Dynamic Model of Arts Labor Supply

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**DYNAMIC MODEL OF ARTS LABOUR SUPPLY**

**Abstract.** In this paper two dynamic models of artist behavior and arts labor supply are developed. Both are based on household production function approach and on the assumption that artists are multiple-job-holders. In the first model proposed here artist is depicted as someone who is hired on the arts labor market and paid for her arts time. In the second model artist is described as someone who sell her products, like paintings for instance, on the market for arts products. In order to make these models dynamic, artist productivity is here supposed to be a function of accumulated human capital of artist. Following the results of existing empirical research, previous experience and previous artistic practice are supposed to be the most important form of human capital accumulation. Once analysis is expended to capture this kind of artist human capital accumulation, the supply of the labor in the arts market appears as a resulting from an inter-temporal process of resources allocation. Both models end with same result: shadow price of producing unit of art commodity in certain year should be equal to the sum of current monetary benefits, current nonmonetary benefits, stream of future monetary benefits, and stream of future nonmonetary benefits generated by production of respected art unit. This result appears to be pretty suitable for formalization of several existing hypotheses aimed at explaining arts labor market peculiarities. Especially, by referring to the stream of expected nonmonetary benefits, models developed here are able to formalize most promising among these hypothesis according to which artist need for self-actualization is driving force in explaining oversupply of arts labor.

**Key words:** arts, household production function, allocation of time, expected benefits

**JEL Code:** Z10, Z11, J22, J24, J31
1. Introduction

It is well known, from earlier works and researches on labor supply, that whenever workers derive satisfaction from the process of work, some traditional results about labor supply are reversed. Work of artists is most famous example of this phenomenon. It is noticed, for example, that whenever non-arts wages increase, relative to arts wages, the supply of labor in the arts market increases relatively to non-arts market. This, obviously, contradict to what one would expect relying on traditional approach.

Explanation offered so far is based on Throsby (1994) model of artist behavior. It explains this phenomenon by the operation of the income effect in the choice between earned income and arts time. Arts time, as we know, provide itself satisfaction to the artists. Throsby model is essentially static. It does not consider inter-temporal aspects of supply of labor. These aspects are especially important if one consider supply of arts labor. Artist wages and prices of their works (paintings, for example) are function of accumulated human capital of artist. This accumulation of human capital, on the other hand, can result both from investment in formal education and from previous art practice and experience of artists. Previous practice and experience is in the case of arts, according to empirical works and casual observation, much more important than investment in formal education.

Once analysis is expended to capture artist human capital accumulation, the supply of the labor in the arts market appears as a resulting from an inter-temporal process of resources allocation that is based on an accumulation of human capital decision. This dynamic extension of basic model allows some other, even more important, peculiarities of arts labor market to be explained (oversupply of arts labor, earning penalties, poverty among artists, and similar). In this article two such dynamic models of artist behavior are developed. Both are based on household production function approach. Both models are also based on the assumption that artists are multiple-job-holders and that they have to decide how much of their time to devote to artistic work and how much time to non-artistic work. It is in accordance with casual observation that artists, especially in the early ages of their career, works at both non-artistic jobs as well as at artistic one. First model that is proposed here is based on the assumption that artist are hired on the arts labor market and paid for their arts time. This approach was once proposed by Caserta and Cuccia (2001) but has not been solved and developed further. Second model is based on the assumption that artists sell their products, like paintings for instance, on the market. Labor supply is in this case derived from artist product supply function.

In the second section of the article short survey of the static model developed by Throsby (1994) is given first. For the sake of simplicity and comparability model is a bit modified. Following two sections are core of the article: in the third section first dynamic model is given, while in the fourth section of article second model is presented. Implications of the models are discussed in the fifth section. Article ends with concluding remarks where some other cases where these models can be applied as well as some possible generalizations are discussed.
2. A Static Approach to Arts Labor Supply

In the Throsby (1994) work preference model it is assumed that artist maximizes following utility function

\[ U = U(L^M, X) \]  \hspace{1cm} (1)

With \( L^M \) we present time devoted to arts activity, which by definition provides pleasure to artist.\(^1\) On the other hand, \( X \) presents quantity of all other market goods. Needless to say, both partial derivatives of this utility function are positive.

Artists are, of course, paid for their time devoted to art. If their hourly wage from this activity is \( w^M \) than their income earned from arts is equal to \( L^M w^M \). Artists, however, have option to devote part of their time to non-artistic activities (\( L^n \)). If their wage rate earned at non-artistic job is \( w^n \), than their income earned from non-artistic activities will be \( L^n w^n \). Artist total incomes earned from both activities will be \( L^M w^M + L^n w^n \). So, their income constraint become

\[ p X = L^M w^M + L^n w^n \]

where \( p \) presents price of market goods. Crucial assumption of the Throsby work preference model is that non-arts wages are higher than arts wages, \( w^n > w^M \).

Artists are also constrained by disposable time: time they devote to artistic (\( L^M \)) and non-artistic (\( L^n \)) activities should be equal to their disposable time (\( L \)). Formally\(^2\)

\[ L^M + L^n = L \]

Time constraint and income constraint can be combined to give one constraint of the following form\(^3\)

\[ L^M w^n + p X = L w^n + L^M w^n \]  \hspace{1cm} (2)

Behavior of artist can now be outlined with expression (1) and (2). Artist chooses value of \( L^M \) and \( X \) in order to maximize (1) subject to constraint given by (2). In order to solve the problem we form Lagrange of the following form

\[ \mathcal{L} = U(L^M, X) - \lambda [L^M w^n + p X - L w^n - L^M w^M] \]  \hspace{1cm} (3)

First order condition requires partial derivative of (3) with respect to \( L^M \) and \( X \) to be equal to zero. Second order condition will be, for the sake of simplicity, skipped. Solutions we get are\(^4\)

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\(^1\) Although in his formal analysis Throsby (1994) uses general utility function, in the graphical presentation he, in fact, applies quite specific kind of quasi linear utility function. By doing so he was able to present typical artist as someone who has absolute preference to artistic work once her basic needs are satisfied.

\(^2\) In this presentation Throsbys’ initial model is somewhat modified. We use amount of disposable time devoted to artistic and nonartistic work while Throsby use their share in disposable time.

\(^3\) From expression for time constraint it follows that \( L^n = L - L^M \). By substituting this for \( L^n \) in income constraint and rearranging we get constraint given by expression (2).

\(^4\)
Consequently, optimal solution requires marginal rate of substitution of artistic time for market goods (MRS) to be equal to

\[
MRS = -\frac{\partial x}{\partial w^n} = \frac{\partial u/\partial w^n}{\partial u/\partial x} = \frac{w^n - w^M}{\lambda p}
\]

(4)

Obviously, any time wage differential \((w^n - w^M)\) changes, either because of change of \(w^n\) or because of change of \(w^M\), it will generate a substitution as well as an income effect. The substitution effect implies that when wage differential is reduced more time will be devoted to preferred artistic activity. It happens because, as expression (4) shows, shifting labor from non-pleasurable to pleasurable artistic activity is now less costly. It may happen for example as a result of increase of wage rate of artistic activity \(w^M\). It is, indeed, something that happens during the artist career as a result of his professional development. If, on the other hand, wage differential increase, as a result of increase of \(w^n\) for example, labor will be shifted from pleasurable to non-pleasurable activities: it is now more costly to shift labor from non-pleasurable to pleasurable activity. This effect may be weakened or even reversed by the income effect. It may easily happen that an increase in wage differential caused by increase of \(w^n\) results in shifting a labor from non-pleasurable to pleasurable activities. In such case higher income, resulting from an increase of wage rate of non-pleasurable activities, is used to “buy” time for pleasurable artistic activities. This is exactly something that characterizes artist behavior according to Throsby.

Apart from substitution and income effect there is price effect as well. Any increase of market goods price will reduce right hand side of expression (4). Conversely, any decrease of price of market goods will increase it. This will produce a substitution as well as an income effect. If price level increases, assuming overall income does not change, more time will be devoted to pleasurable activities: cost of substituting non-pleasurable for pleasurable activities is now smaller. Conversely, if price level decrease, assuming constant income, this cost will be higher and, as a consequence, more time will be devoted to non-pleasurable than to pleasurable activities. Note, however, that if price level decrease and, as a consequence, income increase, this will put in force income effect that may result with increase of pleasurable relative to non-pleasurable activities.

