Inflation and the underground economy

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Abstract

This paper studies the optimal rate of seigniorage in an economy characterized by bilateral trade and a tax-evading underground sector. Optimal inflation depends on which sector, formal or underground, is more congested with buyers. If the underground sector is more congested, the optimal inflation rate in Peru is about 42.69% per annum. This offers a possible motivation for the high rates of inflation observed in that country in the 1980s. A policy that returns this economy to Friedman rule delivers a welfare loss that is equivalent to a 14% drop in consumption for the representative household. If the formal sector is more congested however, optimal inflation falls to 1.48%, close to the rate observed in 2005.

Keywords: Inflation; Market Congestion; Ramsey Equilibrium; Underground Economy

JEL classification: E2, H2, O1, O2

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1 Introduction

One of the traditional arguments advanced for positive inflation is that in the prevalence of a tax-evading underground economy, governments should rely heavily on seigniorage financing. Regular taxes tend to shift economic activity underground and are therefore distortionary. To reduce these distortions, the tax burden must be spread over all goods and services, including the liquidity services that money provides. Inflation tax is particularly convenient since it needs not be legislated. In the presence of a tax-evading sector, the typical result is that Friedman rule is not optimal. This paper provides strong micro foundations for the underground economy. I show that the optimal rate of inflation can be far lower than the rates proposed in the literature, even for countries with the same degree of tax evasion.

The working definition of the underground economy is that traders in this sector evade taxes and that underground goods are of inferior quality.[1] In terms of magnitudes, the underground-to-formal sector *output ratio* is estimated to be about 8.8% in the US, 44% in Peru and 76% in Nigeria [Schneider and Enste (2000)]. The question asked in this paper is as follows: What is the optimal rate of inflation in an economy characterized by bilateral trade and a tax-evading underground sector? To the best of my knowledge, there are three papers in the literature that address the question of optimal inflation with underground production.[2] All three papers consider tax evasion in environments with centralized market clearing

[1] There is no universally accepted definition of the underground economy. For the purpose of this paper, I focus on this narrow definition. Inferior quality can be interpreted to mean that there are no legal guarantees protecting consumers of underground goods.

[2] See Nicolini (1998), Cavalcanti and Villamil (2003) and Koreshkova (2006).

using the Walrasian auctioneer. To the contrary, bilateral exchange (one-on-one anonymous meetings between buyers and sellers) seems to be the more plausible trade arrangement that facilitates tax evasion. Apart from being the natural way to model tax evasion, this paper shows that the mechanism of bilateral trade can have pivotal implications for the optimal rate of inflation. I find that under one set of bilateral trade market conditions, which I explain shortly, the optimal rate of inflation is high and comparable to some of the rates suggested in the literature. Under a different set of bilateral trade market conditions however, the optimal rate of inflation is extremely low, even for an economy with the same output ratio.

In the environment examined, households have buyers. Some buyers are sent to the formal market, while others are sent to the underground market. If underground goods are of poor quality, a household sends relatively more buyers to the formal market. Each household acts similarly and private interest overwhelms the social optimum. There is a tendency for overcrowding of buyers in the formal sector and trade opportunities become few for each buyer in this sector.[3] If the inflation rate increases, households try to spend money faster at current prices rather than at future higher prices. They divert buyers to the underground market, where the overcrowding of buyers is less. The turnover of goods in the underground market increases and underground output increases relative to the formal sector. Since inflation increases tax evasion, seigniorage financing becomes less attractive and

[3] This of course depends on the allocation of sellers as well. For a full description of how I treat sellers, see section 2. Also, one can think of "fewer trade opportunities" as equivalent to a lower probability of finding a match with a seller.

the optimal rate of inflation is low compared to the literature. This result defies conventional wisdom, which claims that in the presence of a significant tax-evading sector, governments should resort to inflation tax.

On the other hand, if underground goods are of considerably good quality, the underground market tends to be more crowded for underground buyers. In response to higher inflation, buyers move to the "less-crowded" formal market to spend money faster. Thus, inflation reduces tax evasion and seigniorage financing becomes more attractive. The optimal inflation rate is high, as observed in some poor countries.

For a given size of the underground economy, optimal inflation depends crucially on market conditions. An environment with market crowding is essential for generating this outcome. In particular, notice that the results are not driven by the extrinsic quality of underground goods, but rather by differences in market crowding. Compare the above analysis to an equivalent economy with Walrasian market clearing, while retaining the assumption that underground goods are of lower quality. In such an economy, higher inflation still brings higher urgency to spend money. However, the distribution of goods from sellers to buyers is fully and equally efficient in both sectors, due to the Walrasian auctioneer in both sector markets. Money can thus be spent equally fast in both sectors and households need not adjust buyer allocations in order to spend money faster. That is, inflation on its own does not affect the sectoral distribution of the economy, even though underground goods are inferior. Since inflation does not increase nor decrease tax evasion, the optimal rate of inflation is unaffected. The crowding effect is unique to search models and is sometimes termed the "extensive margin" or the "market congestion effect".

An interesting property of the results is that depending on the relative congestion of the two sector markets, inflation can either increase or decrease the underground economy. How does this compare with the literature? Koreshkova (2006) introduced an environment in which credit services are produced solely in the formal sector. Inflation causes agents to trade more with credit, thereby increasing the formal sector at the expense of the underground sector. This approach supports a negative relationship between changes in inflation and changes in underground output. Although intuitively coherent, the data on the other hand is far less conclusive. In Figure 4, I compare changes in inflation to changes in underground output for several countries. There is very little if any such negative correlation.[4] Although there may exist an endogeneity problem, this only cements the need for comprehensive modeling of the underground economy to investigate the evidence.

In relation to Figure 4, the results in this paper can be interpreted as follows. At a given point in time, two countries can take opposite positions on the relative congestion of their formal and underground markets for buyers. Inflation hence impacts their underground sectors in opposite directions. Secondly, over time, a single country can switch states in the relative congestion of the two sector markets

[4] Regressing changes in inflation on changes in the output ratio generates coefficients that are statistically not different from zero.

for buyers. Thus, inflation moves underground output in reverse directions over time. Putting these together, one can get data points that wrongly suggest no relationship between changes in inflation and changes in the output ratio, similar to Figure 4.

Wright (2005) identifies four major areas where the existing literature on micro foundations of money needs further extension. Two of these are (i) extensions to include fiscal policy variables to examine their interaction with monetary policy and (ii) quantitative analysis to enable numerical policy proposals. This paper makes a significant contribution towards the integration of elaborate schemes of public finance into the monetary search literature, following recent progress by Aruoba, Waller and Wright (2006). I show that these models are indeed computable to generate numerical results that are relevant for policy. I build strong micro foundations for the underground economy by including anonymity, which directly motivates tax evasion. Finally, I show that the relative congestion of the two sector markets is important for optimal inflation.

This paper adds to the monetary literature on the informal sector, alongside Koreshkova (2006), Cavalcanti and Villamil (2003) and Nicolini (1998). Optimal policy in the presence of externalities follows fundamentals by Sandmo (1975).[5] The next section presents a two-sector monetary search framework, replicating properties of the underground-formal dichotomy. In section 3, I characterize the model and describe the equilibrium. Section 4 derives the price and output ratios

[5] Also see Ng (1980), Goulder (1995) and Bovenberg and van der Ploeg (1998).

and examines how households adjust decisions when inflation changes. In section 5, I calibrate the model to data from Peru and present quantitative estimates of the optimal inflation rate. Section 6 considers robustness and extensions. I conclude in section 7.

2 Economic Environment

I extend the framework introduced by Shi (1999) to allow for two sectors, formal and underground/informal. These are denoted by the subscripts f and i respectively and are assumed to be on separate islands. Goods are perishable between periods, irrespective of the sector in which they are produced. By this, I preclude the emergence of commodity money. Self-produced goods yield no utility and hence trade is essential for worthwhile consumption. Some of these restrictions are standard in monetary search models, as they permit trade and an endogenous role for fiat money.

Time is discrete, denoted t. Money is the sole state variable. The economy is inhabited by a large number of anonymous and infinitely-lived agents who are either buyers or sellers/producers. For tractability, I collect agents into decision-making families or households.[6] A household is constituted by the measure s of sellers and b of buyers; $s \in (0, \infty)$, $b \in (0, s]$. For simplicity, sellers are allocated

[6] A related tractable environment proceeds with agents rather than households [see Lagos and Wright (2005)]. Taber and Wallace (1999) and Zhou (1999) provide search models in which individuals hold non-degenerate inventories of indivisible money. Molico (1998) presents a search model with divisible money and goods without representative households.

exogenously between sectors, with $s_f + s_i = s$ and $s_j \in (0, s)$, $j = f, i$. There are a large number of households, and each household is infinitesimal compared to the aggregate. The focus is on the representative household, who's state and choice variables are in lower-case letters. Capital-case variables represent those of other households and the aggregate economy, which the representative household takes as given. Economy-wide money supply is M_t, of which the representative household has m_t. There is no population growth; the number of households, sellers and buyers being exogenous constants.

2.1 Market Congestion

The key mechanism driving the results in this paper is the potential for differences in market congestion in the two sectors. Hence, I present this mechanism first.

Each household sends a fraction of its buyers to each sector market. Let B_{jt} and S_j be the aggregate number of buyers and sellers entering market j, $j = f, i$. These agents match one-on-one and may trade if the match is successful. A successful match occurs when any buyer meets a seller from a household other than his own. The total number of successful matches, $\mathcal{X}_{jt}$, is derived from the matching function:

$$\mathcal{X}_{jt} = B_{jt}^{\alpha} S_j^{1-\alpha} \ , \ \alpha \in (0, 1) \ , \ j = f, i \ .$$

Also, define $\mathcal{B}_{jt}$ and $\mathcal{S}_{jt}$ as:

$$\mathcal{B}_{jt} = \frac{\mathcal{X}_{jt}}{B_{jt}} = \left(\frac{S_j}{B_{jt}}\right)^{1-\alpha} \quad \text{and}$$

$$\mathcal{S}_{jt} = \frac{\mathcal{X}_{jt}}{S_j} = \left(\frac{B_{jt}}{S_j}\right)^{\alpha} \ , \ j = f, i.$$

Then $\mathcal{B}_{jt}$ and $\mathcal{S}_{jt}$ are the average matching rates per buyer and per seller respectively.[7] These can also be interpreted as the market congestion rates for buyers and sellers respectively. Since each household is infinitesimal, they take congestion rates as given. The larger the number of buyers entering market j, the higher is the market congestion for buyers in that sector and the fewer the trade opportunities for each buyer in that sector.

Suppose there are more trade opportunities for each underground buyer than for each formal buyer: $\mathcal{B}_{it} > \mathcal{B}_{ft}$. In other words, the formal market is more congested for buyers than the underground sector. Then, an increase in inflation moves buyers to the less-congested underground market, given higher urgency to spend money stocks. Buyers are moved underground to take advantage of better trade opportunities there. On the aggregate level, the turnover of goods increase underground relative to the formal sector. Since inflation can increase tax evasion, seigniorage financing is unattractive and the optimal rate of inflation is low. The opposite is the case when $\mathcal{B}_{it} < \mathcal{B}_{ft}$. I focus on the market congestion rate for

[7]Note that $B_{jt}\mathcal{B}_{jt} = S_{jt}\mathcal{S}_{jt}$, $j = f, i$. Since it takes two to trade, one successfully matched seller implies a successfully matched buyer. See Petrongolo and Pissarides (2001) for a survey of related matching functions.

buyers only, since the allocation of sellers is exogenous.

