



Munich Personal RePEc Archive

Occupational Choice and Dynamic Indeterminacy

Mino, Kazuo and Koji, Shimomura and Wang, Ping

Institute of Economics, Kyoto University

November 2003

Online at <https://mpra.ub.uni-muenchen.de/17063/>

MPRA Paper No. 17063, posted 02 Sep 2009 14:23 UTC

Occupational Choice and Dynamic Indeterminacy*

Kazuo Mino[†], Koji Shimomura[‡] and Ping Wang[§]

November 2003

Abstract

This paper constructs a two-sector model of two-period lived overlapping generations with endogenous occupational choice where ability-heterogeneous agents choose whether to become educated when young and henceforth to become skilled when old. We show that endogenous occupational choice in this two-sector framework can result in dynamic indeterminacy without complicate preferences/technologies and without requiring the consumption-good production to be more capital-intensive.

JEL Classification Numbers: D90, J24, O41

Keywords: occupational choice, overlapping generations, indeterminacy of equilibrium

Correspondence: Ping Wang, Department of Economics, Vanderbilt University, Nashville, TN 37235; 615-322-2388 (tel); 615-343-8495 (fax); ping.wang@vanderbilt.edu (e-mail)

*We have benefitted from insightful comments and suggestions from Michele Boldrin, Greg Huffman, Takashi Kamihigashi, Ryoji Ohdoi, Danyang Xie, and Midwest Economic Theory Meetings and the Society for Economic Dynamics Meetings. The third author acknowledges financial support from the Institute of Economics and Business Administration of Kobe University that enables this international collaboration. The usual disclaimer applies.

[†]Faculty of Economics, Osaka University

[‡]Research Institute for Economics and Business Administration, Kobe University

[§]Vanderbilt University and NBER

1 Introduction

Previous studies have established that dynamic indeterminacy can occur in two-sector overlapping-generations (OLG) models with complicate preferences/technologies or the consumption-good sector more capital-intensive than the investment-good sector. In this paper, we argue that if agents select their occupation over skilled versus unskilled jobs according to their learning ability, a two-sector OLG economy may exhibit dynamic indeterminacy under simple preference and technology specifications, regardless of the factor-intensity rankings.

Specifically, we analyze a dynamic general-equilibrium model of occupational choice in a two-sector overlapping-generations model populated with two-period lived agents. In our economy, all agents are endowed with one unit of labor over the entirety of their lifetime. They are identical in every respect except for the ability to learn. It is assumed that the young who desire to be educated must self-finance the schooling expenses. As a result, those with higher ability (or lower disutility in schooling) borrow when young to accomplish higher education and become skilled workers when old. For simplicity, we assume that agents only consume in the second period, which implies forced savings in goods. Thus, under a positive interest rate, those with lower ability (higher disutility) only work when young, whose savings facilitate both education and physical capital investments. Endogenous educational choice, therefore, gives rise to endogenous occupational choice between borrowers (the to-be skilled) and lenders (the unskilled). Without loss of generality, it is assumed that the ratio of the productivity of the skilled to that of the unskilled is a constant exceeding one and that the production technologies of both the consumption and the investment good sectors exhibit capital-skill complementarity.

By use of the analytical framework described above, we show that equilibrium dynamics with occupational choice may feature multiple converging transition paths so that extrinsic uncertainty will affect the dynamic behavior of the economy. In particular, even with linear preferences, constant-returns Cobb-Douglas production technologies and a perfect credit market, we show that dynamic indeterminacy may arise, featuring a one or two-dimensional stable manifold in a system with two jump variables. This result holds regardless of the factor-intensity rankings. Accordingly, occupational choice is the sole source creating dynamic indeterminacy, as long as the two goods are not homogeneous. Particularly, we show that by allowing occupational choice in our two-sector framework, we add an additional choice variable which contributes to stabilize capital

adjustments, due to capital-skill complementarity and the associated changes in net savings. As a result, the conventional condition on the factor intensity ranking is no longer needed for local indeterminacy.

Related Literature

Our paper contributes to the study on complex dynamics in the overlapping generations models. The majority of this literature has derived complex dynamics such as indeterminacy, cycles and chaos depending either on the complicated preference structure or on the specific restrictions on the production technologies. In the context of OLG models with capital accumulation, for example, Boldrin (1992) demonstrates that indeterminacy arises in the presence of external increasing returns, while Reichlin (1986) finds that if labor-leisure choice is allowed in a complex preference structure, indeterminacy and chaotic equilibrium may emerge under constant returns technology.¹ Michel and Venditti (1997) consider an OLG model involving a utility function that is nonseparable over the life cycle and show that dynamic indeterminacy can occur under the standard neoclassical technology. The most related papers are the two-sector OLG frameworks with constant-returns technologies by Galor (1992) and Reichlin (1992). They demonstrate that indeterminacy may occur if the consumption-good sector uses capital more intensively than the investment good sector.² In contrast to all previous OLG studies, dynamic indeterminacy can emerge in our paper even under linear preferences and constant-returns Cobb-Douglas production technologies, without relying on any specific factor-intensity rankings. As a consequence, local indeterminacy in our model mainly stems from the endogenous occupational choice of the young – this alternative source of indeterminacy contributes to the existing literature.³

¹In the exchange economy models with overlapping generations, the possibility of endogenous fluctuations entirely depends on the forms of the utility functions; see, for example, Benhabib and Day (1982) and Grandmont (1985). Jullien (1986) shows that the standard Diamond economy will exhibit cyclical behavior if there are multiple state variables (money and capital).

