Voracity vs. Scale Effect in a Growing Economy

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Abstract

This paper extends the standard model of growth with insecure property rights by introducing variable labor supply and increasing returns to scale. It is assumed that capital stock is jointly owned by multiple interest groups and that each group participates production activities by supplying its labor force. In this setting, there are two opposing factors that affect growth: over consumption in the absence of secure property rights and the scale effect due to the presence of increasing returns. The growth performance of the economy thus depends on which factor dominates.

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1 Introduction

The literature on economic growth without secure property rights has often analyzed growth models in which multiple interest groups exploit commonly accessible resources: see, for example, Tornell and Velasco (1992), Tornell and Lane (1998 and 1999) and Lindner and Strulik (2002). A distinguished feature of this kind of modeling is that due to the common pool problem, a rise in the number of agents participating the exploitation game has a negative impact on growth. In addition, there may exist the 'voracity effect': an increase in productivity would yield a larger consumption growth of each agent than the income growth generated by the technological improvement, so that a higher productivity decelerates long-term growth.

The existing studies mentioned above use Ak growth models to obtain analytically tractable solutions of the dynamic games.[1] Namely, they assume that final goods are produced by a commonly owned capital stock alone. In this setting each agent can consume final goods without participating any production activity. One may conjecture that the main findings in the growth models with insecure property rights may be generated not only by the common pool issue but also by such a simple technological specification. To examine this question, we extend the baseline model by assuming that production needs labor as well as capital and hence the agents should participate production activities by supplying their labor force. To keep the tractability of analysis, we still assume that output linearly depends on capital, which means that the aggregate production technology exhibits increasing returns to scale with respect to capital and labor. In our generalized setting, there are two opposing factors that affect growth performance of the economy. The first is the over-consumption effect generated by the common pool problem and the second is the scale effect produced by the presence of increasing returns. The first factor may yield the voracity effect, while the second one contributes to accelerating growth. The resulting growth performance of the economy with insecure property rights, therefore, depends on which factor dominates in the process of capital accumulation.

[1] Lindner and Strulik (2002) also analyze a model with the standard neoclassical technology.

2 The Model

We use an Ak growth model with variable labor supply. The economy produces a homogenous final good. The production technology is specified as

$$y = Akf(L), \quad A > 0, \tag{1}$$

where y is output, k is capital and L denotes the aggregate labor supply.[2] We assume that function $f(L)$ is positive, monotonically increasing, strictly concave in L and satisfies $f(0) = 0$.The above formulation assumes that the production technology exhibits increasing return to scale with respect to capital and labor.

There are n (≥ 2) interest groups. The i-th group has s_i members, so that the total number of agents in the economy at large is $\Sigma_{i=1}^n s_i = N$. All the agents in the same group are identical. While the capital stock k is jointly owned by the groups, each member supplies its own labor for production. Thus, if an individual agent supplies l_i units of labor, the aggregate labor supply is $L = \Sigma_{i=1}^n s_i l_i$. The instantaneous utility of an individual agent in group i depends positively on consumption, c_i, and negatively on labor supply, l_i. The objective function of group i is its discounted-sum of utilities over an infinite-time horizon:

$$U_i = \int_0^\infty s_i u(c_i, l_i) e^{-\rho t} dt, \quad \rho > 0, \qquad i = 1, 2, ..., n,$$

where $u(.)$ is assumed to be strictly concave in (c_i, l_i). The final good is used for consumption and capital formation. Since we have assumed that the aggregate production technology is commonly owned by all the groups, the capital formation is determined by

$$\dot{k} = Akf(\Sigma_{i=1}^n s_i l_i) - \Sigma_{i=1}^n s_i c_i. \tag{2}$$

Each group maximizes U_i by selecting the sequences of c_i and l_i subject to (2) together with the given initial level of capital, $k_0 (> 0)$.