Above is even more obvious if we watch behavior of different labor shares in disposable time. Once we have solution of the model \((X^*)\) it is easy to calculate share of artistic time in overall disposable time. Substituting solutions in equation (2), dividing it with \(L\) and transforming we get

\[\frac{\partial u}{\partial L} = \lambda(w^n - w^M)\]

\[\frac{\partial u}{\partial x} = \lambda p\]

\[\frac{\partial c}{\partial L} = \frac{\partial u/\partial L}{\partial u/\partial x} - \lambda \left(\frac{w^n - w^M}{\lambda p}\right) = 0\]

\[\frac{\partial c}{\partial x} = \frac{\partial u/\partial x}{\partial u/\partial x} - \lambda p = 0\]

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\[ l^M = \frac{w^n - px^*}{w^n - w^M} \]  
(5)

where \( l^M = \frac{L^M}{L} \), while \( x^* = \frac{x^*}{L} \). This equation has following partials, which describe the responsiveness of artistic labor supply to changes in commodity price, arts wage, and non-arts wage:

\[
\frac{\partial l^M}{\partial p} = \frac{-x^*}{w^n-w^M} < 0
\]

\[
\frac{\partial l^M}{\partial w^M} = \frac{w^n-px^*}{(w^n-w^M)^2} > 0
\]

\[
\frac{\partial l^M}{\partial w^n} = \frac{px^*-w^M}{(w^n-w^M)^2} > 0
\]  
(6)

Again, as we see, model depicts artists as addicted to artistic work. Responsiveness to change in price in equilibrium is consistent with artists’ peculiar behavior. First, the more severe the budget constraints are, the less time artists will devote to artistic activities (first partial). Second, the higher the arts wage, the more time artist will devote to artistic activities (second partial). Finally and most interestingly, the higher the non-arts wage, the more hours artists will devote to their artistic activities (third partial). As Rengers and Madden (2000) noticed, the model is less spectacular for those artists whose arts wages are higher than non-arts wage, \( w^M > w^n \), and who, therefore, works on artistic activity only. In that case all inequalities in expression (6) turn to zero.

Let us now see what happen if we assume that artists sell their artistic products instead of their artistic time. Assume that quantity of artist products is, in that case, determined by artist production function of the following form

\[ M = M(L^M, X^M) \]

where \( X^M \) stands for quantity of market goods purchased for production of artistic products (raw material). If prices of artistic products are given by \( p^A \) and price of raw material \( s \) are given by \( p^M \), than constraint given in expression (2) becomes

\[ L^M w^n + p X + p^M X^M = Lw^n + M(L^M, X^M)p^A \]  
(7)

Accordingly, new Lagrange gets the following form

\[ \mathcal{L} = U(L^M, X) - \lambda [L^M w^n + p X + p^M X^M - Lw^n - M(L^M, X^M)p^A] \]  
(8)

Solution of this problem is

\[
\frac{\partial u}{\partial l^M} = \lambda \left( w^n - \frac{\partial M}{\partial L^M} p^A \right)
\]

\[
\frac{\partial u}{\partial X} = \lambda p
\]

\[
\frac{\partial M}{\partial X^M} p^A = p^M
\]

Since \( \frac{\partial M}{\partial L^M} \) presents marginal product of labor engaged at artistic activities it follows that \( \frac{\partial M}{\partial L^M} p^A \) presents value of marginal product of labor engaged at artistic activities. In
competitive market this should be equal to arts gross wage, \( w^M \). Obviously, this approach gives same solution as previous one.

Described model is static in its nature. This mean that, as Caserta and Cuccia (2001) noticed, “artist has no past and no future and that wage differential is entirely exogenous”. In what follow dynamic model of artist behavior will be developed. Past and future of the artist will be incorporated in it, while her wage rates and prices of her products will be endogenously determined.

3. A Dynamic Model with Arts Labor Hired

In order to develop sophisticated dynamic model of artist behavior we will rely on household production function approach and theory of allocation of time developed by Becker and his colleges.\(^5\) According to this approach consumers run production process using market goods, their own time and other inputs in order to produce commodities for the final consumption. “These commodities include children, prestige and esteem, health, altruism, envy, and pleasure of the senses, and are much smaller in number than the goods consumed” as Becker noticed once (Becker, 1991, p. 24). Meal, for example, according to this approach should be understood as a commodity produced using goods purchased, own time used for purchase of goods and cooking, and ability to cook as a kind of human capital. Similarly, appreciation of music, as a kind of commodity of “pleasure of sense”, is made by combining market goods or services (CDs, instruments, concerts, music lessons), own time devoted to it, and ability to appreciate music, which again depend on specific human capital of individuals. Consequently, our decisions about consumption of certain commodities are governed not solely by market prices of goods and services used in producing certain commodities but by shadow prices of commodities, which also include opportunistic price of our time, price of human capital, and prices of all other household resources involved in production of respected commodity. Therefore changes in pattern of demand of market goods and services may not necessarily be a result of changes in market prices and in our tastes but rather result of changes in the household production technology and / or in the inputs available for production.

More precisely, consumers are, according to this approach, supposed to maximize their utility function subject to the money income constraint, time constraint, and to a household production function constraint (Becker, 1965; Michael and Becker, 1973). Note that traditional theory of consumer behavior take into the account only money income constraint. As a consequence traditional approach gives as a result only allocation of household money income among different goods and services. The solution of new approach, on the other hand, apart from allocation of earning among different goods, provides allocation of time among work and consumption as well as allocation of time among different kind of consumption. Consequently, changes in shadow prices of commodities, which are governed by changes in market goods prices and wage rates,

will give rise to a different allocation of time and different allocation of money income of households. Note, however, that changes in allocations of earning and time may result not only from changes in market prices and wages but also from previous consumption history of individuals. Consumption of certain commodities and experience gained in that way may, in fact, result in increase of human capital that is relevant for future production of respected commodity. In this case, the cost of production and the shadow price of this commodity will be reduced as a result of this human capital accumulation. Consequently, the consumption of the commodity whose shadow prices falls relatively to others will be increased. This effect is now well known as addictive effect.

Stigler and Becker (1977), who first discussed this effect, choose music appreciation as an example of such an addictive commodity. According to this interpretation, consumption of music is never simple consumption of market goods or services, but rater consumption of output of productive process that combine market goods and services, consumers time, human capital and other inputs. Human capital expressed as ability to enjoy music is of a crucial importance. It is increasing function of weighted cumulative of previous consumption of music. The more time someone devote to music consumption, the more knowledgeable and perceptive he become, and in that way more productive he will be in the next time he consume music. This effect will reduce shadow price of music consumption and in that way make music more attractive relative to other commodities. Increased consumption of music will contribute to further increase of human capital, which in turn will further decrease shadow price and increase consumption of music. And so on. Same apply for all other kind of art appreciation: having artistic paintings or visiting artistic galleries and museums, looking dramas or operas in theatres, enjoying movies in cinemas or at homes by using CDs, and similar.

Artists, on the other hand, are prone to the same kind of addictive behavior toward art, but they are also paid for the time they devote to the artistic practicing. They “enjoy” in the time they devote to practicing art. In other words, they produce and consume commodity known as art. Quantity of this commodity in year $j$ of artist career will be presented by $M_j$. They also produce and consume all other commodities. In order to make things simple we will assume that quantity of all other commodities in year $j$ can be presented as commodity $Z_j$. In that case, utility function which is being maximized by artist household is given by

$$\sum_{j=1}^{n} \left( \frac{1}{1+\rho} \right)^j U[M_j, Z_j]$$

(9)

where $\rho$ stands for time preference rate, while $n$ presents remaining years of career of respected artist.

In order to produce commodity of art artists use market goods, their time, and human capital relevant for production of this commodity. This can formally be presented using following artists’ household production function

$$M_j = M[X_j, L_j^M, H_j]$$

(10)
where \( X_j^M \) presents quantity of market goods used in producing arts in year \( j \), \( L_j^M \) stands for time devoted to arts appreciation and production in year \( j \), while \( H_j \) stands for human capital used for that purpose in year \( j \). \( M_j \) is output of this production function, but it is also argument in utility function. This output will be from now on called simply art and will be expressed in some kind of efficiency unit. Each unit of time devoted to art will produce the same amount of art efficiency units as long as the amount of human capital stays the same. When the amount of human capital change, the number of art efficiency units per unit of time change. Note that this household production function differs from one used by Stigler and Becker (1977) to describe behavior of consumer of arts because it explicitly use market goods as argument of function. It also differs, in the same way, from production function proposed by Caserta and Cuccia (2001) for description of artist behavior. Although artistic market goods may be skipped when dealing with production function of arts consumers, it is pretty unrealistic to miss such an important input when dealing with production function of artist.

Production of all other commodities can be presented with household production function of the following form

\[
Z_j = Z[X_j^Z, L_j^Z]
\]

\( Z_j \) stands for the quantity of all other commodities in year \( j \), \( X_j^Z \) presents quantity of market goods purchased for the production of all other commodities in year \( j \) (purchase of food, shoes, clothing, and similar), while \( L_j^Z \) measure time used in production of these commodities in year \( j \) (time to buy goods, time to make meals, to put make up, and similar).

