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Abstract

This paper investigates a two-country model of capital accumulation with country-specific production externalities. The main concern of our discussion is to explore the presence of equilibrium indeterminacy in an open-economy setting. In contrast to the existing studies on equilibrium indeterminacy in small-open economies, the present paper demonstrates that opening up international trade and financial interactions between two counties does not necessarily enhance the possibility of indeterminacy of equilibrium. It is shown that the results depend heavily upon not only on the degree of external increasing returns but also on the preference structures.

Keywords: Financial integration, Two-country growth model, Equilibrium determinacy, Preference structure

JEL classification: F43, O41

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1 Introduction

The equilibrium business cycle theory based on indeterminacy and sunspots has claimed that economic fluctuations caused by extrinsic uncertainty tend to more prominent in open economies than in closed economies. For example, inspecting a small-open economy version of the two-sector model studied by Benhabib and Farmer (1996), Weder (2001) concludes that the small-open economy requires a lower degree of external increasing returns to hold indeterminacy than the closed-economy counterpart. Similarly, Aguiar-Conraria, L. and Wen, Y. (2005), Lahiri (2001) and Meng and Velasco (2003, 2004) also demonstrate that perfect capital mobility may enhance indeterminacy of equilibrium for small-open economies.[1] The main reason for this results is that in the small-open economies the interest rate is fixed in the world financial market, so that consumption behaves as if the utility function were linear. Since a high elasticity of intertemporal substitutability in consumption serves as a cause of indeterminacy, the small-open economies tend to be more volatile than the closed economies with the same technologies and preferences.

The purpose of this paper is to examine whether the volatility of open economies emphasized by the existing studies on expectations-driven business cycles will hold in the world economy model as well. Since the world economy is a closed economy with heterogeneous agents, the present paper may be considered a study on the relationship between heterogeneity and indeterminacy.[2] The analytical framework of this paper is a two-country model of capital accumulation with external increasing returns. The two countries trade consumption goods. In addition, it is assumed that agents in both countries can access to the perfect international bond market. Given such a setting, we compare the indeterminacy conditions in the autarky equilibrium with those in the world economy.[3]

[1] Meng and Velasco (2003, 2004) study two-sector dependent economy models with sector-specific externalities in which investment goods are not traded. Lahiri (2001) examines a two-sector endogenous growth model with capital mobility. It is to be noted that Nishimura and Shimomura (2002b) also consider equilibrium determinacy in a two-sector small-open economy without factor mobility. Their argument follows the traditional Heckscher-Ohlin modelling.

[2] In a different context, Ghiglino and Olszak-Duquenne (2005) also consider the relation between heterogeneity of agents and equilibrium indeterminacy in a closed economy model of economic growth with production externalities.

[3] In the open-macroecomics literature, we have not seen any study on indeterminacy in a two-country model. It should be noted that in the context of the standard trade theory context, Nishimura and Shimomura (2002b and 2006) investigate Heckscher-Ohlin models with externalities that generate multiple equilibria. As for a general discussion concerning

Our main finding is that, as opposed to the results established in the small-open economy models, the global economy does not necessarily show a higher possibility of indeterminacy. In particular, if the preference structures of both countries satisfy additive separability between consumption and labor, opening up international transactions may lower the possibility of indeterminacy. Furthermore, even in the case of non-separable utilities, we do not find unambiguous results indicating that the world economy holds indeterminacy under weaker restrictions on technology and preferences than the closed economies.

The rest of the paper is organized as follows. Section 2 sets up the base model. Section 3 assumes specific functional forms involved in the model and explores indeterminacy conditions under alternative assumptions on the utility functions. Section 4 considers a modified model that can sustain endogenous long-term growth. This section reconfirms the main finding obtained by the base model. Section 5 concludes the paper.

2 The Analytical Framework

2.1 Structure of the Model

There are two countries, country 1 and 2. Each country produces a country-specific, single good. We assume that country 1 specializes in good x and country 2 specializes in good y. Each good can be either consumed or invested for physical capital accumulation. It is assumed that the imported goods can be consumed, but they cannot be used as investment goods. In addition, agents in each country cannot access to the direct ownership of the foreign capital stock. However, the agents in both counties may access to the perfect international bond market so that they can freely lend to or borrow from each other. Since the international bond market is assumed to be perfect, the uncovered interest parity ensures that the nominal interest rates in both countries are equalized in each moment. [4] Although our assumption that imported goods cannot be used for investment is restrictive, it is helpful to determine real investment in each country without introducing additional assumptions such as the presence of adjustment costs of investment.

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the effect of financial integration on macroeconomic volatility, see, for example, Evans and Hanatkovska (2005).

[4] The structure of the base model is close to the two-country model examined by Turnovsky (1997, Chapter 7). Since Turnovsky (1997) does not assume the presence of external increasing returns, indeterminacy of equilibrium is not the issue in his argument. Baxter and Crucini (1995) use a similar model in their study on international real business cycles.

Production

The production technology of each country is described by

$$z_i = f^i\left(k_i, l_i, \bar{k}_i, \bar{l}_i\right), \quad i = 1, 2, \tag{1}$$

where z_i, k_i and l_i respectively denote output, capital and labor input of country i. In addition, $\bar{k}_i$ and $\bar{l}_i$ express external effects associated with the social levels of capital and labor in country i. It is assumed that function $f^i(.)$ is homogenous of degree one in k_i and l_i and that it increases with $\bar{k}_i$ and $\bar{l}_i$. This means that while the private technology under given level of external effects satisfies constant returns, the social technology exhibits increasing returns to scale with respect to the aggregate levels of capital and labor. We assume that those external effects are country specific so that there are no international spillovers of production technologies.

The commodity markets in both countries are competitive. Firms maximize their instantaneous profits under given levels of external effects. Hence, letting r_i and w_i be the real interest rate and the real wage rate in country i, they respectively equal the net marginal product of capital and the marginal product of labor:

$$r_i = f_k^i\left(k_i, l_i, \bar{k}_i, \bar{l}_i\right) - \delta_i, \quad w_i = f_l^i\left(k_i, l_i, \bar{k}_i, \bar{l}_i\right); \quad i = 1, 2. \tag{2}$$

where $\delta_i \in (0, 1)$ denotes the depreciation rate of capital.

Households

The number of households in each country is normalized to one. The objective functional of the representative household in country i is a discounted sum of utilities such that

$$U_i = \int_0^\infty u^i(x_i, y_i, l_i)\, e^{-\rho t} dt, \quad \rho > 0; \quad i = 1, 2,$$

where x_i and y_i denote consumption of x and y goods. By our assumption, y_1 is exported from country 2 to country 1 and x_2 is exported from country 1 to country 2. The instantaneous utility is assumed to be increasing in x_i and y_i, and decreasing in labor l_i. The standard concavity assumption is imposed on $u^i(.)$. We assume that the households in both countries may have different utility functions but their discount rate is the same rate of ρ.[5]

The flow-budget constraint for the households in each country is given by

$$\dot{\omega}_i = r_i \omega_i + w_i l_i - m_i, \quad i = 1, 2, \tag{3}$$

[5] This assumption is introduced only for notational simplicity.

where ω_i is the real asset holding and m_i is real consumption expenditure. For notational convenience, ω_i, w_i and m_i are expressed in terms of the good country i produces. Thus if p denote the price of good y in terms of good x, the consumption spending in both countries are respectively determined by

$$m_1 = x_1 + py_1, \quad m_2 = \frac{x_2}{p} + y_2.$$

The asset consists of capital stock, k_i, and the foreign bond holding, b_i. Therefore, we define

$$\omega_i = k_i + b_i, \quad i = 1, 2.$$

where b_1 and b_2 are evaluated in terms of x good and y good, respectively.

