The Impact of Firm Size and Market Size Asymmetries on National Mergers in a Three-Country Model

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Abstract

This paper studies the impact of firm and market size asymmetries on merger decisions. To do that I consider a model where a small and a large country compete in a third (world) market. Each of the two countries has two firms (with potentially different costs) that supply the domestic market and export to the third market. Merger decisions in the two countries are modeled as a simultaneously move game. The paper finds that firms in the large country have more incentives to merge than firms in the small country. In contrast, the government of the large country has more incentives to block a merger than the government of the small country. Thus, the model predicts that conflicts of interest between governments and firms concerning national mergers are more likely in large countries than in small ones.

JEL Codes: F13, H77, L11, L41.
Keywords: Mergers; International Trade; Merger Policy; Size Asymmetry.

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1 Introduction

In many European countries there is a heated debate over whether governments and competition authorities should favor or oppose the creation of national champions.[1] An argument often put forth in favor of national champions is that bigger firms will be in a better position to compete in world markets.[2] It's true that the emergence of a national champion might improve a country's welfare if it has strong efficiency gains and shifts profits away from competitors in export markets. However, the emergence of a national champion might also reduce a country's welfare if the efficiency gains from cost savings are smaller than the loss from reduced domestic output. A national champion might also not be able to shift profits away from competitors in export markets due to losses in market share.

This paper contributes to this debate by setting up a three-country model in which firms in two countries serve their respective domestic markets and compete in a third (world) market. This market structure captures the situation in many network industries, such as electricity, natural gas, telecommunications or railways. A typical example is the electricity market, where the German duopolists E.ON and RWE compete with other 'national champions' in several European markets. It is also relevant in markets like cement and dairy products where large national players compete in third markets, but less so in the respective home markets of their competitors.

I use the model to study endogenous mergers and mergers that improve national welfare. By comparing the equilibrium outcomes of these two games I am able to clarify which factors contribute to the existence of conflicts of interest between firms and governments about the desirability of national champions.

In this model mergers have efficiency gains and are modeled as a simultaneous move game. Firms compete à la Cournot, markets are segmented, and there are no producers in the third market.[3] The novelty of my approach is that it allows for both firm size and market size asymmetries. Firms can have different costs of production and the three countries can have different market demands. These assumptions make the model more general than previous ones and allow me state new results that show how firm and market size asymmetries influence incentives for mergers to take place and merger equilibrium outcomes.

The questions that this paper addresses have many links with the existing literature on merger and competition policy, specially with papers which extend the analysis to open economies.[4] This literature has taken two different directions. One line of research focuses on nationally optimal merger policies

[1] For example, the French government advocated a merger between the electricity and gas company SUEZ with the firm GAZ DE FRANCE.

[2] A recent example in Germany has been the aproval of the merger between the E.ON and RUHRGAS corporations where the German Minister of Economics argued that size was very important at the onset of the energy market liberalization in Europe.

[3] This set-up captures the idea that domestic markets are less competitive than export markets. See Brander and Spencer (1985).

[4] The traditional analysis of mergers and acquisitions in industrial organization–Salant et al. (1983) and Deneckere and Davidson (1985)–usually neglects the effects of country borders.

and merger profitability when trade policy instruments are available to national governments–e.g., Richardson (1999), Horn and Levinsohn (2001), and Huck and Conrad (2004). The other line of research is based on the concept of "external effects" of a merger to outsiders. An important early contribution to this topic is Farrel and Shapiro (1990). This concept was extended to open economies by Barros and Cabral (1994). This literature has derived rather general conditions under which a merger benefits, or harms, the parties not participating in the merger. It does not, however, explicitly consider that a merger may lead to cost reductions and so it can not provide a complete characterization of post-merger equilibrium.

This paper takes a different approach by analyzing mergers with a three-country model like Haufler and Nielsen (2008) and Sudekum (2008). Both Haufler and Nielsen (2008) and Sudekum (2008) assume that firms have identical costs of production and that the two competitor countries have the same market demand but demand in the export market can be larger than domestic demand. Haufler and Nielsen (2008) find that there is a range of cost reductions for which a merger is in the private interest of domestic firms, but not in the interest of the country has a whole. They also find that when the export market is larger the range for which the merger is blocked decreases. Sudekum (2008) finds that the promotion of national mergers can be in the interest of individual countries if rent extraction possibilities are strong enough when firms compete on all markets and are subject to transport costs.

2 Set-up

Consider three countries: a small country, s, a large country, L, and a third country, t. Initially there are 2 firms in the small country and 2 firms in the large country. There are no firms in the third country. Firms in the small and the large countries sell their product in the domestic markets and export it to the third country. Thus, there is no bilateral trade between the small country and the large country and firms compete in the third (or export) market.

The inverse demand function in the small country is $P_s = a - Q_s$,with $a > 1$. The inverse demand function in the large country is $P_L = a - \gamma Q_L$, with $0 < \gamma \leq 1$. The parameter γ measures the level of market size asymmetry. A decrease in γ implies that for any given price, demand in the large country increases.[5] The inverse demand function in the export market is $P_t = a - \beta Q_t$, with $0 < \beta \leq 1$. The parameter β represents the market size of the export market.

Firms in the small and large countries are fully owned by residents and produce a homogeneous good. There are no fixed costs (this rules out gains from economies of scale in mergers).[6] Marginal costs of firms are given by $c_{v1} = c$,

[5] The reciprocal of γ measures how many times the market of the large country is bigger than the market of the small country. For example, if $\gamma = 0.25$ the market of the large country is four times the market of the small country.

[6] Transportation costs between s and t and between L and t are assumed to be equal to

$c_{v2} = c + \Delta$, where $v = s, L$ and $\Delta \in [0, (a-c)/3]$. I assume $\Delta \leq (a-c)/3$ so that, in the absence of mergers, even the less efficient firm makes nonnegative profits in all markets. It is useful to define $\delta = \Delta/(a-c)$ and use it as a summary measure of cost asymmetry. Following Barros (1998) I assume that if two firms merge, the high-cost firm ceases production and only the low-cost unit produces. Therefore, a merger can be viewed as an acquisition of a high-cost firm by a low-cost firm.[7] I assume that firms play separate Cournot games in each market which implies that each market can be analyzed independently of the other markets.[8] Thus, before any merger has taken place, the problem of firm si is given by

$$\max_{q_{si};q_{si}^t} \left(a - \sum_{k=1}^{2} q_{sk} - c_{si}\right) q_{si} + \left(a - \beta\left(\sum_{k=1}^{2} q_{sk}^t + \sum_{k=1}^{2} q_{Lk}^t\right) - c_{si}\right) q_{si}^t.$$

Similarly, at the start, the problem of firm Li is:

$$\max_{q_{Li},q_{Li}^t} \left(a - \gamma \sum_{k=1}^{2} q_{Lk} - c_{Li}\right) q_{Li} + \left(a - \beta\left(\sum_{k=1}^{2} q_{Lk}^t + \sum_{k=1}^{2} q_{sk}^t\right) - c_{Li}\right) q_{Li}^t.$$

3 Profitability of Conditional Mergers

When a merger takes place there are three effects that the firms involved in the merger need to take into consideration. First, there is an efficiency gain since the high cost firm transfers production to the low cost firm. Second, a merger leads to less competition both in the domestic market as well as in the export market. These two effects allow the merged firm to have a higher mark-up than the highest mark-up of the individual firms. Thus, the market power of the firms involved in the merger increases in both markets. However, in the export market the merger implies that the market share of the merged firm is lower than the sum of the pre-merger market shares of the firms involved in the merger. This third effect reduces the incentive for mergers to occur.[9] Thus, a merger increases profits in the domestic market but it might reduce profits in the export market.

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zero. Transportation cost can be greater than zero (but lower than a) without changing qualitatively the results in the paper.

[7] Barros (1998) approach is also used by Qiu and Zhou (2007). Perry and Porter (1985) and Farrell and Shapiro (1990) use other approaches to model the impact of a merger on an industry's cost structure.

[8] The assumption of Cournot competition is in line with much of the literature on mergers. Theoretical and empirical arguments in defence of the Cournot model are presented by Haufler and Nielsen (2005). The model proposed by Kreps and Scheinkman (1983) in which firms choose capacities in the first period and compete in prices in the second period generates Cournot outcomes.

[9] The fact that a merger always reduces the firms' combined market share in the export market, despite the cost savings, is driven by the assumption that the merger merely eliminates the high cost firm, thereby reshuffling some output to the lower cost merger partner. If the merger resulted in synergies that reduced the marginal cost of the lower cost merging firm, then mergers would become more attractive.

My first result summarizes the impact of market structure, firm size, and market size asymmetries on incentives for firms to merge.

Proposition 1: *The conditions under which a merger of domestic firms is profitable are less restrictive: (i) when foreign firms are merged, (ii) when cost asymmetries are high, (iii) when the export market is small, and (iv) in the country with the largest domestic market.*

A merger in country i makes a merger in country j profitable for a larger set of parameters. This happens because (a) a merger in country i lowers the number of active firms in the export market from three to two if firms in country j are merged and from four to three if the firms in country j are not merged, and (b) a move from three firms to two creates a larger increase in mark-up than a move from four firms to three and both moves lead to the same loss in market share.

