Education, Signaling, and Wage Inequality in a Dynamic Economy

Yuki, Kazuhiro

Faculty of Economics, Kyoto University, Japan

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Kazuhiro Yuki*

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Abstract

Many empirical works suggest that education has a positive effect on earnings not only because it raises human capital but also because it functions as a signal when employers have incomplete information on employees' skills.

The signaling role could have important consequences on the dynamics of education, wages, and wage distribution when there exist intergenerational linkages in educational decisions. This paper examines the dynamic effects in an economy where education has the dual roles and some fraction of individuals is credit constrained from taking education. In particular, it investigates how the number of educated individuals, the importance of the signaling value of education, and the wage inequality between educated and uneducated workers change over time in such economy, and compares the dynamics with those when education does not function as a signal. It also examines whether the signaling role leads to higher aggregate consumption or not in the long run.

Keywords: Human capital; Education; Signaling; Statistical discrimination; Credit constraint
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*Faculty of Economics, Kyoto University, Yoshida-honmachi, Sakyo-ku, Kyoto, 606-8501, Japan. Phone: +81-75-753-3532. E-mail address: yuki@econ.kyoto-u.ac.jp.

1 Introduction

Education has a positive effect on earnings not only because it raises human capital but also because it functions as a signal when employers have incomplete information on employees' skills, i.e. they statistically discriminate workers based on education.

Many empirical works support this view. Bedard (2001) examines the idea that, in an economy where some students are constrained from entering university, increasing the access may raise the high school dropout rate due to the lower signaling value of high school education (since relatively high ability students proceed to university) and finds supportive evidence for US data. Altonji and Pierret (2001) show that, if easily observable characteristics such as education and hard-to-observe correlates of workers' skills are positively correlated and firms statistically discriminate based on the former variables, as firms learn about their productivities over time, coefficients on the former variables fall and those on the latter variables rise in wage regressions, and they confirm this proposition for US data. Positive evidence is found for other developed economies as well, such as Galindo-Rueda (2003) for Britain and Hämäläinen and Uusitalo (2008) for Finland (note, however, negative evidence by Chevalier et al., 2004, for Britain). Evidence for developing countries is very scarce, but Strobl (2003) finds positive evidence for Ghanian manufacturing workers who were hired through formal channels and do not receive on-the-job-training.

In many situations, the signaling role affects education, wages, and wage distribution statically. Further, it could have important consequences on their dynamics when there exist intergenerational linkages in educational decisions. This paper examines the dynamic effects in an economy where education has the dual roles and some fraction of individuals is credit constrained from taking education. In particular, it investigates how the number of educated individuals, the importance of the signaling value of education, and the wage inequality between educated and uneducated workers change over time in such economy, and compares the dynamics with those when education does not function as a signal. It also examines whether the signaling role leads to higher aggregate consumption or not in the long run.

The model is concerned with a small open economy populated by individuals who are heterogeneous in terms of wealth and ability determined (exogenously) outside education, where the ability is high (*type* h) or low (*type* l). Individuals decide whether or not to take education that changes human capital. Since the cost of education has to be self-financed (due to a lack of credit markets financing it), they take education only if it is affordable and profitable. Workers of different levels of human capital are perfectly substitutable. However, the wage of an individual does not equal her human capital but equals a weighted average of her skill and the average skill (wage) of those with the same educational background as her.[1] An interpretation of a positive weight on the average skill is that those who assess her performance at work have incomplete information on her skill and thus sets a part of the wage based on a signal, the average wage of her educational level.[2] Although the information incompleteness is the major explanation for the weight in the paper, other factors such as effects of unions and fairness consideration in the workplace too may be important.

Because of the wage equation, the return to education of an individual can be decomposed into the part reflecting a change in her skill level and the part reflecting the difference in average skills (wages) of educated and uneducated workers, which is named the *signaling value of education*. Thus, she may take education whose social return is negative, if the signaling value is high enough. Under very mild and reasonable restrictions on permissible education technologies (e.g., the return is higher for type h), it is shown that type h individuals who can afford education always take, while decisions of corresponding type l individuals depend on the signaling value.

The dynamic structure of the model is of an OLG variety. An individual lives for two periods. In childhood, she receives a transfer from the parent to invest in assets and education whose cost must be financed by the received transfer. In adulthood, she obtains income from assets and labor supply and spends it on consumption and a transfer to her single child, from which she derives utility (impure altruism). The intergenerational correlation of the exogenous ability is 1, that is, descendants of a type j $(j = h, l)$ individual continue to be the same type, which is a strong but more realistic assumption than the other extreme of zero correlation. Generations go by in this fashion.

The distribution of wealth (transfers) determines the proportion of individuals of each type *accessible* to education, $F_{j,t}$ $(j = h, l)$. The proportions, together with exogenous variables related to the productivity of education, the exogenous ability, and the degree of information incompleteness (the weight on the average skill), in turn determine the return to education and the proportion of each type *taking* education, and thus skill composition

[1] Skill and human capital are used interchangeably in this paper.

[2] For example, the weight may be considered as the fraction of time during which the assessors cannot recognize her skill (and they can precisely identify it after that). This interpretation is consistent with the finding by Altonji and Pierret (2001) cited at the beginning. The wage equation can be derived from profit maximization problems of firms who hire workers and physical capital for production. Further, productivity growth can be introduced into the model without affecting results qualitatively, as long as the cost of education is assumed to grow proportionately.

of educated and uneducated workers and their wages. Hence, by examining the dynamics of $F_{j,t}$, the dynamics of other dimensions of the economy can be analyzed.[3]

Under complete information, since wage equals skill (thus the signaling value is zero and the wage inequality is constant), the dynamics are simple: $F_{j,t}$ increases (decreases) over time if the human capital of uneducated (educated) type j workers [thus transfers to their children] is high (low) enough relative to the cost of education, otherwise, it is time-invariant. By contrast, the dynamics are generally complicated with incomplete information. The analysis focuses on cases in which the dynamics of $F_{j,t}$ are different qualitatively.[4]

Main results on the dynamics of the economy are summarized as follows. First, unlike the complete information case, the dynamics of $F_{h,t}$ and $F_{l,t}$ are interrelated and differ greatly depending on the initial distribution of wealth and the exogenous variables. The interaction arises through the dependence of wages on average skills, which in turn depend on $F_{j,t}$ directly and indirectly: the number of type l individuals taking education is determined not only by 'accessibility', $F_{l,t}$, but also by the return to education, which depends on the signaling value and thus both $F_{h,t}$ and $F_{l,t}$.

Second, related variables too exhibit interesting dynamics under a realistic situation in which the initial distribution of wealth is such that $F_{h,0}$ is not high and $F_{h,t}$ (thus the number of type h educated workers) increases over time at least at low $F_{h,t}$. The skill composition of educated workers changes over time: at first, education is not profitable to the type l and thus all educated workers are type h; after $F_{h,t}$ reaches a certain level, they become indifferent in the educational choice and the ratio of type l to type h educated workers rises over time; after some point, all of the non-poor type l start to take but, unless $F_{l,t}$ keeps growing, which is unlikely, the ratio starts to fall eventually. As for wage-related variables, the signaling value of education increases, uneducated wages decrease, and the wage inequality between educated and uneducated workers rises over time when education is not profitable to the type l; the signaling value is constant, *all* wages decrease, and the wage inequality falls when they are indifferent in the choice; and after all the non-poor type l start to take education, typically, the signaling value increases, educated (uneducated) wages increase (decrease), and the inequality rises again. The inequality and the signaling value fall in this last stage only if $F_{l,t}$ increases and the relative growth of $F_{l,t}$ to $F_{h,t}$ is sufficiently high.

Finally, as for the relationship between the initial distribution of wealth and the long-run outcome, under some conditions, both the higher proportion of educated workers as well as

[3]Examination of the dynamics of the wealth distribution itself is not needed to derive qualitative results.

[4]For example, the case in which both $F_{h,t}$ and $F_{l,t}$ increase over time irrespective of the initial distribution of wealth and all workers become educated in the long run is ruled out.

lower inequality (and signaling value) are realized with *lower* $F_{h,0}$ and higher $F_{l,0}$.

The number of educated workers of each type and aggregate output net of the education cost (thus aggregate consumption) generally differ from the complete information case. The last part of the paper investigates how they (and inequality, although results are not presented here) differ in steady states and examines the effect of lifting the credit constraint.

When education is not socially productive to anybody, the signaling effect leads to *overeducation* and lower net output. Type h individuals take education to mitigate the negative effect of the presence of type l on their wage, while type l individuals take education to benefit from the positive effect from the educated type h. Lifting the credit constraint exacerbates overeducation and lowers net output further in most cases. By contrast, when education is productive only for type h, net output is lower with the signaling effect in most cases, because fewer type h individuals can afford education (due to the negative effect from the uneducated type l) and/or some of the type l take unproductive education. (The effect of the credit constraint on net output could be ambiguous.) Finally, when education is productive for both types, net output could be *higher* with the signaling effect, because a higher proportion of type l individuals can afford education due to the positive effect from type h individuals. Clearly, lifting the credit constraint leads to optimal universal education and maximizes net output in this case.

Several policy implications can be derived from the analyses. First, policies that enable everyone to access education (e.g. free public education) and those that increase access directly (e.g. wealth redistribution and tuition subsidies) or indirectly through promoting skill accumulation outside education and thereby raising earnings of the poor (e.g. childcare programs and job training programs for the poor) may not be desirable in the presence of the signaling effect. Unless education is socially productive for everyone, such policies could lead to overeducation of unproductive individuals and lower net output, whereas under complete information, such policies always (weakly) raise net output. Hence, when the signaling effect is not negligible, it is important to ensure that education is socially productive for everyone (e.g. by raising the effectiveness of education especially for the disadvantaged) or to restrict access only to high ability students (e.g. by implementing competitive entrance exams). The harm from access-widening policies and the need for these complementary measures are greater at later stages of development, since the signaling value of education and thus the return to education of low ability individuals increase over time.

Second, wealth redistribution that raises the accessibility (raises both $F_{h,t}$ and $F_{l,t}$ since the government usually cannot distinguish different types) may not be desirable for a different reason from above: it could have a negative effect on earnings and education of the low-ability poor. This happens if the policy stimulates education of the high-ability poor too much, and as a result, lowers the average skill of the uneducated and thus their wages, and hampers wealth accumulation of the low-ability poor. Such policy is certainly undesirable when education is socially productive. Without the signaling effect, by contrast, the dynamics of different types are unrelated, thus the policy is always (weakly) desirable.

Finally, a positive weight on the average skill may be interpreted as effects of unions or fairness concern in the workplace on wage setting, so the above implications apply when these effects, not the signaling effect, are important as well.

Existing works that examine the dynamics of education, wages, and wage inequality when education has the signaling role are very limited, among which most closely related is Hendel, Shapiro, and Willen (2005). They use a similar model, but several important differences exist: (i) wages equal average skills and thus are same for workers with different abilities; (ii) education has the signaling role only; and (iii) individuals can borrow at a higher interest rate than a lending rate to finance education. Further, they restrict the analysis to the case in which low-ability individuals never take education and thus the skilled wage is constant. The assumption on the credit market may be more realistic than this paper, but it complicates the determination of temporal equilibrium (e.g., multiple temporal equilibria are generic). Due to the simpler assumption, this paper can adopt more realistic assumptions in other respects and examine the case in which both types take education as well. Because both types may take education, the dynamics are much richer: for example, as mentioned before, the proportion of the non-poor type l taking education and the wage inequality exhibit non-monotonic dynamics typically. Further, the present paper examines relations among exogenous variables, the initial distribution of wealth, the dynamics, and the long-run outcome more systematically and clearly (e.g., phase diagrams are used extensively), and investigates long-run welfare consequences of the signaling effect as well.

D'Amato and Mookherjee (2008) is another closely related work whose model differs in: (i) the educated wage (the modern sector wage) equals the average skill, while the uneducated wage (the traditional sector wage) is constant and does not depend on skill; (ii) ability is a continuous variable drawn from an iid distribution and thus the intergenerational correlation of ability is zero; (iii) assets do not exist and thus education is the only measures to transmit wealth intergenerationally; and (iv) the cost of education decreases with ability. The assumptions on the ability distribution and the education cost are more realistic, but, to make the model tractable, less plausible assumptions are adopted in other respects. An ability threshold exists for given parental wage, and children with ability above the threshold receive education, where the threshold is higher for poorer uneducated parents. A unique steady state exists mainly due to no ability correlation and the constant uneducated wage. Not many results are proved analytically (e.g., convergence to steady state is not established) and the model allows numerous qualitatively different dynamics.

This paper is also somewhat related to the theoretical literature on statistical discrimination. In particular, Antonovics (2006), building on Coate and Loury (1993), develops a dynamic model of race-based statistical discrimination in which parents can allocate incomes to human capital investments of their children but firms can observe the investment decisions only imperfectly and set wages based on two signals, race and an imprecise signal of the investment. She examines conditions under which different racial groups that are identical in terms of innate ability and taste may end up in steady states with different investment and wage levels.

The modeling of the educational decision and intergenerational transmission of wealth draws on studies on the dynamics of human capital accumulation and income distribution, such as Galor and Zeira (1993), Ljungqvist (1993), Benabou (1996), and Yuki (2007, 2008). Closely related are Galor and Zeira (1993) and Yuki (2007, 2008), in which the educational decision is constrained by transfers motivated by impure altruism, as in this paper.

The paper is organized as follows. For ease of presentation, in Section 2, the static part of the model is, then, in Section 3, the full-fledged model is presented and analyzed. Results and policy implications are presented in Section 4 and the paper is concluded in Section 5. Proofs of lemmas and propositions are contained in a supplement available from the author upon request.

2 Static model

This section presents and analyzes the static part of the model (all variables are presented without time subscript). The full-fledged model is presented in the next section. Consider a small open economy (interest rate r is exogenous) populated by individuals who are heterogeneous in terms of wealth and ability determined (exogenously) outside education. The ability is high (*type h*) or low (*type l*), and the proportion of type h (type l) individuals is H $(1-H)$. Total (adult) population is 1.

Individuals decide whether or not to take education. Since the cost of education e has to be self-financed, they must have enough wealth to receive education. Education changes the human capital of a type j $(j=h,l)$ individual from h_{ju} (u is for uneducated) to h_{je}.[5] Her wage reflects partly own skill and partly the average skill (wage) of those with the same educational background as her. Specifically, the wage of a type j worker with educational level k $(k=e,u)$ is given by:

$$s_k\widetilde{E}[h_k]+(1-s_k)h_{jk}, \tag{1}$$

where parameter $s_k \in [0,\frac{1}{2}]$, $(s_e,s_u) \neq (0,0)$, measures the importance of the average skill, $\widetilde{E}[h_k]$, in the wage equation. An interpretation of positive s_k is that the employer has incomplete information on the worker's ability (and acquiring more information is costly and not profitable) and thus sets a part of the wage based on a signal, the average wage of her educational level, where the assumption $s_k \leq \frac{1}{2}$ implies that the information incompleteness is not too severe.[6] Although the information incompleteness is the major explanation for positive s_k in the paper, other factors too may be important.[7] This equation can be derived from profit maximization problems of firms who hire workers and physical capital for production.[8] Further, productivity growth can be incorporated into the model without affecting results qualitatively, as long as the cost of education is assumed to grow proportionately.

As detailed in Section 3, an individual can also spend wealth on assets with interest rate r. Thus, she takes education only if it is financially accessible *and* rewarding. Let F_j be the fraction of type j $(j=h,l)$ individuals with enough wealth for education, and let p_{je} be the probability that such individual actually takes education. Then, the wage of educated type j workers *net* of the cost of education and that of uneducated type j workers are:

$$w_{je} = s_e\frac{p_{he}F_hHh_{he}+p_{le}F_l(1-H)h_{le}}{p_{he}F_hH+p_{le}F_l(1-H)}+(1-s_e)h_{je}-(1+r)e, \tag{2}$$

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[5]The assumption that the cost is common to the two types would be a justifiable simplifying assumption, unless admission to school requires significant spending on private tutoring or study materials. Note that e does not include non-material costs, which are irrelevant to the credit constraint. The type l may incur higher disutility from study but this affects the educated wage net of the disutility, not e, and considering the difference in the disutility would not affect qualitative results.

[6]For example, suppose that those who assess her performance cannot recognize her skill during the first s_k fraction of time (and they can precisely identify it after that). This interpretation is consistent with the finding by Altonji and Pierret (2001) cited in the introduction. Alternatively, $(1-s_k)h_{jk}$ may be construed as the fraction of her skill (or her contribution to output) they can recognize precisely, where they do not know that s_k is common to workers with the same ethnic background, let alone the value of s_k. $s_e \neq s_u$ is allowed because workers of different educational levels typically take different kinds of jobs and the difficulty of knowing workers' abilities depends on jobs' types.

[7]Positive s_k, particularly s_u, may capture effects of unions in wage setting as well. Further, when the effect of education on human capital is strong and thus the workplace is segregated by educational levels, positive s_k may reflect fairness consideration in the workplace too.

[8]Suppose that firms with identical technology hire workers and physical capital to produce a final good. Workers of different skills are perfectly substitutable in production. Then, after normalizing the wage rate per unit of skill (which depends on total factor productivity and the interest rate) to 1, (1) is obtained.

$$w_{ju} = s_u \frac{[(1-p_{he})F_h+(1-F_h)]Hh_{hu}+[(1-p_{le})F_l+(1-F_l)](1-H)h_{lu}}{[(1-p_{he})F_h+(1-F_h)]H+[(1-p_{le})F_l+(1-F_l)](1-H)}+(1-s_u)h_{ju}. \quad (3)$$

That is, $\widetilde{E}[h_e]$ ($\widetilde{E}[h_u]$) and thus w_{je} (w_{ju}) increase (decrease) with F_h and p_{he} and decrease (increase) with F_l and p_{le} (note that $h_{hu}>h_{lu}$ and $h_{he}>h_{le}$).