The household maximizes U_i subject to (3) and the initial value of ω_i by controlling consumption levels and labor supply. We impose the no-Ponzi-game condition, and hence the following intertemporal budget constraint holds as well:

$$\begin{aligned} &\omega_i(0) + \int_0^\infty \exp\left(-\int_0^t r_i(s)\,ds\right) w_i(t)\, l_i(t)\, dt \\ = &\int_0^\infty \exp\left(-\int_0^t r_i(s)\,ds\right) m_i(t)\, dt, \quad i = 1, 2. \end{aligned}$$

Market Equilibrium Conditions

Since physical capital stocks are not traded, the market equilibrium conditions for the commodity markets are:

$$z_1 = x_1 + x_2 + \dot{k}_1 + \delta_1 k_1, \tag{4}$$

$$z_2 = y_1 + y_2 + \dot{k}_2 + \delta_2 k_2. \tag{5}$$

The world financial market is assumed to be perfect. This means that the uncovered interest parity yields

$$r_1 = r_2 + \frac{\dot{p}}{p}. \tag{6}$$

The international borrowing and lending in the world economy should be balanced in each moment, implying that the equilibrium condition for the bond market is

$$b_1 + pb_2 = 0. \tag{7}$$

Note that the homogeneity of production functions gives $z_i = r_i k_i + w_i l_i$ $(i = 1, 2)$. Hence, in view of (3), (4) and $\omega_i = k_i + b_i$, the flow budget constraints for the home and the foreign households present the dynamic equations of b_1 and b_2 as follows:

$$\dot{b}_1 = r_1 b_1 + x_2 - p y_1, \tag{8}$$

$$\dot{b}_2 = r_2 b_2 + y_1 - \frac{x_2}{p}. \tag{9}$$

Equations (8) and (9) respectively describe the current accounts of country 1 and 2.[6]

Perfect-Foresight Competitive Equilibrium

To sum up, the perfect-foresight competitive equilibrium (PFCE) of the world economy is defined in the following manner:

Definition: *The PFCE of the world economy is established if the following conditions are satisfied:*

(i) The firms maximize instantaneous profits under given levels of external effects, $\bar{k}_i$. and $\bar{l}_i$.

(ii) The households maximize their discounted sum of utilities under given sequences of prices, $\{r_i(t), w_i(t), p(t)\}_{t=0}^{\infty}$.

(iii) Commodity markets clear and the bonds market is in equilibrium.

(iv) The uncovered interest parity condition (6) is satisfied.

(v) External effects satisfy consistency conditions, that is, $\bar{k}_i = k_i$ and $\bar{l}_i = l_i$.

In what follows, we analyze the corresponding planning economy rather than the decentralized system displayed above.

2.2 A Pseudo-Planning Problem

In order to characterizes the equilibrium dynamics of the world economy, it is convenient to consider the following pseudo-planning problem:

$$\max \int_0^{\infty} \left[\mu_1 u^1(x_1, y_1, l_1) + \mu_2 u^2(x_2, y_2, l_2)\right] e^{-\rho t} dt, \quad \mu_i > 0, \quad \mu_1 + \mu_2 = 1$$

[6] We assume that the solvency conditions for international lending and borrowing are satisfied. Namely, the following hold:

$$b_1(0) + \int_0^{\infty} \exp\left(-\int_0^t r_1(s)\, ds\right) x_2(t)\, dt = \int_0^{\infty} \exp\left(-\int_0^t r_1(s)\, ds\right) p(t)\, y_1(t)\, dt,$$

$$b_2(0) + \int_0^{\infty} \exp\left(-\int_0^t \left[r_2(s) + \frac{\dot{p}(s)}{p(s)}\right] ds\right) y_1(t)\, dt = \int_0^{\infty} \exp\left(-\int_0^t \left[r_2(s) + \frac{\dot{p}(s)}{p(s)}\right] ds\right) \frac{x_2(t)}{p(t)} dt.$$

subject to

$$\dot{k}_1 = f^1\left(k_1, l_1, \bar{k}_1, \bar{l}_1\right) - x_1 - x_2 - \delta_1 k_1, \tag{10}$$

$$\dot{k}_2 = f^2\left(k_2, l_2, \bar{k}_2, \bar{l}_2\right) - y_1 - y_2 - \delta_2 k_2, \tag{11}$$

and given initial levels of $k_1(0)$ and $k_2(0)$. A positive constant, μ_i, denotes a weight on the utility in country i. Taking the sequences of external effects, $\left\{\bar{k}_1(t), \bar{l}_1(t), \bar{k}_2(t), \bar{l}_2(t)\right\}_{t=0}^{\infty}$, as given, the planner solves this problem by selecting the optimal levels of l_i, x_i and y_i.

The Hamiltonian function for the planning problem can be set as

$$\begin{aligned} H &= \mu_1 u^1(x_1, y_1, l_1) + \mu_2 u^2(x_2, y_2, l_2) \\ &\quad + q_1\left[f^1\left(k_1, l_1, \bar{k}_1, \bar{l}_1\right) - x_1 - x_2 - \delta_1 k_1\right] \\ &\quad + q_2\left[f^2\left(k_2, l_2, \bar{k}_2, \bar{l}_2\right) - y_1 - y_2 - \delta_2 k_2\right], \end{aligned}$$

where q_i represents the shadow value of k_i. The necessary conditions for an optimum are:

$$\mu_i u^i_{x_i}(x_i, y_i, l_i) = q_1, \quad i = 1, 2, \tag{12}$$

$$\mu_i u^i_{y_i}(x_i, y_i, l_i) = q_2, \quad i = 1, 2, \tag{13}$$

$$\mu_i u^i_{l_i}(x_i, y_i, l_i) = q_i f^i_{l_i}\left(k_i, l_i, \bar{k}_i, \bar{l}_i\right), \quad i = 1, 2, \tag{14}$$

$$\dot{q}_i = q_i\left[\rho + \delta_i - f^i_{k_i}\left(k_i, l_i, \bar{k}_i, \bar{l}_i\right)\right], \quad i = 1, 2, \tag{15}$$

together with (10), (11) and the transversality conditions:

$$\lim_{t\to\infty} e^{-\rho t} q_i k_i = 0, \quad i = 1, 2. \tag{16}$$

Inserting $\bar{k}_i = k_i$ and $\bar{l}_i = l_i$ into the optimal conditions, we find the necessary conditions for an optimum of this pseudo-planning problem completely characterize PFEC conditions of the world economy defined above. .

Lemma 1 *If μ_1 and μ_2 are appropriately selected, the necessary conditions for an optimum for the planning problem and the PFEC in the world economy are identical.*

Proof. See Appendix 1. ■

Due to this fact, we can focus on the dynamic behavior of the pseudo-planning economy when examining the equilibrium dynamics of the world economy.

2.3 Dynamical Systems

The necessary conditions for the planning problem may be summarized as a complete dynamical system with respect to the state variables, k_1 and k_2, and the costate variables, q_1 and q_2. The structure of the dynamic system is sensitive to the specification of the preference structure. First, consider the general case where the instantaneous utility functions are non separable ones. In this case, by use of (12) and (13), consumption demand x_i and y_i may be expressed as functions of q_1. q_2 and k_i. Substituting those functions into (14) and assuming that $\bar{k}_i = k_i$ and $\bar{l}_i = l_i$, we find that l_i depends on q_1, q_2 and k_i. Therefore, the equilibrium levels of x_i, y_i and l_i are written as:

$$x_i = x^i\left(k_i, q_1, q_2\right), \quad y_i = y^i\left(k_i, q_1, q_2\right), \quad l_i = l^i\left(k_i, q_1, q_2\right); \quad i = 1, 2.$$

Substituting those into (10), (11) and (15) the complete dynamic system may be written as

$$\dot{k}_1 = z^1\left(k_1, q_1, q_2\right) - x^1\left(k_1, q_1, q_2\right) - x^2\left(k_2, q_1, q_2\right) - \delta_1 k_1,$$

$$\dot{k}_2 = z^2\left(k_2, q_1, q_2\right) - y^1\left(k_1, q_1, q_2\right) - y^2\left(k_2, q_1, q_2\right) - \delta_2 k_2,$$

$$\dot{q}_i = q_i\left[\rho - r^i\left(k_i, q_1, q_2\right)\right], \quad i = 1, 2,$$

where

$$\begin{aligned} z^i\left(k_i, q_1, q_2\right) &\equiv f^i\left(k_i, l^i\left(k_i, q_1, q_2\right), k_i, l_i\left(k_i, q_1, q_2\right)\right), \\ r^i\left(k_i, q_1, q_2\right) &\equiv f_k\left(k_i, l^i\left(k_i, q_1, q_2\right), k_i, l^i\left(k_i, q_1, q_2\right)\right) - \delta_i. \end{aligned}$$

Next, assume that the utility function is additively separable between consumption and labor in such a way that

$$u^i\left(x_i, y_i, l_i\right) = \phi_c^i\left(x_{i,} y_i\right) + \phi_L^i\left(l_i\right).$$