Human capital, which is argument in production function of artistic commodity, is itself function of previous artistic experience and production of art. This is how addictive effect enters in our analysis. It can formally be presented by following human capital production function

\[
H_j = h(M_{j-1}, M_{j-2}, M_{j-3}, ..., E_j)
\]

So, we assume that entire work history of artist can have influence on his human capital relevant for production of artistic commodity. In order to allow for influence of formal education on artists' human capital we also introduce \( E_j \) as a measure of artists' years of education.\(^6\)

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\(^6\)Existing researches, although scarce, seems to suggest three interesting conclusions regarding importance of human capital for artists’ productivity and for their earnings. First, years of schooling, as a measure of formal education, have no influence on artists’ earnings from artistic works. In some researches even negative coefficients have been obtained. Second, formal education has significant influence on artists’ earnings from non-artistic works. Finally, artistic earning is found to be influenced by years of artists’ experience, which measure on-the-job-training or learning by doing as it is sometime called. While first two results are surprising, last one is quite in line with findings for other professions. It is probably reason why this result has not been stressed enough in previous researches. Nevertheless, it is very important and it motivates approach taken in this paper. For more detailed survey and discussion of above issues
Important difference between artists and consumers of art is that artists are paid for their artistic work. More experienced they are and more human capital they have, they will be able to produce more art efficiency units. So, hourly wages of artists will be increasing function of their human capital. Formally

\[ w^M_j = w^M(H_j) \]  

(13)

In other words, hourly wage of artist in year \( j \) is increasing function of artists’ human capital attained in that year, \( H_j \). Obviously, their total earning from artistic work in year \( j \) is equal to \( w^M_j L^M_j \). It is important to have in mind that by \( w^M_j \) we do not understand the wage per hour of time in which artist is hired, but the wage per hour of entire time devoted to arts practicing. As a result, changes of \( w^M_j \) are a result of simultaneous changes of wage per hour of time in which artist is hired for her artistic works and consequent changes of share of time artist is being hired in total time devoted to art practicing \( L^M_j \).

Note, however, that artists unlike most of other professions and workers very often make their living by working other non-artistic jobs as well.\(^7\) Income earned by artistic practice is most of the time, especially in the early ages of their career, not enough to support their living and their artistic persuasion. So, they devote \( L^N_j \) units of disposable time in year \( j \) working at non-artistic jobs. Assuming for the sake of simplicity that wage at non-artistic work is constant and equal to \( w^N \), we conclude that their non-artistic income in year \( j \) should be equal to \( w^N L^N_j \). Their total earning from artistic and non-artistic work is therefore equal to \( w^N L^N_j + w^M_j L^M_j \). Above consideration is, of course, based on the assumption that hourly wages of non-artistic jobs are higher than that of artistic job. Reason why in this situation artists devote at all part of their disposable time to art is, of course, in the fact that, apart from gaining certain money income from artistic production, artists receive significant stream of nonmonetary benefits that artistic practice bring itself. If and when, in a later year of their career, artists wages reach level of non-artistic wages or above it artists devote their entire working time to artistic jobs. More generally, the higher the level of \( w^M_j \), the higher proportion of \( L^M_j \) in disposable time will be. Artist career is characterized with pretty stable increase of human capital and, therefore, with ever increasing value of \( w^M_j \), which naturally lead to increase of \( L^M_j \).

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\(^7\) Multiple job-holding is widespread among artists. It has been documented and analyzed by great number of cultural economists. See for example: Throsby (1992, 1994a, 1994b, 2007), Jeffri (1991), Rengers and Madden (2000), Alper and Wassall (2006), Abbing (2004). More subtle analysis would require, as Rengers and Madden (2000) and Throsby (2007) insisted, artistic working time to be divided on non-art, art related (like teacher of art, for example), and pure art working time. Within pure artistic working time it is also possible to make further distinction between those activities that are more artistically rewarding but less financially attractive and those activities that are less artistically rewarding but more financially attractive (Throsby, 2006, 2007). No doubt, even pure artistic jobs differ among themselves by the quantity of artistically rewarding activities in their content. All this can make empirical analysis of this market much more complicated than in other cases.
Relying on previous considerations, we can now introduce two additional constraints encountered by artists. First one is income constraint and it says that total artist income from work and from his wealth in year \( j \) should be equal to his market goods expenditure in respected year. More precisely and formally

\[
\sum_{j=1}^{n} \left( \frac{1}{1+r} \right)^{j} \left[ p^Z X^Z_j + p^M X^M_j \right] = \sum_{j=1}^{n} \left( \frac{1}{1+r} \right)^{j} \left[ w^n L^n_j + w^M L^M_j + b_j \right]
\]  

(14)

where \( r \) stands for interest rate, \( p^Z \) presents prices of market goods used for production of all other commodities, \( p^M \) presents prices of market goods used for production of artistic commodity, while \( b_j \) stands for income earned from artist wealth and other sources (social assistance or artists support programs, for example). Note also that, for the sake of simplicity, we assumed that prices of all market goods are constant in all considered years. Other important constraint that should be taken into the account is time constraint and it says that time used for artistic and non-artistic work and time used for production of all other commodities should be equal to artist disposable time in every year. Formally

\[
L^n_j + L^M_j + L^Z_j = L_j
\]  

(15)

where \( L_j \) presents artist disposable time in year \( j \). These last two constraints can be combined in the one of the following form\(^8\)

\[
\sum_{j=1}^{n} \left( \frac{1}{1+r} \right)^{j} \left[ p^Z X^Z_j + p^M X^M_j + w^n L^n_j + w^M L^M_j \right] = \sum_{j=1}^{n} \left( \frac{1}{1+r} \right)^{j} \left[ w^M L^M_j + w^n L_j + b_j \right]
\]  

(16)

What we got here is, using Becker terminology, “full income constraint” of artist. It differs from same constraints in the case of art consumer by part \( w^M L^M_j \) on the right hand side of the expression. It is quite natural: Artists not only enjoy dealing with art but also earn money income from it; Artists are working at artistic as well as non-artistic jobs.

Artist decision making process is now simplified and presented by expression (9) which should be maximized under constraints given by expressions (10), (11), and (16). Of course, before that, expression (12) should be substituted for \( H_j \) in expression (10), while expression (13) should be substituted for \( w^M L^M_j \) in expression (16). The problem can be further simplified by substituting values of \( X^Z_j, X^M_j, L^Z_j, \) and \( L^M_j \) in expression (16) by values of these variables derived from household production functions (10) and (11). In that case the decision making process that outlines artist behavior can be described by expression (9) which should be maximized subject to new modified constraint (16). In order to solve problem we form following Lagrange

\[
\mathcal{L} = \sum_{j=1}^{n} \left( \frac{1}{1+r} \right)^{j} U[M_j, Z_j] - \lambda \sum_{j=1}^{n} \left( \frac{1}{1+r} \right)^{j} \left[ p^Z X^Z_j + p^M X^M_j + w^n L^n_j + w^M L^M_j - w^M L^M_j - w^n L_j - b_j \right]
\]  

(17)

\(^8\) From expression (15) it follows that \( L^n_j = L_j - L^Z_j - L^M_j \). By substituting this for \( L^n_j \) in income constraint (14) and by rearranging we get expression (16) for full income constraint.
Since $X_j^\pi$ and $L_j^\pi$ are here derived from expression (11) for $Z_j$, it follows that $X_j^K$ and $L_j^K$ are both function of $Z_j$. On the other hand, since $X_j^M$ and $L_j^M$ are both derived from expression (10) for $M_j$, it follows that $X_j^M$ and $L_j^M$ are both function of $M_j, M_{j-1}, M_{j-2}$ and all other previous $M$. As we already know from (13), $w_j^M$ is function of $H_j$, which is, in turn, according to (12), also function of all previous $M$.

Solution of the problem is straightforward. First order condition requires that partial derivatives of Lagrange with respect to $M_j$ and $Z_j$ be equal to zero. For the sake of simplicity, second order condition will be skipped in the following consideration. Using described procedure we get following solution\(^9\) for commodity $Z_j$

\[
\frac{\partial U}{\partial Z_j} = \lambda \left( \frac{1+\rho}{1+\tau} \right)^j \left[ p^Z \frac{aX_j^\pi}{aZ_j} + w^n \frac{dL_j^\pi}{dZ_j} \right] = \lambda \left( \frac{1+\rho}{1+\tau} \right)^j \pi_{Zj} \tag{18}
\]

where $\pi_{Zj} = p^Z \frac{aX_j^\pi}{aZ_j} + w^n \frac{dL_j^\pi}{dZ_j}$ presents shadow price of commodity $Z_j$. This is very known solution for allocation of time and income derived by Becker in his already quoted works. It simply says that marginal utility of commodity $Z_j$ should be equal to marginal cost of all inputs involved in production of that commodity. Needless to say $\lambda$ presents, as usual, marginal utility of money income.