When cost asymmetries are high a merger increases profits in the domestic and export markets and so a merger is profitable no matter the size of the export market. When cost asymmetries are low a merger increases profits in the domestic market but reduces profits in the export market and so a merger is only profitable when the export market is relatively small. Hence, the incentives for a merger of domestic firms are higher when cost asymmetries are high and, if cost asymmetries are low, when the size of the export market is small. Similarly, the incentives for a merger of domestic firms are higher in the country with the largest domestic market because if a merger increases profits in the domestic market but reduces profits in the export market, then the losses in the export market are relatively smaller in the country with the largest domestic market.

Figure 1 illustrates how the incentives for a merger in the small country depend on cost asymmetries and the size of the export market.

Insert Figure 1 here

The thin dotted curve in Figure 1 characterizes incentives for a profitable merger in the small country when firms in the large country are not merged. The thick solid curve characterizes incentives for a profitable merger in the small country when firms in the large country are merged. To the right (left) of each curve firms in the small country merge (do not merge).

4 Merger Game Played by Firms

I will now characterize the incentives to merge assuming that governments do not intervene in markets. As the starting point, I assume that no merger has taken place in either country. The decisions of firms in each country to merge or not to merge are taken simultaneously. The relevant payoffs of this game are summarized in Table I in the Appendix. Propositions 2 and 3 characterize the equilibria of the merger game played by firms.

Proposition 2: *Assume the large country's market is less than or equal to* 1.26 *times the small country's market, that is,* $\gamma \geq 0.794$.
(i) If cost asymmetries are low and the export market is large, that is, $0 < \beta \leq f_s^{L1+L2}(\delta)$, *then firms in either country do not merge;*
(ii) If cost asymmetries are either low or moderate and the size of export market satisfies $\max[0, f_s^{L1+L2}(\delta)] < \beta \leq f_L^{s1,s2}(\delta, \gamma)$, *then there are two pure-strategy equilibria–firms in either country do not merge and firms in either country merge–, and one mixed-strategy equilibrium in which firms in the small country merge with probability*

$$p_s = \frac{(63\gamma - 100\beta) - (738\gamma + 800\beta)\delta + (1647\gamma + 2000\beta)\delta^2}{(13 + 162\delta - 603\delta^2)\gamma}, \tag{1}$$

and firms in the large country merge with probability

$$p_L = \frac{(63 - 100\beta) - (738 + 800\beta)\delta + (1647 + 2000\beta)\delta^2}{13 + 162\delta - 603\delta^2}. \tag{2}$$

(iii) If either cost asymmetries are high or the export market is small when cost asymmetries are low or moderate, that is, $\max[0, f_L^{s1,s2}(\delta, \gamma)] < \beta \leq 1$, *then firms in either country merge.*

Figure 2 illustrates the equilibria of the merger game played by firms when $\gamma = 0.85$ (the predictions of the model are similar for any $\gamma \geq 0.794$).

Insert Figure 2 here

As in Figure 1, the thin dotted and the thick solid curves in Figure 2 characterize incentives for a merger in the small country when firms in the large country are not merged and merged, respectively. The thin solid and the thick dotted curves in Figure 2 characterize incentives for a merger in the large country when firms in the small country are not merged and merged, respectively.

The two solid curves in Figure 2 determine the different equilibria of the merger game played by firms. To the right of the thin solid curve firms in either country merge. To the left of the thick solid curve firms in either country do not merge. Between the two solid curves the game has three equilibria: firms in either country merge, firms in either country do not merge, and firms in the small country merge with probability p_s whereas firm in the large country merge with probability p_L, with $p_s < p_L$.

Thus, Figure 2 tells us that if firm size asymmetries are high there will be mergers in both countries due to the profit gains in the domestic and in the export markets. When firm size asymmetries are moderate mergers are not as attractive since they lead to gains in the domestic market but losses in the export market. In this case we have two possible situations. If the export market is small, then domestic profit gains are larger than the losses in the export market and firms in either country merge. If the export market is big, then we have multiple equilibria. If firm size asymmetries are low, mergers are the least attractive since they generate small profit gains in domestic market and large losses in the export market. In this case we have three outcomes. If the export market is small, firms in either country merge. If the export market is intermediate we have multiple equilibria. If the export market is big firms in either country do not merge.

Proposition 3: *Assume that the large country's market is more than* 1.26 *times the small country's market, that is,* $\gamma < 0.794$.
(i) If cost asymmetries are low and the export market is large, that is, $0 < \beta \leq \min[f_s^{L1+L2}(\delta), f_L^{s1,s2}(\delta, \gamma)]$, *then firms in either country do not merge;*
(ii) If cost asymmetries are low and the size of the export market satisfies $f_L^{s1,s2}(\delta, \gamma) \leq \beta \leq f_s^{L1+L2}(\delta)$, *then firms in the small country do not merge and firms in the large country merge;*
(iii) If cost asymmetries are moderate and the size of the export market satisfies $\max\left(0, f_s^{L1+L2}(\delta)\right) < \beta \leq f_L^{s1,s2}(\delta, \gamma)$, *then there are two pure-strategy equilibria–firms in either country do not merge and firms in either country merge–, and one mixed-strategy equilibrium in which firms in the small country merge with probability (1) and firms in the large country merge with probability (2);*
(iv) If either cost asymmetries are high or the export market is small when cost asymmetries are low or moderate, that is, $\max[0, f_s^{L1+L2}(\delta), f_L^{s1,s2}(\delta, \gamma)] < \beta \leq 1$, *then firms in either country merge.*

Figure 3 illustrates the equilibria of the merger game played by firms when $\gamma = 0.25$ (the predictions of the model are similar for any $\gamma < 0.794$).

Insert Figure 3 here

In Figure 3 the intersection of the area to the right of the thick solid curve with the area to the right of the thin solid curve represents parameter configurations where firms in either country merge. The intersection of the area to the left of the thick solid curve with the area to the left of the thin solid curve represents parameter configurations where firms in either country do not merge. To the right of the thin solid curve and to the left of the thick solid curve firms in the small country do not merge and firms in the large country merge. Finally, to the right of the thick solid curve and to the left of the thin solid curve we have three equilibria: firms in either country merge, firms in either country do not merge, and firms in the small country merge with probability p_s and firms in the large country merge with probability p_L.

The thick solid and the thin dotted curves in Figure 3 are equal to the ones depicted in Figures 1 and 2 since incentives for mergers in the small country do not depend on the market size of the large country. However, an increase in the market size of the large country changes the incentives for mergers in the large country. Comparing Figures 2 and 3 we see that an increase in the market size of the large country moves the thin solid curve and the thick dotted curve closer to the delta axis. This means that for low firm size asymmetries, an increase in the market size of the large country makes mergers increasingly more attractive in the large country than in the small country. This happens because the bigger the market size of the large country, the greater are the domestic gains of a merger of firms in that country. So, when firm size asymmetries are low and the merger leads to losses in the export market, the bigger the size of the domestic market the more likely is that the domestic profit gains exceed the export market losses and the more attractive is becomes for firms to merge.

The fact that an increase in the market size of the large country makes a merger increasingly more attractive in the large country but not in the small country, implies that now there exist equilibria where firms in the large country merge and firms in the small country do not merge. This happens when firm size asymmetries are low and the size of the export market is intermediate.

5 Welfare Impact of Conditional Mergers

This section analyzes incentives for governments to merge national firms. I assume that governments maximize national welfare: the sum of consumer surplus and profits in the domestic and export markets. To find out the impact of a merger on national welfare a government must take into account the merger's impact on consumer surplus and on profits in the domestic and export markets.

A merger of domestic firms reduces domestic output (even with cost savings because we move from asymmetric Cournot duopoly to monopoly with only the low cost firm). Thus domestic consumers lose, while profits rise. Hence, the welfare impact of a domestic merger on the domestic market is ambiguous: with low cost savings, the effect is negative, because the loss in consumer surplus outweights the increase in profits; but with sufficiently high cost savings the effect is positive since the increase in profits outweights the loss in consumer surplus. In the export market the welfare impact of a merger is also ambiguous: with low cost savings the effect is negative, because the loss in market share outweights the gain from restricting total output and raising price; but with high cost savings, profits in the export market can rise.

Proposition 4 summarizes the impact of market structure, firm size, and market size asymmetries on incentives for governments to merge national firms.

Proposition 4: *The conditions under which a merger of domestic firms is welfare improving are less restrictive: (i) when foreign firms are merged, (ii) when cost asymmetries are high, (iii) when the export market is large, and (iv) in the country with the smallest domestic market.*

The first result in Proposition 4 arises here for the same reasons as the first result in Proposition 1. When cost asymmetries are high the government wants to merge domestic firms since the merger leads to a small reduction in consumer surplus and a large increase in profits in the domestic market and also an increase in profits in the export market (the increase in mark-up makes up for the loss of market share). When cost asymmetries are low the government does not want to merge domestic firms since a merger would reduce profits in the export market (the increase in mark-up does not make up for the loss in market share) and the increase in profits in the domestic market is not enough to make up for the reduction in consumer surplus. Hence, governments have more incentives to merge national firms when cost asymmetries are high.

When cost asymmetries are moderate there is a trade-off between welfare losses in the domestic market (profits in the domestic market increase less than the reduction in consumer surplus) and profit gains in the export market. Hence, governments have more incentives to merge national firms when the export market is large. The conditions under which a merger of domestic firms is welfare improving are less restrictive in the country with the smallest domestic market because if a merger reduces welfare in the domestic market and increases profits in the export market, then the losses in the domestic market are relatively smaller in the country with the smallest domestic market.

