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Modeling Social Preferences: A Generalized Model of Inequity Aversion

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Abstract: Taking note of the wide variety and growing list of models in the literature to explain patterns of behavior observed in laboratory experiments, this paper identifies two tests, the Variety Test (ability of a model to explain outcomes under variety or alternative scenarios) and the Psychological Test (ability of a model to conform to psychological intuition), that can be used to judge any model of other-regarding behavior. It is argued that for a mathematical model to qualify as a social welfare function, it must simultaneously pass the two tests. It is shown that none of the models proposed to date passes these two tests simultaneously. The paper proposes a generalized model of inequity aversion which parsimoniously explains interior solution in the dictator game and dynamics of outcomes in other games. The paper postulates that one's idea of equitable distribution is state-dependent, where the state is determined by psychological and structural parameters. The state could be fair, superior or inferior. Individuals in a fair state have zero *equity-bias* and split the pie evenly. Those in a superior (inferior) state have positive (negative) *equity-bias* and value more (less) than fair distribution as equitable distribution. Given psychological tendencies of an individual, every experimental design/structure assigns one of the three states to players which lead to individual-specific valuation of equity. Prediction about outcomes across different experiments and designs can be made through predicting their impact on *equity-bias*. All aspects of an individual's behavior, such as altruism, fairness, reciprocity, self-serving bias, kindness, intentions etc, manifest themselves in the *equity-bias*. The model therefore is all-encompassing.

Key Words: Experimental Economics, Social Preferences, Other-regarding Preferences, Inequity aversion.

1. Introduction:

Economic agents are typically modeled as self-regarding selfish beings whose welfare is unaffected by welfare of others in society. There have been long concerns about validity of the self-regarding assumption and calls to take *other-regarding preferences* seriously in economic theorizing. This assumption has heavily been under attack in recent years and efforts to take *other-regarding preferences* seriously escalated due to laboratory experiments in behavioral economics, particularly the *dictator game* experiment. These experiments suggest that individuals value fairness and most, if not all, behave altruistically and when allowed to do so, some sacrifice their self-interest to supposedly punish unfairness. This, naturally, led theoretical economists to look for theories that could explain the sort of behavior exhibited by subjects in these experiments. The main focus of this area of research has been to identify other-regarding individual specific utility, or social welfare, functions that could explain patterns of behavior, such as fairness and reciprocity, observed in data from laboratory experiments. Literature in this area is expanding rapidly and so is the variety of proposed models. These models include linear models of Fehr and Schmidt (1999), Charness and Rabin (2002), Levine (1998), Rotemberg (2004), and Erlei (2004), and nonlinear models by Bolton and Ockenfels (2000), Cox, Friedman and Gjerstad (2004), Ottone and Ponzano (2005), Cox, and Friedman and Gjerstad (2007).

Noting that altering the utility function allows one to explain just anything, Camerer (2003, p 101) writes

“The goal is not to explain every different finding by adjusting the utility function just so; the goal is to find parsimonious utility functions, supported by psychological intuition, that are general enough to explain many phenomena in one fell swoop, and also make new predictions”

Models proposed in this area of research however haven't been subjected to any formal tests to analyze their ability to explain many phenomenon in one fell swoop and ensure their conformity to psychological intuition. This paper formalizes Camerer's idea in the form of two tests that can be used to judge models of other-regarding preferences. The

two tests are (i) the Variety test (V-Test) and (ii) the Psychological test (P-Test). This paper applies these tests to representative models of the literature and demonstrates that none of the models proposed to date passes these tests simultaneously. They fail to support psychological intuition when put to such a theoretical test, and most are not general enough to explain many phenomena in one fell swoop and do well in experiments of specific designs only. The paper then proposes a generalized model of inequity aversion that passes the two tests simultaneously. This is done through introducing *equity-bias* in the Fehr and Schmidt (1999) model with some other generalization. In the Fehr and Schmidt's model, individuals are inequality averse. This paper highlights the difference between inequity and inequality. It argues that one's idea of an equitable distribution is state-dependent where state is determined by psychological and structural parameters. The state could be fair, superior or inferior. Individuals in a fair state have zero *equity-bias* and split the pie evenly. Those in a superior (inferior) state have positive (negative) *equity-bias* and value more (less) than fair distribution as equitable distribution. *i.e.* bias in state leads to bias, not necessarily with the negative connotation, in equity. Given psychological tendencies of an individual, every experimental design assigns a state to the player which leads individual specific valuation of equity. The model is developed in a two-player environment but results are applicable to games with multiple players competing or otherwise.

Before going into details of the two tests, let me point out that this paper will limit its discussion to the two most frequently used standard games, namely, the *dictator game* (DG) and the *ultimatum game* (UG). In the DG, a player called the *dictator* is given a certain amount of money with the option to share it with another player, the *recipient*, in any proportion, 0 to 100 percent, inclusive. In the UG, the dictator is lowered to the status of a *proposer* and the *recipient* is alleviated to the status of a *responder* who is allowed to either accept or reject a distribution proposed by the proposer. When accepted, each player keeps his/her share and when rejected both players get zero.

(i) The Variety Test (V-Test):

Variety of the laboratory experiments conducted in this area has grown richer and different versions of the games have been experimented with. A reasonable model

should be general enough to perform consistently across (i) different designs of the same game (e.g. across different versions of the dictator game) and (ii) across different games of the same nature (e.g. across the dictator, ultimatum and impunity games). This test has been applied, not under the name V-test though, in the literature, as we will note in our discussion later.

(ii) The Psychological Test (P-Test):

As mentioned earlier, the main focus of the models in this area of research has been to explain patterns of the behavior observed during laboratory experiments. Whether or not a mathematical model that is able to explain outcomes in laboratory experiments conforms to psychological intuition is something that is often ignored and not tested for. The question however to ask is, is there any such test that can be used, together with some other test(s), to give a mathematical model the status of an other-regarding welfare function? Fortunately, there is at least one, Sen's Weak Equity Axiom. Think of two individuals, one normal and the other with a disability. The disabled person is less efficient in converting a dollar into utility relative to the normal person. Psychological intuition tells us that when distributing a given sum, one should be more altruistic towards the disabled person than towards the normal person. This is Sen's Weak Equity Axiom¹. Conformity to Sen's WEA can be checked through the sign of the partial derivative of equilibrium pay-offs with respect to dollar-to-utility conversion efficiency of a player.

Formally, let $V_i = V_i(u(x_i), u(x_j/\chi))$ be the social utility of an individual i where x_i is the pay-off of an individual i , $u(x_i)$ the selfish utility of i from x_i , x_j is the pay-off of individual j and $u(x_j/\chi)$ the selfish utility of individual j from x_j . χ quantifies the inefficiency of individual j in converting x_j into utility relative to i . The larger the value of χ the less efficient j is in converting a dollar into utility. The player's objective is to maximize V_i subject to a constraint (such as $x_i + x_j = N$ where N is size of the pie). Let x_i^* and x_j^* be the equilibrium pay-offs after maximization. Sen's WEA requires $d(x_j^*)/d\chi > 0$.

¹ The disability can be interpreted in general as poor socio-economic status.

In fact the P-test can be thought of as a special case of the V-test. Experiments show that Sen's WEA is satisfied. Garza (2006) performed three different dictator games; the standard dictator game and two versions of the dictator game with poverty where the dictator is informed that their recipients were poor. Garza found that giving in the poverty game was significantly higher than in the standard dictator game.

The rest of the paper is organized as follows. Section 2 provides a review of selected models representative of previous literature and evaluates the performance of the models against the V-test and the P-test. Section 3 provides a discussion on equity vs equality and spells out the concept of *equity-bias*. Section 4 formally introduces the *state-dependent/equity-bias* model of inequity aversion. This section details dynamics of the model in the dictator and ultimatum games. Section 5 discusses application of the model to different versions of the dictator and ultimatum games experiments conducted in the literature. Finally, Section 6 concludes the paper.

2. Review of Previous Models:

This section reviews selected models of other-regarding preferences. These models include The FS model (Fehr and Schmidt (1999)), the BO model (Bolton and Ockenfels 2000) and the CFG model (Cox, and Friedman and Gjerstad 2007). The following justifies selection of the models in this section.

The model in this paper is primarily an extension of the FS model. The paper therefore starts the review with the Fehr and Schmidt (1999) model which is representative of piecewise linear models of inequality aversion. Results of the analysis therefore apply to any model with inequity modeled as inequality, such as Charness and Rabin (2002) and Erlei (2004). Appendix A demonstrates as an example that the FS model, or a restricted version of it, can be derived as a monotonic transformation of Charness and Rabin (2002). The same can be done for other piecewise linear models of the same sort.

As well known, the FS model cannot explain interior solutions in the dictator game. Fehr and Schmidt attribute this flaw to the piecewise linearity of preferences in advantageous inequity. They claimed that modifying their social welfare function to introduce non-

linearity in the advantageous inequity could resolve the issue (Fehr and Schmidt 1999 p. 823). The BO model can be shown to be the FS model with the proposed non-linearity. This is the reason why this model is reviewed as well and will be analyzed in the next section. Although I do not review other nonlinear models, the criticism in this paper is also applicable to other nonlinear models such as Ottone and Ponzano (2005).

Finally, most recently, Cox et al (2007) proposed a parametric model of other-regarding preferences which depends on status, reciprocity, and perceived property rights. This allowed them to make distribution conditional on status, unlike the unconditional distributional preferences in the FS type models. Cox et al (2007) notes a number of recent experiments which compare explanatory power of earlier models, arguing that a large majority of subjects make choices that are inconsistent with unconditional inequality aversion.

Apart from the distributional preference models above, as mentioned by Cox et al (2007), there are alternative models of intention-based reciprocity such as Rabin (1993), Dufwenberg and Kirchsteiger (2004) and Falk and Fischbacher (2001). These alternative models are complex and have many equilibria, and so seem intractable in most applications (Cox. et al 2007). Although I do not review intention-based models, the model proposed in this paper captures dynamics generated by intentions and encompasses this class of models as well. The analysis on Fehr and Schmidt (1999) in this section also applies to Levine (1998) who developed a simple tractable model with intentions where utilities are linear in one's own and the other's pay-offs.

2.1. The FS (Fehr and Schmidt 1999) model:

Fehr and Schmidt (1999) used a simple linear model to explain results of laboratory experiments including the dictator and ultimatum games. They modeled fairness as self-centered inequity- in fact inequality- aversion, whereby people are willing to sacrifice part of their material pay-offs to move in the direction of equality. Assuming two players in the game, their social welfare function of an individual i is linearly increasing in self-pay-off, x_i , and decreasing in advantageous inequality, $x_i - x_j$. *i.e.*

$$V_i = x_i - \beta_i \max(x_i - x_j, 0) - \alpha_i \max(x_j - x_i, 0) \quad 0 \leq \beta_i \leq 1 \quad \text{and} \quad \alpha_i \geq \beta_i \quad (1)$$

α_i is the social marginal utility when disadvantageous inequality ($x_i - x_j < 0$) reduces by 1 unit. Similarly, β_i is the social marginal utility when advantageous inequality ($x_i - x_j > 0$) decreases by 1 unit. $\alpha_i > \beta_i$ implies that a unit increase in disadvantageous inequality hurts i more than a unit increase in advantageous inequality would.