The gross return to education for a type j individual is $s_e\widetilde{E}[h_e]+(1-s_e)h_{je}-\{s_u\widetilde{E}[h_u]+(1-s_u)h_{ju}\}$, out of which $s_e\widetilde{E}[h_e]-s_u\widetilde{E}[h_u]$ is the return unrelated to her skill accumulation and thus named the *signaling value of education*. Two assumptions are imposed on the return.

Assumption 1 (Return to education of type h individuals)

$$h_{he}-(1+r)e > s_u[Hh_{hu}+(1-H)h_{lu}]+(1-s_u)h_{hu}.$$

It states that a type h individual has a minimal incentive to take education: she wants to take it if nobody does and she can receive the highest possible wage with education.[9]

Assumption 2 (Difference between returns to education of two types)

$$[(1-s_e)h_{he}-(1-s_u)h_{hu}]-[(1-s_e)h_{le}-(1-s_u)h_{lu}] = (1-s_e)(h_{he}-h_{le})-(1-s_u)(h_{hu}-h_{lu}) > 0.$$

This states that the return is strictly greater for the high ability type. That is, education enlarges the inter-type wage differential. These assumptions impose mild restrictions on permissible *education technologies* (associations between h_{ju} and h_{je}, $j=h,l$).

2.1 Educational choice

2.1.1 Type h individuals

Lemma 1 (Educational choices of type h individuals) $p_{he}=1$.

That is, type h individuals, if financially accessible, *always* take education.

2.1.2 Type l individuals

Decisions of type l individuals are more complex. When $(F_h,F_l)\neq(1,1)$ or $s_u=0$, $p_{le}=1$ iff

$$s_e\frac{F_hHh_{he}+F_l(1-H)h_{le}}{F_hH+F_l(1-H)}+(1-s_e)h_{le}-(1+r)e\geq s_u\frac{(1-F_h)Hh_{hu}+(1-F_l)(1-H)h_{lu}}{(1-F_h)H+(1-F_l)(1-H)}+(1-s_u)h_{lu}, \quad (4)$$

that is, iff the *net* return to education (net of $(1+r)e$) for type l is non-negative with $p_{le}=1$.

To derive the corresponding condition when $F_h=F_l=1$ and $s_u>0$, firms' beliefs on expected skill of a worker taking an action *not* observed under $p_{le}=1$, i.e. not take education,

[9] The weakest condition for $p_{he}>0$ to be an equilibrium is $h_{he}-(1+r)e > s_uh_{lu}+(1-s_u)h_{hu}$. When this condition but not Assumption 1 holds, $p_{he}=p_{le}=0$ is always an equilibrium, while whether $p_{he}>0$ is an equilibrium or not depends on F_h (thus multiple equilibria are possible). Since $p_{he}=p_{le}=0$ is not an interesting equilibrium from the paper's perspective, a stronger assumption is imposed to simplify the analysis. The essentially same assumption is made in Hendel, Shapiro and Willen (2005) (equation 4).

must be specified. Under the reasonable belief formation explained in the proof of Lemma 1, it can be shown that $p_{le}=1$ iff [10]

$$s_e[Hh_{he}+(1-H)h_{le}]+(1-s_e)h_{le}-(1+r)e \geq h_{lu}. \tag{5}$$

When $(F_h,F_l)\neq(1,1)$ or $s_u=0$, if (4) does not hold, not all of the non-poor type l take education. Some of them do take, i.e. $p_{le}\in(0,1)$, iff

$$s_eh_{he}+(1-s_e)h_{le}-(1+r)e > s_u\frac{(1-F_h)Hh_{hu}+(1-H)h_{lu}}{(1-F_h)H+(1-H)}+(1-s_u)h_{lu}, \tag{6}$$

that is, the net return is positive with $p_{le}=0$, and $p_{le}=0$ otherwise. When $p_{le}\in(0,1)$, they are indifferent in the choice and thus p_{le} is determined by

$$s_e\frac{F_hHh_{he}+p_{le}F_l(1-H)h_{le}}{F_hH+p_{le}F_l(1-H)}+(1-s_e)h_{le}-(1+r)e=s_u\frac{(1-F_h)Hh_{hu}+[(1-p_{le})F_l+(1-F_l)](1-H)h_{lu}}{(1-F_h)H+[(1-p_{le})F_l+(1-F_l)](1-H)}+(1-s_u)h_{lu}. \tag{7}$$

When $p_{le}=1$ $(=0)$, the LHS of (7) is strictly less (greater) than the RHS, because (4) with '$\geq$' replaced by '$<$' and (6) hold. Further, when $s_e>0$ $(s_u>0)$, the LHS decreases (the RHS increases) with p_{le}, hence there exists a unique $p_{le}\in(0,1)$ satisfying the equation. As is clear from the equation, p_{le} increases with F_h and decreases with F_l.

When $F_h=F_l=1$ and $s_u>0$, if (5) is not satisfied, $p_{le}\in(0,1)$ iff

$$s_eh_{he}+(1-s_e)h_{le}-(1+r)e > h_{lu}, \tag{8}$$

which is (6) with $F_h=1$, and $p_{le}=\frac{(s_e(h_{he}-h_{le})-\{h_{lu}-[h_{le}-(1+r)e]\})H}{\{h_{lu}-[h_{le}-(1+r)e]\}(1-H)}$.

It is useful to introduce a figure showing educational choices of the type l for each combination of F_h and F_l (>0). The next lemma describes the shape of the dividing line between $p_{le}=1$ and $p_{le}\in(0,1)$ (equation 4 with '$\geq$' replaced by '$=$') on the (F_h,F_l) plane.

Lemma 2 (Shape of the dividing line between $\mathbf{p_{le}=1}$ and $\mathbf{p_{le}\in(0,1)}$) *The dividing line between $p_{le}=1$ and $p_{le}\in(0,1)$ is continuous and positively sloped on the (F_h,F_l) plane. Further, it is strictly convex (strictly concave) when $F_l<(>)F_h$.*

Together with Lemma 2, the next lemma describes precisely educational choices of type l individuals when $(F_h,F_l)\neq(1,1)$. For notational simplicity, define $E[h_k,\phi]\equiv\phi h_{hk}+(1-\phi)h_{lk}$ $(k=e,u)$, where $\phi\in[0,1]$. For example, $E[h_e,s_eH]$ is a weighted average of human capital of the educated when the weight on h_{he} is s_eH.

Lemma 3 (Educational choices of the type l at $(\boldsymbol{F_h},\boldsymbol{F_l})\neq(\mathbf{1},\mathbf{1})$) *When $(F_h,F_l)\neq(1,1)$,*

(I) If $E[h_e,s_e]\equiv s_eh_{he}+(1-s_e)h_{le}\leq(1+r)e+h_{lu}$ ($<$ when $s_e=0$), $p_{le}=0$ for any F_h,F_l.

(II) If $h_{le}\geq(1+r)e+s_uh_{hu}+(1-s_u)h_{lu}=(1+r)e+E[h_u,s_u]$, $p_{le}=1$ for any F_h,F_l.

(III) Otherwise,

[10] When $s_e=0$ and $h_{le}-(1+r)e=h_{lu}$, $p_{le}=1$, *not* $p_{le}\in[0,1]$, is assumed.

(a) *if* $E[h_e, s_eH] < (1+r)e + E[h_u, s_uH]$, *the dividing line between* $p_{le}=1$ *and* $p_{le} \in (0,1)$ *is located below* $F_l{=}F_h$ *on the* (F_h,F_l) *plane and* $p_{le}<1$ *when* $F_l \geq F_h$.

(i) *If* $E[h_e, s_e] < (1+r)e + E[h_u, s_uH]$, *it crosses* $F_l=0$ *at* $F_h \in (0,1)$, *and* $p_{le}=0$ *for* F_h *lower than the value at the intersection. Otherwise, it approaches* $(F_h,F_l)=(0,0)$.

(ii) *If* $E[h_e,s_eH]<(\geq)(1+r)e+h_{lu}$, *it crosses* $F_h=1$ *at* $F_l \in (0,1)$ (*approaches* $(F_h,F_l)=(1,1)$).

(b) *if* $E[h_e,s_eH]=(1+r)e+E[h_u,s_uH]$, $p_{le}=1$ $(\in(0,1))$ *when* $F_l \leq (>) F_h$.

(c) *if* $E[h_e,s_eH] > (1+r)e+E[h_u,s_uH]$, *it is located above* $F_l{=}F_h$ *and* $p_{le}=1$ *when* $F_l \leq F_h$.

(i) *If* $E[h_e,s_eH]<(\geq)(1+r)e+E[h_u,s_u]$, *it approaches* $(F_h,F_l)=(1,1)$ (*crosses* $F_l=1$ *at* $F_h \in (0,1)$).

(ii) *If* $h_{le} < (\geq)\ (1+r)e+E[h_u,s_uH]$, *it approaches* $(F_h,F_l)=(0,0)$ (*crosses* $F_h=0$ *at* $F_l \in (0,1)$).

The next lemma summarizes educational choices at $(F_h,F_l)=(1,1)$ described earlier.

Lemma 4 (Educational choices of the type l at $(\boldsymbol{F}_\mathbf{h},\boldsymbol{F}_\mathbf{l})=(\mathbf{1},\mathbf{1})$) *At* $(F_h,F_l)=(1,1)$,

(i) $p_{le}=0$ *iff* $s_e h_{he}+(1-s_e)h_{le} \leq (1+r)e+h_{lu}$ ($<$ *when* $s_e=0$).

(ii) $p_{le}=(<)1$ *iff* $s_eHh_{he}+(1-s_eH)h_{le} \geq (<)(1+r)e+h_{lu}$.

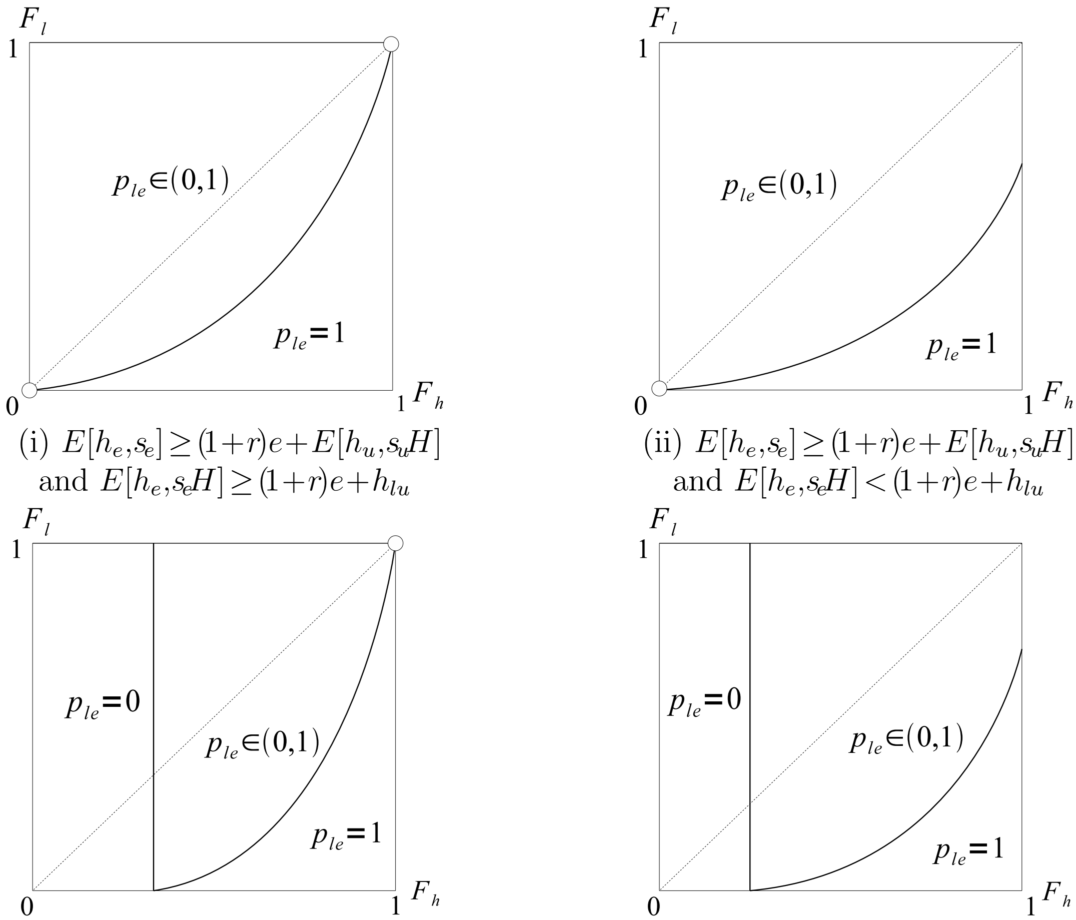
Lemmas 3 (I) and 4 (i) show that $p_{le}=0$ for any F_h and F_l, if education is not profitable for type l even when the average skill of the educated is highest and that of the uneducated is lowest. Similarly, from Lemmas 3 (Ⅱ) and 4 (ii), $p_{le}=1$ always, if education is profitable for them even when the average skill of the educated is lowest and that of the uneducated is highest.[11] Otherwise, p_{le} depends on F_h and F_l, thus it is convenient to introduce figures illustrating their choices.

Figure 1 illustrates the choices when $E[h_e, s_eH] < (1+r)e + E[h_u, s_uH]$ and $E[h_e, s_e] > (1+r)e+h_{lu}$, based on Lemmas 2, 3 (Ⅲ)(a), and 4.[12] In this case, the dividing line between $p_{le}=1$ and $p_{le} \in (0,1)$ is located below $F_l{=}F_h$. For given F_h, when F_l is greater than the value on the dividing line and thus $p_{le} \in (0,1)$, the proportion of the educated among the type l equals F_l on the dividing line.

In general, $p_{le}=1$ tends to hold when F_h is high and F_l is low: as the proportion of the type h (type l) accessible to education is higher (lower), the average skill of the educated (uneducated) is higher (lower), the signaling value of education is higher, and the net return to education is more likely to be positive. If $E[h_e,s_e]<(1+r)e+E[h_u,s_uH]$ (Figure 1 (iii) and (iv)), $p_{le}=0$ when F_h is very low: since education is not very effective in raising skill (given h_{hu} and h_{lu}, h_{he} and h_{le} are low) and the average skill of the uneducated is not low (due to

[11] At $(F_h,F_l)=(1,1)$, all the type h take education, so the lowest possible average skill of the educated is $Hh_{he}+(1-H)h_{le}$.

[12] From Lemma 3, when $s_e=0$ ($s_u=0$), only Figure 1 (iii)[(ii)] is possible.



(i) $E[h_e,s_e]\geq(1+r)e+E[h_u,s_uH]$ and $E[h_e,s_eH]\geq(1+r)e+h_{lu}$

(ii) $E[h_e,s_e]\geq(1+r)e+E[h_u,s_uH]$ and $E[h_e,s_eH]<(1+r)e+h_{lu}$

(iii) $E[h_e,s_e]<(1+r)e+E[h_u,s_uH]$ and $E[h_e,s_eH]\geq(1+r)e+h_{lu}$

(iv) $E[h_e,s_e]<(1+r)e+E[h_u,s_uH]$ and $E[h_e,s_eH]<(1+r)e+h_{lu}$

Figure 1: Educational choices of type l individuals when $E[h_e,s_eH]<(1+r)e+E[h_u,s_uH]$ and $E[h_e,s_e]>(1+r)e+h_{lu}(\geq$ if $s_e=0)$ (Lemma 3 (Ⅲ)(a))

high $1-F_h$), the net return is negative for type l. When $E[h_e,s_eH]\geq(1+r)e+h_{lu}$ (Figure 1 (i) and (iii)), $p_{le}=1$ at $(F_h,F_l)=(1,1)$ from Lemma 4.[13]

Similarly, Figure 2 presents the choices when $E[h_e,s_eH]>(1+r)e+E[h_u,s_uH]$ and $h_{le}<(1+r)e+E[h_u,s_u]$, based on Lemmas 2, 3 (Ⅲ)(c), and 4.[14] The dividing line is above F_l=F_h, i.e. $p_{le}=1$ for $F_l<F_h$, due to the higher net return than in the previous case. When $E[h_e,s_eH]<(1+r)e+E[h_u,s_u]$ (Figure 2 (iii) and (iv)), $p_{le}=1$ at $(F_h,F_l)=(1,1)$ from Lemma 4.

[13]From (5), she *strictly* prefers taking education, unless $E[h_e,s_eH]=(1+r)e+h_{lu}$. In contrast, when $(F_h,F_l)\neq(1,1)$, $p_{le}=1$ on the dividing line but the type l are indifferent in the educational choice from (4).

[14]From Lemma 3, when $s_e=0$ $(s_u=0)$, only Figure 2 (iii)[(ii)] is possible.

