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The economics of early social stratification

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Abstract

We develop an endogenous fertility model of social stratification with two hereditary classes: a warrior elite and a peasantry. Our model shows that the extra cost warriors must incur to raise their children and to equip them for war is the key determinant of (1) the relative sizes of both classes and (2) the warriors' economic privileges in terms of income and consumption. Higher costs of warrior children imply greater economic privileges for warriors and a smaller ratio of warriors to peasants. Historical evidence confirms this prediction. Finally, we identify conditions under which the military function of warriors may legitimise their privileges.

Keywords: Social stratification; income inequality; warfare; military participation ratio; Malthus; economic history; economic sociology; economic anthropology; population economics.
JEL codes: J10, N3, N4, Z1.

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1 Introduction

Most anthropological and sociological theories of early social stratification share three recurrent themes. First, early stratified societies divide labour between warriors, who fight wars, and peasants, who work the land. Second, a society must be able to produce a sizable food surplus (i.e., more food than is needed to feed the peasants and their families) in order to support a non-food-producing warrior elite. Third, social positions in early stratified societies are, for the most part, hereditary (Summers, 2005).

We integrate these three themes into an endogenous fertility model that accounts for the demographic forces that influence social stratification. We investigate two issues: what determines the class composition in a stratified society and what determines the degree of economic privilege of its upper classes.

Our model produces a set of predictions that can be tested against historical evidence. First, in equilibrium, the average warrior enjoys higher income and higher consumption than the average peasant. Second, as the relative cost of warrior children increases, the size of the warrior elite relative to the peasantry falls, and the warriors' economic privileges increase. Third, taking land, technology, and prices as given, total population will be lower in a stratified society.

We also identify conditions under which the military function of warriors may legitimise their privileges. If specialised weapons are very effective, or if training and equipping warriors is cheap, the military contribution of warriors will outweigh the burden they impose on the rest of society.

We depart from previous work by focusing our analysis on individual choice instead of treating social classes as organic units (the traditional organicist approach). The formal tools of economics allow us to identify some of the mechanisms underlying the stratification of human societies.

The rest of this introduction reviews the sociological and anthropological theories of social stratification and presents the main empirical regularities. It also covers the basics of Malthusian population theory. In Section 2 we present our model, discuss its predictions, and test them against the historical evidence. Finally, we summarise the predictions of the model and provide the intuition in Section 3.

1.1 Agriculture and social stratification developed together

Evidence of early social stratification, mostly in the form of artefacts interred with the dead, is first found in the same archaeological stratum as the oldest vestiges of agriculture (Angle, 1986). Ethnographic studies reveal that most contemporary bands of hunter-gatherers, such as the Kung in the Kalahari or the Yolngu in Arnhem Land, are egalitarian and devoid of leadership (Boehm, 1999; Knauft, 1994; Winterhalder, 2001). We do not know how prehistoric hunter-gatherers organised themselves, but the observation of their contemporary remnants and the archaeological evidence indicate that prehistoric hunter-gatherers lacked social stratification. All early agrarian societies, on the contrary, were socially stratified (e.g., Sumer, Ancient Egypt, and Mycenaean Greece). The parallel emergence of agriculture and social stratification suggests that these two developments were related in some way..

The ability to produce a surplus of food lies at the core of many theories of social stratification. The argument runs as follows. Surplus food is required to support a non-food-producing upper class. Agriculture has the potential to yield a surplus, which explains why agrarian societies can be stratified. Hunter-gatherers, on the other hand, are always living on the edge of subsistence: chronically undernourished and constantly threatened by famine [evidence of this is surveyed by Kaplan (2000)].. They are thus unable to afford a class of non-food-producers. Gordon Childe (1942, p. 18; 1954) is the foremost surplus theorist of social stratification. Unsurprisingly, his ideas are very popular among Marxist thinkers (e.g., Beaucage, 1976, pp. 409–410; Mandel, 1962, pp. 26, 43).

Surplus theories of social stratification have several critics. Pearson (1957), for instance, argues that all societies have the potential to produce food in excess of biological necessity. According to Pearson, the social organisation generates a surplus and not the other way around. Sahlins (1972/1998) maintains that the key precondition for social stratification is not the ability to produce surplus food, but the feasibility of food storage [see Cashdan (1980) and Hayden (1995)]. Without storage, Sahlins argues, there is no accumulation of wealth, and without wealth, social inequalities cannot exist. Most hunter-gatherers are nomads. As they quickly deplete local resources, they have no alternative but to keep moving. Nomadism makes storing food, and thus stratification, impossible. It comes as no surprise to Sahlins that the few reported cases of stratified hunter-gatherer communities are all located in exceptionally favourable ecological niches, where the abundance of food allows for permanent settlement and thus for food storage [Testart (1982) surveys the evidence]. The Pomo people of Central California are a classic example of a stratified gathering society. Acorns, the staple of the Pomo diet before modernisation, were only available during one month in autumn. During that month the Pomo gathered the acorns and stored them for the rest of the year. The acorn stores were controlled by the chiefs (Kniffen, 1939).

Whatever the preconditions to stratification may be (surplus, storage, or both), the emergence of an upper class of non-food-producers remains to be explained. Two opposing explanations have been proposed: a conflict-based explanation, advanced by Fried (1967; also see Hayden, 1995), and a functionalist explanation, attributed to Service (1962; also found in Davis, 1949, p. 367). Conflict theorists hold that 'aggrandisers' seized control of the means of production and then used the surplus to obtain a superior standard of living. The functionalists, on the other hand, believe that the upper classes provide goods that benefit society as whole: they lead war parties and organise defence, build and maintain irrigation systems, store food as famine relief, and manage intergroup trade. As a reward for their services, the lower classes allow the upper classes a greater share of society's wealth.

Intermediate positions have emerged between these extremes. For instance, Johnson and Earle (2000) maintain that the intensification of agriculture and consequent population growth pose a number of problems that can only be solved through hierarchy and the centralisation of power: resource competition leading to raids and warfare, the risk of failure in food production, inefficient use of resources that call for major technology investments, and resource deficiencies that can only be made up by foreign trade (pp. 29–32). Once power is acquired by an upper class, that group uses its power to establish privileges for itself (pp. 266–277, 301–303). At the same time, the lower classes face a trade-off between the benefits they derive from the public goods provided by the upper classes and the burden of inequality net of the cost of revolting (Boone, 1992).

1.2 The rise of an hereditary warrior elite

The intensification of agriculture required people to abandon nomadism and become sedentary. Sedentism, in turn, created competition for the most productive soils and the opportunity to ransack the food stores of neighbouring communities. As a result, warfare escalated among early agrarian societies (Johnson and Earle, 2000, p. 252; Rowthorn and Seabright, 2009). The mounting demands of war triggered dramatic enhancements in military technology (Ferguson, 2003), creating the need for professional warriors who could handle it (Carneiro, 1970; Webster, 1975). The increased effectiveness of weaponry increased the fighting advantage of warriors over peasants. It is a common view among anthropologists and sociologists that the monopoly of weapons allowed warriors to prevent upward social mobility and become a hereditary social class (Summers, 2005).

According to Andreski (1968, pp. 31–32), a class of warriors can emerge in two ways: either by gradual differentiation of warriors from the rest of the population, or by conquest and subjugation of another group. Gradual differentiation occurs when a group manages to monopolise arms-bearing in order to secure a privileged position in society or if the professionalisation of warriors is necessary for society to augment its military power. Andreski maintains that conquest was the most common mechanism of social stratification and provides a long list of historical cases to back up his claim: the subjugation of one city by another in Sumer (p. 42), the Dorian invasions in Greece (p. 43–44), and the Norse conquest of Russian Slavic tribes (p. 62), to mention just a few. Perhaps the chemically purest examples of stratification by conquest can be found in East Africa and Sudan. In those regions, ample kingdoms were founded through the conquest of agriculturalists by pastoralists (p. 32). In Ankole, for instance, the pastoralist Hima conquered the agricultural Iru sometime before the British colonisation. The Hima forced the Iru to pay tribute and allowed them no political rights. Only Iru men were allowed to bear arms and participate in war.