2.2 Household's Problem

Household agents are altruistic towards fellow members. Let U_t be instantaneous utility from consumption, net of the disutility of production. $\Phi(Q_{jt}) = Q_{jt}^{\phi}$, $\phi > 1$ is the disutility of producing Q_{jt} units inside a match. Also, let the pair $\{q_{jt}, x_{jt}\}$ be the terms of trade whenever the representative household's buyers engage in purchases and $\{Q_{jt}, X_{jt}\}$ when the sellers engage in sales. Here, q_{jt} (or Q_{jt}) is the quantity to be traded and x_{jt} (or X_{jt}) is the monetary payment in currency. The terms of trade will be discussed later but for now, it suffice to take these values as given. The household's problem is:

$$v(m_t) = \max_{b_{jt}, m_{jt}, m_{t+1}, j=f,i} U_t + \beta E v(m_{t+1}) \quad , \quad \beta \in (0,1) \ ,$$

subject to the terms of trade as well as:

$$U_t = c_{ft} + \eta c_{it} - s_f \mathcal{S}_{ft} \Phi(Q_{ft}) - s_i \mathcal{S}_{it} \Phi(Q_{it}) \ , \tag{1}$$

$$c_{ft} = (1-\tau) b_{ft} \mathcal{B}_{ft} q_{ft} - Q_t^g \ , \tag{2}$$

$$c_{it} = b_{it} \mathcal{B}_{it} q_{it} \ , \tag{3}$$

$$b_{ft} + b_{it} \leq b \ , \tag{4}$$

$$m_{ft} + m_{it} \leq m_t \ , \tag{5}$$

$$m_{t+1} - m_t \leq s_f \mathcal{S}_{ft} X_{ft} + s_i \mathcal{S}_{it} X_{it} + P_t Q_t^g - b_{ft} \mathcal{B}_{ft} x_{ft} - b_{it} \mathcal{B}_{it} x_{it} \ , \tag{6}$$

$$m_{jt}, x_{jt}, c_{jt}, b_{jt} \geq 0 \ \ , \ \ j = f, i \ \text{ and } m_t \geq 0 \ \forall t.$$

Given the market congestion rates, total successful matches for household agents sent to market j are $b_{jt}\mathcal{B}_{jt}$ for buyers and $s_j\mathcal{S}_{jt}$ for sellers. Total purchases are thus $b_{jt}\mathcal{B}_{jt}q_{jt}$, while total disutility is $s_j\mathcal{S}_{jt}\Phi(Q_{jt})$, $j = f, i$. In (1), formal and underground goods are perfect substitutes in consumption but underground goods may be of inferior quality: $\eta \leq 1$.[8] I define composite consumption as $c_t = c_{ft} + \eta c_{it}$, where c_{jt} is consumption of sector j goods. A fraction, τ, of formal sector purchases is paid as a commodity tax. Also, the government buys off the quantity Q_t^g from formal buyers and pays for these units by printing money. Due to perishability, the household consumes all goods instantly. In (6), incoming funds from sales, X_{jt}, arrive simultaneously as outgoing funds, x_{jt}, during purchases. Hence the former cannot be used to finance the latter within the same period. Nominal income from sales to the government is $P_tQ_t^g$, where P_t is the per-unit price paid by the government.

[**Figure 1**]

[8] An alternative formulation is to consider η as representing a less-efficient production technology in the underground sector.

I specify the timing of events next. Starting a period with money holdings m_t, the representative household makes decisions on the allocation of buyers and money. The household also instructs its buyers and sellers on the terms of trade, which include the offers to make and those to accept in all successful matches. Next, the markets open. Formal agents visit only the formal market while informal agents go to the underground market. Once in the market, agents match one-on-one according to the matching function. Anonymity forbids credit transactions and trade is *quid pro quo.* After a bargain is reached, a successfully matched seller produces the desired output and trade is then finalized. As markets close, goods exiting the formal market gates are all taxed. Each formal buyer compulsorily sells some quantity Q_t^g to the government and receives money. Agents return to their respective households where purchased goods and sales receipts are gathered. There is consumption and the period ends.

2.3 Terms of Trade

Notice that the terms of trade, $\{q_{jt}, x_{jt}\}$, essentially establishes the per-unit price, p_{jt}, which is implied by $p_{jt} = \frac{x_{jt}}{q_{jt}}$, $j = f, i$. After the money and buyer allocations, a representative buyer enters his assigned market j with $\frac{m_{jt}}{b_{jt}}$ units of money, $j = f, i$. In each successful match, trade can occur if the offer is acceptable to both sides. For each implementable offer, monetary payments cannot exceed the buyer's money holding upon entering the match: $x_{jt} \leq \frac{m_{jt}}{b_{jt}}$, $j = f, i$. This feasibility constraint is intrinsic to the environment, given that trade is *quid pro quo*.[9]

Let ω_t (or Ω_t) be the value of money. Then, for an offer to be accepted, it must satisfy the seller's individual rationality constraint. This is simply $x_{jt}\Omega_t \geq \Phi(q_{jt})$, $j = f, i$. In both sectors, I allow buyers to hold all the bargaining power and to make take-it-or-leave-it offers. Optimal offers ensure that the individual rationality constraint holds with equality. Combined with the feasibility constraint, we have:

$$\frac{m_{jt}}{b_{jt}} \geq \frac{\Phi(q_{jt})}{\Omega_t} \ , \ j = f, i \ . \tag{7}$$

Inequality (7) is named the cash-and-carry constraint and is the final constraint on the household's problem.

Sellers act as "offer takers" and take the quantity requested as given. Temporarily assume that money is valued, allowing the cash-and-carry constraint to bind in both sectors. Then one can rewrite the level of output-per-trade in each sector as:

$$q_{jt} = \left[\frac{m_{jt}}{b_{jt}}\Omega_t\right]^{\frac{1}{\phi}} \ , \ j = f, i \ . \tag{8}$$

With quantities determined, the *quantity-per-trade ratio*, $\frac{q_{it}}{q_{ft}}$, can be readily derived. I return to this later.

To summarize, the terms of trade is simply $x_{jt} = X_{jt} = \frac{m_{jt}}{b_{jt}}$ and q_{jt} $(= Q_{jt})$

[9] Market clearing models of the underground economy are useful due to the ease of incorporating credit. For ways to include credit in models with anonymous agents, see Berentsen, Camera and Waller (2005).

given by (8). Having established this, I next address the price P_t which the government pays for goods. I allow formal sector agents to charge a premium on all sales to the government to take account of matching costs. Specifically, I assume $P_t = \frac{p_{ft}}{\mathcal{X}_{ft}}$.

2.4 Government

The definition of a sector as "underground" suggests the existence of an authority that makes this distinction. There is a centralized government that implements both monetary and fiscal policies. Money supply, M_t per capita household, grows at the rate γ per period. There is no government debt. Instead, newly printed money, $(\gamma - 1) M_t$, is used by the government in the market as payment for Q_t^g. That is, Q_t^g is real seigniorage income. The real government budget constraint is:

$$G = \tau b_{ft} \mathcal{B}_{ft} q_{ft} + Q_t^g \ , \tag{9}$$

where G is an exogenous expenditure each period. Since part of government revenues are nominal while expenditure is real, the government faces a liquidity constraint much like private households. Following Cooley and Hansen (1991),

$$(\gamma - 1) M_t = P_t Q_t^g \ . \tag{10}$$

Note that the money growth rate and tax rate are endogenous. Consider a reduction in τ. The government's liquidity constraint goes into deficits as consistent with the optimal region of the Laffer curve. This requires an adjustment in transfers to supply the funds necessary to alleviate the fiscal position, which in turn changes γ. Thus, (9) and (10) emphasize the inherent interaction between the fiscal and monetary policy variables τ and γ.

3 Characterizing the equilibrium

This section examines the euler conditions that characterizes the equilibrium. Let λ_{jt}, $j = f, i$, be the Lagrange multiplier on the cash-and-carry constraint in each successful match. m_{jt} is chosen such that the cash-and-carry constraint binds to an equal extent in expectation in each sector: $\mathcal{B}_f \lambda_{ft} = \mathcal{B}_i \lambda_{it}$. The implied euler condition for money is:

$$\frac{\omega_t}{\beta} = \omega_{t+1} + \mathcal{B}_{jt+1}\lambda_{jt+1} \ , \ j = f, i \ . \tag{11}$$

Money kept between periods delivers its discounted value in the next period as well as helps alleviate the cash-and-carry constraint in future trade matches. From (11), it can be shown that both cash-and-carry constraints bind in all successful matches in equilibrium if the return on money is sufficiently low: $\gamma > \beta$. From this point on, I assume this to be the case.

Next, I turn to the optimal quantity of output that is demanded in each trade match. The associated first order conditions are derived as:

$$1-\tau=\lambda_{ft}\frac{\Phi\left(q_{ft}\right)}{\Omega_t}\frac{\phi}{q_{ft}}+\omega_t\frac{dx_{ft}}{dq_{ft}} \text{ and} \tag{12}$$

$$\eta=\lambda_{it}\frac{\Phi\left(q_{it}\right)}{\Omega_t}\frac{\phi}{q_{it}}+\omega_t\frac{dx_{it}}{dq_{it}} \quad . \tag{13}$$

Demanding a higher quantity yields marginal utility from the additional units. The marginal cost is incurred at two levels. At the buyer level, demanding a larger quantity requires of the buyer to pay more money, thus making the corresponding cash-and-carry constraint more binding. The rate at which this constraint becomes more binding depends on how much is required to motivate the seller to deliver the additional quantity, which in turn depends on the seller's production disutility costs on the margin. Secondly, as buyers purchase higher quantities from the market and need more money to do so, the household is pressured to deliver more money to its buyers. This causes the liquidity constraint (6) to become more binding.

The first order condition for b_{ft} is given as:

$$\mathcal{B}_{ft}\left[(1-\tau)\,q_{ft}-\lambda_{ft}\frac{\Phi\left(q_{ft}\right)}{\Omega_t}-\omega_t x_{ft}\right]=\mathcal{B}_{it}\left[\eta q_{it}-\lambda_{it}\frac{\Phi\left(q_{it}\right)}{\Omega_t}-\omega_t x_{it}\right]. \tag{14}$$

Allocating more buyers to the formal sector generates more formal sector purchases and yields the associated marginal benefits in consumption utility. All things being equal, as more buyers visit the formal sector, $\frac{m_{ft}}{b_{ft}}$ declines and the cash-and-carry constraint binds further in this sector. The household is pressured to deliver more money to formal sector buyers, causing the liquidity constraint to become more binding as well. A similar effect pertains to the underground sector. For the marginal buyer, the net benefits must be equal between sectors in expectation.[10]

All households are alike and so I apply symmetry as usual. The only state variable is money. To proceed to describe an equilibrium therefore, it is essential to ensure that this variable evolves at a constant rate. Assuming a fixed inflation rate γ, the euler condition for money holding in steady state reduces to:

$$\lambda_{jt} = \frac{\gamma - \beta}{\beta \mathcal{B}_j} \Omega_t \ , \ j = f, i \ .$$

Substituting this into (12) to (14) gives (15) to (17) below.