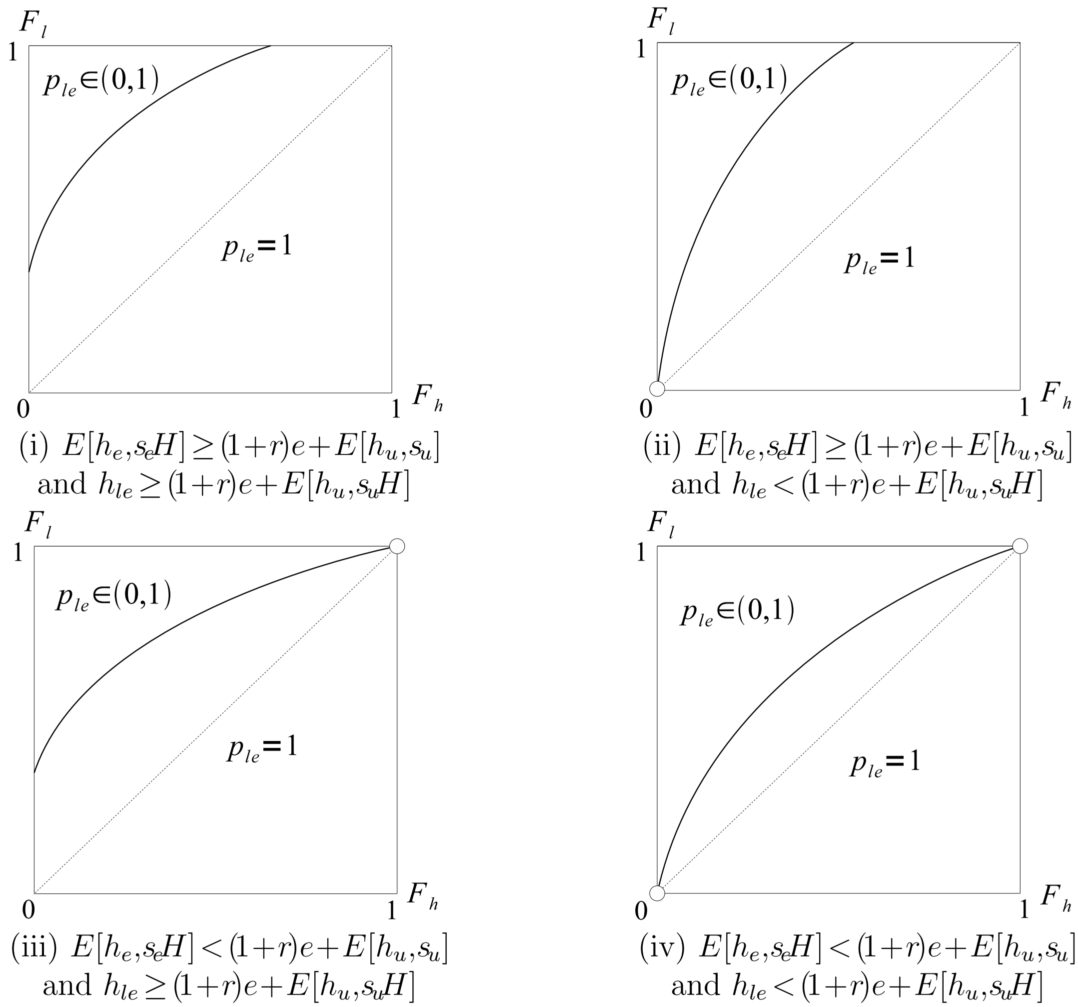


Figure 2: Educational choices of type l individuals when $E[h_e,s_eH]>(1+r)e+E[h_u,s_uH]$ and $h_{le}<(1+r)e+E[h_u,s_u]$ (Lemma 3 (Ⅲ)(c))

2.2 Wages

Equilibrium wages (*net* of the cost of education for educated workers) are derived by plugging values of $p_{he}(=1)$ and p_{le} into (2) and (3). Lemma 5 shows how they depend on F_h and F_l.

Lemma 5 (Relationship between wages and F_h, F_l)

(i) When $p_{le}=1$ and $s_u>0$ ($s_e>0$), w_{ju} (w_{je}) ($j=h,l$) decreases (increases) with F_h and increases (decreases) with F_l.

(ii) When $p_{le}\in(0,1)$ and $s_u,s_e>0$, wages decrease with F_h for $F_l\leq F_h$; for $F_l>F_h$, they decrease (increase) with F_h if $E[h_e,s_eH]<(>)(1+r)e+E[h_u,s_uH]$; and they do not depend on F_l. When $s_u=0$ or $s_e=0$, they are constant.

(iii) When $p_{le}=0$, w_{ju} ($j=h,l$) decreases with F_h if $s_u>0$, while $w_{he}=h_{he}-(1+r)e$.

When $p_{le}=1$ and $s_u>0$ ($s_e>0$), higher F_h and lower F_l are associated with the lower (higher) average skill of the uneducated (educated) and thus lower w_{ju} (higher w_{je}) ($j=h,l$).

When $p_{le}=0$, only type h individuals take education, thus uneducated wages depend only on F_h (when $s_u>0$) and w_{he} is same as under $s_u=s_e=0$. By contrast, when $p_{le}\in(0,1)$, F_h and F_l affect the average skills not only directly but also indirectly through p_{le} so that $w_{le}=w_{lu}$ is maintained, and the indirect effect operates opposite in direction: higher F_h and lower F_l lead to higher p_{le} and thus the higher (lower) average skill of the uneducated (educated). When $s_u=0$ or $s_e=0$, the opposing effects offset each other and the wages are independent of F_h and F_l. The same is true for F_l when $s_u,s_e>0$. As for F_h when $s_u,s_e>0$, the direct effect is stronger (weaker) than the indirect one for w_{ju} (w_{je}) and the wages decrease with F_h for $F_l\leq F_h$, while the same (opposite) is true for $F_l>F_h$ when $E[h_e, s_eH]<(>)(1+r)e+E[h_u, s_uH]$, which corresponds to the case of Figure 1 (Figure 2).

3 Dynamic model

Based on the results in the previous section, this section presents and analyzes the dynamic model. Consider a small open OLG economy populated by a continuum of individuals who are heterogeneous in terms of wealth and ability determined (exogenously) outside education. An individual lives for two periods, first as a child and then as an adult.

3.1 Lifetime of an individual

Childhood: In childhood, an individual receives a transfer from her parent and spends it on two investment options, assets (which yields interest rate r) and education (which costs e), in order to maximize future income. Consider an individual born into lineage i in period $t-1$ (generation t) who receives b_t^i units of transfer and can allocate it between asset a_t^i and education e_t^i. As shown in the previous section, $p_{he,t}=1$, while $p_{le,t}$ can take any value between 0 and 1 depending on $F_{h,t}$, $F_{l,t}$, and exogenous variables and parameters. When her type is j $(j=h,l)$ and $p_{je,t}=1$, the allocation is determined by b_t^i:

$$a_t^i=b_t^i, \qquad e_t^i=0, \quad \text{if} \quad b_t^i<e, \tag{9}$$

$$a_t^i=b_t^i-e, \quad e_t^i=e, \quad \text{if} \quad b_t^i\geq e. \tag{10}$$

By contrast, a type l individual is indifferent in the choice when $p_{le,t}\in(0,1)$ and $b_t^i\geq e_t$, while $a_t^i=b_t^i$ and $e_t^i=0$ otherwise.

Adulthood: In adulthood, she obtains income from assets and labor supply and spends it on consumption c_t^i and a transfer to her single child, b_{t+1}^i. She maximizes the utility

$$u_t^i = \left(c_t^i\right)^{1-\gamma_b}\left(b_{t+1}^i\right)^{\gamma_b}, \quad 0<\gamma_b<1. \tag{11}$$

subject to the budget constraint

$$c_t^i + b_{t+1}^i = w_t^i + (1+r)a_t^i, \tag{12}$$

where w_t^i is her gross wage. By solving the maximization problem, the following consumption and transfer rules are obtained.

$$c_t^i = (1-\gamma_b)\{w_t^i + (1+r)a_t^i\}, \tag{13}$$

$$b_{t+1}^i = \gamma_b\{w_t^i + (1+r)a_t^i\}. \tag{14}$$

Generational change: At the beginning of period $t+1$, current adults pass away, current children become adults, and new children are born into the economy. Since each adult has one child, the population of each generation is time-invariant and normalized to be one.

3.2 Dynamics of individual transfers

The dynamic equation linking the received transfer b_t^i to the transfer given to the next generation b_{t+1}^i is derived from (14). For a current uneducated worker of type j $(j = l, h)$, it is obtained by substituting $w_t^i = w_{ju,t}$ and $a_t^i = b_t^i$ into (14):

$$b_{t+1}^i = \gamma_b\{w_{ju,t} + (1+r)b_t^i\}. \tag{15}$$

The assumption $\gamma_b(1+r) < 1$ is made so that the fixed point of the equation for *given* $w_{ju,t}$, $b^*(w_{ju,t}) \equiv \frac{\gamma_b}{1-\gamma_b(1+r)} w_{ju,t}$, exists. The fixed point becomes crucial in later analyses. For a present educated worker of type j $(j = l, h)$, the dynamic equation is

$$b_{t+1}^i = \gamma_b\{w_{je,t} + (1+r)b_t^i\}, \tag{16}$$

which is obtained by substituting $w_t^i = w_{je,t} + (1+r)e$ ($w_{je,t}$ is the net wage) and $a_t^i = b_t^i - e$ into (14). When $p_{le,t} \in (0,1)$, $w_{le,t} = w_{lu,t}$ holds, so the two equations coincide for type l.

The equations show that the dynamics of transfers within a lineage depend on the time evolution of wage and education levels and thus (from Sections 2.1 and 2.2) the evolution of $F_{h,t}$, $F_{l,t}$, and ability determined outside education. It is assumed that the intergenerational correlation of the ability is 1, that is, descendants of a type j individual continue to be type j. Although this assumption is strong, considering that the ability reflects not only innate ability, which itself shows a high correlation, but also abilities developed through interactions with parents, it would be more realistic than the other extreme of zero correlation.

3.3 Aggregate dynamics

The time evolution of $F_{h,t}$ and $F_{l,t}$, the fractions of type h and l individuals who can afford education, is in turn determined by the dynamics of individual transfers. That is, the individual and aggregate dynamics are interrelated.

Specifically, when $p_{ju}=1$ $(j=l,h)$, if offspring of some of type j uneducated workers become accessible to education through wealth accumulation, $Fr_{j,t+1}>Fr_{j,t}$, while, if some of educated workers cannot leave enough transfers to cover the cost of education, $Fr_{j,t+1}<Fr_{j,t}$. The former occurs iff there exist lineages satisfying $b_t^i<e$ and $b_{t+1}^i\geq e$. From (15), the following condition must hold for such lineages to exist:

$$b^*(w_{ju,t})\equiv\frac{\gamma_b}{1-\gamma_b(1+r)}w_{ju,t}>e. \tag{17}$$

By contrast, the latter occurs iff lineages satisfying $b_t^i\geq e$ and $b_{t+1}^i<e$ exist. From (16), the necessary condition is

$$b^*(w_{je,t})\equiv\frac{\gamma_b}{1-\gamma_b(1+r)}w_{je,t}<e. \tag{18}$$

Since $b^*(w_{je,t})\geq b^*(w_{ju,t})$, the above equations cannot hold simultaneously. If (17) holds, $Fr_{j,t+1}\geq Fr_{j,t}$, while if (18) is true, $Fr_{j,t+1}\leq Fr_{j,t}$; $Fr_{j,t+1}=Fr_{j,t}$ is possible depending on the distribution of transfers over the population, but, if the condition continues to hold, $Fr_{j,t}$ does change at some point. When neither equations are satisfied, $Fr_{j,t+1}=Fr_{j,t}$. The dynamics of $Fr_{l,t}$ when $p_{le,t}\in(0,1)$ are determined by the relative value of $b^*(w_{lu,t})=b^*(w_{le,t})$ to e and the dynamics when $p_{le,t}=0$ depend on the relative value of $b^*(w_{lu,t})$ to e only.

By substituting (3) with $p_{he}=p_{le}=1$ into (17) (with '$>$' replaced by '$\gtreqless$'), and substituting (2) with $p_{he}=p_{le}=1$ into (18) (with '$<$' replaced by '$\gtreqless$') and rearranging, the critical equations when $p_{le}=1$ are given by (time subscripts are suppressed):

$$b^*(w_{hu})\gtreqless e\;\Leftrightarrow\; s_u\frac{(1-F_h)Hh_{hu}+(1-F_l)(1-H)h_{lu}}{(1-F_h)H+(1-F_l)(1-H)}+(1-s_u)h_{hu}\gtreqless\tfrac{1-\gamma_b(1+r)}{\gamma_b}e, \tag{19}$$

$$\Leftrightarrow F_l\gtreqless 1-\frac{\left[h_{hu}-\frac{1-\gamma_b(1+r)}{\gamma_b}e\right]H(1-F_h)}{\left\{\frac{1-\gamma_b(1+r)}{\gamma_b}e-E[h_u,1-s_u]\right\}(1-H)},\quad\text{if } E[h_u,1-s_u]<\tfrac{1-\gamma_b(1+r)}{\gamma_b}e, \tag{20}$$

$$b^*(w_{lu})\gtreqless e\;\Leftrightarrow\; s_u\frac{(1-F_h)Hh_{hu}+(1-F_l)(1-H)h_{lu}}{(1-F_h)H+(1-F_l)(1-H)}+(1-s_u)h_{lu}\gtreqless\tfrac{1-\gamma_b(1+r)}{\gamma_b}e, \tag{21}$$

$$\Leftrightarrow F_l\gtreqless 1-\frac{\left\{E[h_u,s_u]-\frac{1-\gamma_b(1+r)}{\gamma_b}e\right\}H}{\left[\frac{1-\gamma_b(1+r)}{\gamma_b}e-h_{lu}\right](1-H)}(1-F_h),\quad\text{if } h_{lu}<\tfrac{1-\gamma_b(1+r)}{\gamma_b}e, \tag{22}$$

$$b^*(w_{he})\gtreqless e\;\Leftrightarrow\; s_e\frac{F_hHh_{he}+F_l(1-H)h_{le}}{F_hH+F_l(1-H)}+(1-s_e)h_{he}\gtreqless\frac{e}{\gamma_b}, \tag{23}$$

$$\Leftrightarrow F_l\lesseqgtr\frac{h_{he}-\frac{e}{\gamma_b}}{\frac{e}{\gamma_b}-E[h_e,1-s_e]}\frac{H}{1-H}F_h,\quad\text{if } E[h_e,1-s_e]<\frac{e}{\gamma_b}, \tag{24}$$

$$b^*(w_{le})\gtreqless e\;\Leftrightarrow\; s_e\frac{F_hHh_{he}+F_l(1-H)h_{le}}{F_hH+F_l(1-H)}+(1-s_e)h_{le}\gtreqless\frac{e}{\gamma_b}, \tag{25}$$

$$\Leftrightarrow F_l\lesseqgtr\frac{E[h_e,s_e]-\frac{e}{\gamma_b}}{\frac{e}{\gamma_b}-h_{le}}\frac{H}{1-H}F_h,\quad\text{if } h_{le}<\frac{e}{\gamma_b}. \tag{26}$$

When $p_{le}=0$, by substituting $F_l=0$ into (19),[15]

$$b^*(w_{hu}) \gtreqless e \;\Leftrightarrow\; s_u\frac{(1-F_h)Hh_{hu}+(1-H)h_{lu}}{(1-F_h)H+(1-H)}+(1-s_u)h_{hu} \gtreqless \frac{1-\gamma_b(1+r)}{\gamma_b}e, \tag{27}$$

$$\Leftrightarrow F_h \lesseqgtr 1-\frac{\{\frac{1-\gamma_b(1+r)}{\gamma_b}e-E[h_u,1-s_u]\}(1-H)}{[h_{hu}-\frac{1-\gamma_b(1+r)}{\gamma_b}e]H}. \tag{28}$$

Regarding values of h_{he} and h_{lu} relative to e, the following assumption is imposed.

Assumption 3 (Dynamics of $F_{h,t}$ and $F_{l,t}$ when $s_e=s_u=0$)

(i) $w_{he,max}\equiv h_{he}-(1+r)e \geq \frac{1-\gamma_b(1+r)}{\gamma_b}e.$

(ii) $w_{lu,min}\equiv h_{lu} \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e.$

If (i) [(ii)] is not satisfied, $F_{h,t}$ and $F_{l,t}$ decrease [increase] over time and $F_h=F_l=0$ [$=1$] in the long run, as when $s_e=s_u=0$. The assumption is imposed to rule out these obvious situations. From the equations, under complete information, $F_{h,t}$ non-decreases and $F_{l,t}$ non-increases over time, and, in the long run, $F_h=1$ (when $h_{hu}>\frac{1-\gamma_b(1+r)}{\gamma_b}e$) or $F_h=F_{h,0}$ (otherwise), and $F_l=0$ (when $h_{le}<\frac{e}{\gamma_b}$) or $F_l=F_{l,0}$ (otherwise).

The next lemma presents conditions for existence of $b^*(w_{jk})=e$ ($j=h,l;\,k=e,u$) and its position on the (F_h,F_l) plane. (If it does not exist, $b^*(w_{jk})>e$ or $<e$ for any F_h and F_l.)

