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Giordani, Paolo E. and Zamparelli, Luca

Department of Economics and Business, LUISS "Guido Carli" University, Department of Economc Theory, University of Rome 'La Sapienza'

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Paolo E. Giordani* Luca Zamparelli†

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Abstract

In an R&D-driven growth model with asymmetric fundamentals the steady state equilibrium R&D investments are industry-specific and they are such that R&D returns are equalized across industries. Return equalization, however, makes investors indifferent as to where to target research and, hence, the problem of allocation of R&D investments across industries is indeterminate. Agents' indifference creates an ambiguous investment scenario. We assume that agents hold "ambiguous" beliefs on the per-industry profitability of their R&D investments. Investors' aversion towards ambiguity (in the sense of Gilboa-Schmeidler, 1989) eliminates the indeterminacy of the R&D investment problem. In particular, we prove that the asymmetric return-equalizing equilibrium is robust against a however small degree of investors' aversion to ambiguity.

Keywords: R&D driven growth models, symmetry/asymmetry, ambiguity.

JEL Classification: 032, 041, D81.

*Department of Economics and Business, LUISS "Guido Carli" University, Viale Romania 32, 00197 Roma, Italy (e-mail: pgiordani@luiss.it).

†Department of Economic Theory, University of Rome "La Sapienza", Piazzale A. Moro 5, 00185, Roma, Italy (e-mail: l.zamparelli@dte.uniroma1.it).

1 Introduction

R&D driven growth models focus on the role of technical progress as the main source of economic growth. In this class of models, unlike the standard neoclassical growth model, technical change is said to be 'endogenous' as it is the outcome of R&D investment decisions taken by profit maximizing firms. In the neoclassical model, where a perfectly competitive environment is assumed, the endogenous determination of the rate of technical change was problematic because of the difficulty of accounting for the cost of innovation. In fact, when the level of technology is considered as an input in production, the aggregate production function exhibits increasing returns to scale, which implies that total output is not sufficient to pay factors of production according to their marginal productivities.

Since the early 80s economists began to adopt Dixit-Stiglitz technology (or preferences) in order to develop general equilibrium models based on monopolistic competition and increasing returns.[1] Krugman (1979, 1980) provided the first application in the field of international trade. Shortly afterwards, macroeconomics (Akerlof and Yellen 1985a, 1985b; Blanchard and Kiyotaki 1987) and economic geography (Krugman 1991a, 1991b) followed. The introduction of monopolistic competition proved to be fundamental in economic growth theory as well, as it allowed creating the rents necessary to justify a costly research activity thus making endogenous technical change possible. Romer (1987, 1990) produced the seminal contributions by modeling technical change as the increasing number of available goods (*horizontal innovation*). Anant, Dinopoulos and Segerstrom (1990), Grossman and Helpman (1991), Aghion and Howitt (1992) followed by developing models where innovation is aimed at improving the quality, or the productivity, of existing goods (*vertical innovation*).

Both models of horizontal and vertical innovation typically share a common three-sector structure. The research sector produces innovations -or designs, or ideas- which are sold to the intermediate goods sector. Intermediate goods are imperfect substitutes, and each of them is associated to a specific design protected by an infinitely-lived patent granting its owner the right to be the sole producer of that good: this sector is, in turn, monopolistic. Finally, intermediate goods and labor are hired to produce the final good in a competitive environment.[2] In such a framework, monopolistic competition in the

[1] The so-called Dixit-Stiglitz preferences have been developed independently by Spence (1976) and Dixit and Stiglitz (1977). Ethier (1982) provided the first "Dixit-Stiglitz" representation of technology.

[2] In fact, some R&D driven models are based on a two-sector structure where the intermediate

intermediate sector is the key to the feasibility of innovation: profits earned in that sector finance innovation by paying for the cost of patents.

Dixit-Stiglitz technology in the final good sector can be described as $Y = L^{1-\alpha} \sum_{j=1}^{N} (A_j x_j)^{\alpha}$, where $\alpha \in (0,1)$, L is labor, x_j is the quantity of intermediate good of industry j, and A_j is an industry-specific productivity parameter. In the horizontal innovation case $A_j = A$, and innovation consists of increasing the number N of existing intermediate goods. Since the marginal products of intermediate goods are independent of one another, the amount of resources (in fact a measure of capital) employed in the intermediate sector may escape the law of decreasing returns, provided it is spread across an increasing number of industries. In turn, increasing variety is a way of introducing increasing returns in capital and labor. On the other hand, when innovation is vertical, the number N is fixed and innovation consists of improving the industry specific parameters A_js.

R&D driven growth models typically focus on *symmetric equilibria.* Symmetry is to be understood in a twofold way. In the first place, it means equal size of intermediate goods industries. This notion of symmetry is common to both horizontal and vertical innovation models, and it is guaranteed by the symmetry of the economy's *fundamentals* across industries. In the final good production function, the cross partial elasticity of substitution between any two intermediate goods is the same: $\sigma_{i,j} = 1/(1-\alpha)$. Furthermore, it is assumed that technology in the intermediate goods sector and - in the vertical innovation case - in the R&D sectors is the same for each industry. Symmetry in both cost and demand conditions, the fundamentals of the economy, ensures that equilibrium in the intermediate goods sector is symmetric, i.e. $x_i = x$, $\forall$ i.