3.1 The Equilibrium

Definition 1 *A symmetric monetary search equilibrium is defined as the inflation rate γ, the set of household choices $(b_f, m_{ft})_{t=0}^{\infty}$ and the implied value of money $(\omega_t)_{t=0}^{\infty}$ such that given τ, the following requirements are met: (i) each household solves its optimization problem; (ii) the representative household's variables replicate the aggregate equivalents; (iii) prices are positive, though bounded (the value of money is positive and bounded); and (iv) the government budget balances.*

In particular, an equilibrium involves a solution to a system of four equations

[10] The matching rates $\mathcal{B}_{ft}$ and $\mathcal{B}_{it}$ can be interpreted in terms of probabilities.

for b_f, m_{ft}, ω_t and γ:

$$1 - \tau = \left[1 + \frac{\gamma - \beta}{\beta \mathcal{B}_f}\right] \Omega_t \frac{m_{ft}}{b_f} \frac{\phi}{q_f} \ , \tag{15}$$

$$\eta = \left[1 + \frac{\gamma - \beta}{\beta \mathcal{B}_i}\right] \Omega_t \frac{m_{it}}{b_i} \frac{\phi}{q_i} \ , \tag{16}$$

$$\frac{\frac{m_{it}}{b_i}}{\frac{m_{ft}}{b_f}} = \frac{\gamma - \beta + \beta \mathcal{B}_f}{\gamma - \beta + \beta \mathcal{B}_i} \ , \tag{17}$$

$$G = \tau b_f \mathcal{B}_f q_f + (\gamma - 1) \frac{M_t}{P_t} \ . \tag{18}$$

Variables without the time subscript represent equilibrium real values. Those with time subscripts are nominal values that depend on the money stock at date t.

Given τ, there exists an equilibrium. The equations (15), (16) and (17) deliver values for the household variables b_f, m_{ft} and ω_t, all in terms of γ. The required inflation rate that balances the budget, given τ, is then derived from (18). All other variables - such as q_j, c_j, $\mathcal{B}_j$, λ_{jt}, x_{jt}, p_{jt} and P_t - can be derived as functions of the four in the definition.

Equation (17) plays a central role in understanding the implications of the model. First, the sector with the higher buyer congestion rate always has the higher money holding per buyer. If market congestion is worse for formal buyers, each is compensated with higher sums of money. In other words, if $\mathcal{B}_f < \mathcal{B}_i$, households take advantage of the intensive margin when buying from the formal sector and the extensive margin when buying underground goods. Secondly, suppose there is an increase in γ, with $\mathcal{B}_f < \mathcal{B}_i$. All things being equal, more money is diverted to underground buyers per capita and q_{it} increases relative to q_{ft}. That is, the erosive effect of inflation on household money stock increases tax evasion and seigniorage financing becomes less attractive. The reverse is the case when market congestion is worse for buyers in the underground market. A discussion of the effect of inflation follows in the next section.

4 Size, Prices and Inflation

The quantity-per-trade ratio describes trade within an underground match relative to a formal sector match and is denoted $R_I = \frac{q_i}{q_f}$. Summing over all such trade encounters in each sector gives the aggregate output ratio in trades involving all household buyers. This is denoted $R = \frac{b_i \mathcal{B}_i q_{it}}{b_f \mathcal{B}_f q_{ft}}$. The subscript I is used to denote the *intensive margin*.

4.1 Relative Quantities and Relative Price

Since the cash-and-carry constraint binds in both sectors, (8) gives the quantity-per-trade in each sector. Using this outcome together with (17), the equilibrium *quantity-per-trade ratio* becomes:

$$R_I = \left[\frac{\gamma - \beta\left(1 - \mathcal{B}_f\right)}{\gamma - \beta\left(1 - \mathcal{B}_i\right)}\right]^{\frac{1}{\phi}} ,$$

which completely describes the intensive margin. The intensive margin concerns the quantity traded within each successful match, which depends on the amount of money each buyer takes into a match. If the formal market is congested for buyers, households take advantage of each successful formal match to acquire large quantities, which implies the expense of higher sums of money in formal matches compared to underground matches. In other words, high market congestion for formal buyers reduces the intensive ratio.[11]

Next, the aggregate trades equivalent is:

$$R = \frac{b_i \mathcal{B}_i}{b_f \mathcal{B}_f} R_I \equiv \frac{s_i}{s_f} \left[\frac{\eta}{1-\tau} \right]^{\frac{\alpha}{1-\alpha}} R_I^{\frac{1}{1-\alpha}} \ , \tag{19}$$

which is the underground-to-formal sector *output ratio.* Comparing with the intensive ratio, R stresses the effect of the matching rate on aggregate market outcomes. Suppose R_I is given. Then for the representative buyer sent to each island, the congestion of the underground market relative to the formal market, $\frac{\mathcal{B}_i}{\mathcal{B}_f}$, determines the quantity of expected purchases by an underground buyer relative to a formal buyer: $\frac{\mathcal{B}_i}{\mathcal{B}_f} R_I$. Preference and policy parameters η and τ are reflected in R because households are mindful of the effect of their buyer allocation decisions on the eventual mix of goods that they consume. Given the bargaining outcome and market congestion conditions, households employ their buyer allocation decision to edge closer to their preferred mix of goods. The allocation of buyers and its effect on market congestion and aggregate trade outcomes is termed the *extensive margin.* This margin is conclusively captured by R and a search model is essential

[11] One can consider the effect of technology as another dimension of the intensive margin. Superior technology in the formal sector means that even with equal financial compensation, formal sector sellers can deliver higher quantities within each trade meeting.

for separating R from R_I.

Price in each transaction as determined from the terms of trade is $p_{jt} = \frac{m_{jt}}{b_j}\frac{1}{q_j}$, $j = f, i$ in equilibrium. Using (17), the relative price ratio in private trades reduces to $\frac{p_{it}}{p_{ft}} = \frac{1}{R_I}\frac{m_{it}}{b_i} / \frac{m_{ft}}{b_f}$, or:

$$\frac{p_{it}}{p_{ft}} = \left[\frac{\gamma - \beta\left(1 - \mathcal{B}_f\right)}{\gamma - \beta\left(1 - \mathcal{B}_i\right)}\right]^{1-\frac{1}{\phi}} . \tag{20}$$

Similar to (19), the relative price is not only a function of preferences and taxes but also an endogenous outcome of monetary policy, unlike in the earlier papers. With relatively high market congestion for formal buyers, each brings more money into a match and this increases the formal sector price relative to that underground. If $\mathcal{B}_i < \mathcal{B}_f$, it is possible to generate higher prices in the underground sector. It is worth noting however that p_{ft} is price before taxes. The effective price ratio after tax is $\frac{p_{it}}{p_{ft}}\left(1-\tau\right)$, which I report in section 5.

The ratio R has been the subject of virtually all of what is known in the literature on underground economy. The environment presented above enables us to use published empirical estimates of R and back out the micro level ratio R_I as well as the price ratio $\frac{p_{it}}{p_{ft}}\left(1-\tau\right)$ as demonstrated. Some of these results may be particularly useful since empirically, micro level data is unattainable in studies on the underground economy.

4.2 Effect of Inflation

In this subsection, I assume that monetary injections are via lump sum transfers to households and also that $\frac{d\tau}{d\gamma} = 0$.[12] In the equivalent case in Cavalcanti and Villamil (2003) as well as Nicolini (1998), firms and households do not adjust portfolios when inflation increases. In particular, inflation has no effect on sectoral allocations. $\left.\frac{dR}{d\gamma}\right|_{\tau}$ is strictly negative in Koreshkova (2006) since inflation causes agents to use more credit, which is exclusively produced in the official sector. In the model proposed however:

$$\left.\frac{dR_I}{d\gamma}\right|_{\tau} = [\mathcal{B}_i - \mathcal{B}_f + \varphi]\,\frac{R_I}{A}$$

by quotient rule, where

$$\begin{aligned} A &= \frac{\phi}{\beta}\,[\gamma - \beta + \beta\mathcal{B}_f]\,[\gamma - \beta + \beta\mathcal{B}_i] \quad \text{and} \\ \varphi &= [\gamma - \beta + \beta\mathcal{B}_i]\,\left.\frac{d\mathcal{B}_f}{d\gamma}\right|_{\tau} - [\gamma - \beta + \beta\mathcal{B}_f]\,\left.\frac{d\mathcal{B}_i}{d\gamma}\right|_{\tau} . \end{aligned}$$

Notice that $\mathcal{B}_j > 0\ \forall b_j \in [0, b]$ and hence $A > 0\ \forall\gamma \geq \beta$. Secondly, $\left.\frac{d\mathcal{B}_f}{d\gamma}\right|_{\tau} > 0$ and $\left.\frac{d\mathcal{B}_i}{d\gamma}\right|_{\tau} < 0$ whenever $\mathcal{B}_i - \mathcal{B}_f > 0$ and vice versa. Thus, φ is a function of the same sign as $\mathcal{B}_i - \mathcal{B}_f$. Assume that underground buyers have better matching success: $\mathcal{B}_i - \mathcal{B}_f > 0$. When γ increases, households seek to spend nominal balances faster and they divert some buyers from the formal market to the less con-

[12]Specifically, government simply hands money to each buyer, instead of requesting Q_t^g units of output. For now, ignore the effect on the government budget.

gested underground market, as consistent with (17). Since b_i increases, $\left.\frac{d\mathcal{B}_i}{d\gamma}\right|_\tau < 0$ and the household compensates each underground buyer with more money per capita, which increases R_I. Since b_i increases, aggregate matches, $b_i\mathcal{B}_i$, increase underground relative to the formal sector. The effect on the extensive ratio R is therefore in the same direction as R_I.

Even with $\frac{d\tau}{d\gamma} = 0$, monetary policy affects the relative price. If $\mathcal{B}_i - \mathcal{B}_f > 0$, the underground price level rises relative to the formal sector price as the rate of inflation increases. Again, by quotient rule:

$$\left.\frac{d\frac{p_{it}}{p_{ft}}}{d\gamma}\right|_\tau = [\mathcal{B}_i - \mathcal{B}_f + \varphi]\,(\phi - 1)\,\frac{\frac{p_{it}}{p_{ft}}}{A}\ .$$

Intuitively, increased inflation implies that each underground buyer starts to hold more money compared to previously (if $\mathcal{B}_i - \mathcal{B}_f > 0$). Thus, underground buyers begin to demand higher quantities in each trade. They need to pay higher prices to motivate the additional units, owing to the convex cost of production ($\phi > 1$). This change in the relative price implies a marginal decline in R_I, however this effect is of second order and does not reverse the initial rise in R_I and R. When $\mathcal{B}_i - \mathcal{B}_f < 0$, the relative size and relative price ratios respond in the opposite direction of the corresponding effect above as inflation increases.

4.3 The Ramsey Problem (Optimal Inflation)

Bailey (1956) and Phelps (1973) brought the subject of optimal inflation into the fold of public finance. In this seminal contribution, Phelps advocates for a positive tax on the liquidity services that money provides if taxes on other goods and services are distortionary. This argument favours a positive nominal interest rate, or simply, positive inflation. Tax distortions are socially costly while inflation presents the usual welfare consequences. The task facing a benevolent government is to find the best trade-off between the deadweight loss from tax financing and that from seigniorage financing.