The model given above is a differential game in which each player's strategies are its consumption and labor supply, while the state variable of the game is the aggregate stock of capital. Following the existing studies, we focus on the Markov-perfect Nash (feedback Nash) equilibrium. That is, we assume that each group's strategies, c_i and l_i, are functions of the

[2] This specification has been used, for instance, by Benhabib and Farmer (1994) and Pelloni and Waldmann (1998).

current level of the aggregate capital k alone. This means that the value function of the i-th group's optimization problem at time t can be written as

$$V_i(k(t)) \equiv \max \int_t^{\infty} s_i e^{-\rho(\tau - t)} u(c_i(\tau), l_i(\tau)) \, d\tau.$$

This function satisfies the Hamilton-Jacobi-Bellman (HJB) equation such that

$$\rho V_i(k) = \max_{c_i . l_i} \left\{ s_i u(c_i, l_i) + V_i'(k) \left[Akf\left(\Sigma_{i=1}^n s_i l_i\right) - \Sigma_{i=1}^n s_i c_i \right] \right\} \tag{3}$$

for all $t \geq 0$. In solving the maximization problem defined in the right-hand-side of (3), the i-th group takes the other players' strategies, $\{c_j, l_j\}_{j \neq i}$ $(j = 1, 2, .., n)$, as given. The first-order conditions for maximization are:

$$u_c(c_i, l_i) - V_i'(k) = 0, \quad i = 1.2., ..., n, \tag{4}$$

$$u_l(c_i, l_i) + V_i'(k) \, Akf'(L) = 0, \quad i = 1, 2, ..., n. \tag{5}$$

Equations (4) and (5) give the Markov-perfect Nash solutions, $\{c_i(k), l_i(k)\}$. Substituting these solutions into the HJB equation (3), we obtain

$$\rho V_i(k) = s_i u(c_i(k), l_i(k)) + V_i'(k) \left[Akf\left(\Sigma_{j=1}^n s_j l_j(k)\right) - \Sigma_{j=1}^n s_j c_j(k) \right]. \tag{6}$$

Using the envelop theorem, we find that differentiation of both sides of (6) with respect to k yields:

$$\begin{aligned} \rho V_i'(k) \;&=\; V_i'(k) \left[Af\left(\Sigma_{j=1}^n s_j l_j(k)\right) + Ak\left(\Sigma_{j \neq i}^n s_j l_j'(k)\right) f'\left(\Sigma_{j=1}^n s_j l_j(k)\right) - \Sigma_{i \neq j}^n s_j c_j'(k) \right] \\ &\quad + V_i''(k) \left[Akf\left(\Sigma_{j=1}^n l_i(k)\right) - \Sigma_{j=1}^n s_i c_j(k) \right], \quad i = 1, 2, ..., n. \end{aligned} \tag{7}$$

3 Strategic Balanced Growth

In what follows, we restrict our attention to the symmetric equilibrium in which it holds that $s_i = s,\ c_i(k) = c(k)$ and $l_i(k) = l(k)$ for all i. We also focus on the balanced-growth equilibrium where c, k and y grow at a positive, common rate. Since the production technology has the Ak property, these restrictions require that the optimal consumption of each agent is proportional to the aggregate capital stock and that the optimal labor supply is constant over time. Hence, we can set

$$c_i(k) = \phi k, \quad l_i(k) = l, \quad i = 1, 2, ..., n, \tag{8}$$

where ϕ and l are unknown, positive constants.

We now specify the instantaneous utility function as

$$u(c_i, l_i) = \frac{c_i^{1-\sigma}}{1-\sigma} h(l_i), \quad 0 < \sigma < 1, \tag{9}$$

where $h(l_i)$ has a positive value and satisfies $h'(l_i) < 0$ and $h''(l_i) < 0$.[3] Then, in view of (8), condition (4) becomes

$$V'(k) = \phi^{-\sigma} k^{-\sigma} h(l), \tag{10}$$

implying that

$$V''(k) = -\sigma \phi^{-\sigma} k^{-\sigma-1} h(l). \tag{11}$$

Since (8) means that $c_i'(k) = \phi$ and $l_i'(k) = 0$, by use of (8), (10) and (11) we find that (7) can be written as

