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Abstract

This study examines the expectational stability of the rational expectation equilibria (REE) under Taylor rules when trend inflation is non-zero. We find that whether or not a higher (lower) trend inflation makes the REE more (less) unstable depends largely on the data (such as contemporaneous data, forecasts and lagged data) used in the conduct of monetary policy.

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1 Introduction

Many of the New Keynesian analyses have virtually neglected the existence of non-zero trend inflation. However, some point out that non-zero trend inflation greatly alters the implications for monetary policy making. Among them, Kiley (2007) and Ascsari and Ropele (2009) show that, under Taylor rules, the parameter region that satisfies the determinacy of the rational expectation equilibria (REE) substantially depends on the level of trend inflation. Since the determinacy is viewed as a normative requirement for monetary policy rules, their results suggest that the central bank should choose the policy rule parameters by correctly recognizing the relationship between the determinacy and level of trend inflation.

However, REE determinacy is not the sole requirement for the monetary rules. In the literature of adaptive learning, which is based on the framework of Evans and Honkapohja (2001), Bullard and Mitra (2002) propose the expectational stability (E-stability) of REE as another requirement for monetary policy rules. They compute the parameter regions that satisfy the E-stability as well as the determinacy of REE under alternative versions of Taylor rules (such as contemporaneous, forecast-based and lagged-based rules). Their analysis provides a benchmark in the literature, but their study focuses on a relatively specific environment in which the trend inflation is exactly equal to zero. An important extension is to generalize their work introducing non-zero trend inflation.

In this study, we examine the E-stability (as well as the determinacy) of REE in a New Keynesian model under non-zero trend inflation. In doing so, we pay attention not only to positive, but also to negative trend inflation. Although the existing studies have not examined the E-stability and determinacy of REE under negative trend inflation, we consider this issue an important one for recent monetary policy making because deflation has become a more pressing concern in the major developed countries. We show how the parameter combination of the Taylor rule coefficients that guarantee both E-stability and REE determinacy varies with the level of trend inflation.

2 The model

2.1 A New Keynesian model under non-zero trend inflation

Some previous studies, such as Kiley (2007), Sbordone (2007), Cogley and Sbordone (2008) and Ascari and Ropele (2007, 2009), provide alternative expressions of New Keynesian models under non-zero trend inflation. Among them, we employ the model of Sbordone (2007) and Cogley and Sbordone (2008), which is given as follows:

$$y_t = E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n), \tag{1}$$

$$\begin{aligned} \pi_t &= \kappa y_t + b_1 E_t \pi_{t+1} + b_2 \sum_{j=2}^{\infty} \phi_1^{j-1} E_t \pi_{t+j}, & (2) \\ r_t^n &= \rho_r r_{t-1}^n + \varepsilon_t. & (3) \end{aligned}$$

π_t is the percentage deviation of inflation from the (possibly non-zero) rate of trend inflation, which is assumed to be constant. y_t, i_t and r_t^n are the output gap, the nominal interest rate and the natural rate of real interest, respectively.

Since firms take into account the influence of trend inflation on their future relative prices, they are more forward-looking under non-zero trend inflation than under zero trend inflation. As a result, the third term of (2), which is absent under zero trend inflation, arises and thus the parameters such as κ, b_1, b_2 and ϕ_1 are affected by the level of trend inflation.[1] To see how non-zero trend inflation influences firms' forward-lookingness, it would be useful to check the values of b_1 and b_2 for alternative levels of trend inflation. Under our parameter values, (b_1, b_2) takes $(.968, -.009)$, $(.99, 0)$ and $(1.033, .017)$ for the rate of (annualized) trend inflation -1%, 0%, 2%, respectively. Thus, the sum of coefficients on inflation expectations increases with the level of trend inflation. We also find that the sign of b_2 is positive for $\bar{\Pi} > 1$ and negative for $\bar{\Pi} < 1$. This implies that the "additional forward-lookingness" stemming from the presence of non-zero trend inflation works in the opposite direction, depending on whether the trend inflation is positive or negative.