Then it is easy to see that x_i and y_i depend only on q_1 and q_2 alone, while $z^\iota\left(.\right)$ and $r^i\left(.\right)$ involve only k_i and q_i

$$\dot{k}_1 = z^1\left(k_1, q_1\right) - x^1\left(q_{1,} q_2\right) - x^2\left(q_{1,} q_2\right) - \delta_1 k_1,$$

$$\dot{k}_2 = z^2\left(k_2, q_2\right) - y^1\left(q_1, q_2\right) - y^2\left(q_1, q_2\right) - \delta_2 k_2,$$

$$\dot{q}_i = q_i\left[\rho + \delta_i - r^i\left(k_i, q_i\right)\right],. \ i = 1, 2.$$

Finally, consider the case in which the utility function is additively separable for each argument, that is,

$$u^i\left(x_i, y_i, l_i\right) = \phi_X^i\left(x_i\right) + \phi_Y^i\left(y_i\right) + \phi_L^i\left(l_i\right).$$

As demonstrated by Turnovsky (1997, Chapter 7), this assumption simplifies the dynamic system substantially: consumption demand for x goods depends on q_1 alone and that for y good depends on q_2 alone. Thus the dynamic system is described by

$$\dot{k}_1 = z^1 (k_1, q_1) - x^1 (q_1) - x^2 (q_1) - \delta_1 k_1,$$

$$\dot{k}_2 = z^2 (k_2, q_2) - y^1 (q_2) - y^2 (q_2) - \delta_2 k_2,$$

$$\dot{q}_i = q_i \left[\rho + \delta_i - r^i (k_i, q_i)\right], \quad i = 1, 2.$$

Notice that this system consists of two independent sub-system with respect to (k_1, q_1) and (k_2, q_2). Therefore, while the current account depends on the foreign variables (see (8)equations (9)), consumption and investment decisions in each country are not interdependent each other even in the financially integrated world. Since the most general case will not produce tractable results in the base model, we focus on the second and the third cases when examining exogenous growth models in the next section. In section 4 we treats non-separable utility between consumption and labor in the case of endogenous growth.

3 Equilibrium Dynamics

3.1 Specification

In order to obtain more concrete results about determinacy of equilibrium, we now specify the production and the utility functions as the forms that have been frequently employed in the literature on indeterminacy and sunspots in the real business cycle models. The production function of each country is

$$z_i = k_i^{\alpha_i} l_i^{1-a_i} \bar{k}_i^{\alpha_i - a_i} \bar{l}^{\beta_i + a_i - 1}, \quad 0 < a_i < 1, \ \alpha_i, \beta_i > 0, \ \alpha_i + \beta_i > 1.$$

Accordingly, imposing $\bar{k}_i = k_i$ and $\bar{l}_i = l_i$, we obtain the social production function in each country as follows:

$$z_i = k_i^{\alpha_i} l_i^{\beta_i}, \quad i = 1, 2. \tag{17}$$

In what follows we assume that $0 < \alpha_i < 1$, so that capital externalities are not so large that unbounded growth is possible. The rate of return to capital and the real wage rate are respectively given by

$$r_i = f_k^i - \delta_i = a_i k_i^{\alpha_i - 1} l_i^{\beta_i} - \delta_i, \quad i = 1, 2, \tag{18}$$

$$w_i = f_l^i = (1 - a_i) \, k_i^{\alpha_i} l_i^{\beta_i - 1}, \quad i = 1, 2. \tag{19}$$

The utility function is assumed to be additively separable between consumption and labor. More specifically, the instantaneous utility of the households in country i is given by

$$u^i(x_i, y_i, l_i) = \begin{cases} \dfrac{\left(x_i^{\theta_i} y_i^{1-\theta_i}\right)^{1-\sigma_i} - 1}{1-\sigma_i} - \dfrac{l_i^{1+\gamma_i}}{1+\gamma_i}, & 0 < \theta_i < 1, \ \gamma_i > 0, \ \ \sigma_i > 0, \\ \theta_i \ln x_i + (1-\theta_i) \ln y_i - \dfrac{l_i^{1+\gamma_i}}{1+\gamma_i}, & \text{for } \sigma_i = 1. \end{cases}$$

Given those specifications, the optimal conditions (12), (13), (14) and (15) may be written as

$$\mu_i \theta_i x_i^{\theta_i(1-\sigma_i)-1} y_i^{(1-\theta_i)(1-\sigma_i)} = q_1, \quad i = 1, 2, \tag{20}$$

$$\mu_i (1-\theta_i) x_i^{\theta_i(1-\sigma_i)} y_i^{(1-\theta_i)(1-\sigma_i)-1} = q_2, \ \ i = 1, 2, \tag{21}$$

$$\mu_i l_i^{\gamma_i} = (1-a_i) k_i^{\alpha_i} l_i^{\beta_i - 1} q_i, \ \ i = 1, 2, \tag{22}$$

$$\dot{q}_i = q_i \left(\rho + \delta_i - a_i k_i^{\alpha_i - 1} l_i^{\beta_i}\right), \ \ i = 1, 2. \tag{23}$$

3.2 Indeterminacy with Logarithmic Utility Functions

We first examine the case where the utility function has a log-additive form in consumption. Ever since Benhabib and Farmer (1994), this specification has been most popular in the studies on indeterminacy and sunspots in the real business cycle settings. In this case $\sigma_1 = \sigma_2 = 1$, and hence the consumption demand functions become

$$x_i = \frac{1}{\mu_i \theta_i q_1}, \quad y_i = \frac{1}{\mu_i (1-\theta_i) q_2}, \quad i = 1, 2. \tag{24}$$

As pointed out in the previous section, the demand functions for each good depend on its own price alone. On the other hand, from (21) we obtain

$$l_i = \left[\frac{1-a_i}{\mu_i}\right]^{\frac{1}{\gamma_i + 1 - \beta_i}} k_i^{\frac{\alpha_i}{\gamma_i + 1 - \beta_i}} q_i^{\frac{1}{\gamma_i + 1 - \beta_i}}, \quad i = 1, 2. \tag{25}$$

This states that the effects of changes in capital and commodity price on the equilibrium level of labor input depends on the sign of $\gamma_i, +1 - \beta_i$. If the external effect of labor is small enough to satisfy $\gamma_i + 1 > \beta_i$, then the employment level is positively related to q_i and k_i. If $\gamma_i + 1 < \beta_i$, the labor employment decreases with q_i and k_i.

In view of (10) , (11) , (23) , (24) and (25) , the complete dynamic system with logarithmic utility functions is given by the following:

$$\dot{k}_1 = A_1 k_1^{\frac{\alpha_1(\gamma_1+1)}{\gamma_1+1-\beta_1}} q_i^{\frac{\beta_1}{\gamma_1+1-\beta_1}} - \left[\frac{1}{\theta_1\mu_1} + \frac{1}{\theta_2\mu_2}\right]\frac{1}{q_1} - \delta_1 k_1, \tag{26}$$

$$\dot{k}_2 = A_2 k_2^{\frac{\alpha_2(\gamma_2+1)}{\gamma_2+1-\beta_2}} q_2^{\frac{\beta_2}{\gamma_2+1-\beta_2}} - \left[\frac{1}{(1-\theta_1)\,\mu_1} + \frac{1}{(1-\theta_2)\,\mu_2}\right]\frac{1}{q_2} - \delta_2 k_2, \tag{27}$$

$$\dot{q}_1 = q_1\left[\rho + \delta_1 - a_1 A_1 k_1^{\frac{\alpha_1(\gamma_1+1)}{\gamma_1+1-\beta_1}-1} q_1^{\frac{\beta_1}{\gamma_1+1-\beta_1}}\right], \tag{28}$$

$$\dot{q}_2 = q_2\left[\rho + \delta_2 - a_2 A_2 k_2^{\frac{\alpha_2(\gamma_2+1)}{\gamma_2+1-\beta_2}-1} q_2^{\frac{\beta_2}{\gamma_2+1-\beta_2}}\right], \tag{29}$$

where

$$A_i = \left[\frac{1-a_i}{\mu_i}\right]^{\frac{\beta_i}{\gamma_i+1-\beta_i}}, \quad i = 1, 2.$$

