Technological Spillovers and Patterns of Growth with Sector-Specific RD

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Abstract

This paper studies a two-sector model of endogenous technical change in which expansion of each production sector is associated with sector-specific R&D investment. It is shown that the pattern of growth is sensitive to the specification of intersectoral technological spillover as well as to the preference structure. If technological spillovers and preferences of consumers are represented by CES functional forms, the balanced-growth equilibrium may not exhibit a well-behaved saddlepoint property: it is possible that the balanced-growth path is locally indeterminate or unstable. In addition, a slight modification of technological spillover effects easily yields multiple balanced-growth paths. In contrast, Cobb-Douglas specifications present a unique and determinate balanced-growth path.

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1 Introduction

The last decade has witnessed extensive investigations on the R&D-based growth models developed by Romer (1990), Grossman and Helpman (1991a) and Aghion and Howitt (1992). The standard analytical framework of this literature has assumed that the whole economy shares a stock of homogenous technological knowledge. The knowledge stock is enhanced by R&D activities, which in turn raises the total factor productivity of the aggregate production technology of final goods. However, if we consider the presence of heterogenous final goods, it is plausible to assume that producing a specific type of final good needs a specific kind of technological knowledge. This means that R&D investment would be sector specific rather than homogeneous activity in a single R&D sector. In addition, if there are heterogenous knowledge stocks, we should explicitly specify the forms of spillover effects among the different kinds of knowledge. The micro-oriented, empirical literature on technical progress has paid much attention to industry or firm specific R&D and the forms of technological spillovers[1].

The purpose of this paper is to examine the role of technological spillovers among different kinds of knowledge in a multisector economy with sector-specific R&D. We develop a two-sector model of endogenous technical change in which each production sector uses sector-specific technical knowledge. Our main concern is to examine equilibrium dynamics in the presence of sector-specific R&D and interindustry technological diffusion. More specifically, we extend one of the base models of endogenous technical change presented by Grossman and Helpman (1991a, Chapter 3) and Romer (1990) by introducing two types of consumption goods. Each consumption good is produced by use of a specific set of intermediate goods. R&D activities in each sector expand the variety of intermediate goods used in its own sector. We assume that productivity of research scientists in each R&D sector is positively related to the existing stock of knowledge of their own as well as to one accumulated by the other sector. The forms of technological interactions between the two R&D sectors are thus expressed by the manner how knowledge of the one sector affects the other sector's R&D.

Our main finding is that dynamic behavior of the economy is highly sensitive to the pattern of technological diffusion as well as to the preference structure. In the base model we assume that the utility and knowledge production functions are of CES type. We find that the balanced-growth equilibrium can be locally indeterminate so that there may exist a continuum of equilibria around the balanced-growth path. If this is the case, the economy may display sunspot-driven fluctuations around the balanced-growth path. Additionally, under certain conditions, the balanced-growth equilibrium could be locally unstable. Moreover, given a slight modification of form of technological diffusion, the economy may have multiple balanced-growth paths, and hence the global behavior of the economy would be quite different from its local behavior around the long-run equilibrium. Those complexities are not generally observed in the standard R&D-based models of growth with homogeneous technical knowledge.[2] On the other hand, if we assume that the utility and production functions take Cobb-Douglas forms, then there exists a unique balanced-growth equilibrium and it satisfies saddlepoint stability. In this case, the equilibrium path converging to the long-run equilibrium is uniquely determined and the transitional process is monotonic. Therefore, the model of heterogenous R&D activities with Cobb-Douglas functional forms dynamically behaves like the model with homogeneous technical knowledge. Since Cobb-Douglas functions are frequently employed in the growth economics, our findings demonstrate that the well-behaved dynamics established in the standard models may critically depend on the restrictive specifications of the functional forms.

It is to be noted that several authors have investigated the growth models with multiple R&D sectors. Smulders and van de Klundert (1995) examine a growth model in which each monopolistic competitive firm engages in its own R&D. Although their model considers the presence of spillover effect among different R&D activities, in the symmetric, macroeconomic equilibrium, differences among the stocks of technical knowledge disappear and spillover effect depends on the number of firms alone.[3] Li (2000) develops a model of two-dimensional quality upgrades in which each upgrade needs different R&D effort. Additionally, Li (2001) considers n-sector version of the base model. In a similar vein, Segerstorm (1998), Dinopolus and Thompson (1998) and Young (1998) examine the models with two R&D sectors, one expands variety and the other upgrades quality of goods. The main concern of those authors is to explore the conditions for the absence of scale effect in the long-run equilibrium. Therefore, they do not discuss dynamic behavior of the models out of the steady state. Acemoglu (2002) assumes that labor and capital augmenting technical progress are respectively realized by specific R&D activities. Xie (1998), on the other hand, constructs a model with two R&D sectors in which variety expansions in intermediate inputs and consumption goods need different kinds of researches. Those studies, however, do not consider intersectoral technical spillovers.[4]

In the existing literature, Starz (1998) is most closely related to our study. He also examines a two-sector model with sector-specific technical knowledge and shows that the economy may have multiple balanced-growth paths. The main purpose of his contribution is to construct a model that exhibits large fluctuations of long-term growth rate generated by exogenous disturbances. To concentrate on this issue, Starz (1998) uses a simple model where the microeconomic structure of R&D investment is not fully specified. Additionally, the model does not consider forward-looking behaviors of economic agents. In this paper, following the standard modelling, we explicitly specify the optimizing behaviors of agents and analyze the equilibrium dynamics with rational expectations.[5]

The rest of the paper is organized as follows. The next section constructs the base model. Section 3 characterizes the balanced-growth equilibrium and explores dynamic properties of the model. Intuitive implication of the stability results is also given in this section. Section 4 discusses an example in which there exist multiple balanced-growth paths. Section 5 concludes the paper.

2 The Model

2.1 Production

There are two types of consumption goods, C_1 and C_2. We assume that each consumption good is produced by using a set of intermediate goods. The production function of each final good is specified as:

$$C_i = \left(\int_0^{A_i} c_i\left(j\right)^{\frac{\alpha_i - 1}{\alpha_i}} dj \right)^{\frac{\alpha_i}{\alpha_i - 1}}, \quad \alpha_i > 1, \quad i = 1, 2, \tag{1}$$

where $c_i\left(j\right)$ is the intermediate good of type j devoted to produce i-th consumption good and α_i represents the elasticity of substitution among the intermediate inputs. A_i denotes the range of intermediate goods used in the i-th consumption good sector.[6] The consumption good markets are assumed to be competitive. Letting P_i be the price of C_i and $p_i\left(j\right)$ be the price of $c_i\left(j\right)$, profit maximization behavior of producers yields the inverse demand function for $c_i\left(j\right)$:

$$p_i\left(j\right) = P_i \left(\frac{C_i}{c_i\left(j\right)} \right)^{\frac{1}{\alpha_i}}, \quad i = 1, 2, \quad j \in \left[0, A_i\right]. \tag{2}$$

By cost minimization, the relation between P_i and $p_i\left(j\right)$ satisfies

$$P_i = \left(\int_0^{A_i} p_i\left(j\right)^{1-\alpha_i} dj \right)^{\frac{1}{1-\alpha_i}}, \quad i = 1, 2. \tag{3}$$

Each intermediate good is produced by a monopolistically competitive firm. We assume that one unit of intermediate good is produced by use of one unit of physical labor. Denoting the wage rate by w, we assume that the firm producing $c_i(j)$ maximizes its profits

$$\pi_i(j) = p_i(j)\, c_i(j) - w c_i(j)$$

subject to the inverse demand function given by (2). As a result, the optimal price is determined by the simple mark-up formula such that

$$p_i(j) = \frac{\alpha_i}{\alpha_i - 1} w, \quad i = 1, 2, \quad j \in [0, A_i]. \tag{4}$$