Lemma 6 (Positions of $b^*(w_{jk})=e$ $(j=h,l;\,k=e,u)$)

(i) $b^*(w_{hu})=e$ *when* $p_{le}=1$ *exists only if* $h_{hu}>\frac{1-\gamma_b(1+r)}{\gamma_b}e>E[h_u,1-s_u]$*. When* $E[h_u,1-s_u(1-H)]<(>)\frac{1-\gamma_b(1+r)}{\gamma_b}e$*, it intersects with* $F_h=0$ *at* $F_l\in(0,1)$ ($F_l=0$ *at* $F_h\in(0,1)$)*.*

(ii) $b^*(w_{lu})=e$ *exists only if* $E[h_u,s_u]>\frac{1-\gamma_b(1+r)}{\gamma_b}e$*. When* $E[h_u,s_uH]<(>)\frac{1-\gamma_b(1+r)}{\gamma_b}e$*, it intersects with* $F_h=0$ *at* $F_l\in(0,1)$ ($F_l=0$ *at* $F_h\in(0,1)$)*.*

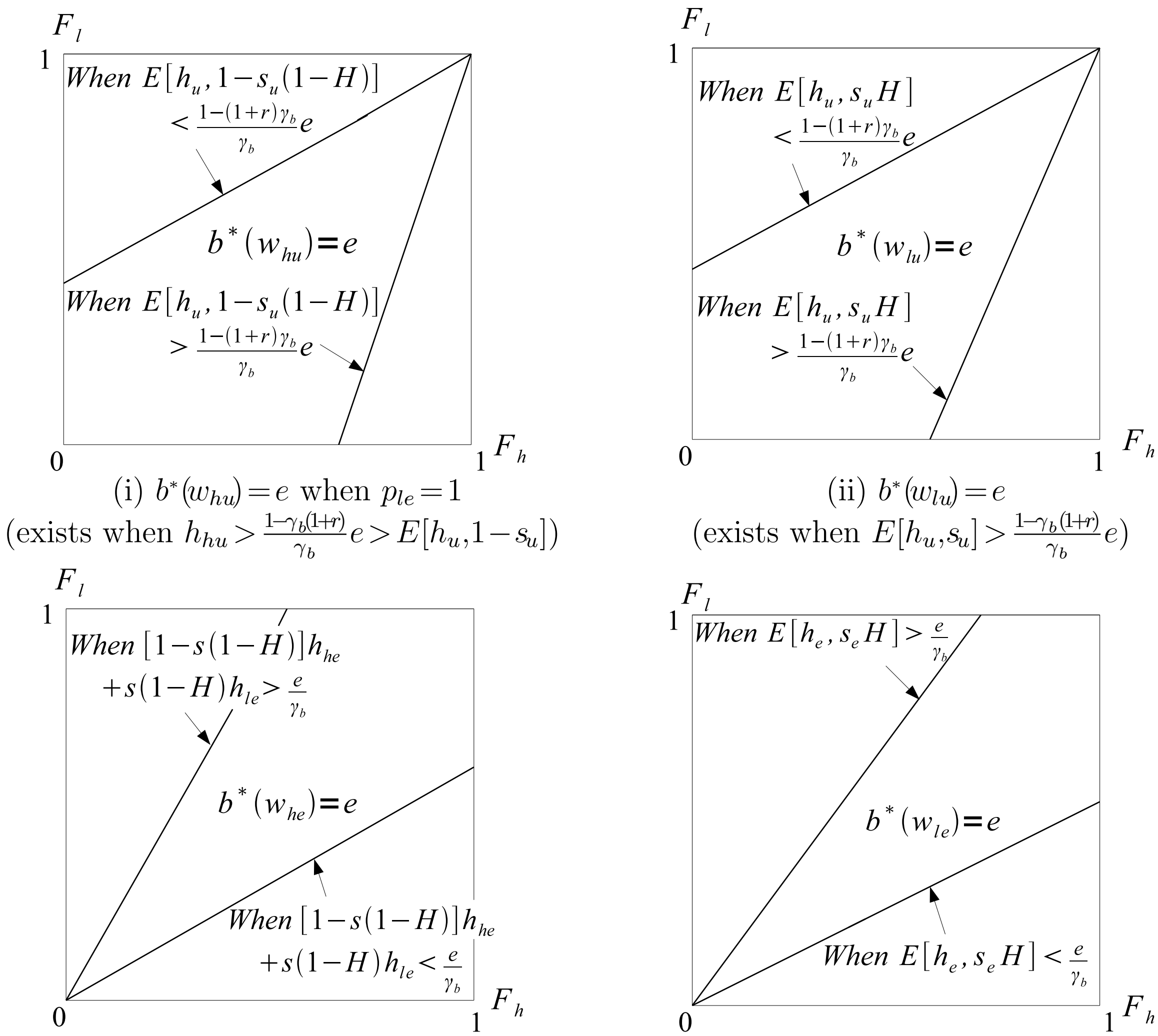
(iii) $b^*(w_{he})=e$ *exists only if* $E[h_e,1-s_e]<\frac{e}{\gamma_b}$*. When* $E[h_e,1-s_e(1-H)]<(>)\frac{e}{\gamma_b}$*, it intersects with* $F_h=1$ *at* $F_l\in(0,1)$ ($F_l=1$ *at* $F_h\in(0,1)$)*.*

(iv) $b^*(w_{le})=e$ *exists only if* $E[h_e,s_e]>\frac{e}{\gamma_b}>h_{le}$*. When* $E[h_e,s_eH]<(>)\frac{e}{\gamma_b}$*, it intersects with* $F_h=1$ *at* $F_l\in(0,1)$ ($F_l=1$ *at* $F_h\in(0,1)$)*.*

(v) $b^*(w_{hu})=e$ *when* $p_{le}=0$ *exists at* $F_h\in(0,1)$ *only if* $E[h_u,1-s_u(1-H)]>\frac{1-\gamma_b(1+r)}{\gamma_b}e>E[h_u,1-s_u]$*.*

Figure 3 illustrates Lemma 6 (i)-(iv) graphically on the (F_h,F_l) plane. As for $b^*(w_{ju})=e$ $(j=h,l)$ [Figure 3 (i) and (ii)], $b^*(w_{ju})>(<)e$ above (below) the locus, since w_{ju} when $p_{le}=1$ increases with F_l and decreases with F_h. As h_{hu} and h_{lu} become greater relative to e, the locus shifts downward and the region satisfying $b^*(w_{ju})>e$ expands. As for $b^*(w_{je})=e$ [Figure 3 (iii) and (iv)], $b^*(w_{je})>(<)e$ *below* (*above*) the locus, and, as h_{he} and h_{le} become greater relative to e, the locus shifts *upward* and the region satisfying $b^*(w_{je})>e$ expands.

[15] $b^*(w_{lu})\gtreqless e$ when $p_{le}=0$ is not presented because it is not used in later analyses. From Assumption 3 (i) just below, $b^*(w_{he})>e$ when $p_{le}=0$.



(i) $b^*(w_{hu}) = e$ when $p_{le} = 1$
(exists when $h_{hu} > \frac{1-\gamma_b(1+r)}{\gamma_b}e > E[h_u, 1-s_u]$)

(ii) $b^*(w_{lu}) = e$
(exists when $E[h_u, s_u] > \frac{1-\gamma_b(1+r)}{\gamma_b}e$)

(iii) $b^*(w_{he}) = e$
(exists when $E[h_e, 1-s_e] < \frac{e}{\gamma_b}$)

(iv) $b^*(w_{le}) = e$
(exists when $E[h_e, s_e] > \frac{e}{\gamma_b} > h_{le}$)

Figure 3: Positions of $b^*(w_{jk}) = e$ $(j = h, l;\ k = e, u)$ (Lemma 6)

Figure 3 has presented positions of the critical loci under the supposition that $p_{le} = 1$ for any F_h and F_l, which is generally *not* true, as proved in Lemmas 3 and 4 and as illustrated in Figures 1 and 2. $b^*(w_{jk}) = e$ $(j = h, l; k = e, u)$ when $p_{le} = 1$ $(= 0)$ is *effective* only in the region $p_{le} = 1$ $(= 0)$ on the (F_h, F_l) plane, i.e. in the region below the dividing line between $p_{le} = 1$ and $p_{le} \in (0, 1)$ (at the left side of the dividing line between $p_{le} = 0$ and $p_{le} > 0$). Thus, the relationship between $b^*(w_{jk}) = e$ and the dividing lines must be known.

Lemmas A1, A2, and A3 of Appendix summarize the relation of the dividing line between p_{le}=1 and $p_{le} \in (0,1)$ with $b^*(w_{he}) = e$, $b^*(w_{hu}) = e$, and $b^*(w_{lk}) = e$ $(k = e, u)$, respectively. Lemmas A1 and A2 are illustrated in Figures A1 and A2 as well. The lemmas are used in proving main results presented in the next section, including the dynamics of $F_{h,t}$ and $F_{l,t}$.

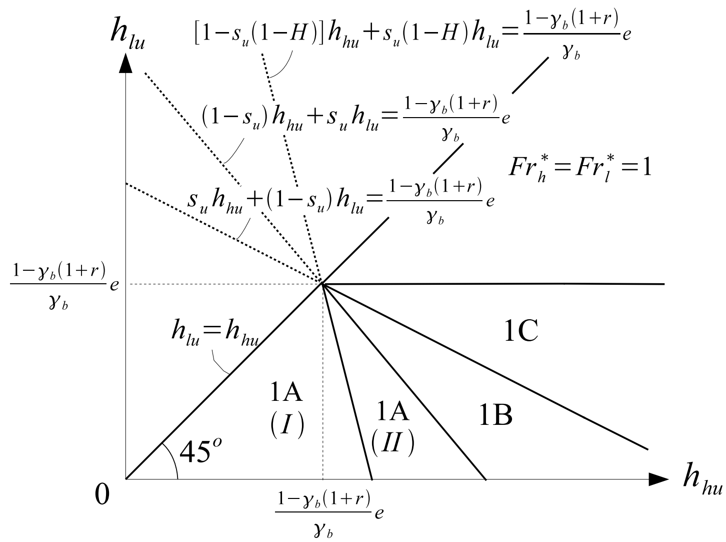


Figure 4: Regions of Propositions 1A, 1B, and 1C on the (h_{hu}, h_{lu}) plane

4 Results

4.1 Dynamics of $F_{h,t}$ and $F_{l,t}$ and of related variables

This subsection analyzes the dynamics of $F_{h,t}$ and $F_{l,t}$ and of related variables such as wages, wage inequality, and the signaling value of education. It also examines how the initial distribution of wealth affects steady-state values of the variables. The dynamics are qualitatively different depending on exogenous variables and parameters. Hence, for ease of presentation, the results are presented in three propositions. Roughly speaking, Propositions 1A, 1B, and 1C below correspond to cases in which h_{hu} and h_{lu} are low (especially h_{hu}), intermediate, and high (especially h_{lu}), respectively (see Figure 4).[16] In terms of the cost of education, they correspond to cases in which e is high, intermediate, and low, respectively. In the figure and in the propositions, F_j^* $(j = h, l)$ denotes a steady-state value of F_j. As mentioned earlier, if $h_{lu} > \frac{1-\gamma_b(1+r)}{\gamma_b}e$ (not considered below), $F_h^* = F_l^* = 1$. In some cases, the results are presented only for $F_{l,t} \leq F_{h,t}$, since it is reasonable to assume $F_{l,0} \leq F_{h,0}$ and, in these cases, $F_{l,t} \leq F_{h,t}$ is satisfied for any t when $F_{l,0} \leq F_{h,0}$.

Proposition 1A presents the dynamics when $E[h_u, 1-s_u] < \frac{1-\gamma_b(1+r)}{\gamma_b}e$. $b^*(w_{hu}) = e$ is the one when $p_{le} = 1$ except in (II)(a), in which it could be the one when $p_{le} = 0$ as well.

Proposition 1A (Dynamics of $\mathbf{F_{h,t}}$ and $\mathbf{F_{l,t}}$, part A) *Suppose* $E[h_u, 1-s_u] < \frac{1-\gamma_b(1+r)}{\gamma_b}e$.

(I) *When* $E[h_u, 1-s_u(1-H)] \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e$

(a) If $E[h_e, s_e] \leq \frac{e}{\gamma_b}$ *(< with* $s_e = 0$*),* $F_{l,t}$ *decreases over time and* $F_l^* = 0$.

[16] Since $s_e, s_u \in [0, \frac{1}{2}]$, the weight on h_{hu} (h_{lu}) is greater than h_{lu} (h_{hu}) in the equation dividing Propositions 1A and 1B (1B and 1C).

(*i*) *If* $E[h_e, 1-s_e(1-H)] < \frac{e}{\gamma_b}$ *and* $(1-s_e)(h_{he}-h_{le})+h_{lu} < \frac{1-\gamma_b(1+r)}{\gamma_b}e$, $F_{h,t}$ *decreases* (*is constant*) *above* (*on or below*) *effective* $b^*(w_{he})=e$.[17]

(*ii*) *Otherwise,* $F_{h,t}$ *is constant for* $F_{l,t} \leq F_{h,t}$ *and* $F_h^* = F_{h,0}$.

(*b*) *If* $E[h_e, s_e] > \frac{e}{\gamma_b}$ ($\geq$ *with* $s_e = 0$), $F_{h,t}$ *is constant for* $F_{l,t} \leq F_{h,t}$ *and* $F_h^* = F_{h,0}$.

(*i*) *If* $E[h_e, s_e H] < \frac{e}{\gamma_b}$, $F_{l,t}$ *decreases above* $b^*(w_{le})=e$ *and is constant otherwise.*

(*ii*) *Otherwise,* $F_{l,t}$ *too is constant for* $F_{l,t} \leq F_{h,t}$ *and* $F_l^* = F_{l,0}$.

(*II*) *When* $E[h_u, 1-s_u(1-H)] > \frac{1-\gamma_b(1+r)}{\gamma_b}e$

(*a*) *When* $E[h_e, s_e] \leq \frac{e}{\gamma_b}$ ($<$ *with* $s_e = 0$), $F_{l,t}$ *decreases and* $F_l^* = 0$. $F_{h,t}$ *increases above* (*or at the left side of*) *effective* $b^*(w_{hu}) = e$, *decreases above effective* $b^*(w_{he}) = e$, *and is constant otherwise.*[18]

(*b*) *When* $E[h_e, s_e] > \frac{e}{\gamma_b}$ ($\geq$ *with* $s_e = 0$)

(*i*) *If* $E[h_e, s_e H] < \frac{e}{\gamma_b}$, $b^*(w_{le})=e$ *and* $b^*(w_{hu})=e$ *intersect below the* 45° *line* (*and below the dividing line between* $p_{le} = 1$ *and* $p_{le} \in (0,1)$, *if it exists*). $F_{l,t}$ *decreases* (*is constant*) *above* (*on or below*) $b^*(w_{le})=e$ *and* $F_{h,t}$ *increases* (*is constant*) *at the left* (*right*) *side of effective* $b^*(w_{hu})=e$.[19]

(*ii*) *Otherwise,* $F_{l,t}$ *is constant for* $F_{l,t} \leq F_{h,t}$ *and* $F_l^* = F_{l,0}$, *and* $F_{h,t}$ *increases* (*is constant*) *at the left* (*right*) *side of* $b^*(w_{hu})=e$.

Figure 5 illustrates the dynamics when $E[h_u, 1-s_u(1-H)] \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e$ (Proposition 1A(I)), that is, when h_{hu} and h_{lu} (especially h_{hu}) are very low, or e is very high (see Figure 4).[20] Horizontal (vertical) arrows represent directions of motion of $F_{h,t}$ ($F_{l,t}$), while, in regions with slanted lines, both $F_{h,t}$ and $F_{l,t}$ are time-invariant. As explained in footnote 20, it is possible that the position of the dividing line and the line's relation with $b^*(w_{he}) = e$ are different from those in the figure, but qualitative results are mostly unchanged.

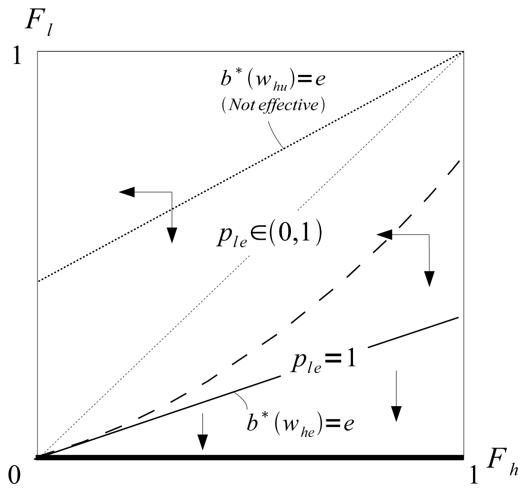
The figure shows that $F_{h,t}$ never increases at $F_{l,t} \leq F_{h,t}$. This is so even when $h_{hu} > \frac{1-\gamma_b(1+r)}{\gamma_b}e$ (and $s_u > 0$), in which $F_{h,t}$ increases over time under $s_u = s_e = 0$. Since h_{hu} and/or h_{lu} are very low (or e is very high), the presence of type l uneducated workers depresses $w_{hu,t}$ below $\frac{1-\gamma_b(1+r)}{\gamma_b}e$ and descendants of the type h uneducated cannot accumulate enough wealth

[17] $b^*(w_{he})=e$ is below the 45° line. If $(1-s_e)(h_{he}-h_{le})+E[h_u, s_u H] > \frac{1-\gamma_b(1+r)}{\gamma_b}e$, it intersects with the dividing line between $p_{le}=1$ and $p_{le} \in (0,1)$, and $F_{h,t}$ is constant when it is smaller than the value at the intersection.

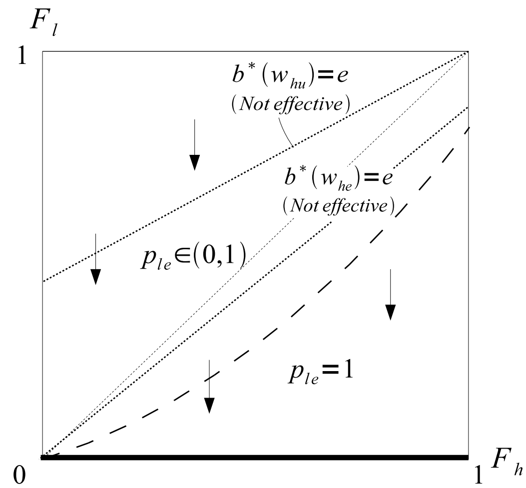
[18] See the proof of the proposition for possible relations among the loci.