Using same procedure for $M_j$ we get following solution\(^10\)

\[
\frac{\partial U}{\partial M_j} = \lambda \left( \frac{1+\rho}{1+\tau} \right)^j \left[ \frac{w^M dX_j^M}{dM_j} + \frac{(w^n - w^M) dL_j^M}{dM_j} \right] - \lambda \sum_{v=1}^{n} \left( \frac{1+\rho}{1+\tau} \right)^j L_v \frac{dw^M_v}{dM_j} \]

\[
+ \sum_{v=j+1}^{n} \left( \frac{1+\rho}{1+\tau} \right)^j \left[ \lambda p^M_v \frac{dX_j^M}{dM_j} + \lambda (w^n - w^M_v) \frac{dL_j^M}{dM_j} \right]
\]

Before discussing obtained result we need to make some further transformations. Notice first that $\lambda p^M$ and $\lambda (w^n - w^M_v)$ in the last part of previous expression can be expressed as\(^11\)

\[\lambda p^M = \lambda (w^n - w^M_v)\]

\[\lambda (w^n - w^M_v) = \lambda p^M \]

---

\(^9\) This follows from the following first order condition for $Z_j$

\[
\frac{\partial L}{\partial Z_j} = \left( \frac{1}{1+\tau} \right)^j \frac{\partial U}{\partial Z_j} - \lambda \left( \frac{1+\rho}{1+\tau} \right)^j \left[ p^Z \frac{aX_j^\pi}{aZ_j} + w^n \frac{dL_j^\pi}{dZ_j} \right] = 0.
\]

\(^10\) This follows from the following first order condition for $M_j$

\[
\frac{\partial L}{\partial M_j} = \left( \frac{1}{1+\tau} \right)^j \frac{\partial U}{\partial M_j} - \lambda \left( \frac{1+\rho}{1+\tau} \right)^j \left[ \frac{w^M dX_j^M}{dM_j} + \frac{(w^n - w^M_v) dL_j^M}{dM_j} \right] - \lambda \sum_{v=1}^{n} \left( \frac{1+\rho}{1+\tau} \right)^j L_v \frac{dw^M_v}{dM_j} \]

\[= \lambda \sum_{v=j+1}^{n} \left( \frac{1+\rho}{1+\tau} \right)^j \left[ \frac{w^M_v}{dM_j} \frac{dX_j^M}{dM_j} + \frac{(w^n - w^M_v) dL_j^M}{dM_j} \right] + \lambda \sum_{v=1}^{n} \left( \frac{1+\rho}{1+\tau} \right)^j L_v \frac{dw^M_v}{dM_j} = 0
\]

\(^11\) In order to get these results we used alternative way of solving above decision making problem. We first substitute production function (10) and (11) in utility function (9). This utility function is supposed to be maximized subject to constraint (16). Lagrange now takes following form

\[
V = \sum_{j=1}^{n} \left( \frac{1}{1+\tau} \right)^j U[M(X_j^M, L_j^M, H_j), Z(X_j^\pi, L_j^\pi)] - \lambda \sum_{j=1}^{n} \left( \frac{1}{1+\tau} \right)^j \left[ p^Z X_j^\pi + p^M X_j^M + w^n L_j^\pi + w^n L_j^M - w_j^M L_j^M - w_j^M L_j - b_j \right]
\]

11
\[ \lambda p^M = \left( \frac{1 + r}{1 + \rho} \right)^j \frac{\partial U}{\partial M_j} \frac{\partial X^M}{\partial M_j} \]

\[ \lambda (w^n - w^M) = \left( \frac{1 + r}{1 + \rho} \right)^j \frac{\partial U}{\partial M_v} \frac{\partial X^M}{\partial M_v} \]

Now, by substituting this in last part of previous expression we get

\[ \frac{\partial U}{\partial M_j} = \lambda \left( \frac{1 + r}{1 + \rho} \right)^j \left[ \frac{p^M}{X^M} \frac{\partial X^M}{\partial M_j} + \frac{w^n - w^M}{X_v} \frac{\partial X_v^M}{\partial M_j} \right] - \lambda \sum_{v=1}^{n} \left( \frac{1 + r}{1 + \rho} \right)^j L_v \frac{d w^M}{d M_j} \]

\[ + \sum_{v=1}^{n} \frac{1}{(1 + \rho)^v} \frac{\partial U}{\partial M_v} \left[ \frac{\partial M_v}{\partial X_v^M} \frac{d X_v^M}{d M_j} + \frac{\partial M_v}{\partial L_v^M} \frac{d L_v^M}{d M_j} \right] \]

It can be further proved that expression in last bracket can be written as\(^{12}\)

\[ \frac{\partial M_v}{\partial X_v^M} \frac{d X_v^M}{d M_j} + \frac{\partial M_v}{\partial L_v^M} \frac{d L_v^M}{d M_j} = - \frac{\partial M_v}{\partial H_v} \frac{d H_v}{d M_j} \]

By substituting this result in previous expression we finally arrive with following solution

\[ \frac{\partial U}{\partial M_j} = \lambda \left( \frac{1 + r}{1 + \rho} \right)^j \left[ \frac{p^M}{X^M} \frac{\partial X^M}{\partial M_j} + \frac{w^n - w^M}{X_v} \frac{\partial X_v^M}{\partial M_j} \right] - \lambda \sum_{v=1}^{n} \left( \frac{1 + r}{1 + \rho} \right)^j L_v \frac{d w^M}{d M_j} \]

\[ - \sum_{v=1}^{n} \frac{1}{(1 + \rho)^v} \frac{\partial U}{\partial M_v} \frac{\partial M_v}{\partial H_v} \frac{d H_v}{d M_j} \]  

(19)

By rearranging it we get equally useful expression

\[ \left[ \frac{p^M}{X^M} \frac{\partial X^M}{\partial M_j} + \frac{w^n}{X_v} \frac{\partial X_v^M}{\partial M_j} \right] = \frac{1}{2} \lambda \left( \frac{1 + r}{1 + \rho} \right)^j \frac{\partial U}{\partial M_j} + \frac{1}{2} \sum_{v=1}^{n} \left( \frac{1 + r}{1 + \rho} \right)^j \frac{\partial U}{\partial M_v} \frac{\partial M_v}{\partial H_v} \frac{d H_v}{d M_j} + W_j^M \frac{\partial L_j^M}{\partial M_j} \]

\[ + \sum_{v=1}^{n} \frac{1}{(1 + \rho)^v} \frac{L_v^M}{L_v^M} \frac{d w^M}{d M_j} \]  

(20)

Expression (20) provides some important results. Left hand side of this expression presents value of all costs committed by artist in year \( j \) for production of unit of arts commodity. As we see this is very similar to shadow price \( \pi_{Z_j} \) obtained previously for commodity \( Z_j \) in expression (18). So, we can say that it presents shadow price of unit of

\[ \text{By solving for first order condition, that is by equating partial derivatives of this Lagrange with respect to } X_j^2, L_j^2, X_j^M, L_j^M, \text{ and } H_j \text{ to zero, we get a set of equations from which we can derive expressions } \]

\[ \lambda p^M = \left( \frac{1 + r}{1 + \rho} \right)^j \frac{\partial U}{\partial M_j} \frac{\partial X^M}{\partial M_j} \text{ and } \]

\[ \lambda (w^n - w^M) = \left( \frac{1 + r}{1 + \rho} \right)^j \frac{\partial U}{\partial M_v} \frac{\partial X_v^M}{\partial M_v} \]

These expressions are valid for every \( j \) and, therefore, for every \( v \), and that is exactly what we need for our further transformations.

\[ ^{12} \text{Since } \frac{d M_v}{d M_j} = 0 \text{ it is obvious that } \]

\[ \frac{d M_v}{d M_j} = 0 = \frac{\partial M_v}{\partial H_v} \frac{d H_v}{d M_j} + \frac{\partial M_v}{\partial L_v^M} \frac{d L_v^M}{d M_j} + \frac{\partial M_v}{\partial X_v^M} \frac{d X_v^M}{d M_j} \]

From it we easily derive above relation.
commodity $M_j$ produced in year $j$. On the right hand side we have four parts. Together, all of them present stream of benefits that artist has from this arts unit production in year $j$. First two elements present monetary equivalent of nonmonetary benefits generated by producing unit of arts, while the last two elements present monetary benefits generated by this production. First and third elements present benefits grasped immediately in year $j$. Second and forth elements, on the other hand, present streams of the benefits that are supposed to be generated from year $j$ to the end of artist career, given as a present value in year $j$. More precisely, first element on the right hand side presents monetary equivalent of nonmonetary benefits gained in year $j$ which is a result of “pleasure” that dealing with art itself provide to artists. Since, however, artist activity in year $j$ increases her future human capital in all years that follows up to the end of her career ($\frac{dH_U}{dM_j}$), it will inevitably contribute to the increase of the productivity of her arts production ($\frac{\partial M_U}{\partial H_U}$) in all years that follows. This, in turn, will contribute to the increase of her future “pleasure” of dealing with art ($\frac{\partial U}{\partial M_U}$) in all years that follow to the end of her career. This future stream of nonmonetary benefits is presented by the second part of the right hand side of expression (20). Third element is easy to understand: it presents wage earned in year $j$ from producing unit of art. However, monetary benefits do not end with this. Since artist activity in year $j$ contribute to the increase of human capital in all years that follows up to the end of artist career, and since artist future wages are influenced by this increase of human capital ($\frac{dw^M}{dM_j}$), we can expect artist activity in year $j$ to produce stream of wage increase in all years that follows up to the end of artist career. This stream of future monetary benefits is given by last part of expression (20). It is interesting to notice here that, from the formal point of view, streams of benefits of artist and consumer of arts differs exactly by last two elements in expression (20): these benefits are specific for artists and do not occur in the case of arts consumer.

