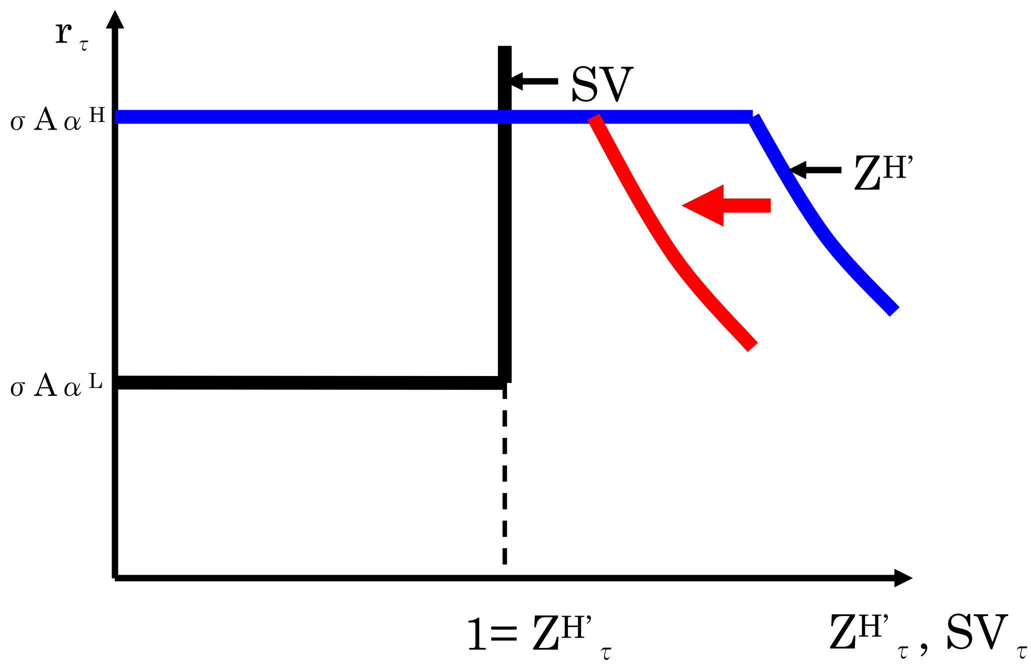


Figure 4—Equilibrium of Credit Market (region 3)

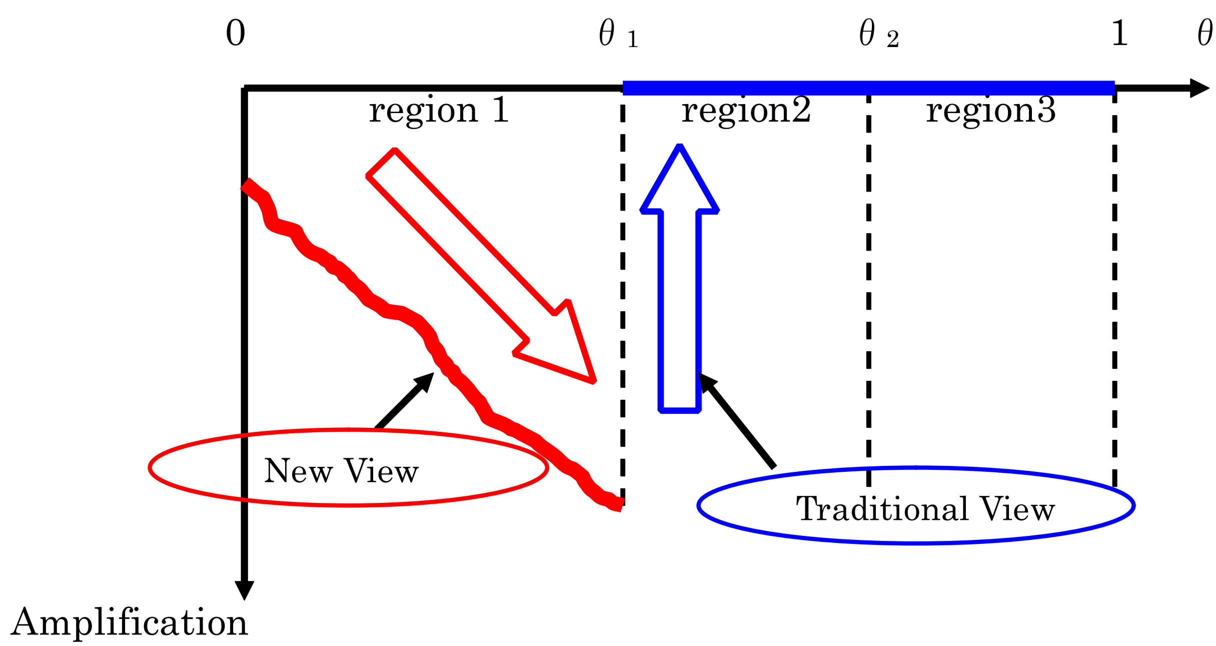


Figure 5－The Relation between Financial Development and Amplification

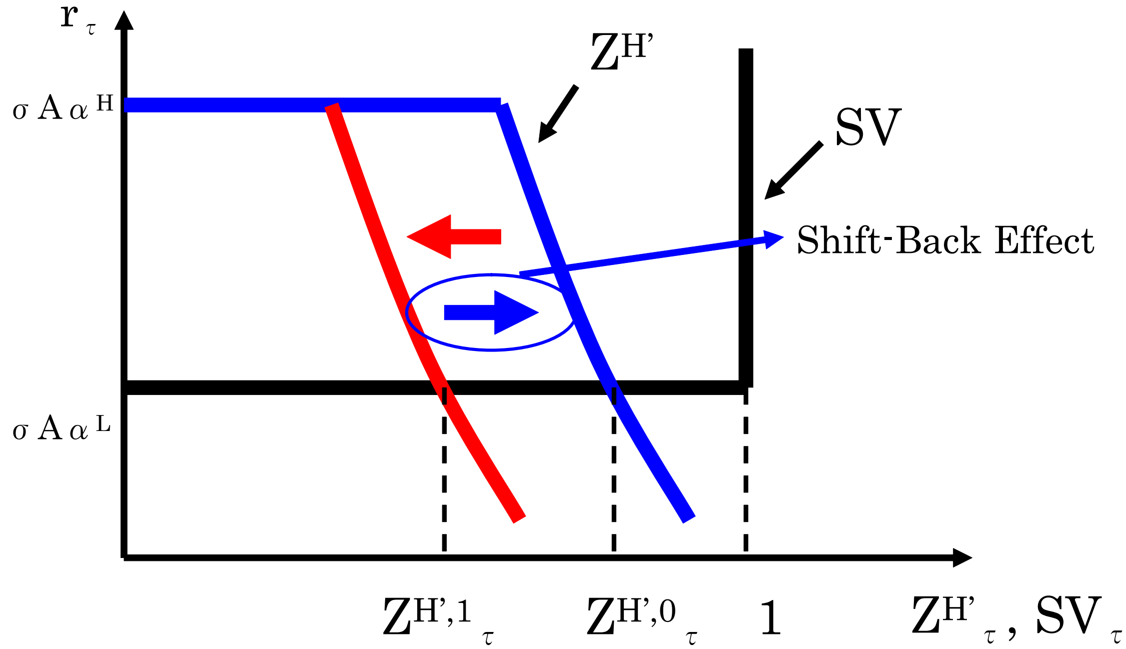


Figure 6－Equilibrium of Credit Market (region 1)