Stressing rating criteria allowing for default clustering: the CPDO case

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Stressing Rating Criteria Allowing for Default Clustering: the CPDO case *

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Abstract

After a brief review of the literature on rating arbitrage for corporate and structured finance, we introduce the standard criteria adopted by rating agencies to assess riskiness of Constant Proportion Debt Obligations (CPDO). Then, we propose a new rating model in order to incorporate a more realistic loss distribution showing a multi-modal shape, which, in turn, is linked to default possibilities for clusters (possibly sectors) of names of the economy. In this framework, we show that the riskiness of CPDOs is substantially increased leading to a decrease of their rating, and in particular, we found that the expected payout of the gap-risk option, embedded in CPDOs, is greatly enhanced.

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1 Introduction

It has been estimated that banks globally created at least $4 billion of CPDOs: structures with typically 10 year maturity, promising an annual interest of as much as 2 percentage points above money-market rates generated by a dynamical leveraged strategy that were usually granted AAA rating. CPDOs were first created by ABN Amro in 2006 but rapidly spread to the rest of the market.

Short term money-market rates could be taken to be close enough to risk free rates, as usually proxied by short term government yields, even though the recent credit crisis proved this assumption could be quite wrong at times. Securities rated AAA are considered as the next safest thing after government bonds.

As such it almost seems like the dynamic trading strategy beneath CPDO was delivering almost without risk a return of 2 percentage points above the risk free rate with a very high likelihood, more than 93.7% probability according to the 10 year minimum survival rate that S&P assigns to AAA rating and 10 year maturity.

As it turned out markets might not be as efficient as argued by some economists but nevertheless are far from being that inefficient. As it has been pointed out by Linden et al. (2007) the rating criteria assigned to these structures were bearing a significant model risk. Put differently, not all the rating criteria used by agencies were bullet proof.

Before analyzing in detail the rating criteria of CPDO we will see through a brief review of the literature, how the scope for rating arbitrage exists already for corporate bond ratings: the bread and butter of the rating agencies business.

With time agencies moved on to assign a rating to securitizations like Residential Mortgage Backed Securities (RMBS), SubPrime RMBS, Commercial Mortgage Backed Securities (CMBS) and Collateralized Debt Obligation (CDO). When assigning a rating to a securitization, agencies have to assess the likelihood of default of different part of the capital structure of a portfolio of credit references. This can still be considered a closely related business to their original core business of assessing the default risk of single credit references.

More recently, during the last 10 years, agencies started to assess the remoteness of the risk of structures whose payoff depends on risks associated to the mark-to-market dynamics of underlying credit portfolios. Thus an area, namely the modeling of the mark-to-market risk of a pool of credit, not so related to their original core business.

In particular we will focus our attention on CPDO rating criteria. We will enlarge the set of criteria stresses performed in Lindet et al. (2007) to include the possibility of cluster of defaults occurring to the pool of underlying names. To do so we will change the pool default simulation engine from a multifactor Gaussian copula to the Generalized Poisson Loss model as in Brigo et al. (2007a).

We will see how allowing for cluster of defaults hitting the pool of credit references the risk of an investor holding a CPDO structure increases dramatically, or equivalently the CPDO rating worsens. From the point of view of a bank arranging a CPDO we will also see how the gap-option reserve provision and capital allocation for the expected and unexpected loss respectively increases dramatically when allowing for cluster of defaults impacting the pool of credit references.

2 Ratings

Ratings were initially thought of as an investment guideline for unsophisticated corporate bond investors. With time their scope has enlarged as regulator set investment limits based on ratings, some investors are constrained to investment grade securities, or as they are used in covenants for financial contracts, for example a company might agree with its creditors to maintain its rating above a certain level.

To assign a rating to a corporate bond rating agencies rely on a series of quantitative variables, such as balancesheet leverage and profitability, and qualitative variables, such as management quality. The outcome is the rating: a letter grade ranging from AAA, highest quality, to C. The more remote the risk for an investor of not receiving in full the due interest and principal, the closest the corporate bond rating will be to AAA.

Cantor and Packer (1994) review the rating industry history and analyze the differences in the rating scales across agencies and across type of issues (corporate bonds vs structured finance). In their review of the historical evolution of the industry, the authors point out how initially ratings where issued free of charge and agencies revenues where linked to the sales of hard copies research material. With the introduction of cheap photocopying and pressures from corporates during the 1970 recession seeking ratings to reassure nervous investors, the rating industry started charging issuers a fee for the rating.

The apparent conflict of interest of agencies to assign an higher rating to please issuers to the expense of unsophisticated investors have until recently been limited by the agencies' opposite incentive to maintain an untarnished reputation. Recent event have tarnished the reputation at a particularly unfortunate time. In fact the use of ratings was expected to increase considerably with the advent of Basel II. During the recent crisis this prospective role has been questioned at times.

In front of these questions the Financial Stability Forum has instead been focused in taking advantage of the current crisis to propose an overhaul of the rating industry publishing a set of best practices, requesting agencies more disclosure about rating criteria and reducing their potential conflict of interest[1].

The authors compared also the ratings assigned by different agencies on a sample of issues that received a rating from multiple agencies. Issuers that received a near investment grade or a split rating (investment grade from one agency and speculative grade from another) are statistically more prone to seek a third rating. Also it appears that agencies disagree more often and by a greatest extent for speculative grade issuers than for investment grade issuers.

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[1]The conflict of interest rating agencies are facing is a serious issue as some analysts estimated that Moody's earned $884 million in 2006, or 43 percent of total revenue, from rating structured notes. See "Moody's, S&P Lose Credibility on CPDOs They Rated", 14 August 2007, Bloomberg News.

Thus overall the evidence presented by the authors goes in the direction of rating arbitrage activity on the part of the issuers. In fact despite the rating process being lengthy and time consuming for both the issuer and the rating agency, an issuer might have an incentive to shop around in view of a group of agencies with a systematically higher rating scale.

Finally the authors review the evolution of the structured finance rating for Mortgage Backed Securities (MBS) and Asset Backed Securities (ABS). In particular they correlate the evolution of the market shares of each agencies for MBS and ABS. The authors show how credit enhancements for MBS have gradually declined and also they point out how in a few instances agencies appeared to modify rating criteria as a response to declining market share.

In a later contribution, Cantor and Packer (1997) investigate further the equivalence of the rating scale across agencies. The higher rating assigned by agencies other than Moody's or S&P could in fact be due to self selection bias. The practice of Moody's and S&P is to assign a rating to any US corporate bond public issue. Other agencies instead assign only solicited rating. As such an issuer that has ground to believe that a third agency will not assign a better rating than Moody's and S&P will not ask for it. The results of their analysis is that self selection bias does not explain by itself the systematically higher rating given by third agencies.

More recently Becker and Milbourne (2008) analyze the effects of competition in the rating industry. They do so investigating the reactions of the two incumbents, Moody's and S&P, to the increased market share gained by Fitch. On one hand the increased competition make the outside threat more real: the loss of reputation resulting in a loss of business. On the other hand reputation is a valuable asset only if it produces future revenues: an increase in competition reduces future revenues thus reducing the incentive of maintaining a reputation.

The Securities and Exchange Commission (SEC) since 1975 has been limiting competition in the rating industry designating the Nationally Recognized Statistical Rating Organization (NRSRO). This was intended to put a limit on rating inflationary pressure in view of increased regulatory rating uses. The unintended consequence was to put a limit to competition creating an oligopoly.

The US congress, concerned with this limitation of competition, enacted the Credit Rating Agency Reform Act in 2006 to improve ratings quality for the protection of investors and in the public interest by fostering accountability, transparency, and competition in the credit rating agency industry. In particular the Credit Rating Agency Reform Act set forth objective criteria to obtain NRSRO recognition.

The authors measure the reaction of the two incumbents, Moody's and S&P, in the face of the increased market share by Fitch. They put forward three pieces of evidence that point to the increased competition actually reducing the quality of ratings.

1. Positive correlation between Moody's and S&P ratings and Fitch market share: Moody's and S&P reacted to increased competition issuing higher ratings on average.

2. Ratings and bond yields have become less correlated. In a regression of bond yields versus a set of bond characteristics including rating, the coefficient linking the bond yield and rating gets smaller when Fitch market share increases. Thus ratings are seen as less informative of an issuer default risk when competition between rating agencies is high.

3. Equity prices react more strongly to downgrades when Fitch market shares increased. Thus when ratings are less informative and more lenient the equity market reaction to a downgrade, and in particular a downgrade to speculative grade, becomes stronger.

2.1 Structured Finance Ratings

Structured finance deals can take a huge variety of shapes and forms. Here, we will only point to two basic features that are relevant for the scope of this article.

1. When rating most structured finance deals, as is the case for example for Arbitrage CDO, agencies have to assess the default risk of a tranche of a portfolio. Tranches are characterized by attachment and detachment pints. The owner of a tranche will keep on receiving the interest and will receive principal at maturity in full unless the portfolio losses reach the attachment point.

2. In some instances, as is the case for CPDOs, rating agencies need also to assess the mark-to-market risk of a pool of credit.

It can be argued that to assign a rating to a structured finance deal agencies have been, until recently, relying primarily on quantitative assessments as opposed to qualitative criteria. For each kind of structured finance deal rating criteria were published to communicate to the public the mechanics of the rating process.

The starting point of the rating criteria would be the quality of the underlying pool of reference credits for which both quantitative and qualitative considerations play a role. From the underlying credit quality through a series of assumptions as much as possible pinpointed to historical evidence eventually the structured finance deal would receive a rating. In doing so investors transparently received the maximum possible disclosure of the rating process. However rating agencies would take some model risk to the extent that the assumptions were not conservative enough or did not have enough historical data to back them.

Weaknesses in the rating criteria or more in general a misalignment between the perception of the rated risk between the agencies and the market could be reversed engineered, and possibly leveraged as for CPDOs, to provide investors with the highest possible spread for a given rating. Even though unsophisticated investors, the main users of ratings, received full disclosure of the quantitative assumptions to rate CDO, they might not have been able to judge their conservativeness.

Skreta and Veldkamp (2008) have been investigating the issue of rating arbitrage in CDO markets. They test the null hypothesis that rating model arbitrage does not exist testing for homogeneity of CDO characteristics across rating agencies and across rating methodology: if S&P and Fitch assign a rating based on the probability that the interest or principal of a tranche are missed, Moody's assign a rating focusing on the expected loss of a tranche.

Based on the significance of the mean equality test a set of multivariate discriminant analysis are performed. The conclusion is that there appear to be rating arbitrage in the sense that some CDO characteristics (in particular currency, maturity and seniority structure) appear to have a significant discriminatory power. If rating arbitrage was not existent then the CDO rated by the various agencies should not show any particular pattern.