It is easy to see that the steady state equilibrium, if it exists, is uniquely determined. Letting k_i^* and q_i^* be the steady-state values of capital stocks and prices., the dynamic system linearized at the steady state is written as

$$\begin{bmatrix} \dot{k}_1 \\ \dot{k}_2 \\ \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} h_{11} & 0 & h_{13} & 0 \\ 0 & h_{22} & 0 & h_{24} \\ h_{31} & 0 & h_{33} & 0 \\ 0 & h_{42} & 0 & h_{44} \end{bmatrix} \begin{bmatrix} k_1 - k_1^* \\ k_2 - k_2^* \\ q_1 - q_1^* \\ q_2 - q_2^* \end{bmatrix},$$

where $\partial\dot{k}_1/\partial k_1$, $h_{13} = \partial\dot{k}_1/\partial q_1$, etc, all of which are evaluated by the steady-state values of k_i^* and q_i^*. Denote the eigenvalue of the coefficient matrix given above by λ. Then the characteristic equation of the coefficient matrix in the above linear system is written as

$$\begin{aligned} &\left[\lambda^2 - (h_{11} + h_{33})\,\lambda + h_{11}h_{33} - h_{13}h_{31}\right] \\ \times &\left[\lambda^2 - (h_{11} + h_{33})\,\lambda + h_{22}h_{44} - h_{24}h_{42}\right] = 0. \end{aligned} \tag{30}$$

Notice that in view of the steady-state condition, $a_i A_i k_i^{\frac{\alpha_i(\gamma_i+1)}{\gamma_i+1-\beta_i}-1} q_i^{\frac{\beta i}{\gamma_i+1-\beta_i}} = \rho + \delta_i$, we obtain:

$$\begin{aligned} h_{ii} &= A_1 \frac{\alpha_i(\gamma_i+1)}{\gamma_i+1-\beta_i} k_1^{\frac{\alpha_1(\gamma_1+1)}{\gamma_1+1-\beta_1}-1} q_i^{\frac{\beta_1}{\gamma_1+1-\beta_1}} - \delta_1 \\ &= \frac{\gamma_i+1}{\gamma_i+1-\beta_i}\left[\rho + \frac{\delta_i\beta_i}{\gamma_i+1}\right], \qquad i = 1, 2, \end{aligned}$$

implying that

$$\text{sign } h_{ii} = \text{sign } \left(\gamma_i + 1 - \beta_i\right), \quad i = 1, 2.$$

We also see that when $\gamma_i + 1 - \beta_i > 0$, it holds that h_{13}, h_{31}, $h_{24,}$ $h_{42} > 0$ and h_{33}, $h_{44} < 0$. As a result, we show that if $\gamma_i + 1 - \beta_i > 0$, then (30) has two positive and two negative real roots. The presence of two-dimensional stable manifold around the steady state means that the initial levels of jump variables, q_1 and q_2 can be uniquely determined. Therefore, we obtain the following result:

Proposition 1 *If the utility function is additively separable between domestic and imported good and production technologies of both countries satisfy* $\gamma_i + 1 - \beta_i > 0$ $(i = 1, 2)$, *then the steady state of the world economy is determinate.*

In contrast, if $\gamma_i + 1 < \beta_i$ $(i = 1, 2)$, it holds that $h_{11},\ h_{22},\ h_{13}, h_{24} < 0$ and $h_{31,}$ $h_{33,}$ h_{42}, $h_{44}, > 0$. Thus it is possible to have the following inequalities:

$$\begin{aligned} h_{11} + h_{33} &< 0, \quad h_{11} + h_{33} < 0, \\ h_{11}h_{33} - h_{13}h_{31} &> 0, \quad h_{22}h_{44} - h_{24}h_{42} > 0 \end{aligned}$$

If this is the case, (30) have four roots with negative real parts, implying that there are a continuum of converging path near the steady state. Hence, the steady state of the world economy is at least locally indeterminate. If $\gamma_1 + 1 > \beta_1$ and $\gamma_2 + 1 < \beta_2$, then the steady state of country 2 will be indeterminate, but that of country 1 is determinate. Since fluctuations of country 2 do not affect investment and price in country 1.

In addition, volatility in country 2 does not fluctuate the current account of country 1 either. To see this, we should note that, as shown in Appendix A, the relative price p in the market economy is proportional to the ratio of shadow values of capital, q_2/q_1. Given this fact, from (24) the current account of country 1 is described by

$$\begin{aligned} \dot{b}_1 &= r^1\left(k_1, q_1\right) b_1 + x_2\left(q_1\right) - \frac{q_2\mu_2}{q_1\mu_1} y_1\left(q_2\right) \\ &= .a_1 A_1 k_1^{\frac{\alpha_1(\gamma_1+1)}{\gamma_1+1-\beta_1}-1} q_1^{\frac{\beta_1}{\gamma_1+1-\beta_1}} b_1 + \frac{1}{\mu_2\theta_2 q_1} - \frac{\mu_2}{\mu_1^2\left(1-\theta_1\right) q_1}. \end{aligned}$$

What is noteworthy is that the current account of country 1 depends on k_1 and q_1 alone. Therefore, even if the equilibrium trajectory of country 2 is indeterminate so that q_2 exhibits volatility, the imported good y in terms of good x is independent of the behavior of q_2.[7] The current account of country 1 is affected by sunspot-driven expectations only when the autarkic equilibrium of country 1 is indeterminate. In sum, we have shown:

Proposition 2 *If the utility functions of both countries are logarithmic in consumption goods, then the possibility of indeterminacy will not enhanced by international trade and financial integration. Indeterminacy in one country will not spillover to the other country in any respect.*

3.3 Indeterminacy with CES Utility Functions

When $\sigma_i \neq 1$ so that the utility functions are of CES forms in consumption, equations (20) and (21) yield the following consumption demand functions:

$$x_i = \Delta_x^i q_1^{\frac{(1-\acute{\sigma}_i)(1-\theta_i)-1}{\sigma_i}} q_2^{-\frac{(1-\sigma_i)(1-\theta_i)}{\sigma_i}} - \delta_1 k_1, \quad i = 1,2 \tag{31}$$

$$y_i = \Delta_y^i q_1^{\frac{(1-\sigma_i)(1-\theta_i)-1}{\sigma_i}+1} q_2^{-\frac{(1-\sigma_i)(1-\theta_i)}{\sigma_i}-1} - \delta_2 k_2, \quad i = 1,2, \tag{32}$$

where

$$\begin{aligned} \Delta_x^i &= \frac{1}{\mu_i\theta_i}\left(\frac{\theta_i}{1-\theta_i}\right)^{(1-\sigma_i)(1-\theta_i)} > 0, \\ \Delta_y^i &= \frac{1}{\mu_i\theta_i}\left(\frac{\theta_i}{1-\theta_i}\right)^{(1-\sigma_i)(1-\theta_i)+1} > 0. \end{aligned}$$

We see that the own price effect is always negative, while the effects of foreign price on consumption demand for the home good depends on the sign of $1-\sigma_i$. If $\sigma_i > 1$, the substitution effect dominate so that a rise in the foreign price increases in consumption demand for the home goods. Conversely, if $\sigma_i < 1$, a higher foreign good price lowers demand for the home goods.

The dynamical equations in the case of non-separable utility between x_i and y_i are thus displayed by

$$\dot{k}_1 = A_1 k_1^{\frac{\alpha_1(\gamma_1+1)}{\gamma_1+1-\beta_1}} q_i^{\frac{\beta_1}{\gamma_1+1-\beta_1}} - \sum_{i=1,2} \Delta_x^i q_1^{\frac{(1-\acute{\sigma}_i)(1-\theta_i)-1}{\sigma_i}} q_2^{-\frac{(1-\sigma_i)(1-\theta_i)}{\sigma_i}} - \delta_1 k_1, \tag{33}$$

$$\dot{k}_2 = A_2 k_2^{\frac{\alpha_2(\gamma_2+1)}{\gamma_2+1-\beta_2}} q_2^{\frac{\beta_2}{\gamma_2+1-\beta_2}} - \sum_{i=1,2} \Delta_y^i q_1^{\frac{(1-\sigma_i)(1-\theta_i)-1}{\sigma_i}+1} q_2^{-\frac{(1-\sigma_i)(1-\theta_i)}{\sigma_i}-1} - \delta_2 k_2, \tag{34}$$

[7] If this is the case, comparative dynamics analysis by Turnovsky (1997, Chapter 7) that assumes determinacy of equilibrium should be re-examined.

together with (26) and (27).

Inspecting the dynamical system derived above, we find that if the closed economies hold determinacy of equilibrium, opening up international trade will not enhance volatility. More formally, we obtain the following result:

Proposition 3 *Suppose that* $\gamma_i + 1 - \beta_i > 0$ *for* $i = 1$ *and* 2 *so that the steady state in the autarkic equilibrium of both countries are locally determinate. Then the world economy holds determinacy of equilibrium for any values of* $\sigma_i \ (> 0)$.

