Strategic Information Transmission: Comment

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Crawford and Sobel (1982) developed a model of strategic information transmission in which a better-informed sender sends a possibly informative signal to a decision-making receiver and studied how strategically transmitted information is related to the analogy between the two players' interests. They adopted the Bayesian Nash equilibrium as their equilibrium concept and showed that the signal by the sender, the transmitted information, is more informative in pareto-superior equilibrium when the players' interests are more analogous. Their analyses, however, are not complete in that they analyzed the model based on partial consideration of the players' behavior, mixed behavior of the sender and pure behavior of the receiver. In the present study, we attempt to complete their analyses by

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analyzing the model based on full consideration of the players' behavior, both pure and mixed behavior. We adopt the Nash equilibrium as our equilibrium concept and conclude that results in our complete analyses are similar to the results in Crawford and Sobel (1982).

We first review the model introduced by Crawford and Sobel (1982), hereinafter referred to as CS. There are two players, a sender and a receiver, and they play a signaling game. So, only the sender observes its type $m \in [0, 1]$ and makes a signal $n \in N$ to the receiver where N is an uncountable borel set of feasible signals. Then, after observing the signal n, the receiver chooses its action $y \in \mathbb{R}$. Here, the sender's type m is a random variable from a differentiable probability distribution function $F(m)$ with a probability density function $f(m)$. In addition, the sender has a twice continuously differentiable von Neumann-Morgenstern utility function $U^S(y, m, b)$ where b is a scalar parameter. The receiver has another twice continuously differentiable von Neumann-Morgenstern utility function $U^R(y, m)$.

Under this setting of the model, CS defined strategies as follows. The sender's strategy was defined as a function $q : N \times [0, 1] \longrightarrow \mathbb{R}_+$ such that for each $m \in [0, 1]$, $\int_N q(n|m)dn$ $= 1$. In the equation $\int_N q(n|m)dn = 1$, the left-hand side is an integral of q with respect to the Lebesgue measure. Hence, this definition requires q to be integrable with respect to the Lebesgue measure and its integral over N to be one for each fixed m, which in turn implies that $q(n|m)$ is a conditional probability density function in n given m. That is, the sender's strategy was defined as a conditional probability density function in signals given its types. Next, the receiver's strategy was defined as a function $y : [0, 1] \longrightarrow \mathbb{R}$. Thus, it was defined as an action plan $y(\cdot)$ such that given a signal n, $y(n)$ specifies a single action for the receiver.

These definitions of the strategies, however, limit the players' behavior, and consequently they leave the analyses in CS incomplete. In the sender's case, a conditional probability density function q can represent only mixed behavior that assigns zero probability to every signal $n \in N$. Accordingly, it cannot express any behavior that assigns positive probability to a single signal n. So, this definition excludes all pure behavior under which each type m of the sender would send only one signal $n(m) \in N$. For example, q cannot describe the sender's behavior of truthful signaling which reveals its type truthfully; that is, $n(m) = m$. In the receiver's case, on the other hand, an action plan $y(\cdot)$ can represent pure behavior, but not mixed behavior. This is because $y(\cdot)$ specifies only one action for each signal[2] n. Since the analyses in CS base on these definitions, they consider only part of the players' behavior, the mixed behavior by the sender and the pure behavior by the receiver. Therefore, those analyses are not complete.

To complete the analyses in CS, we propose new definitions for the players' strategies. First, the sender's strategy is defined as a function $\mu : [0,1] \times \text{ß}(N) \longrightarrow [0,1]$ where $\text{ß}(N)$ is

[2] CS claimed that the receiver would never choose mixed behavior in equilibrium, which might justify their limited consideration on the receiver's behavior. Their claim, however, is only partially true because the receiver would choose pure behavior only with probability one in equilibrium. Thus, it is possible that the receiver chooses mixed behavior in equilibrium, but all such equilibrium outcomes would happen with probability zero. This is because given the sender's signals n, the receiver might not be able to figure out conditional probability density functions in the sender's types m according to Bayes' rule. Then, the receiver's expected utility function would not be well-defined, and thus every action $y \in \mathbb{R}$ could be the best response to those signals n. Consequently, if the receiver would choose mixed actions y' such that those signals n are the sender's best responses to y', then the receiver's mixed actions y' could be induced in equilibrium. However, a set of the sender's signals from which the receiver cannot figure out conditional probability density functions has probability zero to happen, and as a result the receiver's mixed behavior can happen at most with probability zero.