I focus on the Ramsey problem that seeks to find the optimal mix of consumption and inflation taxes when government can commit to the announced policy. Without money, the cash-and-carry constraint (7) cannot be satisfied and the economy described degenerates into autarky. Following Kiyotaki and Wright (1989), money acts as an intermediate commodity that facilitates trade. Diamond and Mirrlees (1971) established a general result emphasizing the undesirability of taxing the intermediate goods sector when *all* final goods and services fall under the tax radar. In application to monetary economics, their conclusion implies that inflationary tax should not be used despite the distortions caused by taxes on the final goods sector.[13] However, where there is a third sector - the underground economy - that evades regular taxes, the optimal policy set may include positive

[13] Also, see Kimbrough (1986), Faig (1988), Guidotti and Vegh (1992), Chari, Christiano and Kehoe (1996).

seigniorage.

The formalized Ramsey problem is to solve the household's problem subject to the government's budget constraints and the first order conditions in section (3). Let the variables with tildes represent the Ramsey allocations: $\left\{\widetilde{b}_f, \widetilde{m}_{ft}, \widetilde{q}_f, \widetilde{q}_i\right\}$. The required inflation and tax rates are respectively:

$$\widetilde{\gamma} = \beta + \beta \left[\frac{\widetilde{m}_{it}}{\widetilde{b}_i} - \frac{\widetilde{m}_{ft}}{\widetilde{b}_f}\right]^{-1} \left[\widetilde{\mathcal{B}}_f \frac{\widetilde{m}_{ft}}{\widetilde{b}_f} - \widetilde{\mathcal{B}}_i \frac{\widetilde{m}_{it}}{\widetilde{b}_i}\right] \text{ and} \tag{21}$$

$$\widetilde{\tau} = \frac{G - (\widetilde{\gamma} - 1)\, M_t/\widetilde{P}_t}{\widetilde{b}_f \widetilde{\mathcal{B}}_f \widetilde{q}_f} \ . \tag{22}$$

Market clearing models of the underground economy have commonly used credit to establish reasons why Friedman rule is suboptimal. Even in the absence of credit, search frictions rule out any possibility that Friedman rule may be optimal, except for the special case where market congestion rates are equal between the two sectors. From (17), one can show that second term on the right hand side of (21) is non-negative. First, $\frac{\widetilde{m}_{it}}{\widetilde{b}_i} = \frac{\widetilde{m}_{ft}}{\widetilde{b}_f}$ whenever $\widetilde{\mathcal{B}}_f = \widetilde{\mathcal{B}}_i$. Further, when $\widetilde{\mathcal{B}}_f \neq \widetilde{\mathcal{B}}_i$, $\frac{\widetilde{m}_{it}}{\widetilde{b}_i} - \frac{\widetilde{m}_{ft}}{\widetilde{b}_f}$ and $\widetilde{\mathcal{B}}_f \frac{\widetilde{m}_{ft}}{\widetilde{b}_f} - \widetilde{\mathcal{B}}_i \frac{\widetilde{m}_{it}}{\widetilde{b}_i}$ are of the same sign, in which case the second term is strictly positive.

The trade-off between taxes and inflation in the presence of congestion externalities warrants further explanation. In the typical environment with market clearing and evadable taxes, optimal policy considers (i) real distortions created by formal sector taxes and (ii) the welfare cost of inflation. Assume that the implied optimal policy set in this case is the pair $\{\widetilde{\tau}_a, \widetilde{\gamma}_a\}$. The additional dimension provided in the framework with bilateral exchange is that there is a role for inflation in correcting any imbalances in market congestion rates in the two sectors.[14]

The Trade-off with market congestion - Case 1

To illustrate, suppose sellers are distributed evenly between the two sectors. Then, to minimize search frictions and maximize aggregate matches, buyers must also be allocated equally between sectors. Suppose instead that the allocation of buyers is skewed towards the underground sector, causing high market congestion for buyers in that sector. The optimal policy set includes low taxes: $\widetilde{\tau}_1 < \widetilde{\tau}_a$ and high seigniorage: $\widetilde{\gamma}_1 > \widetilde{\gamma}_a$. Low taxes edge buyers back into the formal market and improves the coordination problem.

In this illustration, two factors account for the negative relationship between τ and γ. The first is the traditional argument that as γ increases, seigniorage income rises, which finances the government and helps reduce τ. The second is that as γ increases, buyers move to the formal sector via the extensive margin in order to spend money faster. Thus, more goods become taxable, which also means the tax rate can adjust downwards even further. For both of these factors, $\frac{d\tau}{d\gamma} < 0$ when the underground market has the higher market congestion for buyers. Apart from the trade-off between distortionary taxes and the welfare cost of inflation, the Ramsey problem also seeks to even out market congestion rates in the two sectors and improve the coordination problem.

[14] For more on second best taxation in environments with externalities, see Sandmo (1975), Ng (1980), Goulder (1995) and Bovenberg and van der Ploeg (1998).

The Trade-off with market congestion - Case 2

When the market is more congested for formal buyers, the trade-off between these two taxes is less clear. Consider a marginal reduction in inflation. Seigniorage incomes decline, but tax revenues increase even with no change to the tax rate. This is because buyers return to the congested formal market, given a lower urgency to spend money. Depending on the influx of buyers into the formal market, the rise in tax revenues can outweigh losses in seigniorage income. That is, marginal reductions in τ also become affordable. In summary, lower inflation and a lower tax rate are jointly feasibly: $\frac{d\tau}{d\gamma} > 0$.[15]

If government can lower the welfare cost of inflation (by lowering γ) and at the same time lower tax distortions (by lowering τ), then is Friedman rule optimal? Not necessarily, because of market congestion. Since $\mathcal{B}_f < \mathcal{B}_i$, lower taxes and lower inflation both have the same effect of moving buyers to the congested formal market. Thus, for low-enough levels of γ and τ, too many buyers enter the already-crowded formal market and the coordination problem worsens. This hinders aggregate trade and reduces welfare. Optimal policy includes $\{\widetilde{\tau}_2, \widetilde{\gamma}_2\}$ such that $\widetilde{\tau}_1 < \widetilde{\tau}_2 < \widetilde{\tau}_a$ and $\widetilde{\gamma}_2 < \widetilde{\gamma}_a$, but this does not guarantee that Friedman rule becomes optimal. In this case as well, the Ramsey problem finds optimal policy after considering not only tax distortions and the welfare cost of inflation, but also market congestion.

[15] The analysis here is aimed at explaining our simulation results as in section 5, for the case where $\mathcal{B}_f < \mathcal{B}_i$. See the upper right panel of Figure 3.

5 Calibration and Results

This section calibrates the model to match data from Peru and identifies the optimal rate of inflation. I normalize the number of sellers, s, to unity. Using time diary data, Juster and Stafford (1991) estimate that US residents spend on average 23.9 hours on paid work and 6.8 hours shopping per week. b is set to $\frac{6.8}{23.9}$. This value is adopted for Peru, but considered a lower bound for time spent shopping in that country.[16]

[**Table 1**]

Data on tax revenue as a percentage of GDP is retrieved from the World Development Indicators (WDI) database of the World Bank. The average for 2000 to 2004 is used to represent τ. Also collected from the same database is average annual CPI inflation for 2000 to 2005, which is used to represent $\gamma^{12} - 1$. Finally, an estimate of the underground-to-formal sector output ratio is taken from Schneider and Enste (2000) and used to represent R.

Specifically, the equations I calibrate are (15) to (19). Temporarily assume that we know s_f (and hence s_i). Then, given the above values for τ, G, γ, b, β and α, equations (15) to (18) are used to get b_f, m_{ft}, ω_t and Q^g. The remaining requirement is to verify s_f. The model is simulated for the value s_f such that the relative size of the underground economy, R, equals 0.44, as consistent with (19).

[16] The appendix includes sensitivity analysis on b, ϕ and α.

This completes the calibration.[17]

For the first case, I assume that underground goods are just as good as formal sector goods: $\eta = 1$. In Table 2, households send relatively more buyers to the underground sector, causing high market congestion for underground buyers ($\mathcal{B}_i < \mathcal{B}_f$). Each underground buyer is handed a relatively high sum of money: $\frac{m_{it}}{m_t} > \frac{b_i}{b}$. Since each underground buyer holds more money per capita, they can buy more units and the intensive margin ensures that $R_I > 1$. In order to match the *output ratio* of $R = .44$, I assign sufficiently few sellers to the underground market. The value of s_f derived is retained for all other simulations for this first case. For each policy set fed into the model, (19) is then used to evaluate the new level of R, given s_f constant.

[**Table 2**]

For the second case, I assume that $\eta = .85$. Market congestion is reversed, with the formal sector being more congested for buyers. Market congestion is lower for each underground buyer, requiring lower money allocation to these buyers: $\frac{m_{it}}{m_t} < \frac{b_i}{b}$. Since each underground buyer bears lower money stocks, they buy fewer units per capita compared to formal buyers and $R_I < 1$. Here again, the model is simulated to deliver s_f such that $R = .44$. The value of s_f derived is retained for all other simulations for this second case. The quantity Q^g is real government revenue from seigniorage spending. The values are however small compared to the

[17] For the sake of comparison, I also calibrate the US economy for which data is collected similarly and from the same sources, with $R = .088$, $\tau = .1073$ and $\gamma^{12} = 1.028262$.

total government budget, G.

5.1 Optimal Inflation Tax

The optimal policy set is in Table 3. It is important to note that the higher optimal inflation recommended for the economy with $\eta = 1$ is not because that economy has higher tax evasion. In fact, in both economies, I start off with $R = 0.44$ as shown in Table 2. Instead, the economy with $\eta = 1$ has higher optimal inflation because of higher market congestion for buyers in the underground market. Inflation does not only bring seigniorage income, it also reduces tax evasion as buyers start to take advantage of lower market congestion in the formal market. This acts as an additional incentive for seigniorage financing and explains the optimal rate of 42.69% in Peru. This result is robust for marginally inferior underground goods: $\eta = 1 - \varepsilon$; ε being an arbitrarily small positive number. That is, inflation can increase the consumption of higher-quality formal sector goods. This result is new, and opposite to that found in Peterson and Shi (2004), where inflation causes households to compromise on the quality of goods they consume. In this case seigniorage contributing significantly to the government budget.

[**Table 3**] [18]

[18] For different configurations of relative credit-use, Nicolini (1998) finds optimal annual interest rates between 7.34% and 19.17%. In Table 3, I convert these estimates into inflation rates using the Fisher equation as in section 6. The tax rate in that paper is calibrated differently and not compared. For an economy with 40% output ratio, Koreshkova (2006) estimates the optimal rate of inflation to be approximately 60% per annum. Her base economy is calibrated to US data and hence the tax rates are also not comparable.

On the other hand, when the congestion of buyers is higher in the formal sector, inflation increases R, which acts as a disincentive to seigniorage financing. The optimal inflation rate here is 1.48%, despite the large tax-evading sector. In Figure 3, seigniorage income (Q^g/G) rises with γ, as consistent with models with centralized markets. Given G, seigniorage helps alleviate tax financing. However, as γ increases, buyers exit the formal market in search for better matching rates underground $(\mathcal{B}_i > \mathcal{B}_f)$. The turnover of taxable goods decline, along with tax revenues, at the going tax rate. Tax revenues decline at a rate faster than the gains from seigniorage, requiring τ to rise.