Proof. See Appendix B. ■

In this model, the price of foreign good affects capital formation of the home country only through consumption demand. The above proposition indicates that such an interaction between the two countries may not enhance the possibility of indeterminacy. Therefore, in this case, globalization would not produce fluctuations due to extrinsic uncertainty in self-fulfilling expectations. On the contrary, globalization may be a 'stabilizing factor', as the following proposition demonstrates:

Proposition 4 *Suppose that country 1 holds indeterminacy in the autarkic equilibrium, while country 2 satisfies determinacy condition, i.e.* $\gamma_2 + 1 > \beta_2$. *Then if the own price effects in the consumption demand functions dominate their cross price effect, the world economy may establish determinacy of equilibrium.*

Proof. See Appendix C. ■

The intuition behind this result may be stated as follows. From (22) we obtain

$$\frac{\mu_1 l_1^{\gamma_1}}{q_1} = (1 - a_1)\, k_1^{\alpha_1} l_1^{\beta_1 - 1}.$$

Given the price, q_1, and capital, k_1, the left-hand side of the above expresses labor supply curve and the right-hand side is the labor demand curve. If country 1 is a closed economy, $\gamma_1 + 1 < \beta_1$ is a necessary condition for local indeterminacy around the steady state. As discussed by Benhabib and Farmer (1994), in this case the labor demand curve has a positive slope and it is steeper than the labor supply curve. Now suppose that country 1 is in the steady state initially. Then if the household anticipates a reduction in price, q_1, the labor supply curve shifts up, implying that labor employment and production increase. Since a lower price raises the domestic demand for x goods, such an increase in consumption demand may be consistent with a rise in production. This means that the initial anticipation of a lower price will be self fulfilling. In the world economy, the mechanism that may generate indeterminacy is essentially the same as that in the close economy. If the households in the world anticipated a decrease in q_1, the consumption demand for x goods increase. This again makes the labor supply curve in country 1 shifts up, which raises the production level of x. However, in the case of world economy a lower q_1 increases the export for country 2 as well as the domestic consumption demand. If such a demand effect is large enough, a rise in employment in country 1 is insufficient to satisfy the newly created consumption demand. In this case the initial anticipation cannot be self fulfilled. Therefore, the equilibrium is determinate even though labor externalities are sufficiently large to satisfy $\gamma_1 + 1 < \beta_1$.

4 Endogenous Growth

So far, we have considered a model of exogenous growth in which the world economy will not display sustained growth in the steady-state equilibrium. In this section, we consider an endogenously growing world economy by modifying the base model. It is shown that in the case of endogenous growth we may treat a lower dimensional dynamic system than in the base model. This enables us to examine the model with nonseparable utilities between consumption and labor supply in which there is a strong interdependency between the two countries.[8]

In the following, we assume that both countries have Ak technologies in the sense that their production functions are linearly homogenous with respect to the private and the social levels of capital. Hence, we assume that $\alpha_1 = \alpha_2 = 1$, implying that the social production function in each country becomes

$$z_i = k_i^{a_i} l_i^{1-a_i} \bar{k}_i^{1-\alpha_i} \bar{l}_i^{\beta_i + a_i - 1} = l_i^{\beta_i} k_i, \quad i = 1, 2. \tag{35}$$

As a result, the rate of returns to capital are given by

$$r_i = a_i l_i^{\beta_i} - \delta_i, \ \ i = 1, 2. \tag{36}$$

4.1 Separable Utility

We first inspect the model with spareable utility functions. As is well known, if the utility function is additively separable between consumption and labor,

[8]The model in this section is an example of two-country model of endogenous growth. Farmer and Lahiri (2005 and 2006). present alternative formulations of two country models with endogenous growth.

the sub-utility generating from consumption should be logarithmic to hold balanced growth equilibrium with a positive growth rate. Thus in this section we assume that $\sigma_i = 1$, so that

$$u^i(x_i, y_i, l_i) = \theta_i \ln x_i + (1-\theta_i)\ln y_i - \frac{l_i^{1+\gamma_i}}{1+\gamma_i}.$$

Given this function, (14) yields

$$l_i^{\gamma_i+1-\beta_i} = (1-a_i)\, q_i k_i, \quad i = 1, 2.$$

Let us denote $q_i k_i = v_i$. Then the equilibrium level of employment in country i is

$$l_i = \eta_i v_i^{\frac{1}{\gamma_i+1-\beta_i}}, \quad i = 1, 2. \tag{37}$$

where $\eta_i = (1-a_i)^{\frac{1}{\gamma_i+1-\beta_i}}$. The equilibrium employment level in each country increases (resp. decreases) with the value of capital, if $\gamma_i + 1 > \beta_i$.(resp. $\gamma_i + 1 < \beta_i$). In this case, the demand functions for final goods are given by

$$x_i = \mu_i \theta_i / q_i, \quad y_i = \mu_i (1-\theta_i)/q_i, \quad i = 1, 2. \tag{38}$$

By use of (24) , we show that the growth rates of capital stocks in both countries are expressed as follows:

$$\frac{\dot{k}_1}{k_1} = \eta_1 v_1^{\frac{\beta_1}{\gamma_1+1-\beta_1}} - \frac{\mu_1\theta_1 + \mu_2\theta_2}{v_1} - \delta_1,$$

$$\frac{\dot{k}_2}{k_2} = \eta_2 v_2^{\frac{\beta_2}{\gamma_2+1-\beta_2}} - \frac{\mu_1(1-\theta_1) + \mu_2(1-\theta_2)}{v_2} - \delta_2.$$

The price in each good changes according to

$$\frac{\dot{q}_i}{q_i} = \rho - a_i l_i^{\beta_i}, \quad i = 1, 2.$$

By the definition of $v_i\,(= q_i k_i)$, we thus obtain:

$$\frac{\dot{v}_1}{v_1} = (1-a_1)\,\eta_1 v_1^{\frac{\beta_1}{\gamma_1+1-\beta_1}} + \rho - \frac{\mu_1\theta_1 + \mu_2\theta_2}{v_1} - \delta_1, \tag{39}$$

$$\frac{\dot{v}_2}{v_2} = (1-a_2)\,\eta_2 v_2^{\frac{\beta_2}{\gamma_2+1-\beta_2}} + \rho - \frac{[\mu_1(1-\theta_1) + \mu_2(1-\theta_2)]}{v_2} - \delta_2. \tag{40}$$

Differential equations (39) and (40) constitute a complete set of dynamic system.

The balanced-growth of the world economy is attained when $\dot{v}_1 = \dot{v}_2 = 0$. Note that in the balanced growth equilibrium, the real income of both countries need not grow at a common rate. If each country grows at a different rate, the relative price between x and y goods continue to change at a constant rate to keep the uncovered interest parity condition, $r_1 = \dot{p}/p + r_{2.}$ Hence, letting g_i be the steady-growth rate of country i, in the balanced-growth equilibrium, we obtain

$$g_i = a_i l_i^{\beta_i} - \rho - \delta_i = -\dot{q}_i/q_i, \quad i = 1, 2, \tag{41}$$

which means that change in the relative price on the balanced-growth path is

$$\frac{\dot{p}}{p} = \frac{\dot{q}_1}{q^1} - \frac{\dot{q}_2}{q_2} = a_2 l_2^{\beta_2} - a_1 l_1^{\beta_1} + \delta_1 - \delta_2. \tag{42}$$

The rate of change in asset holding in country 1 is given by

$$\frac{\dot{b}_1}{b_1} = g_1 + \rho + \frac{\mu_1 \theta_1}{q_1 b_1} - \frac{p\mu_2 (1 - \theta_2)}{q_2 b_1}.$$

In the balanced-growth equilibrium b_1 changes at the rate of g_1 and thus it holds that

$$\frac{p\mu_2 (1 - \theta_2)}{q_2 b_1} - \frac{\mu_1 \theta_1}{q_1 b_1} = \frac{p y_1 - x_1}{b_1} = \rho.$$

Similarly, the balanced-growth condition in country 2 involves $x_2/p - y_1 = \rho b_2$.

