Generalized Impulse Response Analysis: General or Extreme?

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Abstract

This note discusses a pitfall of using the generalized impulse response function (GIRF) in vector autoregressive (VAR) models (Pesaran and Shin, 1998). The GIRF is *general* because it is invariant to the ordering of the variables in the VAR. The GIRF, in fact, is *extreme* because it yields a set of response functions that are based on extreme identifying assumptions that contradict each other, unless the covariance matrix is diagonal. With an empirical example, the present note demonstrates that the GIRF may yield quite misleading economic inferences.

Keywords: Generalized Impulse Response Function, Orthogonalized Impulse Response Function, Vector Autoregressive Models

JEL Classification: C13, C32, C51

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1 A Pitfall of the GIRF

Notwithstanding its popularity, the orthogonalized impulse response function (OIRF; Sims, 1980) analysis of structural vector autoregressive (VAR) models is subject to the so-called Wold-ordering problem.[1] That is, when one changes the order of the VAR with an alternative identifying assumption, she may obtain dramatically different response functions (Lütkepohl, 1991). Pesaran and Shin (1998) propose an ordering-invariant approach, the generalized impulse response function (GIRF), based on the work of Koop *et al.* (1996). The GIRF has been employed by many researches, to name a few, Boyd *et al.* (2001), Cheung *et al.* (2004), and Huang *et al.* (2008).

However, it is important to recognize that there is a pitfall of using the GIRF. Let $\psi^g_{y_j}(n)$ and $\psi^o_{y_j}(n)$ denote the GIRF and the OIRF at time $t+n$, respectively, when there is one standard error shock at time t to the j^{th} variable in an m-variate VAR with $\mathbf{y}_t = [y_{1,t}\ y_{2,t}\ \cdots\ y_{m,t}]'$. Pesaran and Shin's (1998) Proposition 3.1 implies $\psi^g_{y_1}(n) = \psi^o_{y_1}(n)$.[2] Define $\tilde{\psi}^o_{y_j}(n)$ as the OIRF when $y_{j,t}$ is ordered first in $\mathbf{y}_t$. Then, $\psi^g_{y_1}(n) = \tilde{\psi}^o_{y_1}(n)$. Now re-order the vector so that $\mathbf{y}_t = [y_{2,t}\ y_{1,t}\ \cdots\ y_{m,t}]'$, which yields $\psi^g_{y_2}(n) = \tilde{\psi}^o_{y_2}(n)$ by the proposition and because the GIRF is invariant to the ordering of the variables in $\mathbf{y}_t$. Repeat this procedure until we get $\psi^g_{y_m}(n) = \tilde{\psi}^o_{y_m}(n)$. Collecting these response functions, then, the GIRF for the entire system is,

$$\psi^g(n) = \left\{\tilde{\psi}^o_{y_1}(n)\ \tilde{\psi}^o_{y_2}(n)\ \cdots\ \tilde{\psi}^o_{y_m}(n)\right\}$$

The GIRF, therefore, is *not general* in effect because it employs extreme identifying assumptions that each variable is ordered first. More seriously, $\tilde{\psi}^o_{y_i}(n)$ and $\tilde{\psi}^o_{y_j}(n)$ are not consistent with each other when $i \neq j$ if the covariance matrix is non-diagonal.[3,4] Hence, the GIRFs conflict each other. This result trivially applies to vector error correction models also. In next section, I show that such inconsistency may lead to misleading economic inferences.

[1] The OIRF recursively identifies the structural shocks by using the Choleski decomposition of the covariance matrix, which yields a unique lower triangular matrix. This scheme, therefore, assumes that the variable ordered first in the VAR is contemporaneously unaffected by all other variables.

[2] The GIRF and the OIRF coincide for the first variable in $\mathbf{y}_t$.

[3] $\tilde{\psi}^o_{y_i}(n)$ assumes $y_{i,t}$ is not contemporaneously affected by all other variables including $y_{j,t}$, while $\tilde{\psi}^o_{y_j}(n)$ needs an assumption that $y_{j,t}$ is not contemporaneously affected by all other variables including $y_{i,t}$.

[4] If it is diagonal, there is no gain of using a structural VAR model, because it coincides with a reduced-form VAR, in other words, equation-by-equation least squares estimations.

2 An Empirical Illustration

This section provides an empirical illustration to compare the implications of the GIRF with those of the OIRF. I use a trivariate VAR model of the US per capita investment (i), consumption (c), and real GDP (y), measured in logarithms, as Pesaran and Shin (1998) did. The data frequency is quarterly and observations span from 1947Q1 to 2008Q4, obtained from the Federal Reserve of St. Louis FRED data bank.

Note that the GIRFs to (one standard error) investment shock (Panel 1-a in Figure 1) coincide with the OIRFs to an i-shock when i is assumed to be contemporaneously unaffected by other two variables, c and y (Panel 1-b). Note also that under this assumption, the OIRFs to a y-shock are very different from the corresponding GIRFs. However, the GIRFs to a y-shock are identical to the OIRFs when y is ordered first in the VAR (Panel 1-d) by construction. Again, the other OIRFs under that assumption are quite different from the corresponding GIRFs. Likewise, the GIRFs to a c-shock are identical only to the OIRFs to a c-shock when c is ordered first (Panel 1-c).