2.2 Rating Arbitrage for Arbitrage CDO Rating

In a benign credit environment with steadily decreasing default rates and tightening credit spreads investors started to push the envelope seeking the highest possible spread within their investment guidelines limits. To this end the CDO technology became instrumental. An investor wishing to take exposure to a particular tranche of a pool of credit references with minimum requirements of diversification across sectors and rating classes will select the pool of names with the widest spread within each sector and rating category. In fact given the different risk assessment between the market and the rating agencies this strategy would in general provide the investor with the widest spread for the tranche.

Tejwani et al. (2007) present evidence that between 2004 and 2007 the constituents of the CDX series turned out to experience a faster than average rating downgrade. In their analysis the authors compare the rating action of the CDX.NA.IG S2 to S7 to the expected rating action as computed from the rating transition matrices. Across all CDX series from March 2004 to March 2007 the total downgrade notches minus the total upgrade notches is much above what would have been expected from the transition matrices.

One explanation could be the time inhomogeneity of transition matrices. In fact there is quite some evidence that transition matrices are not time homogeneous: there is evidence of rating momentum, see Altman and Kao (1992) and Lucas and Lonski (1992), and evidence that rating transitions differ depending on the credit cycle, see Nickell et al. (2000) and Bangia et al. (2002). The period the authors considered, March 2004 to March 2007, was a period of expansion though.

An alternative explanation could be the focus of Arbitrage CDO issuance activity on the names with the widest credit spread within each rating category. These in fact would be the names for which the market is betting on a faster rating deterioration.

2.3 Market Value CDO Rating

A bank that originates a pool of loans and keeps them in the balancesheet would need to allocate economic capital until the loans maturity. To free economic capital the bank could securitize the pool of loans via a balance-sheet CDO and keep only some parts of the capital structure.

In a balance-sheet CDO there is a waterfall in place that directs the flows generated by the pool of credit references towards the most senior part of the capital structure in case a set of Over Collateralization (OC) tests and/or Interest Coverage (IC) tests are not passed. This is intended to give senior investors assurances about the safety of their investments.

From the point of view of senior investors there remains the risk that even though currently both OC and IC tests are passed, news might have arrived to the market regarding a disastrous expected performance of the pool of credit. In a balance-sheet CDO senior investors will have to wait until the OC and/or IC tests are not passed before seeing the cash-flows of the collateral being directed to serve their notes first.

In a Market Value CDO instead when the market value of the collateral pool reaches some triggers, the pool cash-flows are directed to senior investors without having to wait the OC and/or IC tests to fail.

When rating Market Value CDOs agencies had to include in their criteria also assumptions regarding the market value dynamics of the collateral.

Market value CDOs were first introduced in the late 90s but never gained prominence in the securitization scene. Moreover in 2001 they also suffered a major drawback due to the CBO crisis. Since then they basically disappeared and never made a significant comeback, not even at the peak of the securitization activity in 2006.

2.4 Leveraged Super Senior Rating

A cash-flow CDO issuer often had the problem of placing the super-senior part of the capital structure. In fact the remoteness of its risk and the consequent meagre credit spread attached to it made super senior tranches particularly unattractive to yield seeking investors.

Leveraged Super Senior tranche appeared in the structured credit market to make the tranche spread attractive to investors. Being a leveraged structure the arrangers typically designed a set of triggers that depended on both the cumulative credit enhancement lost by the Super Senior tranche and the market spread for the underlying Super Senior tranche.

When triggers are reached the leveraged super senior exposure is unwound and the remaining proceeds, if any, are returned to the investor. A super-senior investor only need to worry that the collateral pool loss rate does not reach the super-senior attachment point. A leveraged super-senior investor instead needs also to worry about the mark-to-market of the underlying super-senior tranche. Indeed, when the mark-to-market triggers are reached the proceeds he will receive will be almost certainly below par.

Investors in order to buy the leveraged super-senior structure needed reassurances that the triggers were indeed remote. To accomplish this need rating agencies started to give ratings also to the Leveraged Super Senior triggers. They would give a AAA rating to the super-senior tranche: thus certifying the remoteness of the risk that the pool loss would reach the super-senior attachment. Then they would give a AAA rating to the leveraged super-senior triggers: thus certifying the remoteness of the risk that the mark-to-market triggers of the leveraged super-senior tranche would be reached.

2.5 Rating CPDO

One can see the usefulness for unsophisticated investors of CDO ratings given the extensive datasets of default history available to agencies in order to assess the riskiness of different part of the capital structure. An issuer wishing to sell in the market part of the capital structure of his pool of credits asks a rating agency for an independent risk assessment.

If rating have as their main users unsophisticated investors one can see also the need to rate Market Value CDO. Indeed, the main purpose of Market Value CDO is to give senior investors more comfort that the mark-to-market trigger activation represents a remote risk.

In the case of Leveraged Super Senior tranche we assist to an issuer that wishing to sell a bigger portion of the pool capital structure lures investors with attractive spreads based on the remoteness of the default risk of the underlying pool of credits and also based on the remoteness of the market value triggers. An investor should be quite sophisticated to assess these two different types of risk, default and mark-to-market risk, and their interaction as summarized in the trigger matrix. For this reason arranger turned to rating agencies to help them reassure investors of the remoteness of the mark-to-market risk embedded in the triggers.

CPDO can be considered the latest and more extravagant of the structured credit products that arrived at the end of the longest boom in the credit markets. The anecdotical justification of their existence is to allow institutional investors to take advantage of the mean reverting nature of credit spreads through a mechanic trading strategy. It might be argued that it would be strange for an institutional investor to take a leveraged long exposure to credit on the peak of the credit market. The devil's advocate would suggest an alternative explanation: maximize the spread paid by the notes, targeting a AAA rating, reverse engineering the criteria adopted by agencies to handle the mark-to-market risk of other structured deals.

3 CPDO

As for CPPIs, in CPDOs investors are dynamically selling protection, by leveraging the risk premium contained in credit spreads, see Hull et al. (2005), but with a different strategy. Indeed, in case the case of CPDOs if the spread widens we increase the leverage whereas in the case of CPPIs we decrease the leverage. In this way CPDOs try to exploit the mean reverting bahaviour that corporate spreads have historically displayed.

Rating agencies have introduced stylized criteria to rate CPDO structures. We briefly review in Section 4 the standard assumptions, as can be found also in Linden et al.

(2007). Then, in Section 5, we consider a different choice for the loss dynamics by introducing the General Poisson Cluster Loss model (hereafter GPCL).

The GPCL model leads to a multi-modal risk-neutral loss distribution, which allows us to consider the possibility of a simultaneous default of more than one underlying name. One might argue that the bumps, arising in the risk-neutral loss distribution, are there as a remuneration to bear the highly skewed risk of senior tranches. We will hint at the evidence found in literature, for instance Longstaff and Rajan (2006), that this bumps might be there also in the objective measure.

Our main point will be to show how the rating of a specific CPDO structure changes introducing a loss distribution with bumps. Also we will see how the gap option does indeed have a premium also in the objective measure when modeling the loss with the GPCL model.

Incidentally we note how the changes recently proposed by S&P to the rating criteria of CDOs require that the AAA attachment should be able to withstand the default, with recovery 0%, of the sector in the CDO pool with the highest weight[2].

3.1 Payoff of an hypothetical CPDO structure

Quite suspiciously the "flavours" of a CPDO structure are quite limited: institutional investors, the main investors in these structures, infact usually demand tailor made solutions to better fit their investment view.

We will present below the details of an hypothetical CPDO structure that is extremely similar to most of the CPDOs that were sold in the market in its main characteristics: interest paid, dynamical leverage rule, fees paid, maturity, roll of the leveraged exposure on the 5 year on-the-run series of the Globoxx[3].

A CPDO is a note bond paying the investor an interest until the earliest of:

- Maturity. Usually 10 years to give enough time to the structure to profit via the dynamic strategy from the mean reverting properties of the underlying credit derivatives index spread.
- Cash-in as defined in the section below.
- Cash-out. In case the cumulative loss of the dynamic strategy reach 90% the leveraged credit exposure is unwound and the proceed if any are given back to the investor.

The interest paid is equal to Libor plus a spread I_t varying depending on the leverage β_t at time t. Our hypothetical CPDO structure pays quarterly the following spread over Libor.

[2]See "Request for Comment: Update To Global Methodologies And Assumptions For Corporate Cash Flow CDO And Synthetic CDO Ratings", 18 March 2009, Standard & Poor's.

[3]The Globoxx is a credit derivative index whose constituents are the union of the constituents of the iTraxx and CDX: the two most popular credit derivative indices.

$$I_t = \begin{cases} 1.25\% & \text{if} \quad \beta_t \leq 7.5 \\ 1.5\% & \text{if} \quad \beta_t = 10 \\ 1.75\% & \text{if} \quad \beta_t = 12.5 \\ 2\% & \text{if} \quad \beta_t = 15 \end{cases} \tag{1}$$

An investor will give at inception 100% to the structure. The structure will initially sell leveraged unfunded protection on the on-the-run series of the underlying credit derivative index and will invest the cash in short term money market instruments.

Subsequently the leverage will be updated according to dynamical leverage rule and every six months the leveraged position will be rolled in the on-the-run 5 year Globoxx series. This strategy allows the structure to always execute the leverage adjustments, as outlined in section 3.1, in the most liquid index series, the on-the-run, and maturity, the 5 year.

All cashflows related to interest payments, fees paid, index losses paid, semi-annual index roll cost paid and protection premium received will be against the money market deposit. Thus at each point in time the net value (NAV) of the assets of the structure, V_t, will be the sum of the Euribor deposit and the unrealized mark-to-market of the leveraged credit derivative index position since the last roll date.

Dynamical Leverage rule

The leverage β_t increases with the maximum level reached by the spread of an underlying credit derivatives index S_t up to time t, unless the value of the assets of the structure V_t fall below 40% in which case the leverage is linearly reduced to 0 when $V_t = 10\%$.

$$\beta_t = \begin{cases} 7.5 & \text{if} \quad \max_{s\in[0,t]} S_s < 0.40\% \text{ and } V_t > 0.4 \\ 10 & \text{if} \quad 0.40\% \leq \max_{s\in[0,t]} S_s < 0.50\% \text{ and } V_t > 0.4 \\ 12.5 & \text{if} \quad 0.50\% \leq \max_{s\in[0,t]} S_s < 0.60\% \text{ and } V_t > 0.4 \\ 15 & \text{if} \quad 0.60\% \leq \max_{s\in[0,t]} S_s \text{ and } V_t > 0.4 \\ 7.5(V_t - 0.1)/0.3 & \text{if} \quad V_t \leq 0.4 \end{cases} \tag{2}$$

The reason to begin reducing the leverage when the V_t goes below 40% is to reduce the cost of the gap option. Indeed, CPDOs are sold to investors via so called Special Purpouse Vehicles (SPV). The SPV will buy gap-options to be protected in the event of negative unwinding proceeds in case of a cash-out.