²While Galor (1992) assumes that both production sectors have the standard neoclassical production functions, Reichlin (1992) examines a two-sector model in the absence of factor substitution.

³The literature of occupational choice within the dynamic general-equilibrium framework includes Banerjee and Newman (1993), Aghion and Bolton (1997), Fender and Wang (2003), and many cited therein. This literature builds upon a one-sector framework in which the dynamic properties of the equilibrium have not been completely characterized.

2 The Model

Consider a two-sector model where sector 1 manufactures the investment good (regarded as the numeraire) and sector 2 produces the consumption good. The economy is populated with two-period overlapping generations. There is a continuum of individual agents of unit mass within each generational cohort, who are identical in every respect except for their ability or disutility of acquiring education. Individual agents do not value leisure and consume only in the second period of their lifetime. They have no initial wealth, but each is endowed with one unit of time which can be supplied in one of the two periods of their lifetime as a production input. In the absence of altruism, the utility function is simply assumed linear in consumption when old.⁴

Agents are heterogeneous *ex ante* only in their disutility costs incurred in acquiring education driven by their innate abilities. Consider a particular agent born at time $t - 1$, whose consumption occurs at time t . We denote the disutility costs by α_{t-1} (in units of the numeraire investment good), which are assumed to be uniformly distributed, i.e., $\alpha_{t-1} \in U[-\epsilon, \epsilon]$ with $0 < \epsilon < \infty$. Thus, a more able person will have less disutility from acquiring education. We assume a pecuniary education cost of $\eta \geq \epsilon > 0$ per person (in units of the investment good), where $\eta \geq \epsilon$ ensures that no one will undertake education for fun. Since an individual agent is not endowed with an initial wealth nor provided with a bequest, the pecuniary cost of education must be financed by borrowing against his/her future income. We assume throughout the paper that the credit market is perfect and hence an individual agent who wishes to borrow is always granted with the education loan.

Denote the market (real) interest rate from period $t - 1$ to t as r_t and the corresponding gross rate of interest as $R_t \equiv 1 + r_t$. Given linear utility in second-period consumption, an individual agent born at time $t - 1$ will undertake education to become skilled when old if the benefit from earning a high-skilled wage in the second period, $w_{H,t}$, net of the disutility cost of education, α_{t-1} , outweighs the costs from the sum of the foregone earnings for an unskilled job in the first period, $R_t w_{L,t-1}$, and the (interest payment included) costs of becoming educated, $R_t \eta$. Thus, the optimal schooling decision can be expressed as to undertake education whenever:

⁴The assumption of one unit lifetime-endowment of labor is innocuous. The structure of two-period lived agents who consumes only when old implies a one-to-one relationship between educational choice and occupational choice (borrowers versus lenders). We will discuss the implications for relaxing this assumption in the concluding section.

$w_{H,t} - \alpha_{t-1} \geq R_t(\eta + w_{L,t-1})$. Under proper assumption (to be specified later), there is a critical value of α_{t-1} , denoted α_{t-1}^* , at which an agent is indifferent between undertaking education and remaining unskilled:

$$w_{H,t} - \alpha_{t-1}^* = R_t(\eta + w_{L,t-1}). \quad (1)$$

Should there exists such a critical point, agents of type $\alpha_{t-1} \in [-\epsilon, \alpha_{t-1}^*]$ will undertake education and become skilled (type H) in the second period and those with $\alpha_{t-1} \in (\alpha_{t-1}^*, \epsilon]$ remain uneducated and work as unskilled (type L) in the first period. This gives rise to an endogenous occupational choice under which a nontrivial fraction of agents become borrowers (the educated) and the remainder become lenders (the unskilled). Notably, although those who decide to work as unskilled when young save the entire wage income (forced saving), such a saving decision is endogenously determined by occupational choice.

Within this stylized framework, it is assumed that skilled and unskilled workers are fractional substitutes with one unit of skilled labor equivalent to $\delta > 1$ units of unskilled labor. Thus, letting $\ell_{i,t}$ represent aggregate employment of type i worker in period t ($i = L, H$), the aggregate “effective labor” can be expressed as: $N_t = \ell_{L,t} + \delta\ell_{H,t}$. Free mobility of labor between the two sectors implies that the unskilled and skilled wage rates can be expressed as:

$$w_{H,t} = \delta W_t = \delta w_{L,t}. \quad (2)$$

Denote K_t as the amount of capital available at the beginning of period t . Assume competitive factor markets and 100 percent depreciation of the capital stock. Both consumption and investment goods are produced using labor and capital with Cobb-Douglas technologies that exhibit constant returns. Under the effective labor setup, it is easily seen that our production technologies feature capital-skill complementarity (as an increase in δ leads to a higher marginal product of capital).