Secondly, symmetric equilibrium indicates equal R&D investment in each industry. This notion of symmetry applies to vertical innovation models only, as a horizontal innovation amounts to the creation of an altogether different industry. In this case, however, symmetric fundamentals are not sufficient to justify the focus on symmetric outcomes. The main structural difference between models of horizontal and vertical innovation consists of the permanent versus temporary nature of monopolistic profits.

sector disappears and where innovation occurs in the final (consumption) goods sector (see for example Grossman and Helpman 1991, and the model we develop in Section 2). The difference in the structure, however, is more formal than substantial. In the two-sector models final consumption goods are aggregated into an utility index; the two structures can be reconciled by interpreting the utility index as final good production function, and consumption goods as intermediate inputs.

While in both models each monopolistic firm is granted an infinitely-lived patent, the monopolistic position of a firm introducing a vertical innovation has a temporary nature as it only lasts until the next improvement in the same industry occurs. This distinctive feature of the vertical innovation literature, usually referred to as 'creative destruction' (Schumpeter, 1942 [1975]), is responsible for the role of expectations on *future* R&D investment decisions in determining the amount and the distribution across industries of *current* R&D investment. Since investors anticipate that their monopolistic position will only last up to the next innovation in their product line, their incentive to invest in R&D in a particular industry is negatively affected by the future amount of R&D investment expected in that industry. In turn, in order to focus on symmetric R&D investment, the additional assumption of *symmetric expectations* needs to be made. Only under the joint hypothesis of symmetric expectations and symmetric fundamentals, investors are indifferent as to which industry they target, and hence the model may focus on a symmetric solution to the allocation of R&D efforts. Grossman and Helpman (1991, p.47) recognize the centrality of the assumption of symmetric expected R&D investments in order to justify the selection of the symmetric equilibrium: with the assumption that "the profit flows are the same for all industries [...] an entrepreneur will be indifferent as to the industry in which she devotes her R&D efforts provided that she expects her prospective leadership position to last equally long in each one. We focus hereafter on the symmetric equilibrium in which all products are targeted to the same aggregate extent. In such an equilibrium the individual entrepreneur indeed expects profit flows of equal duration in every industry and so is indifferent as to the choice of industry". Indeed, Cozzi (2005, 2007) shows the existence of multiple asymmetric equilibria triggered by self-fulfilling *asymmetric expectations*.

Both notions of symmetric equilibrium in innovation-driven growth have been criticized. On the one hand, Park (2007) questions the soundness of symmetric equilibrium in the intermediate goods sector of horizontal innovation models. He claims that symmetric technology in the production of intermediate goods is inconsistent with the assumption that intermediate goods are imperfect substitutes as inputs in the final good production function. He argues that goods produced with identical technology are, in fact, the very same good and, at the same time, he denies that intermediate goods can be differentiated thanks to the different design they are associated to. In turn, symmetric Dixit-Stiglitz technology cannot be legitimately used to represent the concept of variety. If, on the contrary, asymmetric technology in the intermediate goods sector is assumed, the model becomes unable to yield balanced growth.

On the other hand, the role of symmetric expected R&D efforts in the characterization of the symmetric equilibrium has also been questioned. Indeed, expecting the same amount of future R&D efforts across industries is not a sufficient condition for investors to choose a symmetric allocation of current R&D efforts: equal future profitability leaves the investor *indifferent* as to which industry to select when deciding R&D efforts across industries. As a result, under the assumption of symmetric expectations the allocation problem of R&D efforts is indeterminate. This indeterminacy in the distribution of R&D investment may generate multiple asymmetric equilibria, analogous to those identified by Cozzi (2005, 2007), each characterized by a different balanced growth path.

In two recent papers Giordani and Zamparelli (2008) and Cozzi et Al. (2007) tackle the weakness of symmetric equilibrium respectively in the intermediate good sector and in the R&D sector. Giordani and Zamparelli (2008) develop an R&D growth model with asymmetric technology and demand conditions, where the resulting steady state equilibrium of the intermediate sector is asymmetric. They do not address, however, the problem raised by Park (2007), since their analysis is carried out within the vertical innovation framework. Cozzi et Al. (2007) solve the indeterminacy of equilibrium in the R&D sector. They prove that the *symmetric equilibrium* is the only rational expectations equilibrium robust to a however small "degree" of investors' ambiguity aversion in the evaluation of R&D returns.

The balanced growth path equilibrium in Giordani and Zamparelli (2008) is characterized by an asymmetric configuration of R&D investments capable of equalizing R&D returns across industries. Notice however that, as in the standard symmetric case, equalization of returns leaves the agent *indifferent* as to which industry to invest in. As a result, the *asymmetric equilibrium* is not uniquely pinned down. In this paper we make the focus on the asymmetric equilibrium compelling. Our basic idea is that the agents' indifference - arising from the equalization of R&D returns across industries - gives them in principle the possibility of adopting a whatever (even randomly chosen) investment strategy. This makes these agents highly uncertain about the configuration of future R&D investment, since that configuration is the result of a decision problem analogous to the one they are currently facing.