[**Figure 2**]

[**Figure 3**]

The large variation in optimal inflation is not driven by differences in the quality of goods but rather by differences in market congestion. In the equivalent economy with market clearing, inflation on its own does not alter the extent of tax evasion, irrespective of the relative quality of underground goods. The optimal rate of inflation is hence unaffected.

To better understand the welfare implications of the simulations, I define the index:

$$\%\Delta c^{\gamma} = \frac{U^{\beta} - U^{\gamma}}{c^{\beta}} \times 100\% \ .$$

U^{γ} is the instantaneous return to the household [see equation (1)] in an equilibrium with inflation rate γ, using a corresponding tax rate that balances the government budget. Similarly, c^{β} is the composite consumption level at $\gamma = \beta$. Starting from Friedman rule, the value $\%\Delta c^{\gamma}$ denotes the proportional increase in consumption required to compensate the representative household for the transition to a new equilibrium with $\gamma > \beta$.[19] In Table 3, the difference in the welfare effects of inflation in the two cases is accounted for by (i) the size of the optimal inflation rate in each case, which affects the extent to which the coordination problem is corrected and (ii) the change in tax distortions that is achieved via the adjustment to the new tax rate.

In the first case ($\eta = 1$), I compare the optimal policy [$\{\widetilde{\gamma} = 42.69\%, \widetilde{\tau} = 9.98\%\}$ and the results in the first column of Table 3] to the actual policy [$\{\gamma = 2.24\%, \tau = 12.71\%\}$ and its associated results in the first column of Table 2]. High optimal inflation goes a long way to (i) improve the coordination problem (reduce $|\mathcal{B}_f - \mathcal{B}_i|$) as well as (ii) reduce tax distortions due to lower taxes. The combined effect is such that a reduction in inflation from this optimal value down to Friedman rule requires $\%\Delta c^{\gamma} = -14.09\%$.

In the second case ($\eta = .85$), optimal policy [$\{\widetilde{\gamma} = 1.48\%, \widetilde{\tau} = 12.47\%\}$] is compared to the actual [$\{\gamma = 2.24\%, \tau = 12.71\%\}$] in the same fashion. Since the optimal inflation rate is lower than the actual, this (i) worsens the coordination problem by increasing $|\mathcal{B}_f - \mathcal{B}_i|$. However, buyers return to the congested formal

[19] Benabou (1991) reconclines a related index of the welfare effects of inflation to the area under the real money demand function.

market given less urgency to spend money, thereby increasing the volume of goods under the tax radar. As this happens, a tax rate marginally lower than the actual becomes feasible, thus (ii) reducing tax distortions. The net effect of these two accounts for $\%\Delta c^{\gamma} = -.049\%$.

The results are worth comparing with those in Nicolini (1998), also calibrated to Peru. Nicolini considers an economy in which an exogenous segment of the formal sector commodity space has trade conducted only with credit, while another segment has trade strictly with cash. A mutually exclusive set of goods are produced in a tax-evading underground sector and traded only with cash. Over-reliance on tax financing widens the tax burden between the formal-cash goods and the underground-cash goods sectors. On the other hand, over-reliance on seigniorage financing widens the tax burden between the formal-credit and formal-cash goods sectors. Optimal policy employs a mix of both sources of financing. For different relative sizes of the formal-credit sector, he documents variations in optimal inflation in the range shown in Table 3.

The results in this paper show that even without any assumptions regarding credit-use, there can be large variations in the optimal rate of inflation if trade is decentralized. This is against evidence provided by Besley and Levenson (1996), which calls to question the role played by credit. They document a high prevalence of Rotating Saving and Credit Associations in Taiwan, allowing informal sector agents access to financial intermediation. Participation rates were found to be as high as 45% at the highest income percentile, which is significant compared to the relative size of the underground economy. The micro foundations of money alone can explain why Friedman rule fails to be optimal. Augmenting the current framework with credit can lead to higher optimal rates of inflation, perhaps close to estimates found in Koreshkova (2006). Her paper finds that a country with 40% underground economy, the optimal rate of inflation is about 60% per annum. In contrast, decentralized market conditions can instead support high or low inflation rates even for closely similar rates of tax evasion.

6 Discussion

The economic environment examined is directly equivalent to one in which households interact with a centralized market for government bonds. Augmenting the household's liquidity constraint with bonds, the euler condition for bonds is $\frac{\omega_t}{\beta} = \omega_{t+1}(1 + r_{t+1})$, where r_t is the net nominal interest rate. Comparing this euler condition with (11), the interest rate is derived as:

$$r_t = \mathcal{B}_f \frac{\lambda_{ft}}{\omega_t} = \mathcal{B}_i \frac{\lambda_{it}}{\omega_t} \equiv \frac{\gamma - \beta}{\beta} .$$

Friedman rule involves setting γ to β, or alternatively, r_t to zero.

In the environment studied, the allocation of sellers between sectors is exogenous. The configuration $\{s_f, s_i\}$ is nevertheless consistent with the equilibrium. Due to take-it-or-leave-it offers by buyers, sellers exit each trade match with zero net surplus in both sectors. Households are therefore indifferent in the allocation of sellers between sectors when I endogenize the seller allocation decision. Using (12) and (13), it is easy to show that the first order condition for s_{ft} holds true for all values of $s_{ft} \in [0, s]$. The result is an infinite set of equilibria, including the point $\{s_f, s_i\}$ used in section 2. Employing Nash bargaining may narrow the set of equilibria. Such an extension is likely to strengthen the results discussed in this paper. I conjecture that in response to changes in the inflation rate, sellers are likely to move in the same direction as buyers, further strengthening the results on the extensive margin.

This paper generate endogenous micro level trade ratios including the quantity-per-trade ratio and the relative price. A somewhat related paper in the literature is McLaren (1998). He considers a non-monetary economy with markets for imported goods. There are several markets, each for a specific class of imported goods. Depending on the tax rate and the concentration of tax inspectors in a given market, traders decide either to import legally and pay the associated taxes or to smuggled at a risk of detection. Quantity per importer is fixed and only the choice of sector is endogenously influenced by policy. In equilibrium, traders in the market for a particular class of good are all simultaneously legitimate importers or all smugglers. This is an outcome of market clearing. Although separate prices can be derived for the two sectors, only one is operational for each commodity class. He then studied the optimal tax and audit rates in a Ramsey-type equilibrium. The current paper on the other hand endogenizes production quantities, prices and sector choice, and these depend on fundamentals as well as economic policy, including money.

A possible extension is to introduce capital into the environment examined in this paper. First, notice that the model presented above can be interpreted as one with constant returns to scale production technology involving labour: $q_{jt} = l_{jt}$, $j = f, i$, where l_{jt} is labour input. In this case, the disutility of production reverts to disutility of labour: $\Phi(l_{jt})$. The introduction of capital simply involves employing a more general production function and an appropriate capital accumulation equation. This extension will facilitate interesting dynamic and business cycle applications. One is however compelled to take a stand on which good(s), formal or underground, can be accumulated into capital, if not both. How exactly are they combined in the constitution of a uniform capital stock?

7 Conclusion

There are two main conclusions to draw from this paper. First, the data fails to support the conventional wisdom that higher inflation strictly reduces the size of the underground economy. There are data points for which decreases (increases) in the underground-to-formal sector *output ratio* were indeed accompanied by higher (lower) inflation. However, there are just as many data points that instead suggest the reverse. I develop a theoretical framework that explains the evidence. The solution I propose is that the relative congestion of the formal and underground markets for buyers can be different across countries. Where the formal market is more congested for buyers, inflation causes households to compromise on the quality of goods they consume and commit more money and more buyers to the underground sector. In this case, underground output increases both on the intensive and extensive margins relative to the formal sector. When the underground sector is more congested for buyers, inflation achieves the opposite result. In short, inflation can move underground output in both directions, as consistent with the data.

The second conclusion is as follows. In the presence of an underground sector, tax distortions are socially costly while inflation presents the usual welfare consequences. If both sector markets are characterized by Walrasian market clearing, the task facing a benevolent government is to find the best trade-off between the deadweight loss from tax financing and that from seigniorage financing. With bilateral trade however, optimal policy also seeks to correct the coordination problem that exists when market congestion is unbalanced between sectors. When the underground market is more congested for buyers, the benevolent government reduces the formal sector tax rate to encourage buyers back into the formal sector. Optimal policy thus involves high seigniorage financing and low taxes. I find optimal inflation rates as high as 42.69% per annum for Peru. Although this rate is lower than the rates observed in that country from the mid 1970s to the mid 1990s, it does offer a general explanation for the high rates of inflation in some poor countries within the context of optimal public finance policy.

When the formal sector is more congested for buyers, optimal policy seeks to reduce the overcrowding of buyers in the formal sector to improve the coordination problem. This requires high taxes combined with low seigniorage spending. For the relevant configuration of the model, I generate an optimal annual inflation rate of 1.48% for Peru, which is close to the rate observed in that country in 2005. In Peru, the size of underground output relative to the formal sector is estimated at 44%. With such high rates of tax evasion, a familiar assertion in the literature calls for high reliance on seigniorage financing. Further, Cooley and Hansen (1991) showed quantitatively that when inflation tax revenue is replaced by revenue from other distortionary taxes, the welfare effect is negative. The results in this paper show that the optimal inflation rate can be far lower than suggested in the literature, even though formal sector taxes are distortionary.

The results in this paper must not be taken to imply that within the range of low to high inflation, only the extreme policies are optimal. The environment examined ignores other important considerations for inflation tax, including the cost of administering alternative forms of taxation, the availability of other stores of value apart from money and the redistributive implications of inflation. On the theory front, I make significant inroads in integrating fiscal policy instruments into the literature on the micro foundations of money. I showed that the model is adaptable for the inclusion of capital, thus allowing the familiar dynamic and business cycle analysis. The environment proposed is flexible and permits applications to other sectoral divisions of the economy such as manufacturing versus services. Instead of matters concerning two-sector economies, further extensions may consider two-country applications.