Denote the relative price of the consumption good in units of the investment good as p . Then, competitive profit conditions that equate the price with the unit cost in each sector yield:

$$1 = R_t^{\theta_1} W_t^{1-\theta_1}, \quad (3)$$

$$p_t = R_t^{\theta_2} W_t^{1-\theta_2}, \quad (4)$$

where θ_i , $i = 1, 2$, are constant capital cost shares that take values between 0 and 1. Assuming that $\theta_1 \neq \theta_2$, we can solve (3) and (4) to obtain a unique pair of factor prices (R_t , W_t) for any

nonnegative p_t , that is,

$$R_t = R(p_t) \quad \text{and} \quad W_t = W(p_t). \quad (5)$$

The effects of the relative price of goods on factor returns depend crucially on the factor-intensity rankings. When the consumption good is produced using capital (labor) more intensively, a higher relative price of the consumption good results in a higher (lower) return on capital and lower (higher) wage rates for both the unskilled and the skilled. This is in fact a straightforward application of the Stolper-Samuelson theorem to a three-factor model with two factors that are fractionally substitutable (skilled labor and unskilled labor).

Let x_{t-1} denote the proportion of the generation born at time $t-1$ who become educated. It follows that in labor market equilibrium, we have:

$$\ell_{H,t} = x_{t-1}, \quad (6)$$

$$\ell_{L,t-1} = 1 - x_{t-1}. \quad (7)$$

It is straightforward to show that, with a uniform distribution of disutilities of education across the population, $x_{t-1} = (\alpha_{t-1}^* + \epsilon)/(2\epsilon)$, from which we obtain:

$$\alpha_{t-1}^* = -(1 - 2x_{t-1})\epsilon, \quad (8)$$

which provides a linear relationship between the proportion of the labor force which becomes educated and the critical value of disutility cost of education.

To close the model, we need to specify the goods market clearing conditions. The aggregate consumption of type i at time t is given by,

$$C_{H,t} = (w_{H,t} - R_t\eta)x_{t-1}/p_t, \quad (9)$$

$$C_{L,t} = w_{L,t-1}R_t(1 - x_{t-1})/p_t. \quad (10)$$

Utilizing (2), (5), (9) and (10), we can write the demand for the consumption good in period t as:

$$C_t^d = \{R(p_t)(1 - x_{t-1})W(p_{t-1}) + [\delta W(p_t)x_{t-1} - \eta x_{t-1}R(p_t)]\} / p_t. \quad (11)$$

Using the duality concepts, the supply of the consumption good at time t is equal to

$$C_t^s = R'(p_t)K_t + W'(p_t)(1 - x_t + \delta x_{t-1}). \quad (12)$$

Thus, the market-clearing condition for the consumption good at period t , $C_t^s = C_t^d$, implies:

$$\begin{aligned} & p_t [R'(p_t)K_t + W'(p_t)(1 - x_t + \delta x_{t-1})] \\ = & R(p_t)(1 - x_{t-1})W(p_{t-1}) + [\delta W(p_t)x_{t-1} - \eta x_{t-1}R(p_t)]. \end{aligned} \quad (13)$$

Finally, investment good market clearing is captured by:

$$K_t + \eta x_{t-1} = W(p_{t-1})(1 - x_{t-1}), \quad (14)$$

which equates the demand for loanable funds (the left-hand side) with the supply of loanable funds (the right-hand side).

3 Steady-State Equilibrium

We are now ready to define dynamic competitive equilibrium and non-degenerate steady-state equilibrium in our two-period OLG economy with endogenous educational choice.

Definition 1 *A dynamic competitive equilibrium (DCE) is a tuple of positive quantities $\{C_{H,t}, C_{L,t}, \ell_{H,t}, \ell_{L,t}, K_t, x_t\}$, a tuple of positive prices $\{w_{H,t}, w_{L,t}, R_t, p_t\}$ and a critical value $\alpha_t^* \in [-\epsilon, \epsilon]$, such that (i) schooling is optimal: type $\alpha_{t-1} \in [-\epsilon, \alpha_{t-1}^*]$ become educated and type $\alpha_{t-1} \in (\alpha_{t-1}^*, \epsilon]$ remain uneducated, where α_{t-1}^* satisfies (1); (ii) aggregate consumption of the skilled and unskilled are determined by (9) and (10), respectively; (iii) competitive profit conditions are given by (2), (3) and (4); (iv) allocation of labor across sectors and labor market equilibrium are given by (6), (7) and (8); (v) goods market equilibrium are achieved as in (13) and (14).*

Definition 2 *A non-degenerate steady-state equilibrium (NSSE) is a DCE represented by a tuple $\{C_H, C_L, \ell_H, \ell_L, K, x, w_H, w_L, R, p, \alpha^*\}$ with all variables being constant over time.*

In case when all variables take on their steady-state values, we drop time subscripts.