As for monetary policy rules, we introduce some versions of Taylor rules in which the central bank responds to (i) the contemporaneous data (y_t, π_t), (ii) the forecast ($E_t y_{t+1}$, $E_t \pi_{t+1}$), and (iii) the lagged data (y_{t-1}, π_{t-1}). The policy rule is generically given as

$$i_t = F_l Y_{t-1} + F_c Y_t + F_f E_t Y_{t+1} \tag{4}$$

where $Y_t = [y_t \ \pi_t]'$, $F_i = [F_{iy} \ F_{i\pi}]$ for $i = c, f, l$. c, f and l represent the contemporaneous rule, the forecast-based rule, and the lagged-based rule, respectively.

2.2 Adaptive learning

We assume that agents estimate the structural parameters by recursive least squares with decreasing gain, which is the most standard algorithm of adaptive learning (Evans and Honkapohja, 2001). The perceived law of motion (PLM) is generally given as

$$Y_t = A_t + C_t Y_{t-1} + D_t r_t^n, \tag{5}$$

[1] Cogley and Sbordone (2008) derive the parameters as follows: $\phi_1 = \alpha\beta\overline{\Pi}^{(\theta-1)}$, $\phi_2 = \alpha\beta\overline{\Pi}^{\theta(1+\omega)}$, $\chi = \frac{1-\alpha\overline{\Pi}^{(\theta-1)}}{\alpha(1+\theta\omega)\overline{\Pi}^{(\theta-1)}}$, $b_1 = (1 + (1+\omega)\theta\chi)\,\phi_2 - (\theta-1)\chi\phi_1$, $b_2 = (\theta-1)\chi(\phi_2 - \phi_1)$, $\kappa = \chi(1-\phi_2)$, where α is the probability of not changing prices, β is the discount factor, θ is the elasticity of substitution among different goods, ω is the responsiveness of real marginal cost to output, and $\overline{\Pi}$ is the trend inflation in gross term. The parameter values used in our numerical exercises follow those of Cogley and Sbordone (2008): $\alpha = .588$, $\theta = 9.8$ and $\omega = .429$. β and σ are set at .99 and 6.25, respectively.

where $a_{i,t}$ and $d_{i,t}$ are the i-th element of A_t and D_t, respectively, and $c_{ij,t}$ denotes the ij-th element of C_t. It follows that

$$\begin{aligned} E_t Y_{t+1} &= A_t + C_t Y_t + D_t \rho_r r_t^n \\ E_t Y_{t+2} &= (I + C_t) A_t + C_t^2 Y_t + (C_t D_t \rho_r + D_t \rho_r^2) r_t^n \\ E_t Y_{t+3} &= (I + C_t + C_t^2) A_t + C_t^3 Y_t + (C_t^2 D_t \rho_r + C_t D_t \rho_r^2 + D_t \rho_r^3) r_t^n, \\ &\vdots \end{aligned}$$

To eliminate the expectation terms from (2), we first express the infinite summation term $\sum_{j=2}^{\infty} \phi_1^{j-1} E_t Y_{t+j}$ as a function of Y_t and r_t^n:

$$\begin{aligned} \sum_{j=2}^{\infty} \phi_1^{j-1} E_t Y_{t+j} &= (1-\phi_1)^{-1} \phi_1 A_t + (1-\phi_1)^{-1} (I - \phi_1 C_t) \phi_1 C_t A_t + (I - \phi_1 C_t)^{-1} \phi_1 C_t^2 Y_t \\ &\quad + (I - \phi_1 C_t)^{-1} [\phi_1 \rho_r C_t D_t + (1 - \phi_1 \rho_r)^{-1} \phi_1 \rho_r^2 D_t)] r_t^n. \end{aligned}$$