Consider application of the model to the DG. Assuming D and R to be pay-offs of the dictator and recipient respectively, we can write social utility of the dictator, based on equation (1), as:

$$V_D = D - \beta_D \max(D - R, 0) - \alpha_D \max(R - D, 0)$$

The dictator's objective is to maximize V_D subject to $D + R = N$. Figure 1 plots V_D at different values of β_D . The following holds

$$V_D = \begin{cases} ABC & \text{when } \beta_D = 0 \\ ABD & \text{when } \beta_D < 0.5 \\ ABE & \text{when } \beta_D = 0.5 \\ ABF & \text{when } \beta_D > 0.5 \end{cases} \quad \begin{matrix} C < D < E \\ F < E \end{matrix}$$

Let D^* and R^* denote the values of D and R where V_D is maximum. The solution is easy to work out from the graph.

$$\left(\frac{D^*}{N}, \frac{R^*}{N} \right) = \begin{cases} (1, 0) & \text{when } \beta_D = 0 & \text{(point C)} \\ (1, 0) & \text{when } \beta_D < 0.5 & \text{(point D)} \\ \in ([0.5, 1], [0.5, 0]) & \text{when } \beta_D = 0.5 & \text{(line BE)} \\ (0.5, 0.5) & \text{when } \beta_D > 0.5 & \text{(point B)} \end{cases} \quad (2)$$

Notice that the dictator solution in (2) is independent of the value of α_D .

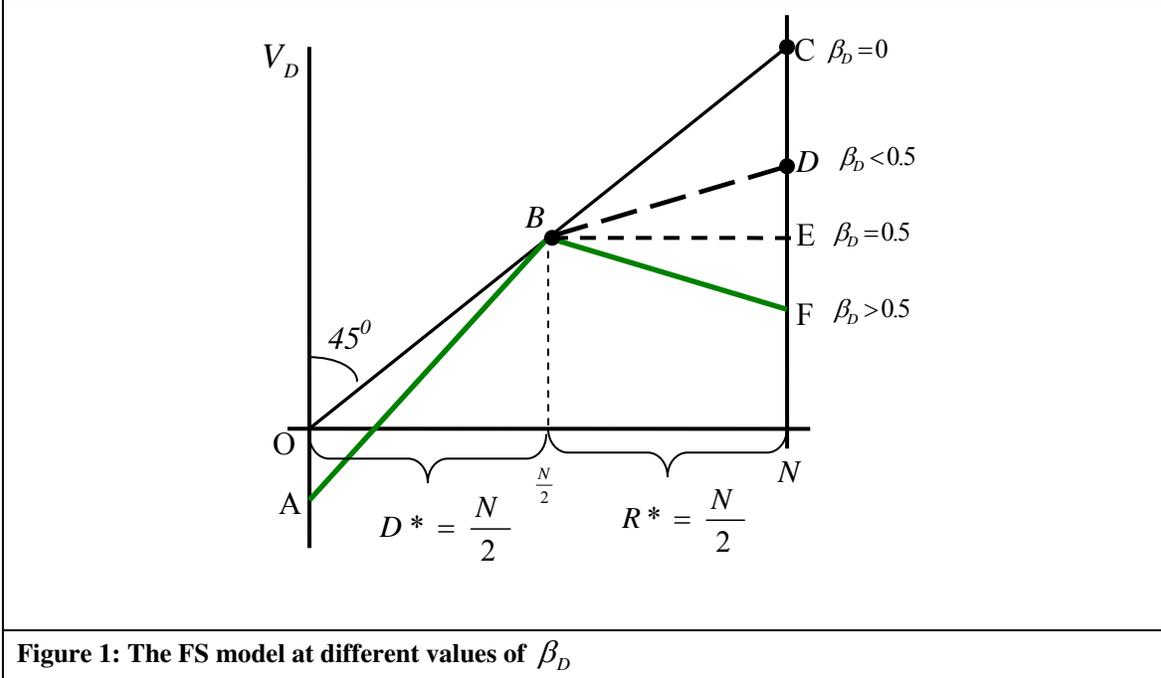


Figure 1: The FS model at different values of β_D

The solution is either $D^*=N$ (with $\beta_D < 0.5$), $D^*=N/2$ (with $\beta_D > 0.5$) or the range $\frac{N}{2} \leq D^* \leq N$ (with $\beta_D = 0.5$) with indifference. The range is not an equilibrium in a strict sense. All it says is that any value within the range is equally good and welfare-maximizing. The FS model can therefore only explain corner solutions ($D=N$ and $N/2$) in the dictator game, leaving the interior unexplained.

Now consider application of the model to the UG. Assuming P and R to be the pay-offs of the proposer and responder respectively, we can write the social utility of the proposer and responder, based on equation (1), as:

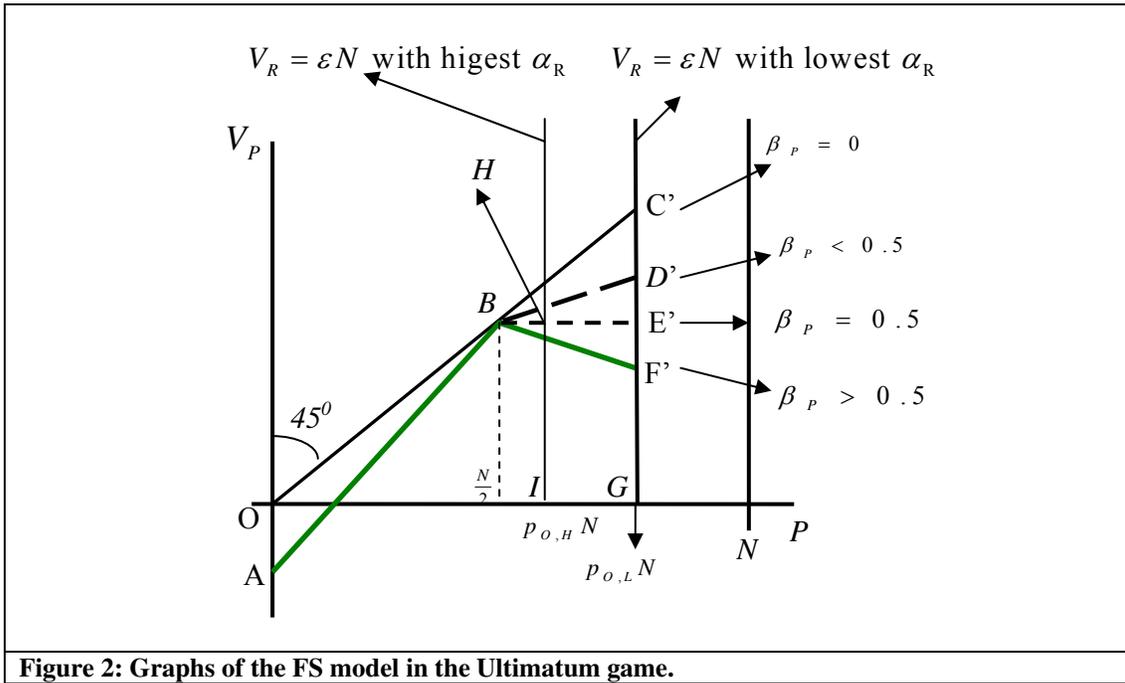
$$V_P = P - \beta_P \max(P - R, 0) - \alpha_P \max(R - P, 0) \quad (3)$$

and
$$V_R = R - \beta_R \max(R - P, 0) - \alpha_R \max(P - R, 0) \quad (4)$$

The proposer's objective is to maximize V_P subject to $P+R=N$ and make an offer to the responder good enough to make the responder's social utility non-negative, *i.e.* the proposer's objective is therefore to maximize V_R subject to $P+R=N$ and $V_R = 0$. This means that in the FS model responders are non-optimizers. They are happy so long as

their social utility is non-negative. $V_R = 0$ requires $R = r_{o,i}N$ or $D = (1 - r_{o,i})N = p_{o,i}N$ where $r_{o,i} = \frac{\alpha_R}{(1 + 2\alpha_R)}$ is the minimum acceptable offer by individual i^2 . Since we may have individuals with different α_R ($0 \leq \alpha_{RH} \leq \alpha_R \leq \alpha_{RH} < \infty$), corresponding to $0 \leq r_{o,L} \leq r_{o,i} \leq r_{o,H} < 0.5$, any offer within the lower and upper bound will be accepted with certain probability. Offers below $r_{o,L}$ will be rejected and those equal to or greater than $r_{o,H}$ will be accepted with certainty. This is depicted in Figure 2. The following holds under different values of β_p when constraints are implemented:

$$V_p = \begin{cases} ABC'GN & \text{when } \beta_p = 0 \\ ABD'GN & \text{when } \beta_p < 0.5 \quad C < D < E \\ ABHIN & \text{when } \beta_p = 0.5 \\ ABF'GN & \text{when } \beta_p > 0.5 \quad F < E \end{cases}$$



² This is derived as follows. Let $r_{o,i}$ be the offer received by the responder, $D = (1 - r_{o,i})N$ and $R = r_{o,i}N$. Substituting this $V_R = 0$ when $P \leq R$, i.e. $R - \alpha_R(P - R) = 0$ gives $r_{o,i} - (\alpha_R - 2\alpha_R r_{o,i}) = 0$ which implies the value of $r_{o,i} = \frac{\alpha_R}{(1 + 2\alpha_R)}$.

The solution lies on the additional constraint $V_R = 0$ when $\beta_p < 0.5$, which implies an equilibrium offer $R^o = r_{o,i}N$, on the line BH with indifference when $\beta_p = 0.5$, and at point B when $\beta_p > 0.5$. The reason why the solution of $\beta_p = 0.5$ excludes points on HE' is that any offer on HE' has some probability of rejection, as a result the individual with $\beta_p = 0.5$ will not be indifferent between BH and HE'. BH will be preferred to HE'.

$$\left(\frac{P^o}{N}, \frac{R^o}{N}\right) = \begin{cases} \left(\in [p_{o,L}, p_{o,H}], \in [r_{o,L}, r_{o,H}]\right) & \text{when } \beta_p \leq 0.5 \\ \left(\in [0.5, p_{o,H}], [0.5, r_{o,H}]\right) & \text{when } \beta_p = 0.5 \quad (\text{line BH with indifference}) \\ (0.5, 0.5) & \text{when } \beta_p > 0.5 \quad (\text{point B}) \end{cases} \quad (5)$$

The solution in UG is similar to the one in the dictator game (compare equation (5) with (2) and Figure 2 with Figure 1). The only difference is that the corners are now determined by the additional constraint $R = r_{o,i}N$ rather than $R=0$. Risk-averse proposers will choose to offer closer to $r_{o,H}$ and risk lovers would offer close to $r_{o,L}$.

Let us call $\beta_p = 0$, selfish-regarding (SR), $0 < \beta_p < 0.5$ not so SR or weakly other-regarding (WOR), $\beta_p = 0.5$ equal regarding (ER) and $\beta_p > 0.5$ other-regarding (OR). The following conclusions inevitably follow from the FS model;

- a) If there was no fear of rejection at $R=0$, the solution according to Fehr and Schmidt's formulation will be exactly the same as the dictator game, the corner solution. This leaves the interior solution unexplained.
- b) When there is fear of rejection the right corner solution on $R = r_{o,i}N$ corresponds to $0 \leq \beta_p < 0.5$ and it is hard to distinguish between SR and WOR individuals. Thus this solution wouldn't tell us whether preferences are weakly other-regarding or selfish.