Although there are 11 endogenous variables in our system, the recursive nature of the model enables us to summarize the system in terms of the sequence of the fraction of the skilled and the relative price (i.e., $\{x_t, p_t\}$) alone. In particular, by combining the market clearing condition for both the consumption and investment goods (13) and (14) to eliminate the sequence of the capital stock $\{K_t\}$, we, on the one hand, have:

$$\begin{aligned} & p_t \{R'(p_t) [(1 - x_{t-1})W(p_{t-1}) - \eta x_{t-1}] + W'(p_t)(1 - x_t + \delta x_{t-1})\} \\ = & R(p_t)(1 - x_{t-1})W(p_{t-1}) + \delta W(p_t)x_{t-1} - \eta x_{t-1}R(p_t). \end{aligned} \quad (15)$$

Under the Cobb-Douglas production technologies, the elasticities,

$$\frac{p_t R'(p_t)}{R(p_t)} = -\frac{(1-\theta_1)}{\theta_1-\theta_2} \equiv \Theta_r \text{ and } \frac{p_t W'(p_t)}{W(p_t)} = \frac{\theta_1}{\theta_1-\theta_2} \equiv \Theta_w, \quad (16)$$

are constant. It is clear that if the consumption good is more capital (resp. labor) intensive than the investment good, i.e., if $\theta_1 >$ (resp. $<$) θ_2 , then $\Theta_r <$ 0 and $\Theta_w >$ 1 (resp. $\Theta_r >$ 1 and $\Theta_w <$ 0). Using Θ_r and Θ_w , we can rewrite (15) to obtain a “goods market equilibrium condition” (referred to as the *EE* locus):

$$(\Theta_r - 1)R(p_t) [(1 - x_{t-1})W(p_{t-1}) - \eta x_{t-1}] + \Theta_w W(p_t)(1 - x_t) = (1 - \Theta_w)\delta W(p_t)x_{t-1}. \quad (17)$$

On the other hand, we can substitute (2), (5), and (8) into (1) to derive an “optimal schooling relationship” (referred to as the *SS* locus):

$$\delta W(p_t) + \epsilon(1 - 2x_{t-1}) = R(p_t)[\eta + W(p_{t-1})]. \quad (18)$$

The *EE* and *SS* loci, (17) and (18), govern the dynamical system in the pair $\{x_t, p_t\}$. In the steady state, $x_t = x$ and $p_t = p$, which satisfy:

$$x = \left\{ 1 + \frac{[(1 - \Theta_w)\delta W(p) + \eta(\Theta_r - 1)R(p)]}{W(p)[R(p)(\Theta_r - 1) + \Theta_w]} \right\}^{-1}, \quad (19)$$

$$x = \frac{1}{2} + \frac{1}{2\epsilon} \{[\delta - R(p)]W(p) - \eta R(p)\}. \quad (20)$$

In general, an NSSE may not exist. However, if both ϵ and η are sufficiently small and if δ is above a critical value that depends on the values of elasticities θ_1 and θ_2 , then the steady-state equilibrium values of (x, p) are uniquely determined by (19) and (20). Substituting these values into (2), (3), (4), (6), (7) and (8), we obtain the steady-state equilibrium values of factor prices (w_H , w_L and R), labor demand for each type (ℓ_H and ℓ_L) and the critical value of the disutility cost of education (α^*). Finally, utilizing (9), (10) and (14), we obtain steady-state equilibrium consumption (C_H and C_L) and capital (K) in a recursive manner. We can establish:

Theorem 1 *The non-degenerate steady-state equilibrium of the dynamical system (17) and (18) exists and is unique, if both ϵ and η are sufficiently small.*

Proof: First, as ϵ converges to zero, the graph of (20) on the (p, x) -plane converges to the vertical line $p = p^e >$ 0, where p^e is the solution to

$$\frac{R(p)\eta}{W(p)} + R(p) = \delta. \quad (21)$$

Since both Θ_r and Θ_w are constant and either $\Theta_r < 0$ and $\Theta_w > 1$, or $\Theta_r > 1$ and $\Theta_w < 0$ holds, p^e uniquely exists that satisfies (21), irrespective of factor-intensity rankings. Substituting p^e into (19), we see that the steady-state x also uniquely exists in the open interval $(0, 1)$. Finally, we note that (21) implies that $0 < R(p^e) < \delta$ and that as η converges to zero, $R(p^e)$ converges to δ which is assumed to be greater than one. Therefore, it is guaranteed that $R(p^e)$ is greater than one as long as both ϵ and η are sufficiently small. Q.E.D.

That is, in order to ensure an *NSSE*, the dispersion of heterogeneity and the pecuniary costs of education cannot be too large whereas the productivity differential between the skilled and the unskilled cannot be too small. One may have thought that these assumptions are restrictive. Standard numerical analyses (summarized in Tables 1A and 1B), however, suggest that, over a large range of plausible parameter values, we can obtain a unique NSSE even by allowing ϵ to be as high as 0.2 (recall that ϵ must be bounded by η) and η to be as high as 0.3 (or more than 35 percent of the unskilled wage), where the value of δ can be taken from 1.25 to 3.5 (or for the skilled wage to exhibit a 25 to 250 percent markup over the unskilled wage).