Equation (4) means that we can focus on the symmetric equilibrium where $c_i(j) = c_i$ and $p_i(j) = p_i$ for all $j \in [0, A_i]$, and hence the profits of intermediate good producers are given by

$$\pi_i = \frac{w c_i}{\alpha_i - 1}, \quad i = 1, 2. \tag{5}$$

2.2 R&D Activities

In formulating R&D, we basically follow Romer (1990): R&D expands the range of available variety of intermediate goods. The R&D activities are assumed to be sector specific and creation of new knowledge is described by

$$\dot{A}_i = \delta_i S_i^{\theta_i} H_i^{1-\theta_i} X_i, \quad 0 < \theta_i < 1, \ \delta_i > 0, \ \ i = 1, 2. \tag{6}$$

In the above, S_i denotes the number of research scientists who work to create new varieties of intermediate goods used for producing i-th consumption good. H_i is the sector-specific human capital that is necessary for developing new intermediate goods used in sector i. The difference between S_i and H_i is that the research scientists employed by sector i can work at sector $j\,(\neq i)$, while the sector-specific human capital can be used by sector i alone. For simplicity we assume that H_i is constant over time, so that it may be considered a fixed input.[7] Finally, X_i expresses external effects on R&D activities. We consider that X_i consists of intersectoral as well as intrasectoral effects of technological diffusion. More specifically, in the base model we set

$$X_i = \left(\phi_i^{\frac{1}{\eta}} A_i^{\frac{\eta-1}{\eta}} + (1-\phi_i)^{\frac{1}{\eta}} A_j^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \ \eta > 0, \ 0 < \phi_i < 1, \quad i \neq j, \ i, j = 1, 2. \tag{7}$$

where $\phi_i^{\frac{1}{\eta}} A_i^{\frac{\eta-1}{\eta}}$ and $(1-\phi_i)^{\frac{1}{\eta}} A_j^{\frac{\eta-1}{\eta}}$ respectively express the intrasectoral and intersectoral effects of knowledge spillovers. Equation (7) means that sector-specific knowledge stocks are substitute each other for $\eta > 1$, while they are complement each other for $\eta < 1$. When $\eta = 1$, the spillover function takes a Cobb-Douglas form, while X_i is a linear function of A_i and A_j if $\eta = \infty$.

The R&D sectors are assumed to be competitive. The instantaneous profits of the research firms are given by $v_i \dot{A}_i - w_S S_i - w_H H_i$, where v_i is the patent price of i-th knowledge and w_S and w_H respectively denote the wage rates for the scientists and the sector-specific researchers. Since H_i is assumed to be fixed, in what follows, we normalize H_i to one. Consequently, the profit maximization condition yields the following:[1]

$$w_S = v_i \theta_i \delta_i S_i^{\theta_i - 1} \left(\phi_i^{\frac{1}{\eta}} A_i^{\frac{\eta-1}{\eta}} + (1-\phi_i)^{\frac{1}{\eta}} A_j^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \quad i, j = 1, 2. \tag{8}$$

On the other hand, if the patent duration is infinite, free-entry to the monopolistic competitive markets of intermediate goods equates the price of patent with the present value of monopoly profits. Hence, letting $r(s)$ be the interest rate, it holds that

$$v_i(t) = \int_t^{\infty} \exp\left(-\int_t^{\tau} r(s)\, ds\right) \pi_i(\tau)\, d\tau, \quad i = 1, 2.$$

As a consequence, the following no-arbitrage condition holds in each moment:

$$r = \frac{\dot{v}_i}{v_i} + \frac{\pi_i}{v_i}, \quad i = 1, 2. \tag{9}$$

2.3 Consumption

The household sector consists of a continuum of identical consumers. The representative household consumes two types of consumption goods and supplies L units of physical labor, S units of research time and 2 units of human capital in each moment[8]. The household maximizes a discounted sum of utilities given by

$$U = \int_0^{\infty} \log u(C_1, C_2)\, e^{-\rho t}\, dt, \quad \rho > 0.$$

We assume that the instantaneous felicity function is of a CES form:

$$u(C_1, C_2) = \left(\gamma^{\frac{1}{\varepsilon}} C_1^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)^{\frac{1}{\varepsilon}} C_2^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \ \varepsilon > 0, \ \ 0 < \gamma < 1, \tag{10}$$

[1] Similarly, we obtain

$$w_H = v_{ii} \delta_i S_i^{\theta_i} \left(\phi_i^{\frac{1}{\eta}} A_i^{\frac{\eta-1}{\eta}} + (1-\phi_i)^{\frac{1}{\eta}} A_j^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}.$$

This condition, however, plays no essential role in the subsequent analysis.

where ε is the elasticity of substitution between the first and second consumption goods. When $\varepsilon = 1$, the function reduces to a Cobb-Douglas one. The optimal choice condition gives

$$\frac{C_2}{C_1} = \left(\mu \frac{P_2}{P_1}\right)^{-\varepsilon}, \tag{11}$$

where $\mu = [(1/\gamma) - 1]^{-1/\varepsilon} (> 0)$.

Let us denote the total consumption expenditure by $E = P_1 C_1 + P_2 C_2$. The optimal choice of each good is as follows:

$$C_1 = \frac{E}{P_1 + P_2 \left(\mu \frac{P_2}{P_1}\right)^{-\varepsilon}}, \quad C_2 = \frac{E}{P_1 \left(\mu \frac{P_2}{P_1}\right)^{\varepsilon} + P_2}. \tag{12}$$

As a result, the instantaneous indirect utility function is written as

$$\hat{u}(E, P_1, P_2) \equiv \log \frac{\varepsilon}{\varepsilon - 1} + \log \left(\gamma^{\frac{1}{\varepsilon}} \left(\frac{E}{P_1 + \mu^{-\varepsilon} P_1^{\varepsilon} P_2^{1-\varepsilon}} \right)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)^{\frac{1}{\varepsilon}} \left(\frac{E}{\mu^{\varepsilon} P_1^{1-\varepsilon} P_2^{\varepsilon} + P_2} \right)^{\frac{\varepsilon-1}{\varepsilon}} \right).$$

The flow budget constraint for the consumer is

$$\dot{V} = rV + wL + w_s S + 2w_H - E,$$

where V denotes the asset holding of the household. Since the household maximizes the discounted sum of indirect utilities by controlling E, the optimal level of consumption spending should satisfy the Euler equation,

$$\frac{\dot{E}}{E} = r - \rho,$$

together with the transversality condition: $\lim_{t \to \infty} e^{-\rho t} (V/E) = 0$.

Following Grossman and Helpman (1991), we take E as a numeraier for analytical simplicity. Thus in what follows, every price is expressed in terms of E. Given this normalization, the interest rate always equals the discount rate:

$$r = \rho, \quad \text{for all } t \geq 0. \tag{13}$$

2.4 The Market Equilibrium Conditions

Since physical labor is only input for intermediate good production, the aggregate labor used to produce C_i is determined by

$$L_i = \int_0^{A_i} c_i(j)\, dj = A_i c_i, \quad i = 1, 2. \tag{14}$$

By use of (1), we find that in the symmetric equilibrium $C_i = A_i^{\alpha_i/(\alpha_i-1)} c_i$, so that the reduced form of the production function of i-th consumption good is

$$C_i = A_i^{\frac{1}{\alpha_i - 1}} L_i, \quad i = 1, 2. \tag{15}$$

Similarly, from (3) and (4) the price of i-th consumption good is

$$P_i = A_i^{\frac{1}{1-\alpha_i}} \frac{\alpha_i w}{\alpha_i - 1}, \quad i = 1, 2. \tag{16}$$

Notice that by (15) and (16), together with the normalization condition, $P_1 C_1 + P_2 C_2 = 1$, it holds that

$$\left(\frac{\alpha_1}{\alpha_1 - 1} L_1 + \frac{\alpha_2}{\alpha_2 - 1} L_2 \right) w = 1. \tag{17}$$

This equation gives the relation between the wage rate of physical labor and employment levels in the final good sectors.

Finally, the full employment conditions for physical labor and research work are respectively given by

$$L_1 + L_2 = L, \tag{18}$$

$$S_1 + S_2 = S, \tag{19}$$

where L and S are assumed to be fixed. In the main text, we assume that production and R&D respectively use different types of labor. This assumption is made only for analytical simplicity. Appendix 1 of the paper shows that the main results will not be altered, when production and R&D employ the same type of labor.