4. A Dynamic Model with Arts Products Sold

In previous consideration we assumed that artists are paid for the time they devote to art practicing. In many cases this is pretty realistic picture of what is really happening on the market. Actors, singers, musicians, dancers and other artist engaged in so called performing arts are, for example, paid for their time being hired. In that case artistic organizations that hire them have its’ own production function and its’ own (profit, artistic quality or other) maximizing goals. Demands for artists’ labor and other inputs are in that case derived from this process of maximization under production function and other constraints encountered by these organizations. Other kinds of artists are, however, paid for their products, that is for what we notified previously with $M_j$. Creative painting is obvious example. Creative painters are paid for their pictures. Creative painting is obvious example. Creative painters are paid for their pictures.

In the case of creative painting it is necessary to make distinction between primary and secondary market of creative pictures. Our focus here is on primary market of picture and on prices of picture at that market. For more detailed exposition of primary and secondary market of creative paintings and of artistic works market in general see Heilbrun and Gray (2004, p. 165-187).
Same apply for sculptors, writers, composers, craftspeople and some other kind of so called creative artists.\textsuperscript{14} Decision making process is in this case somewhat different. Artists are paid for their products. Their household production function directly generates production function of the entire art industry. Labor supply function is derived from the process of maximization under these constraints. In what follows we will try to describe artist behavior in this very common situation.

Like in previous case artist are supposed to maximize utility function given with expression (9). Household production functions are also given by previous expressions (10) and (11). Human capital production function, which is supposed to be substituted in (10), is also like before given with expression (12). Time constraint that artist encounter is also same, expression (15). Their income constraint is, however, different than in previous case. Of course, they are, for the same reason as before, supposed to work at artistic as well as at non-artistic jobs. Their earning from non-artistic jobs is same as in previous case and it is given by $w^nL_j^n$. However, since they are selling their artistic products, their earning from artistic work is now given by following expression

$$p^A M_j = p^A M[X^M_j, L^M_j, H_j].$$

New element here is $p^A$ and it presents price of artistic product measured per efficiency unit. We supposed here that this market price is constant. Note, however, that this does not mean that price of artist works (paintings, for example) does not change during the artist career. On the contrary, prices of artist works will increase as a result of accumulation of human capital, which is given as a function of previous artistic experience of artist. To understand this notice that, as we already said, $M_j$ does not measure quantity of works but quantity of works of same efficiency units. It measures not just number of creative paintings, for example, but their quality as well. And the quality is what increase as a result of human capital accumulation. We may, for example, measure it in efficiency units of artist with no experience that is with zero years of experience. Number of paintings made by artist with $j$ years of experience can be in that case presented by $M[X^M_j, L^M_j, H_0]$. Previous expression for artist earning from artistic work can be transformed in following way

$$p^A M_j = p^A \left( \frac{M[X^M_j, L^M_j, H_j]}{M[X^M_j, L^M_j, H_0]} \right) M[X^M_j, L^M_j, H_0] = p^A M[X^M_j, L^M_j, H_0]$$

Note that

\textsuperscript{14} Although somewhat blurred, distinction between creative and performing artists is very useful one. Rengers and Madden (2000) pointed to seven important differences between them. First, creative artists are self-employed, while performing artists work on short time contracts. Second, and for this analysis most important, is related to previous one: creative artists are paid for their “products”, while performing artists are paid per hour hired. Third, creators are restricted by income constraint, while performers mostly have restriction regarding availability of works and contracts. Forth, creators work individually, while performers work with others. Fifth, work of creators is valued according to its’ innovations, while works of performers is characterized with craftsmanship and technical skill. Sixth, creators have high production costs, while performers have low production costs. Finally, creators are not unionized like performers.
\[
p_j^n = p^A \left( \frac{\frac{M[X_j^n, L_j^n, H_j]}{M[X_j^n, L_j^n, H_j_0]}}{\frac{M[X_j^n, L_j^n, H_j]}{M[X_j^n, L_j^n, H_j_0]}} \right) \tag{21}
\]
presents price of one painting or one artistic work in general of artist with \( j \) years of practice, while
\[
\left( \frac{M[X_j^n, L_j^n, H_j]}{M[X_j^n, L_j^n, H_j_0]} \right)
\]
stands for number of efficiency units per one painting / work of artist with \( j \) years of practice. More precisely it presents quality of the painting measured in efficiency units of artist first picture in career. It is obvious, from expression (21), that price of artist products are not constant and that they have their own time path during artist career. They are a function of artist human capital, and they increase with artist years of experience.\(^{15}\)

This picture is somewhat complicated by the fact that, especially now days with development of new reproducing technologies, in very many cases artists do not sell their products, but rather sell their copy rights to publishing and recording companies. This is case with writers, composers, some singers, and similar. In return they get stream of income, known as a royalty (10 to 15\% of earning), rather than lump sum of money in the form of price of artistic product \( p_j^n \). In order to make analysis simple, we will assume that in these cases artists also receive price \( p_j^n \) for their products. This price will be defined here as equivalent to the net present value of the expected stream of royalties.\(^{16}\)

Having this clarification in mind we can now provide following modified expression for artist income constraint
\[
\sum_{j=1}^n \left( \frac{1}{1+r} \right)^j \left[ p^Z X_j^Z + p^M X_j^M \right] = \sum_{j=1}^n \left( \frac{1}{1+r} \right)^j \left[ w^n L_j^n + p^A M_j + b_j \right] \tag{22}
\]
This constraint can be combined with previous constraints for disposable time, expression (15), to get the new one\(^{17}\)
\[
\sum_{j=1}^n \left( \frac{1}{1+r} \right)^j \left[ p^Z X_j^Z + p^M X_j^M + w^n L_j^n + w^n L_j^M \right] = \sum_{j=1}^n \left( \frac{1}{1+r} \right)^j \left[ p^A M_j + w^n L_j + b_j \right] \tag{23}
\]
Behavior of artists can now be outlined by expression (9) which is supposed to be maximized under constraints given in expressions (10), (11), and (23).

\(^{15}\) Note, however, that apart from time path of painters avarage price of picture, every picture produced in particular year \( j \) has its’ own time path. This time path is determined by the forces that determine movements on the secondary market of pictures (Heilbrun and Gray, 2004).

\(^{16}\) For interesting discussion on the issue of human capital and copy right see Towse (2006). Note also that in some countries even painters receive part of their income in the form of stream of income. This is case in all countries that have adopted so called resale right (droit de suite) according to which authors receive percentage of price every time her picture is resold (3\% in EU, 5\% in California). For more detailed discussion see Heilbrun i Gray (2004, p. 176).

\(^{17}\) From time constraint it follows that \( L_j^n = L_j - L_j^f - L_j^m \). By substituting for \( L_j^n \) in income constraint (22) and rearranging, we get full income constraint (23).
The problem can be further simplified by substituting values of \( X_1^2, X_1^1, L_1^2, \) and \( L_1^M \) in expression (23) by values of these variables derived from household production functions (10) and (11). The decision making process that describes artist behavior can now be described by expression (9) which should be maximized subject to newly modified constraint (23). In order to solve problem we can form following Lagrange

\[
\mathcal{L} = \sum_{j=1}^{n} \left( \frac{1}{1+\rho} \right)^j U[M_j, Z_j] - \lambda \sum_{j=1}^{n} \left( \frac{1}{1+\rho} \right)^j \left[ p^2 X_j^2 + p^M X_j^M + w^n L_j^2 + w^n L_j^M - p^A M_j - w^n L_j - b_j \right]
\]

(24)

Same as before, since \( X_j^2 \) and \( L_j^2 \) are both derived from expression (11) for \( Z_j \), it follows that \( X_j^2 \) and \( L_j^2 \) are both a function of \( Z_j \). Similarly, since \( X_j^M \) and \( L_j^M \) are both derived from expression (10) for \( M_j \), it follows that \( X_j^M \) and \( L_j^M \) are both a function of \( M_j, M_{j-1}, M_{j-2} \) and all other previous \( M \).

