Consumption Externalities and Capital Accumulation in an Overlapping Generations Economy

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Abstract

This paper extends the standard overlapping generations model of capital accumulation by introducing consumption externalities. It is assumed that each generation's felicity depends on the social level of benchmark consumption as well as on its own consumption. Since the benchmark consumption is represented by the average consumption of all agents, the contemporaneous consumption externalities are determined by both intragenerational and intergenerational interactions among the consumers. Given this setting, we show that even in a simple model with a logarithmic utility function, the presence of consumption externalities may significantly affect the dynamic behavior and steady-state characterization of the economy. We also reveal that the same conclusion holds in an endogenous growth model in which production externalities sustain continuing growth.

Keywords: overlapping generations, benchmark consumption, intergenerational externalities, intragenerational externalities

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1 Introduction

Recently, there is a renewed interest in the role of consumption externalities in macroeconomics. Earlier studies on this issue such as Abel (1990) and Galí (1994) introduce consumption external effects into the asset pricing models in order to resolve the discrepancies between theoretical outcomes and empirical findings. Therefore, those studies focus on the external effects of consumption on the individual decision making or on the behavior of asset markets. In contrast, the recent investigations on macroeconomic implications of consumption externalities examine the issue in the dynamic, general equilibrium context and discuss a wider range of topics than those considered by the earlier studies. For example, the recent studies have explored the effect of consumption externalities on optimal taxation (Ljungqvist and Uhlig 2000), equilibrium efficiency (Alonso-Carrera et al. 2003 and Liu and Turnovsky 2003), indeterminacy and sunspots (Weder 2000), as well as on the relationship between savings and long-term economic growth (Carroll et al. 1997 and 2000, and Harbaugh 1996).[1]

A common feature of the existing investigations on consumption externalities in the macroeconomics literature is that most of them employ the representative agent models. In general, introduction of consumption externalities into the standard representative-agent models of growth and business cycles does not produce significant qualitative changes in dynamic behavior of the economy: see Liu and Turnovsky (2003) for a detailed discussion on equilibrium dynamics of the standard representative agent model with consumption externalities. Although consumption externalities may yield large quantitative effects that would be relevant for welfare implications and policy making decisions, the dynamic properties of model economies are usually the same as those of models without consumption external effects. Such a conclusion is in contrast to the effect of production externalities, which may alter the dynamic behavior of the economy in a fundamental manner.

Departing from the recent studies, we examine the role of consumption external effects in an overlapping generations economy. We extend the standard two-period lived OLG model of capital accumulation by introducing ex-

[1] A closely related issue to our discussion is habit formation of consumers that has been widely discussed in the literature (Campbell and Cochrane 1999, Fuhre 2000 and many others). It is also to be noted that a large number of studies have treated both consumption externalities and habit formation at the same time: see, for example, Alvarez-Cuadrado et al. (2004), Abel (1990), Carroll et al. (1997) and (2000). Habit formation seems to be less relevant in our two- period lived OLG framework. Additionally, the models with social status in which utility of the agent is directly affected by the level of wealth holding (e.g. Futagami and Shibata 1998) are also related to our study.

ternal effects among consumption activities of the agents. The key difference between the representative agent and the OLG settings is that the contemporaneous external effects of consumption involve the intergenerational as well as intragenerational externalities in the OLG economy. Unlike the representative agent model, heterogeneity of agents inevitably exists in the OLG model, and hence contemporaneous interactions among consumption activities of the agents would be much more complex in the OLG economy than those in the representative agent economy. This suggests that the presence of consumption externalities generates more fundamental effects on equilibrium dynamics of the economy in an OLG setting than in the representative-agent counterpart. The central purpose of this paper is to confirm this prediction by using a simple model of an OLG economy.

The general conclusion of this paper is that the dynamic behavior of the OLG economy heavily depends on how the consumption external effects are specified. We find that even a simple model with a logarithmic utility and a Cobb-Douglas production function may yield multiple steady states and local indeterminacy of equilibrium. More specifically, if there are positive consumption externalities (i.e. the social level of benchmark consumption positively affects the individual utility), then there generally exists dual steady states. In addition, one of the steady-states exhibits local indeterminacy so that sunspot-driven fluctuations may be observed. In contrast, the external effects are negative in the sense that the utility of each agent decreases with the average consumption levels of other agents, the steady-state equilibrium tends to be uniquely determined. The local dynamics near the steady state, however, may or may not show determinacy of equilibrium. We reveal that those results may hold not only in the standard neoclassical environment in which continuing growth is infeasible without introducing exogenous technical progress but also in the endogenously growing economy where production externalities sustain continuing growth. Although our finding are established in a specified, simple framework, those reveal that consumption externalities would play a relevant role in the OLG economy.

It is to be pointed out that Abel (2003) also introduces consumption externalities into the Diamond model of overlapping generations. The basic model structure of his article is essentially the same as ours. The central concern of Abel (2003) is to characterize the optimal income taxation in the steady state equilibrium. Therefore, equilibrium dynamics out of the steady state is out of touch in his study. Since the main focus of our discussion is to examine dynamic behavior of the model economy, Abel (2003) and our paper may be considered complements rather than substitutes.

The paper is organized as follows. Section 2 sets up the analytical framework with general functional forms. In Section 3, by using a logarithmic utility function and a Cobb-Douglas production function, we investigate the existence and stability of the steady-state equilibrium. Section 4 introduces production externalities into the base model and consider the effects of consumption externalities in an endogenous growth setting. Concluding remarks are given in Section 5.

2 The Base Model

2.1. *Consumption*

We consider a two-period lived overlapping generations economy where in each period only two types of agents are alive: young and old. Agents are identical within the generation. Population is constant over time and the number of agents in each generation is normalized to one. The utility function of agents in cohort born at the beginning of period t is

$$U_t = u(c_t, E_t) + \beta u(x_{t+1}, E_{t+1}), \quad 0 < \beta < 1, \tag{1}$$

where c_t denotes consumption when the agents are young and x_{t+1} is consumption when they are old. E_t and E_{t+1} express the external effects on the felicities in period t and $t+1$, respectively. The instantaneous utility functions, $u(c_t, E_t)$ and $u(x, E_{t+1})$, are assumed to be monotonically increasing and strictly concave in c_t and x_{t+1}.

We assume that the consumption externalities depend on the bench mark levels of average consumption in each period:

$$E_t = E^0(\bar{c}_t, \bar{x}_t),$$

$$E_{t+1} = E^1(\bar{x}_{t+1}, \bar{c}_{t+1}),$$

where $\bar{c}_{t+i}$ and $\bar{x}_{t+i}$ $(i = 01, 2)$ respectively represent the average consumption of the young and old generations. Here, we assume that functions $E^0(.)$ and $E^1(.)$ increase with its each argument. The benchmark consumption in period t, E_t, depends on t-th generation's average consumption as well as on the average consumption of the generation born at the beginning of $t-1$. Similarly, the benchmark consumption in period $t+1, E_{t+1}$, contains the average consumption levels of both t-th and $t+1$-th generations. Therefore, even though we consider the contemporaneous external effects alone, we should deal with the intergenerational externalities[2].

[2] In the representative agent models, it is frequently assumed that the consumption externalities are perceived by the agents with a delay. The studies on "catching up with the Joneses" (e.g. Abel 1990) and "external habit formation" (e.g. Carroll et al. 1997) are

To characterize the external effects on the level of felicity, we follow the taxonomy presented by Dupor and Liu (2003): the consumers' preferences exhibit "jealousy" if the external effects are negative, i.e. $\partial u/\partial E_{t+i} < 0$ $(i = 0, 1)$, while they show "admiration" if positive externalities are present so that $\partial u/\partial E_{t+i} > 0$ $(i = 0, 1)$. In addition, preferences display "keeping up with the Joneses (KUJ)" if $\partial^2 u/\partial c_{t+i}\partial E_{t+i} > 0$ $(i = 0, 1)$, and they show "running away from the Joneses (RAJ)" if $\partial^2 u/\partial c_{t+i}\partial E_{t+i} < 0$ $(i = 0, 1)$. Intuitively, KUJ means that each consumer wants to be similar to others, because an additional consumption yields a larger increment of utility when other agents attain higher levels of consumption. In contrast, RAJ means that each consumer prefers being different from others: her marginal utility is lowered if other consume more[3].