It is interesting to examine that the effects of a more favorable educational environment (captured by either higher δ or lower η). For brevity, consider the simple case with ϵ converging to zero in which the steady state values of (x, p) can be solved in a recursive manner (i.e., p is determined by (21) alone). From (21), a more favorable educational environment leads to a lower (higher) relative price of the consumption good in units of the investment good if the consumption-good sector uses labor (capital) more intensively than the investment-good sector. Regardless of the factor intensity ranking, however, the return on capital always rises while the unskilled wage always falls in response to a favorable change in undertaking education.

4 Equilibrium Dynamics

We are now prepared to characterize the dynamic properties of the steady-state equilibrium. In order to gain intuition, let us start with two degenerate cases, one with no occupational choice and another with only one sector (homogeneous consumption and investment goods).

4.1 The Absence of Occupational Choice

Without occupational choice, we simply set $x_t = 0$ (together with $\epsilon = \eta = 0$). Then the dynamical system reduces to:

$$K_t = W_{t-1} \quad (22)$$

$$(\Theta_r - 1)\tilde{R}(W_t)W_{t-1} = -\Theta_w W_t, \quad (23)$$

where $\tilde{R}(W_t) = (W_t)^{-\frac{1-\theta_1}{\theta_1}}$ is derived from the competitive profit condition (3). Substituting (22) into (23) and manipulating it yield a single dynamic equation:

$$K_{t+1} = \left(\frac{1 - \Theta_r}{\Theta_w} \right)^{\theta_1} (K_t)^{\theta_1}. \quad (24)$$

Since $\theta_1 \in (0, 1)$ and $\frac{1-\Theta_r}{\Theta_w} > 0$ regardless of the factor intensity rankings, capital evolves monotonically and is stable. The existence and the uniqueness of a non-degenerate steady-state equilibrium can thereby be established. Moreover, as K_t is a state variable, we can conclude:

Theorem 2 *If the steady-state equilibrium of the dynamical system (22) and (23) exists, it is always locally determinate.*

The result suggests that endogenous occupational choice plays an essential role for the steady state to be locally indeterminate. The reader should be alerted that our framework differs from Galor (1992) because of the absence of endogenous saving decision once occupational choice is removed. Thus, in this degenerate case, the steady state is always locally determinate.

4.2 The Case of One-Sector Production

The next question to inquire is whether a steady state can be locally indeterminate when the economy degenerates to one sector (i.e., $\theta_i = \theta$ and $p_t = 1$). The competitive profit condition becomes: $1 = R_t^\theta W_t^{1-\theta}$, which can be used to express R_t as a decreasing and convex function of W_t :

$$R_t = \Omega(W_t) \equiv (W_t)^{-\frac{1-\theta}{\theta}}, \quad (25)$$

Moreover, within this one-sector framework, the capital-labor ratio is given by,

$$\frac{K_t}{(1-x_t) + \delta x_{t-1}} = -\frac{1}{\Omega'(W_t)}. \quad (26)$$

We can now rewrite the EE locus as:

$$\Omega'(W_t)[W_{t-1} - (\eta + W_{t-1})x_{t-1}] + [(1 - x_t) + \delta x_{t-1}] = 0, \quad (27)$$

whereas the SS becomes:

$$\delta W_t + \epsilon(1 - 2x_{t-1}) - \Omega(W_t)(\eta + W_{t-1}) = 0. \quad (28)$$

These two equations constitute the dynamical system for $\{x_t, W_t\}$. Finally, the loanable funds equilibrium (or the investment-good market clearing condition) is given by:

$$K_t + \eta x_{t-1} = W_{t-1}(1 - x_{t-1}), \quad (29)$$

which governs the evolution of the capital stock.

It is tedious but straightforward to show that if $\frac{1}{\theta} > \delta > \max\left\{\frac{\theta}{1-\theta}, 1 + \eta - \epsilon\right\}$, then there exists a unique non-degenerate steady-state equilibrium in which a nontrivial fraction of high-agents become unskilled workers and a nontrivial fraction of low-skilled agents undertake education and become skilled workers.⁵ To characterize the dynamics of this one-sector model of occupational choice, we begin by noting that under this one-sector framework, $\{x_t\}$ and $\{W_t\}$ are tied by a unique relationship (26), the factor market equilibrium relationship, for any t . More specifically, for historically given K_0 and x_{-1} , equation (26) implies:

$$K_0 = -\frac{1}{\Omega'(W_0)} [(1 - x_0) + \delta x_{-1}]. \quad (30)$$

It implies that once x_0 is chosen, the associated factor price W_0 is determined by (30). Thus, the system features only one free jump variable. If the steady state is a saddle point, we have a unique equilibrium path converging to the steady state.