- c) Knowing that an offer $r=0.5$ will never be rejected, the OR $\beta_p > 0.5$ solution ($r=0.5$) can only be interpreted as an altruistic solution, which is again a corner solution similar to the OR dictator.
- d) Coming to the ER individual, the solution is again a range $[0.5, r_{O,H}]$ with indifference which is not a unique interior solution.

What points (a) to (d) tell us is that, according to the FS model, a solution $r=0.5$ can be interpreted as a solution with altruism if $r_{O,H} < 0.5$, may possibly be interpreted as a solution with fear of rejection (reciprocity) if for some individuals $r_{O,H} = 0.5$. In the first case preferences can be termed as OR whereas in the second case they may or may not be. Given that in the standard dictator experiment an $r \geq 0.5$ is not always observed, $\beta_p > 0.5$ is not dominantly true. This means that the preferences of the proposer are either SR or WOR. *i.e.* $0 \leq \beta_p < 0.5$. Since SR and WOR have the same solution, the use of other-regarding preferences is irrelevant. The problem with the ER solution is obvious. Indifference means any choice is equally good, which is not an explanation. Thus interior solutions in the UG in the FS model can be interpreted as belonging to a selfish (or WOR) proposer with non-optimizing responder. The UG solution of the FS model explains interior offers as a result of fear of rejection, leaving the altruistic part unexplained.

Coming to application of the V-test and the P-test, the FS model does not withstand the V-test. Firstly, it fails to explain interior solutions in the DG. Secondly, it fails to explain outcomes across different designs of the UG. Camerer (2003, p. 112) mentions that if you modify the UG such that if a responder rejects an offer, 2 units are subtracted from all pay-offs, the model predicts that responders should never reject unequal offers. This is because rejection would reduce the pay-off of the responder without any effect on inequality. When individuals are inequality-averse in pay-offs the model automatically fails the P-test. However when we replace inequality aversion in pay-offs by inequality aversion in utility, the P-test is qualified for a special case of equal distribution of utility. This can be ascertained by replacing x_j in the FS model

with $\frac{x_j}{\chi}$ in equation (1), which becomes a special case of the model ($e_i=1$, implying equality in utility) in this paper and passes the P -test, as we shall see later.

2.2. The BO (Bolton- Ockenfels 2000) Model:

The social welfare function of an individual i in Bolton-Ockenfels model is given (ignoring the $x_i+x_j=0$ case) by

$$V_i = ax_i - \frac{b_i}{2} \left(\frac{x_i}{x_i + x_j} - \frac{1}{2} \right)^2 \quad a_i \geq 0 \text{ and } b_i > 0$$

As $x_i + x_j = N$, it can be reduced to

$$V_i = ax_i - \frac{b_i}{8N^2} (x_i - x_j)^2 \quad (6)$$

Notice that V_i is linearly increasing in x_i and non-linearly decreasing in inequality. Recall that Fehr and Schmidt (1999) blamed linearity of the social welfare function in advantageous inequality for the inability of their model to explain interior outcomes in the dictator game. (6) is nonlinear in inequality. The Bolton-Ockenfels (2000) model is therefore the FS model with nonlinear inequality aversion.

Consider application of the model to the DG. The dictator's objective is to maximize (6) subject to $x_i + x_j = N$, which gives

$$x_i^* = \frac{1}{2}N + \frac{a_i}{b_i}N^2$$

$$x_j^* = \frac{1}{2}N - \frac{a_i}{b_i}N^2$$

The first problem with the solution is obvious. Pay-offs are non-linearly related to the size of the pie. Whereas this may be seen desirable at very high values of N ,

experiments with relatively higher N show that outcomes are not too sensitive to the value of N (Camerer 2003).

Whereas this model can explain interior solutions in the standard dictator game, it cannot adequately explain variety of the dictator games played (the V-test). For example consider the DG where individual i and j are given 10 dollars each. The dictator is given the option of either sharing his/her 10 dollars with the recipient or taking some of the recipient's money. The BO model will give the same solution as the standard DG. These games do not give the same results in practice. Similarly, Camerer (2003, p. 111) notes that if you modify the UG such that when the Responder rejects a proposal, the monetary payoffs are 10 per cent of the original offer, relative shares will be the same no matter whether the responder accepts or rejects. Thus the responder will always accept any offer; no matter how unequal it is (since the utility from rejection will be lower than the utility with acceptance). This shows that the model fails to pass the V-test.

It is a bit hard to work out how to implement the P -test here as the model is defined over pay-offs, not utility. One possibility is to think of x_i and x_j as utilities and replace x_j with x_j/χ . The equilibrium value of x_j^* is ambiguously related to χ but most probably decreasing in χ which is against the spirit of psychological intuition.

2.3. The CFG (Cox, Friedman and Gjerstad 2007) Model:

The model is given by

$$V_i = \frac{1}{\sigma} [x_i^\sigma + \theta_i x_j^\sigma] \quad \text{where } \sigma < 1$$

$$\theta_i = \theta_0 + \theta(r, s) + \varepsilon_i, \quad \theta(0, 0) = 0$$

where r is the reciprocity and s is the state variable. When $r=s=0$, $\theta_i = \theta_0 + \varepsilon_i$ which could be positive, negative or zero, implying a benevolent, malevolent or selfish player. Consider application of the game to the dictator game with status. $\theta_i = \theta_0 - as + \varepsilon_i$. $s=1$

when individuals earn property rights to the sum N and 0 if the sum was *mana* from experimental heaven. The dictator's objective is to maximize

$$V_i = \frac{1}{\sigma} \left[(x_i)^\sigma + \theta_i (N - x_i)^\sigma \right] \text{ which gives } x_i^* = \left[\frac{1}{1 + \theta_i^{\frac{1}{1-\sigma}}} \right] N \text{ and } x_i^* = \left[\frac{\theta_i^{\frac{1}{1-\sigma}}}{1 + \theta_i^{\frac{1}{1-\sigma}}} \right] N.$$

Notice that, depending upon the value of θ_i , the model explains the corner as well as interior solution observed in laboratory experiments.

Empirical estimates of the model in Cox et al (2007) yield $\sigma < 1$ and $a > 0$. This implies that $\frac{dx_i}{ds} > 0$. Intuitively the dictator decreases his/her weight θ_i on utility of the recipient when $s=1$. This implies that the model passes the V-test.

Cox et al (2007) empirically estimate the value of σ and find that it is >0 and <1 in all empirical estimations. One concern, though, is that the estimated value of σ changes in these estimations, which needs to be justified. To see if the model passes the P-test at the at Cox et al (2007) estimated value of σ , let us consider an alternative scenario where the recipient is less efficient in deriving satisfaction from a unit of pay-off than the dictator. The model can be reproduced as:

$$V_i = \frac{1}{\sigma} \left[(x_i)^\sigma + \theta_i \left(\frac{N - x_i}{\chi} \right)^\sigma \right]$$

Maximization yields $x_i^* = \left[\frac{\chi^{\frac{\sigma}{1-\sigma}}}{\chi^{\frac{\sigma}{1-\sigma}} + (\theta_i)^{\frac{1}{1-\sigma}}} \right] N$ and $x_j^* = \left[\frac{(\theta_i)^{\frac{1}{1-\sigma}}}{\chi^{\frac{\sigma}{1-\sigma}} + (\theta_i)^{\frac{1}{1-\sigma}}} \right] N$. Note that

$\frac{dx_j^*}{d\chi} < 0$ when $0 < \sigma < 1$, implying that the dictator will be less altruistic towards the less efficient individuals, which is the opposite of what psychological intuition would

predict³. Thus the model fails the P -test at the estimated value of σ . Qualification of the P -test requires $\sigma < 0$. A special case of $\sigma < 0$ is when $\sigma = -\infty$, the perfect complement case. This is further discussed in Appendix B.

It will become clear later on that the CFG model with $\sigma < 0$ is a special case of the model proposed in this paper.

3. Equity-bias: Equity vs Equality:

Most of the inequity aversion models, with rare exceptions such as CFG, are models of inequality aversion than inequity aversion. Fehr and Schmidt (1999) note:

“The determination of the relevant reference group and the relevant reference outcome for a given class of individuals is ultimately an empirical question. The social context, the saliency of particular agents, and the social proximity among individuals are all likely to influence reference groups and outcomes. Because in the following we restrict attention to individual behavior in economic experiments, we have to make assumptions about reference groups and outcomes that are likely to prevail in this context. In the laboratory it is usually much simpler to define what is perceived as an equitable allocation by the subjects. The subjects enter the laboratory as equals, they do not know anything about each other, and they are allocated to different roles in the experiment at random. Thus, it is natural to assume that the reference group is simply the set of subjects playing against each other and that the reference point, i.e., the equitable outcome, is given by the egalitarian outcome” (Fehr and Schmidt 1999, pp 821-22)

This is where root of the problem lies, as it is assuming too much. Once the experimenter assigns subjects into different roles, through whatever procedure, and gives them unequal property rights, they cease to be equal. For example, a dictator with all the power to give something or nothing to another player is not equal to the passive recipient who has no claim over the sum to be divided. Similarly individuals come from different socio-

³ This points towards issues related to the estimation procedures adopted in the paper, which is worth investigating but is outside the scope of the paper.

economic backgrounds and experiences. Thus it is unreasonable to assume that other-regarding individuals are universally inequality averse irrespective of context. A reasonable postulation would be to think of one's idea of equitable distribution as state-dependent, where the state is determined by psychological and structural parameters. Whereas the psychological parameters include one's valuation (in terms of socio-economic status) of one's self relative to other in the society and to those in the experiment and one's perception about behavior of others in the experiment (kind, selfish etc)⁴, the structural parameters mainly relate to design of the experiment (such as how are different roles and property rights allocated, information about the socio-economic status of players, wording of the experiment, role of the experimenter etc).

Let us define three different states, Fair (F), Superior (S), and inferior (INF). Given individual-specific psychological tendencies/parameters, every experimental design assigns one of the three states to players. When assigned a fair state, one's idea of equity is a fair one (*i.e.* equal distribution); when assigned a superior state (S) or inferior (INF) states, one's idea of equity is a biased one. Let $E_i = x_i/x_j$ be the measure of equity of an individual i over x_i (own pay-off) and x_j (other's pay-off). When i is assigned a fair state, F, his/her idea of an equitable state is a fair one, *i.e.* $E_i=1$. However, when assigned a biased state, S or INF, he/she is emotionally locked into choosing a biased $E_i=1+b_i$ where b_i quantifies *equity-bias*. In state S $b_i > 0$ and i values more than fair distribution as equitable distribution. Similarly, $b_i < 0$ in state INF and i accepts less than fair distributions as equitable. In general $E_i = 1 + b_i$ where

$$b_i = \begin{cases} 0 & \text{when in state F} \\ \infty \geq b_i \geq 0 & \text{when in state S} \\ -1 \leq b_i \leq 0 & \text{when in state INF} \end{cases} \quad (7)$$

For example, in the DG, a dictator is assigned a state superior than the one assigned to the recipient. The dictator owns all the money and is assigned the right to use it as he/she pleases. The recipient is neither a party to the “production” of value nor legally entitled to

⁴ This valuation is of course subject to behavioral instincts that are either hard-wired in human nature (such as self-serving bias) or evolved/learned as part of ones culture or environment.

have any share of the money⁵. Thus, when behaving altruistically, he/she doesn't find it equitable to split the pay-offs, or utility, equally. Competition basically changes the relative location of players, and hence their idea of equitable share. Similarly, a recipient in the ultimatum game is assigned an inferior state, and may accept less than fair offers as equitable. This plays an important role in explaining dynamics of the outcomes in variety of the two games, and other games of the same sort, as we shall see later.