Fees

The fees X_t of the structure are of three kinds:

1. (Management Fees) $X_t^{\mathrm{MF}} = 0.2\%$ per annum on the notional of the notes until the earliest of maturity and the cash-out date.

2. (Gap Fees) $X_t^{\mathrm{GP}} = 0.035\%$ per annum per unit of leverage until the earliest of maturity, the cash-in date and the cash-out date.

3. (Upfront Fees) $X^{\mathrm{UF}} = 1\%$ upfront.

The management and upfront fees represents a running remuneration for the arranger of the structure. The gap fee represents the cost of the gap option the structure will have to buy to cover the risks of a cash injection when liquiditating all risky exposures following a cash-out.

$$X_t = \left(X_t^{\mathrm{MF}} + \beta_t X_t^{\mathrm{GP}}\right) + 1_{\{t=0\}} X^{\mathrm{UF}} \tag{3}$$

Cash-In

In case the cumulative profit of the dynamic strategy reach the bond ceiling B_t the leveraged credit exposure is unwound and the proceeds are invested in a basket of risk free bonds that will guarantee the payment of the remaining fees X_s, interest plus spread and the principal at maturity.

$$B_t = 1 + \int_t^{T_I^{40}} (X_s + I_s) D(t,s) ds \tag{4}$$

where $D(t,s)$ is the risk-free discount rate at time s evaluated[4] at time t. The quarterly interest payment dates over the 10 years of the life of the product are denoted as $\{T_I^1, T_I^2, ..., T_I^{40}\}$.

From the definition of the payoff it is clear that the CPDO structure will not be able to pay the principal at maturity in full unless the profits of the dynamic strategy reach the bond ceiling B_t before maturity.

3.2 Evolution in time of the NAV of the CPDO structure

Having in mind the specific CPDO structure outlined in section 3.1, we will now recap the obligations of the CPDO structure and introduce the equation for the variation of its NAV.

At inception these are the events following the investors' subscription of the CPDO notes originated by a vehicle for a given notional of let us say 1 euro.

- The arranger typically takes an amount in form of upfront fee X^{UF} from the initial investment.

- The vehicles puts the notional minus the upfront fee, $1 - X^{\mathrm{UF}}$, in a short term deposit thus earning the risk-free short term rate: r_t.

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[4]If for simplicity we assume rates to deterministically evolve along the forward curve than the present value of the libor interest plus the principal at maturity is always equal to par.

- The vehicles sells protection[5] on the 5 year Globoxx index for a notional equal to the notional of the CPDO notes times the initial leverage: $\beta_0 = 7.5$ given the initial 5y Globoxx spread of 35 bps.

On any subsequent date the vehicle will:

- Receive libor on the cash invested.
- Pay libor plus spread on the notes notional to the investor.
- Pay the Loss Given Default (LGD) times the leverage for any name defaulted in the underlying credit index to the protection buyer.
- Receive the protection premium times the leverage from the protection seller.
- Pay the fees (management and gap-risk) to the arranger.

On the roll dates all short protection positions are rolled into the new on-the-run series. Thus the Mark To Market (MTM hereafter) accrued since the last roll, or increase in leverage, will have to be liquidated in cash.

Given the initial condition at inception of the NAV, $V_0 = 1 - X^{\mathrm{UF}}$, and the Globoxx 5 year CDS spread, $S_0 = 35$ bps, after inception the NAV process V_t is characterized by the following equation:

$$dV_t = V_t r_t dt - (F_t + I_t)dt - \beta_t dL_t + \left(S_{T_R^i} \beta_{T_R^i} + \int_{T_R^i}^{t} S_s d\beta_s \right) dt - X_t dt + dNPV_t^{T_R^i} \quad (5)$$

where F_t and r_t are respectively the last fixing of the 3-month Libor rate and the overnight rate, I_t is the spread over libor paid by the notes, $\beta_t > 0$ is the leverage of the protection sold, X_t is the fee contributions, NPV_t is the residual (from t to index maturity) Net Present Value (hereafter NPV) of the leveraged protection sold.

We have preferred not to consider explicitly the Cash Account. The only difference is the accrual in between roll dates due to the unrealized market to market, NPV_t, that differentiate the NAV from the Cash Account.

We also consider the recovery rate (R) to be constant in time and equal to 35%, so that the number of defaults C_t and the pool loss L_t processes are linked by the usual relationship $L_t = (1 - R)C_t$.

Let us call T_R^i the last roll date and T_I^j the last interest payment date of the underlying credit index[6].

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[5] For the sake of simplicity and given the negligible impact we will not consider the actual credit index trading conventions but will assume instead the same trading conventions as for single name CDS.

[6] As the roll dates are semiannual and the interest payment dates are quarterly we will either have $T_R^{j/2} = T_I^j$ if j is even or $T_R^{(j/2)-1} = T_I^j$ if j is odd.

The net present value $NPV_t^{T_R^i}$ of the protection sold since the last roll-date T_R^i is:

$$NPV_t^{T_R^i} = \left(S_{T_R^i}\beta_{T_R^i} + \int_{T_R^i}^{t} S_s d\beta_s \right) DV01_t + \beta_t DFLT_t$$

In computing the differential of $NPV_t^{T_R^i}$ we will make use of the following approximation that in fact holds true in case of constant and deterministic risk free rates r and default intensity λ.

$$\begin{aligned} DV01_t^T &= \int_0^{T-t} e^{-s(r+\lambda)} ds \\ dDV01_t^T &= -e^{-(T-t)(r+\lambda)} dt \end{aligned} \tag{6}$$

As soon as the NAV touches the bond-ceiling the entire risky exposure is unwound and the proceeds invested in a risk free Floating Rate Note (that pays libor plus the spread and gives the principal at maturity) and a series of zero coupon bonds that will match the management fees that need to be paid.

As soon as the NAV touches the cash-out threshold (10% in our case), all positions are unwound, the proceeds of the unwinding are given back to the investor, all interest payments on the notes since then on are discontinued and no principal will be given back at maturity.

4 Rating Criteria: Base Case and Stressed Case

From equation (5) we see that to simulate the NAV of a CPDO structure we need to simulate the evolution of interest rates, Globoxx index spread and Globoxx index losses.

Interest Rates Given the low sensitivity of the CPDO structure NAV to interest rates we will assume that the spot zero-coupon curve will move along the forward curve as time goes on.

Protection Premium We will model the Globoxx 5 year CDS spread as an exponential Vasicek (see section 4.1). In line with Linden et al. (2007) we will take as base case the mean parameter estimate and will take as Stressed Case the mean parameter estimates marked adversely up or down, depending on the rating sensitivity, by a quarter the standard deviation.

Pool Losses Following the standard assumptions of rating agencies we will model the number of defaulted names in the underlying pool with a multi-factor Gaussian copula (see section 4.2) in the Base Case. In the stressed case we will model the pool loss with the GPCL as in Brigo et al. (2007b) thus allowing for cluster of defaults.

Figure 1: Red (dark) line and blue (light) line are respectively the 5y Globoxx index and the proxied 5y Globoxx index. Source: Linden et al. (2007).

When rating a structure agencies have to assess the likelihood that the structure can pay in full interest and principal. In doing this they rely on quantitative criteria, as summarized by rating criteria, as well as on qualitative assessment, the rating in fact is delivered by a committee where different areas of expertise are contributing each its unique perspective on the transaction at hand.

When the committee is debating weather to assign the highest possible rating, AAA, the quantitative criteria are usually calibrated versus a stressed historical evolution: for example in case of structured finance related to mortgages the tranche AAA need to withstand some extreme, by historical standard, recessionary scenarios in terms of unemployment and housing prices decline.

As we will see in the case of CPDO this did not seem to be the case. Agencies adopted criteria where most likely the critical variables simulations were calibrated versus an average historical evolutions, i.e. what we will call Base Case, rather than a stressed one.

Also the CPDO is quite exposed to a positive correlation between pool losses and credit spreads whereas rating criteria do not model this joint dependency.

4.1 Credit index spread

Following Fitch's approach described in Linden et al. (2007), we model the on-the-run 5 year Globoxx index spread (S_t) as an exponential Vasicek process.

$$dS_t = \alpha S_t(\theta - \ln S_t)dt + \sigma S_t dW_t \tag{7}$$

Fitch has overcome the limited history available for the iTraxx and CDX indices backfilling the sample with a proxy. In this way they can move back the sample start date to january 1998 (see Figure 1 taken from Linden et al. (2007)) even though iTtraxx and CDX indices are available only since July 2004.

This proxy was built from cash bond indices to match the duration, rating and geographical composition of the Globoxx index. The parameters estimate, and their

	Estimate	1/4 StdDev
α	0.556	0.056
θ	0.80 %	0.25 %
σ	40.7 %	1.3 %

Table 1: Parameters' estimates of the exponential Vasicek process calibrated to the backfilled Globoxx index. Source: Linden et al. (2007).

standard deviations, reported in Linden et al. (2007) is shown in Table 1.

Linden et al. (2007) point out how the standard deviation of the estimators for the mean reversion speed and long term mean are quite large. Also they point out how CPDO ratings turn out to be quite sensitive to these parameters.

In line with their work here we will not only simulate the index spread using the mean estimates of the parameters, hereafter the Mean Parameter Set. We will also stress contemporaneously the parameters in a conservative direction by a quarter the standard deviation of the estimates: increase the long term mean θ by 25 bps, decrease the mean reversion speed α by 0.056 and increase the instantaneous volatility σ by 1.3%, hereafter the Stressed Parameter Set.

4.2 Pool loss simulation

The rating agencies simulation engines for loss-dependent products are based on a multi-factor Gaussian copula (see McGinty and Ahluwalia (2004)) as can be found for example in S&P's *CDO Evaluator* [7] and Fitch's *Vector Model* [8]. These are the same toolkits used to rate synthetic CDOs.

The probabilities of default of the index constituents are derived assuming that the index composition of the Globoxx throughout the 19 rolls during the 10 years of each simulation path is equal to the average rating distribution of the last 4 series of the iTraxx and CDX as summarized in table 2.

The pairwise correlation between the latent factors in the Gaussian copula is derived according to the sector and region of each pair of credit references as in table 3, in line with the assumptions of S&P that could be found in the CDO Evaluator user guide at the time CPDOs were rated.