[19] The dividing line exists when $h_{le} < (1+r)e + E[h_u, s_u]$. $b^*(w_{hu})=e$ intersects with the line if $E[h_e, s_e H] + (1-s_u)(h_{hu}-h_{lu}) < \frac{e}{\gamma_b}$, and $F_{h,t}$ is time-invariant when it is greater than the value at the intersection.

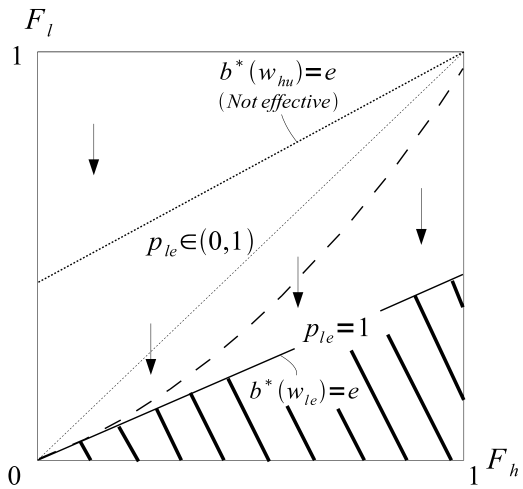
[20] (a)(i) and (b)(i) of the proposition (Figure 5 (a)(i) and (b)(i)) apply only when $s_e > 0$. In (a)(i), the dividing line may intersect with $b^*(w_{he}) = e$ (see footnote 17) or $p_{le} = 1$ at least for any $F_l \leq F_h$ may hold; in (a)(ii), $p_{le} = 0$ for any F_l and F_h, $p_{le} = 0$ for small F_h, and $p_{le} = 1$ for $F_l \leq F_h$ are possible; and in (b)(i), $p_{le} = 1$ at least for $F_l \leq F_h$ may hold.



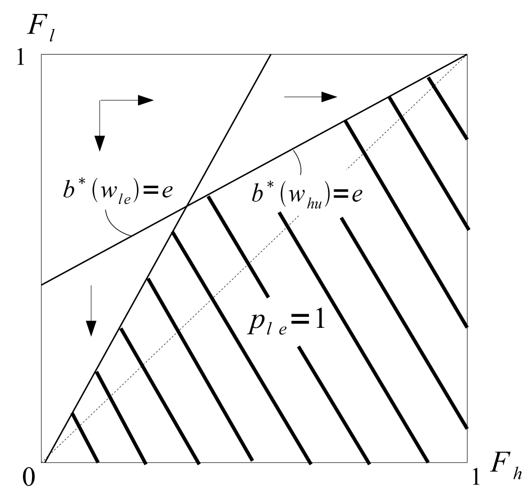
(a)(i) $(E[h_e,s_e]<)E[h_e,1-s_e(1-H)]<\frac{e}{\gamma_b}$ and $(1-s_e)(h_{he}-h_{le})+h_{lu}<\frac{1-\gamma_b(1+r)}{\gamma_b}e$



(a)(ii) $E[h_e,s_e]\leq\frac{e}{\gamma_b}$ ($<$ when $s_e=0$) and the other conditions of (a)(i) are not satisfied



(b)(i) $E[h_e,s_e]>\frac{e}{\gamma_b}>E[h_e,s_eH]$



(b)(ii) $E[h_e,s_eH]\geq\frac{e}{\gamma_b}$

Figure 5: Examples of the dynamics of $F_{h,t}$ and $F_{l,t}$ when $E[h_u,1-s_u(1-H)]\leq\frac{1-\gamma_b(1+r)}{\gamma_b}e$ (Proposition 1A (I))

for education. Further, when h_{he} is particularly low (Figure 5 (a)(i)),[21] $F_{h,t}$ decreases over time above $b^*(w_{he})=e$ (where $b^*(w_{he,t})<e$ holds) due to the low average skill of the educated.

As for the dynamics of $F_{l,t}$, when $E[h_e,s_e]\leq\frac{e}{\gamma_b}$ ($<$ if $s_e=0$), that is, when h_{he} and (particularly) h_{le} are low (see Figure 6),[22] $F_{l,t}$ decreases over time and $F_l^*=0$ (Figure 5 (a)(i) and (a)(ii)). Even with $F_{h,0}>>0$, it is possible that *nobody* can afford education in the long

[21]Since $(1-s_e)(h_{he}-h_{le})+h_{lu}<\frac{1-\gamma_b(1+r)}{\gamma_b}e\Leftrightarrow h_{le}>h_{he}-\frac{1}{1-s_e}[\frac{1-\gamma_b(1+r)}{\gamma_b}e-h_{lu}]$, h_{le} is not particularly low in (a)(i).

[22]In Figure 6, feasible (h_{he},h_{le}) must be at the right side of $h_{he}=(1+r)e+E[h_u,1-s_u(1-H)]$ from Assumption 1 and must be below $(1-s_e)(h_{he}-h_{le})=(1-s_u)(h_{hu}-h_{lu})$ from Assumption 2. The relative position of $(1-s_e)(h_{he}-h_{le})=(1-s_u)(h_{hu}-h_{lu})$ to $(h_{he},h_{le})=(\frac{e}{\gamma_b},\frac{e}{\gamma_b})$ is always as in the figure, and, as $h_{hu}-h_{lu}$ increases, it shifts downward. The corresponding figure for Proposition 1A(I) is slightly different from Figure 6 in that $(1+r)e+E[h_u,1-s_u(1-H)]\leq\frac{e}{\gamma_b}$.

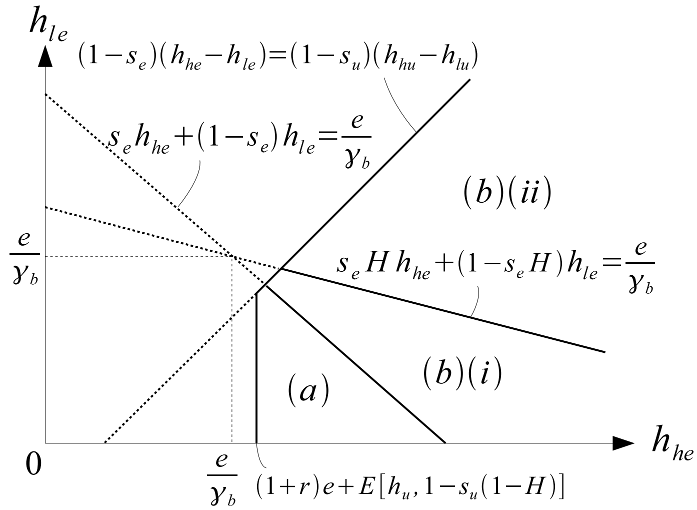


Figure 6: Regions of (a), (b)(i), and (b)(ii) of Propositions 1A(Ⅱ), 1B, and 1C on the (h_{he}, h_{le}) plane

run, i.e. $F_h^* = F_l^* = 0$ (Figure 5 (a)(i)). By contrast, when education is more effective, i.e. h_{he} and/or h_{le} are higher (Figure 5 (b)(i) and (b)(ii)), $F_l^* > 0$ from most initial conditions because of the positive effect of type h educated workers on w_{le}. This is so even when $h_{le} < \frac{e}{\gamma_b}$, in which $F_l^* = 0$ with $s_u = s_e = 0$. When the productivity of education is high, both $F_{h,t}$ and $F_{l,t}$ are time-invariant for any $F_{l,t} \leq F_{h,t}$ (Figure 5 (b)(ii)).

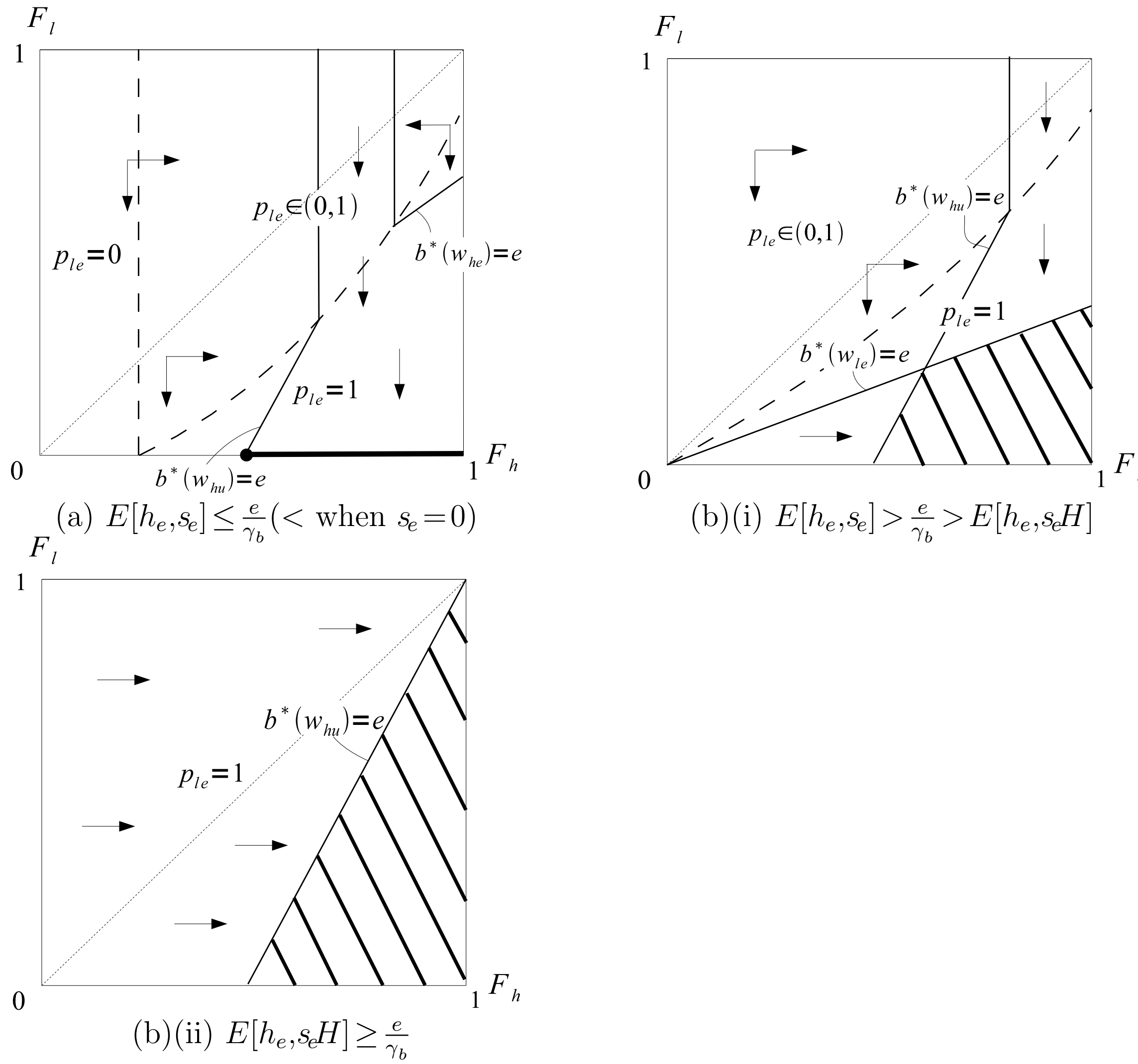
Wage dynamics can be analyzed based on the proposition and Lemma 5. In Figure 5 (a)(ii) and (b)(i) and in the region on or below $b^*(w_{he}) = e$ of Figure 5 (a)(i), since $F_{h,t}$ is constant and $F_{l,t}$ falls over time, when $p_{le,t} = 1$, wages of educated (uneducated) workers increase (decrease) and the inter-group wage inequality rises over time.[23]

The signaling value of education, $s_e \widetilde{E}[h_e] - s_u \widetilde{E}[h_u]$, increases with $F_{h,t}$ and decreases with $F_{l,t}$ when $p_{le,t} = 1$, is constant (equals $(1-s_u)h_{lu} - (1-s_e)h_{le} + (1+r)e$) when $p_{le,t} \in (0,1)$, and increases with $F_{h,t}$ when $p_{le,t} = 0$. Thus, it rises over time when $p_{le,t} = 1$, unless $F_{h,t}$ falls.

Finally, the relationship between the initial distribution of wealth and steady-state values of the variables is discussed. Except in (a)(i), F_j^* $(j = h, l)$ and the proportion of educated workers are higher when $F_{h,0}$ and $F_{l,0}$ (in (b)(i) and (b)(ii)) are higher. In (a)(i), by contrast, if $F_{l,0}$ is high enough that $b^*(w_{he,0}) < e$, the proportion decreases with $F_{l,0}$. Educated wages are lower (in (b)(i) and (b)(ii)), uneducated wages are higher, and the wage differential and the signaling value are lower in the long run, when $F_{h,0}$ is lower and $F_{l,0}$ is higher, that is, when initial wealth is distributed relatively equally over the two types.

Similarly, Figure 7 shows the dynamics when $E[h_u, 1-s_u(1-H)] > \frac{1-\gamma_b(1+r)}{\gamma_b} e > E[h_u, 1-s_u]$,

[23] When $p_{le,t} < 1$, from Lemma 5 (ii) and (iii), wages are constant. As for the region above $b^*(w_{he}) = e$ of Figure 5 (a)(i), since both $F_{h,t}$ and $F_{l,t}$ decrease over time, directions of motion of wages are ambiguous when $p_{le,t} = 1$, whereas they increase over time when $p_{le,t} \in (0,1)$.



(a) $E[h_e, s_e] \leq \frac{e}{\gamma_b}$ ($<$ when $s_e = 0$) (b)(i) $E[h_e, s_e] > \frac{e}{\gamma_b} > E[h_e, s_e H]$

(b)(ii) $E[h_e, s_e H] \geq \frac{e}{\gamma_b}$

Figure 7: Examples of the dynamics of $F_{h,t}$ and $F_{l,t}$
when $E[h_u, 1-s_u(1-H)] > \frac{1-\gamma_b(1+r)}{\gamma_b} e > E[h_u, 1-s_u]$ (Proposition 1A (Ⅱ))

which is possible only when $s_u > 0$ (Proposition 1A (Ⅱ)).[24] Compared to the previous case, (b)(i) and (b)(ii) rather than (a) are more likely because higher h_{ju} ($j = h, l$) leads to higher h_{je} under realistic education technologies. With higher h_{hu} and/or h_{lu} (or lower e), $F_{h,t}$ now increases over time when it is not large and thus the average skill of the uneducated is not low, while the dynamics of $F_{l,t}$ are qualitatively same as before. In regions where $F_{h,t}$ increases, educated (uneducated) wages increase (decrease) and the wage differential (and the signaling value) rises over time when $p_{le,t} = 1$, whereas when $p_{le,t} \in (0,1)$, *all* wages

[24]Unlike Figure 5, ineffective portions of the critical loci are not presented. Figure 7 (b)(i) applies only when $s_e > 0$. In (a), $p_{le} = 0$ for any F_l and F_h (then, $b^*(w_{he}) > e$ and $b^*(w_{hu}) = e$ when $p_{le} = 0$ is effective), $p_{le} > 0$ always, and $p_{le} = 1$ at least for $F_l \leq F_h$ (then, $b^*(w_{he}) > e$) are possible. A portion of $b^*(w_{hu}) = e$ is always effective, while $b^*(w_{he}) = e$ may be ineffective. In (b)(i), $p_{le} = 1$ at least for $F_l \leq F_h$ may hold.

decrease (from Lemma 5) and the *average* wage inequality between educated and uneducated workers *falls*.[25] The relationship between the initial distribution and the long-run outcome in (b)(i) and (b)(ii) is qualitatively same as the corresponding cases of Proposition 1A(I), while, in (a), for high [low] $F_{h,0}$, it is qualitatively same as [similar to] (a)(i) [(a)(ii)] of 1A(I).

The dynamics when $E[h_u,1-s_u] \geq \frac{1-\gamma_b(1+r)}{\gamma_b}e$ ($>$ with $s_u=0$) and $E[h_u,s_uH] \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e$ is presented in the next proposition.

Proposition 1B (**Dynamics of $\mathbf{F_{h,t}}$ and $\mathbf{F_{l,t}}$, part B**) *If* $E[h_u,1-s_u] \geq$ ($>$ *with* $s_u=0$) $\frac{1-\gamma_b(1+r)}{\gamma_b}e \geq E[h_u,s_uH]$, $F_{h,t}$ *grows over time and* $F_h^*=1$.

(*a*) *When* $E[h_e,s_e] \leq \frac{e}{\gamma_b}$ ($<$ *with* $s_e=0$), $F_{l,t}$ *decreases over time and* $F_l^*=0$.

(*b*) *When* $E[h_e,s_e] > \frac{e}{\gamma_b}$ ($\geq$ *with* $s_e=0$)

(*i*) *If* $E[h_e,s_eH] < \frac{e}{\gamma_b}$, $F_{l,t}$ *decreases* (*is constant*) *above* (*on or below*) $b^*(w_{le})=e$.

(*ii*) *Otherwise,* $F_{l,t}$ *is time-invariant for* $F_{l,t} \leq F_{h,t}$ *and* $F_l^*=F_{l,0}$.