Figure 4 illustrates how the incentives for the government of the small country to merge national firms depend on cost asymmetries and the size of the export market.

Insert Figure 4 here

The thin solid curve in Figure 4 characterizes incentives for the government of the small country to merge national firms when firms in the large country are not merged. The thick dotted curve in Figure 4 characterizes incentives for the government of the small country to merge national firms when firms in the large country are merged. To the right (left) of this curve the government of the small country chooses (not) to merge national firms.

6 Merger Game Played by Governments

I now assume that governments determine the market structure in each country and that firms play no active role in merger decisions, that is, if the firms would like to merge but the government opposes the merger, then there is no merger.[10] Like before, I assume that at the start no merger has taken place in either country. The decisions of governments in each country to merge or not to merge firms are taken simultaneously and governments maximize national welfare. The relevant payoffs of this game are summarized in Table II in the Appendix. Propositions 5 and 6 characterize the equilibria of the merger game played by governments.

Proposition 5: *Assume that the large country's market is less than or equal to* 2.15 *times the small country's market, that is,* $0.466 \leq \gamma$.
(i) If either cost asymmetries are low or the export market is small when cost asymmetries are moderate, that is $\max[0, g_L^{s1+s2}(\delta,\gamma)] < \beta \leq 1$*, then governments in either country do not merge firms;*

[10] The case where the government would force a merger even when the firms opposed it does not arise, because whenever a merger raises national welfare it must raise profit but not vice versa (because the merger reduces consumer surplus).

(ii) If cost asymmetries are moderate and the size of the export market satisfies $\max[0, g_s^{L1,L2}(\delta)] < \beta \le \min[g_L^{s1+s2}(\delta, \gamma), 1]$*, then there are two pure-strategy equilibria–governments in either country do not merge firms and governments in either country merge firms–, and one mixed-strategy equilibrium in which the government of the small country merges firms with probability*

$$q_s = \frac{63\gamma + 250\beta - (738\gamma + 1600\beta)\delta + (1647\gamma + 2200\beta)\delta^2}{\left(13 + 162\delta - 603\delta^2\right)\gamma}, \tag{3}$$

and the government of the large country with probability

$$q_L = \frac{63 + 250\beta - (738 + 1600\beta)\delta + (1647 + 2200\beta)\delta^2}{13 + 162\delta - 603\delta^2}. \tag{4}$$

(iii) If either cost asymmetries are high or the export market is large when cost asymmetries are moderate, that is, $0 < \beta \le \min[g_s^{L1,L2}(\delta), 1]$*, then governments in either country merge firms.*

Proposition 5 tells us that if firm size asymmetries are sufficiently low, then governments do not merge national firms since a merger generates welfare losses in both the domestic and export markets. When firm size asymmetries are moderate, then a merger leads to a welfare gain in the export market but a welfare loss in the domestic market. In this case we have multiple equilibria. Finally, if firm size asymmetries are sufficiently high, governments merge national firms since a merger generates welfare gains in both the export and the domestic markets.

Figure 5 illustrates the equilibria of the merger game played by governments when $\gamma = 0.85$ (the predictions of the model are similar for any $\gamma > 0.466$).

Insert Figure 5 here

As in Figure 4, the thin solid and the thick dotted curves in Figure 5 characterize incentives for the government of the small country to merge national firms when firms in the large country are not merged and merged, respectively. The thin dotted and the thick solid curves characterize incentives for the government of the large country to merge national firms when firms in the small country are not merged and merged, respectively. To the right (left) of each curve governments (do not) merge national firms.

The two solid curves in Figure 5 determine the different equilibria of the game. To the right of the thin solid curve governments merge national firms. To the left of the thick solid curve governments do not merge national firms. Between the two solid curves we have three equilibria: governments merge national firms, governments do not merge national firms, and the government of the small country merges national firms with probability q_s and that of the large country with probability q_L, with $q_L < q_s$.

Proposition 6: *Assume that the large country's market is more than* 2.15 *times the small country's market, that is,* $\gamma < 0.466$.

(i) If either cost asymmetries are low or the export market is small when cost asymmetries are moderate, that, is $\max[0, g_L^{s1+s2}(\delta,\gamma), g_s^{L1,L2}(\delta)] < \beta \leq 1$*, then governments in either country do not merge firms;*
(ii) If cost asymmetries are moderate and the size of the export market satisfies $g_L^{s1+s2}(\delta,\gamma) \leq \beta \leq \min[g_s^{L1,L2}(\delta), 1]$*, then the government in the small country merges firms and the government in the large country does not merge firms;*
(iii) If cost asymmetries are moderate and the size of the export market satisfies $\max[0, g_s^{L1,L2}(\delta)] < \beta \leq g_L^{s1+s2}(\delta,\gamma)$*, then there are two pure-strategy equilibria–governments in either country do not merge firms and governments in either country merge firms–, and one mixed-strategy equilibrium in which the government of the small country merges firms with probability (3) and the government of the large country with probability (4);*
(iv) If cost asymmetries are high or the export market is large when cost asymmetries are moderate, that is, $0 < \beta \leq \min[g_L^{s1+s2}(\delta,\gamma), g_s^{L1,L2}(\delta), 1]$*, then governments in either country merge firms.*

Figure 6 illustrates the equilibria of the merger game played by governments when $\gamma = 0.25$ (the predictions of the model are similar for any $\gamma < 0.466$).

Insert Figure 6 here

In Figure 6 the intersection of the area to the right of the thick solid curve with the area to the right of the thin solid curve represents equilibria where governments choose to merge national firms. The intersection of the area to the left of the thick solid curve with the area to the left of the thin solid curve represents equilibria where governments choose not merge national firms. In the area to the right of the thin solid curve and to the left of the thick solid curve the government of the small country merges national firms and the government of the large country does not merge national firms. In the area to the right of the thick solid curve and to the left of the thin solid curve we have multiple equilibria.

7 Conflicts of Interest

This section discusses the implications of the model regarding conflicts of interest between firms and governments about merger decisions. I show that the model predicts that if firms of a small and of a large country compete in a third country, then *the conditions under which conflicts of interest occur are less restrictive in the large country than in the small country.*

This result is driven by the asymmetric equilibria of the two merger games. Propositions 7, 8 and 9 in the Appendix compare the equilibria of the two games. They show that when one of these two types of asymmetric equilibria occur, firms in the large country prefer to merge but the government of the large country opposes a merger. Thus the model predicts that, everything else constant, competition authorities should be less actively involved in the regulation of export industries in small countries than in large ones.

Figures 7, 8 and 9 display the curves that determine the equilibria of the two merger games when $\gamma = 0.85$ (the predictions of the model regarding conflicts of interest are similar for any $\gamma > 0.794$), $\gamma = 0.55$ (the predictions of the model are similar for any $\gamma \in (0.466, 0.794)$), and $\gamma = 0.25$ (the predictions of the model are similar for any $\gamma < 0.466$), respectively.

Insert Figures 7, 8 and 9 here

In Figures 7, 8 and 9 the negatively sloped curves determine the set of equilibria of the merger game played by firms. To the right of the thin solid negatively sloped curve firms in either country merge. To the left of the thick solid negatively sloped curve firms in either country do not merge. The positively sloped curves determine the set of equilibria of the merger game played by governments. To the right of the thin solid positively sloped curve governments in either country merge national firms. To the left of the thick solid positively sloped curve governments in either country do not merge national firms.

We can see from Figures 7, 8 and 9 that if firm size asymmetries are high and the export market is large, then there are no conflicts of interest between national firms and governments: all favor a merger. The interests of national firms and governments are also aligned if firm size asymmetries are low and the export market is large: all are against a merger. However, if firm size asymmetries are moderate or low and the export market is relatively small, then a conflict of interest arises: firms wish to merge but governments oppose the mergers.

Comparing Figures 7, 8 and 9 we also see that an increase in market size asymmetry between the large and the small countries–a decrease in γ–increases the likelihood of conflicts of interest in the large country. In Figure 8 the market size asymmetry is moderate and we find parameter configurations where firms in the large country want to merge, firms in the small country do not want to merge, and governments oppose mergers. In Figure 9 the market size asymmetry is high and there are (i) parameter configurations where firms in the large country want to merge, firms in the small country do not want to merge, and governments oppose mergers, and (ii) parameter configurations where the government of the large country opposes a merger, the government of the small country favors a merger, and firms in either country want to merge. Hence, the conditions under which conflicts of interest occur in the large country are less restrictive than those in the small country.

8 Extensions

There are many possible directions in which one could extend this model. For example, one could relax the assumption that exports are only to the third country. Bilateral trade between the small and the large country has two effects. First, each exporting country has one more export market (the rival's market). Second, it increases the number of firms selling in domestic markets.