In our simulations we will assume the Globoxx rating composition on each roll date is constant and equal to the average of the rating composition of the last four series of the iTraxx and CDX indices as in table 2.

From each rating category we can lookup the average historical default rates for 1 year horizon in the default rate tables provided by rating agencies. Given that a maturity

[7] See `www.sp.cdointerface.com` site.

[8] See `www.fitchratings.com` site.

	iTraxx				CDX			
	S5	S6	S7	S8	S6	S7	S8	S9
AAA	00.8%	00.0%	00.0%	00.0%	03.3%	03.3%	03.2%	03.2%
AA	09.3%	13.1%	11.7%	16.9%	02.5%	02.5%	02.4%	02.4%
A	40.7%	36.9%	40.0%	36.3%	34.2%	35.8%	36.0%	38.4%
BBB	49.2%	50.0%	48.3%	46.8%	60.0%	58.3%	58.4%	56.0%

Table 2: iTraxx and CDX compositions at inception of the series by whole letter ratings for the 4 index series.

	Between Sectors	Within Sectors	Sector Type
Within Countries	0.05	0.15	local
		0.15	regional
		0.15	global
Within Regions	0.05	0.05	local
		0.15	regional
		0.15	global
Between Regions	0.00	0.00	local
		0.00	regional
		0.15	global

Table 3: Correlation assumptions of the S&P's CDO Evaluator. For example, if two entities are in the same sector but in different regions the correlation will be higher (15% rather than 0%) only if the sector is global (technology) rather than local (regulated utilities) or regional. Source: `www.standardandpoors.com`.

shorter than 1 year is not available in these tables, how do we calculate the 6 months average default rate we need in order to measure the average riskiness of the Globoxx in between any two roll dates?

We could assume a constant default intensity for the first year $\lambda_{1y} = -\log(1 - \mathrm{PD}_{1y})$. With $\mathrm{PD}_{1y} = 0.11\%$ for the Globoxx, given its rating composition at inception and Moody's rating transitions, we get a 6 month default rate of $\mathrm{PD}_{6m} = 0.05\% = 1 - \exp(-0.5\lambda_{1y})$.

Assuming a constant default intensity could be quite an overestimation. Lando and Skodeberg (2002) use continous data to estimate the generator of the transition matrix and with this we estimated the 6 month transition matrix. Given the assumed Globoxx rating distribution at inception of table 2 and the generator consistent with the 1 year rating transition they report, this would result in a 6 month default rate of 0.016%. If we were instead to assume a constant default intensity over 1 year, alwasy using their transition matrix, we would obtain 0.027%: a difference of more than 60% between the two estimates.

This are apparently small numbers and it is debatable what is the appropriate 6 month default rate to use. Nevertheless given the highly leveraged strategy beneath CPDO which of the two different estimates to use turn out to make quite a difference as pointed out in Linden et al. (2007).

Another issue arises with the adverse selection of Globoxx constituents. How correct is it to compute the average 6 months default rate of the Globoxx given its initial rating composition? We have already highlighted the evidence presented in Tejwani et al. (2007) suggesting that the CDX pool of names on average experience a faster rating deterioration than expected given its initial rating composition.

In Figure 2 taken from Tejwani et al. (2007) it can be seen how adjusting for the adverse selection of the Globoxx constituents can result in a 5 years cumulative loss rate twice as much the 5 years cumulative loss rate without the adjustment. Thus one would need to stress somewhat the default rate as obtained from the Globoxx rating composition.

It is difficult to assess the relative importance of the two effects on the 6 month default rate assumption:

- overestimation due to the assumption of constant default rate intensity in contrast with the evidence of Lando and Skodeberg (2002)
- underestimation because of the adverse selection due to arbitrage CDO activity as documented in Tejwani et al. (2007)

For this reason we assumed the two effects cancel out and we computed the 6 months default rate from the average of the last 4 series of the Globoxx rating distribution as reported in table 2 assuming a constant default intensity for the first year.

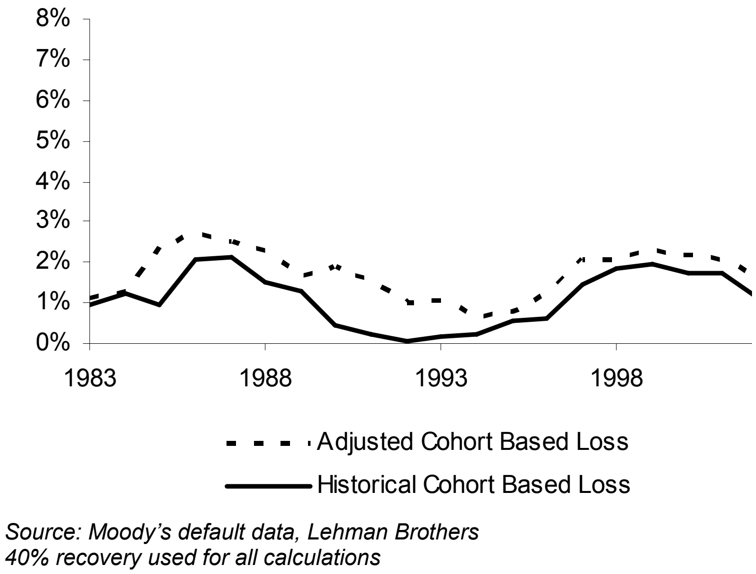

Figure 2: 5-years Rolling Buy-and-Hold Loss for a Portfolio with Rating Distribution Similar to CDX.NA.IG S8. Source: Tejwani et al. (2007).

4.3 Roll down benefit

The Roll Down Benefit (RDB) is the difference between the 5 year and 3 months maturity Globoxx index spread at index inception and the 4 year and 9 months maturity Globoxx index spread on the next index roll-date. If this difference is assumed to be positive on average, as agencies did when rating CPDOs, then the structure will benefit when rolling the Globoxx leveraged short protection exposure.

The rationale of this assumption being that investment grade curves are positively sloped and the iTraxx and CDX indices are indeed refreshed every six months to include only investment grade names.

One problem with this argument is the rating deterioration that the Globoxx might have suffered during the 6 months. In figure 3 we report the different slope of the default intensities of A and Ca rated companies by Moody's for increasing time horizons.

The positive slope of investment grade curves can be explained by the option held by the management to change the capital structure of the company, also with time adverse and unforeseen changes can occur to the sector to which the company belongs.

Conversely the default intensity desumed from the default rate of lowly rated companies is usually inverted as these companies face a "fly or die" situation. If the overhaul of the company is successful and the adverse contingency is passed the likelihood of a default decreases substantially.

It is true that iTraxx and CDX during their short time history displayed curves always positively sloped. Nevertheless, as pointed out in Lindet et al. (2007), the same was not true for the back-filled Globoxx index where there were periods when the proxied Globoxx curve was negatively sloped.

We argue that part of the positive slope of investment grade curve is a remuneration

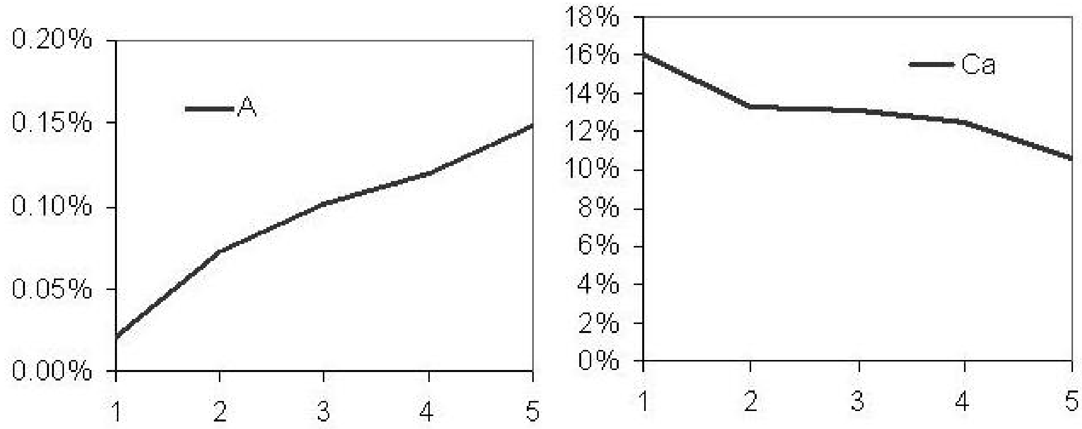

Figure 3: Default intensity for A rated (left chart) and Ca rated (right chart) corporate credit references.

for the risk of rating downgrade. Only a conservative estimate of the remainder should be recognized in the CPDO rating criteria. This turns out to be quite difficult to estimate and is outside the scope of this article.

We will consider as Base Case a roll-down benefit of "only" 3%, substantially below the assumption of rating agencies, and as Stressed Case a roll-down benefit of 0%.

5 Modification of the standard assumptions

So far we have introduced the standard criteria adopted by rating agencies to simulate the NAV of a CPDO structure. The standard criteria is to model independently the spread and the pool loss. The spread is modeled with a mean reverting process whereas the pool loss is modeled with a multi-factor Gaussian copula.

Here we want to introduce a crucial modification to the standard assumptions: we will change the simulation engine for the loss of the pool of names underlying the credit index from the multi-factor Gaussian copula generally adopted by rating agencies to a simulation engine that allows many defaults at the same time instant: the Generalized Poisson Cluster Loss model (hereafter GPCL).

5.1 Bumps in the multifactor Gaussian copula

The average correlation used by rating agencies are so low that they do not produce loss distributions with bumps in the right tail[9].

To show this we imagine 250 homogeneous entities with the same ten year default probability of 5%. In Figures 4 and 5 we plot the 10 year loss distribution obtained with a 0% recovery rate, where the default time of the single entities are correlated through

[9]For instance, in the case of the CDO Evaluator we considered in Table 3 the maximum correlation was 15%.