3 Equilibrium Dynamics

3.1 The Dynamic System

In this section we derive a complete dynamic system that summarizes the behavior of the base model. It is to be noted that since we use the CES utility function, we should assume that $\alpha_1 = \alpha_2$ to obtain a feasible balanced-growth equilibrium. To see why this condition is necessary, notice that if the preference has a CES structure, both consumption goods, C_1 and C_2, should grow at a common rate on the balanced-growth equilibrium: otherwise, the utility level cannot grow at a constant rate on the balanced-growth equilibrium. Hence, considering that from (11) the consumption demand for each good depends on the relative price, we see that P_2/P_1 should stay constant on the balanced-growth path. The equations in (16) give

$$\frac{P_2}{P_1} = \frac{\alpha_2 (\alpha_1 - 1) A_2^{\frac{1}{\alpha_2 - 1}}}{\alpha_1 (\alpha_2 - 1) A_1^{\frac{1}{\alpha_1 - 1}}},$$

which means that $A_2^{\frac{1}{\alpha_2-1}}/A_1^{\frac{1}{\alpha_1-1}}$ should be constant as well. As a result, since the balanced-growth requires that A_1 and A_2 grow at the same rate, we must assume that $\alpha_1 = \alpha_2$. In what follows, we denote $\alpha_1 = \alpha_2 = \alpha\ (> 1)$.[9]

Form (15) and (16) the optimal condition (11) yields:

$$\frac{L_2}{L_1} = \mu^{-\varepsilon} \left(\frac{A_2}{A_1} \right)^{\frac{\varepsilon - 1}{\alpha - 1}} = \mu^{-\varepsilon} x^{\frac{\varepsilon - 1}{\alpha - 1}}, \tag{20}$$

where $x = A_2/A_1$. Thus L_2/L_1 increases (resp. decreases) with x if $\varepsilon > 1$ (resp. $\varepsilon < 1$). As shown above, if the consumption goods are complement each other (i.e. $\varepsilon < 1$), a rise in $x\,(= A_2/A_1)$ lowers L_2/L_1. A larger x enhances C_2. Since C_1 and C_2 are complementary, a rise in C_2 requires an increase in product of C_1 as well. This yields a relative increase in L_1 to raise C_1.

In view of the R&D functions, the growth rates of stocks of knowledge are given by

$$\frac{\dot{A}_1}{A_1} = \delta_1 s^{\theta_1} S^{\theta_1} \left(\phi_1^{\frac{1}{\eta}} + (1 - \phi_1)^{\frac{1}{\eta}} x^{\frac{\eta - 1}{\eta}} \right)^{\frac{\eta}{\eta - 1}}, \tag{21}$$

$$\frac{\dot{A}_2}{A_2} = \delta_2 (1 - s)^{\theta_2} S^{\theta_2} \left(\phi_2^{\frac{1}{\eta}} + (1 - \phi_2)^{\frac{1}{\eta}} x^{\frac{1 - \eta}{\eta}} \right)^{\frac{\eta}{\eta - 1}}, \tag{22}$$

where $s \equiv S_1/S$ and $x \equiv A_2/A_1$. For analytical simplicity, we assume that $\theta_1 = \theta_2 = \theta$. This simplification is not critical for obtaining the main results shown below. Denoting $z \equiv v_2/v_1$, we find that (8) presents the following condition:

$$\left(\frac{1 - s}{s} \right)^{\theta - 1} = \frac{\delta_1}{x z \delta_2} \left[\frac{\phi_1^{\frac{1}{\eta}} + (1 - \phi_1)^{\frac{1}{\eta}} x^{\frac{\eta - 1}{\eta}}}{\phi_2^{\frac{1}{\eta}} + (1 - \phi_2)^{\frac{1}{\eta}} x^{\frac{1 - \eta}{\eta}}} \right]^{\frac{\eta}{\eta - 1}}. \tag{23}$$

Since the left-hand side of (23) monotonically increases with s, we can express the temporary equilibrium level of s as a function of $x\,(= A_2/A_1)$ and $z\,(= v_2/v_1)$ in such a way that $s = s(x, z)$. It is shown that $s(x, z)$ satisfies

$$\text{sign } s_x = \text{sign} \left\{ 1 - \phi_1^{\frac{1}{\eta}} - \phi_2^{\frac{1}{\eta}} \right\} \text{ and } s_z < 0. \tag{24}$$

Using (21), (22) and $\dot{x}/x = \dot{A}_2/A_2 - \dot{A}_1/A_1$, we obtain:

$$\dot{x} = \hat{\Delta}(x,z)\left(\frac{1}{s(x,z)} - 1 - xz\right), \tag{25}$$

where

$$\hat{\Delta}(x,z) = x\delta_2 s(x.z)(1-s(x,z))^{\theta-1} S^{\theta}\left[\phi_2^{\frac{1}{\eta}} + (1-\phi_2)^{\frac{1}{\eta}} x^{\frac{1-\eta}{\eta}}\right]^{\frac{\eta}{\eta-1}} > 0.$$

Equation (9) shows that the relative patent prices $z (= v_2/v_1)$ changes according to $\dot{z}/z = \pi_1/v_1 - \pi_2/v_2$. Thus denoting $v_1 A_1 = q$, we see that (5), (14) and (15) yield

$$\dot{z} = z\hat{\Lambda}(x,q)\left[1 - \frac{\mu^{-\varepsilon} x^{\frac{\varepsilon-\alpha}{\alpha-1}}}{z}\right]. \tag{26}$$

where $\hat{\Lambda}(x,q) = wL_1/(\alpha-1)q$. Note that the normalization condition, $P_1C_1 + P_2C_2 = 1$, and (20) present

$$wL_1 = \frac{\alpha-1}{\alpha}\left[1 + \mu^{-\varepsilon}x^{\frac{\varepsilon-\alpha}{\alpha-1}}\right]^{-1} \equiv \beta(x).$$

Namely, the total wage paid for producing the first consumption good, wL_1, can be expressed as a function of x. This is why $wL/(\alpha-1)q$ is written as a function of x as well as q. Finally, the dynamic equation of q is given by

$$\frac{\dot{q}}{q} = \delta_1 s(x,z)^{\theta} S^{\theta}\left(\phi_1^{\frac{1}{\eta}} + (1-\phi_1)^{\frac{1}{\eta}} x^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} + \rho - \frac{\beta(x)}{(\alpha-1)q}. \tag{27}$$

To sum up, a complete dynamic system consists of (25), (26) and (27). The system is locally block recursive in the sense that motions of x and z are independent of the level of q around the balanced-growth equilibrium where $\dot{x} = \dot{z} = \dot{q} = 0$.