The additional export market leads to a larger increase in profits in the foreign markets following a merger when cost asymmetries are high and a smaller increase when cost asymmetries are low. An increase in competition in the domestic market leads to a smaller increase in profits in the domestic market following a merger when cost asymmetries are high and a reduction in profits in the domestic market when cost asymmetries are low. Thus, allowing for bilateral trade makes a merger less attractive to firms for low cost asymmetries since profits in the domestic and in the export markets are lower. For high cost asymmetries the result depends on the relative sizes of the markets. Let's now turn to the impact of bilateral trade on welfare. An increase in competition in the domestic market leads to a lower reduction in consumer surplus following a merger since a move from four to three firms (when foreign firms are not merged) or from three to two (when foreign firms are merged) reduces consumer surplus by less than a move from two to one. This implies that bilateral trade makes a merger more attractive to governments when cost asymmetries are high since domestic welfare and profits in the export market are higher than without bilateral trade. When cost asymmetries are low the impact of bilateral trade on the attractiveness of a merger to governments depends on the relative size of the markets. Thus, bilateral trade between the small and the large country reduces conflicts of interest between firms and governments when cost asymmetries are high and its impact on conflicts of interest is ambiguous when cost asymmetries are low.

Another possible extension is to assume that exports from firms in the small and large countries must compete against local production from firms in the third country. The impact of competition from firms in the third country on incentives for firms to merge in either the small or the large country depends on the cost savings induced by the merger. Mergers would be less (more) attractive to firms if cost asymmetries are high (low) because the gains (losses) in the export market are lower. Competition from firms of the third country market makes mergers less attractive to governments since the gains from a merger in the export market are smaller. Thus, the presence of additional competitors in the export market should increase the likelihood of conflicts of interest between firms and governments when cost asymmetries are low but not when they are high.

Yet another extension of the model would be to break the assumption that the firm size asymmetries in the small and the large country are the same. For example, one could assume that firms in the large country are uniformly more (or less) efficient than firms in the small country. This extension complicates the analysis since it is no longer possible to find closed form solutions for market size thresholds that define the set of equilibria of the model. However, it is possible to parameterize the model numerically to study this possibility.

One could also relax the assumption that there are only two national firms in each country. Doing that makes the analysis of the merger game considerably more complicated. For example, if there are three national firms in each country we would need to consider all possible merger combinations. We would need to state not only individually rational constraints for mergers to be viable but also stability conditions under which the firms outside the mergers would not make a better offer to one of the participants in the merger.[11]

If competition were in prices with differentiated products mergers would always be profitable since in that case mergers increase mark-ups both domestically and in the export market but there would be no loss of market share effect of a merger. However, the effects on a country's welfare would still be ambiguous due to the price increases domestically, and the conflicts of interest between governments and firms would still depend on the relative sizes of the markets.

An interesting extension of the model would be to study explicitly a game between national firms and competition authorities where firms propose mergers and competition authorities accept or reject mergers proposed by firms. This extension introduces a dynamic aspect to merger analysis in open economies that has not yet been sufficiently explored.[12] This would be a middle ground between the merger game played by firms and the one played by governments.

9 Conclusion

This paper studies incentives for national mergers in a model where firms of two countries compete in a third country market. The main novelty of the paper is that it characterizes incentives for national firms to merge and for governments to promote national mergers when firms can have different cost of production and countries can have different market demands.

The paper finds that firms in the large country have more incentives to merge than firms in the small country. In contrast, the government of the large country has more incentives to block a merger than the government of the small country. Thus, the model predicts that conflicts of interest between governments and firms concerning national mergers are more likely in large countries than in small ones.

[11] See Barros (1998) and Horn and Persson (2001) for closed economy models of mergers in markets with two or more firms.

[12] See Motta and Vasconcelos (2005) for an example of this type of model in a closed economy.

10 References

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11 Appendix

Lemma 1:
(i) If $\frac{9}{100}\frac{7-82\delta+183\delta^2}{1+8\delta-20\delta^2} = f_s^{L1,L2}(\delta) \leq \beta \leq 1$, *and firms in the large country are not merged, then a merger in the small country is profitable;*
(ii) If $\frac{1}{2}\frac{1-18\delta+45\delta^2}{1+8\delta-20\delta^2} = f_s^{L1+L2}(\delta) \leq \beta \leq 1$, *and firms in the large country are merged, then a merger in the small country is profitable;*
(iii) If $\frac{9\gamma}{100}\frac{7-82\delta+183\delta^2}{1+8\delta-20\delta^2} = f_L^{s1,s2}(\delta,\gamma) \leq \beta \leq 1$, *and firms in the small country are not merged, then a merger in the large country is profitable;*
(iv) If $\frac{\gamma}{2}\frac{1-18\delta+45\delta^2}{1+8\delta-20\delta^2} = f_L^{s1+s2}(\delta,\gamma) \leq \beta \leq 1$, *and firms in the small country are merged, then a merger in the large country is profitable.*

Proof of Lemma 1: I start the proof by deriving the conditions under which a merger in s is profitable conditional on a given market structure in L. If s firms are not merged they sell $q_{s1} = (a-c+\Delta)/3$ and $q_{s2} = (a-c-2\Delta)/3$ in the s market. In this case, profits of s firms in the s market are given by $\pi_{s1}^s = (a-c+\Delta)^2/9$ and $\pi_{s2}^s = (a-c-2\Delta)^2/9$. If L firms are not merged they sell $q_{L1} = (a-c+\Delta)/3\gamma$ and $q_{L2} = (a-c-2\Delta)/3\gamma$ in the L market. Profits of L firms in the L market are $\pi_{L1}^L = (a-c+\Delta)^2/9\gamma$ and $\pi_{L2}^L = (a-c-2\Delta)^2/9\gamma$. The market equilibrium in t is given by:

$$\begin{aligned}
q_{s1}^t &= \frac{a-c}{2\beta} - \frac{1}{2}\left(q_{s2}^t + q_{L1}^t + q_{L2}^t\right) \\
q_{s2}^t &= \frac{a-c-\Delta}{2\beta} - \frac{1}{2}\left(q_{s1}^t + q_{L1}^t + q_{L2}^t\right) \\
q_{L1}^t &= \frac{a-c}{2\beta} - \frac{1}{2}\left(q_{s1}^t + q_{s2}^t + q_{L2}^t\right) \\
q_{L2}^t &= \frac{a-c-\Delta}{2\beta} - \frac{1}{2}\left(q_{s1}^t + q_{s2}^t + q_{L1}^t\right)
\end{aligned}$$

Solving this system we obtain $q_{s1}^t = q_{L1}^t = (a-c+2\Delta)/5\beta$ and $q_{s2}^t = q_{L2}^t = (a-c-3\Delta)/5\beta$. The profits of s and L firms in t are given by $\pi_{s1}^t = \pi_{L1}^t = (a-c+2\Delta)^2/25\beta$ and $\pi_{s2}^t = \pi_{L2}^t = (a-c-3\Delta)^2/25\beta$.
If s firms merge the s market becomes a monopoly and the equilibrium quantity is $q_{s_1+s_2} = (a-c)/2$. The monopoly profits are $\pi_{s1+s2} = (a-c)^2/4$. If s firms merge and L firms are not merged, then the equilibrium in t is given by

$$\begin{aligned}
q_{s1+s2}^t &= \frac{a-c}{2\beta} - \frac{1}{2}\left(q_{L1}^t + q_{L2}^t\right) \\
q_{L1}^t &= \frac{a-c}{2\beta} - \frac{1}{2}\left(q_{s1+s2}^t + q_{L2}^t\right) \\
q_{L2}^t &= \frac{a-c-\Delta}{2\beta} - \frac{1}{2}\left(q_{s1+s2}^t + q_{L1}^t\right)
\end{aligned}$$

Solving this system we obtain $q_{s1+s2}^t = q_{L1}^t = (a-c+\Delta)/4\beta$ and $q_{L2}^t = (a-c-3\Delta)/4\beta$. The profits of the merged s firm in t are $\pi_{s1+s2}^t = (a-c+\Delta)^2/16\beta$. A merger of s firms is profitable when L firms are not merged if the total profits of the merged s firm are greater than the sum of the profits of the s firms before the merger, that is,

$$\frac{(a-c)^2}{4}+\frac{(a-c+\Delta)^2}{16\beta}\geq\frac{(a-c+\Delta)^2}{9}$$
$$+\frac{(a-c-2\Delta)^2}{9}+\frac{(a-c+2\Delta)^2}{25\beta}+\frac{(a-c-3\Delta)^2}{25\beta}.$$

Solving this inequality with respect to β we obtain $\beta\geq f_s^{L1,L2}(\delta)$ which proves part (i). If s firms are not merged but L firms are, the equilibrium in t is given by:

$$\begin{aligned}
q_{s1}^t &= \frac{a-c}{2\beta}-\frac{1}{2}\left(q_{s2}^t+q_{L1+L2}^t\right)\\
q_{s2}^t &= \frac{a-c-\Delta}{2\beta}-\frac{1}{2}\left(q_{s1}^t+q_{L1+L2}^t\right)\\
q_{L1+L2}^t &= \frac{a-c}{2\beta}-\frac{1}{2}\left(q_{s1}^t+q_{s2}^t\right)
\end{aligned}$$