Due to the assumption of log-additive utility functions, there is no interaction between the dynamic behaviors of v_1 and v_2. It is easy to confirm that the steady-state value of v_i is uniquely given if $\gamma_i + 1 > \beta_i$. We also find that if $\gamma_i + 1 < \beta_i$, then either there is no steady state or there are dual steady states. Here we assume that both countries have dual balanced-growth paths when $\gamma_i + 1 < \beta_i$. Additionally, we can confirm that when there are two steady state, one with a lower value of v_i is locally indeterminate and the other with a higher value of v_i is locally determinate.[9] Consequently, we may state:

Proposition 5 *If $\gamma_i + 1 - \beta_i > 0$ for $i = 1, 2$, the world economy has a unique balanced growth path that satisfies global determinacy. On the contrary, if $\gamma_i + 1 - \beta_i < 0$ for $i = 1, 2$, then there may exist four steady states: one in which both country grow at the lower rate is locally determinate, while the other three are locally indeterminate.*

[9] Denote the steady state values of v_i as v_i^* and v_i^{**} $(v_i^* < v_i^{**})$. It is seen that that $d\dot{v}_i/dv_i < 0$ for $v_i = v_i^*$ and $d\dot{v}_i/dv_i > 0$ for $v_i = v_i^{**}$. Since v_i is not a predetermined variable, indeterminacy emerges around $v_i = v_i^*$.

Figure 1-a depicts the phase diagram of (39) and (40) for $\gamma_i + 1 > \beta_i$ $(i = 1, 2)$ (so that the world economy is globally determinate). If this is the case, the world economy stays on the balanced-growth path and it has no transitional dynamics. In contrast, if $\gamma_i + 1 < \beta_i$ $(i = 1, 2)$, then the world economy involves four steady states: the balanced-growth path on which both countries attain lower growth rates will not display indeterminacy, while other three are locally indeterminate. As Figure 1-b shows, the steady state where both countries attain higher growth rates is a sink. This suggests that the behaviors of the terms of trade and current accounts of both countries may be totally indeterminate around on the high-growth steady state of the world economy.

It is worth emphasizing that in our setting each country may attain a different rate of balanced growth. In particular, in the case of Figure 1-b, the balanced-growth rate of each country may differ from each other even though both countries have the identical production and preference structures. This is because the balanced growth path where country 1 (country 2) grows faster than country 2 (country 1) may be attained as a steady state equilibriums the world economy even in the presence of free trade and a well-organized international financial market.

4.2 Non-Separable Utility

We now assume that the utility functions are non-separable between consumption and labor. We specify the instantaneous utility function is given by

$$u^i\left(x_i, y_i, l_i\right) = \frac{\left(x_i^{\theta_i} y_i^{1-\theta_i} \Lambda_i\left(l_i\right)\right)^{1-\sigma_i} - 1}{1-\sigma_i}, \quad i = 1, 2., \quad \sigma_i > 0, \quad \sigma_i \neq 1,$$

where

$$\Lambda_i\left(l_i\right) = \exp\left(-\frac{l_i^{1+\gamma_i}}{1+\gamma_i}\right), \quad \gamma_i > 0.$$

When $\sigma_i = 1$, the utility function takes a log-additive form used before. The necessary conditions for an optimum for the planning problem involve the following conditions:

$$\mu_i \theta_i x_i^{(1-\sigma_i)\theta_i - 1} y_i^{(1-\sigma_i)(1-\theta_i)} \Lambda_i\left(l\right)^{1-\sigma_i} = q_1, \tag{43}$$

$$\mu_i\left(1-\theta_i\right) x_i^{\theta_i(1-\sigma_i)} y_i^{(1-\theta_i)(1-\sigma_i)-1} \Lambda_i\left(l_i\right)^{1-\sigma_i} = q_2, \tag{44}$$

$$\mu_i x_i^{(1-\sigma_i)\theta_i} y_i^{(1-\sigma_i)(1-\theta_i)} \Lambda_i'\left(l_i\right) \Lambda_i\left(l_i\right)^{-\sigma_i} = -q_i\left(1-a_i\right) l_i^{\beta_i - 1} k_i, \tag{45}$$

$$\dot{q}_i = q_i \left[\rho + \delta_i - a_i l_i^{\beta_i}\right]. \tag{46}$$

From (43), (44) and (45), the consumption demand functions are given by

$$x_1 = \theta_1 (1 - a_1) l_1^{\beta_1 - \gamma_1 - 1} k_1$$
$$x_2 = \frac{q_2}{q_1} \theta_2 (1 - a_2) l_2^{\beta_2 - \gamma_2 - 1} k_2$$
$$y_1 = \frac{q_1}{q_2} (1 - \theta_1)(1 - a_1) l_1^{\beta_1 - \gamma_1 - 1} k_1$$
$$y_2 = (1 - \theta_2)(1 - a_2) l_2^{\beta_2 - \gamma_2 - 1} k_2$$

Given the non-separable utilities, consumption demand in each country is directly related to capital and labor input as well as to the relative price. Observe that substituting those demand functions into (43) and (44) yields

$$\Phi_1 k_1^{-\sigma_1} q_1^{(1-\sigma_1)(1-\theta_1)} q_2^{-(1-\sigma_1)(1-\theta_1)} l_1^{\sigma_1(1+\gamma_1-\beta_1)} \exp\left(-(1-\sigma_1)\frac{l^{\gamma_1+1}}{\gamma_1+1}\right) = q_1, \tag{47}$$

$$\Phi_2 k_2^{-\sigma_2} q_1^{-(1-\sigma_2)\theta_2} q_2^{(1-\sigma_2)\theta_2} l_2^{\sigma_2(1+\gamma_2-\beta_2)} \exp\left(-(1-\sigma_2)\frac{l^{\gamma_2+1}}{\gamma_2+1}\right) = q_2, \tag{48}$$

where Φ_1 and Φ_2 are positive constants. Those equations show that the equilibrium level of l_1 depends on k_1, q_1 and q_2, while that of l_2 depends on k_2, q_1 and q_2.

Logarithmic differentiation of both sides of (47) and (48) with respective to time yields the following relations:

$$\begin{aligned}\frac{\dot{l}_1}{l_1} &= \frac{1}{\sigma_1(1+\gamma_1-\beta_1) - (1-\sigma_1) l_1^{\gamma_1+1}} \left[\sigma_1 \frac{\dot{k}_1}{k_1} + [1 - (1-\sigma_1)(1-\theta_1)] \frac{\dot{q}_1}{q_1}\right. \\ &\quad \left. + (1-\sigma_1)(1-\theta_1)\frac{\dot{q}_2}{q_2}\right],\end{aligned}$$

$$\begin{aligned}\frac{\dot{l}_2}{l_2} &= \frac{l_2}{\sigma_2(1+\gamma_2-\beta_2) - (1-\sigma_2) l_2^{\gamma_2+1}} \left[\sigma_2 \frac{\dot{k}_2}{k_2} + (1-\sigma_2)\theta_2 \frac{\dot{q}_1}{q_1}\right. \\ &\quad \left. + (1-\theta_2+\sigma_2\theta_2)\frac{\dot{q}_2}{q_2}\right].\end{aligned}$$

On the other hand, from (10) and (11) we obtain:

$$\frac{\dot{k}_1}{k_1} = l_1^{\beta_1} - \theta_1(1-a_1) l_1^{\beta_1-\gamma_1-1} - m\theta_2(1-a_2) l_2^{\beta_2-\gamma_2-1} - \delta_1 \equiv \kappa^1(l_1, l_2, m) \tag{49}$$

$$\frac{\dot{k}_2}{k_2} = l_2^{\beta_2} - (1-\theta_1)(1-a_1)\frac{l_1^{\beta_1-\gamma_1-1}}{m} - (1-\theta_2)(1-a_2)\, l_2^{\beta_2-\gamma_2-1} - \delta_2 \equiv \kappa^2(l_1, l_2, m), \tag{50}$$

where $m = q_2 k_2 / q_1 k_1$. Using those relations, we obtain the following differential equations:.

$$\begin{aligned} \dot{l}_1 &= \frac{l_1}{\Delta_1(l_1)}\Big[\sigma_1 \kappa^1(l_1, l_2, m) + [1-(1-\sigma_1)(1-\theta_1)]\left(\rho - a_1 l_1^{\beta_1}\right) \\ &\quad + (1-\sigma_1)(1-\theta_1)\left(\rho - a_2 l_2^{\beta_2}\right)\Big], \end{aligned} \tag{51}$$

$$\begin{aligned} \dot{l}_2 &= \frac{l_2}{\Delta_2(l_2)}\Big[\sigma_2 \kappa^2(l_1, l_2, m) + (1-\sigma_2)\theta_2\left(\rho - a_1 l_1^{\beta_1}\right) \\ &\quad + (1-\theta_2+\theta_2\sigma_2)\left(\rho - a_2 l_2^{\beta_2}\right)\Big], \end{aligned} \tag{52}$$

$$\dot{m} = m\left[\kappa^2(l_1, l_2, m) - \kappa^1(l_1, l_2, m) + a_1 l_1^{\beta_1} - a_2 l_2^{\beta_2}\right], \tag{53}$$

where

$$\Delta_i(l_i) = \sigma_i(1+\gamma_i-\beta_i) - (1-\sigma_i)\, l_i^{\gamma_i+1}, \quad i = 1, 2.$$

Dynamic equations (51), (52) and (53) present a complete system of l_1, l_2 and m $(= q_2 k_2/q_1 k_1)$.