We assume that agents work when they are young and they do not work when old. Each young agent supplies one unit of labor. Hence, the budget constraints in period t and $t + 1$ are respectively given by the following:

$$w_t = c_t + s_t,$$

$$x_{t+1} = R_{t+1} s_t.$$

where R_{t+1} denotes the gross rate of interest in period $t + 1$, w_t is the real wage rate and s_t is saving of the young agent. The intertemporal budget constraint for the household is thus written as

$$c_t + \frac{x_{t+1}}{R_{t+1}} = w_t. \tag{2}$$

The agents born at the beginning of period t selects c_t and x_{t+1} to maximize U_t subject to the life-time budget constraint of (2). When solving their optimization problem, the agents take the external effects, E_t and E_{t+1}, as

typical examples in this direction of research. It is worth noting that, by using an OLG model with capital formation, de la Croix (1996) assumes that the young agents' felicity is affected by the consumption level of their parents in their young age. In our notation, the felicity function used by de la Croix is:

$$U_t = u\left(c_t, \bar{c}_{t-1}\right) + \beta u\left(x_{t+1}\right).$$

This kind of intergenerational taste inheritance is not considered in our discussion. (See also Chapter 5 in de la Croix and Michel (2002) for a further discussion on habit formation in the OLG models.)

[3] As emphasized by Dupor and Liu (2003), jealousy (admiration) and KUJ (RAJ) are different concepts and they may or may not coincide with each other, depending on the parametric specification of the utility function. See also Chugh (2003) for a moralization of Dupor and Liu (2003).

given. The first-order conditions for an optimum yields

$$\frac{\beta u_1\left(x_{t+1}, E_{t+1}\right)}{u_1\left(c_t, E_t\right)} = \frac{1}{R_{t+1}}. \tag{3}$$

By use of (2) and (3) , we may express the optimal levels of consumption in such a way that

$$c_t = \phi^c\left(w_t, R_{t+1}, E_t, E_{t+1}\right),$$
$$x_{t+1} = \phi^x\left(w_t, R_{t+1}, E_t, E_{t+1}\right).$$

Since we have assumed that population of each cohort is one, in the symmetric equilibrium we can set $\bar{c}_t = c_t$ and $\bar{x}_t = x_t$ for all t. Hence, c_t and x_{t+1} satisfy

$$c_t = \phi^c\left(w_t, R_{t+1}, E^0\left(c_t, x_t\right), E^1(x_{t+1}, c_{t+1})\right),$$
$$x_{t+1} = \phi^x\left(w_t, R_{t+1}, E^0\left(c_t, x_t\right), E^1(x_{t+1}, c_{t+1})\right).$$

As a result, the equilibrium levels of c_t and x_{t+1} may be written as

$$c_t = C\left(w_t, R_{t+1}, x_t, c_{t+1}\right), \tag{4}$$

$$x_{t+1} = X\left(w_t, R_{t+1}, x_t, c_{t+1}\right). \tag{5}$$

2.2 *Production*

In the base mode, we assume that the economy has a standard neoclassical production technology. Firms produce a single commodity in a competitive market. The production function satisfies constant returns to scale with respect to capital and labor. Thus the production technology is described by a strictly concave and increasing function, $y_t = f\left(k_t\right)$, where k_t is capital-labor ratio and y_t is labor productivity. Denoting the capital depreciation rate by δ, the net rate of return to capital is thus given by $f'\left(k_t\right) - \delta$. Due to the arbitrage condition, the gross rate of interest satisfies $R_t = f'\left(k_t\right) - \delta + 1$. In what follows, we assume that capital fully depreciates in one period, that is, $\delta = 1$. (This is a plausible assumption in the two-period lived OLG economy in which one period can be 30 years long.) Therefore, we have

$$R_t = f'\left(k_t\right). \tag{6}$$

Additionally, the competitive rate of real wage is given by

$$w_t = f\left(k_t\right) - k_t f'\left(k_t\right). \tag{7}$$

2.3 *Dynamic System*

Since only young agents save, capital accumulation is determined by the savings of young generation alone. Thus it holds that $k_{t+1} = s_t$ so that from (7) we obtain:

$$k_{t+1} = w_t(k_t) - c_t. \tag{8}$$

Consequently, (4), (5) and (8) constitute a first-order dynamic system with respect to k_t, c_t and x_t. Notice that, in view of the budget constraint the household faces, $x_{t+1} = R_{t+1}(w_t - c_t) = R_{t+1}k_{t+1}$, which yields $x_t = R_t k_t$. This means that (**??**) is written as

$$R(k_{t+1})k_{t+1} = X(w(k_t), R(k_{t+1}), R(k_t)k_t, c_{t+1}). \tag{9}$$

Substituting (8) into (21) gives the following:

$$\begin{aligned} & R(w_t(k_t) - c_t)[w_t(k_t) - c_t] \\ = \; & X(w(k_t), R(w_t(k_t) - c_t), R(k_t)k_t, c_{t+1}). \end{aligned} \tag{10}$$

In sum, the dynamic behavior of the overlapping generations economy with intergenerational as well as intragenerational consumption externalities can be examined by inspecting (8) and (10) which describe motions of capital-labor ratio, k_t, and the young generation's consumption, c_t. It is worth emphasizing that introduction of intergenerational consumption externalities yields a two-dimensional dynamic system. In the standard Diamond model of OLG economy can be reduced to a one-dimensional dynamic system of capital stock, so that specification of the initial level of capital generally pins down the equilibrium path of the economy. In contrast, in our model the initial level of capital, k_t, alone cannot determines the equilibrium trajectory. We need an additional condition, the initial level of c_t (or k_{t+1}) to select a unique path. In this respect, the dynamic behavior of OLG model with consumption externalities would be similar to that of the representative agent economy.[4]

[4] It is to be noted that if intergenerational externalities are specifiedIn our notation, de la Croix and Michel (1996) assume

$$U_t = u(c_t, \bar{c}_{t-1}) + \beta u(x_{t+1}).$$

Namely, the parents' consumption experices affect childrens' felicity. In this case demand functions are given by

$$\begin{aligned} c_t &= C(w_t, R_{t+1}, c_{t-1}) \\ x_{t+1} &= X(w_t, R_{t+1}, c_{t-1}). \end{aligned}$$

Letting $z_t = c_{t-1}$, the dynamic system is expressed as

$$k_{t+1} = w(k_t) - C(w(k_t), R(k_{t+1}), z_t)$$

3 A Model with Specific Functional Forms

Although the dynamic system derived in the previous section is rather simple, it is difficult to conduct a precise analysis without further specification of the functional forms involved in the model. In this section, we specify utility, production and external effect functions in a simple manner. Such a specification turns out to be helpful in order to confirm how the consideration of consumption externalities may alter the dynamic behavior as well as the steady-state characterization of the standard OLG model of capital formation.