$$\rho = Af(Nl) - s(n-1)\phi - \sigma\left[Af(Nl) - N\phi\right],$$

which yields

$$\phi = \frac{\rho + (\sigma - 1) Af(Nl)}{N/n + (\sigma - 1) N}, \tag{12}$$

where $N = sn$. Note that under the symmetric condition, (4) and (5) give $\phi h'(l) = (\sigma - 1) Af'(Nl)$. Hence, from (12) we obtain

$$\frac{\rho/A + (\sigma - 1) f(Nl)}{N/n + (\sigma - 1) N} = \frac{(\sigma - 1) f'(Nl)}{h'(l)}. \tag{13}$$

This equation determines the equilibrium level of individual labor supply, l, on the balanced-growth path.[4] Letting the steady-state level of l be l^*, from (12) the balanced-growth rate, which is given by $g = \dot{k}/k = Af(Nl^*) - Nc/k = Af(Nl^*) - N\phi$, can be expressed as

$$g = \frac{Af(Nl^*) - n\rho}{1 + (\sigma - 1) n}. \tag{14}$$

If we set $f(Nl^*) = 1$ in (1) and $h(l) = 1$ in (9), we obtain the standard modelling that ignores labor input. In this case the balanced-growth rate is

$$g = \frac{A - n\rho}{1 + (\sigma - 1) n}. \tag{15}$$

This shows that if $A < n\rho$ and $\sigma < (n-1)/n$, then the balanced-growth rate is positive and it is negatively related to the total factor productivity, A. In words, a rise in the total factor

[3] Our assumptions ensure that $u(c_i, l_i)$ is strictly concave in (c_i, l_i).

[4] Equation (13) may have multiple solutions. Here, we restrict our attention to the case where (13) has a unique solution.

productivity yields a higher consumption share of income, which depresses the balanced-growth rate. According to Tornell and Lane (1999), this counter intuitive result may be called voracity effect in the absence of secure property rights. If the voracity effect is present, a technological improvement will not contribute to enhancing growth. Additionally, we see that in the standard modelling without labor an increase in the number of interest groups reduces the long-term growth rate, because $dg/dn = -\left[\rho + (1-\sigma) A\right] / \left[1 + (\sigma - 1) n\right]^2 < 0$.

In our generalized setting, (14) shows that the effect of a change in productivity on growth is:

$$\frac{dg}{dA} = \frac{1}{1 + (\sigma - 1) n} \left[f(Nl^*) + Af' N \frac{dl^*}{dA} \right], \tag{16}$$

where from (13) dl^*/dA is given by

$$\frac{dl^*}{dA} = \frac{\rho h'^2}{A^2 (\sigma - 1) \left[h'^2 f' N - s \left(1 + (\sigma - 1) n\right) (N f'' h' - f' h'')\right]}.$$

In the above, if $\sigma < (n-1)/n$ (i.e.$1 + (\sigma - 1) n < 0$), then $dl^*/dA < 0$. As a result, if a rise in A yields a sufficient reduction of individual labor supply, (16) shows that dg/dA has a positive value even under $\sigma < (n-1)/n$. This means that introducing endogenous labor supply and increasing returns to scale reduces the possibility that the voracity effect prevails. We may also confirm that the sign of dg/dn cannot be uniquely determined without imposing further specification on the functional forms and the magnitudes of parameters involved in the model.[5]

4 The Case of Separable Utility

Now assume that the utility is additively separable one:

$$u(c_i, l_i) = \log c_i + \Lambda(l_i), \quad \Lambda' < 0, \quad \Lambda'' < 0,$$

In this case, conditions (10) and (11) respectively become $V'(k) = 1/\phi k$ and $V''(k) = -1/\phi k^2$. Thus (7) shows that $\phi = \rho/s$. The first-order conditions (4) and (5) yield

$$-\rho \Lambda'(l) = Af'(Nl). \tag{17}$$

[5] It should be noted that if n increases under a given s, the total number of agents $N\ (= sn)$ rises as well. If N stays constant, a rise in n reduces $s\,(= N/n)$. In both cases, the sign of dg/dn is ambiguous. See Section 4 for a further discussion on this point.