We assume that the agent's beliefs on the future (per industry) distribution of R&D investments are characterized by uncertainty (or ambiguity), in the sense that information about that distribution is too imprecise to be represented by a (single additive) probability measure. The traditional distinction between 'risk' and 'uncertainty' traces back to Frank Knight (1921), and states that risk is associated with ventures in which an objective probability distribution of all possible events is known, while uncertainty characterizes choice settings in which that probability distribution is not available to the decision-maker. As is well known, the axiomatization of the subjective expected utility (SEU) model, provided among others by Savage (1954), contributed to undermine any meaningful distinction between risk and uncertainty. In recent years a number of attempts have been made to extend the SEU model in order to substantiate that distinction.[3] Here we follow the maxmin expected utility (MMEU) theory axiomatized by Gilboa and Schmeidler (1989). In representing subjective beliefs, it suggests to replace the standard single (additive) prior with a closed and convex set of (additive) priors. The choice among alternative acts is determined by a maximin strategy. For each act the agent first computes the expected utilities with respect to each single prior in the set and picks up the minimal value. Finally she compares all these values and singles out the act associated with the highest (minimal) expected utility. According to this model, the agent is said to be *uncertainty* (or *ambiguity*) *averse* if the given set of priors is not a singleton. In particular, we use the "$\varepsilon-$contamination of confidence" argument, recently axiomatized by Nishimura and Ozaki (2006). In our framework the decision maker is assumed to maximize her expected pay-off with respect to the R&D investment decision, while singling out the worst choice scenario, that is, the minimizing probability distribution over the future configuration of R&D investments. We show that a however small degree of uncertainty in the expectations of the future investment's allocation (an "$\varepsilon-$contamination of confidence") eliminates agents' indifference and makes the configuration where R&D returns are equalized across industries arise as the unique equilibrium.

Giordani and Zamparelli (2008) have proved that, in R&D driven growth economies with asymmetric fundamentals, a costless tax/subsidy scheme reallocating resources towards industries with more productive fundamentals raises the long-run growth rate and the social welfare of the economy. Since their results are based on the R&D return-equalizing equilibrium discussed above, establishing the robustness of such an equilibrium against the introduction of uncertainty in agents' beliefs improves the confidence in the policy implications of the standard asymmetric model.

The rest of the paper is organized as follows. In Section 2 we introduce the model. In Section 3 we explain the core of our argument, enunciate and prove our main result.

[3] Seminal contributions in this respect are Bewley (1986) and Schmeidler (1989).

In Section 4 we conclude with some remarks.

2 The Model

In this section we build a vertical innovation (or "quality ladder") growth model with asymmetric fundamentals.[4] Let us assume a continuum of industries producing final goods indexed by $\omega \in [0,1]$. In each industry firms are distinguished by the quality index j of the goods they supply, with the quality of their goods being increasing in the integer j. At time $t=0$ in each industry some firm knows how to produce a $j=0$ quality product and no other firm can offer a better one. In order to develop higher quality versions of any product firms engage in R&D races. The winner of an R&D race becomes the sole producer of a good whose quality is one step ahead of the previous quality leader.

There exists a fixed number of dynastic households (normalized to one) whose members grow at constant rate $n>0$. Each member shares the same intertemporally additively separable utility $\log u(t)$ and is endowed with a unit of labor she supplies inelastically. Therefore each household chooses her optimal consumption path by maximizing the discounted utility

$$U \equiv \int_0^\infty L(0)e^{-(\rho-n)t}\log u(t)dt \tag{1}$$

where $L(0)\equiv 1$ is the initial population and $\rho > n$ is the common rate of time preferences.

The instantaneous utility function is a logarithmic Cobb-Douglas. We let the utility weights $(\alpha(\omega))$ vary across industries, so as to represent a possible heterogeneity of consumers' preferences among the set of commodities. As the $\alpha(\omega)$'s represent the relative weights of the goods in the utility function, we can normalize them in such a way that $\int_0^1 \alpha(\omega)d\omega = 1$. If we define $\lambda(\omega)$ as the size of quality improvements (the so-called "quality jump"), assumed to be industry-specific to allow for asymmetry in the technical evolution of each line, $j^{\max}(\omega,t)$ as the highest quality reached by product

[4] The model developed in this section is in many respects similar to the one in Giordani and Zamparelli (2008), the main substantial difference being that here we adopt the "TEG specification" to capture the increasing complexity of the innovation process, in contrast with the "PEG specification" adopted in that paper (see below for details).