1.3 Social mobility

Betzig (1986) and Summers (2005) argue that the members of the upper classes use their privileged access to resources in order to further their own reproduction and that of their relatives. However, if this "reproductive skew" is too extreme, it may eventually cause society to collapse. The lower classes may no longer be able to support the demands placed upon them by the mushrooming upper classes, and the latter may fragment as their members scrabble for an ever smaller share of the available food surplus.

The need to preserve a stable class composition puts limits on the extent to which the upper classes can outbreed the lower classes. There are various ways in which a stable class composition can be maintained. The members of the upper classes may voluntarily have fewer children, perhaps in response to their impoverishment as they become too numerous; or some upper-class individuals may become celibate and not reproduce at all; or enough of them may be killed in war before they get an opportunity to reproduce. All of these mechanisms help reduce the degree of reproductive skew. Some cultures have devised quite ingenious practices to restrain the growth of their upper classes. For instance, a newly appointed Ottoman sultan was obliged by law to kill all of his brothers. In the Central African kingdoms of Ankole and Kitara, the sons of a dead king had to fight for the throne until only one of them was left alive (Andreski, 1968, p. 19).

The high reproduction rate of the upper classes can also be offset by inducing their excess members to leave. This could be achieved by some social rule, such as primogeniture, by virtue of which the eldest son inherits the whole family estate. The disinherited sons may sink down into the lower classes or be forced to seek their fortunes elsewhere. For example, the descendents of the king of Siam, except for his successors, were lowered in rank after each generation. After the fifth they became commoners (p. 19). In medieval Europe, noble scions roaming the country in search of fiefs were a common sight. This surplus of nobles was the principal source of knights for the Crusades (p. 137). The Austranesian stress on primogeniture forced the chiefs' younger sons to find and colonise uninhabited islands (Finney, 1996).

Note that the permanent inflow of dispossessed aristocrats requires members of the lower classes to reproduce below the replacement rate (Eberhard, 1962, pp. 264–265; Lenski, 1984, p. 190). Otherwise, the lower classes would grow beyond the point where they can provide for subsistence, and society would starve to death.

1.4 Malthusian principles of social stratification

The size and quality of the professional army that a society can support depends on the size of the surplus that the society can generate. In turn, the size of the surplus depends on three factors: the number of workers, the productivity of the average worker, and the amount consumed by the average worker and his dependents. From Malthus onward, there has been a lively debate on the interplay between these factors (Coleman and Schofield, 1986; Ashraf and Galor, 2008). The classical tradition, exemplified by Malthus and Ricardo, assumed a perfectly elastic supply of population at a constant subsistence wage rate together with diminishing returns to labour. If wages rise above subsistence, the population will expand, leading to more employment. That will force down the marginal product of labour and thus wages. This process will only come to a halt when the marginal product of labour equals the subsistence wage, at which point the labouring population will stop growing. This is also the point at which the surplus product, in the form of rent, will be maximised.

To the extent that the adoption of agriculture involves the development of a more costly military technology and the emergence of a class of specialist warriors, not all of the extra output produced by agriculture can be translated into support for more producers and their families. A fraction of total production must be used to maintain and equip the warrior elite. This point was made by Sauvy (1999), who argued that achieving a "power optimum" requires maximising the surplus available to support the military and the government.

How can population be regulated so as to generate a surplus in view of the limits set by technology and the environment? The growth of the population within a given territory is determined by a combination of fertility, mortality, and migration, the relative importances of which have varied widely across time and geography. The role of migration is obvious and uncontroversial, so we shall focus on the other factors.

The original Malthusian theory assumed that population is automatically regulated through some kind of homeostatic mechanism. If the population gets too large relative to the amount of available resources, malnutrition, famine, disease, and warfare will cause premature deaths. This formulation is based on the biological analogy that an animal species will blindly multiply up to the limits set by the carrying capacity of its habitat. Other formulations rely on conscious choice or social convention to limit the population. In his later writings, Malthus himself suggested prudential restraint involving late marriage or celibacy (Malthus 1820, pp. 248–252). Abortion, infanticide, and prolonged breast-feeding may also serve to space out births or get rid of unwanted children. All of these social practices were common among pre-modern societies (Douglas, 1966; Cashdan, 1985; Macfarlane, 1997). Some practices were deliberately designed to limit the population, whereas others were followed without any such objective in mind. However, even non-deliberate social practices may have a homeostatic effect. Societies compete with each other and those with practices that most effectively regulate their populations may triumph over their rivals. Thus, group selection may lead to the emergence of population practices that are well-adapted to the prevailing environment (Wrigley, 1978).

A controversial notion that needs to be clarified at this point is that of "subsistence consumption." Some versions of the Malthusian theory interpret this notion in biological terms, equating it to the minimum food intake that allows a human being to survive and produce an average of one offspring (e.g., Wolf, 1966, p. 6). The later Malthus regarded such an idea as simplistic, and stressed the influence of socially-conditioned preferences on reproductive behaviour (Malthus, pp. 248–252; Costabile and Rowthorn, 1985). This was a common view among classical economists, such as Ricardo (1821, p. 91).

2 The model

2.1 Overview

The main features of our model are as follows.

Two rival societies divide up a given area of land according to the balance of military power. These societies may be *stratified* or *unstratified*.

There are two hereditary social classes in a stratified society: a warrior elite and a peasantry. The warriors own the land and hire labour from the peasants. The peasants cultivate the land to produce the food necessary to support both classes. Only warriors fight wars, using specialised weapons that only they can handle. The purpose of war is to secure and extend the frontiers of the society.

Unstratified societies do not have warriors. The peasants own and work their own land. They also perform the military duties that warriors perform in stratified societies. Peasants do not use specialised weapons, so a peasant militia is less powerful than a warrior army of the same size.

Warriors and peasants maximise utility by choosing how many children to have and how much food to consume, constrained by their food income and the cost of children in units of food. Warrior children are more expensive than peasant children because they must be trained and equipped for war. Children and food consumption are normal goods.

There are diminishing returns to labour in food production. In the long run, population must adapt itself to the carrying capacity of the land available.

2.2 Definitions

2.2.1 Social structure

An agrarian society called Home is composed of $N > 0$ adults. The adults are divided into $N_w \geq 0$ warriors and $N_p > 0$ peasants, where $N = N_w + N_p$. Social positions are hereditary: the children of warriors become warriors, and the children of peasants become peasants.[1] If $N_w > 0$ we say the society is *stratified*. If $N_w = 0$, the society is *unstratified*.

[1] We assume strong social immobility for expositional ease. We generalise our model to allow for downward social mobility in Appendix A.1. Our results still hold in the generalised version of the model.

2.2.2 War and land holding

If Home is stratified, only warriors carry arms and the military power of the society is proportional to the number of warriors. If Home is unstratified, a peasant militia defends the society and its military power is proportional to the number of peasants. Let P represent Home's military power:

$$P = \begin{cases} \phi N_w & \text{if } N_w > 0, \\ N_p & \text{if } N_w = 0, \end{cases}$$

where $\phi > 1$ is the relative effectiveness of a specialised military technology that only warriors can handle. In line with historical accounts, the expression above assumes that peasants of stratified societies do not participate in war, or play an insignificant role (Gat, 2006, pp. 298–299).