First order condition for the solution of the problem requires that partial derivatives of Lagrange with respect to \( M_j \) and \( Z_j \) be equal to zero. For the sake of simplicity, we again skip consideration of second order condition. Using described procedure we get solution for commodity \( Z_j \) which is exactly the same as one obtained previously in expression (18). As we know, marginal utility of commodity \( Z_j \) should be equal to shadow price of commodity \( Z_j \). Using same procedure for \( M_j \) we get following solution

\[
\frac{\partial U}{\partial M_j} = \lambda \left( \frac{1+\rho}{1+r} \right)^j \left[ p^M \frac{\partial X_j^M}{\partial M_j} + w^n \frac{\partial L_j^M}{\partial M_j} - p^A \right] + \lambda \sum_{v=j+1}^{n} \frac{1}{(1+\rho)^v} \left[ \frac{\partial p^M}{\partial M_j} \frac{dx^M_j}{dm_j} + \lambda w^n \frac{dx^M_j}{dm_j} \right]
\]

In order to make this solution more understandable we will further transform it. First, we can prove that \( \lambda p^M \) and \( \lambda w^n \) from second bracket of previous expression can be expressed as

\[18\]

\[
\frac{\partial L}{\partial M_j} = \lambda \left( \frac{1+\rho}{1+r} \right)^j \left[ \frac{\partial U}{\partial M_j} - \lambda \left( \frac{1}{1+r} \right)^j p^M \frac{\partial X_j^M}{\partial M_j} - \lambda \left( \frac{1}{1+r} \right)^j w^n \frac{\partial L_j^M}{\partial M_j} + \lambda \right] + \lambda \sum_{v=j+1}^{n} \frac{1}{(1+\rho)^v} \left[ \frac{\partial p^M}{\partial M_j} \frac{dx^M_j}{dm_j} + \lambda w^n \frac{dx^M_j}{dm_j} \right] = 0.
\]

\[19\]

To get this result we, again, used alternative way of solving above decision making problem. We first substitute production function (10) and (11) in utility function (9). This utility function is supposed to be maximized subject to constraint (23). Lagrange now takes new form

\[
V = \sum_{j=1}^{n} \left( \frac{1}{1+\rho} \right)^j U[M(X_1^M, L_1^M, H_j), Z_j(X_j^2, L_j^2)] - \lambda \sum_{j=1}^{n} \left( \frac{1}{1+\rho} \right)^j \left[ p^2 X_j^2 + p^M X_j^M + w^n L_j^2 + w^n L_j^M - p^A M_j - w^n L_j - b_j \right]
\]

By solving for first order condition, that is by equating partial derivatives of this Lagrange with respect to \( X_j^2, L_j^2, X_j^1, L_j^M, \) and \( H_j \) to zero, we get a set of equations from which we can derive expressions

\[
\lambda p^M = \left( \frac{1+r}{1+\rho} \right)^j \frac{\partial U}{\partial M_j} + \lambda \frac{\partial p^M}{\partial M_j} \text{ and }
\]

\[
\lambda w^n = \left( \frac{1+r}{1+\rho} \right)^j \frac{\partial U}{\partial M_j} + \lambda \frac{\partial w^n}{\partial M_j}.
\]
\[ \lambda p^M = \left( \frac{1+r}{1+\rho} \right)^V \frac{\partial U}{\partial M} \frac{\partial M}{\partial X^V} + \lambda p^A \frac{\partial M}{\partial X^M} \]

\[ \lambda w^n = \left( \frac{1+r}{1+\rho} \right)^J \frac{\partial U}{\partial M} \frac{\partial M}{\partial X^P} + \lambda p^A \frac{\partial M}{\partial X^W} \]

Now, by substituting these equations in last part of previous expression we get

\[ \frac{\partial U}{\partial M_j} = \lambda \left( \frac{1+r}{1+\rho} \right)^{j} \left[ p^M \frac{\partial X^M_j}{\partial M_j} + w^n \frac{\partial M}{\partial M_j} - p^A \right] + \sum_{v=j+1}^{\infty} \frac{(1+\rho)^v}{(1+\rho)^v} \frac{\partial M_v}{\partial M_j} \frac{\partial M}{\partial X^M_j} \]

\[ + \lambda p^A \sum_{v=j+1}^{\infty} \frac{(1+\rho)^v}{(1+\rho)^v} \frac{\partial M_v}{\partial M_j} \frac{\partial M}{\partial X^M_j} \]

Since expressions in second and third brackets are same and equal to\(^{20}\)

\[ \frac{\partial M_v}{\partial M_j} + \frac{\partial M_v}{\partial X^M_j} = \frac{-\frac{\partial M_v}{\partial H_v}}{\frac{\partial H_v}{\partial M_j}} \]

by substituting we finally get

\[ \frac{\partial U}{\partial M_j} = \lambda \left( \frac{1+r}{1+\rho} \right)^{j} \left[ p^M \frac{\partial X^M_j}{\partial M_j} + \left( \frac{w^n-p^A \frac{\partial M}{\partial M_j}}{\frac{\partial M}{\partial X^M_j}} \right) \right] \]

\[ -\lambda p^A \sum_{v=j+1}^{\infty} \frac{(1+\rho)^v}{(1+\rho)^v} \frac{\partial M_v}{\partial M_j} \frac{\partial M}{\partial X^M_j} - \sum_{v=j+1}^{\infty} \frac{(1+\rho)^v}{(1+\rho)^v} \frac{\partial U}{\partial M_v} \frac{\partial M_v}{\partial H_v} \frac{\partial H_v}{\partial M_j} \]

(25)

By rearranging we get following useful expression

\[ \left[ p^M \frac{\partial X^M_j}{\partial M_j} + w^n \frac{\partial M}{\partial M_j} \right] = \]

\[ \frac{1}{\lambda} \left( \frac{1+r}{1+\rho} \right)^{j} \frac{\partial U}{\partial M_j} + \frac{1}{\lambda} \sum_{v=j+1}^{\infty} \frac{(1+\rho)^v}{(1+\rho)^v} \frac{\partial U}{\partial M_v} \frac{\partial M_v}{\partial H_v} \frac{\partial H_v}{\partial M_j} + p^A \left( \frac{\partial M_v}{\partial M_j} \frac{\partial M}{\partial X^M_j} \right) + p^A \sum_{v=j+1}^{\infty} \frac{(1+\rho)^v}{(1+\rho)^v} \frac{\partial M_v}{\partial H_v} \frac{\partial H_v}{\partial M_j} \]

(26)

By comparing expressions (25) and (26) with previously derived expressions (19) and (20), we notice striking similarity among them. Notice first that element \( p^A \frac{\partial M}{\partial M_j} \) in the first bracket in expression (25) presents value of marginal product of labor, which increases with \( j \), and which is equal to wage paid for artistic work. Formally

\[ p^A \frac{\partial M}{\partial M_j} = w^j \]

Having that in mind we conclude that first part of expression (25) is equal to the first part of expression (19). It is intuitively clear, but it can be formally proved, that same equality exists among second part of expression (25) and second part of expression (19). They present expected value of increases of earnings from \( j + 1 \) year to the end of artist career, caused by unit production in year \( j \) and given in present value in year \( j \). Finally, third parts of compared equations are equal by definition, and they present expected streams of nonmonetary benefit increase that unit production in year \( j \) generates to

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These expressions are valid for every \( j \), and therefore for every \( v \), and that is exactly what our solution is.

\(^{20}\) See footnote 12.
the end of artist career. Looking at expression (26) we see that its’ left hand side again present shadow price of unit of commodity $M_j$ produced in year $j$. Right hand side of this expression presents as before stream of all benefits generated by unit of commodity $M_j$ up to the end of artist career. To conclude, expressions (25) and (26) have same meaning as already discussed expressions (19) and (20) and their implication will be considered together in the next section.

5. Implications

As we already noticed, expressions (20) and (26), stream of benefits generated by producing commodity of art is much larger than that of ordinary goods, $Z_j$, implied by expression (18). As a consequence, artists will be motivated to allocate relatively much greater part of their resources in the art than in ordinary commodities, $Z_j$. Same can be seen by looking at the expression (19) and (25) and comparing them with expression (18). As we see, expression (19) differs from expression (18) for ordinary commodities by three additional parts. First, value of labor resources used in production of art ($M_j$) given in the bracket is reduced by the value of artist earning from artistic work ($M_j$). Similar effect is present in Throsby (1994) model. Second, shadow price of producing commodity of art is further reduced by the value of stream of increases of future wages caused by arts unit production in year $j$, $\sum_{v=j}^{n} \frac{(1+\rho)^v}{(1+r)^v} \lambda M_j \frac{dM_j}{dM_j}$. Finally, it is also reduced by the value of stream of increases of future nonmonetary benefits caused by arts unit production and consequent human capital creation in year $j$, $\sum_{v=j+1}^{n} \frac{(1+\rho)^v}{(1+r)^v} \lambda \frac{dM_j}{dM_j} \frac{dH_v}{dM_j}$. Last two effects are not present in Throsby work preference model. Similar conclusions follow from examination of expression (25).