The second element of this vector can be written as

$$\sum_{j=2}^{\infty} \phi_1^{j-1} E_t \pi_{t+j} = a_{3,t} + c_{31,t} y_t + c_{32,t} \pi_t + d_{3,t} r_t^n \tag{6}$$

By inserting (6) into (2), we obtain the following actual law of motion (ALM):

$$Q_t Y_t = S E_t Y_{t+1} + N i_t + U_t r_t^n + P_t, \tag{7}$$

where

$$\begin{aligned} Q_t &= \begin{bmatrix} 1 & 0 \\ -\kappa - b_2 c_{31,t} & 1 - b_2 c_{32,t} \end{bmatrix}, S = \begin{bmatrix} 1 & \sigma \\ 0 & b_1 \end{bmatrix}, N = \begin{bmatrix} -\sigma \\ 0 \end{bmatrix}, \\ U_t &= \begin{bmatrix} \sigma \\ b_2 d_{32,t} + 1 \end{bmatrix}, P_t = \begin{bmatrix} 0 \\ b_2 a_{3,t} \end{bmatrix}. \end{aligned}$$

It follows that

$$Y_t = \Gamma_t^{-1} [(S + N F_f) A_t + P_t + N F_l Y_{t-1} + ((S + N F_f) \rho_r D_t + U_t) r_t^n], \tag{8}$$

where $\Gamma_t \equiv Q_t - N F_c - (S + N F_f) C_t$. The T-maps from the PLM to the ALM are then given as

$$\begin{aligned} T(A_t) &= \Gamma_t^{-1} [(S + N F_f) A_t + P_t], & (9) \\ T(C_t) &= \Gamma_t^{-1} N F_l, & (10) \\ T(D_t) &= \Gamma_t^{-1} [(S + N F_f) \rho_r D_t + U_t]. & (11) \end{aligned}$$

Here, let $DT(A, C, D)$ be the Jacobian matrix of the T-maps evaluated at the corresponding RE values:

$$DT(A, C, D) = \frac{\partial vec(T(A), T(C), T(D))}{\partial (vec(A, C, D))'}.$$

Then the E-stability of REE can be attained if and only if all of the eigenvalues of DT have real parts less than one.

3 The E-stability under non-zero trend inflation

The combinations of the Taylor rule coefficients, $F_{i\pi}$ and F_{iy}, $i = c, f, l$, that ensure the E-stability and the determinacy of REE are presented in Figures 1, 2 and 3. In all figures, the upper-right panel corresponds to the case of zero trend inflation, which is equivalent to the situation analyzed by Bullard and Mitra (2002). The other panels show the E-stable and the determinate regions under non-zero trend inflation. Although the determinate regions (except for the case of negative trend inflation) are essentially the same as those presented by Ascari and Ropele (2009), the E-stable regions are novel in our study.

Our main finding is that the relationship between the E-stability of REE and the level of trend inflation largely depends on the specification of Taylor rule. Under the contemporaneous rule, the E-stable region, which is exactly the same as the determinate region, shrinks as the rate of trend inflation increases. Under the forecast-based rule, the E-stable region is broader than the determinate region. Nevertheless, the E-stable region is just the same as it is in the case of the contemporaneous rule, so that the implications about the E-stability of REE are the same for these two rules: higher trend inflation makes the REE more unstable. However, this is not necessarily true for the lagged-based rule because the relationship between the E-stability and level of trend inflation is more complex. When the responsiveness to output gap (F_{ly}) is below some threshold value (around 0.3 in our numerical example), then the higher trend inflation makes the REE more unstable. On the other hand, when F_{ly} is larger than the threshold, the E-stable region becomes broader for higher values of trend inflation. This finding is parallel to Ascari and Ropele's (2009) result that a higher trend inflation always narrows the determinate region under the contemporaneous rule and forecast-based rule but either narrows or broadens the determinate region under the lagged-based rule, depending on the value of F_{ly}.