The total area of available land is L_0. The balance of military power determines the distribution of land between Home and Foreign:

$$L = \frac{P^\beta}{P^\beta + (P^{\mathrm{F}})^\beta} L_0, \tag{1}$$

$$L^{\mathrm{F}} = \frac{(P^{\mathrm{F}})^\beta}{P^\beta + (P^{\mathrm{F}})^\beta} L_0,$$

where L is the land area controlled by Home and L^{F} is the land area controlled by Foreign. Variable P^{F} represents Foreign's military power. (Throughout the text, we will use superscript F to identify Foreign variables.) Parameter β measures the effectiveness of military power in acquiring or defending land. We assume decreasing returns to military power: $\beta < 1$.

2.2.3 Food production and allocation

Home's food production is given by

$$Y = AL^\alpha N_p^{1-\alpha}, \tag{2}$$

where $A > 0$ is the total factor productivity and $\alpha \in (0, 1)$ measures the intensiveness of land in production.

If Home is unstratified, peasants own and work their own land and earn their average product. If Home is stratified, warriors own the land in equal shares. Total food production must be divided between Home warriors and Home peasants. We assume that peasants earn a fraction $1 - \alpha$ of total food production and warriors take the rest. This allocation can be justified in two ways.

First, it could result from a crop-sharing rule established by social convention. Crop-sharing rules were frequent among early agriculturalists (Raper and Reid, 1941, pp. 35–36). Of all possible crop-sharing rules, dividing production between warriors and peasants in fractions α and $1-\alpha$ maximises N_w and hence the military power of the stratified society (see proof in Appendix A.2). In a warlike environment, we expect group selection to favour those social conventions that maximise military power.

Second, the same allocation will result if labour markets are competitive. Competitiveness implies that peasants will earn their marginal product of $(1-\alpha)Y$. The assumption of a competitive labour market may seem unrealistic at first glance, since peasants were usually bound to the land in the past. In practice, escape was often easy, and peasants deserted their lord and sought a new one when they felt mistreated (North and Paul, 1973, pp. 30, 79, 200). In China, for instance, massive desertions of peasants were not only possible but indeed frequent, turning the tide of war against the deserted lord and in favour of the new one (Andreski, 1968, p. 48).

Let $\sigma_p \in (0,1]$ be the probability of a peasant reaching reproductive age. Assuming that the incomes of peasants who die before reproducing are inherited by their relatives, the income of the typical surviving peasant is given by

$$y_p = \begin{cases} \dfrac{(1-\alpha)Y}{\sigma_p N_p} & \text{if } N_w > 0, \\ \dfrac{Y}{\sigma_p N_p} & \text{if } N_w = 0. \end{cases} \tag{3}$$

Surviving warriors earn a rent from land:

$$y_w = \frac{\alpha Y}{\sigma_w N_w}, \tag{4}$$

where $\sigma_w \in (0,1]$ is a warrior's probability of reaching reproductive age.

2.2.4 Consumption, reproduction, and utility

People who die prematurely do not eat and do not have children. The utility of an adult is given by

$$u = \frac{\sigma^{1+\delta} c^{\theta} n^{1-\theta}}{\theta^{\theta}(1-\theta)^{1-\theta}},$$

where $c \geq 0$ is his consumption, and $n \geq 0$ is the number of his children. The parameter $\theta \in (0,1)$ represents the weight of consumption in utility, σ is the probability of reaching reproductive age, and $\delta > 0$ is a measure of death aversion. Consumption may not consist exclusively of food. Both warriors and peasants may use part of their incomes to purchase manufactured goods (e.g., weapons) that are obtained at a fixed relative price by trading with the outside world. We do not explore this issue and assume that imports enter into the utility function in terms of their food equivalent.

Historical evidence suggests that fertility among pre-modern peoples depended on the income available to them. The methods they used to limit the number of their children included abstinence, celibacy, prolonged breast-feeding, abortion, and infanticide (Douglas, 1966; Cashdan, 1985; Macfarlane, 1997). Recourse to such methods was more frequent when times were hard than in good times. To capture the link between fertility and income, we shall assume that each adult solves

$$\begin{aligned} \max_{\{c,n\}} \quad & \sigma^{1+\delta} c^{\theta} n^{1-\theta} \\ \text{s.t.} \quad & c + \kappa n = y, \\ & c, n > 0, \end{aligned}$$

where $\kappa > 0$ is the price of a child in units of food and y is the adult's income.

The solution to an adult's problem is given by

$$c = \theta y, \tag{5}$$

$$n = \frac{(1-\theta)\, y}{\kappa}, \tag{6}$$

$$u = \frac{\sigma^{1+\delta} y}{\kappa^{1-\theta}}. \tag{7}$$

This is the standard result of the consumer problem with a Cobb-Douglas utility function. Expenditures on consumption and children are constant fractions of income, and indirect utility is increasing in food income and decreasing in the price of children.

Children are cheaper for peasants than for warriors: $\kappa_p < \kappa_w$. This difference can be read as the extra cost that warriors face in order to train and equip their children for war. As an example, consider the case of Spartans, who were taken away from their mothers to start their military training as young as seven years old (O'Connell, 2002, p. 42). Another example is given by Prestwich (1996), who reports that a complete suit of armour in the Middle Ages would cost the equivalent of a 1939 light tank. According to Andreski (1968, p. 58), the total cost of equipping one knight amounted to the annual income of a whole village, making knighthood a heavy financial burden.

There is some evidence that, before modern times, warriors had a lesser chance than peasants of reaching reproductive age. Griffith (1970, p. 26) affirms that one third of Norwegian kings died in battle during the Viking era. According to Wrigley (1997, p. 206), the life expectancy at birth among the English aristocracy lagged behind that of the population as a whole until the 18th century, among other reasons, because the children of aristocrats were weaned earlier than the children of commoners. Hollingsworth (1957) reports that, during the 14th and 15th centuries, 46% of the sons of English dukes died violent deaths. The local peasants, on the other hand, were free from the hazards of continual combat (although they were occasionally prey to marauding lords). In line with this evidence, we assume that $\sigma_w < \sigma_p$, although our key results will not depend on this assumption.

2.2.5 Population dynamics

There is no migration and no mobility between social classes. Therefore, population dynamics are governed by the following laws of motion:

$$N_w^{\text{next}} = \underbrace{\sigma_w n_w N_w}_{\substack{\text{Children of} \\ \text{surviving warriors}}}, \tag{8}$$

$$N_p^{\text{next}} = \underbrace{\sigma_p n_p N_p}_{\substack{\text{Children of} \\ \text{surviving peasants}}}, \tag{9}$$

where N_w^{next} and N_p^{next} are the sizes of the warrior elite and of the peasantry in the next generation. Survival probabilities σ_p and σ_w are exogenously determined parameters. These assumptions imply that any endogenous changes in population growth must come about through variations in the birth rates n_w and n_p.

2.3 Symmetric equilibria

Assume that Home and Foreign are identical with regard to preferences, technology, prices, and survival rates. Under these conditions, the dynamics outlined above will lead to a stationary equilibrium, the nature of which depends on the structure of the two societies. If both societies are stratified or both are unstratified, they will end up dividing the land equally:

$$L = L^{\text{F}} = \frac{L_0}{2}. \tag{10}$$

The existence of a rough equilibrium among competing societies can be traced back to the early stages of the adoption of agriculture (O'Connell 2002, p. 32).

If one society is stratified and the other is not, the distribution of land will be asymmetric. We shall discuss in detail only the symmetric equilibria. Towards the end of the paper, we touch briefly on the issue of asymmetry when we discuss the warriors' contribution to society, which is to secure and expand its frontiers. When analysing the comparative statics, we shall assume that parameters change simultaneously in both societies.