Theorem 3 (Characterization of the Dynamics) *For sufficiently small ϵ or η , the steady-state equilibrium of the dynamical system (27) and (28) is locally determinate, featuring a unique one-dimensional saddle path.*

⁵This can be done by manipulating (27) and (28) in the steady state to obtain a single equation in terms of W :

$$F(W) = \frac{1}{2}\left(1 + \frac{\delta W}{\epsilon}\right) - \left[\frac{1 - \Omega(W)(1 - \theta)/\theta}{1 - \Omega(W)(1 - \theta)/\theta + \eta\Omega'(W) - \delta} + \frac{\Omega(W)(\eta + W)}{2\epsilon}\right] = 0.$$

We can then use the mean value theorem to prove the existence of a unique root of $F(W) = 0$ under the required conditions. The detailed proof is available upon request.

Proof: Totally differentiating (27) and (28), the Jacobian matrix of the one-sector model evaluated at the steady-state value of (x, W) is given by:

$$\tilde{J} = \begin{bmatrix} \frac{2\epsilon}{Z}[(1-x)W - \eta x]\Omega'' + Z & \frac{\Omega}{Z}[(1-x)W - \eta x]\Omega'' + (1-x)\Omega' \\ \frac{2\epsilon}{Z} & \frac{\Omega}{Z} \end{bmatrix}.$$

where $Z \equiv \delta - (\eta + W)\Omega' > \Omega > 1$ (noting that $\Omega' < 0 < \Omega''$). In the neighborhood of the steady state, $(1-x)W - \eta x = -[(1-x) + \delta x]/\Omega' > 0$. Straightforward manipulations yields the trace and the determinant:

$$\begin{aligned} Tr(\tilde{J}) &= \frac{2\epsilon}{Z}[(1-x)W - \eta x]\Omega'' + Z + \frac{\Omega}{Z} > 0, \\ Det(\tilde{J}) &= \Omega - \frac{2\epsilon}{Z}(1-x)\Omega' > \Omega > 1. \end{aligned}$$

Evaluating the characteristic function at $\{-1, 1\}$, we have:

$$\begin{aligned} \Lambda(-1) &= 1 + Tr(\tilde{J}) + Det(\tilde{J}) > 0 \\ \Lambda(1) &= -\frac{1}{Z} \{ (Z - \Omega)(Z - 1) + 2\epsilon[(1-x)W - \eta x]\Omega'' \}. \end{aligned}$$

Therefore, if ϵ is sufficiently small or if either η is not too large, then $\Lambda(1) < 0$ because $Z > \Omega > 1$. If this is the case, $\Lambda(\tilde{\lambda}) = 0$ has two positive real roots, $\tilde{\lambda}_1$ and $\tilde{\lambda}_2$, which satisfy $0 < \tilde{\lambda}_1 < 1 < \tilde{\lambda}_2$. As a result, the dynamical system holds a local saddle-point property and, since only one of $\{x_t, W_t\}$ can jump freely, the steady state is locally determinate. Q.E.D.

Although the above theorem relies on small values of ϵ or η , our numerical exercises reported in Table 2 suggest that even with fairly large values ϵ or η over a large range of plausible parameter values, the saddle-point stability property is quite robust. Therefore, despite endogenous occupational choice, dynamic indeterminacy in general cannot arise if the OLG economy features only one sector that produces a single homogeneous good.

4.3 The General Setup

We now turn to examining the stability properties in the general two-sector setup with endogenous occupational choice. To begin, we claim that given the initial value K_0 , both x_0 and p_0 can be chosen freely in our two-sector dynamical system. To see this, notice that at $t = 0$, equation (14) does not restrict the choice of x_0 and p_0 (because only x_{-1} and p_{-1} are involved).⁶ By expressing

⁶At $t = 0$, the behavior of the initial old is passive where the good market equilibrium condition and the optimal schooling relationship are not well-defined.

equations (14), (17) and (18) at $t = 1$, we have:

$$\begin{aligned} K_1 + \eta x_0 &= W(p_0)(1 - x_0), \\ (1 - \Theta_w)\delta W(p_1)x_0 &= (\Theta_r - 1)R(p_1)[(1 - x_0)W(p_0) - \eta x_0] + \Theta_r W(p_1)(1 - x_1), \\ \delta W(p_1) + \epsilon(1 - 2x_0) &= R(p_1)[\eta + W(p_0)]. \end{aligned}$$

Obviously, there are five endogenous variables, x_0 , p_0 , x_1 , p_1 and K_1 , implying that these three equations do not restrict the choice of x_0 and p_0 , either. Throughout $t = 2, 3, \dots$, this argument continues to hold true. In other words, x and p are “jump variables” whose initial values can be chosen freely.

Then, denote J as the Jacobian matrix of the linearized 2×2 dynamical system evaluated at the steady-state value of (x, p) and J_{ij} as the (i, j) 's element of J . Total differentiation of (19) and (21) gives:

$$\begin{aligned} J_{11} &= \frac{1}{\Theta_w W} \left[\frac{2\epsilon}{B} Q - (1 - \Theta_w)\delta W - (\Theta_r - 1)(\eta + W)R \right], \\ J_{12} &= \frac{1}{\Theta_w W} \frac{RW'}{B} [Q + B(\Theta_r - 1)(1 - x)], \\ J_{21} &= \frac{2\epsilon}{B}, \\ J_{22} &= \frac{RW'}{B}, \end{aligned}$$

where $Q \equiv (\Theta_r - 1)[(1 - x)W - \eta x]R' + [\Theta_w(1 - x) - (1 - \Theta_w)\delta x]W' > 0$ and $B \equiv \delta W' - (\eta + W)R' < (>) 0$ if $\theta_1 < (>) \theta_2$.