4. The Model:

With the above discussion in mind, let us write a generalized model of inequity aversion.

Let

x_i = pay-off of individual i

$u_i(x_i)$ = idiosyncratic selfish utility function of individual i from own pay-off x_i

$u_{j,i}(x_j)$ = selfish utility of individual j as perceived by individual i from pay-off of individual j x_j .

e_i = the equitable distribution of utility as perceived by individual i

Social utility of the individual i , V_i , is assumed to be increasing in $u_i(x_i)$ and decreasing in advantageous inequity $I_i = u_i(x_i) - e_i u_{ji}(x_j)$. i.e.

$$V_i = u_i(x_i) - \beta_i \max[u_i(x_i) - e_i u_{ji}(x_j), 0] \quad (8)$$

where $u'_i > 0$, $u'_{ij} > 0$, $u''_i \leq 0$, $u''_{ij} \leq 0$, $\beta_i = \frac{\gamma_i}{1 + e_i \left(\frac{u'_{ij}}{u'_i} \right)}$ and $\gamma_i > 1$.

The restrictions on the first and second derivatives of the idiosyncratic selfish utilities imply diminishing/constant marginal utility of money. β_i is the social marginal utility of a

⁵ Even though it is possible for him/her to think of the sum as a result of the experiment to which the other player is also a party to some extent, which morally entitles him/her to some share.

unit decrease in advantageous inequity referred to as the *social beta* of individual i . The logic behind the value of β_i and the restriction on γ_i will be justified later in the paper. It might however be helpful to point out that this restriction ensures that social utility is maximum at the equitable distribution of pay-offs. In the FS model this restriction was $\beta_i > 1/2$, which in our notation is equivalent to $\beta_i = \frac{\gamma_i}{2}$ with $\gamma_i > 1$. The restriction on β_i

could be replaced with $\beta_i > 1$ (as $\beta_i > 1$ always implies $\beta_i > \frac{1}{1 + e_i \left(\frac{u_{ij}}{u_i} \right)}$, see Appendix D

for proof). In order to understand the dynamics generated by the value of social beta and its interpretation, I will stick to the general value of social beta as above.

Let us further assume:

$$u_i(x_i) = f(x_i) = x_i^k \quad (9)$$

$$u_{ji}(x_j) = g(x_j) = f\left(\frac{x_j}{\chi_i}\right) = \left[f\left(\frac{1}{\chi_i}\right)\right] \left[f(x_j)\right] = \left(\frac{x_j}{\chi_i}\right)^k \quad (10)$$

where $\chi_i > 0$ and $0 < k \leq 1$

$f(x_i)$ is the preference technology of individual i , which converts x_i into utility and $g_i(x_j)$ is the preference technology of individual j , as perceived by individual i , which converts x_j into the utility of j . The fact that the preference technology f may not necessarily be equal to g captures dynamics of differences in socio-economic status of players that leads to *equity-bias* in pay-offs. In the specification in equation (10), χ_i embodies individual i 's belief about the socio-economic status of individual j relative to his/her own socio-economic status. For example, when individual i believes he/she is socially better off than individual j (say because of j 's disability or because j is poor relative to i), χ_i is greater than 1. $\chi_i > 1$ implies that i believes that j is less efficient in deriving satisfaction from a dollar than i him/herself or a dollar given to j generates lesser

utility than it does to i (vise-versa for $\chi_i < 1$). Appendix C derives the utility function in equation (8) as a monotonic transformation of a social utility function that explicitly models differences in socio-economic status⁶.

Substituting (9) and (10) in (8) gives

$$V_i = (x_i)^k - \beta_i \max \left[(x_i)^k - E_i (x_j)^k, 0 \right] \quad \beta_i = \frac{\gamma_i}{1 + (E_i)^{\frac{1}{k}}}, \gamma_i > 1 \quad (11)$$

$$\text{where } E_i = e_i \left(\frac{1}{\chi_i} \right)^k$$

Recall that the restriction on β_i ensures that social utility has a maximum, at the state-dependent equitable distribution. Notice that the *social beta* is inversely related to E_i , implying that for a relatively larger *equity-bias*, a relatively lower *social beta* is required for the social utility to have a maximum.

The players' objective is to maximize (11) subject to $x_i + x_j = N$. With the restriction on γ_i (see Appendix D) the problem reduces to minimization of the deviation of the inequity gap from zero, $I_i = (x_i)^k - E_i (x_j)^k$, which occurs at $I_i = 0 \Rightarrow x_i = (E_i)^{\frac{1}{k}} x_j$. Thus equilibrium is the solution to the following two equations:

$$I_i = 0 \text{ equation} \quad x_i = (E_i)^{\frac{1}{k}} x_j \quad (12)$$

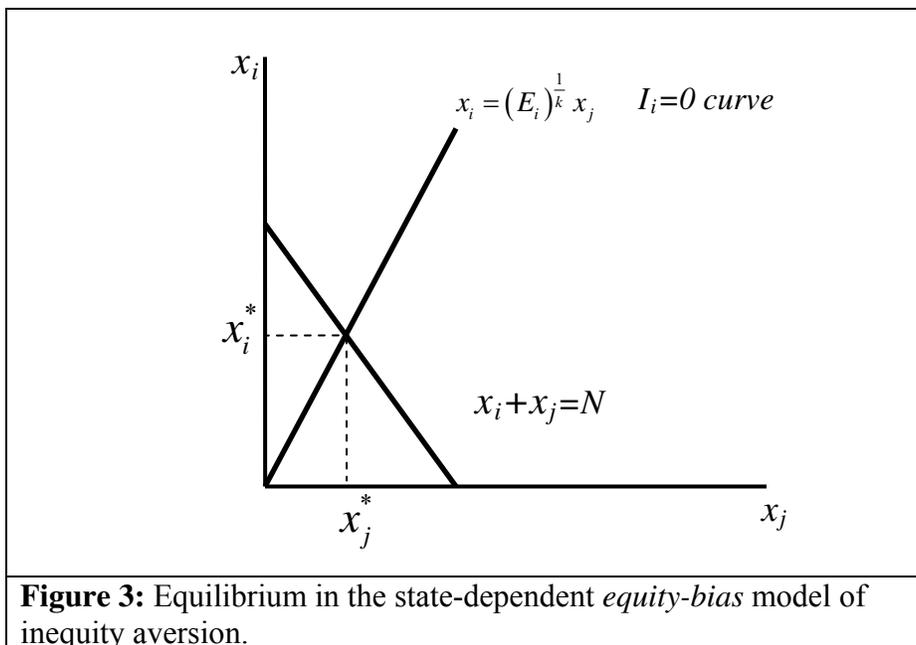
$$\text{and the budget constraint} \quad x_i + x_j = N \quad (13)$$

$$\text{Solving (12) and (13) gives} \quad x_i^* = \left[\frac{(E_i)^{\frac{1}{k}}}{1 + (E_i)^{\frac{1}{k}}} \right] N \text{ and } x_j^* = N - x_i^* = \left[\frac{1}{1 + (E_j)^{\frac{1}{k}}} \right] N$$

⁶ Note that laboratory experiments are conducted in a controlled environment; it is possible for the impact of these problems to be minimized, if not completely avoided. When experiments are blind or double blind, self-serving bias may still be at play.

The solution is simply the intersection of $I_i = 0$ curve and the budget constraint, as shown in Figure 3 below.

This equilibrium solution can also be arrived at using social utility or welfare functions taking a CES (Constant Elasticity of Substitution) form under conditions required to ensure conformity to the P -test. Appendix B derives and discusses the issue in detail.



$(E_i)^{\frac{1}{k}}$ is the equitable distribution of pay-offs (not utility) as perceived by individual i . The value of this is determined by *equity-bias*. When an individual is assigned a fair state, the value of E_i is equal to 1, when assigned a superior state $E_i > 1$ and when assigned an inferior state $E_i < 1$. E_i can be related to *equity-bias*, b_i , thus:

$$E_i = 1 + b_i$$

$$b_i = e_i \left(\frac{1}{\chi_i} \right)^k - 1$$

Without loss of generality k can be assumed to be equal to 1, which implies a constant marginal utility of money. It might be worth mentioning, however, that if desirable, one

can make k a function of N where k declines over income, continuously or continually, which will lead to decrease in offers when a player is in a superior state ($E_i < I$), and increase in offers when a player is in inferior state ($E_i < I$) when size of the pie increases.

From here onwards, I will assume $k=I$ without loss of generality.

Speaking of the two tests, the model passes the V-test as every design manifest itself in a unique value of *equity-bias*, which leads to a design or variety specific solution. Thus the model is general enough to accommodate variety in experiments and design. Also notice that $\frac{dx_j^*}{d\chi_i} > 0$, which is consistent with prediction of the P-Test.

As a passing note, sometimes it might be easier to work with another version of the model as below (assume $k=1$):

$$V_i = x_i - \frac{\gamma_i}{2} \max[x_i - x_j - \tilde{b}_i N, 0] \quad \gamma_i > 1$$

$$\tilde{b}_i \in [-1, 1]$$

Maximization subject to $x_i + x_j = N$ gives $x_i = 0.5(1 + \tilde{b}_i)N$ and $x_j = 0.5(1 - \tilde{b}_i)N$, where \tilde{b}_i captures *equity-bias*. $\tilde{b}_i = 0$ means no *equity-bias*, $\tilde{b}_i > 0$ means positive *equity-bias* and $\tilde{b}_i < 0$ means negative *equity-bias*. The paper will mainly stick to the specification in (11)

4.1 Application to the dictator game:

Let D be pay-off of the dictator, and R be pay-off of the recipient. Based on equation (11), social utility of the dictator can be written as ($k=I$)

$$V_D = D - \beta_D \max[D - E_D R, 0] \quad (14)$$

where $E_D = 1 + b_i$, $\beta_D = \frac{\gamma_D}{1 + E_D}$ and $\gamma_D > 1$.

The dictator's objective is to maximize (14) subject to $D+R=N$. Equilibrium occurs at

$$d^* = \frac{D^*}{N} = \left[\frac{E_D}{1+E_D} \right]$$

Figure 4 plots V_D at different values of γ_D . The following holds for the upper panel:

$$V_D = \begin{cases} OAB & \text{when } \gamma_D = 0 \\ OAC & \text{when } 0 < \gamma_D < 1 \quad D < C < B \\ OAD & \text{when } \gamma_D = 1 \\ OAE & \text{when } \gamma_D > 1 \quad E < D \end{cases}$$

The solution can be summarized thus:

$$\left(\frac{D^*}{N}, \frac{R^*}{N} \right) = \begin{cases} (1, 0) & \text{when } \gamma_D = 0 \quad (\text{point B}) \\ (1, 0) & \text{when } 0 < \gamma_D < 1 \quad (\text{point C}) \\ (\in [d^*, 1], \in [1-d^*, 0]) & \text{when } \gamma_D = 1 \quad (\text{on line AD with indifference}) \\ (d^*, 1-d^*) & \text{when } \gamma_D > 1 \quad (\text{on point E}) \end{cases}$$

where $d^* = \frac{E_D}{1+E_D}$.