The balanced-growth equilibrium is attained when l_i and m stay constant over time, which means that the growth rate of income in each country is constant and the relative price satisfies $(.42)$. We assume that the dynamic system involves a feasible stationary solution. As shown by (47) and (48), once the initial values of k_i and q_i are selected, the initial values of l_1 and l_2 are determined as well. This indicates that under a given level of $k_2(0)/k_1(0)$, the initial levels of l_1, l_2 and m cannot be chosen independently: only two of them are unpredetermined. Therefore, if the linearized system of (51), (52) and (53) involves one stable and two unstable roots, then the balanced-growth equilibrium is locally determinate. If the approximated system has at least two stable roots, then the balanced-growth path is locally indeterminate.

When the utility function is not additively separable between consumption and labor, dynamic behavior of the closed-economy of country i is obtained by setting $\theta_1 = 1$ and $\theta_2 = 0$, respectively. Thus the dynamic system of closed economy is

$$\dot{l}_i = \frac{l_i}{\sigma_i(1+\gamma_i-\beta_i) - (1-\sigma_i)\, l_i^{\gamma_i+1}}\left[(\sigma_i - a_i)\, l_i^{\beta_i} + \rho - (1-a_i)\, l_i^{\beta_i-\gamma_i-1}\right].$$

We now find that either if

$$\gamma_i + 1 > \beta_i \text{ and } \sigma_i > 1, \;\; i = 1, 2, \tag{54}$$

or if

$$\gamma_i + 1 < \beta_i \text{ and } \sigma_i < a_i, \quad i = 1, 2, \tag{55}$$

then each country has a unique balanced-growth path that satisfies global determinacy. Inspection of the world economy dynamics given by (51) , (52) and (53) shows that if (54) holds, it is hard to establish indeterminacy in the world economy. However, if conditions in (55).are fulfilled, the world economy tends to have indeterminate equilibria.

Proposition 6 *If $\gamma_i + 1 > \beta_i$ and $\sigma_i > a_i$ then both the closed and the world economies may hold determinacy of equilibrium. If $\gamma_i + 1 < \beta_i$ and $\sigma_i < a_i$, then the world economy has indeterminate equilibria, while each closed economy does never have indeterminacy.*

Proof. See Appendix D. ■

Although the non-separable preferences increase interdependency between two counties, the above proposition suggests that such a stronger relationship does not necessarily produce complex dynamics. This result is an example showing that internationalization of markets under complex preference structures may or may enhance the possibility of multiple equilibria and sunspots fluctuations. Unlike small-open economies where openness usually presents a higher possibility of indeterminacy, such an unambiguous relationship may not exist in the global economic system.

5 Conclusion

The central message of this paper is that the relation between indeterminacy of equilibrium and openness of the economy is sensitive not only to production technologies but also to preference structure. By use of a two-country models with financial integration, we have examined whether or not financial interactions and international trade enhance the possibility of indeterminacy. We have found that, as opposed to the results shown by the existing studies on small-open economies, opening up international transactions does not necessarily yield a higher possibility of indeterminacy. On the contrary, in some cases globalization may serve as a 'stabilizing factor' in the sense that the equilibrium path of the world economy can be determinate even though the autarky equilibrium of each country is indeterminate. Since our finding depends heavily upon the model structure we use in the paper, it may not be appropriate to claim that the global economy is in general less volatile than closed economies. However, our analysis has suggested that the relation between the sunspot fluctuations and openness of the economy may not be so straightforward as the small-open economy models demonstrate, if we consider the general equilibrium of the world economy.

Appendices

Appendix A: Proof of Lemma 1

First consider the optimization behavior of the households in country i. The Hamiltonian function for the household's problem is

$$H_i = u^i(x_i, y_i, l_i) + v_i(r_i\Omega_i + w_i l_i - m_i), \quad i = 1, 2,$$

where $m_1 = x_1 + py_1$ and $m_2 = x_2/p + y_2$. The necessary conditions for the optimization problems are summarized as follows:

$$u_x^1(x_1, y_1, l_1) = v_1, \ \ u_x^2(x_2, y_2, l_2) = v_2/p, \tag{A1}$$

$$u_y^1(x_1, y_1, l_1) = pv_1, \ \ u_y^2(x_2, y_2, l_2) = v_2, \tag{A2}$$

$$u_l^i(x_i, y_i, l_i) = v_i w_i, \quad i = 1, 2, \tag{A3}$$

$$\dot{v}_i/v_i = \rho - r_i, \quad i = 1, 2, \tag{A4}$$

$$\lim_{t\to\infty} v_i e^{-\rho t}\omega_i = 0, \ \ i = 1, 2. \tag{A5}$$

Keeping the non-arbitrage condition (6) in mind, the equations in (**??**) present

$$\frac{\dot{p}}{p} = r_1 - r_2 = \frac{\dot{v}_2}{v_2} - \frac{\dot{v}_1}{v_1}.$$

This shows that the relation between the relative price, p, and the marginal utility of capital, v_i, must satisfy

$$p = \mu\frac{v_2}{v_1}, \tag{A6}$$

where μ is a positive constant. From (**??**) μ satisfies

$$\mu = \frac{u_x^2(x_2, y_2, l_2)}{u_x^1(x_1, y_1, l_1)}.$$

Thus if the economy involves a steady state in which each variable stays constant over time, substituting the steady-state values of x_i, y_i and l_i into the above equation determines the magnitude of μ.

From (A1), (A2) and (A6) we obtain

$$\frac{u_y^1}{u_x^1} = \frac{u_y^2}{u_x^y} = p = \mu\frac{v_2}{v_1}, \quad \frac{u_x^2}{u_x^1} = \frac{u_y^2}{u_y^1} = \frac{v_2}{v_1 p} = \mu. \tag{A7}$$

Now remember that the optimal conditions (12) and (13) for the planning problem yield:

$$\frac{u_y^1}{u_y^1} = \frac{q_1}{q_2} = \frac{u_y^2}{u_x^2}, \quad \frac{u_x^2}{u_y^1} = \frac{u_y^2}{u_y^2} = \frac{\mu_1}{\mu_2}. \tag{A8}$$

Therefore, if we set $q_1/q_2 = p$ and $\mu_1/\mu_2 = \mu$, (A7) and (A8) receptively correspond to (12) and (13) for the planning problem, so that the optimal conditions for the consumption decisions in the market economy and those in the planning economy are exactly the same. In addition, the optimal conditions for labor supply (A3) is identical to (14) in the planning problem and the commodity market equilibrium conditions (4) and (5) are described by (10) and (11), repetitively.

Finally, the solvency conditions for international lending and borrowing requires that

$$\lim_{t\to\infty} b_1(t) \exp\left(-\int_0^t r_1(t)\, ds\right) = \lim_{t\to\infty} b_2(t) \exp\left(-\int_0^t r_2(s)\, ds\right) = 0.$$

Thus the transversality conditions (A5) give $\lim_{t\to\infty} e^{-\rho t} v_{it} k_{it} = 0 \ (i = 1, 2)$. Since we may set $q_i = \mu_i v_i$, conditions in (A5) are equivalent to (16) for the planning problem.