ω at time t, and $d(j,\omega,t)$ as the consumption of product ω of quality j at time t, then the instantaneous utility function can be written as

$$\log u(t) \equiv \int_0^1 \alpha(\omega) \log \sum_{j=0}^{j^{\max}(\omega,t)} \lambda^j(\omega) d(j,\omega,t) d\omega, \tag{2}$$

and the static maximization problem can be represented as

$$\max_d \int_0^1 \alpha(\omega) \log \sum_{j=0}^{j^{\max}(\omega,t)} \lambda^j(\omega) d(j,\omega,t) d\omega \tag{3}$$

$$s.t. \quad E(t) = \int_0^1 \left[\sum_{j=0}^{j^{\max}(\omega,t)} p(j,\omega,t) d(j,\omega,t) \right] d\omega$$

where $p(j,\omega,t)$ denotes the price of product ω of quality j at time t, and $E(t)$ is the total expenditure at time t.

At each point in time consumers maximize static utility by spreading their expenditure across industries proportionally to the utility contribution of each product line ($\alpha(\omega)$), and by only purchasing in each line the product with the lowest price per unit of quality. As usual in quality-ladder models with Bertrand competition in the manufacturing sector, this product is the one indexed by $j^{\max}(\omega,t)$. As a result, the individual static demand functions are

$$d(j,\omega,t) = \begin{cases} \dfrac{\alpha(\omega)E(t)}{p(j,\omega,t)} & \text{for } j = j^{\max}(\omega,t) \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Moreover, since the only $j^{\max}(\omega,t)$ quality product is actually purchased, in what follows it will be

$$\sum_{j=0}^{j^{\max}(\omega,t)} \lambda^j(\omega) = \lambda^{j^{\max}(\omega,t)}(\omega).$$

Substituting (4) into (2) and (2) into (1) we get the intertemporal maximum problem as

$$\begin{aligned} \max_E U &= \int_0^\infty e^{-(\rho-n)t} [\log E(t) + \int_0^1 \alpha(\omega) [\log \alpha(\omega) + \log [\lambda(\omega)]^{j^{\max}(\omega,t)} \\ &\quad - \log p(j,\omega,t)] d\omega] dt \end{aligned}$$

$$\text{s.t.} \int_0^\infty e^{-\int_0^t [r(s)-n]ds} E(t)dt \leq W(0),$$

where $r(s)$ is the instantaneous interest rate at time s and $W(0)$ is the present value of the stream of incomes plus the value of initial wealth at time $t = 0$. The solution to this problem obeys the differential equation

$$\frac{\dot{E}(t)}{E(t)} = r(t) - \rho. \tag{5}$$

Each good is produced by only employing labor through a constant return to scale technology: in order to produce one unit of good ω firms hire l_ω units of labor regardless of quality. The Bertrand competition assumption implies that the quality leader monopolizes her relative market and that the limit price she can charge is $p\left[j^{\max}(\omega, t), \omega, t\right] = \lambda(\omega) w l_\omega$. Thus the profit flows in each industry are

$$\pi(\omega, t) = \frac{\lambda(\omega) - 1}{\lambda(\omega)} \alpha(\omega) E(t) L(t).$$

Firms can engage in R&D to develop better versions of the existing products in order to displace the current monopolists. We assume free entry and perfect competition in each R&D race. Firms employ labor and produce, through a constant returns technology, a Poisson arrival rate of innovation in the product line they target. The R&D technology is industry-specific. In particular, any firm hiring l_k units of labor in industry ω at time t acquires the instantaneous probability of innovating $A(\omega) l_k / X(\omega, t)$, where $X(\omega, t)$ is the R&D difficulty index. Since independent Poisson processes are additive, the specification of the innovation process implies that the industry-wide instantaneous probability of innovation (or research intensity) is

$$\frac{A(\omega) L_I(\omega, t)}{X(\omega, t)} \equiv i(\omega, t) \tag{6}$$

where $L_I(\omega, t) = \sum_k l_k(\omega, t) dk$. As R&D proceeds, its difficulty index $X(\omega, t)$ is supposed to increase over time in order to rule out the "scale effect" (Jones, 1995), that is, to rule out explosive growth in the presence of a growing population. With reference to Segerstrom (1998), we model the increasing complexity hypothesis according to what is usually called ‘TEG specification’:[5]

$$\frac{\dot{X}(\omega,t)}{X(\omega,t)} = \mu i(\omega,t),$$

where μ is a positive constant.

Whenever a firm succeeds in innovating, it acquires the uncertain stream of profit flows that accrues to a monopolist, that is, the stock market valuation of the firm, $v(\omega,t)$. Thus, the problem faced by an R&D firm is that of choosing the amount of labor input in order to maximize its expected profits[6]

$$\max_{l_k} \left[\frac{v(\omega,t)A(\omega)}{X(\omega,t)} l_k - l_k\right].$$

The problem above provides a finite, positive solution for l_k only when the arbitrage equation

$$\frac{v(\omega,t)A(\omega)}{X(\omega,t)} = 1$$

is satisfied. Efficient financial markets require that the stock market valuation of the firm yields an expected rate of return equal to the riskless interest rate $r(t)$. The firm's market valuation is

$$v(\omega,t) = \frac{\pi(\omega,t)}{r(t) + \frac{A(\omega)L_I(\omega,t)}{X(\omega,t)} - \frac{\dot{v}(\omega,t)}{v(\omega,t)}},$$

that is, the present value of profits discounted at the obsolescence-adjusted interest rate (see Grossman and Helpman, 1991). Finally, the R&D equilibrium condition is

$$\frac{\pi(\omega,t)A(\omega)}{X(\omega,t)\left[r(t) + \frac{A(\omega)L_I(\omega,t)}{X(\omega,t)} - \frac{\dot{v}(\omega,t)}{v(\omega,t)}\right]} = 1. \tag{7}$$