2.3.1 Unstratified equilibrium

In equilibrium, warrior and peasant populations remain constant through time:

$$\begin{aligned} N_w^* &= \sigma_w n_w^* N_w^*, \\ N_p^* &= \sigma_p n_p^* N_p^*, \end{aligned}$$

where an asterisk indicates the equilibrium value of a variable. The first of these conditions is immediately satisfied in an unstratified society, where $N_w^* = 0$. The second condition implies that

$$\sigma_p n_p^* = 1,$$

or equivalently,

$$n_p^* = \frac{1}{\sigma_p}.$$

The average peasant must have, in expected terms, exactly one child. Since a fraction $1 - \sigma_p$ of peasants does not survive to reproduce, the remaining peasants must compensate by having more than one child: $n_p^* > 1$. Plugging the equilibrium value of n_p into the solutions of the adult problem, given in equations (5), (6), and (7), we get the equilibrium levels of income, consumption, and utility:

$$\begin{aligned} y_p^* &= \frac{\kappa_p}{(1-\theta)\,\sigma_p}, \qquad (11) \\ c_p^* &= \frac{\theta\kappa_p}{(1-\theta)\,\sigma_p}, \\ u_p^* &= \frac{\sigma_p^\delta \kappa_p^\theta}{1-\theta}. \end{aligned}$$

The equilibrium value of y_p is our model's equivalent to subsistence income. No matter how much the production technology improves (i.e., how much A increases), the income of peasants will always return to y_p^*. Equilibrium income y_p^* fits nicely into Malthus and Ricardo's view of subsistence: instead of being a biological minimum, equilibrium income is determined by preferences and prices.

In the long run, the population adjusts to keep income at subsistence level. Combining equations (2), (3), (10), and (11), we get the equilibrium level of population:

$$N^* = N_p^* = \left(\frac{(1-\theta)\,A}{\kappa_p}\right)^{\frac{1}{\alpha}} \frac{L_0}{2}. \qquad (12)$$

Observe that population is the only variable affected by technology, represented by parameter A. Technological improvements translate into nothing but larger populations:

$$\frac{\partial \ln N^*}{\partial \ln A} = \frac{1}{\alpha} > 0.$$

Initially, a higher value of A allows more food to be produced by a given amount of labour. Peasants consume part of that extra food and use the rest to have more children. As a result, the population begins to grow. However, diminishing returns to labour imply that population growth will eventually offset the gains from the productivity improvement. In the end, more peasants will be employed on the land, and the average amount of food produced by each of them will return to its original level. Consumption will also fall back to where it started as the price of children remains unaltered through the whole process. This is the typical Malthusian result when there are diminishing returns to labour in production.

The effects of an increase in the price of children, represented by κ_p, are more interesting. Both long-run consumption and utility increase when children become more expensive:

$$\begin{aligned}\frac{\partial \ln c_p^*}{\partial \ln \kappa_p} &= 1 > 0,\\ \frac{\partial \ln u_p^*}{\partial \ln \kappa_p} &= \theta > 0.\end{aligned}$$

Though it may seem paradoxical at first, this result can be explained as follows. An increase in κ_p leads to an initial decrease in fertility and in the long run to a reduction in population, so that fewer adults are employed on the land and hence labour productivity (y_p^*) is higher:

$$\begin{aligned}\frac{\partial \ln N_p^*}{\partial \ln \kappa_p} &= -\frac{1}{\alpha} < 0,\\ \frac{\partial \ln y_p^*}{\partial \ln \kappa_p} &= 1 > 0.\end{aligned}$$

When κ_p first increases, adults are no longer able to afford their current mix of children and consumption, so they experience a reduction in their utility. But when the system reaches the new equilibrium, adults end up with more income but the same number of children they had before the shock. This implies that food consumption, and therefore utility, must be higher. The existence of diminishing returns implies that the short-run and long-run impacts of an increase in the price of children must run in opposite directions. Figure 1 illustrates the process.[2]

[2] Under the plausible assumption that children are normal goods (Tzannatos and Symons, 1989), this comparative statics does not depend on the particular production and utility functions we use throughout this paper. We provide a general proof in Appendix A.3.

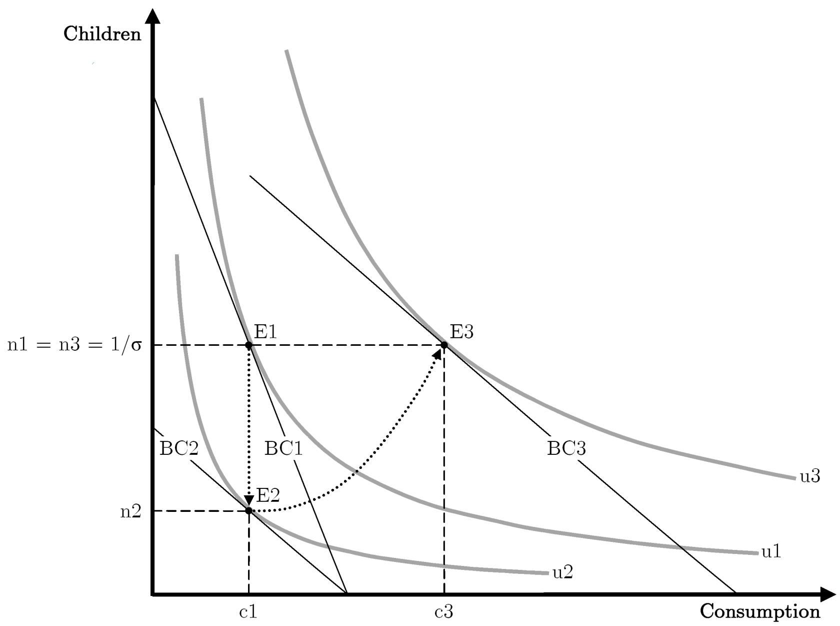

Figure 1: An increase in the cost of children reduces welfare in the short run but increases it in the long run. Point E1 represents the initial equilibrium. The income of a surviving adult is just enough to induce him to have $1/\sigma$ children given the prices he faces. That level of fertility ensures that the population will remain constant. All of a sudden, the price of children rises, so the adult's budget constraint rotates inward from BC1 to BC2. As a result, the adult moves to point E2, where his fertility and his utility are lower than before. This short-run reduction in fertility causes the population to decline through the generations. The decline in population increases the returns to labour, pushing the budget constraint to the right from BC2 to BC3. Equilibrium is re-established at point E3, where income is just enough to induce the descendents of the original adult to have $1/\sigma$ children given their new, higher price. In the new equilibrium, fertility is the same but consumption is higher than it was in the beginning. Consequently, utility is also higher.

2.3.2 Stratified equilibrium

Both N_w and N_p are positive in a stratified society. In the short run, the values of N_w and N_p are fixed. Using the symmetry between Home and Foreign, we obtain the short-run equilibrium:

$$\begin{aligned} y_w &= \frac{\alpha A L^\alpha N_p^{1-\alpha}}{\sigma_w N_w}, & y_p &= \frac{(1-\alpha) L^\alpha N_p^{1-\alpha}}{\sigma_p N_p}, \\ c_w &= \frac{\theta\alpha A L^\alpha N_p^{1-\alpha}}{\sigma_w N_w}, & c_p &= \frac{\theta(1-\alpha) L^\alpha N_p^{1-\alpha}}{\sigma_p N_p}, \\ n_w &= \frac{(1-\theta)\alpha A L^\alpha N_p^{1-\alpha}}{\sigma_w \kappa_w N_w}, & n_p &= \frac{(1-\theta)(1-\alpha) L^\alpha N_p^{1-\alpha}}{\sigma_p \kappa_p N_p}, \\ u_w &= \frac{\sigma_w^\delta \alpha A L^\alpha N_p^{1-\alpha}}{\kappa_w^{1-\theta} N_w}, & u_p &= \frac{\sigma_p^\delta (1-\alpha) A L^\alpha N_p^{1-\alpha}}{\kappa_p^{1-\theta} N_p}. \end{aligned}$$

(Short-run stratified equilibrium)

where $L = L_0/2$. In the long run, surviving warriors and surviving peasants must have just enough children to keep their numbers constant:

$$\begin{aligned} N_w^* &= \sigma_w n_w^* N_w^*, \\ N_p^* &= \sigma_p n_p^* N_p^*. \end{aligned}$$

The conditions above imply that the typical warrior and the typical peasant must have, in expected terms, the same number of children:

$$\sigma_w n_w^* = \sigma_p n_p^*. \tag{13}$$

If peasants were to reproduce faster than warriors, the ratio of warriors to peasants would contract until warriors became extinct and the population was composed solely of peasants. Conversely, if warriors were to reproduce faster than peasants, the ratio of warriors to peasants would expand until peasants were no longer able to support the warrior elite. Eventually, the warrior elite would collapse under its own weight. Hence, condition (13) is necessary to obtain a stratified equilibrium.