3.2 Balanced-Growth Equilibrium

We first consider the existence of the balanced-growth path. Since the balanced-growth equilibrium is established when $x (= A_2/A_1)$, $z (= v_2/v_1)$ and $q (= v_1 A_1)$ stay constant over time, it is easy to see that A_1 and A_2 grow at a common rate, while v_1 and v_2 decrease at the same rate of technical progress. A sufficient condition for the existence of unique balanced-growth equilibrium is the following:

Proposition 1 *If $\varepsilon \leq 1$, there is a unique balanced-growth equilibrium.*

Proof. In (26) $\dot{z} = 0$ means that

$$z = \mu^{-\varepsilon} x^{\frac{\varepsilon-\alpha}{\alpha-1}}. \tag{28}$$

When $\dot{x} = 0$, (25) yields

$$xz = \frac{1-s}{s} = \left\{ \frac{\delta_1}{xz\delta_2} \left[\frac{\phi_1^{\frac{1}{\eta}} + (1-\phi_1)^{\frac{1}{\eta}} x^{\frac{\eta-1}{\eta}}}{\phi_2^{\frac{1}{\eta}} + (1-\phi_2)^{\frac{1}{\eta}} x^{\frac{1-\eta}{\eta}}} \right]^{\frac{\eta}{\eta-1}} \right\}^{\frac{1}{\theta-1}}.$$

The above equation is rewritten as

$$z = \left(\frac{\delta_1}{\delta_2}\right)^{\frac{1}{\theta}} \frac{1}{x} \left[\frac{\phi_1^{\frac{1}{\eta}} + (1-\phi_1)^{\frac{1}{\eta}} x^{\frac{\eta-1}{\eta}}}{\phi_2^{\frac{1}{\eta}} + (1-\phi_2)^{\frac{1}{\eta}} x^{\frac{1-\eta}{\eta}}} \right]^{\frac{\eta}{\theta(\eta-1)}}. \tag{29}$$

Now define

$$F(x) \equiv \mu^{-\varepsilon} x^{\frac{\theta(\varepsilon-1)}{\alpha-1}} - \frac{\delta_1}{\delta_2} \left[\frac{\phi_1^{\frac{1}{\eta}} + (1-\phi_1)^{\frac{1}{\eta}} x^{\frac{\eta-1}{\eta}}}{\phi_2^{\frac{1}{\eta}} + (1-\phi_2)^{\frac{1}{\eta}} x^{\frac{1-\eta}{\eta}}} \right]^{\frac{\eta}{\eta-1}}.$$

If $F(x) = 0$ has a positive solution, there exists a steady-state value of x. Once the steady-state level of x is given, we can confirm that the steady-state levels of z and q are also uniquely determined. If $\varepsilon \leq 1$, then $F(x)$ monotonically decreases with x. In addition, $F(0) = +\infty$ and $\lim_{x\to\infty} F(x) = -\infty$ for $\varepsilon \leq 1$, and hence there is a unique level of $\bar{x}$ satisfying $F(\bar{x}) = 0$.

■

When $\varepsilon > 1, F(0) = 0$ and $F(x)$ is not monotonic. Hence, the balanced-growth path may not exist or there may be multiple long-run equilibria. We examine the presence of multiple balanced-growth paths in Section 4. Proposition 1 states that if we use the Cobb-Douglas utility function (i.e. $\varepsilon = 1$), there is a unique balanced-growth equilibrium. It is also to be noted that if $\varepsilon = 1$, the balanced-growth equilibrium does not need the assumption that $\alpha_1 = \alpha_2$. In contrast to the model with the CES utility function. if the utility function takes the Cobb-Douglas form, the level of utility may grow at a constant rate even though C_1 and C_2 grow at different rates on the balanced growth path. Therefore, we can assume that $\alpha_1 \neq \alpha_2$. Appendix 2 of the paper discusses the balanced-growth characterization for the case of $\varepsilon = 1$ and $\alpha_1 \neq \alpha_2$.

3.3 Patterns of Dynamics

As shown below, in this model local dynamics of the economy around the balanced-growth equilibrium may not satisfy well-behaved saddle-point stability. Before discussing various patterns of dynamics, we present the necessary and sufficient conditions under which the balanced-growth equilibrium is saddle stable and the equilibrium path is uniquely determined.

Proposition 2 *Suppose that the dynamic system consisting of (25), (26) and (27) has a feasible steady state. Then the balanced-growth equilibrium is locally saddle stable, if and only if*

$$\frac{\bar{z}\theta\left(\varepsilon-\alpha\right)}{\left(\alpha-1\right)\left(1-\theta\right)}<\frac{s_x}{s^2}+\bar{z},$$

where s and $s_x=\partial s\left(\bar{x},\bar{z}\right)/\partial x$ are evaluated at the steady state.

Proof. The dynamic system linearized at the steady state is:

$$\begin{bmatrix}\dot{x}\\ \dot{z}\\ \dot{q}\end{bmatrix}=\begin{bmatrix}\Delta\left(\bar{x},\bar{z}\right) & 0 & 0\\ 0 & \bar{z}\Lambda\left(\bar{q}\right) & 0\\ 0 & 0 & \bar{q}\end{bmatrix}\begin{bmatrix}-\frac{s_x}{s^2}-\frac{\theta_2}{\theta_1}\bar{z} & -\frac{s_z}{s^2}-\frac{\theta_2}{\theta_1}\bar{x} & 0\\ -\left(\frac{\varepsilon-\alpha}{\alpha-1}\right)\frac{1}{\bar{x}} & 1/\bar{z} & 0\\ \frac{\partial\dot{q}}{\partial x} & \frac{\partial\dot{q}}{\partial z} & \frac{\beta(\bar{x})}{(\alpha-1)\bar{q}}\end{bmatrix}\begin{bmatrix}x-\bar{x}\\ z-\bar{z}\\ q-\bar{q}\end{bmatrix},$$

where $(\bar{x},\bar{z},\bar{q})$ denote the steady-state values of (x,z,q). It is easy to see that the coefficient matrix of the right hand side of the above has a positive eigenvalue, $\beta\left(\bar{x}\right)/\left(\alpha-1\right)\bar{q}$. In addition, due to the block recursiveness of the system, we may examine the local behavior of x and z around the steady state without considering the motion of q. Denoting the coefficient matrix of the subsystem with respect to x and z by $\bar{J}$, we obtain

$$\bar{J}=\begin{bmatrix}\hat{\Delta} & 0\\ 0 & \bar{z}\hat{\Lambda}\end{bmatrix}\begin{bmatrix}-\frac{s_x}{s^2}-\bar{z} & -\frac{s_z}{s^2}-\bar{x}\\ -\left(\frac{\varepsilon-\alpha}{\alpha-1}\right)\frac{1}{\bar{x}} & \frac{1}{\bar{z}}\end{bmatrix}. \tag{30}$$

Note that (23) gives

$$\frac{\partial}{\partial z}\left(\frac{1-s}{s}\right)=\left(\frac{1}{1-\theta}\right)\frac{1-s}{s\bar{z}}.$$

Hence, in the steady state the following holds:

$$-\frac{s_z}{s^2}-\bar{x}=\frac{1}{\bar{z}}\left[\left(\frac{1}{1-\theta}\right)\frac{1-s}{s}-\bar{x}\bar{z}\right]=\frac{\theta\left(1-s\right)}{\bar{z}s\left(1-\theta\right)}>0.$$

Using the above, we find

$$\text{sign }\det\bar{J}=\text{sign }\left\{-\left(\frac{s_x}{s^2}+\bar{z}\right)\frac{1}{\bar{z}}+\frac{\theta\left(\varepsilon-\alpha\right)}{\left(\alpha-1\right)\left(1-\theta\right)}\right\}.$$

Thus $\bar{J}$ has one stable and one unstable eigenvalues, if and only if

$$\frac{\bar{z}\theta\left(\varepsilon-\alpha\right)}{\left(\alpha-1\right)\left(1-\theta\right)}<\frac{s_x}{s^2}+\bar{z}. \tag{31}$$

Since one of the eigenvalues of the entire system is $\beta\left(\bar{x}\right)/\alpha\bar{q}^2>0$, the above argument shows that there is one stable and two unstable roots. Accordingly, considering that only

x $(= A_2/A_1)$ is non-jumpable variable in our system, we confirm that the balanced-growth equilibrium is locally determinate. ■

For example, (31) is satisfied if $\phi_1^{1/\eta} + \phi_2^{1/\eta} < 1$ ($s_x > 0$ from (24)) and $\varepsilon < \alpha$. That is, there may exist a unique, stable path around the balanced-growth equilibrium, if both ε and η have low values.[10]

By use of above result, we can easily display the necessary and sufficient conditions under which the balanced growth path is locally indeterminate so that there is a continuum of equilibria.