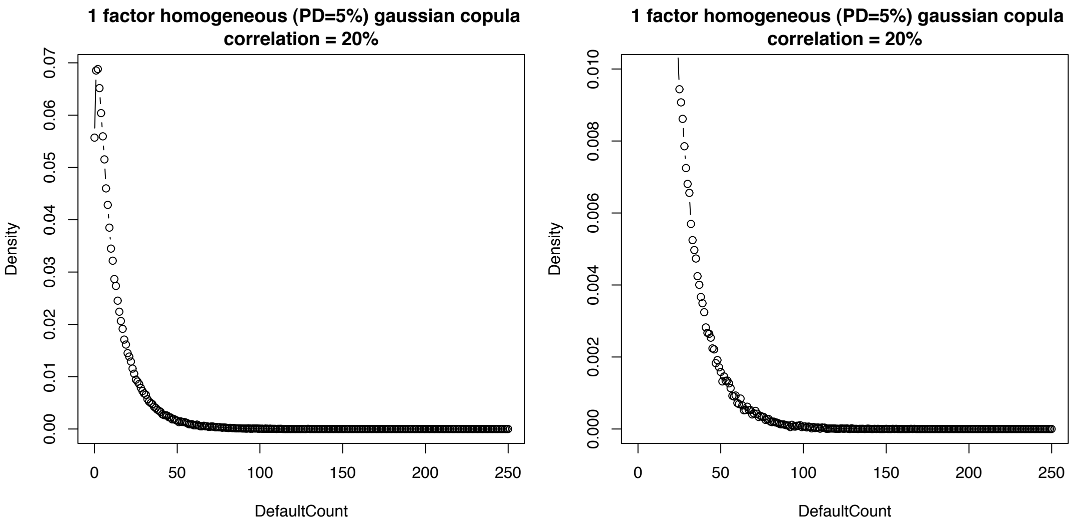

Figure 4: Ten year loss distribution obtained correlating the default times of 250 homogeneous reference entities with default probability of 5% with a one-factor Gaussian copula assuming 0% recovery with flat correlation equal to 20%. In the right chart the y-axis is limited to the [0; 1%] range.

a one-factor Gaussian copula with the flat correlation parameter equal to 20% and 90% respectively.

Both distributions have the same expected loss (5%) but the shape, in particular the tail, is quite different. Even with a Gaussian copula we can obtain a bump for the extreme loss scenario (loss close to 100%) but with a very high flat correlation parameter.

Conversely right-tail bumps arise naturally in many statical and dynamical loss models. Among others, multi-modal loss distributions are predicted in Albanese et al. (2005), in Hull and White (2005) or Torresetti et al. (2006b), in Longstaff and Rajan (2006) and in the GPL and GPCL models by Brigo et al. (2007a, 2007b).

5.2 Bumps in the GPCL

How reasonable is it to assume a loss distribution in the objective measure with bumps? Historical observations of default clusters are rare events even though recently we have seen a sequence of defaulted names belonging to the same sectors in a relatively short amount of time: airlines in 2001-2002 and more recently autos and financials. Thus it is true that we rarely see clusters of default right at the same instant even though we usually see several defaults in a relatively short time window interesting specific sectors.

The evidence presented by Longstaff and Rajan (2006) might point in the direction of allowing the bump feature to be present also in the objective measure. In fact they calibrated to the CDX a dynamical loss model where the loss count is the summation of three independent Poisson processes each one governing a different amplitude of the loss jump.

The economic interpretation the authors give to the three jump sizes are:

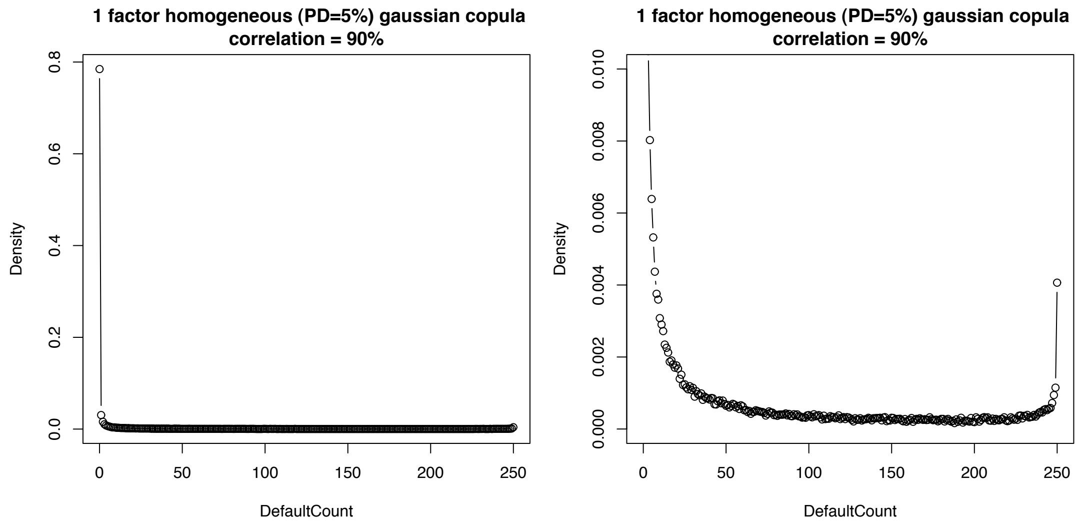

Figure 5: Ten year loss distribution obtained correlating the default times of 250 homogeneous reference entities with default probability of 5% with a one-factor Gaussian copula assuming 0% recovery with flat correlation equal to 90%. In the right chart the y-axis is limited to the $[0; 1\%]$ range.

- idiosyncratic shock: the calibrated amplitude size for the loss jumps being roughly equal[10] to 0.4%.
- industry shock: the calibrated amplitude size for the loss jumps being roughly equal[11] to 6%.
- systematic shock: the calibrated amplitude size for the loss jumps being between 30% and 50%.

The authors then try to see if this economic interpretation is corroborated by a Principal Component Analysis on the panel of daily absolute variations of the CDS spread of the CDX constituents. They find that the first component is a shock affecting all CDS (systematic shock), the remaining significant components can be interpreted as shocks affecting only one sector at a time (industry shock).

This would seem to suggest that the industry shock has an economic meaning as can be seen from the newsflow affecting the CDS spread of the CDX constituents. Thus the bumps that can be found in the implied loss distribution, see Torresetti et al. (2006a), from CDO quotes could be attributed not just to the risk premium investors ask as a remuneration for bearing the skewed risk of senior tranches.

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[10]A figure close to the loss given default of one credit reference in the pool assuming deterministic recovery of 40% ($0.48\% = (1 - 0.4)/125$)

[11]A figure close to the loss size of the average Fama and French sector in the iTraxx or CDX universe of $5\% = (1 - 0.4)/12$.

We think that the evidence of Longstaff and Rajan (2006) might be relevant enough to point in the direction of bumps being present also in the objective measure. Accordingly in the following we consider the loss distribution coming from the GPCL model, and we check the impact of bumps in the CPDO rating.

The works of Brigo et al. (2007a, 2007b) show calibrations of the GPL and GPCL dynamical loss models against iTraxx and CDX tranche and index quotes on several market observation dates. In the GPL and GPCL models the default counting process is modeled by means of a summation of many independent Poisson processes whose intensity depends, in turn, on the number of defaults. In particular the GPCL model is built by taking into account the consistency with single-name dynamics.

5.3 The GPCL model at work

Our goal is to use the GPCL model to estimate the probability distribution in the objective measure preserving the multi-modal features that the model predicts for the probability distribution in the risk-neutral measure. Thus, we follow a calibration strategy rather different from the one proposed in Brigo et al. (2007b) and rather similar to the one proposed by Longstaff and Rajan (2006). The dynamics of the pool loss process L_t and of the default counting process C_t are:

$$dL_t = (1-R)dC_t\,, \quad C_t = \sum_{j=1}^{M} j\, Z_j(t)\,, \quad dZ_j(t) \sim \text{Poisson}\left(\binom{M - C_{t^-}}{j}\tilde{\lambda}_j(t)dt\right) \quad (8)$$

with boundary conditions $Z_j(s) = 0$.

We have designed a calibration for the tranches of the last five iTraxx and CDX 5 year series. For the CDX we have considered all series from S4 to S8, while for the iTraxx we have considered all series from S3 to S7.

We fixed the jump sizes in order to evenly space the logarithmic default-rate from 1/125 to 125/125. The intensities $\tilde{\lambda}_j$ with $j \notin J := \{1, 2, 5, 11, 25, 56, 125\}$ are set to zero, namely the counting process can jump only with an amplitude which is listed in J.

Then, for each series we consider the weekly market quotes for dates when the series are on-the-run (a six month period). The associated intensities $\tilde{\lambda}_j$ are constant and depend on the calibration time s only via a common multiplicative factor $\varphi(s)$ allowed to change daily:

$$\tilde{\lambda}_j(t; s) = \varphi(s)\psi_j(t-s) \quad (9)$$

We set $\varphi(T_R^{i-1}) = 1$ where T_R^{i-1} is the last roll date, i.e. the first trading date of the new on-the-run series.

We have on average 120 market quotes for each series, 6 months times 4 weeks times 5 tranches, but we only calibrate $30 = 7 + 6 \cdot 4 - 1$ parameters for each series (7 jump intensities plus 23 scaling factors: one for each weekly date in the sample minus one).

In Figure 6 we plot the risk neutral 6 months distribution of the number of default for the iTraxx and CDX where $\varphi(s)$ is rescaled in order to match the index spread on 20-sep-2005..