The solution to this system is $q_{L1+L2}^t=q_{s1}^t=(a-c+\Delta)/4\beta$ and $q_{s2}^t=(a-c-3\Delta)/4\beta$. The profits of $s1$ in t are $\pi_{s1}^t=(a-c+\Delta)^2/16\beta$ and the profits of $s2$ are $\pi_{s2}^t=(a-c-3\Delta)^2/16\beta$. If s firms merge and so do L firms we have a duopoly in the t. In this case the equilibrium quantities in t are $q_{s1+s2}^t=q_{L1+L2}^t=(a-c)/3\beta$ and profits of the merged s firm by $\pi_{s1+s2}^t=(a-c)^2/9\beta$. Thus, a merger of s firms is profitable when L firms are merged if

$$(a-c)^2\left[\frac{1}{4}+\frac{1}{9\beta}\right]\geq(a-c+\Delta)^2\left[\frac{1}{9}+\frac{1}{16\beta}\right]+\frac{(a-c-2\Delta)^2}{9}+\frac{(a-c-3\Delta)^2}{16\beta}.$$

Solving this inequality with respect to β we obtain $\beta\geq f_s^{L1+L2}(\delta)$ which proves part (ii). Similarly, a merger of L firms is profitable when s firms are not merged if

$$\frac{(a-c)^2}{4\gamma}+\frac{(a-c+\Delta)^2}{16\beta}\geq\frac{(a-c+\Delta)^2}{9\gamma}$$
$$+\frac{(a-c+2\Delta)^2}{25\beta}+\frac{(a-c-2\Delta)^2}{9\gamma}+\frac{(a-c-3\Delta)^2}{25\beta}.$$

Solving this inequality with respect to β we obtain $\beta\geq f_L^{s1,s2}(\delta,\gamma)$ which proves part (iii). A merger of L firms is profitable when s firms are merged if

$$(a-c)^2\left[\frac{1}{4\gamma}+\frac{1}{9\beta}\right]\geq(a-c+\Delta)^2\left[\frac{1}{9\gamma}+\frac{1}{16\beta}\right]+\frac{(a-c-2\Delta)^2}{9\gamma}+\frac{(a-c-3\Delta)^2}{16\beta}.$$

Solving this inequality with respect to β we obtain $\beta\geq f_L^{s1+s2}(\delta,\gamma)$ which proves part (iv). *Q.E.D.*

Proof of Proposition 1: (i) We know from Lemma 1 part (i) that firms in s wish to merge when firms in L are not merged when $f_s^{L1,L2}(\delta) \leq \beta \leq 1$. We know from Lemma part (ii) that firms in s wish to merge when firms in L are merged when $f_s^{L1+L2}(\delta) \leq \beta \leq 1$. The conditions under which firms in s decide to merge are less restrictive when firms in L are merged since $f_s^{L1,L2}(\delta) - f_s^{L1+L2}(\delta) = \frac{1}{100}\frac{13+162\delta-603\delta^2}{1+8\delta-20\delta^2} > 0$ for $\delta \in [0, 1/3]$.
(ii) Suppose that firms in L are not merged. From Lemma 1 part (i) we have $f_s^{L1,L2}(\delta) \leq 0$ for $\delta \in [7/61, 1/3]$. This means that if $\delta \in [7/61, 1/3]$, then firms in s wish to merge when firms in L are not merged for any β. However, if $\delta \in [0, 7/61)$, firms in s only wish to merge when firms in L are not merged as long as $\beta \geq f_s^{L1,L2}(\delta)$.
(iii) Suppose that firms in L are not merged. From Lemma 1 part (i) we have $f_s^{L1,L2}(0) = 0.63$. This means that if $\beta \in [0.63, 1]$, then firms in s wish to merge when firms in L are not merged for any δ. However, if $\beta \in (0, 0.63)$, firms in s only wish to merge when firms in L are not merged as long as $\beta \geq f_s^{L1,L2}(\delta)$.
(iv) We know from Lemma 1 part (i) that firms in s wish to merge when firms in L are not merged when $f_s^{L1,L2}(\delta) \leq \beta \leq 1$. We know from Lemma 1 part (iii) that firms in L wish to merge when firms in s are not merged when $f_L^{s1,s2}(\delta, \gamma) \leq \beta \leq 1$. The conditions under which firms in L decide to merge when firms in s are not merged are less restrictive than the conditions under which firms in s decide to merge when firms in L are not merged since $f_L^{s1,s2}(\delta, \gamma) \leq f_s^{L1,L2}(\delta)$ for all (γ, δ). *Q.E.D.*

Table I displays the strategies and payoffs in the merger game played by firms. The upper left part of each cell displays the profits of the merged firm in s (if there is a merger) or the sum of profits of the two firms in s (if there is no merger). The lower right part of each cell displays the profits of the merged firm in L (if there is a merger) or the sum of profits of the two firms in L (if there is no merger). Denote the firms' strategies in the small country as m (merger) and n (no merger) and in the large country as M (merger) and N (no merger). Denote the game played by firms by $F_{2,\gamma}$ and its Nash equilibria by $NE(F_{2,\gamma})$.

Table I

$s \backslash L$	M	N
m	$\frac{(a-c)^2}{4} + \frac{(a-c)^2}{9\beta}$, $\frac{(a-c)^2}{4\gamma} + \frac{(a-c)^2}{9\beta}$	$\frac{(a-c)^2}{4} + \frac{(a-c+\Delta)^2}{16\beta}$, $\frac{(a-c+\Delta)^2+(a-c-2\Delta)^2}{9\gamma} + \frac{(a-c+\Delta)^2+(a-c-3\Delta)^2}{16\beta}$
n	$\frac{(a-c+\Delta)^2+(a-c-2\Delta)^2}{9} + \frac{(a-c+\Delta)^2+(a-c-3\Delta)^2}{16\beta}$, $\frac{(a-c)^2}{4\gamma} + \frac{(a-c+\Delta)^2}{16\beta}$	$\frac{(a-c+\Delta)^2+(a-c-2\Delta)^2}{9} + \frac{(a-c+2\Delta)^2+(a-c-3\Delta)^2}{25\beta}$, $\frac{(a-c+\Delta)^2+(a-c-2\Delta)^2}{9\gamma} + \frac{(a-c+2\Delta)^2+(a-c-3\Delta)^2}{25\beta}$

Lemma 2: *Let* $\delta^*(\gamma) = \frac{50-63\gamma}{750-549\gamma}$.
(i) If (γ, δ) *satisfy* $0 < \gamma < 0.794$ *and* $0 \leq \delta \leq \delta^*(\gamma)$*, then* $f_L^{s1+s2}(\delta,\gamma) < f_L^{s1,s2}(\delta,\gamma) < f_s^{L1+L2}(\delta) < f_s^{L1,L2}(\delta)$.
(ii) If $\delta = \delta^*(\gamma)$*, then* $f_L^{s1+s2}(\delta,\gamma) < f_L^{s1,s2}(\delta,\gamma) = f_s^{L1+L2}(\delta) < f_s^{L1,L2}(\delta)$.
(iii) If (γ, δ) *satisfy* $0 < \gamma \leq 0.794$ *and* $\delta^*(\gamma) < \delta \leq 0.0(6)$*, then* $f_L^{s1+s2}(\delta,\gamma) < f_s^{L1+L2}(\delta) < f_L^{s1,s2}(\delta,\gamma) < f_s^{L1,L2}(\delta)$.
(iv) If $0.794 < \gamma < 1$*, then* $f_L^{s1+s2}(\delta,\gamma) < f_s^{L1+L2}(\delta) < f_L^{s1,s2}(\delta,\gamma) < f_s^{L1,L2}(\delta)$.
(v) If $\gamma = 1$*, then* $f_L^{s1+s2}(\delta,\gamma) = f_s^{L1+L2}(\delta) < f_L^{s1,s2}(\delta,\gamma) = f_s^{L1,L2}(\delta)$.

Proof of Lemma 2: $\delta^*(\gamma)$ is obtained by solving $f_s^{L1+L2}(\delta) = f_L^{s1,s2}(\delta,\gamma)$ with respect to δ. Now, $0 < \gamma < 0.794$ implies $0 \leq \delta^*(\gamma) \leq 0.0(6)$. So, if $0 < \gamma < 0.794$ and $0 \leq \delta \leq \delta^*(\gamma)$, then $f_L^{s1,s2}(\delta,\gamma) < f_s^{L1+L2}(\delta)$. However, if $0 < \gamma \leq 0.794$ and $\delta^*(\gamma) < \delta \leq 0.0(6)$, then $f_s^{L1+L2}(\delta) < f_L^{s1,s2}(\delta,\gamma)$. The definitions of $f_L^{s1+s2}(\delta,\gamma)$ and $f_s^{L1+L2}(\delta)$ imply that $f_L^{s1+s2}(\delta,\gamma) < f_s^{L1+L2}(\delta)$ for $\gamma \in (0,1)$ and $f_L^{s1+s2}(\delta,\gamma) = f_s^{L1+L2}(\delta)$ when $\gamma = 1$. Similarly, the definitions of $f_L^{s1,s2}(\delta,\gamma)$ and $f_s^{L1,L2}(\delta)$ imply that $f_L^{s1,s2}(\delta,\gamma) < f_s^{L1,L2}(\delta)$ for $\gamma \in (0,1)$ and $f_L^{s1+s2}(\delta,\gamma) = f_s^{L1+L2}(\delta)$ when $\gamma = 1$. These results imply (i)-(v). *Q.E.D.*