Next, let us check the case of negative trend inflation. As in the cases of positive trend inflation, we find that whether or not a higher trend inflation makes the REE more likely to be E-stable and determinate depends on the versions of Taylor rules. Under the contemporaneous rule, the E-stable region, which is the same as the determinate region, is quite large under negative trend inflation. However, things are different in the cases of the forecast-based rule and the lagged-based rule. When using the forecast-based rule, the E-stable region is broad, but the determinate region is quite narrow under negative trend inflation. As a result, the REE under negative trend inflation is likely to be E-stable but indeterminate for a wide range of policy parameters. As for the lagged-based rule, the region in which both the E-stability and determinacy are satisfied is quite narrow, and the REE is likely to be explosive for wide combinations of the Taylor rule coefficients.

Therefore, when the trend inflation is negative, the central bank can more easily guarantee both the E-stability and the determinacy of REE by adopting the contemporaneous rule instead of the other two rules. Although the existing studies have not reported this point, we consider this property to have an important implication for modern monetary policy making because, in an environment of low trend inflation, the degree of freedom for the central bank to control the nominal interest rate is inevitably small due to the presence of the zero lower bound (ZLB) of the nominal interest rate. Our result suggests that as long as the central bank adopts the contemporaneous rule, the REE is likely to be E-stable and determinate under negative trend inflation even if the coefficients of the Taylor rule are relatively small. It means that the necessity of cutting interest rates against downward shocks is removed to some extent when trend inflation is negative. This will allay the fear of ZLB that the central banks have in an era of very low inflation.

4 Concluding remarks

We have shown that the relationship between the E-stability of REE and the level of trend inflation depends largely on the data used in the conduct of monetary policy. The REE is more likely to be both E-stable and determinate under the contemporaneous rule than under the other two alternative rules when the trend inflation is very low. This result implies that the availability of current economic data for the central bank is especially important in a low inflation environment.

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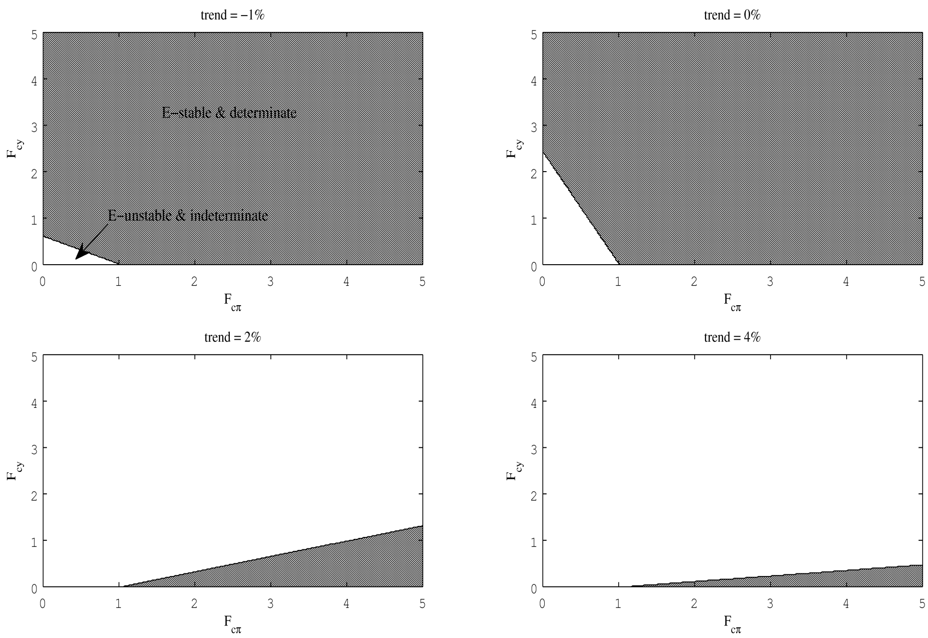