Appendix B: Proof of Proposition 3

The dynamic behavior of the closed economy is described by

$$\dot{k}_i = k_i^{\frac{\alpha_i(\gamma_i+1)}{\gamma_i+1-\beta_i}} q_i^{\frac{\beta_i}{\gamma_i+1-\beta_i}} - \frac{1}{q_i} - \delta_i, \tag{B1}$$

$$\dot{q}_i = q_i \left[\rho + \delta_i - a_i k_i^{\frac{\alpha_i(\gamma_i+1)}{\gamma_i+1-\beta_i}-1} q_i^{\frac{\beta_i}{\gamma_i+1-\beta_i}}\right]. \tag{B2}$$

Linearizing (B1) and (B2) at the steady-state equilibrium, it is easy to see that the trace of the coefficient matrix equals to $\rho\,(>0)$. On the other hand, the coefficient matrix of the linearized system of (33), (34), (26) and (27)is given by:

$$\Lambda(\lambda) = \begin{vmatrix} \lambda - h_{11} & 0 & -h_{13} & -h_{14} \\ 0 & \lambda - h_{22} & -h_{23} & -h_{24} \\ -h_{31} & 0 & \lambda - h_{33} & 0 \\ 0 & -h_{42} & 0 & \lambda - h_{44} \end{vmatrix} = 0,$$

where $h_{13} = \partial x_1/\partial q_1 + \partial x_2/\partial q_1$, $h_{14} = \partial x_1/\partial q_1 + \partial x_2/\partial q_1$, $h_{23} = \partial y_1/\partial q_1 + \partial y_2/\partial q_1$, .and $h_{24} = \partial y_1/\partial q_2 + \partial y_2/\partial q_2$. All of the elements are evaluated at the steady state. The characteristic equation is thus written as

$$\begin{aligned} \Lambda(\lambda) &= \left[\lambda^2 - (h_{11} + h_{33})\lambda + h_{11}h_{33} - h_{13}h_{31}\right] \\ &\times \left[\lambda^2 - (h_{22} + h_{44})\lambda + h_{22}h_{44} - h_{24}h_{42}\right] - h_{14}h_{23}h_{31}h_{42} = 0. \end{aligned} \tag{B3}$$

Given our assumptions, if $h_{14} = h_{23} = 0$, then $\Lambda(\lambda) = 0$ has two positive and two negative roots. Thus if the number of negative roots of $\Lambda(\lambda) = 0$ is higher than three for $h_{14} \neq 0$ and $h_{23} \neq 0$, then the steady state of the world economy is locally indeterminate. Notice that

$$\Lambda(0) = (h_{11}h_{33} - h_{13}h_{31})(h_{22}h_{44} - h_{24}h_{42}) - h_{14}h_{23}h_{31}h_{42},$$

$$\Lambda'(0) = -(h_{11} + h_{33})(h_{22}h_{44} - h_{24}h_{42}) - (h_{22} + h_{44})(h_{11}h_{33} - h_{13}h_{31}).$$

As well as the closed system, if $\gamma_i + 1 > \beta_i$ $(i = 1, 2)$ holds, $h_{11} + h_{33} > 0$ and $h_{22} + h_{44} > 0$. In addition, the saddle-point properties of the closed economies mean that $h_{22}h_{44} - h_{24}h_{42} < 0$ and $h_{11}h_{33} - h_{13}h_{31} < 0$. Hence, $\Lambda'(0) > 0$ and $\Lambda(0) > 0$ for $h_{14} = h_{23} = 0$. This shows that $\Lambda'(\lambda) = 0$ has two positive and one negative roots, and thus when $h_{14} = h_{23} = 0$, the graph of the characteristic equation may be depicted as Figure A1. Therefore, regardless of the sign of $h_{14}h_{23}h_{31}h_{42}$, the characteristic equation, $\Lambda(\lambda) = 0$, has at most two negative roots.

Appendix C: Proof of Proposition 4

Since country 1 has an indeterminate steady state and country 2 has a determinate one, (B3) involves three negative and one positive roots when $h_{14} = h_{23} = 0$. Namely, $\Lambda(0)$ in Appendix B has a negative value if $h_{14} = h_{23} = 0$. Observe that $\Lambda(0)$ can be written as

$$\begin{aligned} \Lambda(0) \;=\; & h_{11}h_{22}h_{33}h_{44} - h_{11}h_{42}h_{24}h_{33} - h_{31}h_{13}h_{22}h_{44} \\ & + h_{31}h_{42}(h_{13}h_{24} - h_{14}h_{23}). \end{aligned}$$

Under our assumption, $\gamma_1 + 1 < \beta_1$ and $\gamma_2 + 1 > \beta_2$, it holds that $h_{31} > 0$ and $h_{42} > 0$. Therefore, if

$$h_{13}h_{24} - h_{14}h_{23} < 0,. \tag{C1}$$

then $\Lambda(0)$ may have a positive sign. As Figure A2 shows, if this is the case, the characteristic equation has two positive and two negative roots so that the steady state of the world economy is determinate. Condition (C1) is equivalent to

$$\left(\sum_{i=1,2} \frac{\partial x_i}{\partial q_1}\right)\left(\sum_{i=1,2} \frac{\partial y_i}{\partial q_2}\right) - \left(\sum_{i=1,2} \frac{\partial x_i}{\partial q_2}\right)\left(\sum_{i=1,2} \frac{\partial y_i}{\partial q_1}\right) > 0,$$

which states that the own price effects on consumption demand dominates the cross price effects.

Appendix D: Proof of Proposition 6

If $\gamma_i + 1 > \beta_i$ and $\sigma_i > 1$, then $\Delta_i(l_i) = (\gamma_i + 1 - \beta_i) - (1 - \sigma_i) l_i^{\gamma_i + 1} > 0$. Assume that the dynamic system has a feasible steady-state equilibrium. Linealized system has the coefficient matrix such that

$$J = \begin{bmatrix} l_1/\Delta_1 & 0 & 0 \\ 0 & l_2/\Delta_2 & 0 \\ 0 & 0 & m \end{bmatrix} \begin{bmatrix} J_{11} & J_{12} & \sigma_1 \kappa_m^1 \\ J_{21} & J_{22} & \sigma_2 \kappa_m^2 \\ \kappa_1^2 - \kappa_1^1 - a_1\beta_1 l_1^{\beta_1} & \kappa_2^2 - \kappa_2^1 + a_2\beta_2 l_2^{\beta_2 - 1} & \kappa_m^2 - \kappa_m^1 \end{bmatrix}, \tag{C1}$$

where

$$\kappa^1(l_1, l_2, m) = l_1^{\beta_1} - \theta_1(1 - a_1) l_1^{\beta_1 - \gamma_1 - 1} - m\theta_2(1 - a_2) l_2^{\beta_2 - \gamma_2 - 1},$$

$$\kappa^2(l_1, l_2, m) = l_2^{\beta_2} - (1 - \theta_1)(1 - a_1)\frac{l_1^{\beta_1 - \gamma_1 - 1}}{m} - (1 - \theta_2)(1 - a_2) l_2^{\beta_2 - \gamma_2 - 1},$$

and $\kappa_i^j = \partial\kappa^j/\partial q_i > 0 \ (i, j = 1, 2)$, $\kappa_m^1 = \partial\kappa^1/\partial m < 0$, $\kappa_m^2 = \partial\kappa^2/\partial m > 0$.and

$$\begin{aligned} J_{11} &= \sigma_1\kappa_1^1 - a_1\beta_1\left[1 - (1 - \sigma_1)(1 - \theta_1)\right] l_1^{\beta_1 - 1}, \\ J_{12} &= \sigma_1\kappa_2^1 - a_2\beta_2(1 - \sigma_1)(1 - \theta_1) l_2^{\beta_2 - 1} > 0, \\ J_{21} &= \sigma_2\kappa_1^2 - a_1\beta_1(1 - \sigma_2)\theta_2 l_1^{\beta_1 - 1} > 0, \\ J_{22} &= \sigma_2\kappa_2^2 - a_1\beta_2(1 - \theta_2 + \theta_2\sigma_2) l_2^{\beta_2 - 1}. \end{aligned}$$

From the determinacy condition for the closed economy, it may hold that $J_{11} > 0$ and $J_{22} > 0$. In view of those sign patterns, we find that

$$\det J < 0 \text{ and } \operatorname{trace} J > 0.$$

This confirms that the dynamic system involves one stable and two unstable roots. Hence, the world economy also satisfies determinacy.

If $\gamma_i + 1 < \beta_i$ and $\sigma_i < a_i$, then $J_{11} > 0$ and $J_{22} > 0$ still may hold. Since $\kappa_j^i \ (i \neq j) < 0$, we obtain $J_{12} < 0$ and $J_{21} < 0$. Remembering that in this case $\Delta^i(l_i) < 0$, we see that

$$\det J > 0 \text{ and } \operatorname{trace} J < 0.$$

This demonstrate that the world economy may observe indeterminate equilibria, even though each closed economy has a unique converging path.

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