3.1 *Preference Structure and Production Technology*

We use one of the simplest forms of the utility function. The felicity function of generation born at the beginning of period t is

$$U_t = \log\left(c_t + \theta E_t\right) + \beta \log\left(x_{t+1} + \theta E_{t+1}\right)$$

The external effects in each period are described by

$$E_t = \bar{c}_t + \eta \bar{x}_t, \tag{11}$$

$$E_{t+1} = \bar{x}_{t+1} + \eta \bar{c}_{t+1}, \tag{12}$$

where $\theta > -1$ and $\eta > 0$. Parameter η expresses the relative magnitude between the intragenerational and intergenerational consumption externalities. Notice that, given those functional forms, $\partial u / \partial E_t = \theta / \left(c_t + \theta E_{t+i}\right)$ and $\partial^2 u / \partial c_t \partial E_{t+i} = -\theta / \left(c_t + \theta E_{t+i}\right)^2$. Therefore, if $\theta < 0$, the functional structure displays jealousy as well as KUJ. In contrast, if $\theta > 0$, preferences exhibit both admiration and RAJ.[5] Obviously, the assumption of logarithmic

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$$z_{t+1}(= c_t) = C\left(w\left(k_t\right), R\left(k_{t+1}\right), z_t\right)$$

Thus we have a two-dimensional dynamic system of $\left(k_t, z_t\right)$. Note that in this case, the initial values of k_t and $z_t\left(= c_{t-1}\right)$ are specified, equilibrium is in general,l determined uniquely.

[5]A slight generalization of the log-utility function is

$$U_t = \frac{\left(c_t + \theta E_t\right)^{1-\sigma} - 1}{1-\sigma} + \beta \frac{\left(x_{t+1} + \theta E_{t+1}\right)^{1-\sigma} - 1}{1-\sigma}, \quad \sigma > 0, \quad \sigma \neq 1.$$

In this case it still holds that $\theta < 0$ means jealousy and KUJ, while $\theta > 0$ yields admiration and RAJ.

utility function is highly restrictive. However, we show that even in this simple case, the presence of consumption externalities may give rise to complex outcomes that cannot be observed in the standard framework.[6]

Give our specifications, the optimization condition (3) is written as

$$\frac{\beta(c_t+\theta E_t)}{x_{t+1}+\theta E_{t+1}}=\frac{1}{R_{t+1}}.$$

Solving (2) and (3) with respect to c_t and x_{t+1} yields:

$$c_t=\frac{1}{1+\beta}w_t+\frac{\theta}{1+\beta}\left(\frac{E_{t+1}}{R_{t+1}}-\beta E_t\right), \tag{13}$$

$$x_{t+1}=\frac{\beta R_{t+1}}{1+\beta}w_t-\frac{\theta R_{t+1}}{1+\beta}\left(\frac{E_{t+1}}{R_{t+1}}-\beta E_t\right). \tag{14}$$

If there is no external effect $(\theta=0)$, then we obtain $c_t=w_t/(1+\beta)$ and $x_{t+1}=\beta R_{t+1}w_t/(1+\beta)$. Since the utility function is logarithmic, the income and substitution effects of a change in the (expected) interest rate cancel each other. In the presence of consumption externalities, however, a change in the rate of interest in period $t+1$ affects c_t and x_{t+1} in such a way that

$$\frac{\partial c_t}{\partial R_{t+1}}=-\frac{\theta E_{t+1}}{(1+\beta)R_{t+1}^2},$$

$$\frac{\partial x_{t+1}}{\partial R_{t+1}}=\frac{\beta}{1+\beta}\left(w_t+\theta E_t\right).$$

Since $w_t>c_t$ and $c_t+\theta E_t>0$, we find that $\partial x_{t+1}/\partial R_{t+1}$ has a positive value. The sign of $\partial c_t/\partial R_{t+1}$ depends on the sign of θ. In words, under the given levels of external effect, a rise in the interest rate increases (decreases) consumption of the young period, if preferences exhibit jealousy and KUJ, while it lowers c_t if admiration and RAJ prevail.

In the symmetric equilibrium, it holds that $\bar{c}_t=c_t$, $\bar{x}_t=x_t$ and $\bar{c}_{t+1}=c_{t+1}$. Thus, using $E_t=c_t+\eta x_t$ and $E_{t+1}=x_{t+1}+\eta c_{t+1}$ we obtain

$$c_t=\frac{1}{1+\beta}w_t+\frac{\theta\eta}{(1+\theta)(1+\beta)}\left(\frac{c_{t+1}}{R_{t+1}}-\beta x_t\right), \tag{15}$$

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[6] In this example, we assume that the external effects are symmetric in both periods. More generally, we may assume that

$$U_t=\log\left(c_t+\theta_0 E_t\right)+\beta\log\left(x_{t+1}+\theta_1 E_{t+1}\right)$$

and

$$E_{t+i}=\bar{c}_{t+i}+\eta_i\bar{x}_{t+i},\quad i=0,1.$$

To avoid unnesseary classification, we focus on the case of symmetry where $\theta_0=\theta_1=\theta$ and $\eta_0=\eta_1=\eta$.

$$x_{t+1} = \frac{\beta R_{t+1}}{1+\beta} w_t - \frac{\theta \eta R_{t+1}}{(1+\theta)(1+\beta)} \left(\frac{c_{t+1}}{R_{t+1}} - \beta x_t \right). \tag{16}$$

Notice that due to the assumption of $-1 < \theta$, the signs of $\partial c_t / \partial R_{t+1}$ and $\partial x_{t+1} / \partial R_{t+1}$ are still the same as shown in (13) and (15), even though the intragenerational externalities are taken into account in calculating the symmetric equilibrium levels of c_t and x_{t+1}. It is also to be noted that (15) and (16) demonstrate that if there is no intergenerational externalities $(\eta = 0)$, the external effect on the equilibrium levels of c_t and x_{t+1} disappears. This shows that the intergenerational consumption externalities may play a pivotal role in our economy.

In this section, we assume that the production technology satisfies the standard neoclassical properties and hence there is no external effect in production activities. We use a Cobb-Douglas function such that

$$y_t = A k_t^{\alpha}, \quad 0 < \alpha < 1.$$

Therefore, the gross rate of interest and the real wage rate are respectively given by

$$R_t = \alpha A k_t^{\alpha - 1}, \tag{17}$$

$$w_t = (1-\alpha) A k_t^{\alpha}. \tag{18}$$

Notice that the above relations give

$$\frac{w_t}{k_t} = \left(\frac{1}{\alpha} - 1 \right) R_t. \tag{19}$$

3.2 *Dynamic system and statedly state equilibrium*

As pointed out in Section 2.2, we can obtain the following complete dynamic system by use of (8), (15), (17) and (18):

$$k_{t+1} = (1-\alpha) A k_t^{\alpha} - c_t,$$

$$c_t = \frac{1}{1+\beta} (1-\alpha) A k_t^{\alpha} + \frac{\theta \eta}{(1+\theta)(1+\beta)} \left(\frac{c_{t+1}}{\alpha A k_{t+1}^{\alpha-1}} - \beta \alpha A k_t^{\alpha} \right),$$

which describe dynamic behaviors of k_t and c_t. The second equation yields

$$c_{t+1} = \alpha A k_{t+1}^{\alpha - 1} \left[\frac{c_t}{D} + \left(\alpha \beta - \frac{F}{D} \right) A k_t^{a} \right],$$

where

$$F = \frac{1-\alpha}{1+\beta} > 0, \quad D = \frac{\eta\theta}{(1+\theta)(1+\beta)}.$$

As a result, a complete system of (k_t, c_t) is presented by the following:

$$k_{t+1} = (1-\alpha) A k_t^{\alpha} - c_t, \tag{20}$$

$$c_{t+1} = \alpha A \left[(1-\alpha) A k_t^{\alpha} - c_t\right]^{\alpha-1} \left[\frac{c_t}{D} + \left(\alpha\beta - \frac{F}{D}\right) A k_t^{a}\right]. \tag{21}$$

Equations (20) and (21) respectively correspond to the general expressions (8) and (10).