As shown in Figure 4 (b), equilibrium requires $I_D=0$. Figure 6 plots the equilibrium in I_D and D space. The indifference curves have slope $=\beta_D$ and the budget constraint $D+R=N$ is rewritten in I_D and D space, which gives $(1+E_D)D - I_D = E_D N$. Since, $\gamma_D > 1$, indifference curves are steeper than the budget constraint and the equilibrium is at the left corner at D^* where $I_D=0$. Notice that D^* varies with E_D . Similarly, when $\gamma_D < 1$ (not shown in the figure) indifference curves will be flatter than the budget constraint and the solution will lie on the right corner, where $D^*=N$. $\gamma_D=1$ is the solution with indifference.

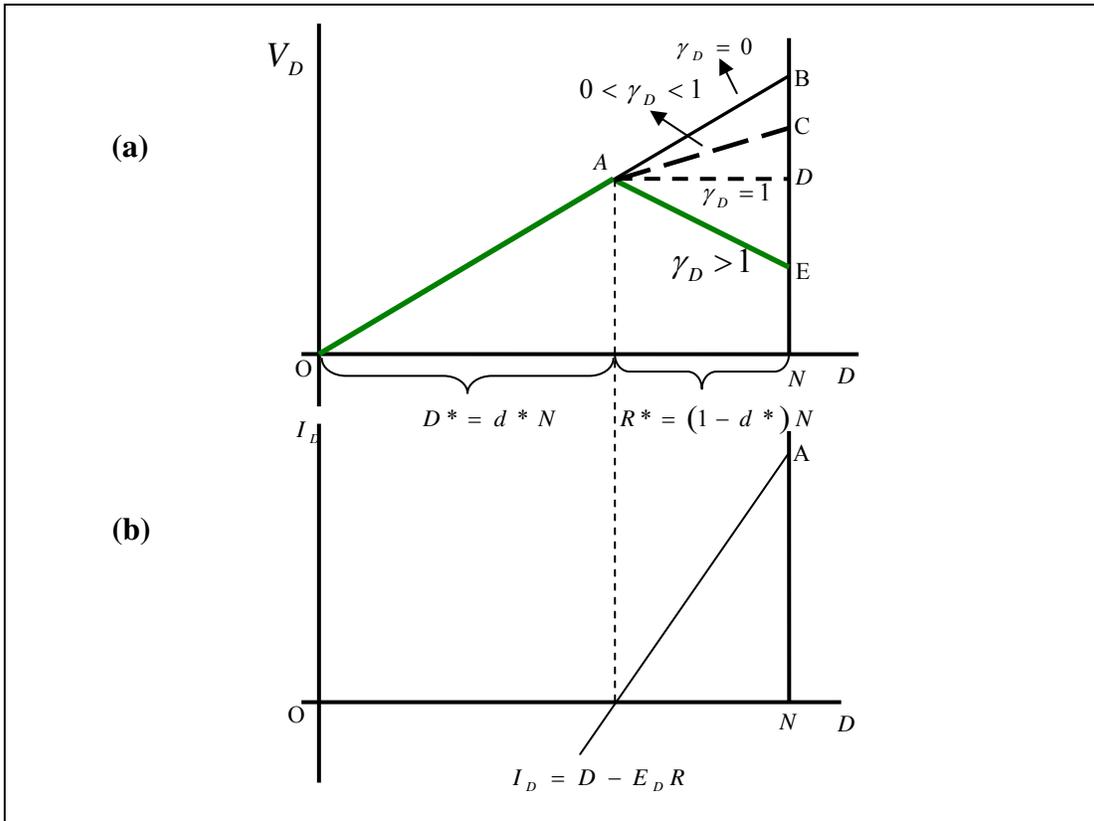


Figure 4 : (a) plot of V_D at different values of α_D . The following holds

$$V_D = \begin{cases} OAB & \text{when } \gamma_D = 0 \\ OAC & \text{when } 0 < \gamma_D < 1 \\ OAD & \text{when } \gamma_D = 1 \\ OAE & \text{when } \gamma_D > 1 \end{cases} \quad \begin{matrix} D < C < B \\ \\ \\ E < D \end{matrix}$$

(b) Plot of the dictator's inequity $I_D = D - E_D R$.

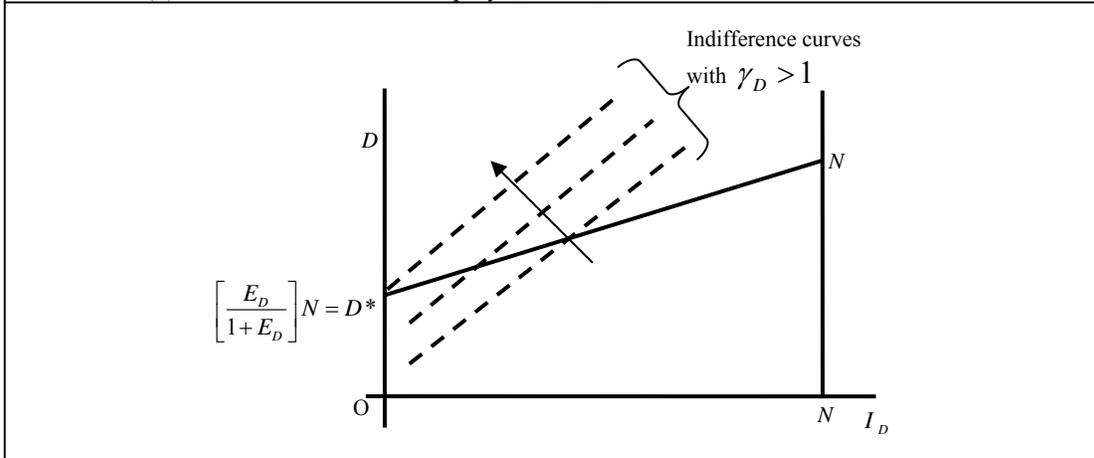


Figure 5 : Equilibrium depicted in $I_D = D - E_D R$ and D space with $\gamma_D > 1$. D^*N is the budget constraint in I_D and D space and the dotted lines are indifference curves.

Theoretically γ_D can assume any value. Recall that we restricted it to be greater than 1 ($\gamma_D > 1$) in the model. This is because it ensures unique interior solutions observed in the laboratory experiments at all values of k^7 . In fact, uniqueness of the solution requires $\beta_D > \frac{1}{1+(E_D)^{\frac{1}{k}}}$, which is ensured by $\gamma_D > 1$ when $\beta_D = \frac{\gamma_D}{1+(E_D)^{\frac{1}{k}}}$ ($\beta_D = \frac{\gamma_D}{1+E_D}$ when $k=1$). This is why we assumed this value of *social beta* in the model and restricted γ_D to be greater than unity. This is proved in proposition 2 below.

Proposition 1: *The equilibrium offer, $1-d^*$, made by the dictator varies with state-dependent equity-bias and can be anywhere between 0 and 1 inclusive.*

Proof: As $1-d^* = \frac{1}{1+E_D} = \frac{1}{2+b_D}$. Theoretically, the *equity-bias* can take any positive or negative value. Thus, when the dictator's valuation of equity is infinitely biased, the offer will be equal to zero (i.e. $b_D = \infty \Rightarrow (1-d^*) = 0$). When the dictator's valuation is negatively biased, say $b_D = -1$, the offer is equal to 1; $b_D = 0$ means the dictator is in a fair state and splits the sum fairly, i.e. half-half. The less than infinitely *equity-biased* dictator in a superior state (i.e. $0 < b_D < \infty$) will give offers in the interior $0.5 > (1-d^*) > 0$ and the dictator in an inferior state with $-1 < b_D < 0$ will lead to the interior solution in $0 > (1-d^*) > 0.5$. Notice that I mainly define states and bias in terms of pay-offs. It is possible for these states to be different in terms of utility. For example, consider a fair state in utility (i.e. $e_i = 1$), which implies $E_i = \left(\frac{1}{\chi_i}\right)^k$ (Since $E_i = e_i \left(\frac{1}{\chi_i}\right)^k$). E_i could be less than, equal to or > 1 (depending upon whether χ_i is greater, equal to, or less than one) which corresponds to positive, zero, and negative equity bias respectively. Thus one can

⁷ $\gamma_D > 1$ will give unique solution for all values of k , but $\gamma_D = 1$ will also give a unique solution when $0 < k < 1$. This is formally derived in Appendix D.

think of a dictator belonging to a superior state in utility and possibly belonging to any of the three states in pay-offs.

Proposition 2: Any $0 \leq d^* \leq 1$ is a unique equilibrium with $\beta_D > (1 - d^*)$

Proof: For d^* to be a unique equilibrium γ_D has to be greater than 1. Recall that

$$\beta_D = \frac{\gamma_D}{1 + E_D}. \quad \gamma_D > 1 \Rightarrow \beta_D > \frac{1}{1 + E_D} \Rightarrow \beta_D > (1 - d^*). \quad \text{The interpretation of this condition}$$

is straight forward. The dictator will keep reducing his/her own pay-offs so long as it increases his/her social-utility more than it hurts his/her selfish utility.

The model clearly explains the corner as well as interior solutions observed during experiments. This model acknowledges that individuals are heterogeneous and differ in their valuation of equity; hence their social utility is maximized at different offers. This leads them to optimally offer different amounts.

4.2 Application of the model to the Ultimatum Game:

Now consider application of the model to the standard ultimatum game experiment. Let P be pay-off of the proposer and R be pay off of the responder. The utility function of the proposer and the responder can be written (using equation (11) with $k=1$) as:

$$V_p = P - \beta_p \text{Max}(P - E_p R, 0) \quad (15)$$

$$V_r = R - \beta_r \text{max}(R - E_r P, 0) \quad (16)$$

The proposer's objective is to maximize (15) subject to $P+R=N$. The proposer's offer

will be equal to $R^o = \left(\frac{1}{1 + E_p} \right) N$, leaving the responder an amount $P^o = \left[\frac{E_p}{1 + E_p} \right] N$. If

this offer is accepted by the responder, both will keep the positive sum; if rejected both will end up getting zero. The responder will accept the offer if it is greater than or equal to his/her social maximum. His/her social utility is maximum where his/her inequity $I_R = R - E_r P$ is =0. Thus the responder's social utility will be maximum at

$R^A = \left[\frac{E_R}{1 + E_R} \right] N$, implying $P^A = \left[\frac{1}{1 + E_R} \right] N$, where the superscript A is added to denote the minimum acceptable offer level.⁸ Unlike the FS model (where responders are non-optimizers and are happy with any offer that makes their social utility non-negative), here responders optimize their social utility and accept offer R^O if and only if $R^O \geq R^A$, where R^A comes from maximization of his/her social utility. In case the offer is below his/her equitable minimum, he/she will reject the offer. This value could be the FS minimum acceptable offer or more than that depending upon the responder's valuation of equitable distribution).

Thus

$$s = \begin{cases} 1 & \text{Accept offer when } R^O \geq R^A \\ 0 & \text{Reject offer when } R^O < R^A \end{cases}$$

The solution will therefore be

$$\left(\frac{P^A}{N}, \frac{R^A}{N} \right) = \begin{cases} (p^*, 1 - p^*) & \text{when } R^O \geq R^A (s = 1) \\ 0 & \text{when } R^O < R^A (s = 0) \end{cases}$$

where $0 \leq p^* = \left[\frac{E_p}{1 + E_p} \right] \leq 1$.