Substituting the short-run equilibrium values of n_w and n_p into condition (13), we obtain the long-run equilibrium level of population and the class composition in Home:

$$N^* = \left(\frac{(1-\theta)(1-\alpha)A}{\kappa_p}\right)^{\frac{1}{\alpha}} \left[1 + \frac{\alpha}{1-\alpha}\left(\frac{\kappa_w}{\kappa_p}\right)^{-1}\right] \frac{L_0}{2}, \tag{14}$$

$$\frac{N_w^*}{N_p^*} = \frac{\alpha}{1-\alpha}\left(\frac{\kappa_w}{\kappa_p}\right)^{-1}. \tag{15}$$

Following Andreski (1968, p. 33), we will refer to N_w/N_p as Home's *military participation ratio*, or MPR.

Finally, plugging N^* and N_w^*/N_p^* into the short-run equilibrium values of income, consumption, fertility, and utility, we get the long-run equilibrium values of these variables:

$$\begin{aligned} y_w^* &= \frac{\kappa_w}{(1-\theta)\,\sigma_w}, & y_p^* &= \frac{\kappa_p}{(1-\theta)\,\sigma_p}, \\ c_w^* &= \frac{\theta\kappa_w}{(1-\theta)\,\sigma_w}, & c_p^* &= \frac{\theta\kappa_p}{(1-\theta)\,\sigma_p}, \\ n_w^* &= \frac{1}{\sigma_w}, & n_p^* &= \frac{1}{\sigma_p}, \\ u_w^* &= \frac{\sigma_w^\delta \kappa_w^\theta}{1-\theta}, & u_p^* &= \frac{\sigma_p^\delta \kappa_p^\theta}{1-\theta}. \end{aligned}$$

(Long-run stratified equilibrium)

The effect of social stratification on economic privilege

The average warrior or peasant must support exactly one child. These averages include people who have no children because they die prematurely and survivors with more than one child. Because warrior children are more expensive than peasant children, equilibrium requires that the average warrior has a higher income than the average peasant. Let $\overline{y}_w$ be the expected per capita income of a warrior, and let $\overline{y}_p$ be the expected per capita income of a peasant. These quantities are obtained by averaging over individuals who survive and those who do not. In equilibrium we have:

$$\overline{y}_w^* = \sigma_w y_w^* = \frac{\kappa_w}{1-\theta} > \frac{\kappa_p}{1-\theta} = \sigma_p y_p^* = \overline{y}_p^*,$$

since $\kappa_w > \kappa_p$. Note that income inequality is entirely determined by the additional costs warriors must incur in training and equipping their children.

As a result of $\overline{y}_w^* > \overline{y}_p^*$, warriors will have higher per capita consumption than peasants:

$$\overline{c}_w^* = \sigma_w c_w^* = \frac{\theta\kappa_w}{1-\theta} > \frac{\theta\kappa_p}{1-\theta} = \sigma_p c_p^* = \overline{c}_p^*.$$

The economic inequality between surviving warriors and surviving peasants is more pronounced than the economic inequality between pre-war warriors and pre-war peasants:

$$\frac{y_w^*}{y_p^*} = \frac{\kappa_w/\sigma_w}{\kappa_p/\sigma_p} > \frac{\kappa_w}{\kappa_p} = \frac{\overline{y}_w^*}{\overline{y}_p^*}$$

$$\frac{c_w^*}{c_p^*} = \frac{\kappa_w/\sigma_w}{\kappa_p/\sigma_p} > \frac{\kappa_w}{\kappa_p} = \frac{\overline{c}_w^*}{\overline{c}_p^*}.$$

This is because warriors have a lower survival rate than peasants. Thus, a surviving warrior must finance σ_w^{-1} children, whereas a surviving peasant must finance σ_p^{-1} $(< \sigma_w^{-1})$ children.

The warriors' degree of privilege in terms of utility is given by:

$$\frac{u_w^*}{u_p^*} = \underbrace{\left(\frac{\sigma_w}{\sigma_p}\right)^{\delta}}_{<1} \cdot \underbrace{\left(\frac{\kappa_w}{\kappa_p}\right)^{\theta}}_{>1} \gtrless 1.$$

Assuming that warriors face a larger risk of death than peasants ($\sigma_w < \sigma_p$), the warrior's degree of privilege could be lower than 1, meaning that the peasantry would be the privileged class. This is possible, but unlikely, as warrior children would probably shun the military career. Some form of compulsion or indoctrination would thus be required to keep the warrior elite from disbanding.

If, on the contrary, warriors enjoy more utility than peasants, coercion may be needed to enforce property rights or, *in extremis*, to prevent a peasant revolution. This is probably why the vast majority of warrior nobilities kept their peasants disarmed. Before the westernisation of Japan, for example, the bearing of arms was a strict prerogative of the nobles (with the very brief exception of the Taikwa reforms period during the 7th century). It was no coincidence that no peasant rebellion ever succeeded in Japan (Andreski, 1958, p. 50). Some authors suggest other mechanisms that would allow a privileged upper class to subsist without coercing the lower classes. For example, if groups are segregated and investments in human capital generate positive externalities within-groups, then individual choices may lead to self-perpetuating economic differences between groups (Lundberg and Startz, 1998). In addition, upper-class propaganda could deceive the lower classes into believing that economic inequalities are in their best interest (Cronk, 1994; DeMarrais et al., 1996). Yet another possibility is that, if people tend to be influenced by members of their own social classes, lower-class people could just learn to play their disadvantaged role in society without further questioning (Henrich and Boyd, 2007).

The impact of stratification on total population and military power

Maximal peasant population cannot be an equilibrium for a stratified society. When the peasant population is at its maximum, the peasantry does not produce the surplus of food that is needed to support a warrior elite. To produce a surplus, a stratified society must employ fewer peasants per unit of land and therefore produce less food than would be possible with the existing technology. Everything else being equal (land area under cultivation, technology, and peasant subsistence income), total population will be lower in a stratified society than in an unstratified society for two reasons. First, stratified societies produce less food than unstratified societies per unit of land, and less food means that fewer people can be fed. Second, the per capita income of warriors is higher than the per capita income of peasants, so even the same amount of food would support fewer people in a stratified society. In algebraic terms: from equations (12) and (14), it follows that population will be lower with than without stratification if and only if

$$\frac{\left[(1-\alpha)\,\kappa_w + \alpha\kappa_p\right](1-\alpha)^{\frac{1-\alpha}{\alpha}}}{\kappa_w} \quad < \quad 1,$$

which is always true since $\alpha \in (0,1)$ and $\kappa_p < \kappa_w$.