Figure 8 illustrates the proposition graphically.[26] The major difference from the previous cases is that $F_{h,t}$ increases over time and $F_h^*=1$, as when $s_u=s_e=0$ (note $h_{hu} > \frac{1-\gamma_b(1+r)}{\gamma_b}e$ in this case). With even higher h_{hu} and/or h_{lu} (or lower e), descendants of type h uneducated workers manage to accumulate enough wealth for education despite the negative effect of the presence of the type l uneducated (on $w_{hu,t}$). The dynamics of $F_{l,t}$ are qualitatively same as before and depend on h_{he}, h_{le}, and e. Thus, when the efficiency of education, particularly for type l, is very low (Figure 8 (a)), $F_{h,t}$ increases and $F_{l,t}$ decreases over time, and the two types are completely segregated by educational levels in the long run, i.e. $F_h^*=1$ and $F_l^*=0$, as under $s_u=s_e=0$ (note $h_{le} < \frac{e}{\gamma_b}$ in this case). By contrast, when education is more effective (Figure 8 (b)(i) and (b)(ii)), $F_{l,t}$ is constant for low or any $F_{l,t}$, thus F_l^* and the long-run proportion of educated workers increase with $F_{l,0}$. The dynamics of wages, the wage inequality, and the signaling value are qualitatively same as regions of increasing $F_{h,t}$ of the previous case. In the long run, since $F_h^*=1$, $w_{lu}=h_{lu}$ as under $s_u=s_e=0$, while educated wages decrease with F_l^*. Hence, both lower wage inequality (and lower signaling value) and higher educational level are realized with higher $F_{l,0}$ in (b)(i) and (b)(ii).

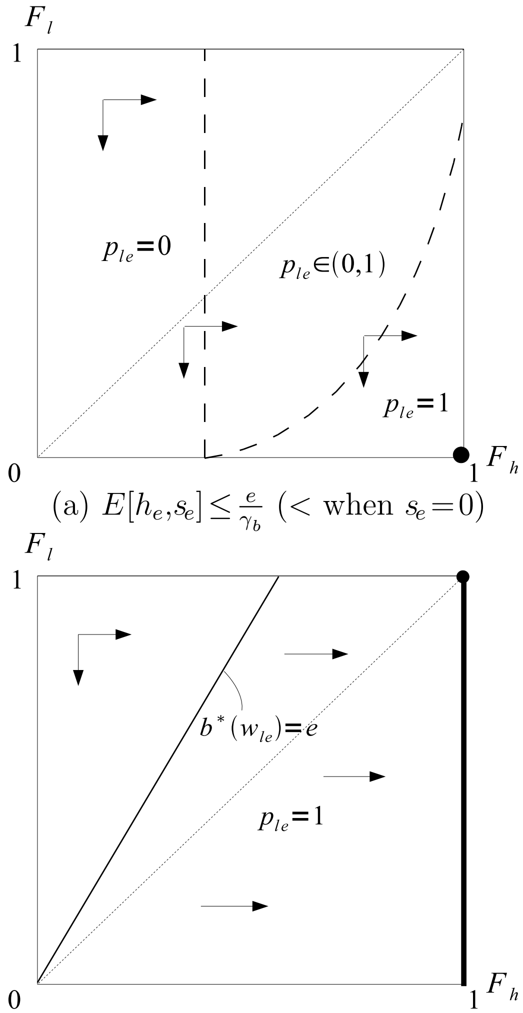
Finally, Proposition 1C presents the dynamics when $E[h_u,s_uH] > \frac{1-\gamma_b(1+r)}{\gamma_b}e$.

Proposition 1C (**Dynamics of $\mathbf{F_{h,t}}$ and $\mathbf{F_{l,t}}$, part C**) *When* $E[h_u,s_uH] > \frac{1-\gamma_b(1+r)}{\gamma_b}e$, $F_{h,t}$ *increases over time and* $F_h^*=1$.

(*a*) *When* $E[h_e,s_e] \leq \frac{e}{\gamma_b}$ ($<$ *with* $s_e=0$), $F_{l,t}$ *decreases over time and* $F_l^*=0$.

[25] Although *intra-type* wage differentials are constant, the ratio of type l to type h educated workers, which is proportional to the slope of the dividing line between $p_{le}=1$ and $p_{le} \in (0,1)$, *increases* over time.

[26] (b)(i) of the proposition (Figure 8 (b)(i)) applies only when $s_e>0$. In (a), $p_{le}=0$ for any F_l and F_h, $p_{le}>0$ always, or $p_{le}=1$ at least for $F_l \leq F_h$ may hold; and in (b)(i), $p_{le}=1$ at least for $F_l \leq F_h$ is possible.



(a) $E[h_e, s_e] \leq \frac{e}{\gamma_b}$ ($<$ when $s_e = 0$) (b)(i) $E[h_e, s_e] > \frac{e}{\gamma_b} > E[h_e, s_e H]$

(b)(ii) $E[h_e, s_e H] \geq \frac{e}{\gamma_b}$

Figure 8: Examples of the dynamics of $F_{h,t}$ and $F_{l,t}$
when $E[h_u, 1-s_u] \geq$ ($>$ with $s_u = 0$) $\frac{1-\gamma_b(1+r)}{\gamma_b} e \geq E[h_u, s_u H]$ (Proposition 1B)

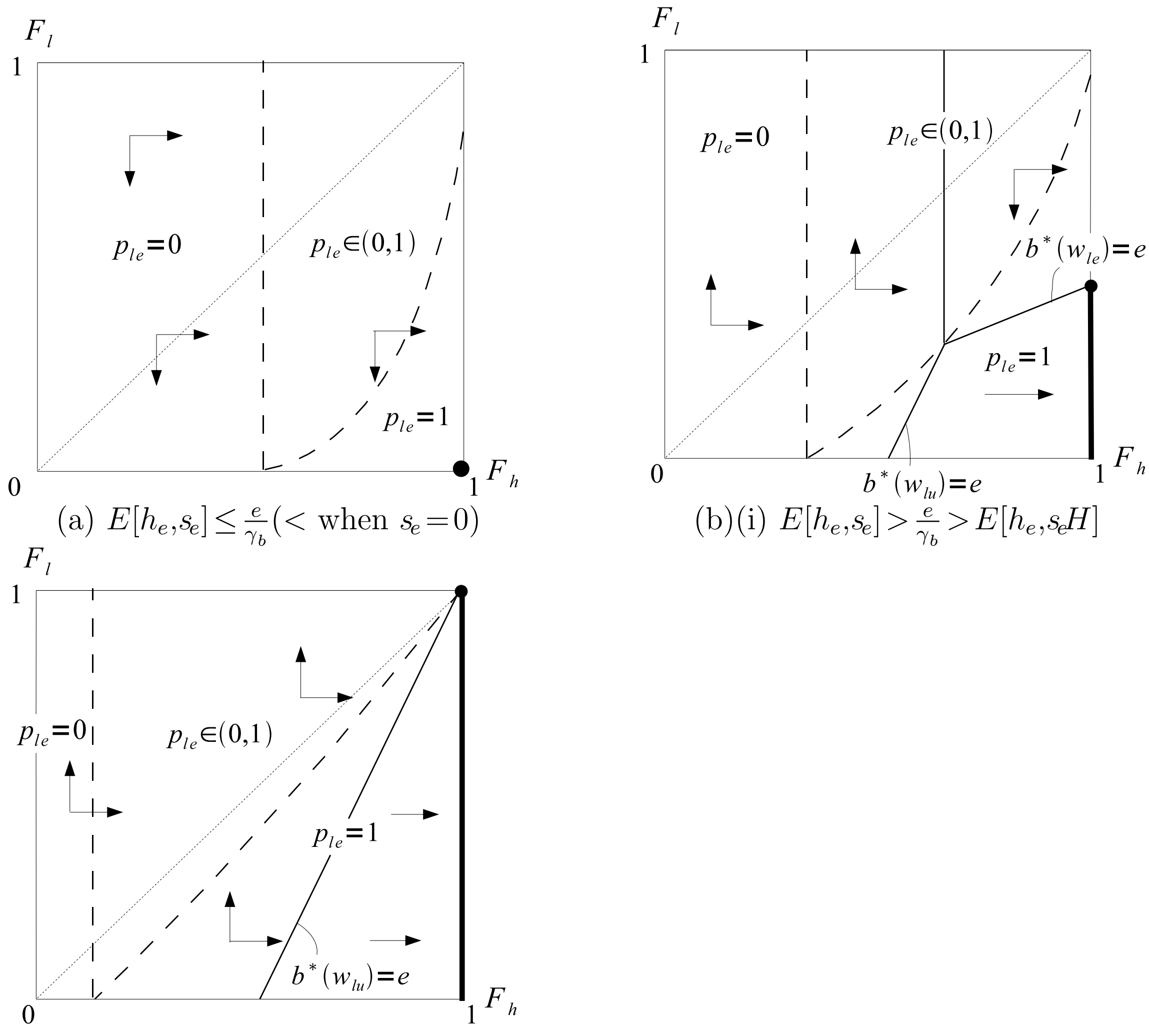
(*b*) *When* $E[h_e, s_e] > \frac{e}{\gamma_b}$ ($\geq$ *with* $s_e = 0$)

(*i*) *If* $E[h_e, s_e H] < \frac{e}{\gamma_b}$, $b^*(w_{le}) = e$ *and* $b^*(w_{lu}) = e$ *intersect on the dividing line between* $p_{le} = 1$ *and* $p_{le} \in (0,1)$ *below the* $45°$ *line.* $F_{l,t}$ *increases (decreases) above effective* $b^*(w_{lu}) = e$ ($b^*(w_{le}) = e$)*, and is constant otherwise.*

(*ii*) *Otherwise,* $F_{l,t}$ *increases (is constant) above (on or below)* $b^*(w_{lu}) = e$ *and* $F_l^* \in [F_{l,0}, 1]$.

The proposition, which applies only when $s_u > 0$ from Assumption 3 (ii), is illustrated in Figure 9.[27] As in the previous case, F_h increases over time and $F_h^* = 1$. What is new is that, when the productivity of education is not low, $F_{l,t}$ *increases* over time at the left side of effective $b^*(w_{lu}) = e$ (Figure 9 (b)(i) and (b)(ii)). Because of the positive effect of the

[27]Proposition 1C (b)(i) (Figure 9 (b)(i)) applies only when $s_e > 0$. In (a), $p_{le} = 0$ for any F_l and F_h may hold; and in (b)(ii), $p_{le} = 1$ at least for $F_l \leq F_h$ is possible. In (b)(i) and (b)(ii), $p_{le} > 0$ always may hold.



(a) $E[h_e,s_e]\le\frac{e}{\gamma_b}$(< when $s_e=0$)

(b)(i) $E[h_e,s_e]>\frac{e}{\gamma_b}>E[h_e,s_eH]$

(b)(ii) $E[h_e,s_eH]\ge\frac{e}{\gamma_b}$

Figure 9: Examples of the dynamics of $F_{h,t}$ and $F_{l,t}$ when $E[h_u,s_uH]>\frac{1-\gamma_b(1+r)}{\gamma_b}e$ (Prop. 1C)

presence of the type h uneducated on w_{lu}, children of some of the type l uneducated can access education, which is not possible with $s_u=s_e=0$.

When the efficiency of education is not very high (Figure 9 (b)(i)), however, the growth of $F_{l,t}$ ceases at some point, and even a temporary decline of it is possible. Such scenario shows interesting dynamics and thus is explained in detail. Suppose that $F_{h,0}$ is not high and thus $p_{le,t}=0$ is satisfied at first. With high $1-F_{h,t}$, the average skill of uneducated workers is high and thus education is not rewarding for type l. As $F_{h,t}$ increases (thus a larger portion of the type h take education) over time, uneducated wages fall, the signaling value of education ($s_e\widetilde{E}[h_e]-s_u\widetilde{E}[h_u]$) rises, and education becomes profitable at some point. Since $w_{lu,t}$ is still relatively high, only some of the non-poor type l take education, i.e. $p_{le,t}\in(0,1)$, and the net wages are equal, i.e. $w_{le,t}=w_{lu,t}$. While $p_{le,t}\in(0,1)$ is satisfied, the ratio of type l to type h educated workers rises, and all wages and the overall wage inequality between educated and uneducated workers *fall* over time.[28] If the growth of $F_{l,t}$ relative to $F_{h,t}$ is high, after $F_{h,t}$ reaches a certain level, $F_{l,t}$ starts to fall and $p_{le,t}$ increases. At some point, the net return to education for type l becomes positive, i.e. $p_{le}=1$, and thereafter, with $F_{h,t}$ growing and $F_{l,t}$ falling, the ratio of type l to type h educated workers falls,[29] educated (uneducated) wages increase (decrease), and the wage inequality (and the signaling value of education) rises over time. Eventually, $w_{le,t}$ recovers the level at which $F_{l,t}$ does not fall, but with even lower $w_{lu,t}$, $F_{l,t}$ never increases again. Hence, $F_h^*=1$ and $F_l^*<1$ in the long run.

The dynamics when h_{he} and/or h_{le} are higher (Figure 9 (b)(ii)) is similar to the previous case, but $F_{l,t}$ never decreases, and when $p_{le,t}=1$, it increases over the longer term. Further, if $F_{l,t}$ grows sufficiently faster than $F_{h,t}$, educated (uneducated) wages decrease (increase) and the inequality and the signaling value fall when $p_{le,t}=1$, and $F_h^*=F_l^*=1$ or close to it is possible even from small $F_{h,0}$ and $F_{l,0}$. With more effective education, the growth of $F_{l,t}$ does not depress $w_{le,t}$ to the level that $b^*(w_{le,t})<e$ holds, while it has a positive effect on $w_{lu,t}$ and promotes the upward mobility of the uneducated type l.

Finally, as for the relationship between the initial distribution of wealth and the long-run outcome, in both (b)(i) and (b)(ii), the proportion of educated workers is higher and the wage inequality (and the signaling value of education) is lower in the long run, as $F_{h,0}$ is *lower* and $F_{l,0}$ is higher. This is because $F_{h,0}$ has a negative effect on the wage and wealth accumulation of the uneducated type l.

4.1.1 Summary

The analysis has shown that the dynamics of $F_{h,t}$ and $F_{l,t}$ are interrelated, can be complicated, and differ greatly depending on the initial distribution of wealth and exogenous variables related to the productivity of education, ability determined outside education, and the degree of information incompleteness. This contrasts with the complete information case, in which the dynamics of the two variables are independent and simple. The interaction arises through the dependence of wages on average skills, which in turn depend on $F_{j,t}$ directly and indirectly: the proportion of type l individuals taking education, $p_{le,t}F_{l,t}$, is affected not only by 'accessibility', $F_{l,t}$, but also by $p_{le,t}$, which depends on the signaling

[28] As noted in footnote 25, the ratio of type l to type h educated workers is proportional to the slope of the dividing line between $p_{le}=1$ and $p_{le}\in(0,1)$. The increasing ratio leads to the falling overall wage inequality.

[29] The ratio rises and then falls in (a) and (b)(i) of Figures 7 and 8 and in (a) and (b)(ii) of Figure 9 as well, if the economy starts from the region $p_{le}<1$ (and if $F_{h,t}$ increases initially in Figure 7, and if the economy transits to $p_{le}=1$ in (b)(ii) of Figure 9).

value and thus both $F_{h,t}$ and $F_{l,t}$. Since wages of type l (type h) workers can exceed (fall short of) their skill levels, unlike under $s_e = s_u = 0$, $F_h^* = F_l^* = 1\,(=0)$, or close to it, is possible from $F_{h,0}, F_{l,0} < 1(>0)$ when both the ability and the productivity are high (low).

Related variables too exhibit interesting dynamics. Consider a realistic situation in which the exogenous ability is not extremely low and thus $F_{h,t}$ increases over time at least at low $F_{h,t}$ (Propositions 1A (Ⅱ), 1B, and 1C) and the initial distribution of wealth is such that $F_{h,0}$ is not high. Then, the skill composition of educated workers changes over time as follows: at first, education is not profitable to the type l and thus all educated workers are type h; after $F_{h,t}$ reaches a certain level, they become indifferent in the educational choice and the ratio of type l to type h educated workers rises and the average skill of the educated falls over time; after some point, all of the non-poor type l start to take education but, unless $F_{l,t}$ keeps growing, which is unlikely, the ratio starts to fall and the average skill begins to rise eventually. As for wage-related variables, the signaling value of education increases, uneducated wages decrease, and the wage inequality between educated and uneducated workers rises over time when $p_{le,t} = 0$; the signaling value is constant, all wages decrease, and the wage inequality falls when $p_{le,t} \in (0,1)$; and when $p_{le,t} = 1$, typically, the signaling value increases, educated (uneducated) wages increase (decrease), and the inequality rises again. The inequality and the signaling value fall when $p_{le,t} = 1$ only if $F_{l,t}$ increases and the relative growth of $F_{l,t}$ to $F_{h,t}$ is sufficiently high.

When skill accumulation outside education (h_{hu} and/or h_{lu}) is low (Proposition 1A), the relationship between the initial distribution of wealth and the long-run outcome is intuitive: in steady states, the wage inequality and thus the signaling value are lower, as $F_{h,0}$ is lower and $F_{l,0}$ is higher, and the proportion of educated workers is higher, as $F_{h,0}$ (in some cases) and $F_{l,0}$ are higher, unless $F_{h,t}$ decreases initially. By contrast, when it is higher (Propositions 1B and 1C), the relationship differs depending on skill accumulation outside education and the productivity of education: in particular, when the productivity is low ((a) of the propositions), the initial distribution has no effect, while when both of them are high ((b)(i) and (ii) of Proposition 1C), lower $F_{h,0}$ and higher $F_{l,0}$ lead to the *higher* proportion of educated workers as well as lower inequality (and signaling value).

4.2 Steady-state welfare analyses

The preceding analysis suggests that the number of educated workers of each type, aggregate output net of the education cost (thus aggregate consumption), and wage inequality generally differ from the complete information case. This subsection investigates how they differ in steady states and how they are affected by the credit constraint.

First, the effect of the credit constraint on education in steady states is examined. Denote the steady-state number of type j ($j=h,l$) educated workers in the original economy and the one in the economy without the credit constraint by N^*_{je} and $\widetilde{N}_{je}$, respectively.