Proof of Proposition 2: (i) If $0.74 \leq \gamma \leq 1$ and $\beta \leq f_s^{L1+L2}(\delta)$, then Lemma 2 (iv) implies $\beta \leq f_s^{L1+L2}(\delta) < f_s^{L1,L2}(\delta)$. Prop. 1 (i) and (ii) together with $\beta \leq f_s^{L1+L2}(\delta) < f_s^{L1,L2}(\delta)$ imply that m is a dominated strategy for firms in s. Thus, firms in s choose n. If $\beta \leq f_s^{L1+L2}(\delta)$, then Lemma 2 (iv) also implies $\beta < f_L^{s1,s2}(\delta,\gamma)$. Prop. 1 (iii) together with $\beta < f_L^{s1,s2}(\delta,\gamma)$ imply that the best response of firms in L to n is N. So, firms in L will play N. Thus, $NE(F_{2,\gamma}) = (n, N)$ for $0.794 \leq \gamma \leq 1$ and $\beta \leq f_s^{L1+L2}(\delta)$.
(ii) If $0.794 \leq \gamma \leq 1$ and $f_s^{L1+L2}(\delta) < \beta \leq f_L^{s1,s2}(\delta,\gamma)$, then Lemma 2 (iv) implies $f_L^{s1+s2}(\delta,\gamma) < f_s^{L1+L2}(\delta) < \beta \leq f_L^{s1,s2}(\delta,\gamma) < f_s^{L1,L2}(\delta)$. If $f_s^{L1+L2}(\delta) < \beta < f_s^{L1,L2}(\delta)$, then Prop. 1 (i) implies that the best response of firms in s to N is n and Prop. 1 (ii) implies that the best response of firms in s to M is m. If $f_L^{s1+s2}(\delta,\gamma) < \beta \leq f_L^{s1,s2}(\delta,\gamma)$, then Prop. 1 (iii) implies that the best response of firms in L to n is N and Prop. 1 (iv) implies that the best response of firms in L to m is M. Thus, (n, N) and (m, M) are pure-strategy Nash equilibria (PSNE from now on) of $F_{2,\gamma}$ when $0.794 \leq \gamma \leq 1$ and $f_s^{L1+L2}(\delta)\} < \beta \leq f_L^{s1,s2}(\delta,\gamma)$. There exists also a mixed-strategy Nash equilibrium (MSNE from now on) where firms in s randomize between m and n to make firms in L indifferent between M and N:

$$p_s \left(\frac{(a-c)^2}{4\gamma} + \frac{(a-c)^2}{9\beta} \right) + (1-p_s) \left(\frac{(a-c)^2}{4\gamma} + \frac{(a-c+\Delta)^2}{16\beta} \right)$$

$$= p_s \left(\frac{(a-c+\Delta)^2}{9\gamma} + \frac{(a-c+\Delta)^2}{16\beta} + \frac{(a-c-2\Delta)^2}{9\gamma} + \frac{(a-c-3\Delta)^2}{16\beta} \right)$$

$$+(1-p_s) \left(\frac{(a-c+\Delta)^2}{9\gamma} + \frac{(a-c+2\Delta)^2}{25\beta} + \frac{(a-c-2\Delta)^2}{9\gamma} + \frac{(a-c-3\Delta)^2}{25\beta} \right),$$

where p_s is the probability that firms in s choose m. Solving this equation for p_s we obtain (1). Firms in L randomize between M and N to make firms in s indifferent between m and n. Let p_L denote the probability that firms in L choose M. Setting $\gamma = 1$ in (1) we obtain (2). Thus, for $0.794 \leq \gamma \leq 1$ and $f_s^{L1+L2}(\delta) < \beta \leq f_L^{s1,s2}(\delta,\gamma)$, we have $NE(F_{2,\gamma}) = \{(n,N), (m,M), (p_s, m; p_L, M)\}$.
(iii) If $0.794 \leq \gamma \leq 1$ and $f_L^{s1,s2}(\delta,\gamma) < \beta$, then Lemma 2 (iv) implies $f_L^{s1+s2}(\delta,\gamma) < f_L^{s1,s2}(\delta,\gamma) < \beta$. Prop. 1 (iii) and (iv) together with $f_L^{s1+s2}(\delta,\gamma) < f_L^{s1,s2}(\delta,\gamma) < \beta$ imply that N is a dominated strategy for firms in L. Thus, firms in L choose M. If $f_L^{s1,s2}(\delta,\gamma) < \beta$, then Lemma 2 (iv) also implies $f_s^{L1+L2}(\delta) < \beta$. Prop. 1 (ii) together with $f_s^{L1+L2}(\delta) < \beta$ imply that the best response of firms in s to M is m. So, firms in s choose m. Thus, for $0.794 \leq \gamma \leq 1$ and $\beta \leq f_s^{L1+L2}(\delta)$, we have $NE(F_{2,\gamma}) = (m, M)$. *Q.E.D.*

Proof of Proposition 3: (i) If $0 < \gamma < 0.794$ and $\beta \leq \min[f_s^{L1+L2}(\delta), f_L^{s1,s2}(\delta,\gamma)]$, then Lemma 2 (i) or (iii) imply $\beta \leq f_s^{L1+L2}(\delta) < f_s^{L1,L2}(\delta)$. Prop. 1 (i) and (ii) together with $\beta \leq f_s^{L1+L2}(\delta) < f_s^{L1,L2}(\delta)$ imply that m is a dominated strategy for firms in s. Thus, firms in s choose n. If $\beta \leq f_L^{s1,s2}(\delta,\gamma)$, then Prop. 1 part (iii) implies that the best response of firms in L to n is N. So, firms in L will play N. Thus, for $0 < \gamma < 0.794$ and $\beta \leq \min\{f_s^{L1+L2}(\delta), f_L^{s1,s2}(\delta,\gamma)\}$, we have $NE(F_{2,\gamma}) = (n, N)$.
(ii) If $0 < \gamma < 0.794$ and $f_L^{s1,s2}(\delta,\gamma) \leq \beta \leq f_s^{L1+L2}(\delta)$, then Lemma 2 part (i) implies $f_L^{s1+s2}(\delta,\gamma) < f_L^{s1,s2}(\delta,\gamma) \leq \beta \leq f_s^{L1+L2}(\delta) < f_s^{L1,L2}(\delta)$. Prop. 1 parts (iii) and (iv) together with $f_L^{s1+s2}(\delta,\gamma) < f_L^{s1,s2}(\delta,\gamma) \leq \beta$ imply that N is a dominated strategy for firms in L. Thus, firms in L choose M. Prop. 1 parts (i) and (ii) together with $\beta \leq f_s^{L1+L2}(\delta) < f_s^{L1,L2}(\delta)$ imply that m is a dominated strategy for firms in s. So, firms in s choose n. Thus, for $0 < \gamma < 0.794$ and $f_L^{s1,s2}(\delta,\gamma) \leq \beta \leq f_s^{L1+L2}(\delta)$ we have $NE(F_{2,\gamma}) = (n, M)$. The proofs of parts (iii) and (iv) are similar to those of parts (ii) and (iii) of Prop. 2, respectively. *Q.E.D.*

Lemma 3:
(i) If $0 < \beta \leq g_s^{L1,L2}(\delta) = \frac{9}{50}\frac{-7+82\delta-183\delta^2}{5-32\delta+44\delta^2}$, *and firms in the large country are not merged, then a merger in the small country improves that country's welfare;*
(ii) If $0 < \beta \leq g_s^{L1+L2}(\delta) = \frac{-1+18\delta-45\delta^2}{5-32\delta+44\delta^2}$, *and firms in the large country are merged, then a merger in the small country improves that country's welfare;*
(iii) If $0 < \beta \leq g_L^{s1,s2}(\delta,\gamma) = \frac{9\gamma}{50}\frac{-7+82\delta-183\delta^2}{5-32\delta+44\delta^2}$, *and firms in the small country are not merged, then a merger in the large country improves that country's welfare;*
(iv) If $0 < \beta \leq g_L^{s1+s2}(\delta,\gamma) = \gamma\frac{-1+18\delta-45\delta^2}{5-32\delta+44\delta^2}$ *and firms in the small country are merged, then a merger in the large country improves that country's welfare.*

Proof of Lemma 3: I start the proof by stating conditions under which a domestic merger improves national welfare for a given market structure in L. Consumer surplus at s is given by $CS_s = (a-p_s)Q_s/2 = Q_s^2/2$,where Q_s is total output produced by s firms. If s firms do not merge, then $Q_s = (2a-2c-\Delta)/3$

and $CS^s_{s1,s2} = (2a-2c-\Delta)^2/18$. If s firms merge, then $Q_s = (a-c)/2$ and $CS^s_{s1+s2} = (a-c)^2/8$. Thus, a merger of firms in s improves national welfare when L firms are not merged if

$$\frac{(a-c)^2}{8} + \frac{(a-c)^2}{4} + \frac{(a-c+\Delta)^2}{16\beta} \geq \frac{(2a-2c-\Delta)^2}{18}$$
$$+\frac{(a-c+\Delta)^2}{9} + \frac{(a-c+2\Delta)^2}{25\beta} + \frac{(a-c-2\Delta)^2}{9} + \frac{(a-c-3\Delta)^2}{25\beta}. \quad (5)$$

Solving (5) with respect to β we obtain $\beta \leq g_s^{L1,L2}(\delta)$ which proves (i). A merger of firms in s improves national welfare when L firms are merged if

$$(a-c)^2\left(\frac{1}{8} + \frac{1}{4} + \frac{1}{9\beta}\right) \geq \frac{(2a-2c-\Delta)^2}{18}$$
$$+\frac{(a-c+\Delta)^2}{9} + \frac{(a-c+\Delta)^2}{16\beta} + \frac{(a-c-2\Delta)^2}{9} + \frac{(a-c-3\Delta)^2}{16\beta}. \quad (6)$$