the class of the Borel subsets[3] of N such that $i)$ for each $m \in [0,1]$, $\mu(m;\cdot)$ is a probability measure on $\mathcal{B}(N)$ and $ii)$ for each $A \in \mathcal{B}(N)$, $\mu(\cdot;A)$ is $\mathcal{B}[0,1]$ measurable. Here, $\mu(m;A)$ denotes probability assigned to a set of signals A given a state m. Next, the receiver's strategy is defined as another function $\nu : N\times\ \mathcal{B}(\mathbb{R}) \longrightarrow [0,1]$ where $\mathcal{B}(\mathbb{R})$ is the class of the Borel subsets of $\mathbb{R}$ such that $i)$ for each $n \in N$, $\nu(n;\cdot)$ is a probability measure on $\mathcal{B}(\mathbb{R})$ and $ii)$ for each $Y \in \mathcal{B}(\mathbb{R})$, $\nu(\cdot;Y)$ is $\mathcal{B}(N)$ measurable[4] . Again, $\nu(n;Y)$ denotes probability assigned to a set of actions Y given a signal n.

In these definitions, the conditions $i)$ require $\mu(m;\cdot)$ and $\nu(n;\cdot)$ to specify what to play at every information set m in $[0,1]$ and n in N, respectively. The conditions $ii)$ require the strategies μ and ν to well-define expected utilities. Then, it is easy to see that these definitions of the players' strategies can properly describe both the pure and the mixed behavior, and as a result the analyses in CS can be completed through them. Let Π^S be the set of the strategies for the sender and let Π^R be the set of the strategies for the receiver. Note that these definitions originated from Balder (1988) and Jung (2009) and are adapted for the model by CS.

The solution concept in CS also needs to be changed for the complete analyses because it is not compatible with our new definitions of the strategies. CS adopted the Bayesian Nash equilibrium under which the players formed conditional probabilistic beliefs about each other's actions and types and maximized their expected utilities with respect to their

[3] Given a Borel set X, the class of the Borel subsets $\mathcal{B}(X)$ is the smallest class of subsets of X such that $i)$ $\mathcal{B}(X)$ contains all open subsets of X and $ii)$ $\mathcal{B}(X)$ is closed under countable unions and complements.

[4] These strategies μ and ν are also known as transition probabilities. For information on the transition probability, please refer to Neveu (1965, III) and Ash (1972, 2.6).

beliefs. Here, the receiver was assumed to use Bayes' rule to update its belief. Bayes' rule, however, would not properly condition the receiver's belief with respect to our strategies. This is because Bayes' rule defines a belief as proportion and our strategies do not always allow such proportion. Consequently, we adopt the Nash equilibrium concept, which is not related to Bayes' rule.

The Nash equilibrium concept is known to be weak in that it does not properly condition actions off the equilibrium path, and the equilibrium concept used by CS also has this weakness in common. A Nash equilibrium is a profile of strategies that consist of rational actions on the equilibrium path. Thus, it might contain irrational actions off the equilibrium path. The problem with these irrational actions is that they could newly rationalize actions on the equilibrium path, which cannot be rationalized by only rational actions. In this case, since we cannot actually rationalize these irrational actions off the equilibrium path, we cannot properly rationalize those actions on the equilibrium path. Accordingly, some of the Nash equilibrium outcomes could not be rationalized properly. The equilibrium in CS shows the same weakness as the Nash equilibrium does. In their equilibrium, the receiver's conditional expected utility functions are not well-defined off the equilibrium path[5] . Since the receiver's rational actions base on its conditional expected utility functions, they are not well-defined off the equilibrium path either. As a result, the equilibrium concept in CS does not properly condition actions off the equilibrium path.

[5] The receiver's conditional expected utility functions are defined based on its conditional probability density functions. The conditional probability density functions base on Bayes' rule. However, off the equilibrium path, Bayes' rule cannot be applied. Thus, the receiver's conditional probability density functions are not well-defined off the equilibrium path. Therefore, the receiver's conditional expected utility functions are not well-defined off the equilibrium path.

This weakness of the Nash equilibrium, however, is innocuous in the current model because it does not affect outcomes in equilibrium. In this model, the sender's signals are irrelevant to the players' utilities. They just function as a means of information transmission. Thus, each individual signal can be rationalized by the receiver's rational actions in equilibrium, either off the equilibrium path or on the equilibrium path. That is, no signal is newly rationalized by the receiver's irrational actions off the equilibrium path, which accordingly means that these irrational actions do not affect equilibrium outcomes. Therefore, the weakness of the Nash equilibrium, inability to condition actions off the equilibrium path, has no effect on equilibrium outcomes[6] .