Since in each industry the market demands, $D(\omega,t) = \left[\alpha(\omega)E(t)L(t)\right]/\lambda(\omega)l_\omega$, require $D(\omega,t)l_\omega$ units of labor in order to be produced, the total employment in the manufacturing sector is

$$\int_0^1 \frac{\alpha(\omega)E(t)L(t)}{\lambda(\omega)} d\omega.$$

[5] The acronym TEG stands for ‘Temporary effects on growth’ of policy measures such as subsidies and taxes. Useful surveys on the scale effect problem and the way it has been solved are Dinopoulos and Thompson (1999) and Jones (1999 and 2003).

[6] We consider labor as numerarie and normalize the wage rate to 1.

As a result, the labor market-clearing condition implies

$$L(t) = \int_0^1 \frac{\alpha(\omega)E(t)L(t)}{\lambda(\omega)} d\omega + \int_0^1 L_I(\omega, t) d\omega. \tag{8}$$

where $\int_0^1 L_I(\omega, t)d\omega$ is the total employment in the research sector.

We now focus on the steady state equilibrium, where all variables grow at constant rates. Along the steady state $\dot{E}(t)/E(t) = 0$ and hence, from the Euler equation, $r(t) = \rho$. Moreover, from the definition of $v(\omega, t)$ it follows that its steady state growth rate is $\dot{v}(\omega, t)/v(\omega, t) = n$. By solving the system made up of (7) and (8), we obtain the steady-state values of expenditure E^*, and of current and expected R&D efforts $L_I^*(\omega, t)$ - which coincide in the rational expectations equilibrium - as

$$E^* = \frac{\frac{\mu}{n}\rho + 1 - \mu}{\left(\frac{\mu}{n}\rho - \mu\right) \int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)} d\omega + 1}$$

and

$$L_I^*(\omega, t) = L(t) \frac{\alpha(\omega) - \frac{\alpha(\omega)}{\lambda(\omega)}}{\left(\frac{\mu}{n}\rho - \mu\right) \int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)} d\omega + 1}. \tag{9}$$

Notice that the steady state research investments are industry-specific and that, by construction, they equalize R&D returns across industries.

3 The Robustness of the Return-Equalizing Equilibrium

The equalization of R&D returns leaves the investor indifferent as to how to allocate resources across industries. As we have argued in the Introduction, this indifference justifies the assumption we make in this section, that is, the investors' aversion against uncertainty. We characterize the agents' R&D investment strategy, and we show that the return-equalizing equilibrium is robust against uncertainty aversion.

Importantly, our assumption on the agents' attitude towards uncertainty does not concern any fundamental of the economy and is to be interpreted as a way of treating the *extrinsic uncertainty* (Cass and Shell, 1983) associated to the future configuration of R&D investments across industries. Moreover, uncertainty does not affect expectations on the *aggregate* amount of research. In fact, we introduce uncertainty to eliminate indeterminacy arising from situations where agents are indifferent among a set of choices. This is not the case for the total amount of research: if agents expect the equilibrium aggregate amount of research, their choice between consumption and savings, which are channelled to the research sector, is uniquely determined and confirms their expectations; there is no indifference, which is the source of the uncertainty in the agents' beliefs.

Assume that the investor is $(1-p)100\%$ sure to face in the future the return-equalizing configuration of R&D investment, and that with a however small probability p any other possible configuration can occur. We can call this situation a "$p-$contamination of confidence".[7] Aversion to ambiguity in this context implies that with probability p the agent expects the worst configuration of future R&D investment, that is, the one which minimizes her expected returns.[8] Since the minimizing configuration is a function of the agent's investment choice, this choice can then be formalized as the result of a "two-player zero-sum game" characterized by

- the minimizing behavior of a "malevolent Nature", which selects the worst possible configuration of *future* R&D efforts and

- the maximizing behavior of the agent, who selects the best possible configuration of *current* R&D efforts.

We denote with $l_m(t)+\gamma(\omega,t)$ the agent's investment in industry ω at time t, and with $L_I^e(t)+\varepsilon(\omega,t)$ the aggregate expected research in industry ω, at time t. l_m and L_I^e are, respectively, the agent's average investment per industry and the average expected research per industry. $\varepsilon(\cdot)$ and $\gamma(\cdot)$ represent deviations from the averages satisfying

$$\int_0^1 \varepsilon(\omega,t)d\omega=0;\quad \int_0^1 \gamma(\omega,t)d\omega=0;\quad \varepsilon(\omega,t)>-L_I^e(t);\quad \gamma(\omega,t)>-l_m(t).$$

The presence of the two functions $\gamma(\cdot)$ and $\varepsilon(\cdot)$ is intended to allow for asymmetry across industries both in the agent's investment and in expected research.