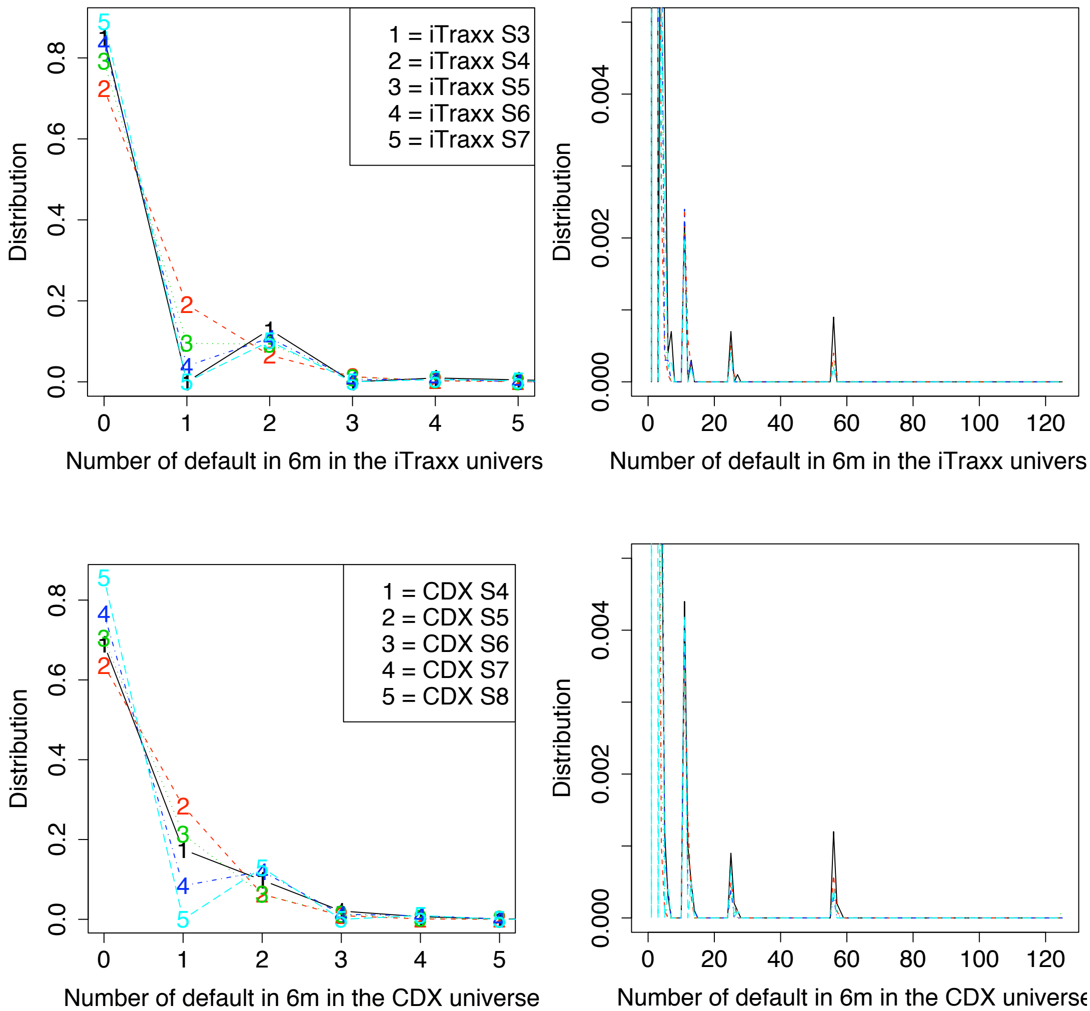

Figure 6: Risk Neutral distributions of the number of default over a 6 month horizon for the iTraxx and CDX pools calibrated to the 5 year iTraxx and CDX tranches.

	0-3%	3-6%	6-9%	9-12%	12-22%
2-Aug-2006	2062.5 bps	77 bps	22 bps	10.5 bps	3.75

Table 4: iTraxx 5y Tranche Spreads Market on 2nd August 2006. The equity tranche is quoted as an upfront premium, all the other tranches are quoted as a running premium. Source: Banca IMI.

series	Roll Date	0%-3%	3%-6%	6%-9%	9%-12%	12%-22%
iTraxx S1	20-Mar-2004	1.7	0.8	1.9	1.1	0.8
iTraxx S2	20-Sep-2004	0.4	0.5	0.8	2.0	1.1
iTraxx S3	20-Mar-2005	5.2	6.0	1.9	1.4	1.1
iTraxx S4	20-Sep-2005	1.6	1.6	1.5	0.8	0.9
iTraxx S5	20-Mar-2006	3.0	3.2	1.6	1.4	0.6
iTraxx S6	20-Sep-2006	1.3	1.9	1.9	1.8	0.6
iTraxx S7	20-Mar-2007	3.2	3.1	3.6	3.8	1.5
series	Roll Date	0%-3%	3%-7%	7%-10%	10%-15%	15%-30%
CDX S2	20-Mar-2004	0.8	0.6	1.5	1.3	0.5
CDX S3	20-Sep-2004	1.6	1.2	2.3	2.4	0.7
CDX S4	20-Mar-2005	4.7	4.1	1.7	1.6	1.9
CDX S5	20-Sep-2005	1.0	0.7	1.6	1.1	2.3
CDX S6	20-Mar-2006	1.0	0.7	2.3	1.6	2.0
CDX S7	20-Sep-2006	1.4	1.2	2.6	2.6	2.7
CDX S8	20-Mar-2007	2.0	1.4	1.9	4.6	5.0

Table 5: Average Absolute Standardized Mispricings for the iTraxx and CDX series. The averages are calculated on all weekly market data when the series where on-the-run.

The distribution in the objective measure is computed with the intensities calibrated to the 5y tranches, $\tilde{\lambda}_j(t;s) = \varphi(s)\psi_j(t-s)$, and rescaled seeking the φ_{obj} to match the probability of default of the underlying pool of names in between roll dates: 0.11%.

The target function in the calibration is the sum of squares of the standardized mispricing: theoretical spread minus market spread divided by half the bid-ask spread. A mispricing of 3 bps in the 0-3% iTraxx tranche on 2nd August 2006 is quite different from the same 3 bps error for the 12-22% tranche.

We will crudely average across the ten CDX and iTraxx distributions in Figure 6 throughout the five series to get the 6 month loss distribution in the objective measure for the Globoxx. In the CPDO structure NAV simulations under the GPCL assumption, the pool losses will be sampled from the averaged distribution on each roll date: i.e. every six months.

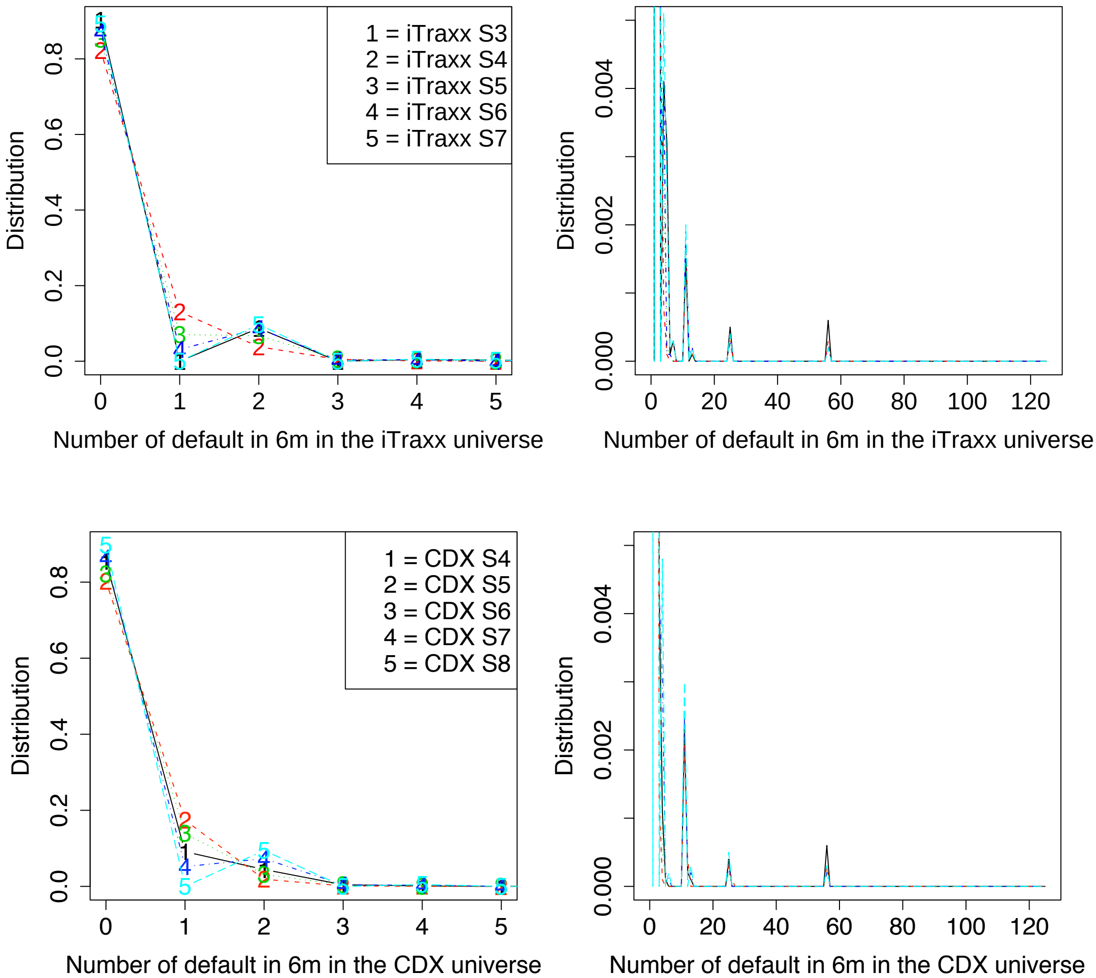

Figure 7: Distribution in the objective measure of the number of default in 6 months for the iTraxx and CDX pools.

6 Numerical results

So far we have introduced the payoff of an hypothetical CPDO structure with underlying the Globoxx index.

We have then analyzed the details of the risk factors affecting the evolution of the NAV of the structure, thus identifying the critical variables that needs to be simulated when assigning a rating to CPDO structures: the Globoxx credit spread and the Globoxx loss rate.

For both credit spread and credit losses simulation we have identified a set of Base Case and Stressed case assumptions. The Stressed Case assumptions are a set of conservative assumptions across both the credit spread and credit loss parametrization: the sort of assumptions an investor would expect from a rating agency when assigning a AAA rating. The parametrization of the credit spread simulation in turn can be stressed across both the roll-down benefit and the parametrization of the Exponential Vasicek.

The results of the stress on the parametrization of the credit spread on the CPDO rating can be found in Linden et al. (2007). Here we want to highlight the increased riskiness of CPDO structures when stressing the parametrization of the loss rate simulation. Rating agencies generally adopt a multi-factor Gaussian copula, the same approach that was used in Linden et al. (2007). We advocate instead the GPCL: a loss simulation engine that admits clusters of defaults.

Here we want to compare these two approaches (multifactor Gaussian copula versus GPCL). Under the hypothesis of each approach we will run the NAV simulations for the CPDO intersecting two sets of assumptions regarding:

- Roll Down Benefit. We will simulate the NAV of the CDPO assuming that between roll dates the index spread decreases (rolls down on the index CDS term structure) by 0% and 3% respectively.
- Mean Reversion Parameters for the process of the credit index spread (Exponential Vasicek). We will simulate the NAV of the CPDO under two different settings of mean reversion parameters: one where they are set to their mean estimate (hereafter Mean Parameters Set) and one where they are set to their mean estimate stressed by a quarter of the standard deviation (hereafter Stressed Parameters Set).

The case that best proxy the criteria used by rating agencies to rate first generation CPDOs can be considered the case of Mean Parameter Set and Roll Down Benefit set to 3%.

We will simulate the CDS Index spread monthly. We will simulate instead the loss semi-annually on the roll dates. Bid-ask spreads for the Globoxx 5 year credit spread are assumed to be 1bps.

6.1 Default and loss rate

The default and loss rates of the CPDO structure simulated under the various assumptions of roll down benefit and mean reversion parameters are presented in figures 8 and

9 for the multi-factor Gaussian copula and GPCL loss simulation engines respectively.

The plots in the left (right) column of each figure are the simulations run assuming a roll down benefit of 3% (0%). In the plot in the top (bottom) row of each figure are the simulations run with the Mean Parameters Set (Stressed Parameters Set).