Appendix

The household solves:

$$
\begin{aligned}
v(m_t) &= \max_{b_{jt},m_{jt},q_{jt},m_{t+1},j=f,i} c_{ft} + \eta c_{it} - s_f \mathcal{S}_{ft} \Phi(Q_{ft}) - s_i \mathcal{S}_{it} \Phi(Q_{it}) \\
&\quad + \beta E v(m_{t+1}) + b_{ft} \mathcal{B}_{ft} \lambda_{ft} \left[\frac{m_{ft}}{b_{ft}} - \frac{\Phi(q_{ft})}{\Omega_t} \right] + b_{it} \mathcal{B}_{it} \lambda_{it} \left[\frac{m_{it}}{b_{it}} - \frac{\Phi(q_{it})}{\Omega_t} \right] \\
&\quad + \omega_t \left[m_t + s_f \mathcal{S}_{ft} X_{ft} + P_t Q_t^g + s_i \mathcal{S}_{it} X_{it} - b_{ft} \mathcal{B}_{ft} x_{ft} - b_{it} \mathcal{B}_{it} x_{it} - m_{t+1} \right].
\end{aligned}
$$

The Euler conditions (11) to (14) follow direct from the above set-up. (15), (16) and (17) are arrived at as follows. First, note that if money is valued, $\lambda_{jt} \geq 0$, $j = f, i$ and hence $x_{jt} = \frac{\Phi(q_{jt})}{\Omega_t}$, with $\frac{dx_{jt}}{dq_{jt}} = \frac{\Phi(q_{jt})}{\Omega_t} \frac{\phi}{q_{jt}}$, $j = f, i$. This substituted into (12), (13) and (14) yield:

$$1 - \tau = [\lambda_{ft} + \omega_t] \frac{\Phi(q_{ft})}{\Omega_t} \frac{\phi}{q_{ft}} \ ,$$

$$\eta = [\lambda_{it} + \omega_t] \frac{\Phi(q_{it})}{\Omega_t} \frac{\phi}{q_{it}} \quad \text{and}$$

$$\mathcal{B}_f \left[(1-\tau) q_{ft} - (\lambda_{ft} + \omega_t) \frac{\Phi(q_{ft})}{\Omega_t} \right] = \mathcal{B}_i \left[\eta q_{it} - (\lambda_{it} + \omega_t) \frac{\Phi(q_{it})}{\Omega_t} \right].$$

With a constant money growth rate $m_{t+1} = \gamma m_t$, the value of money declines at the growth rate of money: $m_{t+1}\omega_{t+1} = m_t \omega_t$. Thus the euler for money gives $\gamma \omega_t m_t = \beta \omega_{t+1} m_{t+1} + \beta \mathcal{B}_f \lambda_{ft+1} m_{t+1}$. Rearranging,

$$\mathcal{B}_f \lambda_{ft} = \mathcal{B}_i \lambda_{it} = \frac{\gamma - \beta}{\beta} \omega_t \ . \tag{23}$$

Due to the restriction $\gamma \geq \beta$, $\lambda_{jt} \geq 0$, $j = f, i$ and the cash-and-carry constraints bind in all transactions:

$$\frac{m_{jt}}{b_{jt}}\omega_t = q_{jt}^{\phi} \quad , \ j = f, i \ . \tag{24}$$

Substituting (23) and (24) in the three conditions and imposing symmetry ($\omega_t = \Omega_t$ and $Q_{jt} = q_{jt}$, $j = f, i$ etc), we have:

$$1 - \tau = \frac{\gamma - \beta\left(1 - \mathcal{B}_f\right)}{\beta \mathcal{B}_f}\omega_t \frac{m_{ft}}{b_{ft}} \frac{\phi}{q_{ft}} \tag{15}$$

$$\eta = \frac{\gamma - \beta\left(1 - \mathcal{B}_i\right)}{\beta \mathcal{B}_i}\omega_t \frac{m_{it}}{b_{it}} \frac{\phi}{q_{it}} \tag{16}$$

$$\mathcal{B}_f\left[\left(1 - \tau\right) q_{ft} - \frac{\gamma - \beta\left(1 - \mathcal{B}_f\right)}{\beta \mathcal{B}_f}\omega_t \frac{m_{ft}}{b_{ft}}\right] = \mathcal{B}_i\left[\eta q_{it} - \frac{\gamma - \beta\left(1 - \mathcal{B}_i\right)}{\beta \mathcal{B}_i}\omega_t \frac{m_{it}}{b_{it}}\right].$$

Simplifying this last condition using (15) and (16) gives:

$$\frac{\frac{m_{it}}{b_i}}{\frac{m_{ft}}{b_f}} = \frac{\gamma - \beta + \beta \mathcal{B}_f}{\gamma - \beta + \beta \mathcal{B}_i} \quad . \tag{17}$$

At the government side (18) follows easily from (9) and (10). The derivation of the ratios are explained in the paper. The ratio of (15) and (16) gives:

$$\frac{\mathcal{B}_i}{\mathcal{B}_f} = \frac{1 - \tau}{\eta} \frac{q_{ft}}{q_{it}} \ . \tag{25}$$

Simplifying further gives $\left(\frac{s_i}{b_i}\right)^{1-\alpha} / \left(\frac{s_f}{b_f}\right)^{1-\alpha} = \frac{1-\tau}{\eta}\frac{q_{ft}}{q_{it}}$; or:

$$\frac{b_i}{b_f} = \frac{s_i}{s_f}\left[\frac{\eta}{1-\tau}\frac{q_{it}}{q_{ft}}\right]^{\frac{1}{1-\alpha}} . \tag{26}$$

Notice that $R = \frac{b_i}{b_f}\frac{\mathcal{B}_i}{\mathcal{B}_f}\frac{q_{it}}{q_{ft}}$, which involves the product of (25), (26) and R_I. The outcome is (19). Finally, (21) follows from (17) and (22) from (18).

In the equivalent model with centralized market clearing and cash-in-advance, the household's problem is

$$\begin{aligned} v(m_t) \quad = \quad & \max_{c_{jt}, l_{jt}, m_{jt}, m_{t+1}, j=f,i} (1-\tau)\, c_{ft} + \eta c_{it} - \Phi(l_{ft}) - \Phi(l_{it}) + \beta E v(m_{t+1}) \\ & + \lambda_{ft}\left[m_{ft} - p_{ft}c_{ft}\right] + \lambda_{it}\left[m_{it} - p_{it}c_{it}\right] \\ & + \omega_t\left[m_t + W_{ft}l_{ft} + W_{it}l_{it} - p_{ft}c_{ft} - p_{it}c_{it} + \Pi_{ft} + \Pi_{it} - m_{t+1}\right] , \end{aligned}$$

where l_{jt} is labour, W_{jt} the wage rate and Π_{jt} is firm profit in sector j. Firms solve $\Pi_{jt} = \max_{l_{jt}} p_{jt} q(l_{jt}) - W_{jt} l_{jt}$, $j = f, i$, where $q(l_{jt}) = l_{jt}$. The government budget constraint is $G = \tau c_{ft} + Q_t^g$, where $Q_t^g = (\gamma - 1)\frac{M_t}{p_{ft}}$. Market clearing requires that $q_{ft} = c_{ft} + Q_t^g$ and $q_{it} = c_{it}$. Given the linear nature of the firm's problem, the auctioneer sets $p_{jt} = W_{jt}$, while the first order conditions of the household's problem are $\lambda_{ft} = \lambda_{it}$, $1-\tau = (\lambda_{ft} + \omega_t)\, p_{ft}$, $\eta = (\lambda_{it} + \omega_t)\, p_{it}$ and $\omega_t W_{jt} = \Phi'(l_{jt})$. In equilibrium:

$$\begin{aligned} \frac{p_{it}}{p_{ft}} &= \frac{\eta}{1-\tau} , \\ \frac{q_{it}}{q_{ft}} &= \left[\frac{\eta}{1-\tau}\right]^{\frac{1}{\phi-1}} . \end{aligned}$$

A summary comparison of the models is in Table 4.

[**Table 4**]

Data

1. Data on the relative size of the underground economy (R) was retrieved from Schneider and Enste (2000). The other ratios, R_I and $\frac{p_i}{p_f}$, are derived using these values of R and formulas outlined in section 4.

2. Figure 3 does not show country names against the data points due to overcrowding. The data is available upon request. The regression $\Delta UE = \beta_0 + \beta_1 \Delta\gamma$ gives $\beta_0 = 1.2486$, $\beta_1 = 0.0872$, $R^2 = 0.0007$ and p-values of 0.7913 and 24.9586 respectively.

3. In Table 5, simulations with $p_{it} > p_{ft}$ must be read with caution. As explained in section 4, the relevant price ratio is $\frac{p_{it}}{p_{ft}}(1-\tau)$, which is less than unity in all cases.

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TABLE 1

PERU

Parameters							Economic Indicators		Target
Period	β	ϕ	α	s	b	M_t	τ	$\gamma^{12}-1$	R
1 Month	.997	1.2	.5	1	$\frac{6.8}{23.9}$	1	12.71%	2.24%	44%

TABLE 2

CALIBRATION OUTCOMES

η	1		.85	
	Peru	US	Peru	US
$\frac{b_i}{b}$	.3351	.0897	.2999	.0773
$\frac{s_i}{s}$	.0920	.0253	.3699	.1303
$\frac{m_{it}}{m_t}$	.5285	.1609	.2680	.0590
$\mathcal{B}_i$	.9824	.9947	2.0820	2.4335
$\mathcal{B}_f$	2.1908	1.9400	1.7786	1.8202
q_i	.3921	.3913	.3972	.3975
q_f	.2014	.2247	.4528	.5060
R_I	1.9466	1.7411	.8773	.7855
R	.4400	.0880	.4400	.0880
$\frac{p_{it}}{p_{ft}}(1-\tau)$	.9973	.9974	.8503	.8506
Q^g	.0001	.0001	.0001	.0002
G	.0107	.0122	.0205	.0261
		In percentages		
$\frac{Q^g}{G}$	.39	.66	.58	.60
$\frac{G}{b_f \mathcal{B}_f q_f}$	12.78	10.8	12.76	10.79
$\frac{G}{b_f \mathcal{B}_f q_f + b_i \mathcal{B}_i q_i}$	8.88	9.93	8.86	9.92

Table 3

Peru: Optimal Policy

η	1	.85
	%	%
$\widetilde{\gamma}^{12}-1$	42.69	1.48
$\widetilde{\tau}\left(\widetilde{\gamma}\right)$	9.98	12.47
$\%\Delta c^{\gamma}$	-14.09	-.049
$\frac{Q^g}{G}$	9.18	.266
$\frac{Q^g}{b_f\mathcal{B}_f q_f+b_i\mathcal{B}_i q_i}$	.78	.023
R	28.53	42.72
Matching Rates (not in %)		
B_f	2.0501	1.7701
B_i	1.1592	2.1057
Inflation Data	%	
1976-1995 (average)	525	
2005	1.6	
Optimal Inflation by:	%	
Nicolini (1998)	14.95 to 3.54	
Koreshkova (2006)	60	

TABLE 4

COMPARISON OF MODELS

	Centralized	**Decentralized**
Price Ratio		
$\frac{p_{it}}{p_{ft}}$	$\frac{\eta}{1-\tau}$	$\left[\frac{\gamma-\beta\left(1-\mathcal{B}_f\right)}{\gamma-\beta(1-\mathcal{B}_i)}\right]^{1-\frac{1}{\phi}}$
Quantity Ratios		
R_I	n.a	$\left[\frac{\gamma-\beta\left(1-\mathcal{B}_f\right)}{\gamma-\beta(1-\mathcal{B}_i)}\right]^{\frac{1}{\phi}}$
R	$\left[\frac{\eta}{1-\tau}\right]^{\frac{1}{\phi-1}}$	$\frac{s_i}{s_f}\left[\frac{\eta}{1-\tau}\right]^{\frac{\alpha}{1-\alpha}} R_I^{\frac{1}{1-\alpha}}$