Solving (6) with respect to β we obtain $\beta \leq g_s^{L1+L2}(\delta)$ which proves (ii). I will now state conditions under which a L merger improves L welfare conditional on a given market structure in s. Consumer surplus in L is given by $CS_L = (a-p_L)Q_L/2 = \gamma Q_L^2/2$,where Q_L is total output produced by L firms. If L firms do not merge, then $Q_L = (2a-2c-\Delta)/3\gamma$ and $CS^L_{L1,L2} = (2a-2c-\Delta)^2/18\gamma$. If L firms merge, then $Q_L = (a-c)/2\gamma$ and $CS^L_{L1+L2} = (a-c)^2/8\gamma$. So, a merger of firms in L improves national welfare when s firms are not merged if

$$\frac{(a-c)^2}{8\gamma} + \frac{(a-c)^2}{4\gamma} + \frac{(a-c+\Delta)^2}{16\beta} \geq \frac{(2a-2c-\Delta)^2}{18\gamma}$$
$$+\frac{(a-c+\Delta)^2}{9\gamma} + \frac{(a-c+2\Delta)^2}{25\beta} + \frac{(a-c-2\Delta)^2}{9\gamma} + \frac{(a-c-3\Delta)^2}{25\beta}. \quad (7)$$

Solving (7) with respect to β we have $\beta \leq g_L^{s1,s2}(\delta,\gamma)$ which proves (iii). A merger of firms in L improves national welfare when s firms are merged if

$$(a-c)^2\left(\frac{1}{8\gamma} + \frac{1}{4\gamma} + \frac{1}{9\beta}\right) \geq \frac{(2a-2c-\Delta)^2}{18\gamma}$$
$$+\frac{(a-c+\Delta)^2}{9\gamma} + \frac{(a-c+\Delta)^2}{16\beta} + \frac{(a-c-2\Delta)^2}{9\gamma} + \frac{(a-c-3\Delta)^2}{16\beta}. \quad (8)$$

Solving (8) with respect to β we have $\beta \leq g_L^{s1+s2}(\delta,\gamma)$ which proves (iv). *Q.E.D.*

Proof of Proposition 4: The proof follows from Lemma 3 just as the proof of Prop. 1 follows from Lemma 1. *Q.E.D.*

Table II displays the strategies and payoffs in the merger game played by governments. The upper left part of each cell displays the sum of consumer surplus and profits of the merged firm in s (if there is a merger) or with profits of the two firms in s (if there is no merger). The lower right part of each cell displays the sum of consumer surplus and profits of the merged firm in L (if there is a merger) or with profits of the two firms in L (if there is no merger). Denote the governments' strategies in the small country as m (merger) and n (no merger) and in the large country as M (merger) and N (no merger). Denote the merger game played by governments by $G_{2,\gamma}$ and its set of Nash equilibria by $NE(G_{2,\gamma})$.

Table II

$s \backslash L$	M	N
m	$\frac{(a-c)^2}{8}+\frac{(a-c)^2}{4}+\frac{(a-c)^2}{9\beta}$, $\frac{(a-c)^2}{8\gamma}+\frac{(a-c)^2}{4\gamma}+\frac{(a-c)^2}{9\beta}$	$\frac{(a-c)^2}{8}+\frac{(a-c)^2}{4}+\frac{(a-c+\Delta)^2}{16\beta}$, $\frac{(2a-2c-\Delta)^2}{18\gamma}+\frac{(a-c+\Delta)^2+(a-c-2\Delta)^2}{9\gamma}+\frac{(a-c+\Delta)^2+(a-c-3\Delta)^2}{16\beta}$
n	$\frac{(2a-2c-\Delta)^2}{18}+\frac{(a-c+\Delta)^2+(a-c-2\Delta)^2}{9}+\frac{(a-c+\Delta)^2+(a-c-3\Delta)^2}{16\beta}$, $\frac{(a-c)^2}{8\gamma}+\frac{(a-c)^2}{4\gamma}+\frac{(a-c+\Delta)^2}{16\beta}$	$\frac{(2a-2c-\Delta)^2}{18}+\frac{(a-c+\Delta)^2+(a-c-2\Delta)^2}{9}+\frac{(a-c+2\Delta)^2+(a-c-3\Delta)^2}{25\beta}$, $\frac{(2a-2c-\Delta)^2}{18\gamma}+\frac{(a-c+\Delta)^2+(a-c-2\Delta)^2}{9\gamma}+\frac{(a-c+2\Delta)^2+(a-c-3\Delta)^2}{25\beta}$

Lemma 4 *Let* $\hat{\delta}(\gamma) = \frac{63-50\gamma}{549-750\gamma}$.
(i) If (γ, δ) *satisfy* $0 < \gamma \leq 0.466$ *and* $0.11475 < \delta \leq \hat{\delta}(\gamma)$*, then* $g_L^{s1,s2}(\delta,\gamma) < g_s^{L1,L2}(\delta) < g_L^{s1+s2}(\delta,\gamma) < g_s^{L1+L2}(\delta)$.
(ii) If $\delta = \hat{\delta}(\gamma)$ *then* $g_L^{s1,s2}(\delta,\gamma) < g_s^{L1,L2}(\delta) = g_L^{s1+s2}(\delta,\gamma) < g_s^{L1+L2}(\delta)$.
(iii) If (γ, δ) *satisfy* $0 < \gamma \leq 0.466$ *and* $\hat{\delta}(\gamma) \leq \delta \leq 0.1991$*, then* $g_L^{s1,s2}(\delta,\gamma) < g_L^{s1+s2}(\delta,\gamma) < g_s^{L1,L2}(\delta) < g_s^{L1+L2}(\delta)$.
(iv) If $0.466 < \gamma < 1$*, then* $g_L^{s1,s2}(\delta,\gamma) < g_s^{L1,L2}(\delta) < g_L^{s1+s2}(\delta,\gamma) < g_s^{L1+L2}(\delta)$.
(v) If $\gamma = 1$*, then* $g_L^{s1,s2}(\delta,\gamma) = g_s^{L1,L2}(\delta) < g_L^{s1+s2}(\delta,\gamma) = g_s^{L1+L2}(\delta)$.

Proof of Lemma 4: The expression for $\hat{\delta}(\gamma)$ is obtained by solving $g_s^{L1,L2}(\delta) = g_L^{s1+s2}(\delta,\gamma)$ with respect to δ. Now, $0 < \gamma \leq 0.466$ implies $0.11475 < \hat{\delta}(\gamma) \leq 0.1991$. So, if $0 < \gamma \leq 0.466$ and $\hat{\delta}(\gamma) \leq \delta \leq 0.1991$, then $g_L^{s1+s2}(\delta,\gamma) < g_s^{L1,L2}(\delta)$. However, if $0 < \gamma \leq 0.466$ and $0.11475 < \delta \leq \hat{\delta}(\gamma)$, then $g_s^{L1,L2}(\delta) < g_L^{s1+s2}(\delta,\gamma)$. The definitions of $g_L^{s1+s2}(\delta,\gamma)$ and $g_s^{L1+L2}(\delta)$ imply that $g_L^{s1+s2}(\delta,\gamma) < g_s^{L1+L2}(\delta)$ for $\gamma \in (0,1)$ and $g_L^{s1+s2}(\delta,\gamma) = g_s^{L1+L2}(\delta)$ when $\gamma = 1$. Similarly, the definitions of $g_L^{s1,s2}(\delta,\gamma)$ and $g_s^{L1,L2}(\delta)$ imply that $g_L^{s1,s2}(\delta,\gamma) < g_s^{L1,L2}(\delta)$ for $\gamma \in (0,1)$ and $g_L^{s1+s2}(\delta,\gamma) = g_s^{L1+L2}(\delta)$ when $\gamma = 1$. It is straightforward to show that these results imply (i) though (v). *Q.E.D.*