Since in practice we may have agents with different *equity-biases*, let us capture this heterogeneity by expressing P^A of an individual of type n ($=1, \dots, n$) by P^{An} , which is distributed with support P^{AH} and P^{AL} . P^{AH} belongs to a responder with the highest minimum acceptable offer (R^{AH}) and P^{AL} belongs to a responder with the lowest minimum acceptable offer (R^{AL}). Thus the outcome of the experiment will depend on whom the proposer is playing with. Consider Figure 6, where the proposer's utility is maximum at point A, where $P^O = OE$ and $R^O = NE$. The equilibrium is:

⁸ It is possible for some players to find $R^O > R^A$ offensive and reject them. I ignore such a possibility in discussions but the model does allow for such interpretations and solution.

$$\left(\frac{P^*}{N}, \frac{R^*}{N}\right) = \begin{cases} (p^*, 1-p^*) & \text{when } R^A \in [NF, NE] \\ (0,0) & \text{when } R^A \in [NE, NC] \text{ excluding point E} \end{cases}$$

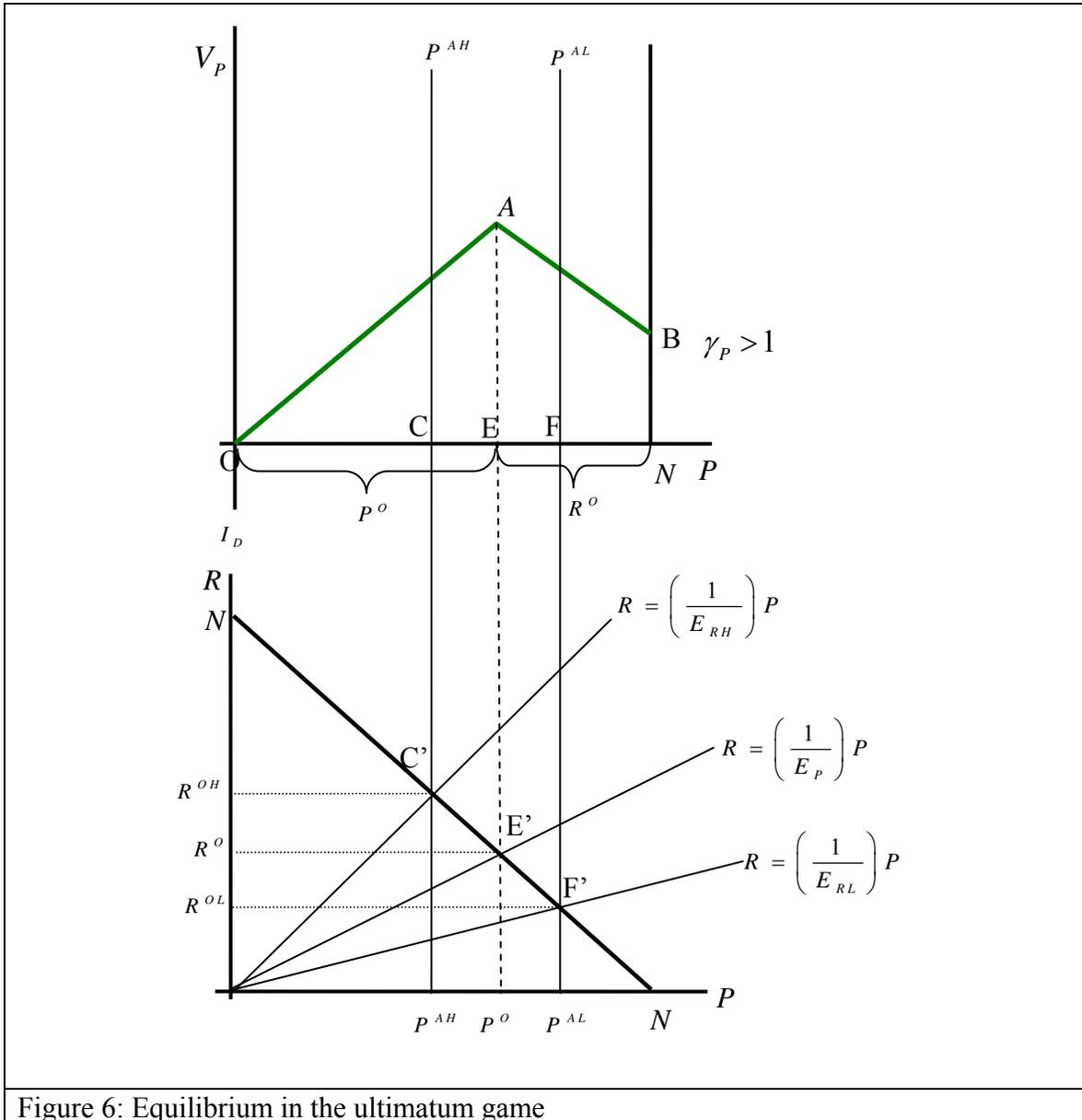


Figure 6: Equilibrium in the ultimatum game

Of course, when $E=C$ the proposer's equitable offer will always be accepted irrespective of type of the responder. The $(0,0)$ equilibrium implies that the proposer does not offer more than NE because he/she thinks it is inequitable and should be accepted by any responder; if not, he/she is happy to face the consequences. The responder, on the other hand, rejects the offer, believing that it is not equitable, and chooses to be worse-off than

accepting the inequitable distribution. The main reason for this rejection is that the offer is less than the minimum acceptable threshold. This rejection could be motivated by reciprocity, intentions, social punishment, self-assertiveness or any other reason. When rejections inflict monetary loss to proposers (as in the ultimatum game), all of these reasons could be in play. Impunity games narrow down the list to social punishment and self-assertiveness. See Appendix E for impunity and private impunity games.

Proposition 3: Positive equilibrium offers will always occur where

- (i) a proposer locked into a superior state (positive *equity-bias*) is matched with a responder who is locked into an inferior state (negative *equity-bias*) and vice versa or
- (ii) both are locked into a fair state.

Proof: Using our concept of *equity-bias* we postulated that $E_i = 1 + b_i$ with $b_i > 0$ in a superior state, < 0 in an inferior state and $= 0$ in a fair state. Substituting this in the solution, we get (assume $k=1$ for simplicity)

$$1 - p^* = \frac{R^O}{N} = \left(\frac{1}{2 + b_p} \right) N$$

$$1 - p^A = \frac{R^A}{N} = \left(\frac{1 + b_R}{2 + b_R} \right) N$$

Equilibrium requires $1 - p^* \geq 1 - p^A \Rightarrow b_R \leq -\frac{b_p}{(1 + b_p)}$. Thus when the proposer is in

superior state ($b_p > 0$), the responder will be in an inferior state ($b_R < 0$) which is what is implied by proposition 3(i). Similarly when the proposer is locked into a fair state $b_p = 0$ the responder must also be locked into a fair state ($b_R = 0$) for an equilibrium pay-off to be positive. Intuitively, the proposer's bias is defined over D/R ; thus a superior state leads to a D/R greater than 1. The responder bias is defined over R/D . For $D/R > 1$, R/D must be < 1 , which implies negative bias for the responder. Equilibrium also of course requires that for b_p of proposer's *equity-bias*, the responder's bias must be at least $-\frac{b_p}{(1 + b_p)}$.

Proposition 3 assumes that this condition holds.

This implies that every distribution in the ultimatum game with acceptance can be explained by a proposer and responder match that satisfies $b_R \leq \left(\frac{-b_P}{1+b_P} \right) \Rightarrow (1-p^O) \geq (1-p^A)$, and any equilibrium with rejection ($P=R=0$) can be explained as a punishment by the responder whose $b_R > \left(\frac{-b_P}{1+b_P} \right) \Rightarrow (1-p^O) < (1-p^A)$.

5. Application of the Model:

As mentioned earlier, there are a variety of dictator and ultimatum games experimental economists have tested in the laboratory. Even though solutions of the model in this paper were derived for the standard dictator and ultimatum games, they can easily be applied to different designs, contexts and multiple player versions of these games. All we need to do is work out what impact would a particular innovation to an experimental design have on the relative state of players and for that matter their *equity-bias* and predictions about the solution follow intuitively⁹. The notion of *equity-bias* is general in nature and embodies all information related to socio-economic status, intentions, reciprocity, social distance, design of the experiment role of the experimenter, etc. The model therefore provides a unified framework to understand the outcomes of research in a broader context.

In the standard dictator game, the state assigned to a dictator is superior than the one assigned to the receiver. The recipient is neither a party to the creation of value, nor legally entitled to any share in the sum. Thus it seems natural for the dictator to have a notion of equity that is biased towards his/her welfare and make offers in the interior. The ultimatum game introduces two changes to the dictator game. Firstly, it lowers the status of the dictator to a relatively inferior position by assigning him/her the role of a proposer. Secondly, it alleviates the status of the recipient to that of a responder who becomes an active partner to the creation of value. This arrangement leads to a reduction in *equity-*

⁹ Understandably, it may not be possible in some cases to predict what impact a particular innovation to an experimental design would have on the relative state of a player beforehand; experimental results could be used to understand the dynamics of such innovations on *states*.

bias of the dictator (now the proposer) and an improvement in *equity-bias* of the recipient (now responder). In a one-shot ultimatum game the proposer is in a relatively superior state than the responder (by just being the first mover), which allows for the possibility of less than fair splits as equitable offers. In a repeated ultimatum game their relative states converge to a fair state and the offers converge to an even split. Rejections in the ultimatum game occur when proposers make offers less than what is minimally acceptable to the responders as an equitable solution. There could be many reasons behind these rejections (to punish the proposer, reciprocity, social punishment, symbolic expression of anger, self-assertiveness etc), which can be isolated through experiments.

The wording of the experiment and instruction list (Gary E. Bolton *et al.* 1998; Elizabeth Hoffman *et al.* 1999), identity of the experimenter, social distance, and design of the experiment seem to have an impact on the outcomes (Hoffman *et al.* (1994a, b). All these factors change the relative positions of players in the game, and hence their *equity-bias*, which show up in their offers and changes in the threshold for accepting or rejecting offers. Hoffman *et al.* (1994), for example, found reduction in offers by dictators when anonymity was increased, implying that anonymity increases *equity-bias*. This means that part of the altruism in the dictator game is to avoid being labeled as too selfish, to look good or not to look bad. Similarly, the experiment by Garza (2006) cited earlier, which introduces poverty in the dictator game, concludes that informing the dictator about the poverty of the recipient leads to more altruism. Again, this is because information about the socio-economic status of players leads to changes in relative states and *equity-biases*, which leads to different outcomes.

Thaler (1995 p 216) mentioned that when instead of giving the dictator a sum as “manna from heaven”, if the dictator is made to feel as if he earned the right to the sum, then sharing shrinks. Making the dictator feel as if he/she earned the right to the sum basically changes his/her location to a relatively more superior (from superior to more superior) state, which leads to an increase in his emotional state of *equity-bias*. Similarly, Schotter *et al.* (1996) introduced property rights in two-stage-survival dictator and ultimatum games. In the first stage proposers were competing with each other in offering higher amounts to a single responder. They earn property rights to the sum when a proposer

accepts the responder's offer. They move to the second stage with property rights they earned in the first state. In the second stage they offered lesser amounts and responders rejected smaller amounts less often. The offers were still significantly higher than zero and considered to be fair by player 2. This is because earning property rights in the second state increased the *equity-bias* of the proposers. The responders also made note of that and respected it by revising their lowest threshold (less biased).