Proposition 3 *The steady-state equilibrium of (25), (26) and (27) is locally indeterminate, if and only if*

$$\frac{\hat{\Lambda}}{\hat{\Delta}} < \frac{s_x}{s^2} + \bar{z} < \frac{\bar{z}\theta\left(\varepsilon - \alpha\right)}{\left(\alpha - 1\right)\left(1 - \theta\right)}.$$

Proof. If the above conditions are met, $\bar{J}$ in (30) has a positive determinant and a negative trace, so that all the characteristic roots of $\bar{J}$ have negative real parts. Thus the entire system have two stable and one unstable roots, implying that the initial levels of q and z cannot be determined uniquely under a given level of x. ■

Consequently, a set of necessary conditions for indeterminacy are $\phi_1^{1/\eta} + \phi_2^{1/\eta} < 1$ ($s_x > 0$) and $\varepsilon > \alpha$. This means that high substitutability between the final goods increases the possibility of multiple converging equilibria. Such kind of indeterminacy of equilibrium may produce expectation driven, sunspot fluctuations: the economy exhibits cyclical growth even in the absence of exogenous disturbances to the fundamentals.

Finally, let us consider two special cases. First, suppose that utility and knowledge production functions take the Cobb-Douglas forms, that is, $\varepsilon = \eta = 1$. In this case we obtain the following simple result:

Corollary 1 *If* $\varepsilon = \eta = 1$, *then the dynamic system is locally saddle stable.*

Proof. It is easy to see that when $\varepsilon = 1$, the determinant of submatrix (30) satisfies

$$\operatorname{sign} \det \bar{J} = \operatorname{sign} \left\{ s_z \bar{z} - s_x \bar{x} \right\}.$$

In addition, when $\eta = 1$, (23) is replaced with

$$\frac{\left(1 - s\right)^{\theta - 1}}{s^{\theta - 1}} = \frac{\delta_1 x^{1 - (\phi_1 + \phi_2)}}{\delta_2 z}.$$

As a result, we find

$$s_z\bar{z} - s_x\bar{x} = \frac{s^{\theta+2}\delta_1\bar{x}^{1-(\phi_1+\phi_2)}}{(1-\theta)(1-s)^{\theta}\delta_2\bar{z}}(\phi_1+\phi_2-2) < 0.$$

This implies that $\bar{J}$ has one stable and one unstable roots. Therefore, the entire system has one stable and two unstable roots, which ensures that there is a locally unique stable path converging to the steady state. ■

Proposition 1 has shown that the balanced-growth path is uniquely given if $\varepsilon = 1$. Therefore, Corollary 1 states that if both utility and knowledge production functions are Cobb-Douglas ones, we obtain a well-behaved dynamic system in which a unique equilibrium path monotonically converges to the uniquely determined balanced-growth equilibrium.

Next, assume that there is no technological spillovers, so that $\phi_1 = \phi_2 = 1$ in (7). If this is the case, dynamic behavior of the economy depends entirely on the preference structure:

Corollary 2 *Suppose that technological spillovers are intrasectoral alone. Then the balanced-growth equilibrium is saddle stable if* $\varepsilon < 1$*, while it is totally unstable if* $\varepsilon > 1$.

Proof. When $\phi_1 = \phi_2 = 1$, (23) becomes

$$\frac{1-s}{s} = \left(\frac{\delta_1}{xz\delta_2}\right)^{\frac{1}{\theta-1}}.$$

Using this relation, we can rewrite (25) as

$$\dot{x} = \hat{\Delta}(x,z)\left[\left(\frac{\delta_1}{xz\delta_2}\right)^{\frac{1}{\theta-1}} - xz\right], \tag{32}$$

where $\hat{\Delta}(x,z) = x\delta_2 s(x.z)(1-s(x,z))^{\theta-1}S^{\theta} > 0$. The sub-dynamical system of x and z consists of (26) and (32). Hence, in view of the steady-state conditions, the coefficient matrix of the linealized sub-system is expressed as

$$\hat{J} = \begin{bmatrix} \hat{\Delta} & 0 \\ 0 & \hat{\Lambda} \end{bmatrix} \begin{bmatrix} \frac{1}{\bar{x}}\left(\frac{\theta}{1-\theta}\right) & \frac{1}{\bar{z}}\left(\frac{\theta}{1-\theta}\right) \\ -\left(\frac{\varepsilon-\alpha}{\alpha-1}\right)\frac{1}{\bar{x}} & \frac{1}{\bar{z}} \end{bmatrix}.$$

The determinant of $\hat{J}$ satisfies

$$\text{sign } \det \hat{J} = \text{sign } \left\{\frac{\varepsilon-1}{\alpha-1}\right\}.$$

As a consequence, if $\varepsilon < 1$, matrix $\hat{J}$ has a negative determinant and thus the entire system has one stable and two unstable roots, which implies that the local saddle stability holds.

Since the trace of $\hat{J}$ has a positive value, both eigenvalues of $\hat{J}$ have positive real parts when $\varepsilon > 1$. This means that the entire system has three unstable roots, and thus there is no converging path towards the balanced-growth equilibrium. ■

3.4 Intuitive Discussion

As for the stability conditions shown above, we can give intuitive implications. To obtain economic intuition, it is useful to focus on the two elasticity parameters, ε and η. As shown in the proof of Proposition 2, the balanced-growth path is locally determinate if and only if

$$-\left(\frac{s_x}{s^2}+\bar{z}\right)\frac{1}{\bar{z}}+\frac{\theta\left(\varepsilon-\alpha\right)}{\left(\alpha-1\right)\left(1-\theta\right)}<0,$$

where sign $s_x =$ sign $\left\{1-\phi_1^{1/\eta}-\phi_2^{1/\eta}\right\}$, $\alpha > 1$ and $0 < \theta < 1$. A set of sufficient conditions for saddle-point stability thus include $\varepsilon < \alpha$ and $s_x > 0$. Since $0 < \phi_i < 1$ $(i = 1, 2)$, s_x tends to be positive if η has a small value. In words, when both consumption goods and stocks of technical knowledge have low elasticities of substitution, the economy may exhibit a well-behaved, saddlepoint property. For example, let us assume that $\varepsilon < 1$ and $\eta < 1$ and that own effect of technological spillovers is small enough to satisfy $s_x > 0$. Now suppose that the economy initially stays on the balanced-growth path and there is an unanticipated increase in $x\ (= A_2/A_1)$. Such a disturbance takes the economy out of the balanced-growth equilibrium. Since η is low, the stocks of technical knowledge, A_1 and A_2, are complement each other. Additionally, the own effect of technical diffusion is also low due to small values of ϕ_1 and ϕ_2. Therefore, a rise in x (that is, a relative increase in A_2) will enhance R&D activity in the first sector so that the researchers shift from the second to the first R&D sector. At the same time, (20) shows that if $\varepsilon < 1$, an increase in x reduces L_2/L_1. Thus the labor forece engaging in production also shifts from the second to the first final good sector. Therefore, in the presence of low substitutability among the consumption goods as well as among the knowledge stocks, an increase in A_2/A_1 enhances both production and innovation activities in the first sector. As a result, accumulation of A_1 is accelerated and hence $x\,(= A_2/A_1)$ starts decreasing. Namely, x displays self-stabilizing motion. On the other hand, (26) shows that $d\dot{z}/dz > 0$ around the steady state, implying that the relative patent price, z, exhibits self-destabilizing behavior. This means that stable behavior of x serves to sustain the saddle stability of the sub-dynamical system with respect to x and z.