What I claim here is that one has to estimate and report response functions based on her economic model. For example, if one interprets y-shocks as an output (supply) shock, while i-shocks and c-shocks are treated as expenditure (demand) shocks, she may employ an ordering $[y\ i\ c]$ assuming that y does not contemporaneously respond to demand shocks. Then, she will report the response functions to an i-shock, for instance, that are very different from the GIRFs both quantitatively and qualitatively. If one believes that i is primarily driven by *animal spirit*, she may employ $[i\ y\ c]$ instead and reports quite smaller responses of i to a y-shock than the corresponding GIRF. The responses of i to a c-shock are again a lot different in Panels 1-b and 1-d as i exhibits delayed overshooting for a year, while the GIRF produces bigger responses of i to a c-shock and delayed overshooting persists only for a half year.

I am not claiming that the OIRF is better than the GIRF because there are many other alternative options available.[5] It seems more reasonable to me to use an identifying assumption that consistently describes the underlying economic models rather than to use the GIRF with a combination of extreme assumptions that conflict with each other.

[5] For example, one may employ over-identified or partially identified systems. A just-identified non-recursive system can also be considered.

3 Conclusion

This note points out that there is a pitfall of using the GIRF. Economic inferences based on the GIRF can be misleading because the GIRF employs a set of extreme identifying assumptions that contradict each other unless the covariance matrix is diagonal. Our empirical example demonstrates that this is by no means a negligible matter.

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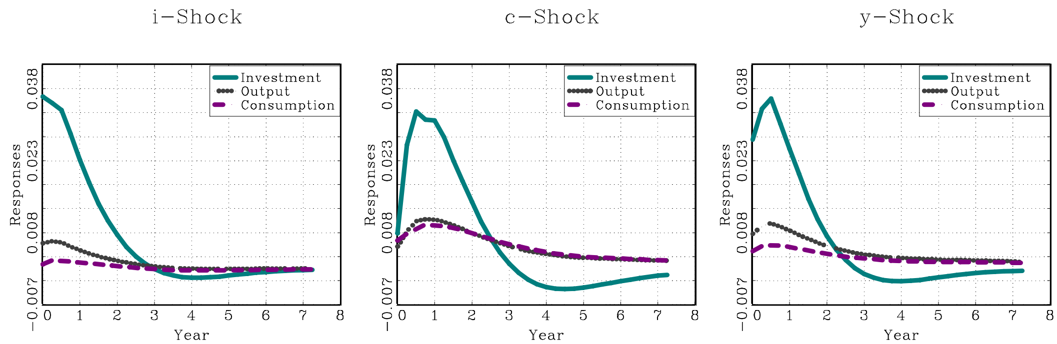
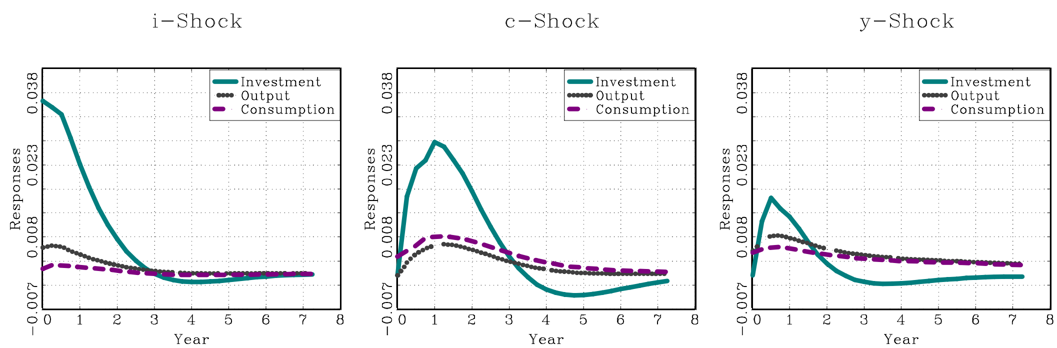
Firgue 1. Impulse Response Function to One Standard Error Shock

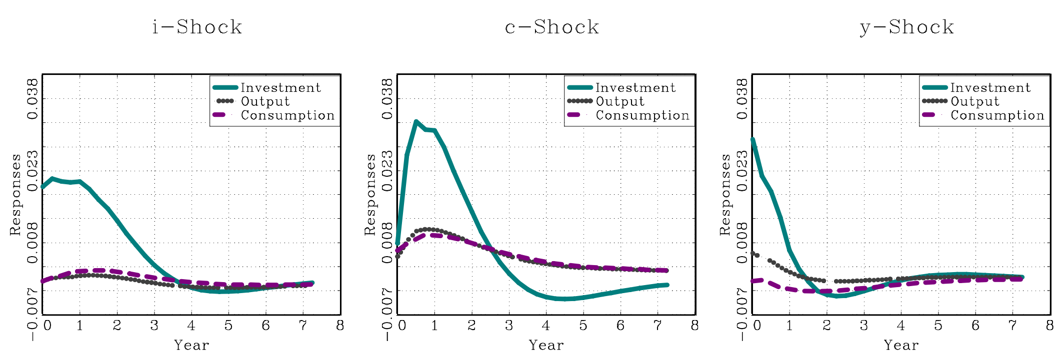
1−a. Generalized Response Function



1−b. Orthogonalized Response Function: (i y c)



1−c. Orthogonalized Impulse Response Function: (c y i)



1−d. Orthogonalized Impulse Response Function: (y i c)