As a consequence, marginal rate of substitution ($MRS$) between commodity $Z_j$ and commodity $M_j$, given by

$$MRS = -\frac{\Delta z_j}{\Delta M_j} = \frac{\frac{du}{dz_j}}{\frac{du}{dz_j}} = \frac{\lambda (\frac{1+\rho}{1+r})}{(\frac{1+\rho}{1+r})} \frac{\frac{dM_j}{dM_j}}{\frac{dM_j}{dM_j}} \left[ 1 + \sum_{v=j+1}^{n} \frac{(1+\rho)^v}{(1+r)^v} \lambda \frac{dM_j}{dM_j} \frac{dH_v}{dM_j} \right]$$

will be at the point of optimum much smaller than that of two ordinary goods. As a result, artists will devote much higher share of their resources to art than what will be the case if we suppose that $M_j$ is another ordinary good. In fact, they will devote much more of their labor and other resources to arts than what Throsby (1994) model would suggest (see expression (4)). Two additional effects, not captured by Throsby’s model, are responsible for it. First is effect of expected stream of future monetary benefits (second term in numerator). Second is effect of expected stream of future nonmonetary benefits (third term in numerator). Although not present in Throsby’s model, these effects are, in fact, in line with Throsby sort of argument. They further strengthen
importance of nonmonetary benefits in explaining artists’ pure market performance and earning penalties evidenced in numerous researches on artists earning.\(^{21}\)

Note, however, that it is not easy to say what would be time pattern of \(\frac{\partial U}{\partial M_j}\) and \(MRS\) during the artist career. There is no reason to believe that it will be ever decreasing by the passage of time, as one might be prompted to conclude. Human capital creation resulting from art practicing has, in fact, two contradicting course of influence on these two values. On the one hand, it has ever decreasing influence on the first part of expressions (19) and (25) given in bracket. This decreasing effect is twofold. First, by the increase of \(j\) it is natural to expect increase of artist human capital and consequent increase of artist wage from artistic job, \(w^M_j\). This, in turn, will cause decrease of shadow price of commodity \(M_j\), that is decrease of the first part of expressions (19) and (25). Second, constant increase of human capital by the passage of time will increase both marginal productivity of arts labor \(\frac{\partial M_j}{\partial L^M_j}\) and marginal productivity of arts market goods \(\frac{\partial M_j}{\partial X^M_j}\). This, in turn, will contribute to the further decrease of the first part of expressions (19) and (25). As a consequence, older artist are by passage of time motivated to devote more resources to art commodity and art production. Specifically, owing to this effect at the certain point of time in their career artists devote their entire working time to the artistic jobs. Formally speaking \(L^M_j\) becomes equal to zero, artists stop working two kinds of jobs, and their total earning becomes equal to \(L^M_j\ w^M_j\). There is plenty of empirical evidence to support this effect.

On the other hand, younger artist are also motivated to invest much of their resources in practicing of art. Their shadow prices of art commodity are reduced by the second and third element of expressions (19) and (25). These two parts, as already told, present expected streams of monetary and nonmonetary benefits increases that practicing of arts in year \(j\) bring to artist up to the end of their career. Quite naturally, the younger the artist the higher this effect is. In the case of very old and experienced artist this effect can vanish indeed. This effect explains why young artists have such a strong drive for their profession, even though their wages from artistic work are very low. It seems safe to say that young artists have almost absolute preferences toward practicing of art once their basic needs are satisfied. This effect explain why young artists usually work two jobs, artistic as well as non-artistic, and why great number of the young artists experience poverty during their career. Paradigmatic, in that sense, is story reported by Abbing (2004) according to which different financial subsidies and other programs of Dutch government to reduce poverty among creative painters had not been followed by reduction of poverty as expected but by two unexpected results both supporting Throsby (1994) idea that young artists have almost absolute preference toward art practicing once their basic needs are satisfied. First, introduction of different subsidies was followed by reduction of time young artist devote to non-artistic works. Second, number of artists had increased by the passage of time after introduction of programs.

Both phenomena can, obviously, be explained by income effect. Surprising is, however, sharp influence of this effect at such a low level of income.

So, we have two contradicting sets of factors that have influence on movement of $\frac{\partial U}{\partial M_f}$ and $MRS$ during artist career. One of them, current arts wage and productivity of resources used in art production, which are result of artists “history”, have decreasing influence on movement of above variables. In other words, they make substitution effect stronger with passage of artist career. Second set of factors, expected monetary and nonmonetary benefits, which present artist “future”, on the other hand has increasing influence on their movement. In other words, they make substitution effect weaker with passage of artist career. Answer to the question which of these two effects is stronger is empirical one. Empirical facts seem to support thesis that first effect is stronger and that overall substitution effect increase with passage of artist career. Most important in that respect is the fact that young artist devote much more of their working time to non-artistic jobs than older artists. Older artists, in fact, very often, at the certain point of time, stop working non-artistic jobs. Needless to say, answer to this question will be different for different kind of arts and to different artists due to the specific characteristics and circumstances.

There are several striking peculiarities of arts labor market that have attracted attentions of researchers in last several decades. First, artists pay significant earning penalties and have lower level of average earning than ocupacions with similar level of education. Second, there are huge variations in artists earnings and huge unequalities among artists themselves. Third, there is constant excess supply of arts labor and related constant unemployment at arts labor market. Finally, as a consequence, artists are more likely to be multiple-job-holders than other professios. Several hypotheses have been offered so far in explaining these peculiarities. Let us see where theoretical models developed here belong and how they mach with existing hypotheses.

First hypothesis is one proposed by Throsby (1994) according to which dealing with arts, apart from monetary benefits, bring huge amount of nonmonetary benefits to artists. Models developed here, as already noticed strengthen this argument even more by adding effect of expected stream of future nonmonetary benefits as a motive for dealing with arts. Expected stream of utility derived from artistic practice can, of course, take form of pure pleasure derived from artistic work as such, as Throsby insists, but can also take form of excitement from expected recognition by peers and artistic public in general, and more generally form of expected nonmonetary benefits derived from self-discovery and self-actualization.

Second hypothesis is one proposed long ago by Adam Smith and advocated recently as a possible explanation by Towse (2006). According to this explanation young people tend to enter artistic labor market too frequently because they overestimate their talent and likelihood of their future success. In other words oversupply of artists is a result of special kind of myopia that is inherent to young people. Models offered in this paper

\[22\] For more detailed survey and analysis of arts market peculiarities see Alper and Wassall (2006), Towse (2006), and Menger (2002, 2006).
also allow for such an explanation. Once we understand future monetary and
nonmonetary benefits not as a exact one, but rather as expected one, and once we
allow that these expectations can be, for some reasons, systematically overestimated,
models proposed here offer a room for such interpretation as well. Weather there is
ground for such systematic overestimations of expectations is debatable question,
however (Alper and Wassall. 2006).

Third hypothesis is offered by Santos (1976), who claims that artists belong to a class of
risk-taking workers who are willing to trade off a small chance of huge financial rewards
for a much larger chance of low earnings. By dealing with expected streams of earnings
models offered in this paper can be used as a framework for this kind of thinking as well.
Santos’s hypothesis can explain high variations in earning of artists, as well as excess
supply of arts labor. However, it cannot explain earning penalties that characterize
artists earning. Even more importantly, Santos does not explain why artistic occupations
would attract such disproportionate number of risk-takers.

Forth hypothesis can be interpreted as one which insists that artistic occupations do not
attract disproportionate number of risk-takers but disproportionate number of self-
actualization-seekers. Self-actualization and self-discovery is characterized with
permanent learning by doing and permanent search for innovations. This is not possible
within routine activities that characterize ordinary jobs. Uncertainty is *sine qua non* for
such kind of persuasion. As we know, there is plenty of uncertainty and plenty of
possibilities for satisfaction of this motive in all kind of artistic occupations. It is,
therefore, neither myopia nor risk-seeking behavior that explain artist occupational
choice, but rather a fact that uncertain artistic occupations offer plenty of possibilities
for self-actualization. This idea, which also has long history - Marx, Hegel, Aristotle - has
recently been advocated most prominently by Menger (2006). Models developed here
fit perfectly with this important explanation: what we call expected stream of future
non-monetary benefits refers mostly to artists need for self-actualization and self-
discovery.