At the bottom of each chart can be found the probability of default (PD) and average loss (Exp loss) of the CPDO structure. The histograms in the chart is the distribution of the cash-in times. A path where the structure has not cashed-in before maturity (thus including the paths where a cash-out has occurred) will count as a default of the CPDO structure. Thus the percentage of paths falling in the last bin in Figure 8 and 9, the bin of the CPDO maturity, tells us the percentage simulation paths that resulted in a CPDO default.

The results contained in figures 8 to 9 are summarized in tables 7 and 8.

We see that introducing a loss distribution with bumps worsens considerably the default rate of the structure and thus reduces substantially the rating. This is particularly true in the Base Case for both the roll-down benefit and the Mean Parameter Set where the structure had a very low probability of default in the multi-factor Gaussian copula case.

6.2 Draw-down

In figures 10 and 11 we show the distribution of the maximum draw-down of the NAV. The draw-down DD_t is defined as the maximum drop of the NAV from the so called high-watermark HW_t.

$$HW_t = \max_{s\in[0,t]} NAV_s$$
$$DD_t = \max_{t\in[0,T]} \{HW_t - NAV_t\}$$

In fact the structure might be cashed-in before maturity but nevertheless can suffer considerable losses.

Put in another way, the histogram in figures 10 and 11 gives us an idea of the volatility of the NAV[12].

At the bottom of each plot is the average draw-down. We see that stressing the roll-down-benefit and the mean reversion parameters the average draw-down increases from 32% to as much as 48%. Moving from the multi-factor Gaussian copula framework to the GPCL framework to simulate the loss instead does not increase the average draw-down.

[12] Rating agencies have enlarged the notion of rating from corporate and sovereign, that was already assimilated by the investors community, to the structured finance space. Nevertheless structures might have the same default probability of an equivalent, in terms of rating, corporate or sovereign but their NAV can have a much higher volatility with respect to an equivalently rated, let us say AAA, corporate bond price volatility.

	AAA	AA+	AA	AA-	A+	A	A-	BBB+	BBB	BBB-
10Y PD	0.7%	1.0%	1.5%	1.9%	2.3%	2.7%	3.6%	4.8%	7.1%	12.3%

Table 6: Probability of default for structured finance deals of Standard and Poor's. Source: www.standardandpoors.com

	Copula		GPCL	
	RDB=3%	RDB=0%	RDB=3%	RDB=0%
Mean Parameter Set α=0.55,θ=0.80%,σ=40.0%	1.12%	3.52%	2.04%	7.16%
Stressed Parameter Set α=0.49,θ=1.05%,σ=38.7%	2.24%	9.76%	4.08%	11.32%

Table 7: CPDO average default rate under the various assumptions in terms of Roll Down Benefit (RDB) and Mean reversion parameters governing the dynamic of the index spread.

6.3 Rating

When assessing the rating of a structured finance issue S&P and Fitch take into consideration the Default Rate of the structure simulated under a set of scenario whereas Moody's take into consideration the loss rate.

Rating agencies will look up the rating from the average simulated default or loss rate table.

In Table 9 we report the CPDO rating based on the CPDO default rates of Table 7 and the default rates assigned by S&P for 10 years maturity structured finance deals reported in Table 6.

For both the Copula and the GPCL case we note that stressing the roll-down benefit assumption has a much greater impact, a difference of two notches, than stressing the parametrization of the credit spread simulation.

When stressing both the parametrization of the credit spread simulation and the roll-down benefit assumption both the Copula and the GPCL case give the same rating (BBB-). Instead when using the base case for the both the credit spread simulation and the roll-down benefit assumption, simulating the loss with the GPCL worsens the rating of the CPDO by 2 notches with respect to the multifactor Gaussian Copula (A+ instead of AA).

We see that on both cases, Gaussian Copula and GPCL, when moving from the Base Case to the stressed case for both the roll-down benefit and the credit spread process parametrization, we get on the verge of the investment grade scale.

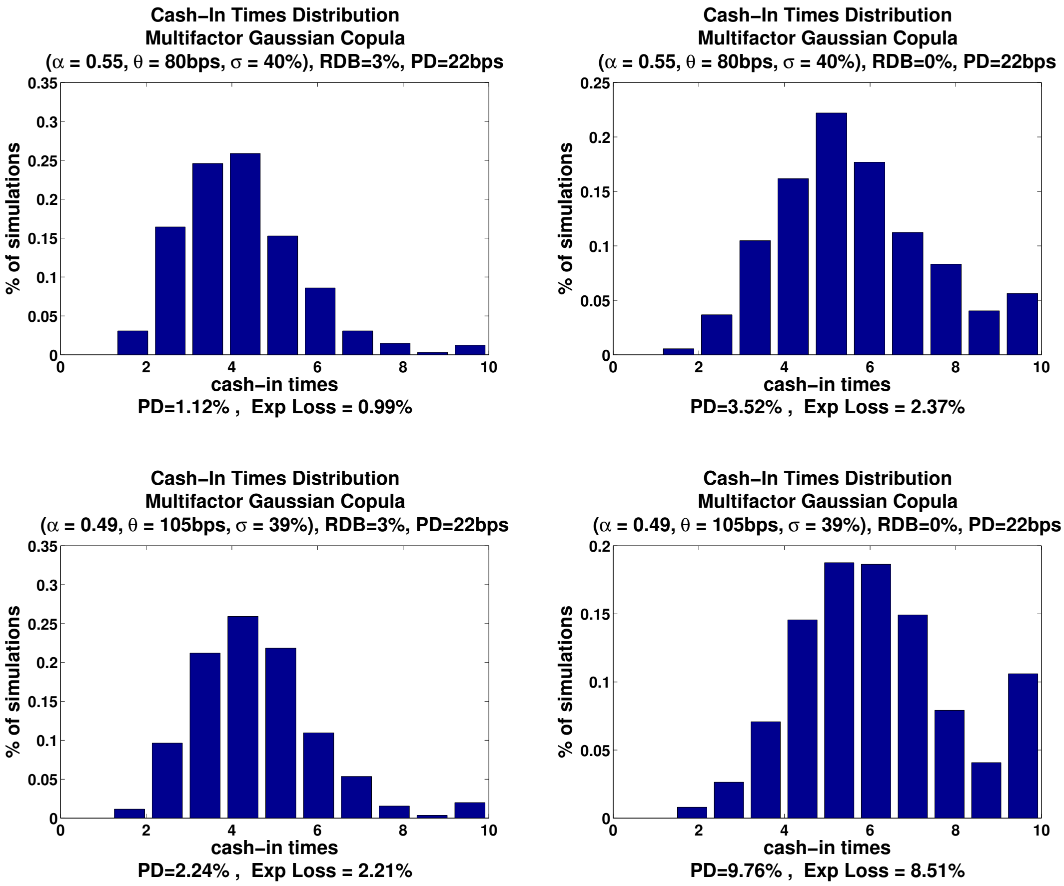

Figure 8: Distribution in the cash-in times under the multi-factor Gaussian copula approach.

	Copula		GPCL	
	RDB=3%	RDB=0%	RDB=3%	RDB=0%
Mean Parameter Set α=0.55,θ=0.80%,σ=40.0%	0.99%	2.37%	2.36%	6.52%
Stressed Parameter Set α=0.49,θ=1.05%,σ=38.7%	2.21%	8.51%	4.79%	10.44%

Table 8: CPDO average loss rate under the various assumptions in terms of Roll Down Benefit (RDB) and Mean reversion parameters governing the dynamic of the index spread.

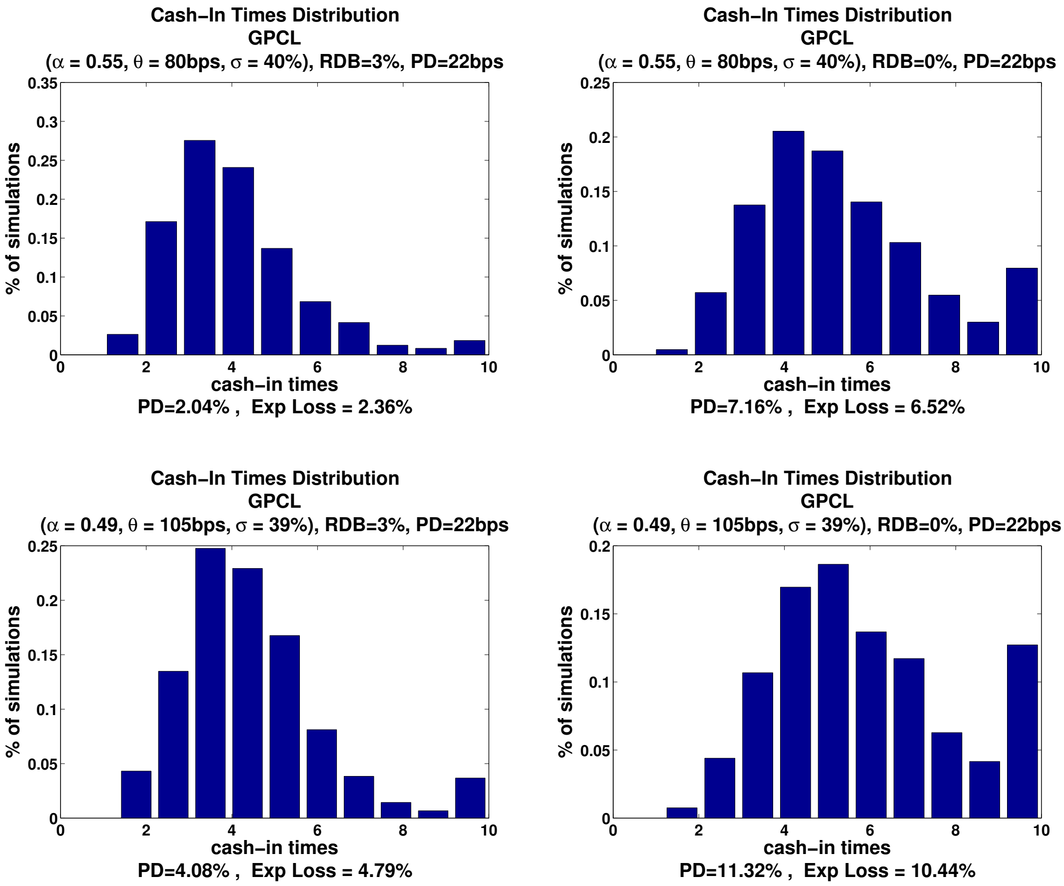

Figure 9: Distribution in the Cash-in times under the GPCL Approach.