Next, consider the case of multiple converging equilibria. The necessary conditions for the local asymptotic stability of the sub-dynamical system are:

$$-\left(\frac{s_x}{s^2}+\bar{z}\right)\frac{1}{\bar{z}}+\frac{\theta\left(\varepsilon-\alpha\right)}{\left(\alpha-1\right)\left(1-\theta\right)}>0 \text{ and } \frac{s_x}{s^2}+\bar{z}>0.$$

Those conditions are met, if $\varepsilon > \alpha$, $s_x > 0$ (i.e. η is small) and if $s_x/s^2 + \bar{z}$ is not so large as the first condition above fails to hold. Therefore, multiplicity of converging paths tends to emerge, when the substitutability among the consumption goods is high and that among the knowledge stocks is low. As in the case of saddlepoint stability, an increase in x expands the R&D activities in the first sector by shifting researchers from the second to the first R&D sector. On the other hand, since $\varepsilon > \alpha\,(> 1)$, an increase in x raises L_2/L_1 so that production of the second final good rises. Hence, the initial increase in x yields two opposing effects on the final goods production: the reallocation of researchers between the R&D sectors increases the variety of intermediate goods devoted to the first consumption good, while the reallocation of production workers raises the second consumption good. If the second consumption good sector grows faster than the first one, then the intermediate goods production used by the second final good sector also increases. As a consequence, $x\ (= A_2/A_1)$ rises further. In contrast, if the first consumption good sector grows faster than the second consumption good sector, then x will decrease and thus it shows self-stabilizing behavior. During the transition process, x may display cyclical motion depending on which consumption good sector grows faster than the other. Since x eventually converges to its steady-state level, the initial value of the relative price of patents, z_0, can be selected arbitrarily at least around the balanced-growth path. Hence, we can find a continuum of converging trajectories near the balanced-growth equilibrium. This implies that sunspot-driven, non-fundamental shocks may enhance economic fluctuations.

Finally, assume that $\eta > 1$, $\varepsilon > \alpha\,(> 1)$ and $s_x < 0$. Under those conditions, (31) may fail to hold. In particular, if s_x is small enough to yield $s_x/s^2 + \bar{z} < 0$, then the trace of $\bar{J}$ is positive and thus the balanced-growth path is totally unstable. This is because the motion of x is unstable when $\varepsilon > \alpha$ and $s_x < 0$. In this case, both consumption goods and stocks of technical knowledge have high substitutability. Again, suppose that there is an unanticipated rise in x in the economy that initially stays in the steady state. Since A_1 and A_2 are highly substitutable, a rise in $x\,(= A_2/A_1)$ shifts researchers from the first to the second R&D sector, which reduces $s\,(= S_1/S)$. In addition, the high substitutability between C_1 and C_2 yields a relative increase in C_2, because from (20) a rise in x yields a shift of the production labor form the first to the second consumption good sector. Thus R&D as well as production activities expand in the second sector, which produces a further increase in x. Considering unstable behavior of z, the diverging motion of x establishes total instability of the balanced-growth path. Notice that if $\varepsilon = +\infty$ and $\eta = +\infty$, both A_1 and A_2 as well as C_1 and C_2 are perfectly substitutable each other, so that heterogeneity in our model disappears. In this case, (31) cannot be satisfied and the trace of $\bar{J}$ is positive, that is, the balanced-growth path is unstable. In fact, if $\varepsilon = +\infty$ and $\eta = +\infty$, then the model becomes the same one studied by Grossman and Helpman (1991a, Chapter 3). In the absence of technological heterogeneity, the model satisfies the Ak property and the total instability means that the economy always stays in the balanced-growth equilibrium.

4 Multiple Balanced-Growth Paths

As pointed out by Proposition 1, if $\varepsilon > 1$, the economy may have multiple balanced-growth equilibria. A more detailed inspection of function $F(x)$ defined in the proof of the proposition reveals there is little possibility of the presence of multiple steady states under plausible parameter values. However, it is not difficult to obtain multiple steady states in our framework, if we slightly modify the form of knowledge production functions. For example, suppose that $\phi_i = 1$ in (7) and that there are additional external effects in an additive form. More specifically, following Starts (1998), let us assume:

$$\dot{A}_i = \delta_i S_i^{\theta} A_i + \varphi_i A_i + \lambda_i A_j, \ \varphi_i > 0, \ \lambda_i > 0, \ \ i, j = 1, 2, \ \ i \neq j. \tag{33}$$

Namely, R&D effort represented by research work is magnified by own knowledge stock, but there are additive effects of knowledge spillovers.[11] By use of (33), we obtain:

$$\begin{aligned}\frac{\dot{A}_1}{A_1} &= \delta_1 S_1^{\theta} + \lambda_1 x + \varphi_1, \\ \frac{\dot{A}_2}{A_2} &= \delta_2 S_2^{\theta} + \lambda_2 \frac{1}{x} + \varphi_2.\end{aligned}$$

In this case, (23) becomes

$$\frac{1-s}{s} = \left(\frac{\delta_1}{xz\delta_2}\right)^{\frac{1}{\theta-1}}.$$

Given (33), the model is close to the case of no intersetoral technical spillovers examined above. Only key difference is that the dynamic behavior of x is replaced with

$$\dot{x} = S^{\theta} \left[\delta_2 \left(1 - s(x, z)\right)^{\theta} - \delta_1 s(x, z)^{\theta}\right] x + \lambda_2 - \lambda_1 x^2 + \left(\varphi_2 - \varphi_1\right) x, \tag{34}$$

where

$$s(x,z) = \frac{1}{1+\left(\frac{\delta_1}{xz\delta_2}\right)^{\frac{1}{\theta-1}}}.$$

We can examine local dynamics by use of (26) and (34). The steady-state values of x and z satisfy $\dot{x} = \dot{z} = 0$ in (26) and (34) so that we obtain the following conditions:

$$S^{\theta}\left[\delta_2\left(\frac{\left(\frac{\delta_1}{xz\delta_2}\right)^{\frac{1}{\theta-1}}}{1+\left(\frac{\delta_1}{xz\delta_2}\right)^{\frac{1}{\theta-1}}}\right)^{\theta} - \delta_1\left(\frac{1}{\bar{1}+\left(\frac{\delta_1}{xz\delta_2}\right)^{\frac{1}{\theta-1}}}\right)^{\theta}\right] + \frac{\lambda_2}{x} - \lambda_1 x + \varphi_2 - \varphi_1 = 0, \tag{35}$$

$$z - \mu^{-\varepsilon} x^{\frac{\varepsilon-\alpha}{\alpha-1}} = 0. \tag{36}$$

Condition (36) gives $xz = \mu^{-\varepsilon} x^{\frac{\varepsilon-1}{\alpha-1}} = \frac{1-\gamma}{\gamma} x^{\frac{\varepsilon-1}{\alpha-1}}$. Substituting this into (35) presents

$$\begin{array}{l} S^{\theta}\delta_2\left[\frac{(\delta_1\gamma/\delta_2(1-\gamma))^{\frac{1}{\theta-1}} x^{\frac{\varepsilon-1}{(\alpha-1)(1-\theta)}}}{1+(\delta_1\gamma/\delta_2(1-\gamma))^{\frac{1}{\theta-1}} x^{\frac{\varepsilon-1}{(\alpha-1)(1-\theta)}}}\right]^{\theta} - S^{\theta}\delta_1\left[\frac{1}{1+(\delta_1\gamma/\delta_2(1-\gamma))^{\frac{1}{\theta-1}} x^{\frac{\varepsilon-1}{(\alpha-1)(1-\theta)}}}\right]^{\theta} \\ -\lambda_1 x + \frac{\lambda_2}{x} - \varphi_1 + \varphi_2 = 0. \end{array} \tag{37}$$

The positive roots of this equation present the steady-state levels of x.

To examine a numerical example, we use the following parameter values:

$$\begin{aligned} \alpha &= 8,\ \varepsilon = 10,\ \delta_1 = 0.012,\ \delta_2 = 0.022,\ \varphi_1 = 0.03,\ \varphi_2 = 0.08, \\ \lambda_1 &= 0.05,\ \lambda_2 = 0.03,\ \theta = 0.8,\ \gamma = 0.5,\ S = 75. \end{aligned}$$

In this example, the elasticity of substitution among intermediate goods is high $(\alpha = 8)$, and thus the mark up ratio $\alpha/(1-\alpha)$ is sufficiently low as 1.125. Given the above parameter values, (37) has three roots. One of them that gives the highest value of x yields an implausibly high rate of balanced growth, so that it violates the transversality condition for the household optimization under a plausible level of time discount rate, ρ. Hence, we focus on the other two roots whose values are: $x = 0.0178$ and 0.5693. The balanced-growth rates of consumption corresponding to those steady-state values of x are $g = 0.0469$ and 0.0704, respectively. Moreover, the steady state with a higher growth rate exhibits local saddlepoint property, while the steady state with a lower growth is locally unstable.[12] This means that the stable saddle path that converges to the high-growth steady state does not span the entire (x, z) space. Hence, the economy whose initial value of x is sufficiently small cannot converges to any balanced-growth path.