We can now state the R&D investment problem as

[7]To avoid confusion let us remark that in the literature this situation is usually called $\varepsilon-$contamination (which is also the phrase used in the Introduction). However, as we will see, in our context ε stands for the extension of the state space.

[8]See the representation theorem (theorem 1) in Nishimura and Ozaki (2006) for an axiomatization of the choice behavior assumed here.

$$\max_{\gamma(.)} \left[\min_{\varepsilon(.)} \int_0^1 [l_m(t) + \gamma(\omega, t)] \left(p\frac{A(\omega)v(\omega,t)}{X(\omega)} + (1-p)q(t) \right) d\omega \right]$$

s.t. (i), (ii) $\int_0^1 \gamma(\omega, t)d\omega = \int_0^1 \varepsilon(\omega, t)d\omega = 0$; (iii) $\varepsilon(\omega, t) > -L_I^e(t)$; (iv) $\gamma(\omega, t) > -l_m(t)$

where

$$v(\omega, t) \equiv \frac{\pi(\omega, t)}{r(t) + (1-\mu)\frac{A(\omega)}{X(\omega,t)}\left[L_I^e(t) + \varepsilon(\omega, t)\right]},$$

and where $q(t)$ is defined as the expected R&D return which, with probability $1-p$, is constant across industries.

In Appendix A we solve the maxmin problem above via the calculus of variations and characterize the agent's investment strategy as

$$l_m(t) + \gamma(\omega, t) = l_m(t)\frac{\pi(\omega, t)}{\int_0^1 \pi(\omega, t)d\omega} \qquad \forall \omega \in [0, 1], \tag{10}$$

and the distribution of expected R&D investments as

$$L_I^e(t)+\varepsilon(\omega, t) = \frac{\pi(\omega, t)}{\int_0^1 \pi(\omega, t)d\omega}\left(L_I^e(t) + \frac{r(t)}{(1-\mu)}\int_0^1 \frac{X(\omega, t)}{A(\omega)}d\omega\right) - \frac{r(t)}{(1-\mu)}\frac{X(\omega, t)}{A(\omega)} \quad \forall \omega \in [0, 1]. \tag{11}$$

We can now turn to the steady state and enunciate the following

Proposition 1 *For a however small probability (p) of deviation ($\varepsilon(\omega)$) from the return-equalizing expectations on the future R&D investment, decision makers adopting a maxmin strategy to solve their investment allocation problem choose a steady state investment strategy which equalizes R&D returns across industries. The values of these investments coincide with those in (9).*

Proof. See Appendix B. ■

We have shown that, even under $\varepsilon(\cdot)$ and p however small, the return-equalizing equilibrium arises as the unique optimal investment allocation. That is to say, even though the agent is 'almost sure' ($p \to 0$) to face in the future the return-equalizing configuration of R&D investment (which would leave her in a position of indifference in her current allocation problem), the mere possibility of a slightly different future configuration (as captured by $\varepsilon(\omega)$) makes her strictly prefer to choose the return-equalizing

R&D investment strategy. This occurs because, whenever the agent evaluates any different allocation of her current investments, she will always be induced to expect the worst configuration of future investments inside the ε-generated set.

4 Concluding Remarks

The fact that R&D investment decisions are taken under conditions of severe uncertainty about their returns has long been recognized in the economics literature (see among others Rosenberg (1994) and Freeman and Soete (1997)): innovations are "unique" events, and the process aimed at producing them is an uncertain and largely unpredictable economic activity. The concept of "Knightian uncertainty" (as opposed to "risk") appears to be essential in any attempt to analyze the evolution of the innovation process in modern economies. Recent studies on ambiguity (and ambiguity attitude) have tried to give an "operational" meaning to Knightian uncertainty. We have adopted the multiple-prior approach pioneered by Gilboa and Schmeidler (1989). In particular, in a vertical innovation growth model with asymmetric fundamentals we have explored the relationship between ambiguity and *extrinsic uncertainty*, that is, uncertainty not related to the economy's fundamentals but lying in the current evaluation of R&D investments to be carried out by future investors.[9] We have shown that a however small degree of ambiguity aversion eliminates the indeterminacy in the R&D investment allocation problem. As a result, and in contrast with horizontal innovation growth models, the family of vertical innovation models can be meaningfully extended to more realistic asymmetric frameworks where the return-equalizing equilibrium is univocally identified as the unique robust rational expectations equilibrium.

References

[1] Aghion, P. and P. Howitt (1992). "A Model of Growth Through Creative Destruction". *Econometrica,* 60, 323-351.

[2] Akerlof, G. A. and Yellen, J. (1985a). "Can small deviations from rationality make significant differences to economic equilibria?", *American Economic Review*, 75: 708–721.

[9] For an analysis of the relationship between ambiguity and "intrinsic uncertainty" in the innovation process see for instance Cozzi and Giordani (2007).

[3] Akerlof, G. A. and Yellen, J. (1985b). “A near-rational model of the business cycle, with wage and price inertia”, *Quarterly Journal of Economics, Supplement*, 100: 823–838.