Similarly, structure/design of the experiment also plays a role. Consider two types of dictator games. (i) First, assume that the dictator is given \$20 to share with another anonymous recipient. (ii) Second, consider the same two players now given 10 dollars each, with the dictator having the option of giving or taking away up to 10 dollars. Both experiments involve sharing 20 dollars but individuals are not located on the same position on the bias-chart. The dictator in (i) is in a more superior state than in (ii). Thus we will expect the share of the dictator in (ii) to be lower than, or at least as much as, in (i). Similarly, in ultimatum game we would expect the responder in (ii) to reject offers higher than in (i); this is because he/she is not in as inferior state in (ii) as he is in (i). The model also gives insight into why the hypotheses in experiments such as Bardsley (2005) are erroneous and provide a context for explaining results of his experiment. Bardsley hypothesized that the standard dictator game would give the same solution if the sum was instead distributed and the dictator was given the option of (giving and) taking money from the recipient. Arguing that the dictator is facing a similar problem of allocating the same budget, optimal allocation should be the same. The key assumption in Bardsley's argument is that preferences of the dictator are the same in the two experiments. The model in this paper postulates that preferences are state-dependent and the two models belong to two different states; expecting it to give the same solution is simply wrong.

An increase in competition on the proposer's side is expected to reduce the proposer's *equity-bias*, leading to relatively larger offers and increasing the responder's *equity-bias*, leading to rejection of relatively larger offers. However, if proposers were to compete for lowest offers, the proposer's *equity-bias* is expected to increase, leading to smaller offers, and that of the responders is expected to move further away from fair

offers, leading them to accept relatively lower offers. This holds only if the responders are aware of the nature of the competition. Thaler (1995) writes

A good general theory of fairness predicts that fair-minded players behave self-interestedly in some situations. Two experiments show that competition can push ultimatum offers closer to zero, in ways consistent with fairness. Schotter et al. (1994) created competition among Proposers. Eight Proposers made offers in a first stage. The four who earned the most in the first-stage game could then play a second-stage game (with a different player). Sensible fairness theories would say that Proposers now have an excuse for making low offers—they must compete for the right to play again—so low offers are not as unfair, and Responders will accept them more readily. That is what happened.

Information asymmetry may also play its role in positioning players on the state/bias chart and so may intentions. About the role of intentions in rejections in ultimatum games, Fehr and Schmidt (2005) conclude “Taken together, the evidence from Blount (1995), Kagel, Kim and Moser (1996), Offerman (1999), Brandts and Sola (2001) and Falk, Fehr and Fischbacher (2000a, 2000b, 2003) supports the view that subjects want to punish unfair intentions or unfair types. Although the evidence provided by the initial study of Blount was mixed, the subsequent studies indicate a clear role of these motives.”

Similarly, in experiments by Kagel, Kim and Moser (1996) where subjects had to divide 100 chips in an UG, chips were convertible to monetary pay-offs at different prices across players. For example in one treatment responders’ chips were valued 10 cents each and that of proposers 30 cents each. Players knew their own conversion rate but not necessarily that of others. When the proposer is aware that his/her chips are valued three times more than that of the responder, an equal monetary split would require the proposer to give 75 chips to the responder. When the information was available to responders, they rejected unequal money splits more frequently than when they were not aware of the difference in chips’ money value. Thus unequal proposals were rejected at higher rates than unintentional unequal proposals (Fehr and Schmidt 2005). Similarly, another important insight of the experiment was that proposers offered close to 50 percent when there was information asymmetry. This implies that proposers prefer to seem fair than be fair. Thaler (1995) believes this is an important reminder that self-interested behavior is

alive and well, even in ultimatum games. This also points towards an important distinction between altruism/fairness as a natural instinct and altruism/fairness as a code or altruism as an instinct complemented by religious or moral affiliations.

This could intuitively be predicted by the model as well. Information asymmetry in this experiment positions proposers in a relatively superior state and they make offers closer to 50% than 75%. This is when they prefer to seem fair than act fair. What the model also predicts is that individuals with commitment to moral codes through say religious or cultural affiliations would push offers closer to the 75%. Information asymmetry should as such not change their relative state. It will be interesting to investigate whether or not individuals with different religious affiliation would behave differently.

Similarly it can be shown that the model framework in this paper explains the wide variety of data in laboratory experiments such as Blount (1995) and Falk, Fehr and Fischbacher (2003). The framework also provides a rationale for the three-player games by Guth and Damme (1998), Kagel and Wolf (2001), and Berby-Meyer and Nuedereke (2005) by invoking player and design-specific *equity bias*.

6. Conclusions:

This paper takes note of the variety and multiplicity of models of other-regarding preferences proposed in experimental economics and points towards the need for certain criteria to judge these models. The paper identifies two tests, the Variety Test (ability of a model to explain outcomes under variety or alternative scenarios) and the Psychological Test (ability of a model to conform to psychological intuition), that can be used to judge any model of other-regarding preferences. It is argued that, for a mathematical model to qualify as a social welfare function, it must simultaneously pass the two tests. It is shown that none of the models proposed to date passes the two tests simultaneously with the exception of the Cox et al (2007) model which simultaneously passes the two test when some additional restrictions are imposed.

This paper then proposes a generalized model of inequity aversion. The paper introduces the concept of *equity-bias* and postulates that one's idea of equitable distribution is state-

dependent where state is determined by psychological and structural parameters. The state could be fair, superior or inferior. When assigned a fair state, one's valuation of equity is a fair one (even split), and when assigned a biased-state (superior or inferior) one's valuation of equity is a biased one. *i.e.* bias in state leads to bias in equity. Individuals in a fair state have zero *equity-bias* and split the pie evenly. Those in a superior (inferior) state have positive (negative) *equity-bias* and value more (less) than a fair distribution as an equitable distribution. Given the psychological tendencies/state of an individual, every experimental design assigns one of the three states to the player, which leads to individual-specific valuation of equity. Predictions about outcomes in different experiments, or the same experiment with different designs, can be made through predicting its impact on *equity-bias*.

The model is more general than its previous counterparts. It parsimoniously explains interior solutions in the dictator game and provides a framework to understand outcomes in other experiments. It provides a framework to understand why outcomes change with design of the experiment and across different experiments of the same nature. All we need to do is work out what impact would a particular innovation to an experimental design have on the relative states of players, and for that matter their *equity-bias*, and prediction about its impact on outcomes intuitively follow. The notion of *equity-bias* is general in nature and embodies all information related to socio-economic status, intentions, reciprocity, social distance, design of the experiment, role of the experimenter, etc. The model therefore is all-encompassing and provides a unified framework to understand outcomes of research in a broader context. For example, in the standard dictator game, the state assigned to the dictator is superior than the one assigned to the receiver. This is because the recipient is neither a party to the creation of value, nor legally entitled to any share in the sum. Thus it seems natural for the dictator to have a notion of equity that is biased towards his/her welfare. The standard ultimatum game introduces two changes to the dictator game. Firstly, it assigns the dictator the role of the proposer, which reduces his/her superiority and secondly, it alleviates the status of the recipient to that of a responder who becomes an active partner to the creation of value. This arrangement leads to reduction in the *equity-bias* of the dictator (now the proposer)

and increase in the *equity-bias* of the recipient (now the responder). Thus the fear of rejection, along with other factors, changes *equity-bias* of the players, hence equitable distribution. This is the main reason why offers in the dictator game are positive but lower than in the ultimatum game. Competition changes the relative position of the players and their valuation of equitable distribution. So does the design of experiments, such as the way property rights are assigned (e.g. earned or manna from heaven), the wording of the experiment/instructions, role of the experimenter etc.

The framework in this paper is simple and doesn't require individuals to process complex information. It rationalized all kind of choices, smart or otherwise, as state-dependent other-regarding utility maximizing outcomes. Policy makers can benefit from understanding the evolution of relative states and equity biases. Research therefore should be directed to unfold the dynamics and evolution of equitable states relevant to policy debates. The framework can also be used to understand the outcomes of other games, such as *public good* and *trust* games.

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APPENDIX A: Another example of linear-model with inequality

Charness-Rabin (*quasi-maximin preferences*) can be written as under (in a two player environment)

$$U_i = (1-\gamma)x_i + \gamma W$$

$$W = \delta \min(x_1, x_2) + (1-\delta)(x_1 + x_2)$$

This implies

$$U_i = \begin{cases} (1-\gamma)x_i + \gamma\delta x_j + \gamma(1-\delta) & \text{if } x_i \geq x_j \\ \lceil 1-\gamma(1-\delta) \rceil x_i + \gamma(1-\delta) & \text{if } x_i < x_j \end{cases}$$

$$U_i - \gamma(1-\delta) = \begin{cases} \lceil 1-\gamma(1-\delta) \rceil x_i - \gamma\delta(x_i - x_j) & \text{if } x_i \geq x_j \\ \lceil 1-\gamma(1-\delta) \rceil x_i & \text{if } x_i < x_j \end{cases}$$

$$\frac{U_i - \gamma(1-\delta)}{\lceil 1-\gamma(1-\delta) \rceil} = \begin{cases} x_i - \frac{\gamma\delta}{\lceil 1-\gamma(1-\delta) \rceil} (x_i - x_j) & \text{if } x_i \geq x_j \\ x_i & \text{if } x_i < x_j \end{cases}$$

Assume $V_i = \frac{U_i - \gamma(1-\delta)}{\lceil 1-\gamma(1-\delta) \rceil}$ and $\beta_i = \frac{\gamma\delta}{\lceil 1-\gamma(1-\delta) \rceil} < 1$. The model reduces to Fehr and

Schmidt's 1999 model with $\alpha_i = 0$. *i.e.*

$$V_i = x_i - \beta_i \max[(x_i - x_j), 0]$$

The fact that $\alpha_i = 0$ makes the model inferior under the specification as it would give corner solution in both the dictator game as well as the ultimatum game.

APPENDIX B: CES Welfare Function VS the base case model.

In general a CES welfare function takes the following form

$$V_i = \frac{1}{\sigma} [\gamma_i x_i^\sigma + (1-\gamma_i) x_j^\sigma] \quad (17)$$

Maximizing (17) subject to $x_i + x_j = N$ gives $x_i^* = \left[\frac{\left(\frac{\gamma_i}{1-\gamma_i}\right)^{\frac{1}{1-\sigma}}}{1 + \left(\frac{\gamma_i}{1-\gamma_i}\right)^{\frac{1}{1-\sigma}}} \right] N$ and $x_j^* = \left[\frac{1}{1 + \left(\frac{\gamma_i}{1-\gamma_i}\right)^{\frac{1}{1-\sigma}}} \right] N$.

When $\left(\frac{\gamma_i}{1-\gamma_i}\right)^{\frac{1}{1-\sigma}} = e_i \Rightarrow \gamma_i = \frac{(e_i)^{1-\sigma}}{1 + (e_i)^{1-\sigma}}$, the solution will be exactly the same as implied

by equations (13), (12) with $k = \chi_i = 1$.

Let us now consider if this function passes the P -test.