A formal definition of the Nash equilibrium is as follows. For simplicity, we define a probability measure $P : \mathcal{B}[0,1] \longrightarrow [0,1]$ as $P(M) = \int_M f(m)dm$ for $M \in \mathcal{B}[0,1]$. In the definition, the integrals are well-defined for each strategy profile $(\mu, \nu) \in \Pi^S \times \Pi^R$ according to Ash (1972, 2.6) because the utility functions U^S and U^R are assumed to be continuous and bounded above[7] .

Definition 1 (Nash equilibrium) *A strategy profile (μ^*, ν^*) is a Nash equilibrium if they solve*

$$\max_{\mu \in \Pi^S} \int_{[0,1]} \int_N \int_{\mathbb{R}} U^S(y, m, b)\nu^*(n; dy)\mu(m; dn)P(dm)$$

and

$$\max_{\nu \in \Pi^R} \int_{[0,1]} \int_N \int_{\mathbb{R}} U^R(y, m)\nu(n; dy)\mu^*(m; dn)P(dm).$$

[6] Likewise, we can see that the weakness of the Nash equilibrium is innocuous in cheap talk games in which *i*) there are two players, a sender and a receiver, and *ii*) the sender's actions are irrelevant to the players' utilities.

[7] CS assumed that for each $m \in [0,1]$, there are a and a' in $\mathbb{R}$ such that $\frac{\partial U^S(a,m,b)}{\partial a} = 0$ and $\frac{\partial U^R(a',m)}{\partial a'} = 0$. Also, they assumed that U^S and U^R are strictly concave in a. These two assumptions imply that U^S and U^R are bounded above.

Finally, our complete analyses lead to results similar to the results in CS. Concretely, if the players' interests differ, then Nash equilibria derived under the complete definitions of the strategies are all partition equilibria[8] that Crawford and Sobel (1982) derived under the partial definitions of the strategies, which can represent only the sender's mixed behavior and the receiver's pure behavior, with an exception of an action set that happens with probability zero. Consequently, every theorem in Crawford and Sobel (1982) stays true almost surely under the complete definitions of the players' strategies, and therefore the sender's signals are more informative in pareto-superior equilibrium when the players' interests are more analogous. Here, the exception in equilibrium happens because the Nash equilibrium concept cannot condition actions that do not affect expected utilities. In a Nash equilibrium $(\mu, \nu) \in \Pi^S \times \Pi^R$, a set of actions $Y \subset \mathbb{R}$ happens with probability zero if and only if we have $\int_{[0,1]} \int_N \int_Y \nu(n; dy)\mu(m; dn)P(dm) = 0$. Hence, those actions in the set Y do not affect expected utilities, and so they could be part of a Nash equilibrium.

This similarity between the results in CS and the results in our complete analyses is due to a hypothesis on the players' interests and an assumption on the receiver's utility function. In this model, the sender's signaling is an informational activity. So, the receiver could infer the sender's types from its signals. Thus, if the sender's interests differ from the receiver's interests, then the sender would not make perfectly informative signals from which the receiver can completely figure out the sender's types because the perfectly informative

[8] A partition equilibrium in CS is an equilibrium in which *i*) signals sent by the sender partition its type space $[0, 1]$ into a finite number of intervals and *ii*) actions chosen by the receiver in equilibrium are monotonically associated with the intervals. In this definition, monotonicity implies that higher actions are associated with higher intervals in the partition.

signals would lead to the receiver's favorite outcomes that contradict to the sender's interests. Accordingly, the sender would make only imperfectly informative signals in equilibrium. Here, these imperfectly informative signals can be represented both by the pure behavior and by the mixed behavior while the perfectly informative signals can be represented only by the pure behavior. Note that the definition of the sender's strategies in CS admits the mixed behavior. Therefore, under the hypothesis that the sender's interests differ from the receiver's interests, the sender's signals in equilibrium can be described according to the partial definition of the sender's strategies in CS as well as according to the complete definition in the current study[9] .

In addition, CS assumed that the second-order partial derivative of the receiver's utility function, U^R_{11}, satisfies $U^R_{11}(y, m) < 0$ for every y and m. Then, U^R has a unique maximum in actions y for each state m, and thus this assumption induces the pure behavior by the receiver almost surely in equilibrium. Note that the definition of the receiver's strategies in CS allows the pure behavior. Therefore, under the assumption of $U^R_{11} < 0$, the receiver's actions in equilibrium can be expressed almost surely according to the partial definition of the receiver's strategies in CS as well as according to the complete definition in the current study. Consequently, our complete analyses end up with the same results as the results in CS almost surely.

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[9] If the players' interests coincide, then one of Nash equilibria contains perfectly informative signals by the sender. However, this Nash equilibrium is not an equilibrium in CS because the perfectly informative signals cannot be represented according to the partial definition of the sender's strategies in CS. Therefore, if the players' interests coincide, then the results in CS differ from the results in the current study.

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