Sauvy (1952–54/1969, pp. 51–59) argues that maximising military power requires maximising the surplus that is available to support a professional army. In our model, the assumption that peasants get their marginal product entails that the surplus, and hence the warrior population, will be maximised in equilibrium (see proof in Appendix A.1). For Sauvy's proposition to be correct, it must also be the case that a stratified society is more powerful than its unstratified equivalent. This requires the following condition to be satisfied:

$$\underbrace{\frac{\phi\alpha}{1-\alpha}\left(\frac{\kappa_w}{\kappa_p}\right)^{-1}\left(\frac{(1-\theta)(1-\alpha)A}{\kappa_p}\right)^{\frac{1}{\alpha}}\frac{L_0}{2}}_{\text{Military power if stratified}} \quad > \quad \underbrace{\left(\frac{(1-\theta)A}{\kappa_p}\right)^{\frac{1}{\alpha}}\frac{L_0}{2}}_{\text{Military power if unstratified}},$$

which reduces to

$$\phi\left(\frac{\kappa_w}{\kappa_p}\right)^{-1} \quad > \quad \frac{(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{\alpha}.$$

Therefore, the stratified society will be more powerful than the unstratified society if specialised weapons are very effective (ϕ is high) or warriors are relatively cheap (κ_w/κ_p is low).

In the long-run, technological progress in food production will only increase population

Just as in the unstratified equilibrium, a technological improvement in food production has no effect but to increase the total population:

$$\frac{\partial \ln N^*}{\partial \ln A} \quad = \quad \frac{1}{\alpha} \quad > \quad 0.$$

This result explains why the shift from hunting-and-gathering to agriculture (8,000 to 3,000 BC) was accompanied by the first demographic explosion ever to be recorded (Bocquet-Appel, 2002). The yield of the land steadily increased as a result of a series of technological improvements that can be read as increases in parameter A: fertilisers, fallowing, irrigation systems, the plough, and crop rotation (Vasey, 1992, pp. 44-57, 111). Suddenly, food was available to our ancestors in quantities they never dreamt of before (Price and Gebauer, 1995). Just a few centuries after adopting agriculture, typical communities saw their numbers grow from about 30 people to 300 or more. Population densities increased in those places where agriculture was adopted, from less than one hunter-gatherer per square mile to 20 or more farmers in the same area (Johnson and Earle, 2000, pp. 43, 125, 246).

The model also explains why the adoption of agriculture was not accompanied by improvements in our ancestors' nutrition, as studies of the human fossil record have revealed (Armelagos et al., 1991; Cohen, 1989):

$$\frac{\partial \ln c_p^*}{\partial \ln A} = \frac{\partial \ln c_w^*}{\partial \ln A} = 0.$$

More expensive weapons imply a lower MPR and higher income inequality

An increase in the costs of weapons and military training is represented in our model by an increase in the ratio κ_w/κ_p. Our model predicts that an increase in κ_w/κ_p will reduce the MPR, reduce total population, and sharpen income inequality in favour of warriors:

$$\begin{aligned}
\frac{\partial \ln \left(N_w^*/N_p^*\right)}{\partial \ln \left(\kappa_w/\kappa_p\right)} &= -1 && < 0, \\
\frac{\partial \ln \left(N^*\right)}{\partial \ln \left(\kappa_w\right)} &= \frac{-\alpha\kappa_p}{\left(1-\alpha\right)\kappa_w + \alpha\kappa_p} && < 0, \\
\frac{\partial \ln (\overline{y}_w^*/\overline{y}_p^*)}{\partial \ln \left(\kappa_w/\kappa_p\right)} &= 1 && > 0.
\end{aligned}$$

Observe that a decrease in N_w^*/N_p^* will always be accompanied by an increase in $\overline{y}_w^*/\overline{y}_p^*$. This is precisely the main stylised fact detected by Andreski (1968): a negative correlation between the MPR and the warriors' degree of economic privilege (pp. 40–41, 73). He observes that increases in the cost of weapons tend to reduce the MPR and to increase the economic advantage of warriors, exactly as our model predicts. He provides historical examples from a wide array of civilisations (pp. 39–72). We reproduce three of Andreski's examples to give the reader a taste of the historical evidence.

Persia— In the times of the Achaemenid Empire, the Persian army consisted of nobles and freemen. The MPR was high since the freemen were very numerous. Most of the army battled on foot, supported by a minimal cavalry. The main weapons in use were the bow and the long spear. Protective armour was scanty and uncommon. When the Sassanid dynasty rose to power in the third century A.D., it introduced a series of effective but very expensive military innovations; most importantly, the stirrup and heavy protective armour. As a result, the freemen disappeared, the warrior nobility shrunk while its privileges expanded, and the peasants were reduced to harsh servitude (pp. 46–47).

Poland— The original Polish kingdom was despotic. Freemen and the king's personal guard, the Druzhina, comprised the army. Both groups were armed with primitive weapons and did not wear body armour. Gradually, the army incorporated more advanced equipment. Heavily armed horsemen were the mainstay of the Polish forces that repelled the Teutonic Knights in Grünewald (1410 A.D.). The modernisation of the army was accompanied by a reduction of the MPR and an increase in social inequalities: peasants were reduced to the status of serfs, and military service was restricted to the nobility (pp. 59–60).

England— The Norman conquest of England, which introduced heavy cavalry to the country, sharpened social inequalities relative to the preceding Anglo-Saxon period. This process began to be reversed during the wars against the Welshmen, when English warriors learned how to use the long bow. An inexpensive yet formidable weapon, the long bow was far superior to any other type of bow. In combination with cavalry, it was able to inflict enormous damage on enemy forces. The adoption of the long bow forced profound changes in military tactics and organisation. As a consequence of these changes, serfdom virtually disappeared from England, yeomen thrived, the MPR increased, and social inequality became much less pronounced (pp. 64–65).

2.4 The social contribution of warriors

In our model, warriors provide the public good of extending the frontiers of the society and defending it against foreign predators. In real life, these military functions may legitimise the warriors' privileges.

Suppose that Home and Foreign are in symmetric, long-run equilibrium. Also suppose there is a peasant revolution in Home that occurs just after the end of the harvest. All of Home's warriors are killed off during the revolution. To make up for the lack of warriors, the emancipated peasants form a militia to which everyone belongs. This militia makes no use of specialised weapons, so each peasant is equal in fighting capacity to $1/\phi$ warriors (where $\phi > 1$). Membership in this militia does not interfere with agricultural production.

The immediate impact of the revolution is to increase the standard of living of the peasants, who no longer have to pay a tribute to the warrior elite. From equation (3) it follows that

$$\widehat{y}_p \quad = \quad \frac{y_p}{1-\alpha} \quad > \quad y_p,$$

where $\widehat{y}_p$ is the post-revolution peasant income.

The revolution in Home will affect fertility and the balance of military power. Fertility will rise because peasants use part of their extra income to have more children, thereby increasing the number of peasant mouths to be fed in the next generation. The effect of the revolution on the military capacity of Home is ambiguous. Because of population growth, Home will have a larger peasant militia in the next generation. However, Home will no longer have specialised warriors to fight on its behalf. If Home emerges militarily stronger because of the peasant revolution, it will seize part of Foreign's land to the benefit of its own population. Conversely, if Home is weakened militarily by the revolution, it will be forced to give up some land to its neighbour.