As ϵ converges to zero, so does J_{21} , implying that the characteristic equation converges to:

$$\Lambda(x) \equiv \begin{vmatrix} \lambda - J_{11} & -J_{12} \\ 0 & \lambda - J_{22} \end{vmatrix} = 0. \quad (31)$$

In view of (31), we find the following:

Theorem 4 *If a non-degenerate steady-state equilibrium of the dynamical system (17) and (18) exists, then for sufficiently small ϵ , it is locally indeterminate, regardless of the factor intensity rankings.*

Proof: From (31), as ϵ converges to zero, one root converges to J_{11} and another to J_{22} . Using

the definition of Θ_w and Θ_r and applying the steady-state SS locus, we can derive:

$$\begin{aligned} J_{22} &= \frac{\Theta_w p R W}{\delta \Theta_w p W - (\eta + W) \Theta_r p R} \\ &= \frac{\theta_1 R W}{\theta_1 \delta W - (\theta_2 - 1)(\eta + W) R} \\ &= \frac{\theta_1 W}{\eta + W} \in (0, 1). \end{aligned}$$

Thus, one root (denoted λ_1) is positive and within the unit circle. Similarly, taking $\epsilon \rightarrow 0$, we can manipulate J_{11} to obtain:

$$\begin{aligned} J_{11} &= \frac{-[(1 - \Theta_w)\delta W + (\Theta_r - 1)(\eta + W)R]}{\Theta_w W} \\ &= \frac{\theta_2 \delta W + (1 - \theta_2)(\eta + W)R}{\theta_1 W} \\ &= \frac{\delta}{\theta_1} > 1, \end{aligned}$$

implying another root (denoted λ_2) is positive but outside the unit circle. Since both x and p are jump variables, the dynamical system is locally indeterminate. Q.E.D.

When ϵ is not small, the underlying dynamic properties become too complicated to be characterized analytically. Accordingly, we perform numerical exercises to show the robustness of our results. Although we have examined many cases with various parameter values, we report the most informative ones in Tables 1A and 1B, for the two cases where the consumption-good production is more capital- or labor-intensive.⁷ In the benchmark cases, we select the skill markup as $\delta = 2.5$, and the ability differential and the pecuniary cost of education as $2\epsilon = \eta = 0.2$. The capital cost shares in the two sectors are chosen as 0.2 and 0.3, respectively. These parameter values give a steady-state fraction of the skilled as $x = 0.53$ (0.58) and the steady-state gross rate of interest as 2.01 (1.96) for the case where the consumption-good production uses capital (labor) more intensively. Again, we impose the constraints: $\eta \geq \epsilon$ and $\delta > R > 1$. Thus, we perturb 2ϵ from 0.1 to 0.4, η from 0.1 to 0.3 (with $\eta \geq \epsilon$) and δ from 1.25 to 3.5 (consistent with dynamic efficiency). Our results indicate that for these plausible parameters, the dynamical system is always locally indeterminate, with a positive stabilizing root and another positive root outside the unit circle.

⁷For example, we have experimented a wide range of $\{\theta_1, \theta_2\}$, with $|\theta_1 - \theta_2|$ as small as 0.001 and as large as 0.5, and found that our results are robust.

In summary, we have established the local indeterminacy property regardless of the factor intensity rankings, contrasting sharply with the stability condition obtained in the standard two-sector OLG model without occupational choice where the steady state may be indeterminate only when the consumption-good sector is more capital intensive than the investment good sector (cf. Galor 1992 and Reichlin 1992). A natural question arises: what are the underlying forces giving rise to dynamic indeterminacy?

From Theorems 2 and 3, it is clear that in a standard OLG framework without complicate preference/technology specifications, dynamic indeterminacy cannot arise if occupational choice is absent or if it features only one sector that produces a single homogeneous good. In the one sector case, there are lack of reinforcing forces in which the price dynamics (factor prices) are purely driven by the dynamics of the capital stock and the steady-state becomes a saddle. Under a two-sector setup with independent price dynamics, the conventional condition on the factor intensity ranking (for the consumption-good sector to be more capital intensive) ensures that not only the price but the quantity dynamics are stable in the absence of occupational choice.

By allowing occupational choice in such a two-sector overlapping-generations setting, we add an additional choice variable which contributes to stabilize capital adjustments. More specifically, due to capital-skill complementarity, an increase in the proportion of the population who are skilled raises the capital rental, which is a stabilizing force. Moreover, as the mass of the skilled increases, the aggregate costs of education increase whereas the loanable funds supply decreases. Both of these reduce net savings and hence capital investment, which again help stabilize capital adjustments. As a result, the conventional condition on the factor intensity rankings is no longer needed for local indeterminacy.