TABLE 5

SENSITIVITY ANALYSIS

	$\eta = 1$							$\eta = .85$						
ϕ	2		1.2		1.2		1.2	2		1.2		1.2		1.2
α	.5		.75		.9		.5	.5		.75		.9		.5
b	$\frac{6.8}{23.9}$		$\frac{6.8}{23.9}$		$\frac{6.8}{23.9}$		1	$\frac{6.8}{23.9}$		$\frac{6.8}{23.9}$		$\frac{6.8}{23.9}$		1
	USA	Peru	USA	Peru	USA	Peru	Peru	USA	Peru	USA	Peru	USA	Peru	Peru
$\frac{b_i}{b}$	.0897	.3351	.0897	.3351	.0897	.3351	.3351	.0773	.2999	.0773	.2999	.0773	.2999	.2999
$\frac{s_i}{s}$	.0590	.2267	.0069	.0204	.0001	.0002	.0939	.0925	.3227	.2103	.4451	.5981	.6713	.3691
$\frac{m_{it}}{b_i}$	4.3064	4.1718	6.2719	5.5243	6.2481	5.5082	1.5669	3.2113	3.3859	2.6872	3.1427	2.6911	3.1443	.8949
$\frac{m_{it}}{m_t}$	.1099	.3978	.1601	.5268	.1595	.5252	.5251	.0706	.2890	.0591	.2682	.0592	.2683	.2684
p_{it}	8.6430	8.3700	16.193	14.244	16.261	14.340	4.0815	6.4393	6.7888	6.7888	7.9467	6.8259	7.9802	2.2761
p_{ft}	7.7211	7.3116	14.507	12.482	14.579	12.579	3.5806	6.7610	6.9708	7.1226	8.1563	7.1597	8.1898	2.3357
$\mathcal{B}_i$	1.5201	1.5418	.7210	.6805	.5920	.5356	.5295	2.0502	1.9445	1.7583	1.5112	1.3914	1.2291	1.1094
$\mathcal{B}_f$	1.9061	2.0219	1.3994	1.5085	1.1446	1.1812	1.1674	1.8593	1.8441	1.3170	1.2919	1.0435	1.0514	.9493
c_i	.0193	.0733	.0071	.0252	.0058	.0196	.0681	.0225	.0828	.0153	.0510	.0121	.0413	.1308
c_f	.1960	.1453	.0723	.0499	.0589	.0389	.1347	.2280	.1641	.1552	.1011	.1223	.0819	.2590
R_I	1.1194	1.1448	1.7327	1.9351	1.7261	1.9252	1.9246	.9524	.9739	.7866	.8779	.7877	.8785	.8788
R	.0880	.4400	.0880	.4400	.0880	.4400	.4400	.0880	.4400	.0880	.4400	.0880	.4400	.4400

Figure 1

Timing of Events

t			$t+1$
Decisions	Markets Open	Markets Close	Pooling
$\longrightarrow$	$\longrightarrow$	$\longrightarrow$	$\longrightarrow$
b_{jt}, m_{jt}	Buyers $\rightarrow \frac{m_{jt}}{b_{jt}}$	Taxes Paid	Consumption
m_{t+1}	Match, Bargain	Govt. Purchases: Q_t^g	
Terms of trade	Produce, Trade		

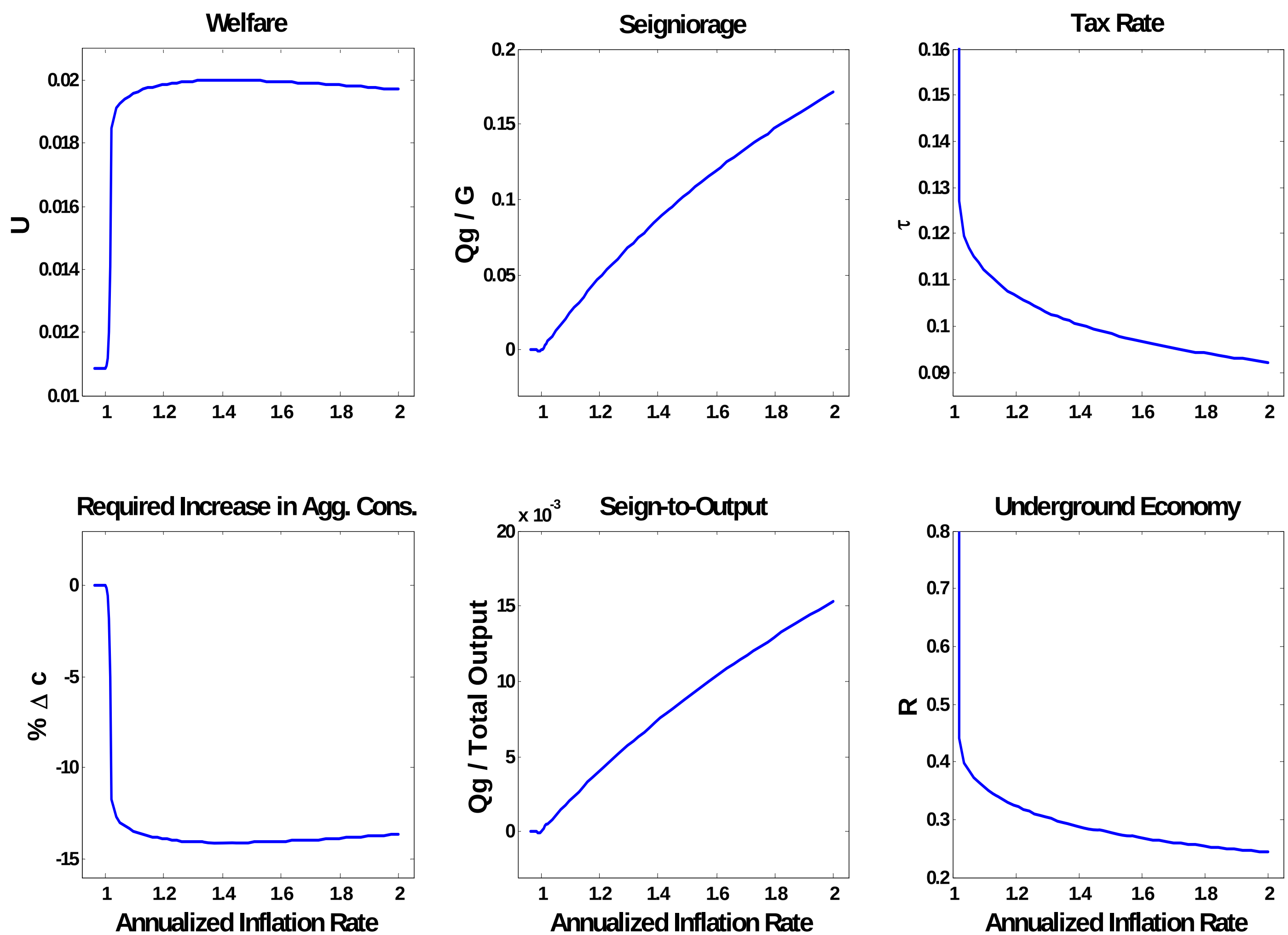


Figure 2: Peru ($\eta = 1$)

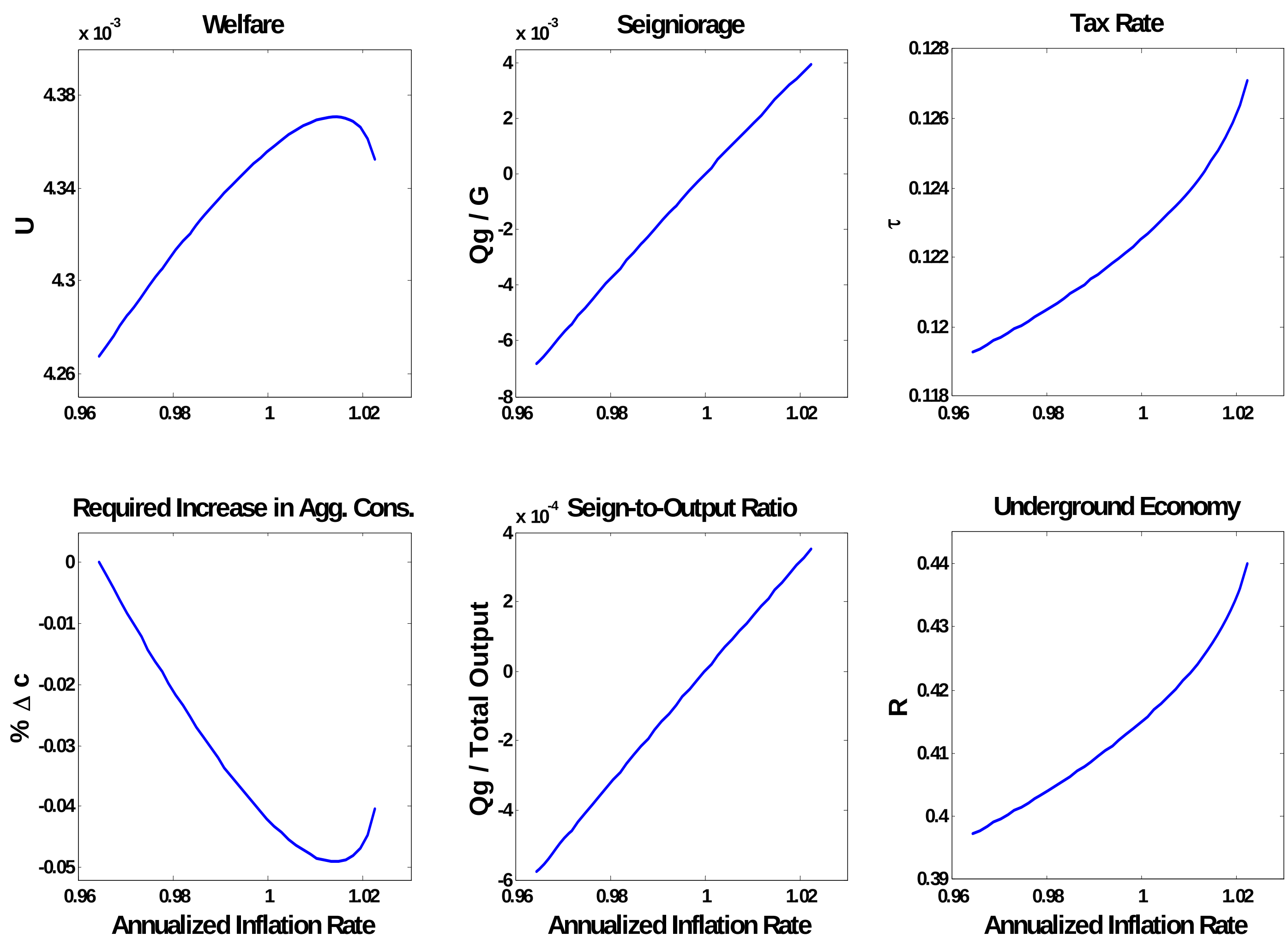
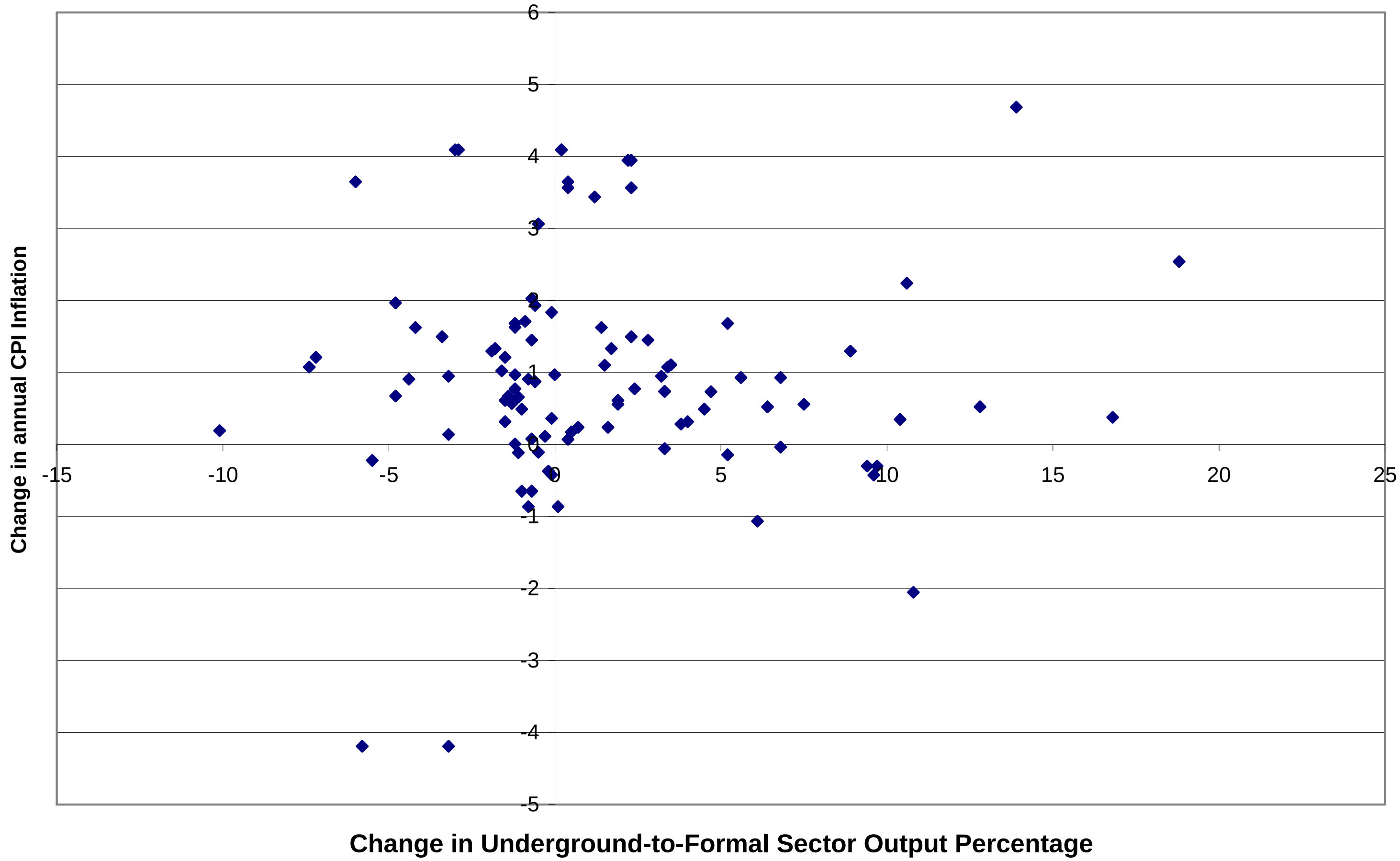