Proof of Proposition 5: (i) If $0.466 \le \gamma \le 1$ and $g_L^{s1+s2}(\delta,\gamma) < \beta$, then Lemma 4 part (iv) implies $g_s^{L1,L2}(\delta) < g_s^{L1+L2}(\delta) < \beta$. Prop. 4 parts (i) and (ii) together with $g_s^{L1,L2}(\delta) < g_s^{L1+L2}(\delta) < \beta$ imply that m is a dominated strategy for the government of s. Thus, the government of s chooses n. If $g_L^{s1+s2}(\delta,\gamma) < \beta$, then Lemma 4 part (iv) also implies $g_L^{s1,s2}(\delta,\gamma) < \beta$. Prop. 4 part (iii) together with $g_L^{s1,s2}(\delta,\gamma) < \beta$ imply that the best response of the government of L to n is N. So, the government of L plays N. Thus, for $0.466 \le \gamma \le 1$ and $g_L^{s1+s2}(\delta,\gamma) < \beta$ we have $NE(G_{2,\gamma}) = (n, N)$.
(ii) If $0.466 \le \gamma \le 1$ and $g_s^{L1,L2}(\delta) < \beta \le g_L^{s1+s2}(\delta,\gamma)$, then Lemma 4 part (iv) implies $g_L^{s1,s2}(\delta,\gamma) < g_s^{L1,L2}(\delta) < \beta \le g_L^{s1+s2}(\delta,\gamma) < g_s^{L1+L2}(\delta)$. If $g_s^{L1,L2}(\delta) < \beta < g_s^{L1+L2}(\delta)$, then Prop. 4 part (i) implies that the best response of the government of s to N is n and Prop. 4 part (ii) implies that the best response of the government of s to M is m. If $g_L^{s1,s2}(\delta,\gamma) < \beta \le g_L^{s1+s2}(\delta,\gamma)$, then Prop. 4 part (iii) implies that the best response of the government of L to n is N and Prop. 4 part (iv) implies that the best response of the government of L to m is M. Thus, (n, N) and (m, M) are PSNE of $G_{2,\gamma}$ when $0.466 \le \gamma \le 1$ and $g_s^{L1,L2}(\delta) < \beta \le g_L^{s1+s2}(\delta,\gamma)$. There also exists a MSNE where the government of s randomizes between m and n to make the government of L indifferent between M and N:

$$\begin{aligned}
q_s(a-c)^2\left(\frac{1}{8\gamma}+\frac{1}{4\gamma}+\frac{1}{9\beta}\right)+(1-q_s)\left(\frac{(a-c)^2}{8\gamma}+\frac{(a-c)^2}{4\gamma}+\frac{(a-c+\Delta)^2}{16\beta}\right) \\
= q_s\left(\frac{(2a-2c-\Delta)^2}{18\gamma}+\frac{(a-c+\Delta)^2+(a-c-2\Delta)^2}{9\gamma}\right) \\
+ q_s\frac{(a-c+\Delta)^2+(a-c-3\Delta)^2}{16\beta}+(1-q_s)\frac{(2a-2c-\Delta)^2}{18\gamma} \\
+(1-q_s)\left(\frac{(a-c+\Delta)^2+(a-c-2\Delta)^2}{9\gamma}+\frac{(a-c+2\Delta)^2+(a-c-3\Delta)^2}{25\beta}\right),
\end{aligned}$$

where q_s is the probability that the government of s chooses m. Solving this equation for q_s we obtain (3). The government of L randomizes between M and N to make the government of s indifferent between m and n. Let q_L denote the probability that the government of L chooses M. Setting $\gamma = 1$ in (3) we obtain (4). Thus, for $0.466 \le \gamma \le 1$ and $g_s^{L1,L2}(\delta) < \beta \le g_L^{s1+s2}(\delta,\gamma)$, we have $NE(G_{2,\gamma}) = \{(n,N), (m,M), (q_s, m; q_L, M)\}$.
(iii) If $0.466 \le \gamma \le 1$ and $\beta \le g_s^{L1,L2}(\delta)$, then Lemma 4 part (iv) implies $\beta < g_s^{L1,L2}(\delta) < g_s^{L1+L2}(\delta)$. Prop. 4 parts (iii) and (iv) together with $\beta < g_s^{L1,L2}(\delta) < g_s^{L1+L2}(\delta)$ imply that n is a dominated strategy for the government of s. Thus, the government of s chooses m. If $\beta \le g_s^{L1,L2}(\delta)$, then Lemma 4 part (iv) also implies $\beta < g_L^{s1+s2}(\delta,\gamma)$. Prop. 4 part (iv) together with$\beta < g_L^{s1+s2}(\delta,\gamma)$ imply that the best response of the government of L to m is M. So, the government of L plays M. Thus, $NE(G_{2,\gamma}) = (m, M)$ for $0.466 \le \gamma \le 1$ and $\beta \le g_s^{L1,L2}(\delta)$. *Q.E.D.*

Proof of Proposition 6: (i) If $0 < \gamma < 0.466$ and $\max\{g_L^{s1+s2}(\delta,\gamma), g_s^{L1,L2}(\delta)\} < \beta$, then Lemma 4 (i) or (iii) imply $g_L^{s1,s2}(\delta,\gamma) < g_L^{s1+s2}(\delta,\gamma) < \beta$. Prop. 4 (iii) and (iv) together with $g_L^{s1,s2}(\delta,\gamma) < g_L^{s1+s2}(\delta,\gamma) < \beta$ imply that M is a dominated strategy for the government of L. If $g_s^{L1,L2}(\delta) < \beta$, then Prop. 4 (ii) implies that the best response of the government of s to N is n. Thus, $NE(G_{2,\gamma}) = (n, N)$ for $0.466 \leq \gamma \leq 1$ and $\max\{g_L^{s1+s2}(\delta,\gamma), g_s^{L1,L2}(\delta)\} < \beta$.
(ii) If $0 < \gamma < 0.466$ and $g_L^{s1+s2}(\delta,\gamma) \leq \beta \leq g_s^{L1,L2}(\delta)$, then Lemma 4 (iii) implies $g_L^{s1,s2}(\delta,\gamma) < g_L^{s1+s2}(\delta,\gamma) \leq \beta \leq g_s^{L1,L2}(\delta) < g_s^{L1+L2}(\delta)$. Prop. 4 (iii) and (iv) together with $g_L^{s1,s2}(\delta,\gamma) < g_L^{s1+s2}(\delta,\gamma) \leq \beta$ imply that M is a dominated strategy for the government of L. Prop. 4 (i) and (ii) together with $\beta \leq g_s^{L1,L2}(\delta) < g_s^{L1+L2}(\delta)$ imply that n is a dominated strategy for the government of s. Thus, $NE(G_{2,\gamma}) = (m, N)$ for $0 < \gamma < 0.466$ and $g_L^{s1+s2}(\delta,\gamma) \leq \beta \leq g_s^{L1,L2}(\delta)$. The proofs of (iii) and (iv) are similar to those of (ii) and (iii) of Prop. 5. *Q.E.D.*

Proposition 7: *Let* $0.794 \leq \gamma \leq 1$.
(i) If $0 < \beta \leq f_s^{L1+L2}(\delta)$*, then* $NE(F_{2,\gamma}) = NE(G_{2,\gamma}) = (n, N)$*;*
(ii) If $\max\{f_L^{s1,s2}(\delta,\gamma), g_L^{s1+s2}(\delta,\gamma)\} \leq \beta \leq 1$*, then* $NE(F_{2,\gamma}) = (m, M) \neq (n, N) = NE(G_{2,\gamma})$*;*
(iii) If $0 < \beta \leq \min\{g_s^{L1,L2}(\delta), 1\}$*, then* $NE(F_{2,\gamma}) = NE(G_{2,\gamma}) = (m, M)$.

Proof of Proposition 7: The proof follows from Propositions 2 and 5. *Q.E.D.*

Proposition 8: *Let* $0.466 \leq \gamma < 0.794$.
(i) If $0 < \beta \leq \min\{f_s^{L1+L2}(\delta), f_L^{s1,s2}(\delta,\gamma)\}$*, then* $NE(F_{2,\gamma}) = NE(G_{2,\gamma}) = (n, N)$*;*
(ii) If $f_L^{s1,s2}(\delta,\gamma) \leq \beta \leq f_s^{L1+L2}(\delta)$*, then* $NE(F_{2,\gamma}) = (n, M) \neq (n, N) = NE(G_{2,\gamma})$*;*
(iii) If $\max\ \{f_s^{L1+L2}(\delta), f_L^{s1,s2}(\delta,\gamma), g_L^{s1+s2}(\delta,\gamma)\} \leq \beta \leq 1$*, then* $NE(F_{2,\gamma}) = (m, M) \neq (n, N) = NE(G_{2,\gamma})$*;*
(iv) If $0 < \beta \leq \min\{g_s^{L1,L2}(\delta), 1\}$*, then* $NE(F_{2,\gamma}) = NE(G_{2,\gamma}) = (m, M)$.

Proof of Proposition 8: The proof follows from Propositions 3 and 5. *Q.E.D.*

Proposition 9: *Let* $0 < \gamma < 0.466$.
(i) If $0 < \beta \leq \min\{f_s^{L1+L2}(\delta), f_L^{s1,s2}(\delta,\gamma)\}$*, then* $NE(F_{2,\gamma}) = NE(G_{2,\gamma}) = (n, N)$*;*
(ii) If $f_L^{s1,s2}(\delta,\gamma) \leq \beta \leq f_s^{L1+L2}(\delta)$*, then* $NE(F_{2,\gamma}) = (n, M) \neq (n, N) = NE(G_{2,\gamma})$*;*
(iii) If $\max\{f_s^{L1+L2}(\delta), f_L^{s1,s2}(\delta,\gamma), g_L^{s1+s2}(\delta,\gamma), g_s^{L1,L2}(\delta)\} \leq \beta \leq 1$*, then* $NE(F_{2,\gamma}) = (m, M) \neq (n, N) = NE(G_{2,\gamma})$*;*
(iv) If $g_L^{s1+s2}(\delta,\gamma) < \beta \leq g_s^{L1,L2}(\delta)$*, then* $NE(F_{2,\gamma}) = (m, M) \neq (m, N) = NE(G_{2,\gamma})$*;*
(v) If $0 < \beta \leq \min\{g_s^{L1,L2}(\delta), g_L^{s1+s2}(\delta,\gamma), 1\}$*, then* $NE(F_{2,\gamma}) = NE(G_{2,\gamma}) = (m, M)$.

Proof of Proposition 9: The proof follows from Propositions 3 and 6. *Q.E.D.*