5 Conclusion

This paper has examined a model of endogenous technical change with two R&D sectors. We have assumed that each final good needs sector-specific technical knowledge so that each R&D sector produces heterogenous knowledge. The central message of our analysis is that dynamic property of the economy heavily depends on the form of intersectoral technical spillovers as well as on the preference structure. This is in contrast to the standard R&D based growth models with homogenous technical knowledge in which well-behaved saddle-point stability generally holds. In our model economy, if creation of new knowledge in each R&D sector is subject to a Cobb-Douglas function of stocks of technical knowledge and if the instantaneous utility of the representative family is logarithmic, then the economy exhibits well-behaved dynamics: there is a unique balanced-growth equilibrium and it is at least locally determinate in the sense that there is a unique converging path towards the balanced-growth equilibrium. Such a well-behaved dynamic pattern may not hold, if we assume the CES forms of preferences and knowledge production functions. In the generalized model, we have shown that the balanced-growth equilibrium may be locally indeterminate or it would be unstable. In the former case, we may have a continuum of converging paths, and hence sunspot fluctuations may emerge. In the latter, there is no converging path towards the balanced-growth equilibrium.

A limitation of our discussion is that we have treated technical diffusion as external effects. As Romer (1990) claims, such a specification captures nonexcludability of technical knowledge. However, in reality, at least part of technical knowledge is traded in the market thorough transfers of patents and our formulation (and the formulations in the majority of R&D based growth models) does not consider this aspect. Dynamic analysis on the models with heterogenous R&D with a more detailed microeconomics structure of technical spillovers deserves further investigation.

Appendix 1

In the main text we have assumed that production and R&D respectively use different types of labor. In this Appendix, we show that the main conclusions of our analysis still hold when the production and R&D activities employ homogenous labor. This appendix assumes that the utility and knowledge production functions are of Cobb-Douglas types. The main conclusion does not change, if we use the CES forms of utility and knowledge production functions.

If the homogenous labor is allocated between the production and the research sectors, the labor market equilibrium condition is

$$L_1 + L_2 + S_1 + S_2 = N, \tag{A1}$$

where N is the aggregate labor supply that is assumed to be fixed. As a result, we may set $w = w_S$. As shown in Section 3.1, if the utility function satisfies log-linearity, the labor allocation satisfies $L_2 = \zeta L_1$, where $\zeta = \frac{(1-\gamma)\alpha_1(\alpha_2-1)}{\gamma(\alpha_1-1)\alpha_2}$. In addition, when we assume that $\theta_1 = \theta_2$ to simplify the algebra, we obtain:

$$\frac{S_2}{S_1} = \left(\frac{\theta_1\delta_1}{\theta_2\delta_2}\frac{x^{1-(\phi+\phi_2)}}{z}\right)^{\frac{1}{\theta_1-1}} = \chi(x,z),$$

$$\frac{S_2^{\theta_2-1}}{S_1^{\theta_1-1}} = \frac{\theta_1\delta_1}{\theta_2\delta_2}\left(\frac{x^{1-(\phi+\phi_2)}}{z}\right).$$

Hence, (A1) may be written as

$$(1+\zeta) L_1 + (1+\chi(x,z)) S_2 = N. \tag{A2}$$

By use of $E = P_1C_1 + P_2C_2$, together with (12) and (13) in the main text, we obtain

$$E = \frac{\alpha w}{\alpha - 1}L_1 + \frac{\alpha_2 w}{\alpha_2 - 1}L_2 = \left(\frac{\alpha_1}{\alpha_1 - 1} + \frac{\alpha_2\zeta}{\alpha_2 - 1}\right)L_2 w.$$

Thus if we set $E = 1$, the above gives

$$L_1 = \frac{1}{w}\left(\frac{\alpha_1}{\alpha_1 - 1} + \frac{\alpha_2\zeta}{\alpha_2 - 1}\right)^{-1} = L_1(w), \quad L_1' < 0.$$

This means that from (A2) S_1 is determined by

$$S_1 = \frac{1}{1+\chi(x,z)}(N - (1+\zeta) L_1(w)). \tag{A3}$$

Keeping in mind that we have assumed that $\theta_1 = \theta_2$, we can derive the following dynamic equation of x :

$$\frac{\dot{x}}{x} = x^{1-\phi}\delta_1 S_1^{\theta}\left(\frac{S_2}{S_1} - xz\right) = \Delta(x,z,w)(\chi(x,z) - xz), \tag{A4}$$

where from (A3) $\Delta(.)$ is given by

$$\begin{aligned}\Delta(x,z,w) &= \delta_1 x^{1-\phi} S_1^{\theta} \\ &= \delta_1 x^{1-\phi}\left[\frac{1}{1+\chi(x,z)}(N - (1+\xi) L_1(w))\right]^{\theta}.\end{aligned}$$

The behavior of z does not change so that

$$\frac{\dot{z}}{z} = \frac{wL_2}{(\alpha_2 - 1)\, v_2 A_2} (1 - \xi xz) = \Lambda(x, z, w)(1 - \xi xz), \tag{A5}$$

where

$$\begin{aligned}\Lambda(x, z, w) &= \frac{wL_2}{(\alpha_2 - 1)\, v_2 A_2} \\ &= \frac{\theta_1 \delta_2}{\alpha_2 - 1} \left[\frac{\chi(x, z)}{1 + \chi(x, z)} (N - (1 + \zeta) L_1(w)) \right]^{\theta - 1} \xi L_1(w).\end{aligned}$$

Finally, by use of $w = \theta_1 \delta_1 v_1 S_1^{\theta_1 - 1} A_1^{\phi} A_2{}^{1-\phi}$, we obtain

$$\frac{\dot{w}}{w} = \frac{\dot{v}_1}{v_1} + (\theta_1 - 1) \frac{\dot{S}_1}{S_1} + \phi \frac{\dot{A}_1}{A_1} + (1 - \phi) \frac{\dot{A}_2}{A_2}.$$

Since $E = 1$ means that $r = \rho$ for all $t \geq 0$, the dynamic behavior of v_1 is described by $\dot{v}_1 / v_1 = \rho - \pi_1 / v_1$. Hence, substituting (21) and (22) into the above and using (A3), it is easy to see that dynamic equation of the real wage, w, can be expressed as

$$\dot{w} = \Omega(x, z, w). \tag{A6}$$

The complete dynamic system with respect to x, z and w is thus given by (A4), (A5) and (A6). Although function $\Omega(.)$ in ($A6$) is complex, as well as in the base model, the dynamic behaviors of x and z near the steady state are independent of the motion of w. Therefore, the patterns of growth in the case of homogenous labor are essentially the same as those in the case of heterogenous labor examined in the main text.