Figure 3: Peru ($\eta = .85$)

Figure 4: Change in Inflation and Change in Underground Economy



See point 2 on page 42 for comments on countries associated with these data points.

Figure 5: Inflation and the Underground Economy

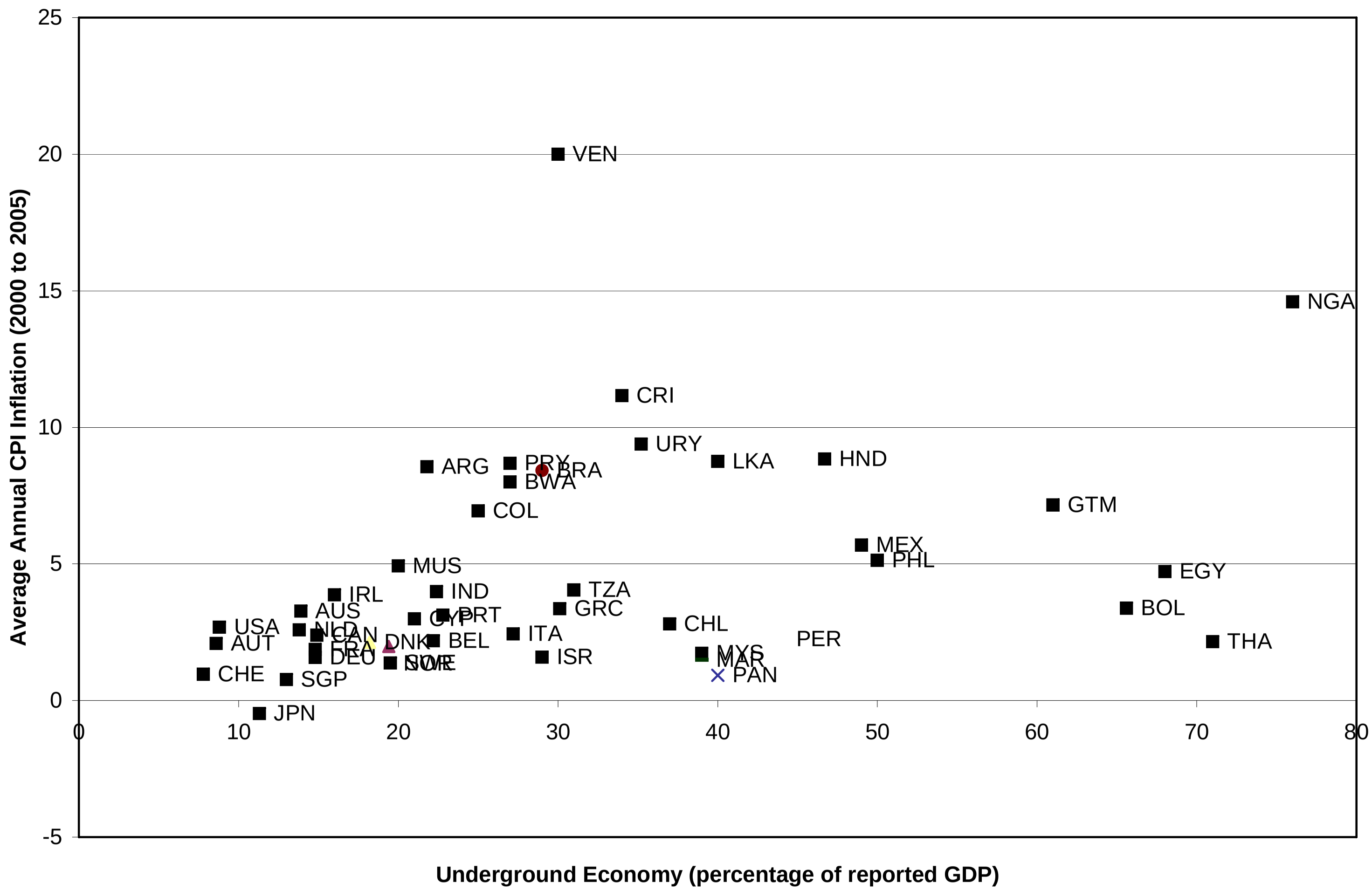


Figure 6: Government Spending and Underground Economy

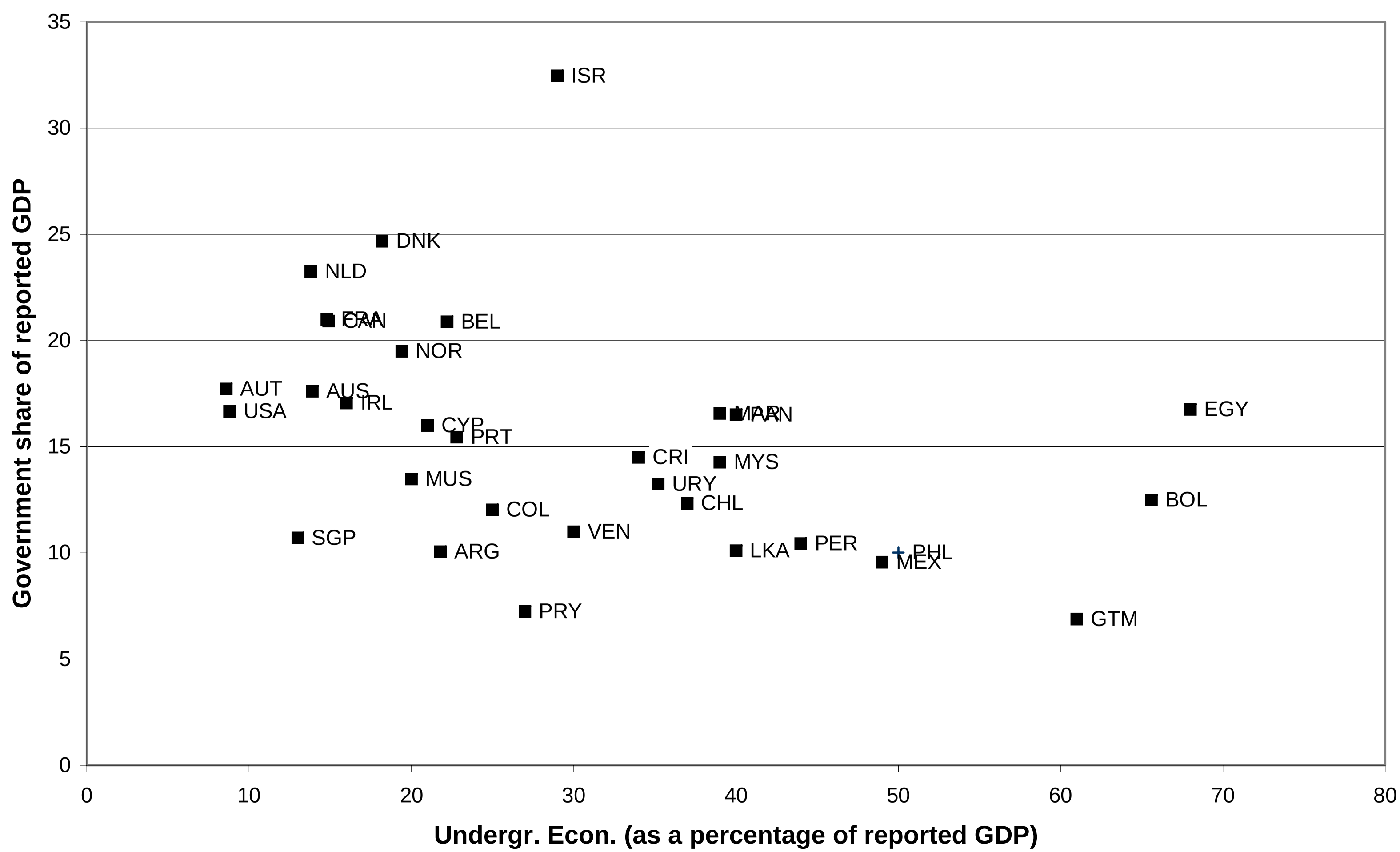


Figure 7: Taxation and the Underground Economy