In the steady state equilibrium, k_t and c_t stay constant over time. Thus the steady-state conditions are given by

$$c = (1-\alpha) A k^{\alpha} - k,$$

$$c = \alpha A k^{\alpha-1} \left[\frac{c}{D} + \left(\alpha\beta - \frac{F}{D}\right) A k^{a}\right].$$

In order to characterize the steady-state equilibrium, it is convenient to use $R_t = \alpha A k_t^{\alpha-1}$ and rewrite the steady state conditions in the following way:

$$\frac{c}{k} = \frac{1-\alpha}{\alpha} R - 1,$$

$$\frac{c}{k} = R \left[\frac{c/k}{D} + \left(\beta - \frac{F}{\alpha D}\right) R\right].$$

Eliminating c/k from the above two equations, we obtain

$$\frac{1-\alpha}{\alpha} R - 1 = R \left[\frac{1}{D}\left(\frac{1-\alpha}{\alpha} R - 1\right) + \left(\beta - \frac{F}{\alpha D}\right) R\right]$$

or

$$\left[\frac{\beta(1+\theta)(1-\alpha)}{\alpha\theta\eta} + \beta\right] R^2 - \left[\frac{1-\alpha}{\alpha} + \frac{(1+\beta)(1+\theta)}{\eta\theta}\right] R + 1 = 0. \tag{22}$$

The solutions of this equation gives the steady state rate of interest rate and it determines the steady-state values of k_t, c_t, w_t and x_t as well.

By inspecting (22), we find the following:

Proposition 1 *The steady-state equilibrium is uniquely determined if and only if*

$$\frac{1+\theta}{\eta\theta} < -\frac{1-\alpha}{\alpha}. \tag{23}$$

There may exist dual steady states if

$$\frac{1+\theta}{\eta\theta} > \max\left\{-\frac{\alpha}{1-\alpha}, -\frac{1-\alpha}{\alpha(1+\beta)}\right\}. \tag{24}$$

Proof. See Appendix A

■

If $\theta > 0$, then condition (24) holds. As a result, if preferences show admiration and RAJ and if the economy has a steady-state equilibrium, there exist dual steady states. In addition, even if $\theta < 0$, condition (24) could be satisfied and thus dual steady state may emerge in the economy where jealousy and KUJ prevail. By contrast, if $\theta < 0$ and condition (23) is satisfied, then the economy with jealousy and KUJ has a unique steady state. To understand conditions (23) and (24) graphically, let us rewrite (22) in the following way:

$$\frac{1}{R} = \left[\frac{1-\alpha}{\alpha} + \frac{(1+\beta)(1+\theta)}{\eta\theta}\right] - \left[\frac{\beta(1+\theta)(1-\alpha)}{\alpha\theta\eta} + \beta\right] R, \tag{25}$$

Figure 1 (a) depicts the graphs of both sides of the above for the case where condition (24) is satisfied. Figure 1 (b) shows the situation when condition (23) is met. A change in external effect parameter θ or η shifts the graph of RHS of (25). The effect of a change in θ or η on the steady-state level of R is, however, ambiguous in the analytical sense. We will conduct comparative statics in the steady state numerically in Section 3.5.

3.3 *Equilibrium Dynamics*

In order to examine the local dynamics, let us linealize (20) and (21) at the steady state equilibrium. Letting (k, c) be the steady state values of (k_t, c_t), we obtain:

$$\begin{bmatrix} k_{t+1} - k \\ c_{t+1} - c \end{bmatrix} = J \begin{bmatrix} k_t - k \\ c_t - c \end{bmatrix},$$

in which the coefficient matrix J is given by

$$J = \begin{bmatrix} (1-\alpha) R, & -1 \\ J_{21} & J_{22} \end{bmatrix},$$

where

$$J_{21} = \left[\left(\alpha\beta - \frac{(1+\theta)(1-\alpha)}{\theta\eta} - \frac{(1-\alpha)^3}{\alpha} \right) R + (1-\alpha)^2 \right] R, \tag{26}$$

$$J_{22} = \left[\frac{(1+\theta)(1+\beta)}{\theta\eta} + \frac{(1-\alpha)^2}{\alpha} \right] R + \alpha - 1. \tag{27}$$

In the above, each element of the coefficient matrix, J, is evaluated at the steady state where R denotes the steady-state value of R_t determined by (22). (See Appendix B for the evaluation of elements in J.)

The trace and determinant of J are given by $\mathrm{Tr}J = (1-\alpha) R + J_{22}$ and $\det J = (1-\alpha) R J_{22} + J_{21}$, respectively. If one of the eigenvalues is less than one and the other is greater than one, the saddle point property is satisfied. The condition for saddle stability is

$$(1 + \mathrm{Tr}\ J + \det J)(1 - \mathrm{Tr}\ J + \det J) < 0. \tag{28}$$

Additionally, if the following two conditions are met

$$(1 + \mathrm{Tr}\ J + \det J)(1 - \mathrm{Tr}\ J + \det J) > 0. \tag{29}$$

$$(\det J)^2 < 0, \tag{30}$$

then the absolute values of the both characteristic roots are less than one. In this case, the system satisfies asymptotic stability. Thus if (29) and (30) hold, the economy displays local indeterminacy of equilibrium. Since the coefficient matrix shown above contains the steady-state value of R_t that is a complex function of given parameters of θ, η, α and β, stability of the dynamic system is highly sensitive to the parameter values involved in the model. Although it is possible to express (28), (29) and (30) in terms of $(\theta, \eta, \alpha, \beta)$ alone, they are too complex to present useful economic interpretation. Therefore, in what follows, we present numerical examples to examine the local behavior of the model around the steady state.

3.4 *Numerical Examples*

We assume that the length of one period in our economy spans 30 years. Thus if the annual discount rate is about 3.5%, the discount factor $\beta = (0.965)^{30} \simeq 0.343$. In addition, according to the conventional specification, we assume that the income share of capital, α, is 0.35. First, suppose that preferences show admiration and RAJ. If $\theta = 0.2$ and $\eta = 1.0$, then the solution of (22) are $R = 0.1056$ and 2.280.Letting the annual interest rate be

r, the gross rate of interest in one period is $(1+r)^{30}$. Since $R = (1+r)^{30} - 1$, the annual rate of interest corresponding to the steady-state values $R = 0.1056$ and 2.28 are respectively $r \simeq 0.003$ and $r \simeq 0.04$. Evaluating the coefficient matrix J by the steady state value of $R = 0.1056$, we find that the characteristic roots of J are 0.343 and 0.483. This means that the steady state with a lower value of R (so that a higher value of the steady state level of k) is a sink. Since the initial value of c_t is not predetermined in our system, the steady state with a higher capital stock is locally indeterminate and thus sunspot-driven fluctuations may be observed. On the other hand, at the steady state with $R = 2.28$, the characteristic roots of J are 0.202 and 21. 422. Therefore, the steady state with a lower capital stock has a saddlepoint property, which means that the equilibrium path is locally determinate.

If $\theta = 0.4$ and $\eta = 1.0$, the steady-state levels of R are 0.162 and 2.345. In this case, the characteristic roots of J are 0.695 and 0.346 for $R = 0.162$, while they are 0.35 and 14. 43 for $R = 2.345$. Again, the steady state with a high level of capital is locally indeterminate. In a similar manner, assuming that $\alpha = 0.35$ and $\beta = 0.341$, we change the values of θ and η to examine the local stability of each steady state. We find that for a wide range of values of θ and η, the stability of each steady state mentioned above still holds: the steady state with a higher capital stock is locally indeterminate, while the steady state with a lower capital is determinate.