Finally, last hypothesis is one that can be derived from theory on the earning of
superstars (Rosen, 1981; Adler, 1985, 2006). Unlike second and third hypothesis this one
is able to provide explanation for earning penalties as well as for huge variation in artists
earning and excess supply of arts labor. Superstars in arts, sports etc. are individuals
who are able to attain enormous success in their profession and whose earnings are
significantly greater than that of their competitors. Rosen (1981) claims this
phenomenon to be result of interaction of two factors, one on the demand side and
other on the supply for arts side. On the demand side of the story there is hierarchy of
talent and preference of consumers to consume most talented artist. On the supply side
there is nearly perfect (costless) reproducibility of art (especially performing arts) that
occurs as a result of technological advancement (CDs for example). In that
circumstances every consumer is able to consume best art, while most talented artist is
able to capture whole market. This is known as winner-take-all situation. Adler (1985,
2006) proved that existence of superstars cannot be explained solely by differences in
talent. According to his explanation there are a lot of artists who poses stardom-quality
talent. What produce superstars is the need on the side of consumers to consume the
same kind of arts that other consumers do. This need develops from the fact that consumption of art is not momentary experience but a dynamic process that is based on previous artistic experience and knowledge accumulated in that way. This accumulation of knowledge is, however, not a result of consumers own ability to learn and judge about intrinsic value of artistic products. Rather, it is a result of complex social interaction and social processes able to induce path dependency phenomenon in consumption of art. Owing to Adler amendments theory of superstar becomes relevant not only for performing but also for creative arts as well. No doubt, theory of superstar has path-breaking importance in explaining peculiarities of arts labor market. It, however, do not contradict to the explanation offered here. On the contrary, two explanations are complementary. Stardom explanation is about demand side of arts labor market. Explanation offered here is about supply side of that market: it explains why some people indulge in such peculiar market at all.

6. Concluding Remarks

Before analyzing some other cases where proposed models can be applied, let us note at the beginning of this section that above given models can be used to describe behavior of arts consumers as well. In that particular case current and expected monetary benefits, given by the third and forth part of expression (20), vanish and disappear from the equations. Consumers of arts get only current and expected nonmonetary benefits from arts. More precisely, they get current and expected pleasure of consuming arts, given by first and second part of expression (20). This is very similar to the result provided by Stigler and Becker (1977) except that models developed here capture all resources used for arts consumption. It capture not only consumer own time but market goods and services that should be purchased (pictures, gallery tickets, CDs, concert tickets) for this consumption as well. Having that in mind and knowing that these models can describe behavior of artist as well as behavior of arts consumers, we can say that models proposed in this article are bit more general than that proposed by Stigler and Becker (1977).

Models proposed in this paper can also be regarded as more general because they describe not only behavior of artists but also behavior in all those cases where work itself bring pleasure to workers, as well as in all those cases where previous consumption has influence on current shadow prices of commodities. Art is only paradigmatic case in which these widespread phenomena are most obvious and easy to understand. Work of scientists is also very obvious case although they rarely experience poverty stage during their career as artists do. Their time earning profiles can prove, however, that expected stream of monetary benefits, last part of expression (20), plays important role in explaining their behavior in the early ages of their career. More importantly, their readiness to accept much lower wage rates compared to that in consulting or R&D activities within companies can be easily explained by the fact that their stream of nonmonetary benefits, first and second part of expression (20), is significant indeed. In fact, it is so significant that it becomes decisive for their decision to deal with science. No doubt, a lot of scientific results, which are crucial for growth of our standard of living, are paid by the mere pleasure that scientists derive from their works.
Same applies for journalists especially those dealing with investigative journalism. Main motivation for their work comes not from monetary benefits but from current and expected nonmonetary benefits that their work provides to them. There are, no doubt, a lot of other professions that can and should be analyzed in similar manner. What is more important, it seems that, as a result of technological advancement, number of such professions is growing. Technological progress has dramatically increased, and it is expected to increase even more, demand for so called creative works. Models developed here can be used for analysis of creative worker behavior in general.

Another interesting phenomenon that can be explained using above models is nonpaid work of volunteers. In many cases volunteers’ readiness to work for free can be simply explained by the stream of nonmonetary benefits that such engagements bring to them. In most of the cases, however, other reasons may be even more important. Work of volunteers is very often explained by the fact that working as volunteers in the field of your own profession, while living from the income earned working some other job can help you develop your profession and your resume to the level that can help you to get position in your own preferred field of work. This in turn is supposed to increase your future monetary earnings as well as your future nonmonetary benefits. More formally, current volunteers wages, part three of expression (20) \( w_j^M \frac{\partial u_j}{\partial M_j} \), are equal to zero, but their expected stream of monetary benefits, last part of expression (20)

\[
\sum_{j=0}^{n} \left( \frac{1+r}{1+r} \right)^j L_j \frac{w_j^M}{M_j} \frac{\partial dM_j}{dM_j} \]

as well as their expected stream of nonmonetary benefits, first and second part of expression (20)

\[
\frac{1}{\lambda (1+r)} \left( \frac{1+r}{1+r} \right)^j \frac{\partial u_j}{\partial M_j} + \frac{1}{\lambda} \sum_{j=0}^{n} \left( \frac{1+r}{1+r} \right)^j \frac{\partial u_j}{\partial M_j} \frac{\partial dH_j}{dH_j} \frac{\partial dM_j}{dM_j}
\]

large to make such an engagement very profitable indeed, and to motivate young professionals to spent a good deal of their disposable time working for free.

Currently volunteering work of professionals is not so widespread phenomenon as it might become in the future according to some analysts. On the contrary, what we experienced in last three decade within developed countries is constant increase of wage premium paid to skilled labor (college graduates and above). This has occurred in spite of the fact that share of skilled labor has increased dramatically. Most convincing explanations offered so far is one according to which due to skill-biased technological progress demand for skilled workers has increased even faster than its’ supply (Krusell, at al. 1997). If supply of skilled labor continues to grow at existing rate and if, due to creative-labor-biased technological progress that seems started by nineties, demand for labor shifts more toward creative than simply skilled labor, we may easily come in situation to experience excess supply of skilled labor. In that case volunteering work among professionals might become widespread indeed. Volunteering may become important screening mechanism for unveiling creative abilities of young professionals. In general, whole market for professionals might take characteristics that are now regarded to be exclusive peculiarities of arts labor market.

Even more interestingly, technological progress is, at the same time, making all types of jobs easier to work. Galor and Weil (1993) developed growth model that stylize the facts that since the end of sixties female participation in labor force and female wage rates
have been growing relative to that of males. Those processes are explained by the fact that technological progress in the last half of twenty century reshaped requirements for almost all kind of jobs by reducing dramatically “masculine” requirements and by increasing “brain” requirements, making, in that way, almost all jobs affordable to females and, consequently, increasing supply of labor in developed economies. To say that “masculine” requirements are reduced is somehow same as to say that disutility of work is reduced. Disutility of work, on the other hand, is nothing but negative value of what we call pleasure from work. More formally it is negative value of first part of our expression (20) and (26), \[
\frac{1}{\lambda} \left( \frac{1+r}{1+\rho} \right) \frac{\partial U}{\partial M_j}.
\]
By allowing this element to be negative as well as positive models developed here become even more general and able to explain much wider span of economic and social phenomena than what its’ title suggest. Needles to say, apart from heaviness of work, there are a lot of other sources of disutility of work, like working conditions, ecological environment and similar. Of course, in these particular cases wages of non pleasurable occupations should be larger than wages of alternative less difficult occupations. \( M_j \) can and should, in that case, be treated as “bad” or “discommodity”. Difference between wages of more unpleasant and less unpleasant works presents equalizing differences in sense explained by Rosen (1986). Note, however, that in our case wage differences can compensate not only for current differences in disutility of work, but also for differences in expected stream of disutility, second part of expressions (20) and (26), that may be caused by current work. Expected stream of disutility may, for example, take form of deteriorated quality of life resulting from health problems induced by inadequate working conditions.

Finally, it is well known that great number of the companies invest in the development of cultural and social environment within company, which is supposed to increase productivity of their workers. It is even more important now that, due to IT revolution, hierarchical structure of company become flatter and replaced with team work structure. On the other hand, in the light of the fact that in modern times we spend much more of our time in working place than at home, it is obvious that such kind of behavior can increase our welfare dramatically indeed. This phenomenon can also be captured by the models developed here. We can say that investment in companies’ cultural and social environment increase stream of utility (or decrease stream of disutility) we get from working in company with healthy interaction and interpersonal relationship among employees. Formally, it increases first and second part of our expressions (20) and (26). Again, since we spent most of our time at working place, competition among companies for labor force by usage of this kind of investment, can be of enormous importance for the welfare of whole society. Unfortunately, due to internal competition among employees and to a lack of leadership, a lot of times companies’ cultural and social environment develops toward quite unpleasant one. In that respect, we can mention increasing number of reports on mobbing, gossips and rumors within company, and all forms of pervasive competition among employees within companies.
References