Taking all of the above into account, the per capita income of the next generation of peasants will be:

$$y_p^{\text{next}} \quad = \quad \frac{y_p}{(1-\alpha)^{1-\alpha}} \left(\frac{2(\kappa_w/\kappa_p)^{\beta}}{(\kappa_w/\kappa_p)^{\beta} + (\alpha\phi)^{\beta}} \right)^{\alpha}.$$

Therefore, peasants will be worse off than before the revolution if and only if

$$\phi \left(\frac{\kappa_w}{\kappa_p} \right)^{-1} \quad > \quad \frac{1}{\alpha} \left(\frac{2-(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{(1-\alpha)^{\frac{1-\alpha}{\alpha}}} \right)^{\frac{1}{\beta}}. \tag{16}$$

If this inequality holds, the social contribution of warriors will outweigh the burden they impose on society. This requires specialised weapons to be very effective (a high ϕ) or training and equipping warriors to be relatively cheap (a low κ_w/κ_p). See the full derivation of these results in Appendix A.4.[3,4]

[3] In this example, the role of the warrior elite is similar to that of the "king" in Grossman (2002). In his model, a privileged elite provides a public service whose benefits to the rest of the population outweigh the cost of supporting the elite. There are, however, two differences between our model and Grossman's. In Grossman's model, the elite defends producers against internal predators who would otherwise steal part of their output. In our model, the elite defends producers against external predators. A second difference concerns the duration of the benefits. In Grossman's model, the policing activities of the elite ensure a permanently higher standard of living for producers, whereas in our model endogenous population change will eventually bring the standard of living back to the subsistence level.

[4] In a related paper, Acemoglu and Robinson (2001) model the transition from a nondemocratic society controlled by a rich elite to a democracy. They find that the poor will threaten to revolt when the cost of revolting is low; for example, during recessions. This threat may force the elite to democratise. On the other hand, the redistributive nature of democratic regimes may encourage the elite to mount a coup. The more unequal the distribution of resources, the more likely it is that society will end up oscillating between democratic and nondemocratic regimes. Unlike our model, Acemoglu and Robinson's work does not account for the effects of demographic forces.

3 Concluding remarks

Our model produces three main predictions:

1. *The average warrior enjoys higher income and higher consumption than the average peasant.* In the long run, the average warrior and the average peasant must each have exactly one child. Otherwise, the absolute and relative sizes of the two classes will not be stable. Because warrior children are more expensive than peasant children, warriors must receive extra income in order to persuade them to have the same expected number of children as peasants. Since children and food are normal goods, warriors spend some of their extra income on food and hence on average enjoy a higher level of consumption than peasants.

2. *As the relative cost of warrior children increases, the size of the warrior elite relative to the peasantry falls and the warriors' economic privileges increase.* An increase in the price of warrior children will initially push warrior fertility below one. In order to restore equilibrium, the per capita income of warriors must rise to the point where they are again willing to replenish their numbers. This is accomplished by reducing the size of the warrior elite: the reduction in warrior fertility induces a decline in warrior population, and as land is divided among fewer warriors than before, each will earn a bigger rent. Since consumption is a normal good, part of this bigger rent will be used to finance the more expensive children and part will be destined to finance more consumption.

3. *Taking land, technology, and prices as given, total population will be lower in a stratified society than in an unstratified society.* There are two reasons for this. First, a stratified society employs fewer people per unit of land and therefore produces less food on a given land area than its unstratified equivalent. Less food means fewer people can be fed. Second, the per capita income of warriors is higher than the per capita income of peasants, so even the same amount of food would support fewer people in a stratified society.

These predictions hold even in the presence of downward social mobility. See Appendix A.1 for a formal proof.

Our model also identifies conditions under which the peasantry benefits from the existence of a warrior elite. Consider a stratified society in which a peasant revolution eliminates the warriors. The immediate impact of the revolution will be to increase the per capita income of peasants, who no longer have to pay a tribute to the warriors. However, without warriors to defend them, the emancipated peasants may become vulnerable to external predation and be forced to give up part of their land to the predators. Under certain parameter combinations, the resulting loss will be so great that peasants become worse off than before the revolution. This will be the case if specialised weapons are very effective or if training and equipping warriors is cheap. In the long run, however, the Malthusian population dynamics will undo any change in the peasant standard of living brought about by a revolution or, conversely, by stratification.

A Appendices

A.1 Robustness of our results to downward social mobility

The assumption of strong social immobility is a good approximation to the state of affairs in early stratified societies (Kautsky 1997, p. 95). Nevertheless, some degree of downward social mobility was always observed (Lenski 1984, p. 289–291; Eberhard 1962, pp. 264–265). Here we introduce downward social mobility into our model. It turns out that our results are robust to this extension.

Assume that adults can raise two kinds of children: some destined to be warriors, others to be peasants. Define the adult's utility as follows:

$$u = \frac{\sigma^{1+\delta} c^{\theta} \left[\varphi_w m^{\rho} + \varphi_p n^{\rho}\right]^{\frac{1-\theta}{\rho}}}{\theta^{\theta} (1-\theta)^{1-\theta}}.$$

Parameters $\varphi_w, \varphi_p > 0$ represent the weights of both types of children in utility. We set $\rho > 1$, which means that the adult values child variety.

When maximising his utility, a warrior faces the following budget constraint:

$$y_w = c_w + \kappa_w (m_w + n_w)$$

In line with the historical evidence (Lenski 1984, p. 289–91), we preclude upward social mobility: $m_p = 0$. Thus, the peasant's budget constraint is given by:

$$y_p = c_p + \kappa_p n_p,$$

Finally, let population dynamics be governed by the following laws of motion:

$$N_w^{\text{next}} = m_w \sigma_w N_w, \tag{17}$$

$$N_p^{\text{next}} = n_w \sigma_w N_w + n_p \sigma_p N_p. \tag{18}$$

The solution of a peasant's optimisation problem is the same as in the case of strong social immobility:

$$c_p = \theta y_p, \tag{19}$$

$$m_p = 0, \tag{20}$$

$$n_p = \frac{(1-\theta)\, y_p}{\kappa_p}. \tag{21}$$

The solution of a warrior's optimisation problem, on the other hand, is given by

$$c_w = \theta y_w, \tag{22}$$

$$m_w = \frac{Z\,(1-\theta)\, y_w}{\kappa_w}, \tag{23}$$

$$n_w = \frac{(1-Z)\,(1-\theta)\, y_w}{\kappa_w}, \tag{24}$$

where

$$Z = \varphi_w^{\frac{1}{1-\rho}} \left(\varphi_w^{\frac{1}{1-\rho}} + \varphi_p^{\frac{1}{1-\rho}} \right)^{-1} \in [0,1]$$

Strong social immobility can be modelled as the limiting case in which $Z = 1$.

Combining equations (3), (4), and (17)–(24), we obtain the equilibrium values of the MPR and economic inequality:

$$\frac{N_w^*}{N_p^*} = \frac{Z\alpha}{(1-Z)\,\alpha + (1-\alpha)\,\frac{\kappa_w}{\kappa_p}},$$

$$\frac{\overline{y}_w^*}{\overline{y}_p^*} = \frac{(1-Z)\,\alpha + (1-\alpha)\,\frac{\kappa_w}{\kappa_p}}{(1-\alpha)\, Z}.$$

It follows directly that

$$\frac{\partial \ln\left(N_w^*/N_p^*\right)}{\partial \ln\left(\kappa_w/\kappa_p\right)} < 0,$$

$$\frac{\partial \ln(\overline{y}_w^*/\overline{y}_p^*)}{\partial \ln\left(\kappa_w/\kappa_p\right)} > 0,$$

as desired.