Figure 1: The E-stability and determinacy regions under the contemporaneous rule

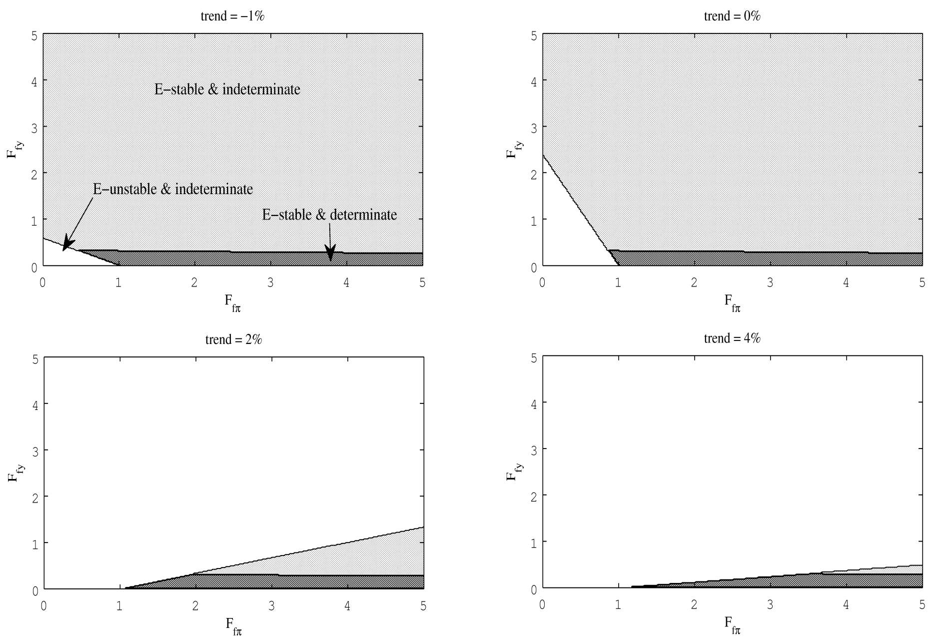

Figure 2: The E-stability and determinacy regions under the forecast-based rule

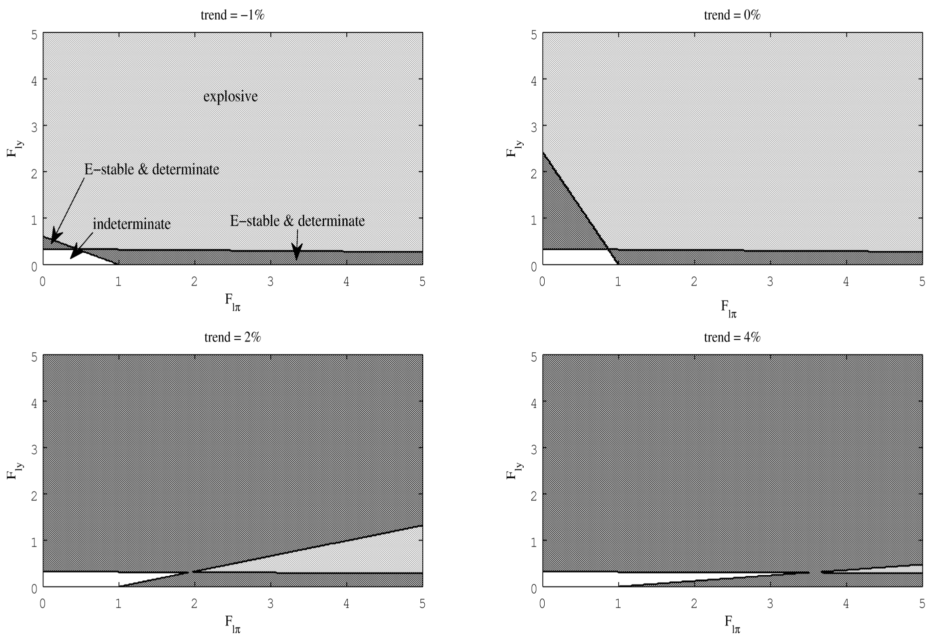

Figure 3: The E-stability and determinacy regions under the lagged-based rule