Lemma 7 (Effect of the credit constraint on education in steady states)

(i) If $E[h_u, 1-s_u] < \frac{1-\gamma_b(1+r)}{\gamma_b}e$ ($\leq$ *with* $s_u=0$), $N^*_{he} \leq \widetilde{N}_{he} = H$ ($<$ *almost always*). *Otherwise,* $N^*_{he} = \widetilde{N}_{he} = H$.

(ii) If $E[h_e, s_e] \leq (1+r)e + h_{lu}$ ($<$ *with* $s_e=0$), $N^*_{le} = \widetilde{N}_{le} = 0$. *Otherwise,* $N^*_{le} \leq \widetilde{N}_{le}$ ($<$ *mostly*).

With the credit constraint, fewer type h individuals take education except when h_{hu} and/or h_{lu} is not low and thus the constraint does not prevent them from accessing education. The constraint also lowers the number of type l educated workers unless the productivity of education is so low that it is not profitable to them.

The next lemma examines the effect of positive s_e and/or s_u on steady-state education. The steady-state number of type j ($j=h,l$) educated workers under $s_e=s_u=0$ is denoted $N^*_{je}(s=0)$. When $s_e, s_u=0$ and $h_{je}=(1+r)e+h_{ju}$, $p_{je}=1$ is assumed to hold.

Lemma 8 (Effect of positive s_e and/or s_u on education in steady states)

(I)(a) When $h_{he} < (1+r)e + h_{hu}$, *if* $E[h_u, 1-s_u] < \frac{1-\gamma_b(1+r)}{\gamma_b}e$ ($\leq$ *with* $s_u=0$), $N^*_{he} \geq N^*_{he}(s=0) = 0$ ($>$ *almost always*); *otherwise,* $N^*_{he} = H > N^*_{he}(s=0) = 0$.

(b) When $h_{he} \geq (1+r)e + h_{hu}$, *if* $h_{hu} \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e$, $N^*_{he} \leq N^*_{he}(s=0) = F_{h,0}H$; *if* $h_{hu} > \frac{1-\gamma_b(1+r)}{\gamma_b}e > E[h_u, 1-s_u]$, $N^*_{he} \leq N^*_{he}(s=0) = H$ ($<$ *mostly*); *otherwise,* $N^*_{he} = N^*_{he}(s=0) = H$.

(II)(a) When $h_{le} < (1+r)e + h_{lu}$, *if* $E[h_e, s_e] \leq \frac{e}{\gamma_b}$ ($<$ *with* $s_e=0$), $N^*_{le} = N^*_{le}(s=0) = 0$; *otherwise,* $N^*_{le} \geq N^*_{le}(s=0) = 0$ ($>$ *mostly*).

(b) When $h_{le} \geq (1+r)e + h_{lu}$:

(i) When $E[h_e, s_e] \leq \frac{e}{\gamma_b}$ ($<$ *if* $s_e=0$), $N^*_{le} = N^*_{le}(s=0) = 0$.

(ii) When $E[h_e, s_e] > \frac{e}{\gamma_b} > h_{le}$, $N^*_{le} \geq N^*_{le}(s=0) = 0$ ($>$ *mostly*).

(iii) When $h_{le} \geq \frac{e}{\gamma_b}$, *if* $E[h_u, s_u H] \leq \frac{1-\gamma_b(1+r)}{\gamma_b}e$, $N^*_{le} = N^*_{le}(s=0) = F_{l,0}(1-H)$; *otherwise,* $N^*_{le} \geq N^*_{le}(s=0) = F_{l,0}(1-H)$.

When education is not socially productive for type h (Lemma 8 (I)(a)), positive s_e and/or s_u result in *overeducation* almost always. They take education to mitigate the negative effect of the presence of type l on their wage: education lowers the discrepancy between the wage and their skill, i.e. $s_e(h_{he} - \widetilde{E}[h_e]) < s_u(h_{hu} - \widetilde{E}[h_u])$. When it is productive for them (Lemma 8 (I)(b)), fewer of them can afford and take education due to the negative effect from the uneducated type l, if h_{hu} and/or h_{lu} are low and thus $F^*_h < 1$ (cases of Proposition 1A).

As for type l, when education is not productive for them (Lemma 8 (Ⅱ)(a)), overeducation occurs as long as some of them can afford education (cases (b) of Propositions 1A−1C), because they can benefit from the positive effect from the educated type h. (The positive effect from the uneducated type h is not large and thus $p_{le}=1$ when $F_l^*>0$.) By contrast, when education is productive (Lemma 8 (Ⅱ)(b)), more of them take education with the signaling effect, unless either $F_l^*=0$ (cases (a) of the propositions) or $F_{l,t}$ is always time-invariant (case (b)(ii) of Propositions 1A−1B). When both h_{hu} and/or h_{lu} and h_{le} are high (Lemma 8 (Ⅱ)(b)(iii)), the positive effect from the type h uneducated stimulates wealth accumulation of the type l uneducated and some of them become accessible to education in the long run, while when the efficiency is intermediate (Lemma 8 (Ⅱ)(b)(ii)), the same mechanism works on educated individuals.[30]

Finally, based on these lemmas, the next proposition examines effects of positive s_e and/or s_u and of the credit constraint on steady-state net aggregate output. In the proposition, Y^*, $Y^*(s=0)$, $\widetilde{Y}$, and $\widetilde{Y}(s=0)$ are net output of the original economy, of the economy with $s_e=s_u=0$, of the economy without the credit constraint, and of the complete information economy without the constraint, respectively. Clearly, $\widetilde{Y}(s=0)$ is highest and optimal.

Proposition 2 (Steady-state net aggregate output) [31]

(i) *When* $h_{he}<(1+r)e+h_{hu}$ *and* $h_{le}<(1+r)e+h_{lu}$, $\widetilde{Y}\leq Y^*\leq Y^*(s=0)=\widetilde{Y}(s=0)$ $(Y^*<Y^*(s=0)$ *almost always and* $\widetilde{Y}<\widetilde{Y}(s=0))$.[32]

(ii) *When* $h_{he}\geq(1+r)e+h_{hu}$ *and* $h_{le}<(1+r)e+h_{lu}$,

(a) If $E[h_u,1-s_u]<\frac{1-\gamma_b(1+r)}{\gamma_b}e$ $(\leq$ *with* $s_u=0)$, $Y^*\leq Y^*(s=0)=(<)\widetilde{Y}(s=0)$ *when* $h_{hu}>(\leq)\frac{1-\gamma_b(1+r)}{\gamma_b}e$. *If* $E[h_e,s_e]\leq(1+r)e+h_{lu}$ $(<$ *when* $s_e=0)$ *as well,* $Y^*\leq\widetilde{Y}(<$ *almost always*$)=\widetilde{Y}(s=0)$; *otherwise, the relative size is ambiguous.*

(b) Otherwise, $\widetilde{Y}\leq Y^*$ *and* $Y^*(s=0)=\widetilde{Y}(s=0)$.[33] *If* $E[h_e,s_e]\leq\frac{e}{\gamma_b}$ $(<$ *with* $s_e=0)$ *as well,* $Y^*=Y^*(s=0)$; *otherwise,* $Y^*\leq Y^*(s=0)(<$ *mostly*$)$.

(iii) *When* $h_{he}\geq(1+r)e+h_{hu}$ *and* $h_{le}\geq(1+r)e+h_{lu}$, $Y^*\leq(<$ *mostly*$)$ $\widetilde{Y}=\widetilde{Y}(s=0)$ *and* $Y^*(s=0)<\widetilde{Y}(s=0)$.

[30] As for effects of a change in the *degree* of information incompleteness, i.e. a change in (positive) s_e or s_u, clear-cut results are not obtained because the effects on wealth accumulation of the two types and their education depend on exogenous variables and the initial distribution of wealth in a complicated manner. The exception is the effect of s_e on N_{le}^*, which is mostly positive. Hence, higher s_e mitigates undereducation (worsens overeducation) of the type l when education is productive (unproductive) for them. A detailed analysis is available from the author upon request.

[31] The situation in which education is productive only for type l, i.e. $h_{he}<(1+r)e+h_{hu}$ and $h_{le}\geq(1+r)e+h_{lu}$, is possible but empirically unlikely, thus the result is not presented.

[32] If $E[h_u,1-s_u]\geq\frac{1-\gamma_b(1+r)}{\gamma_b}e$ ($>$ with $s_u=0$) and $E[h_e,s_e]\leq(1+r)e+h_{lu}$ ($<$ with $s_e=0$), $\tilde{Y}=Y^*$; otherwise, $\tilde{Y}\leq Y^*$ ($<$ mostly).

[33] If $E[h_e,s_e]\leq(1+r)e+h_{lu}$ ($<$ when $s_e=0$), $\tilde{Y}=Y^*$.

(a) *If* $E[h_u, 1-s_u] < \frac{1-\gamma_b(1+r)}{\gamma_b}e$ $(\leq$ *with* $s_u = 0)$*,* $Y^* \leq Y^*(s=0)$ *when* $E[h_e, s_e] \leq \frac{e}{\gamma_b}$ $(<$ *with* $s_e = 0)$ *or* $h_{le} \geq \frac{e}{\gamma_b}$*; in other cases, the relative size is ambiguous.*

(b) *Otherwise,* $Y^* \geq Y^*(s=0)$ *when* $E[h_e, s_e] > \frac{e}{\gamma_b} > h_{le}$ *or when* $E[h_u, s_u H] > \frac{1-\gamma_b(1+r)}{\gamma_b}e$ *and* $h_{le} \geq \frac{e}{\gamma_b}$*;* $Y^* = Y^*(s=0)$ *in other cases.*

From the proposition and the lemmas, effects on net output and several dimensions of inequalities are summarized as follows. When education is not productive for anybody, the signaling effect leads to overeducation, intra-type inequality, and lower net output. Lifting the credit constraint exacerbates overeducation and lowers net output further in most cases, although intra-type equality is attained.[34]

When education is productive only for type h, net output is lower with the incomplete information in most cases, because fewer type h individuals take education and/or some of the type l take unproductive education. Inequality among the type l is clearly higher under incomplete information, whereas inter-type inequality and the inequality between educated and uneducated workers are lower, because type h (type l) wages are lower (higher) and a lower (higher) proportion of the type h (type l) take education. (These results on inter-type and inter-education inequalities hold when education is productive for both types as well.) The effect of the credit constraint on net output now depends on whether the constraint affects education of the type h or not: if it does not prevent them from accessing education in the long run (Proposition 2 $(ii)(b)$), lifting the constraint just exacerbates overeducation of the type l and lowers net output; otherwise, the effect could be ambiguous.

Finally, when education is productive for both types, net output can be *higher* with the signaling effect. This is the case if skill accumulation outside education is high enough that all of the type h and the higher proportion of the type l can access education in the long run because of either higher upward mobility of the uneducated (when the efficiency of education is high) or lower downward mobility of the educated (when the efficiency is intermediate). Lifting the credit constraint leads to optimal universal education and maximizes net output.

4.3 Policy Implications

Several policy implications can be derived from the analyses. First, policies that enable everyone to access education (e.g. free public education) and those that increase access directly (e.g. wealth redistribution and tuition subsidies) or indirectly through promoting

[34] The effect on inter-type inequality is ambiguous, because while the return to education falls as the ratio of type l to type h educated workers increases, inter-type inequality among educated workers is higher than among uneducated workers from Assumption 2.

skill accumulation outside education and thereby raising earnings of the poor (e.g. child nutrition programs, childcare programs, and job training programs for the poor) may not be desirable in the presence of the signaling effect. Unless education is socially productive for everyone, such policies could lead to overeducation of unproductive individuals and lower net output. By contrast, under complete information, such policies always (weakly) raise net output, since social and individual returns to education coincide. Hence, when the signaling effect is not negligible, it is important to ensure that education is socially productive for everyone (e.g. by raising the effectiveness of education especially for the disadvantaged) or to restrict access only to high ability students (e.g. by implementing competitive entrance exams), although the latter kind of policies would raise equity concerns. The harm from policies increasing the access and the need for these complementary measures are greater at later stages of development, since the signaling value of education and thus the return to education of low ability individuals increase over time.

Second, wealth redistribution that raises the accessibility (raises both $F_{h,t}$ and $F_{l,t}$ since the government usually cannot distinguish different types) may not be desirable for a different reason from above: it could have a negative effect on earnings and education of the low-ability poor. This happens if the policy stimulates education of the type h poor too much, and as a result, lowers the average skill of the uneducated and thus their wages, and hampers wealth accumulation of the type l poor (see Figure 9 (b)(i) and (ii)). Such policy is certainly undesirable when education is socially productive. Without the signaling effect, by contrast, the dynamics of different types are unrelated, thus the policy is always (weakly) desirable. The interrelated dynamics also imply that policies that focus too much on raising skills of a particular type are not very effective in the long term.

Finally, positive s_e and/or s_u may reflect effects of unions or fairness concern in the workplace on wage setting (see footnote 7), so the above implications apply when these effects, not the signaling effect, are important as well.

5 Conclusion

Many empirical works suggest that education not only raises human capital but also acts as a signal when employers have incomplete information on employees' skills. This paper has examined how the number of educated individuals, the importance of the signaling value of education, and the wage inequality between educated and uneducated workers change over time in an economy where education has the dual roles and some fraction of individuals is credit constrained from taking education, and has compared the dynamics with those under complete information. It also examines whether the signaling role leads to higher aggregate consumption or not in the long run.

In the present paper, weights s_e and s_u of wage equations, i.e. the degree of information incompleteness, are assumed to be exogenous and held constant. However, it would be more realistic to consider them as endogenous because, in actual economy, the degree of information incompleteness would be different depending on types of jobs and the composition of jobs changes over the course of development. The important extension of endogenizing the weights is left for future work.

Appendix Lemmas A1–A3

Lemma A1 (Relation of $\mathbf{b^*(w_{he})=e}$ with the dividing line between $\mathbf{p_{le}=1}$ and $\mathbf{p_{le}\in(0,1)}$) *Suppose that the dividing line and $b^*(w_{he})=e$ exist, i.e. conditions of Lemma 3 (III) and of Lemma 6 (iii) hold.*

(A) When $E[h_e,1-s_e(1-H)]>\frac{e}{\gamma_b}$

(i) If $E[h_u,s_u]+(1-s_e)(h_{he}-h_{le})\leq\frac{1-\gamma_b(1+r)}{\gamma_b}e$, $b^*(w_{he})=e$ is on or below the dividing line.

(ii) If $E[h_u,s_u]+(1-s_e)(h_{he}-h_{le})>\frac{1-\gamma_b(1+r)}{\gamma_b}e>E[h_u,s_uH]+(1-s_e)(h_{he}-h_{le})$, $b^*(w_{he})=e$ intersects with the line at $F_l\in(0,1)$. Only a segment of $b^*(w_{he})=e$ on or below the line is effective.

(iii) Otherwise, $b^*(w_{he})=e$ is above the line and thus $b^*(w_{he})>e$ always.

(B) When $E[h_e,1-s_e(1-H)]<\frac{e}{\gamma_b}$

(i) If $E[h_u,s_uH]+(1-s_e)(h_{he}-h_{le})\leq\frac{1-\gamma_b(1+r)}{\gamma_b}e$ ($<$ with $s_u=0$), $b^*(w_{he})=e$ is below the line.

(ii) If $E[h_u,s_uH]+(1-s_e)(h_{he}-h_{le})>\frac{1-\gamma_b(1+r)}{\gamma_b}e>h_{lu}+(1-s_e)(h_{he}-h_{le})$, $b^*(w_{he})=e$ intersects with the line at $F_h\in(0,1)$ and only a segment of $b^*(w_{he})=e$ on or below the line is effective.

(iii) Otherwise, $b^*(w_{he})\geq e$ always ($b^*(w_{he})=e$ is on or above the line).

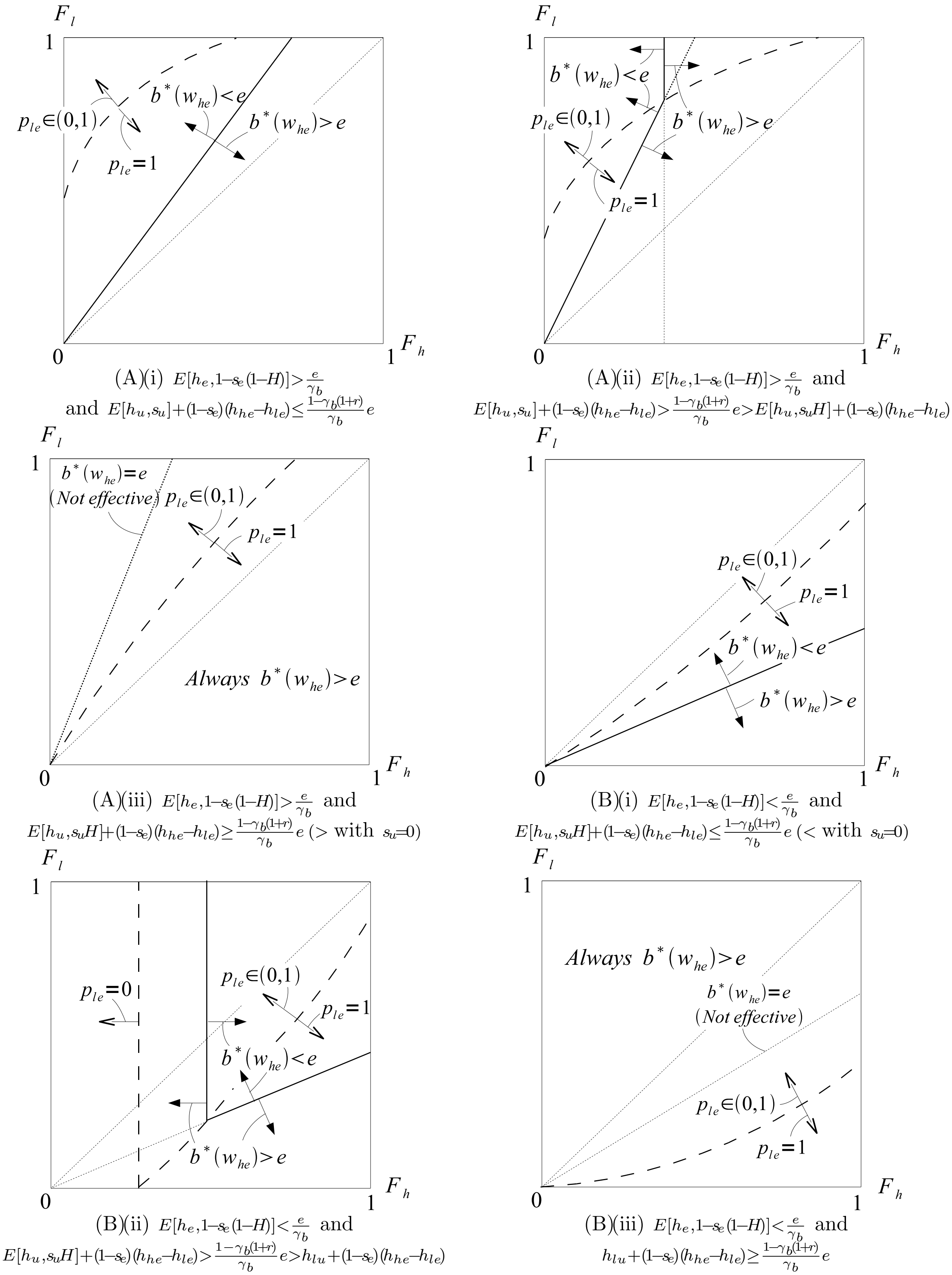
(C) When $E[h_e,1-s_e(1-H)]=\frac{e}{\gamma_b}$, $b^*(w_{he})=e$ is the 45° line.