	Copula		GPCL	
	RDB=3%	RDB=0%	RDB=3%	RDB=0%
Mean Parameter Set α=0.55,θ=0.80%,σ=40.0%	AA	A-	A+	BBB-
Stressed Parameter Set α=0.49,θ=1.05%,σ=38.7%	A+	BBB-	BBB+	BBB-

Table 9: CPDO rating based on the CPDO default rate of Table 7 and the probability of default estimated by S&P for structured finance deals reported in Table 6 under the various assumptions in terms of Roll Down Benefit (RDB) and Mean reversion parameters governing the dynamic of the index spread.

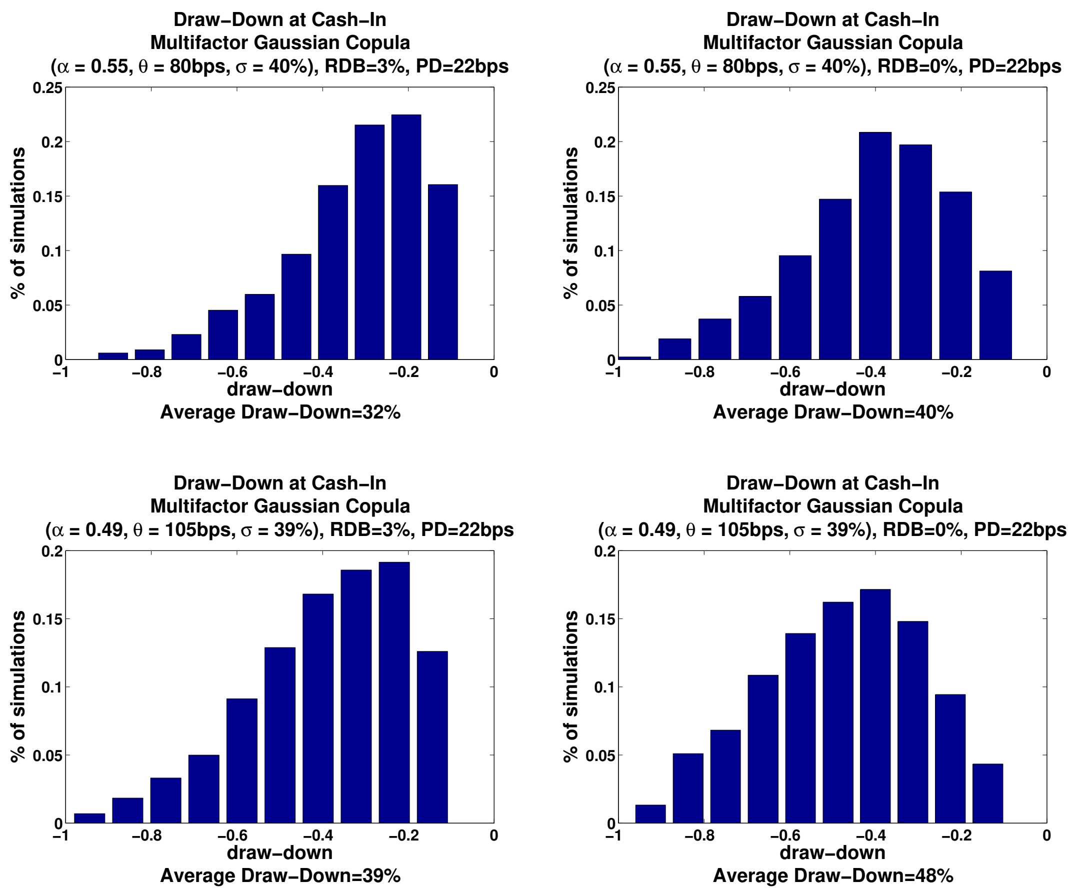

Figure 10: Draw-down distribution under the multi-factor Gaussian copula approach.

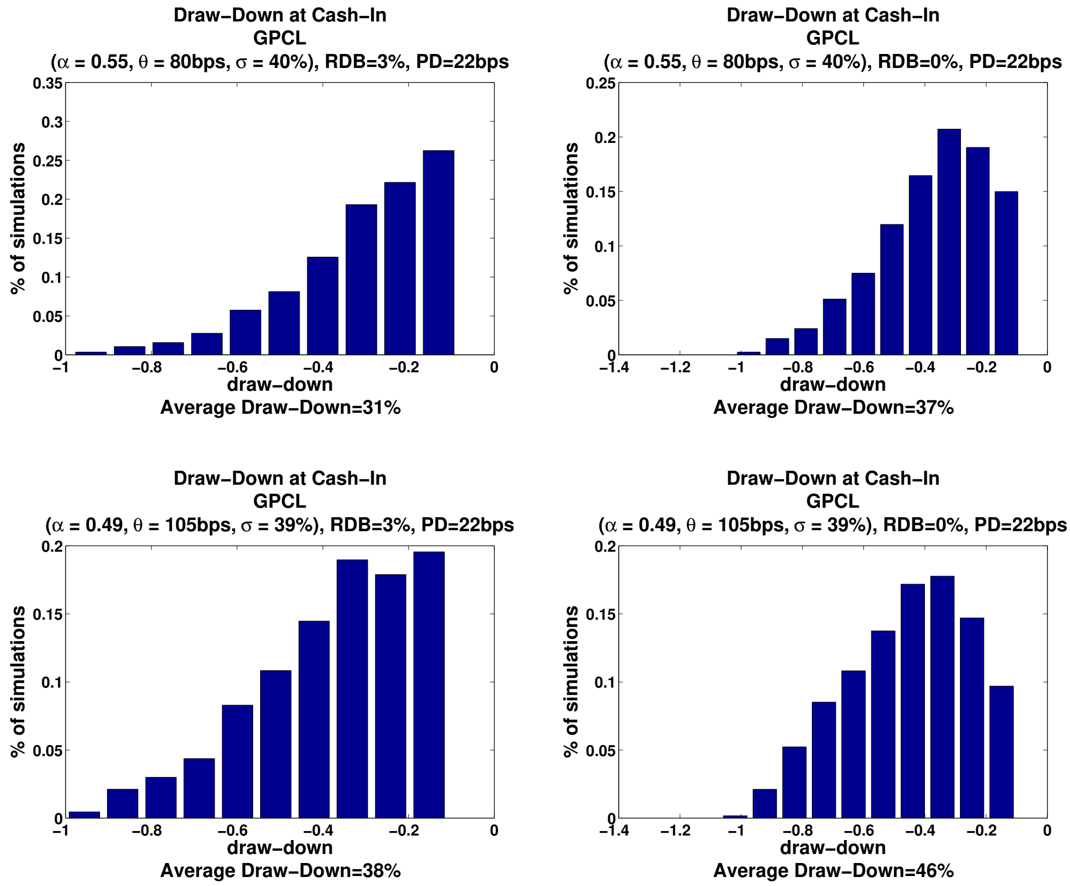

Figure 11: Draw-Down Distribution under the GPCL approach.

	Copula		GPCL	
	RDB=3%	RDB=0%	RDB=3%	RDB=0%
Mean Parameter Set α=0.55,θ=0.80%,σ=40.0%	0.02 bps	0.08 bps	4.32 bps	1.63 bps
Stressed Parameter Set α=0.49,θ=1.05%,σ=38.7%	0.12 bps	0.35 bps	2.51 bps	2.08 bps

Table 10: CPDO average gap risk fee in bps under the various assumptions in terms of Roll Down Benefit (RDB) and Mean reversion parameters governing the dynamic of the index spread.

6.4 Gap-Risk

The specific CPDO structure we have analyzed here pays a substantial fee for the gap risk option: 3.5bps per leverage per annum.

In case the leverage goes to its maximum, 15, since then on the structure would pay 52.5 bps per annum (15 times 3.5 bps) as a gap-risk fee.

Thus the market pays quite a considerable premium for the risk that when unwinding all positions the structure might need a cash injection.

In Table 10 we report the average gap-option fee (bps per unit of leverage per annum) across simulations: we will divide the simulated gap protection payments, the absolute value of the average NAV when negative or 0 otherwise at cash-out, by the average leverage to the earliest between maturity, cash-in or cash-out.

Note that when stressing the parameter of the mean reversion process in the GPCL framework we actually reduce the average gap risk fee.

This happens because the average leverage is actually reduced and thus the expected loss we suffer when a cluster of defaults arrives is considerably reduced as can be seen in Table 11 where we report the average leverage at cash-out under the various assumptions of Roll Down Benefit (RDB) and Mean reversion parameters.

When we stress the mean reversion parameters we make it more likely that a cluster of defaults arrive at a time when the NAV was already deeply depressed due to an adverse movement in spreads and thus reduce the impact of the cluster of defaults.

Finally we want to point out how the results of our simulation provide an insight also with respect to the reserves that the bank selling the gap-option will need to book. Indeed, the bank selling the gap-option would have to make a provision for the expected loss and allocate capital for the unexpected loss. The bank provision for the expected loss could be calculated as the Base Case with the Gaussian Copula: 0.02 bps out of 3.5 bps. The capital allocated to sustain the unexpected loss could instead be calculated with the GPCL with the Stressed Parameters Set and RBD=0: 2.08 bps out of 3.5 bps.

	Copula		GPCL	
	RDB=3%	RDB=0%	RDB=3%	RDB=0%
Mean Parameter Set α=0.55,θ=0.80%,σ=40.0%	5.45	5.18	11.30	13.2
Stressed Parameter Set α=0.49,θ=1.05%,σ=38.7%	3.8	4.5	7.9	7.37

Table 11: CPDO average leverage at cash-out times under the various assumptions in terms of Roll Down Benefit (RDB) and Mean reversion parameters governing the dynamic of the index spread.

7 Conclusions

We have introduced the details of a specific CPDO transaction. We have then reviewed the standard criteria generally adopted by rating agencies to rate first generation CPDO transactions. We have highlighted the importance of the roll down benefit hypothesis and the difficulty in measuring it historically.

We have then introduced one important modification to the standard criteria: namely the adoption of the GPCL framework when simulating the index pool loss. We have then performed a batch calibration to both the iTraxx and CDX series. In this way we obtained for each series of each index an average estimate of the importance of the bumps in the right tail of the loss distribution.

The risk neutral loss distributions thus obtained were mapped to the objective measure rescaling the cluster default intensities in such a way to obtain the target average default rate in the objective measure.

In the same spirit of Linden et al. (2007) we run our simulations under a set of assumptions regarding the roll down benefit and the mean reversion parameters. For each set of assumptions we consider both loss simulations frameworks: multi-factor Gaussian copula and GPCL.

We show the increase in riskiness, decrease in rating, of the CPDO when simulating the loss with the GPCL framework instead of the multifactor Gaussian copula.

Finally we hint at the dramatic increase of the fair gap risk fee under the objective measure in the GPCL framework with respect to the multi-factor Gaussian copula.

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