This equation determines the steady-state level of individual labor supply for the case of additively separable utility. Since $\phi = \rho/s$, the balanced-growth rate is given by

$$g = Af\left(Nl^*\right) - N\phi k = Af\left(Nl^*\right) - n\rho. \tag{18}$$

First, consider the effect of a rise in A. Using (17) and (18), we find:

$$\frac{dg}{dA} = f\left(Nl^*\right) - \frac{AN\left(f'\right)^2}{Af''N + \rho\Lambda''} > 0.$$

Thus if the agents utility functions are additively separable between consumption and labor, there is no voracity effect. Next, consider a change in the number of interest groups. If the total population, $N\,(= sn)$, is constant, a rise in n (so a decrease in the number of agents in each group, s) unambiguously lowers the balanced growth rate, because form (17) the steady-state level of l does not depend on n under a given N. In contrast, if the number of groups stays constant but the number of members of each group increases (so the total population N rises), we obtain

$$\left.\frac{dg}{ds}\right|_{n=\text{constant}} = \frac{\rho n l^* A f' \Lambda''}{ANf'' + \rho\Lambda''} > 0.$$

Thus if the number of players is constant, a rise in population stimulates growth because there is only scale effect. However, if the population rises due to an increase in the number of groups, we obtain:

$$\left.\frac{dg}{dn}\right|_{s=\text{constant}} = \frac{\rho s l^* A f' \Lambda''}{ANf'' + \rho\Lambda''} - \rho.$$

The sign of this is ambiguous.

To specify the sign of $dg/dn|_{s=\text{constant}}$, let us assume that $f(L) = L^\beta$ and $\Lambda(l) = -l^{1+\chi}/(1+\chi)$, where $0 < \beta < 1$ and $\chi > 0$. Then from (17) the steady-state level of l is given by

$$l^* = \left(\frac{A\beta}{\rho}\right)^{\frac{1}{\chi+1-\beta}} N^{\frac{\beta-1}{\chi+1-\beta}}.$$

Hence, the balanced growth rate determined by (18) is

$$g = A^{\frac{\chi+1}{\chi+1-\beta}} \left(\frac{\beta}{\rho}\right)^{\frac{\beta}{\chi+1-\beta}} (sn)^{\frac{\beta\chi}{\chi+1-\beta}} - \rho n.$$

Since the right-hand side of the above is strictly concave in n, the the growth effect of a change in the number of agents is:

$$\left.\frac{dg}{dn}\right|_{s=\text{constant}} > 0 \text{ for } n < \hat{n}, \quad \left.\frac{dg}{dn}\right|_{s=\text{constant}} < 0 \text{ for } n > \hat{n},$$

where $\hat{n}$ satisfies

$$\left(\frac{\beta\chi}{\chi+1-\beta}\right)A^{\frac{\chi+1}{\chi+1-\beta}}\left(\frac{\beta}{\rho}\right)s^{\frac{\beta\chi}{\chi+1-\beta}}\hat{n}^{\frac{(\beta-1)(\chi+1)}{1+\chi-\beta}}=\rho.$$

Namely, when the number of interest groups is smaller than $\hat{n}$, the scale effect due to the presence of increasing returns dominates the negative effect of common pool problem caused by an increase in the number of players. However, if n exceeds $\hat{n}$, the scale effect is not large enough to cancel the common pool effect and thus a lager number of interest groups depresses the long-term growth. This example demonstrates that, unlike the representative-agent economy with increasing returns, a rise in the scale of economy may have a negative impact on growth if property rights are insecure.

5 Conclusion

In this paper we have extended the standard model of growth with insecure property rights by introducing variable labor supply and increasing returns to scale. We have assumed that capital stock is jointly owned by multiple interest groups and that each group participates production activities by supplying its labor force. Given these assumptions, there are two opposing factors that affect growth: over consumption in the absence of secure property rights and the scale effect due to the presence of increasing returns. We have revealed that the growth performance of the economy thus depends on which factor dominates.

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