Appendix 2

In this appendix, we briefly discuss how the main results of the model with Cobb-Douglas functions would be modified if $\alpha_1 \neq \alpha_2$ and $\theta_1 \neq \theta_2$. Using (12), (14) and (15), we obtain:

$$\frac{L_2}{L_1} = \frac{A_1^{1/(\alpha_1 - 1)} C_2}{A_2^{1/(\alpha_2 - 1)} C_1} = \frac{(1 - \gamma) A_1^{1/(\alpha_1 - 1)} P_1}{\gamma A_2^{1/(\alpha_2 - 1)} P_2} = \frac{\alpha_1 (1 - \gamma)(\alpha_2 - 1)}{\alpha_2 \gamma (\alpha_1 - 1)}.$$

Since on the balanced-growth path $x\,(= A_2/A_1)$ and $z\,(= v_2/v_1)$ stay constant, A_1 and A_2 grow at the same rate and the market value of each knowledge, $v_i A_i$, does not change. Therefore, denoting the steady rate of technical change by $\bar{g}\left(= \dot{A}_i / A_i\right)$, we see that $\dot{v}_i / v_i = -g$ in the balanced-growth equilibrium. Additionally, from (15) and (16) the balanced-growth rates of final goods and the relative price, P_2/P_1, are respectively given by

$$\frac{\dot{C}_i}{C_i} = \frac{\bar{g}}{\alpha_i - 1} \text{ and } \frac{\dot{P}_2}{P_2} - \frac{\dot{P}_1}{P_1} = \left(\frac{1}{\alpha_2 - 1} - \frac{1}{\alpha_1 - 1} \right) \bar{g}.$$

Since the relative share of consumption expenditure, P_2C_2/P_1C_1, is constant on the balanced-growth path, the difference in the growth rates of the final goods is offset by the steady change in the relative price. It is easy to see that the steady-state value of x and the balanced-growth rate of technical progress are respectively given by

$$\bar{x} = \left(\frac{\theta_2\xi}{\theta_2\xi + \theta_1}\right)^{\frac{\theta_2 - 1}{2-(\phi_1+\phi_2)}} \left(\frac{\theta_1}{\theta_2\xi + \theta_1}\right)^{\frac{1-\theta_1}{2-(\phi_1+\phi_2)}} \left(\frac{\delta_2}{\delta_1}\xi\right)^{\frac{1}{2-(\phi_1+\phi_2)}} S^{\frac{\theta_2-\theta_1}{2-(\phi_1+\phi_2)}},$$

$$\bar{g} = \frac{\theta_1\delta_1}{\theta_1 + \theta_2\xi} \left(\frac{\xi\delta_2\left(1-\bar{s}\right)^{\theta_2}}{\delta_1\bar{s}^{\theta_1}}\right)^{\frac{1-\phi_1}{2-(\phi_1+\phi_2)}} S^{\frac{(\theta_2-\theta_1)(1-\phi_1)}{2-(\phi_1+\phi_2)}+\theta_1} \bar{s}^{\theta_1 - 1}.$$

Footnotes

1. See, for example, Grilichas (1992), Caballero and Jaffe (1993) and Branstetter (2001).

2. Grossman and Helpman (1991a, Chapter 3) shows that the variety expansion model of technical change without physical capital may have the Ak property and hence the economy has no transitional dynamics and it always stays on the balanced-growth path. Arnold (2000) proves that Romer's (1990) model, which involves physical capital and transition processes, can display well behaved saddlepoint stability under mild restrictions on the model. Note that if the intermediate inputs in the final good production are complement each other, even a model with homogenous technical knowledge could produce multiple steady states and complex dynamics: see Benhabib et al. (1994) and Evans et al. (1998).

3. See also Krusell (1998).

4. Although the number of studies on technical spillovers in closed economies is rather small, there are many studies on the relationship between growth and technological spillovers in the world economy. Well cited studies in this field include Rivera-Batiz and Romer (1991 and 1992), Grossman and Helpman (1991b), and Barro and Sala-i-Martin (1997). Most of those studies focus on the balanced-growth equilibrium analysis. In addition, many of them treat one way technical spillovers from advanced economies to developing ones rather than mutual spillovers.

5. In Section 4 we discuss Start's (1998) formulation of R&D functions.

6. Alternatively, we can consider that $c_i(j)$ denotes a consumption good of type j in group i and C_i is a composite good that consists of a variety of goods ranging from 0 to A_i.

7. In the standard modelling where there is only one R&D sector, it is usually assumed that S and H are perfectly substitute each other, so that its production function is $\dot{A} = \delta SX$: see, for example, Romer (1990). If we assume that $\dot{A}_i = \delta_i S_i X_i$ in our two-sector setting, the research scientists generally work only one of the two sectors during the transition process. Although the balanced-growth characterization is essentially the same as that of our formulation, the bang-bang behavior of the model makes stability analysis difficult. Introduction of the sector-specific human capital, H_i, is helpful for avoiding such kind of analytical difficulty.

8. Remember that we have assumed that $H_1 = H_2 = 1$.

9. This assumption can be dropped, if the utility function is of a Cobb-Douglas type ($\varepsilon = 1$): see Section 3.2 and Appendix 2.

10. Remember that $\phi_1^{1/\eta} + \phi_2^{1/\eta}$ increases with η, because $0 < \phi_i < 1$ $(i = 1, 2)$.

11. As pointed out in Section 1, the following analysis presents a microfundation for Start's (1998) modelling.

12. It is possible to confirm this fact by analyzing the phase diagram of (26) and (34) in (x, z) space.

References

[1] Acemoglu, D. "Directed Technical Change", *Review of Economic Studies 69* (2002): *781-809.*

[2] Aghion, P. and Howitt, P. "A Model of Growth through Creative Destruction." *Econometrica* 60 (1992): 323-351.

[3] Arnold, L. "Stability of the Market Equilibrium in Romer's Model of Endogenous Technical Change: A Complete Characterization", *Journal of Macroeconomics* 21(2000): 69-84.

[4] Barro, R.J. and Sala-i-Martin, X. "Technological Diffusion, Convergence, and Growth", *Journal of Economic Growth* 2 (1997): 1-26.

[5] Benhabib, J., Perli, R. and Xie, D. "Monopolisitic Competition, Indeterminacy and Growth", *Ricerche Economiche* 48 (1994):113-142.

[6] Branstetter, L.G. "Are Knowledge Spillovers International or International in Scope", *Journal of International Economics* 53 (2001): 53-79.

[7] Caballero, R.J. and Jaffe, A.B. "How High are the Giant's Shoulders: An Empirical Assessment of Knowledge Spillovers and Creative Destruction in a Model of Economic Growth", *NBER Macroeconomics Annual* 1993, MIT Press, Cambridge, MA.,(1993): 15-74.

[8] Dinopoulos, E. and Thompsion, P. "Schumpeterian Growth without Scale Effects", *Journal of Economic Growth* 3, (1998): 135-154.

[9] Evans, G.W., Honkapohja, S. and Romer, P.M. "Growth Cycles." *American Economic Review* 88 (1998): 595-515.

[10] Grilichas, Z. "The Search for R&D Spillovers." *Scandinavian Journal of Economics*, 94 (1992): 29-47.

[11] Grossman, G.M. and Helpman, E. *Innovation and Growth in the Global Economy*, Cambridge, MA., MIT Press(1991a).

[12] Grossman, G.M. and Helpman, E. "Trade, Knowledge Spillovers and Growth." *European Economic Review* 35 (1991b): 515-526.

[13] Krusell, P. “Investment-Specific R&D and the Decline in the Relative Price of Capital.” *Journal of Economic Growth* 3 (1998): 131-141.

[14] Li, C-W. “Endogenous vs. Semi-Endogenous Growth in a Two-R&D-Sector Model.” *Economic Journal* 110 (2000): 109-122.

[15] Li, C-W. ”Growth and Scale Effects: the Role of Knowledge Spilliovers”, *Economics Letters* 74, (2002): 177-185.

[16] Revera-Batiz, L. and Romer, P. “Economic Integration and Endogenous Growth.” *Quarterly Journal of Economics* 106 (2) (1991): 531-555.

[17] Revera-Batiz, and Romer, P.M. “International Trade with Endogenous Technical Change.” *European Economic Review* 35 (1992): 971-1004.

[18] Romer. P. M. “Endogenous Technical Change.” *Journal of Political Economy* 95 (1990): s71-s102.

[19] Segerstrom, P. ”Endogenous Growth without Scale Effects”, *American Economic Review*, 88 (1998): 1290-1310.

[20] Smulders, S. and van de Klundert, T. ”Imperfect Competition, Concentration and Growth with Firm-Specific R&D”, *European Economic Review* 39 (1995): 139-160.

[21] Startz, R. “Growth States and Shocks.” *Journal of Economic Growth* 3 (1998): 203-216.

[22] Young, A. ”Growth without Scale Effects”, *Journal of Political Economy* 106 (1998): 41-63.

[23] Xie, D. “An Endogenous Growth Model with Expanding Ranges of Consumer Goods and of Producer Durables” *International Economic Review* 39 (1998): 439-460.