[4] Anant, T.C.A., E. Dinopoulos e P. Segerstrom (1990). “A Schumpeterian Model of the Product Life Cycle”, *American Economic Review*, 80, 1077-1091.

[5] Bewley, T. (1986). “Knightian Decision Theory: part I”, Cowles Foundation Discussion Paper No. 807, Yale University.

[6] Blanchard, O. and Kiyotaki,N. (1987). “Monopolistic competition and the effects of aggregate demand”, *American Economic Review*, 77: 647–666.

[7] Cass, D. and K. Shell (1983). “Do Sunspots Matter?”, *Journal of Political Economy*, 91, 193-227.

[8] Cozzi, G. (2005). "Animal Spirits and the Composition of Innovation", *European Economic review*, 49, 627-637.

[9] Cozzi, G. (2007). "Self-fulfilling Prophecies in the Quality Ladders Economy", *Journal of Development Economics*, 84, 445-464.

[10] Cozzi, G. and P.E. Giordani (2007). "Ambiguity Attitude, R&D Investments and Economic Growth". EUI and University of Glasgow, Mimeo.

[11] Cozzi, G., P.E. Giordani and L. Zamparelli (2007). "The refoundation of the symmetric equilibrium in Schumpeterian growth models", *Journal of Economic Theory*, 136(1), 788-797

[12] Dinopoulos, E. and P. Thompson (1999). “Scale-Effects in Schumpeterian Models of Economic Growth”, *Journal of Evolutionary Economics 9,* 157-185.

[13] Dixit, A. K. and J. E. Stiglitz, (1977). “Monopolistic competition and optimum product diversity”, *American Economic Review*, 67: 297–308.

[14] Ethier, W. (1982). “National and international returns to scale in the theory of international trade”, *American Economic Review*, 72: 389–405

[15] Freeman, C. and L. Soete (1997). *The Economics of Industrial Innovation* (Third Edition). The MIT Press.

[16] Gilboa, I. and D. Schmeidler (1989). “Maxmin Expected Utility with Non-Unique Prior”, *Journal of Mathematical Economics* 18, 141-153.

[17] Giordani, P.E. and L. Zamparelli (2008). "The Importance of Industrial Policy in Quality-Ladder Growth Models", *Topics in Macroeconomics, Berkeley Electronic Press*, 8(1), Article 1.

[18] Grossman, G.M. and E. Helpman (1991). “Quality Ladders in the Theory of Growth”, *Review of Economic Studies* 58, 43-61.

[19] Jones, C.I. (1995). “Time Series Tests of Endogeneous Growth Models”, *Quarterly Journal of Economics*, 110, 495-525.

[20] Jones, C.I. (1999). “Growth: With or Without Scale Effects?”, *American Economic Association Papers and Proceedings 89*, 139-144.

[21] Jones, C.I. (2005). “Growth and Ideas”, in P. Aghion and S. Durlauf (eds.) *Handbook of Economic Growth,* Elsevier, Volume 1B: 1063-1111.

[22] Knight, F. (1921). *Risk, Uncertainty and Profit*. Houghton Mifflin Company.

[23] Krugman, P. (1979). “Increasing returns, monopolistic competition, and international trade”. *Journal of International Economics*, 9: 469–480.

[24] Krugman, P. (1980). “Scale economies, product differentiation, and the pattern of trade”. *American Economic Review*, 70: 950–959.

[25] Krugman, P. (1991a). *Geography and Trade.* Cambridge, MA, MIT Press.

[26] Krugman, P. (1991b). “Increasing returns and economic geography. *Journal of Political Economy*, 99: 483–499.

[27] Nishimura, K.J. and H. Ozaki (2006). “An Axiomatic Approach to $\varepsilon-$Contamination”, *Economic Theory*, 27, 333-340.

[28] Park, M.S. (2007) “Homogeneity Masquerading as Variety: the Case of Horizontal Innovation Models”. *Cambridge Journal of Economics*, 31: 379 - 392.

[29] Romer, P.M. (1987). “Growth Based on Increasing Returns Due to Specialization”, *American Economic Review*, 77(2): 56-62.

[30] Romer, P.M. (1990). "Endogeneous Technological Change", *Journal of Political Economy* 98, S71-S102.

[31] Rosenberg, N. (1994). *Exploring the Black Box*. Cambridge, Massachusetts: Cambridge University Press.

[32] Savage, L.J. (1954). *The Foundations of Statistics*. John Wiley and Sons.

[33] Schmeidler, D. (1989). "Subjective Probability and Expected Utility without Additivity". *Econometrica*, 57, 571-587.

[34] Schumpeter, J.A. (1942 [1975]). *Capitalism, Socialism and Democracy*, New York: Harper.

[35] Segerstrom, P.S. (1998). "Endogenous Growth without Scale Effect", *American Economic Review* 88, 1290-1310.

[36] Spence, A. M. (1976). "Product selection, fixed costs, and monopolistic competition", *Review of Economic Studies*, 43: 217–235.