Now suppose that preferences exhibit jealousy and KUJ. If we set $\theta = -0.2$ and $\eta = 1.0$, condition (23) holds so that the feasible steady state is unique. In this case, we find that the steady state value of R is 1.85 and that characteristic roots of J are -7.52 and 0.369. Thus the unique steady state is a saddle point and local determinacy holds. If $\theta = -0.4$ and $\eta = 1.0$, then $R = 1.456$ and characteristic roots of J are -1.257 and 0.367. Again, the saddle point property holds. However, if $\theta = -0.5$ and $\eta = 1.0$, then $R = 1.254$. In this case characteristic roots of J are -0.5007 and 0.486, so that the steady state is a sink. At the same time, if $\theta = -0.5$ but $\eta = 0.5$, then $R = 1.72$ and the characteristic roots of J are -2.571 and 0.487. These examples indicate that the possibility of indeterminacy increases with the magnitude of external effect.

Table 1 summarizes typical examples. Here, H and L respectively indicate that the steady state with a higher and lower rate of return to capital.

θ	η	R	Local determinacy
0.4	1.0	$R^H = 2.345, \quad R^L = 0.162,$	H determinate, L indeterminate
0.2	1.0	$R^H = 2.281, \quad R^L = 0.105$	H determinate, L indeterminate
0.2	0.5	$R^H = 2.008, \quad R^L = 0.057$	H determinate, L indeterminate
0.1	1.0	$R^H = 2.208, \quad R^L = 0.062$	H determinate, L indeterminate
-0.1	1.0	2.002	determinate
-0.2	1.0	1.850	determinate
-0.5	0.5	1.702	determinate
-0.4	1.0	1.465	indeterminate
-0.42	1.0	1.375	indeterminate
-0.45	1.0	1.299	indeterminate

Table 1

3.5 *Intuition*

To obtain intuitive implication of the stability results shown above, assume that the economy initially stays in the steady-state equilibrium. Now, suppose that there is a sunspot-driven expectation change and the agents born at the beginning of period t anticipate that the consumption demand of young agents in the next period (period $t+1$) will increase. Remember that from (15) and (16) the consumption demand functions of the agents born in period t are written as

$$c_t = \frac{1}{1+\beta} w_t + \frac{\theta\eta}{(1+\theta)(1+\beta)} \frac{c_{t+1}}{R_{t+1}} - \frac{\beta\theta\eta}{(1+\theta)(1+\beta)} x_t, \tag{31}$$

$$x_{t+1} = \frac{\beta R_{t+1}}{1+\beta} w_t - \frac{\theta\eta}{(1+\theta)(1+\beta)} c_{t+1} + \frac{\beta\theta\eta R_{t+1}}{(1+\theta)(1+\beta)} x_t. \tag{32}$$

Therefore, if admiration and RAJ prevail ($\theta > 0$), an anticipated rise in c_{t+1} increases c_t so that capital formation in period $t+1$ falls. A decrease in capital then raises the marginal product of capital and thus R_{t+1} will increase. Since there are two steady states in the case of $\theta > 0$, the magnitude of impact of the rise in R_{t+1} depends on which steady state the economy initially stays. If the economy initially is in the steady state with a lower level of capital (so a higher rate of R), a rise in k_{t+1} makes a large increment of R_{t+1}, because the productivity function satisfies strict concavity. This means that in view of (31) , the positive effect of the increase in R_{t+1} on x_{t+1} may dominate the negative external effect created by a rise in c_{t+1}. In this case, both c_{t+1} and x_{t+1} are anticipated to rise, which contradicts the fact that a rise in c_t reduces capital accumulation so that product in $t+1$ will fall. Consequently, the initial anticipation cannot be self-fulfilled and the sunspot-driven fluctuations will not emerge.

On the other hand, if the economy initially stays in the steady state with a lower level of capital, the initial rate of return is high. Again, the anticipated increase in c_{t+1} lowers k_{t+1} and increases R_{t+1}. Since the initial capital stock is sufficiently large, a fall in k_{t+1} yields a small increase in R_{t+1}. Hence, the positive effect of a rise in R_{t+1} on x_{t+1} is relatively small, so that the anticipated rise in c_{t+1} may decrease x_{t+1}. As a result, the total consumption in period $t+1$ may fall, which makes the initial expectation change self-fulfilled. The local indeterminacy is thus observable around the steady state with a larger level of capital.

Next, consider the case of jealousy and KUJ ($\theta < 0$). When θ is negative, an anticipated increase in c_{t+1} raises c_t, and hence k_{t+1} will increase. This lowers R_{t+1} so that there is a negative impact on x_{t+1}. In addition, if θ and η are small, (32) shows that a rise in c_{t+1} produces a small increment in x_{t+1}. Consequently, the increase in total consumption in period $t+1$ is not large enough to meet the rise in output generated by the increase in savings in period t. Conversely, if θ and η are high enough, a rise in c_{t+1} will increase x_{t+1} despite the presence of negative effect of the fall in the interest rate. If this is the case, the initial anticipation can be self fulfilled: there may exist a continuum of stable paths around the steady state.

4 Endogenous Growth

In this section we introduce production externalities into the base model in order to re-examine the main findings in the previous section in the context of endogenous growth. We now assume that the production function of each firm is

$$y_t = Ak_t^{\alpha}\bar{k}_t^{1-\alpha}, \quad 0 < \alpha < 1.$$

Here, following Romer (1986), $\bar{k}_t$ denotes the average capital-labor ration that represents the external effects generated by technological diffusion. Therefore, in the absence of consumption externalities, the model structure in this section is essentially the same as one used by Grossman and Yanagawa (1994) who examine the welfare effect of bubble in an endogenously growing economy with overlapping generations. If we normalize the number of firms to one, the symmetric equilibrium requires that $\bar{k}_t = k_t$. Using this condition, the social technology internalizing the external effect becomes $y_t = Ak_t$ and the gross rate of return to capital and the real wage rate are respectively given by

$$R_t(k_t) = \alpha A = R, \tag{33}$$

$$w(k_t) = (1-\alpha) Ak_t. \tag{34}$$

4.1 *Dynamics with Endogenous Growth*

In our endogenous growth setting, the real interest rate is fixed. As a result, only difference between the neoclassical and endogenous growth versions of the dynamic system is that the latter assumes that R_t constant over time. Hence, in the case of endogenous growth, the dynamic system with the neoclassical technology (20) and (21) are reduced to:

$$k_{t+1} = (1-\alpha) Ak_t - c_t,$$

$$c_{t+1} = \alpha A \left[\frac{c_t}{D} + \left(\alpha\beta - \frac{F}{D} \right) Ak_t \right].$$

Denoting the gross rate of capital formation by $k_{t+1}/k_t = G_t$, we see that the above can be rewritten as

$$G_t = (1-\alpha) A - z_t,$$

$$z_{t+1} = \frac{\alpha A}{G_t} \left[\frac{z_t}{D} + \left(\alpha A - \frac{F}{D} \right) A \right].$$

Using these equations, we may drive a complete dynamic system with a single variable, G_t, as follows:

$$G_{t+1} = \left(\frac{1}{\alpha} - 1 \right) R + \frac{R(1+\theta)(1+\beta)}{\theta\eta} - \left[1 + \frac{(1+\theta)\left(\frac{1}{\alpha} - 1\right)}{\theta\eta} \right] \frac{\beta R^2}{G_t}. \tag{35}$$

This equation presents a reduced dynamic system that summarizes an endogenous growth version of our base model.

4.2. *The Balanced-Growth Path and Equilibrium Dynamics*

In the balanced-growth equilibrium, k_t, c_t and x_t grow at a common rate. The balanced-growth rate is given by the solution of the following equation:[7]

$$G = \left(\frac{1}{\alpha} - 1 \right) R + \frac{R(1+\theta)(1+\beta)}{\theta\eta} - \left[1 + \frac{(1+\theta)\left(\frac{1}{\alpha} - 1\right)}{\theta\eta} \right] \frac{\beta R^2}{G}. \tag{36}$$

Using (36), we find:

[7] Note that if we set $G = 1$ in the following equation (36), we obtain the steady-state equation (22) for the exogenous growth model.