Maximizing $V_i = \frac{1}{\sigma} \left[\gamma_i x_i^\sigma + (1-\gamma_i) \left(\frac{x_j}{\chi_i}\right)^\sigma \right]$ subject to $x_i + x_j = N$ gives (after substituting γ_i)

$$x_i^{**} = \left[\frac{e_i (\chi_i)^{\frac{\sigma}{1-\sigma}}}{1 + e_i (\chi_i)^{\frac{\sigma}{1-\sigma}}} \right] N$$

$$x_j^{**} = \left[\frac{1}{1 + e_i (\chi_i)^{\frac{\sigma}{1-\sigma}}} \right] N$$

$$\frac{d(x_j^{**})}{d\chi_i} = -\frac{\sigma}{1-\sigma} \left[\frac{1}{1 + e_i (\chi_i)^{\frac{\sigma}{1-\sigma}}} \right]^2 e_i (\chi_i)^{\frac{2\sigma-1}{1-\sigma}} N$$

Passing P -test requires $\frac{d(x_j^{**})}{d\chi_i} > 0$ which is true only when $\frac{\sigma}{1-\sigma} < 0$ which requires

$\sigma > 1$, or $\sigma < 0$. $\sigma > 1$ is irrelevant as the solution only applies to $\sigma < 1$ (when preferences are convex). Thus, the relevant restriction on the CES utility function would be $\sigma < 0$.

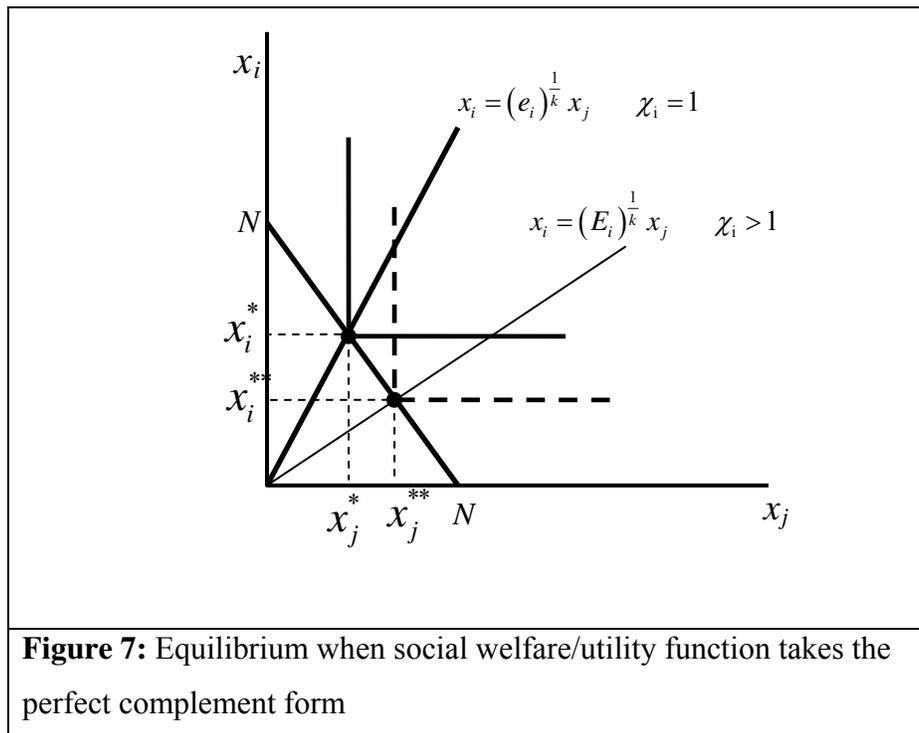
A special case of $\sigma < 0$ is when $\sigma = -\infty$. This is the perfect complement case.

In general the following social welfare function with Leontief preferences technology will generate exactly the same solution as the original model.

$$V_i = \min \left[(x_i)^k, e_i \left(\frac{x_j}{\chi_i} \right)^k \right]$$

Maximization in this case subject to budget constraint requires $x_i = (e_i)^{\frac{1}{k}} \left(\frac{x_j}{\chi_i} \right) = (E_i)^{\frac{1}{k}} x_j$

which is exactly the same as the $I_i=0$ equation. This together with the budget constraint will give you exactly the same solution the base case model. The figure below plots equilibrium in this case and shows that it satisfies the P -test.



When $\sigma \rightarrow 0$, $V_i \rightarrow \gamma_i \ln(x_i) + (1 - \gamma_i) \ln(x_j)$ which is log transformation of the Cobb-

Douglas function. The Cobb-Douglas function fails to pass the P -test as $\frac{d(x_j^{**})}{d\chi_i} = 0$.

APPENDIX C: Derivation of the main model.

Assume the individual i has the following social utility function

$$\tilde{V}_i = u_i \left(\frac{x_i}{\delta_i w_i} \right) - \beta_i \max \left[u_i \left(\frac{x_i}{\delta_i w_i} \right) - e_i u_{ij} \left(\frac{x_j}{\delta_{ij} w_{ij}} \right), 0 \right]$$

$$u_i' > 0, u_{ij}' > 0, u_i'' \leq 0, u_{ij}'' \leq 0, \beta_i = \frac{\gamma_i}{1 + e_i \left(\frac{u_{ij}'}{u_i'} \right)} \text{ and } \gamma_i > 1.$$

w stands for wealth and δ quantifies the efficiency/inefficiency with which individuals convert pay-offs into utility. The subscript ij on represents individual i belief about individual j since i doesn't have perfect information about j 's utility, wealth and efficiency.

Notice that selfish utility is determined by pay-off relative to wealth which basically acknowledges the fact that a dollar received by a wealthier person generates lesser utility than when it is received by a poor person. The parameter δ is there to capture heterogeneity in preference technology. When $\delta_i > \delta_{ij}$ individual i is more efficient in converting a dollar into utility than individual j (because of some socio-economic feature other than wealth, say disability). This specification therefore acknowledges socio-economic status as one of the determinant of selfish utility generated by pay-offs.

Let us assume the selfish utility functions are homogeneous of degree k in its arguments.

We can re-write the social utility function as under

$$\tilde{V}_i = \left(\frac{x_i}{\delta_i w_i} \right)^k - \beta_i \max \left[\left(\frac{x_i}{\delta_i w_i} \right)^k - e_i \left(\frac{x_j}{\delta_{ij} w_{ij}} \right)^k, 0 \right]$$

Multiplying both sides by $(\delta_i w_i)^k$ and substitute $(\delta_i w_i)^k \tilde{V}_i = V_i$ and $\chi_i = \left(\frac{\delta_{ij}}{\delta_i} \right) \left(\frac{w_{ij}}{w_i} \right)$, we get

$$V_i = (x_i)^k - \beta_i \max \left[(x_i)^k - e_i \left(\frac{x_j}{\chi_i} \right)^k, 0 \right]$$

Which is the model in the paper. As argued, χ_i in the model captures individual i 's valuation of his/her socio-economic status relative to that of individual j .

APPENDIX D: Derivation of the Restriction on Social Beta

As

$$V_i = u_i(x_i) - \beta_i \max[u_i(x_i) - e_i u_{ij}(x_j), 0]$$

Differentiating w.r.r x_i , we get (assume $u'_i = \left(\frac{du_i}{dx_i}\right)$ and $u'_{ij} = \left(\frac{du_{ij}}{dx_j}\right)$)

$$\frac{d(V_i)}{dx_i} = \begin{cases} u'_i & \text{when } u_i \geq e_i u_{ij} \\ u'_i - \beta_i \left[u'_i - e_i u'_{ij} \left(\frac{dx_j}{dx_i} \right) \right] & \text{when } u_i \leq e_i u_{ij} \end{cases}$$

Since, $\left(\frac{dx_j}{dx_i}\right) = -1$, $\frac{d(V_i)}{dx_i} = \begin{cases} u'_i & \text{when } u_i \leq e_i u_{ij} \\ u'_i - \beta_i (u'_i + e_i u'_{ij}) & \text{when } u_i \geq e_i u_{ij} \end{cases}$

We know $u'_i > 0$, so the social utility has positive slope when $u_i \geq e_i u_{ij}$. For the point $u_i = e_i u_{ij}$ to be the unique maximum, the social utility must decline when $u_i < e_i u_{ij}$. Thus

$\frac{d(V_i)}{dx_i}$ has to be < 0 when $u_i > e_i u_{ij}$.i.e. $u'_i - \beta_i (u'_i + e_i u'_{ij}) < 0$. This implies

$\beta_i > \frac{1}{1 + e_i \left(\frac{u'_{ij}}{u'_i}\right)}$. Thus, if we assume $\beta_i = \frac{\gamma_i}{1 + e_i \left(\frac{u'_{ij}}{u'_i}\right)}$, γ_i has to be greater than 1 for the

condition to be satisfied. γ_i can also be equal to 1 when $u''_i < 0$ and $u''_{ij} < 0$ (i.e. diminishing marginal utility in pay-offs). γ_i must be greater than 1 in case of constant marginal utilities i.e. $u''_i = u''_{ij} = 0$.

The condition can further be simplified as well. Proposition 2 shows that this condition is equivalent to $\beta_i > 1 - x_i^*$ which will always hold when $\beta_i > 1$. Thus the model will still

give the same solution if we replace $\beta_i > \left[1 + e_i \left(\frac{u'_{ij}}{u'_i}\right) \right]^{-1}$ condition with $\beta_i > 1$ for

simplicity.

APPENDIX E: Application to Impunity and Private Impunity Games:

Rejections in the ultimatum game are normally interpreted as the responder's punishment to the proposer. This behaviour is explicitly modeled in many studies in the form of "inequality aversion" by Fehr and Schmidt (1999), Reciprocity (Falk & Fischbacher, 2006), intentions by Rabin (1993). Whether or not rejections in the ultimatum games are punishment can be ascertained through impunity game. Impunity game is similar to the ultimatum game but rejections reduce pay-off of the responders to zero only, the proposer keeps his/her share. This game basically restrains the responder's ability to punish the proposer's inequitable and unkind offers. Rejections in this case lead to increase in inequality. Inequality aversion and reciprocity therefore cannot explain rejection in the impunity game. This game is not very well researched and need some more attention. Recently Horita and Yamagishi (2007) conducted experiments on impunity game and concluded that a substantial proportion of participants rejected extremely unfair offers which was about half of the rejections in ultimatum game. (Bolton & Zwick (1995): nearly 0 %, Fukuno & Ohbuchi (2001): 30.8%, read these references as well). Horita, and Yamagishi (2007) interpret rejections in the impunity game as a "social punishment" or symbolic expression of anger with reference to the the experiments of Fukuno& Ohbuchi, 2001; Güth & Huck(1995); Xiao and Houser (2005).

Horita and Yamagishi (2007) introduced another innovation to the impunity game. In the impunity game the proposer is informed of the responder's rejection. Horita and Yamagishi (2007) makes the responders decision private (the proposer is not informed) and call it private impunity game. The proposer is unaware that the responder has any rejection power. The proposer believes as if he/she is a dictator. Responders know that proposers will not be informed of their decision. Rejections in the private impunity game cannot be used as "social punishment". Horita and Yamagishi (2007) experiments conclude that a substantial proportion of responders rejected unfair offers even in the private impunity game. They interpret these rejections as "Self-assertiveness" based on post-experiment questionnaire responses. Given that the experimenter is made aware of the responder's decision, the rejections in the private impunity game can also be

interpreted as showing symbolic expression of anger through the experimenter or not to belittle oneself in the eyes of the experimenter.

Assuming the evidence in impunity or private impunity game is robust, can the model in this paper explain rejections in these games. The answer is, yes it does. The responder reject offers if they are less than their acceptable minimum. This rejection could be motivated by reciprocity, intentions, social punishment, self-assertiveness or any other. When rejections inflict monetary loss to proposers (as in the ultimatum game) all of these reasons could be in play. Impunity games reduce narrow down the list to social punishment and self-assertiveness.