A.2 A crop-sharing rule that maximises military power

Suppose that warriors take a share γ of total food production, Y. This leaves $(1-\gamma)Y$ to be divided among $\sigma_p N_p$ surviving peasants. The income of a typical peasant is therefore:

$$y_p = \frac{(1-\gamma)Y}{\sigma_p N_p}.$$

From equation (6) we know that

$$n_p = \frac{(1-\theta)y_p}{\kappa_p}.$$

Also recall that total production is $Y = AL^\alpha N_p^{1-\alpha}$. Hence,

$$\sigma_p n_p = \frac{(1-\theta)(1-\gamma)AL^\alpha N_p^{-\alpha}}{\kappa_p}.$$

Long-run equilibrium requires $\sigma_p n_p^* N_p^* = N_p^*$. From this condition and the above equations, it follows that

$$N_p^* = \left(\frac{(1-\theta)(1-\gamma)AL^\alpha}{\kappa_p}\right)^{\frac{1}{\alpha}}.$$

Rearranging terms, we get the amount of food taken by the warrior elite:

$$\text{Surplus} = \gamma Y = \gamma(1-\gamma)^{\frac{1-\alpha}{\alpha}}\left(\frac{1-\theta}{\kappa_p}\right)^{\frac{1-\alpha}{\alpha}} LA^{\frac{1}{\alpha}}.$$

The right-hand side of the above equation is maximised by making $\gamma = \alpha$. Since N_w^* is proportional to the available surplus, N_w^* will also be maximal.

A.3 Comparative statics with a general utility function

Assume that the utility of a typical adult is given by function $u(c, n)$, where $c \geq 0$ is the adult's consumption and $n \geq 0$ the number of his children. Function u is strictly increasing in both its arguments and also quasi-concave. The adult maximises his utility by choosing c and n, subject to the budget constraint $c + \kappa n \leq y$. Parameter $\kappa \geq 0$ represents the price of children, and $y \geq 0$ is the adult's income. Utility maximisation yields the following first order condition:

$$\frac{u_n}{u_c} = \kappa, \tag{25}$$

where the subscripts denote partial differentiation.

The model in the text assumes that fertility must converge in the long run to a certain level $\overline{n}$ $(= 1/\sigma)$ that keeps the population constant. This demographic equilibrium is achieved through the interplay of two forces: demographic pressures and diminishing returns to labour. These two forces will carry y to a value that induces adults to choose $n = \overline{n}$, given price κ. That means that the long-run value of c is fully determined by the following expression:

$$\frac{u_n(c,\overline{n})}{u_c(c,\overline{n})} = \kappa,$$

What happens if the price of children increases? Differentiating the above expression with respect to c and rearranging yields

$$\frac{\partial c}{\partial \kappa} = \left[\frac{\partial}{\partial c}\left(\frac{u_n(c,\overline{n})}{u_c(c,\overline{n})}\right)\right]^{-1}.$$

Hence, long-run consumption will be an increasing function of κ if and only if

$$\frac{\partial}{\partial c}\left(\frac{u_n(c,\overline{n})}{u_c(c,\overline{n})}\right) > 0. \tag{26}$$

Condition (26) can be interpreted in terms of indifference curves, as shown in Figure 1. The condition states that the slope of the indifference curve must be flatter at E3 than at E1. Observe that, if condition (26) holds, long-run utility will also rise when κ increases, simply because $u_c > 0$.

A bit of algebra proves that condition (26) will hold if children are normal goods. First, we use first order condition (25) to reformulate condition (26) as follows:

$$\frac{\partial}{\partial c}\left(\frac{u_n}{u_c}\right) = \frac{u_{cn}}{u_c} - \frac{u_n u_{cc}}{u_c^2} = \frac{u_{nc} - \kappa u_{cc}}{u_c} > 0.$$

Since $u_c > 0$, the above inequality reduces to

$$u_{nc} - \kappa u_{cc} > 0.$$

On the other hand, the effect of income on the adult's demand for children is given by

$$\frac{dn}{dy} = \frac{u_{cn} - \kappa u_{cc}}{-\kappa^2 u_{cc} + 2\kappa u_{cn} - u_{nn}}. \tag{27}$$

Since u is quasi-concave and $\kappa = u_n/u_c$, the denominator of the right-hand side of equation (27) must be positive. Therefore, children will be normal if $u_{cn} - \kappa u_{cc} > 0$, a condition that subsumes condition (26).

A.4 Derivation of inequality 16

Home and Foreign are in symmetric long-run equilibrium:

$$N_p^{\mathrm{F}} = N_p^* \tag{28}$$

$$N_p^{\mathrm{F}} = N_w^* = \frac{\alpha}{1-\alpha}\left(\frac{\kappa_w}{\kappa_p}\right)^{-1} N_p^*, \tag{29}$$

where we have used equation (15). Immediately following the revolution, the per capita income of peasants increases to:

$$\widehat{y}_p = \frac{y_p}{1-\alpha} > y_p,$$

where the hat denotes the post-revolution value of a variable [here we have used eq. (3)]. Since fertility is proportional to per capita income [see eq. (6)], it follows that immediately after the revolution peasant fertility rises to

$$\widehat{n}_p = \frac{n_p}{1-\alpha} > n_p,$$

where n_p is pre-revolutionary fertility. From equation (9), it follows that the population in the next period will be

$$N_p^{\mathrm{next}} = \sigma_p \widehat{n}_p N_p = \frac{\sigma_p n_p}{1-\alpha} N_p$$

But Home was in long-run equilibrium before the revolution, so $\sigma_p n_p = 1$. Hence,

$$N_p^{\mathrm{next}} = \frac{N_p}{1-\alpha} > N_p. \tag{30}$$

Foreign did not experience a revolution, so there is no initial change in the per capita incomes or population sizes of either of its social classes. Thus, in the next period

$$N_w^{\mathrm{F,next}} = N_w^{\mathrm{F}} = \frac{\alpha}{1-\alpha}\left(\frac{\kappa_w}{\kappa_p}\right)^{-1} N_p. \tag{31}$$

Since each peasant is equal in fighting capacity to $1/\phi$ warriors, relative military power in the next period will be:

$$\frac{P^{\mathrm{F,next}}}{P^{\mathrm{next}}} = \frac{\phi N_w^{\mathrm{F,next}}}{N_p^{\mathrm{next}}} = \alpha\phi\left(\frac{\kappa_w}{\kappa_p}\right)^{-1},$$

where we have used equations (29), (30), and (31). The area of land controlled by Home in the next period is given by equation (1):

$$L^{\text{next}} = \frac{(P^{\text{next}})^{\beta}}{(P^{\text{next}})^{\beta} + (P^{\text{F,next}})^{\beta}} L_0.$$

Before the peasant revolution, the area of land controlled by Home was $L = L_0/2$. Substituting into the two previous equations yields:

$$\frac{L^{\text{next}}}{L} = \frac{2(\kappa_w/\kappa_p)^{\beta}}{(\kappa_w/\kappa_p)^{\beta} + (\alpha\phi)^{\beta}}. \tag{32}$$

Home's per capita income in the next period will be:

$$y_p^{\text{next}} = A\left(\frac{L^{\text{next}}}{N_p^{\text{next}}}\right)^{\alpha}, \tag{33}$$

while before the revolution it was:

$$y_p = (1-\alpha)A\left(\frac{L}{N_p}\right)^{\alpha}. \tag{34}$$

Dividing equation (33) by equation (34) yields:

$$\frac{y_p^{\text{next}}}{y_p} = (1-\alpha)^{-1}\left(\frac{L^{\text{next}} N_p}{L N_p^{\text{next}}}\right)^{\alpha},$$

and from equations (30) and (32), it follows that:

$$\frac{y_p^{\text{next}}}{y_p} = \frac{1}{(1-\alpha)^{1-\alpha}}\left(\frac{2(\kappa_w/\kappa_p)^{\beta}}{(\kappa_w/\kappa_p)^{\beta} + (\alpha\phi)^{\beta}}\right)^{\alpha}$$

Therefore, $y_p^{\text{next}} < y_p$ if and only if

$$\phi\frac{\kappa_p}{\kappa_w} > \frac{1}{\alpha}\left(\frac{2-(1-\alpha)^{\frac{1-\alpha}{\alpha}}}{(1-\alpha)^{\frac{1-\alpha}{\alpha}}}\right)^{\frac{1}{\beta}}.$$

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