Proposition 2 *When preferences show admiration and RAJ* $(\theta > 0)$, *then there may exist dual balanced-growth paths. When preferences exhibit jealousy and KUJ* $(\theta < 0)$, *then there exists a unique balanced-growth equilibrium if*

$$1 + \frac{(1+\theta)\left(\frac{1}{\alpha} - 1\right)}{\theta\eta} < 0, \tag{37}$$

while there may exist dual balanced-growth paths if

$$1 + \frac{(1+\theta)\left(\frac{1}{\alpha} - 1\right)}{\theta\eta} > 0 \quad \text{and} \quad \left(\frac{1}{\alpha} - 1\right) R + \frac{R(1+\theta)(1+\beta)}{\theta\eta} > 0. \tag{38}$$

Proof. See Appendix C. ■

Employing the same numerical examples as those in Section 3.4, we find that (37) tends to hold when $\theta < 0$. (If $\alpha = 0.35$, (37) holds even if $\theta = -0.5$ and $\eta = 1.0$.) Thus, given plausible parameter values, the endogenously growing economy with jealousy and KUJ generally has a unique balanced-growth equilibrium.

It is easy to examine the local behavior of the economy around the balanced-growth equilibrium. Figures 2 (a) and 2 (b) are the phase diagrams of (35). Figure 2(a) corresponds to the case where conditions in (38) hold. As figure shows, the stationary point with a lower growth rate is unstable and that with a higher growth rate is stable. Therefore, since the initial value of k_{t+1} (so G_t) is not predetermined in this system, the low-growth steady state is locally determinate, while the high-growth steady state is locally indeterminate. This conclusion is similar to the stability results shown in the numerical examples for the case of exogenous growth: the steady state with high economic activities displays indeterminacy, while the steady state with low economic activities holds determinacy of equilibrium in the case of admiration and RAJ.

Figure 2(b) is for the case of jealousy and KUJ where (37) is satisfied. The balanced-growth path is uniquely given and the local stability can be determined by inspecting the following:

$$\left.\frac{dG_{t+1}}{dG_t}\right|_{G_t = G} = \left[1 + \frac{(1+\theta)\left(\frac{1}{\alpha} - 1\right)}{\theta\eta}\right] \frac{\beta R^2}{G^2} \tag{39}$$

If the absolute value of the RHS of the above is higher than one, the stationary point is locally unstable so that the economy always stays on the balanced-growth path and local determinacy of equilibrium holds. If the absolute value of RHS of (39) is strictly less than one, then the growth rate cyclically converges to its steady-stage value. Thus indeterminacy holds around the balanced-growth path. Equation (39) means that, other things being equal, the absolute value of the right hand side becomes small when G is large. However, G is a function of other parameters, the magnitude of $\left.\frac{dG_{t+1}}{dG_t}\right|_{G_t=G}$ depends totally on the values of $(\alpha, \beta, \theta, \eta)$. To sum up, we find:

Proposition 3 *If there are dual balanced-growth equilibria, the balanced-growth path with a lower growth rate is locally determinate, while that with a higher growth rate is locally indeterminate. If preferences show jealousy and KUJ and if the balanced-growth path is uniquely given, then determinacy of equilibrium depends on the parameter values involved in the model.*

In order to examine the stability conditions more specifically, we consider some numerical examples in the next subsection.

4.3. *Numerical Examples*

Since in our endogenous growth setting, the rate of return to capital $R = \alpha A$ is fixed, we should specify the value of productivity parameter A. To do so, we set a bench mark long-term growth rate that the economy without consumption externalities realizes. First, notice that if there is no consumption external effect, the dynamic behavior of capital is determined by $k_{t+1} = \beta w_t / (1+\beta)$ so that

$$k_{t+1} = \frac{\beta (1-\alpha) A}{1+\beta} k_t.$$

This means that the economy stays on the balanced growth path at the outset and the balanced growth rate is given by

$$G = \frac{\beta (1-\alpha) A}{1+\beta}. \tag{40}$$

As a benchmark example, we assume that the balanced growth rate in the absence of consumption externalities is 2% per year. Thus G in (40) equals $(1.02)^{30} = 1.8114$.

As before, we set $\alpha = 0.35$ and $\beta = 0.341$, so that $\beta (1-\alpha) / (1+\beta) = 0.165$. Thus we should assume that $A = 1.811/0.165 = 10.976$. If this is the case, the rate of return to capital $R = \alpha A = (0.35)(10.976) = 3.8416$, which means that the annual rate of return is about 4.5%. Given those values, suppose that $\theta = -0.2$ and $\eta = 1.0$. This case ensures that there is a unique balanced-growth path and the balanced growth rate is $G = 2.057$. Given this condition, (39) shows that

$$\left.\frac{dG_{t+1}}{dG_t}\right|_{G_t=2.057} = -9.34.$$

Hence, the dynamic system is locally unstable and the equilibrium is determinate. Similarly, if $\theta = -0.4$ and $\eta = 1.0$, we have $G = 2.606$ and $dG_{t+1}/dG_t = -1.224$, so that determinacy still holds. However, if we set $\theta = -0.5$ and $\eta = 1.0$, then $G = 3.257$ and $dG_{t+1}/dG_t = -0.394$. In this case, the balanced-growth path is locally stable, which means that the equilibrium path around the balanced-growth equilibrium is indeterminate. That is, sunspot-driven growth cycle may be observed. As well as the case of exogenous growth, we see that a higher degree of external effect (i.e. a larger absolute value of θ) increases both the balanced-growth rate and possibility of sunspots and indeterminacy. More specifically, the minimum magnitude of θ that generates intermediacy is around 0.42 if $\eta = 1.0$. Actually, when $\theta = -0.42$ and $\eta = 1.0$, then $G = 2.785$ and $dG_{t+1}/dG_t = -0.992$, which ensures the local stability of the balanced-growth path, which generates local intermediacy.

Finally, as for the case of admiration and RAJ, the balanced-growth rates are $G = 6.95$ and 30.67 when $\theta = 0.2$ and $\eta = 1.0$, that is, the higher balanced growth rate is about 12% per year and the lower one is about 6.05% per year. If $\theta = 0.1$ and $\eta = 0.5$, then $G = 6.886$ and 56.225. In the latter economy annually grows at 15%. When $\theta = 0.4$ and $\eta = 1.0$, we obtain $G = 7.12$ and 17.77 (the annual growth rates are about 6.5% and 10%, respectively). Consequently, under our specification, a higher degree of externality enhances growth in the low-growth steady state, while it lowers the growth rate in the high growth steady state. It is also to be noted that in the case of admiration and RAJ, introducing a small external effect will give rise to a drastic change in the long-term growth performance of the economy.

Table 2 summarizes typical examples under the assumption that $\alpha = 0.35$ and $\beta = 0.341$. In this table H means the balanced-growth path with a higher growth rate, while L shows one with a lower growth rate.

θ	η	G	Local determinacy
0.4	1.0	$G^H = 17.772,\ G^L = 7.121$	H indeterminate, L determinate
0.2	1.0	$G^H = 30.674,\ G^L = 6.957$	H indeterminate, L determinate
0.2	0.5	$G^H = 56.225,\ G^L = 6.885$	H indeterminate, L determinate
-0.1	1.0	1.900	determinate
-0.2	1.0	2.057	determinate
-0.5	0.5	2.301	determinate
-0.4	1.0	2.686	determinate
-0.42	1.0	2.785	indeterminate
-0.45	1.0	2.949	indeterminate

Table 2

5 Concluding Remarks

This paper has extended the standard OLG model of capital accumulation by introducing consumption externalities. By using a model with simple functional forms, we have shown that the presence of consumption external effects may produce fundamental effects on the dynamic behavior as well as on the steady-state characterization of the model economy. Our findings have demonstrated that, consumption externalities may play a more fundamental role in an overlapping generations economy than in the representative-agent counterpart. The main reason for such a difference is that in the OLG setting, the contemporaneous external effects involve not only intragenerational but also intergenereationa interactions among the consumers. We have confirmed that the intergenreational externalities would be pivotal in generating complex dynamics in the standard OLG model of capital accumulation.