(i) If $E[h_u,s_uH]+(1-s_e)(h_{he}-h_{le})=\frac{1-\gamma_b(1+r)}{\gamma_b}e$, the dividing line and $b^*(w_{he})=e$ coincide.

(ii) When $E[h_u,s_uH]+(1-s_e)(h_{he}-h_{le})<(>)\frac{1-\gamma_b(1+r)}{\gamma_b}e$, the line is above (below) $b^*(w_{he})=e$.

Figure A1 illustrates the relation when $E[h_e,1-s_e(1-H)]\neq\frac{e}{\gamma_b}$, based on Lemma A1 (A) and (B).[35] Figure A1 (A)(ii) shows an example of the relation when the conditions of Lemma A1 (A)(ii) hold. The broken line is the dividing line and $p_{le}\in(0,1)$ ($p_{le}=1$) in the region above (below) the line, while $b^*(w_{he})=e$ holds on the solid line and $b^*(w_{he})<(>)e$ at the left

[35]When $s_e=0$, $b^*(w_{he})=e$ does not exist and the lemma does not apply. When $s_u=0$, cases (A)(ii) and (B)(ii) of the lemma (Figure A1 (A)(ii) and (B)(ii)) do not occur.



(A)(i) $E[h_e,1-s_e(1-H)]>\frac{e}{\gamma_b}$ and $E[h_u,s_u]+(1-s_e)(h_{he}-h_{le})\leq\frac{1-\gamma_b(1+r)}{\gamma_b}e$

(A)(ii) $E[h_e,1-s_e(1-H)]>\frac{e}{\gamma_b}$ and $E[h_u,s_u]+(1-s_e)(h_{he}-h_{le})>\frac{1-\gamma_b(1+r)}{\gamma_b}e>E[h_u,s_uH]+(1-s_e)(h_{he}-h_{le})$

(A)(iii) $E[h_e,1-s_e(1-H)]>\frac{e}{\gamma_b}$ and $E[h_u,s_uH]+(1-s_e)(h_{he}-h_{le})\geq\frac{1-\gamma_b(1+r)}{\gamma_b}e$ (> with $s_u=0$)

(B)(i) $E[h_e,1-s_e(1-H)]<\frac{e}{\gamma_b}$ and $E[h_u,s_uH]+(1-s_e)(h_{he}-h_{le})\leq\frac{1-\gamma_b(1+r)}{\gamma_b}e$ (< with $s_u=0$)

(B)(ii) $E[h_e,1-s_e(1-H)]<\frac{e}{\gamma_b}$ and $E[h_u,s_uH]+(1-s_e)(h_{he}-h_{le})>\frac{1-\gamma_b(1+r)}{\gamma_b}e>h_{lu}+(1-s_e)(h_{he}-h_{le})$

(B)(iii) $E[h_e,1-s_e(1-H)]<\frac{e}{\gamma_b}$ and $h_{lu}+(1-s_e)(h_{he}-h_{le})\geq\frac{1-\gamma_b(1+r)}{\gamma_b}e$

Figure A1: Examples of the relation of $b^*(w_{he})=e$ with the dividing line between $p_{le}=1$ and $p_{le}\in(0,1)$ when $E[h_e,1-s_e(1-H)]\neq\frac{e}{\gamma_b}$ (Lemma A1 (A) and (B)) (right) side of the line. Note that the solid line is vertical at the intersection of the two loci, because $b^*(w_{he})=e$ when $p_{le}=1$ is not effective above the dividing line and the proportion of type l individuals with education, $p_{le}F_l$, is same on the vertical line. Unlike the figure, the dividing line may approach $(F_h,F_l)=(1,1)$ but the relation is qualitatively same.

In Figure A1 (B)(ii), the dividing line intersects with $F_l=0$ at $F_h\in(0,1)$ and thus $p_{le}=0$ at small F_h. Unlike Figure A1 (A)(ii), $b^*(w_{he})=e$ is not effective when F_h is small, and $b^*(w_{he})<e$ when F_h and F_l are high. In Figure A1 (A)(iii) and (B)(iii), $b^*(w_{he})=e$ is always above the dividing line and is not effective, thus $b^*(w_{he})>e$ holds for any F_h and F_l. Finally, in Figure A1 (A)(i) and (B)(i), $b^*(w_{he})=e$ is always below the dividing line.

Similarly, the next lemma examines the relation of $b^*(w_{hu})=e$ with the dividing line.

Lemma A2 (Relation of $\mathbf{b^*(w_{hu})=e}$ with the dividing line between $\mathbf{p_{le}=1}$ and $\mathbf{p_{le}\in(0,1)}$) *Suppose that the dividing line and $b^*(w_{hu})=e$ when $p_{le}=1$ exist (conditions of Lemma 3 (III) and of Lemma 6 (i) hold).*

(A) When $E[h_u,1-s_u(1-H)]>\frac{1-\gamma_b(1+r)}{\gamma_b}e$

(i) If $E[h_e,s_e]+(1-s_u)(h_{hu}-h_{lu})\leq\frac{e}{\gamma_b}$, $b^*(w_{hu})=e$ when $p_{le}=1$ is above the dividing line and $b^*(w_{hu})<e$ always when $p_{le}=1$, while $b^*(w_{hu})=e$ when $p_{le}=0$ is effective.

(ii) If $E[h_e,s_e]+(1-s_u)(h_{hu}-h_{lu})>\frac{e}{\gamma_b}>E[h_e,s_eH]+(1-s_u)(h_{hu}-h_{lu})$, $b^*(w_{hu})=e$ (when $p_{le}=1$) intersects with the line at $F_l\in(0,1)$ and a segment of it on or below the line is effective.

(iii) If $E[h_e,s_eH]+(1-s_u)(h_{hu}-h_{lu})\geq\frac{e}{\gamma_b}$ ($>$ with $s_e=0$), $b^*(w_{hu})=e$ is below the line.

(B) When $E[h_u,1-s_u(1-H)]<\frac{1-\gamma_b(1+r)}{\gamma_b}e$

(i) If $E[h_e,s_eH]+(1-s_u)(h_{hu}-h_{lu})\leq\frac{e}{\gamma_b}$ ($<$ with $s_e=0$), $b^*(w_{hu})<e$ always ($b^*(w_{hu})=e$ is above the line).

(ii) If $E[h_e,s_eH]+(1-s_u)(h_{hu}-h_{lu})>\frac{e}{\gamma_b}>h_{le}+(1-s_u)(h_{hu}-h_{lu})$, $b^*(w_{hu})=e$ intersects with the line at $F_h\in(0,1)$ and only a segment of $b^*(w_{hu})=e$ on or below it is effective.

(iii) Otherwise, $b^*(w_{hu})=e$ is on or below the line.

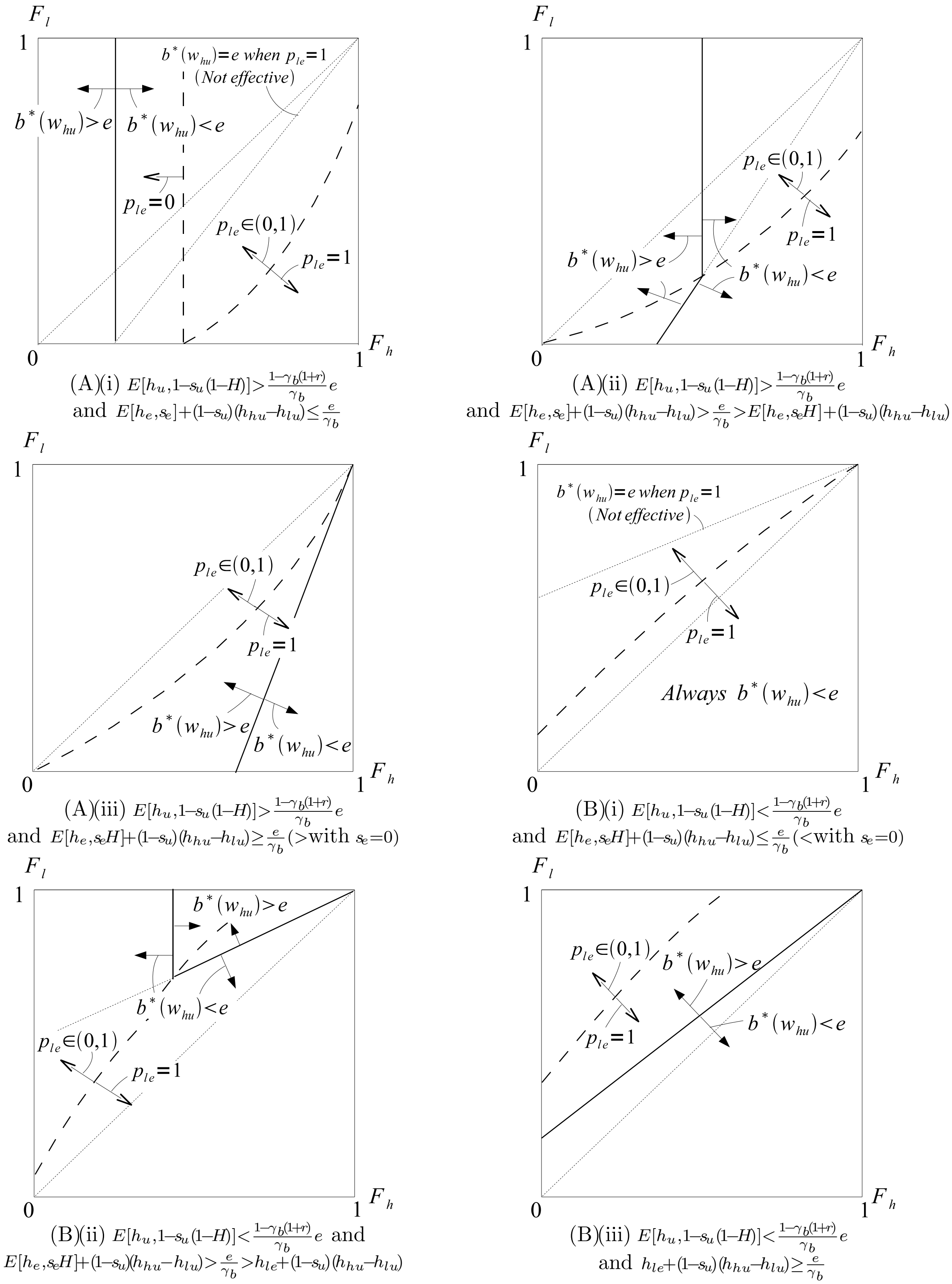
(C) When $E[h_u,1-s_u(1-H)]=\frac{1-\gamma_b(1+r)}{\gamma_b}e$, $b^*(w_{hu})=e$ is the 45° line.

(i) If $E[h_e,s_eH]+(1-s_u)(h_{hu}-h_{lu})=\frac{e}{\gamma_b}$, $b^*(w_{hu})=e$ and the dividing line coincide.

(ii) When $E[h_e,s_eH]+(1-s_u)(h_{hu}-h_{lu})<(>)\frac{e}{\gamma_b}$, the line is below (above) $b^*(w_{hu})=e$.

Figure A2 illustrates the relation when $E[h_u,1-s_u(1-H)]\neq\frac{1-\gamma_b(1+r)}{\gamma_b}e$, based on Lemma A2 (A) and (B).[36] Figure A2 (A)(i) corresponds to the case in which the conditions of Lemma A2 (A)(i) hold. Since the dividing line intersects with $F_l=0$ at $F_h\in(0,1)$, $p_{le}=0$ at small F_h.

[36]When $s_u=0$, $b^*(w_{hu})=e$ does not exist and the lemma does not apply. When $s_e=0$, cases (A)(ii) and (B)(ii) of the lemma (Figure A2 (A)(ii) and (B)(ii)) do not occur.



(A)(i) $E[h_u,1-s_u(1-H)]>\frac{1-\gamma_b(1+r)}{\gamma_b}e$ and $E[h_e,s_e]+(1-s_u)(h_{hu}-h_{lu})\le\frac{e}{\gamma_b}$

(A)(ii) $E[h_u,1-s_u(1-H)]>\frac{1-\gamma_b(1+r)}{\gamma_b}e$ and $E[h_e,s_e]+(1-s_u)(h_{hu}-h_{lu})>\frac{e}{\gamma_b}>E[h_e,s_eH]+(1-s_u)(h_{hu}-h_{lu})$

(A)(iii) $E[h_u,1-s_u(1-H)]>\frac{1-\gamma_b(1+r)}{\gamma_b}e$ and $E[h_e,s_eH]+(1-s_u)(h_{hu}-h_{lu})\ge\frac{e}{\gamma_b}$ ($>$with $s_e=0$)

(B)(i) $E[h_u,1-s_u(1-H)]<\frac{1-\gamma_b(1+r)}{\gamma_b}e$ and $E[h_e,s_eH]+(1-s_u)(h_{hu}-h_{lu})\le\frac{e}{\gamma_b}$ ($<$with $s_e=0$)

(B)(ii) $E[h_u,1-s_u(1-H)]<\frac{1-\gamma_b(1+r)}{\gamma_b}e$ and $E[h_e,s_eH]+(1-s_u)(h_{hu}-h_{lu})>\frac{e}{\gamma_b}>h_{le}+(1-s_u)(h_{hu}-h_{lu})$

(B)(iii) $E[h_u,1-s_u(1-H)]<\frac{1-\gamma_b(1+r)}{\gamma_b}e$ and $h_{le}+(1-s_u)(h_{hu}-h_{lu})\ge\frac{e}{\gamma_b}$

Figure A2: Examples of the relation of $b^*(w_{hu})=e$ with the dividing line between $p_{le}=1$ and $p_{le}\in(0,1)$/between $p_{le}=0$ and $p_{le}>0$, when $E[h_u,1-s_u(1-H)]\neq\frac{1-\gamma_b(1+r)}{\gamma_b}e$ (Lemma A2 (A)(B))

When $p_{le} > 0$, $b^*(w_{hu}) = e$ is always above the line and is not effective, while $b^*(w_{hu}) = e$ when $p_{le} = 0$ (the solid vertical line) is effective (effective only in this case). Thus, $b^*(w_{hu}) > (<) e$ at the left (right) side of the line.

In Figure A2 (A)(ii), $b^*(w_{hu}) = e$ (when $p_{le} = 1$) intersects with the dividing line and thus a portion of it is not effective. $b^*(w_{hu}) > e$ holds above effective $b^*(w_{hu}) = e$, while $b^*(w_{hu}) < e$ holds below it and in the region with F_h greater than the value at the intersection. The two loci intersect in Figure A2 (B)(ii) too, but the relationship between them is opposite to Figure A2 (A)(ii). Thus, $b^*(w_{hu}) < e$ when F_h is smaller than the value at the intersection. In Figure A2 (B)(i), $b^*(w_{hu}) = e$ is always above the dividing line and $b^*(w_{hu}) < e$ for any F_h and F_l, and in Figure A2 (A)(iii) and (B)(iii), $b^*(w_{hu}) = e$ is always below the line.

As for $b^*(w_{lk}) = e$ $(k = e, u)$, the following result is enough to examine the dynamics.

Lemma A3 (Relation of $\mathbf{b^*(w_{lk}) = e}$ with the dividing line between $\mathbf{p_{le} = 1}$ and $\mathbf{p_{le} \in (0,1)}$)

(i) *If $b^*(w_{lu}) = e$ or $b^*(w_{le}) = e$ intersects with the dividing line between $p_{le} = 1$ and $p_{le} \in (0,1)$, they intersect on the dividing line.*

(ii) *If $b^*(w_{le}) > e$ ($b^*(w_{lu}) < e$) holds on the dividing line, $b^*(w_{lu}) = e$ ($b^*(w_{le}) = e$), when it exists, is located below the line on the (F_h, F_l) plane.*

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