Finally, one may inquire whether the factor intensity rankings matter at all for dynamic adjustments. Let focus on the case with sufficiently concentrated distribution of ability (ϵ small). In this case, the local dynamics feature a one-dimensional stable manifold over two jump variables (which can be expressed as $x_t = \phi(p_t)$). Consider that, at a particular period, agents expect the long-run relative price of consumption to be higher (p increases). When the consumption sector is more capital intensive, the Stolper-Samuelson theorem implies that the returns to capital must go up more than proportionately whereas the returns to labor must go down. As a result, the capital stock rises and the occupational choice is in favor of being unskilled (x decreases). Thus, along the transition path, there will be downward adjustments in x associated with upward adjustments

in p (i.e., $\phi' < 0$). When the consumption sector is more labor intensive, the transition path will feature upward adjustments in both x and p (i.e., $\phi' > 0$). Summarizing, although the factor intensity rankings do not affect the stability properties, they matter for the configuration of the underlying dynamic adjustments. Moreover, the configuration of such dynamic adjustments in the relative price and occupational choice help justify why dynamic indeterminacy can arise, driven by self-fulfilling prophecies.⁸

5 Concluding Remarks

This paper has presented a two-sector overlapping-generations model with endogenous occupational choice where borrowing is required for investment in education. We have demonstrated that the economy may involve indeterminacy of converging paths, implying that expectations-driven, endogenous fluctuations can emerge. In our model the utility function is linear in consumption in the old age, the production technologies take the Cobb-Douglas functional forms with constant returns, and the credit market is perfect. Therefore, dynamic indeterminacy in our economy mainly stems from the presence of occupational choice behavior of the agents, though the two-sector structure is essential for the result.

A natural question to inquire is what if we generalize the borrower-lender relationship. In our benchmark economy, we have assumed that the agents to be educated are borrowers and the agents who do not plan to be educated are lenders. This is because all agents are assumed to be two-period lived and to consume only in their old age. If we consider instead three-period lived agents who consume in both the second and the third periods, there need not be a one-to-one relationship between educational decision (educated versus uneducated) and occupational choice (borrowers versus lenders). Such a generalization involves a more complicate preference structure and intertemporal reinforcing forces, thus increasing the likelihood of dynamic indeterminacy as one would expect.

⁸More precisely, updating the SS locus by one period and applying the stable manifold relationship, we get: $[\eta + W(p_t)]R(p_{t+1}) - \delta W(p_{t+1}) = \epsilon(1 - 2\phi(p_t))$. When the consumption sector is more capital intensive ($\theta_1 < \theta_2$), an expected increase in p_{t+1} (sunspot driven) raises the LHS. Since $\phi' < 0$, an raise in p_t can change occupational choice to restore the equilibrium relationship. Since $dp_{t+1}/dp_t > 0$, such an increase in p_t leads to an increase in p_{t+1} and hence the expectations are fulfilled. The case of $\theta_1 > \theta_2$ can be worked out by similar arguments, where $R' < 0$, $W' > 0$ and $\phi' > 0$.

References

- [1] Aghion, P. and Bolton, P. (1997), "A Theory of Trickle-Down Growth and Development", *Review of Economic Studies*, 64, 151-172.
- [2] Banerjee, A. and Newman, A. (1993), "Occupational Choice and the Process of Development", *Journal of Political Economy*, 101, 274-298.
- [3] Benhabib, J. and Day, R. (1982), "A Characterization of Erratic Dynamics in the Overlapping Generations Model", *Journal of Economic Dynamics and Control*, 4, 37-55.
- [4] Boldrin, M. (1992), "Dynamic Externalities, Multiple Equilibria, and Growth", *Journal of Economic Theory*, 58, 198-218.
- [5] Fender, J. and Wang, P. (2003) "Educational Policy in a Credit Constrained Economy with Skill Heterogeneity," *International Economic Review*, 44, 939-964.
- [6] Galor, O. (1992), "A Two-Sector Overlapping-Generations Model: A Characterization of the Dynamical System", *Econometrica*, 60, 351-386.
- [7] Grandmont, J. M. (1985), "On Endogenous Competitive Business Cycles", *Econometrica*, 53, 995-1045.
- [8] Jullien, B. (1986), "Competitive Business Cycles in an Overlapping Generations Economy with Productive Investment", *Journal of Economic Theory*, 46, 45-65.
- [9] Michel, P. and Venditti, A. (1997), "Optimal Growth and Cycles in Overlapping Generations Models", *Economic Theory*, 9, 511-528
- [10] Reichlin, P. (1986), "Equilibrium Cycles in an Overlapping Generations Model with Production", *Journal of Economic Theory*, 40, 89-102.
- [11] Reichlin, P. (1992), "Endogenous Fluctuations in a Two-Sector Overlapping Generations Models", in *Cycles and Chaos in Economic Equilibrium*, edited by J. Benhabib, Princeton University Press, 158-179.