In this paper we have restrict our attention to analyze working of the simplified model. As for possible extension of our argument, two topics seem to deserve further investigation. First, it would be interesting to investigate various policy issues in our framework. A sample may include: transitional effects of fiscal policies, the optimal taxation scheme that considers transitional process as well as steady state, social security, fiscal policy and government debt, and the relation between efficiency and bubbles.

Second, since the dynamic behavior of model economy is sensitive to the preference structure, it is useful to re-examine our findings by using alternative specification of preferences. For example, following Galí (1994) and Abel (2003), we may assume that the externalities are introduced in a multiplicative form such that

$$U_t = \frac{c_t^{1-\sigma} E_t^{\theta\sigma}}{1-\sigma} + \beta \frac{x_{t+1}^{1-\sigma} E_{t+1}^{\theta\sigma}}{1-\sigma}, \quad \sigma > 0$$

where $E_t = \bar{c}_t \bar{x}_t^{\eta}$ and $E_{t+1} = \bar{x}_{t+1} \bar{c}_{t+1}^{\eta}$. In this formulation, we have four combinations: (i) admiration and RAJ if $\theta > 0$ and $\sigma > 1$; (ii) admiration and KUJ if $\theta > 0$ and $0 < \sigma < 1$; (iii) jealousy and RAJ if $\theta < 0$ and $\sigma > 1$, and; (iv) jealousy and KUJ if $\theta < 0$ and $0 < \sigma < 1$. Although in this case only numerical experiments can deal with dynamics of the model, examining alternative preference structures will be useful to check robustness of the results obtained in this paper.

Appendices

A. Proof of Proposition 1

Define

$$\begin{aligned}\Lambda(R) &= \frac{\beta}{1+\beta}\left(\frac{1-\alpha}{\alpha}+\frac{\eta\theta}{1+\theta}\right)R^2 \\ &\quad -\left[1+\frac{(1-\alpha)}{(1+\beta)\alpha}\left(\beta+\frac{\eta\theta}{1+\theta}\right)\right]R+\frac{\eta\theta}{\alpha(1+\theta)(1+\beta)}\end{aligned}$$

The root of $\Lambda(R)=0$ gives the steady state value of the gross rate of return to capital. It is to be note that

$$\begin{aligned}\Lambda(0) &= \frac{\eta\theta}{\alpha(1+\theta)(1+\beta)}, \\ \Lambda(1) &= \frac{\theta\eta}{1+\theta}-1<0\end{aligned}$$

It is easy to confirm that if $\theta>0,$ (so that $\Lambda(0)>0$), then $\Lambda(R)=0$ has two positive roost, one is in between 0 and 1, and the other is higher than 1. Since R should be larger than 1, the economy has a unique, feasible steady state. On the other hand, if $\theta<0$, then $\Lambda(0)<0$. Therefore, when $\frac{1-\alpha}{\alpha}+\frac{\eta\theta}{1+\theta}>0$, equation $\Lambda(R)=0$ has one positive and one negative roots. Since $\Lambda(1)<0$, the positive root is higher than 1, which means that the feasible steady state is uniquely determined. However, if $\frac{1-\alpha}{\alpha}+\frac{\eta\theta}{1+\theta}<0$, $\Lambda(R)=0$ has two positive roots. In this case, both roots are larger than 1, if and only if

$$\frac{\beta}{1+\beta}\left(\frac{1-\alpha}{\alpha}+\frac{\eta\theta}{1+\theta}\right)>1+\frac{(1-\alpha)}{(1+\beta)\alpha}\left(\beta+\frac{\eta\theta}{1+\theta}\right).$$

B. Evaluation of the coefficient matrix J.

Using the steady-state conditions (), () and $R=\alpha Ak^{\alpha}$, each element in the coefficient matrix of the linearized dynamic system evaluated at the steady state are written as follows:

$$\frac{dk_{t+1}}{dk_t}=\alpha(1-\alpha)Ak^{\alpha-1}=(1-\alpha)R,\qquad \frac{dk_{t+1}}{dc_t}=-1$$

$$
\begin{aligned}
J_{21} &= \frac{dc_{t+1}}{dk_t} = \alpha A\left(1-\alpha\right)\left[\alpha\left(\alpha-1\right)Ak^{\alpha-1}\right]\left[\left(1-\alpha\right)Ak^{\alpha}-c\right]^{\alpha-2} \\
&\quad \times\left[\frac{c}{D}+\left(\alpha\beta-\frac{F}{D}\right)Ak^{\alpha}\right]+\alpha A\left[\left(1-\alpha\right)Ak^{\alpha}-c\right]^{\alpha-1}\left(\alpha\beta-\frac{F}{D}\right)\alpha Ak^{\alpha-1} \\
&= -\left(1-\alpha\right)^2 R\frac{c}{k}+\left(\alpha\beta-\frac{F}{D}\right)R^2 \\
&= \left[\left(\alpha\beta-\frac{F}{D}-\frac{\left(1-\alpha\right)^3}{\alpha}\right)R+\left(1-\alpha\right)^2\right]R.
\end{aligned}
$$

$$
\begin{aligned}
J_{22} &= \frac{dc_{t+1}}{dc_t} = -\alpha A\left(1-\alpha\right)\left[\left(1-\alpha\right)Ak^{\alpha}-c\right]^{\alpha-2}\left[\frac{c}{D}+\left(\alpha\beta-\frac{F}{D}\right)Ak^{\alpha}\right] \\
&\quad +\alpha A\left[\left(1-\alpha\right)Ak^{\alpha}-c\right]^{\alpha-1}\frac{1}{D} \\
&= \left(1-\alpha\right)\frac{c}{k}+\frac{R}{D} \\
&= \left(\frac{1}{D}+\frac{\left(1-\alpha\right)^2}{\alpha}\right)R+\alpha-1.
\end{aligned}
$$

In calculations dc_{t+1}/dk_t and dc_{t+1}/dc_t, we use $\left[\frac{c}{D}+\left(\alpha\beta-\frac{F}{D}\right)Ak^{\alpha}\right]=c/R$ and $c/k=\frac{1-\alpha}{\alpha}R-1$, both of which come from () and () .

C. Proof of Proposition 3

Define the following function:

$$
\Omega\left(G\right)=G^2-\left[\left(\frac{1}{\alpha}-1\right)R+\frac{R\left(1+\theta\right)\left(1+\beta\right)}{\theta\eta}\right]G+\beta R^2\left[1+\frac{\left(1+\theta\right)\left(\frac{1}{\alpha}-1\right)}{\theta\eta}\right].
$$

Equation $\Omega\left(G\right)=0$ may have two positive roots if

$$
\left(\frac{1}{\alpha}-1\right)R+\frac{R\left(1+\theta\right)\left(1+\beta\right)}{\theta\eta}>0 \quad \text{and} \quad 1+\frac{\left(1+\theta\right)\left(\frac{1}{\alpha}-1\right)}{\theta\eta}>0.
$$

Since those conditions are satisfied if $\theta>0$, the case of admiration and RAJ would yield dual balanced growth rates. On the other hand, if $\theta<0$ and

$$
1+\frac{\left(1+\theta\right)\left(\frac{1}{\alpha}-1\right)}{\theta\eta}<0,
$$

then $\Omega\left(G\right)=0$ has one positive root. Thus in this case there is unique balanced-growth equilibrium.

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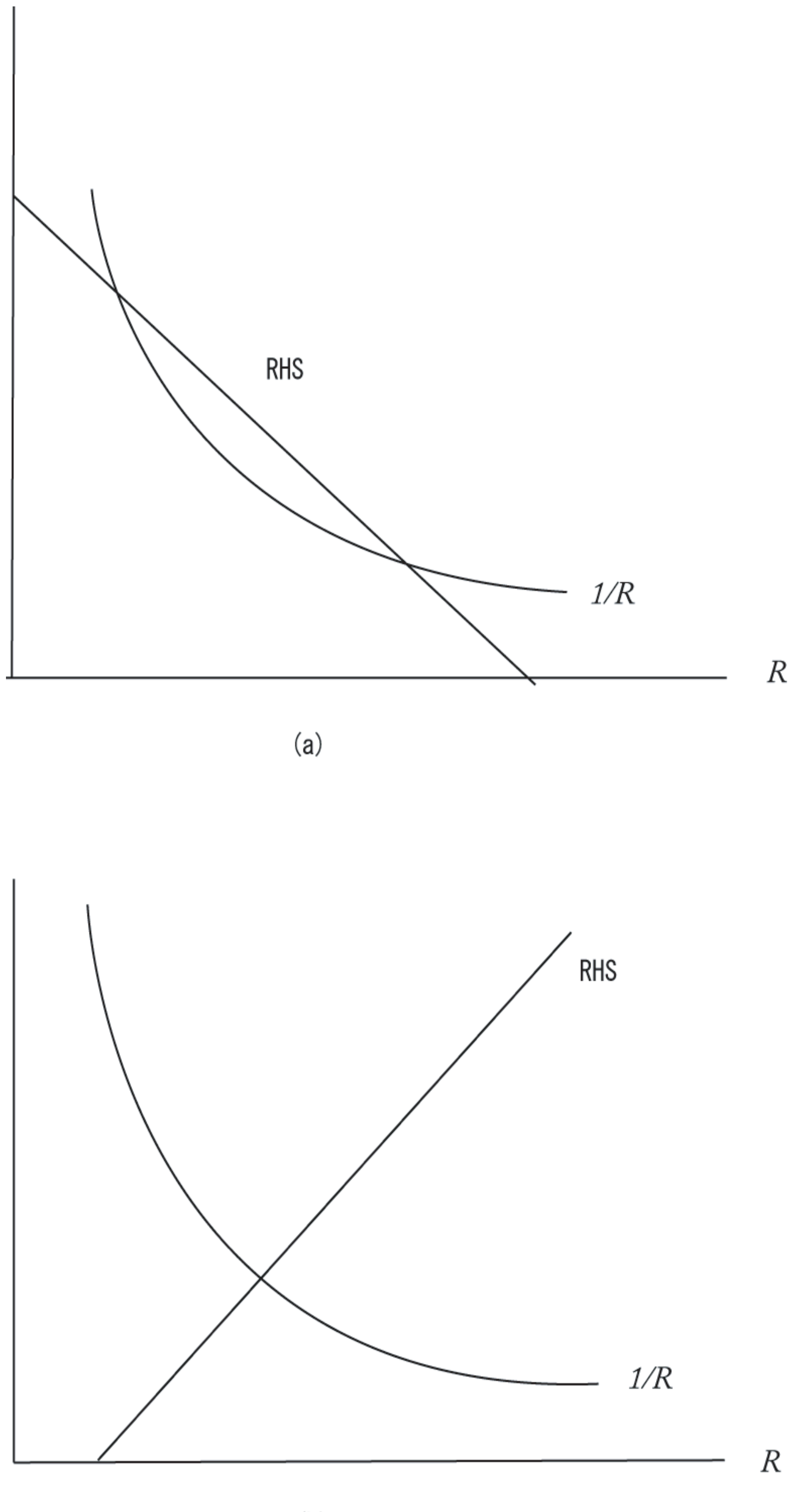
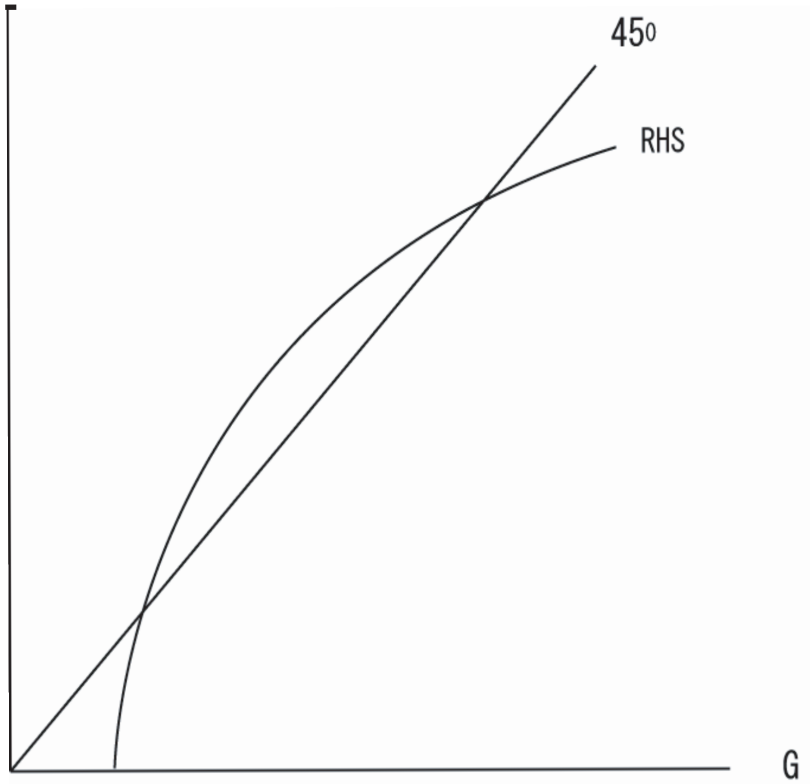


Figure 1



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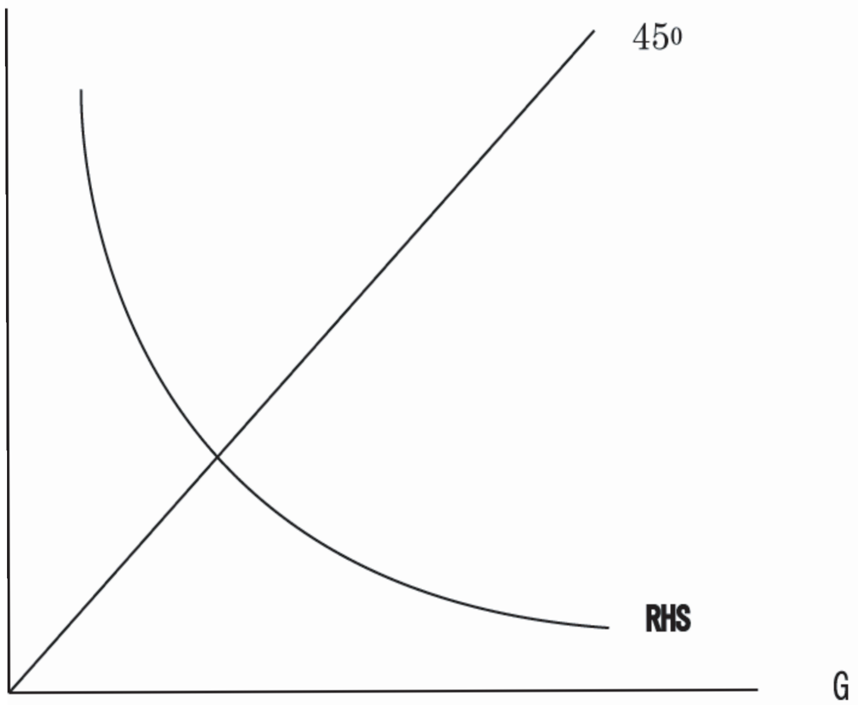

(b)

Figure 2