A The Maxmin Problem

$$\max_{\gamma(\cdot)}\left[\min_{\varepsilon(\cdot)}\int_0^1 \left[l_m(t)+\gamma(\omega,t)\right]v(\omega,t)\frac{A(\omega)}{X(\omega,t)}d\omega\right]$$

$$\text{s.t.}\quad \int_0^1 \gamma(\omega,t)d\omega=\int_0^1 \varepsilon(\omega,t)d\omega=0;\quad \varepsilon(\omega,t)>-L_I^e(t);\quad \gamma(\omega,t)>-l_m(t).$$

where

$$v(\omega,t)\equiv\frac{\pi(\omega,t)}{r(t)-\frac{\dot v(\omega,t)}{v(\omega,t)}+\frac{A(\omega)}{X(\omega,t)}\left[L_I^e(t)+\varepsilon(\omega,t)\right]}.$$

Under TEG specification

$$\frac{\dot X(\omega,t)}{X(\omega,t)}=\mu\frac{A(\omega)}{X(\omega,t)}\left[L_I^e(t)+\varepsilon(\omega,t)\right].$$

Moreover, as by differentiating (7) with respect to time, we obtain $\dot v(\omega,t)/v(\omega,t)=\dot X(\omega,t)/X(\omega,t)$, then

$$v(\omega,t)\equiv\frac{\pi(\omega,t)}{r(t)+(1-\mu)\frac{A(\omega)}{X(\omega,t)}\left[L_I^e(t)+\varepsilon(\omega,t)\right]}.$$

From the definition of probability p the return from any investment is industry specific $(v(\omega,t)A(\omega)/X(\omega,t))$ with probability p, while it is constant across industries with probability $(1-p)$ (let us define this constant value as $q(t)$). Then the problem is equivalent to

$$\max_{\gamma(.)}\left[\min_{\varepsilon(.)}\int_0^1 [l_m(t)+\gamma(\omega,t)]\left(p\frac{A(\omega)\pi(\omega,t)}{X(\omega)\left(r(t)+(1-\mu)\frac{A(\omega)}{X(\omega,t)}\left[L_I^e(t)+\varepsilon(\omega,t)\right]\right)}+(1-p)q(t)\right)d\omega\right]=$$

$$=\quad(1-p)q(t)+p\max_{\gamma(.)}\left[\min_{\varepsilon(.)}\int_0^1 [l_m(t)+\gamma(\omega,t)]\frac{A(\omega)\pi(\omega,t)}{X(\omega)\left(r(t)+(1-\mu)\frac{A(\omega)}{X(\omega,t)}\left[L_I^e(t)+\varepsilon(\omega,t)\right]\right)}d\omega\right],$$

which admits the same solution as

$$\max_{\gamma(.)}\left[\min_{\varepsilon(.)}\int_0^1 [l_m(t)+\gamma(\omega,t)]\frac{A(\omega)\pi(\omega,t)}{X(\omega)\left(r(t)+(1-\mu)\frac{A(\omega)}{X(\omega,t)}\left[L_I^e(t)+\varepsilon(\omega,t)\right]\right)}d\omega\right].$$

Notice that this is valid for a however small probability p. Given these conditions, we first solve for the minimization problem

$$\min_{\varepsilon(\cdot)}\int_0^1 \frac{\left[l_m(t)+\gamma(\omega,t)\right]\pi(\omega,t)}{\frac{X(\omega,t)}{A(\omega)}r(t)+(1-\mu)(L_I^e(t)+\varepsilon(\omega,t))}d\omega$$

$$\text{s.t.}\ \int_0^1 \varepsilon(\omega,t)d\omega=0.$$

We set $e(\omega,t)=\int_0^\omega \varepsilon(s,t)ds$; then $e'(\omega,t)=\varepsilon(\omega,t)\ \forall\omega\in[0,1]$ and the minimization problem ($\mathrm{P}_{\min}$) can be expressed as

$$\min_{e'(\cdot)}\int_0^1 G(e')d\omega$$

$$\text{s.t.}\ e(0)=0;\quad e(1)=0$$

where

$$G(e')=\frac{\left[l_m(t)+\gamma(\omega,t)\right]\pi(\omega,t)}{\frac{X(\omega,t)}{A(\omega)}r(t)+(1-\mu)(L_I^e(t)+\varepsilon(\omega,t))}$$

This is the simplest problem of calculus of variations. Since under the conditions specified above $G(e') \in C^2$, we can apply the Euler theorem stating that, if $G(e, e', \omega) \in C^2$ and e^* is optimal and C^1, then e^* must necessarily solve

$$G_e - \frac{d}{d\omega} G_{e'} = 0 \tag{12}$$

As in our case G does not depend on e, $G_e = 0$, and hence (12) becomes $(d/d\omega)\, G_{e'} = 0$, implying that

$$G_{e'} \equiv G_\varepsilon = -\frac{\pi(\omega,t)[l_m(t)+\gamma(\omega,t)]}{\left[\frac{X(\omega,t)}{A(\omega)} r(t) + (1-\mu)(L_I^e(t)+\varepsilon(\omega,t))\right]^2}$$

be constant with respect to ω. Hence

$$\frac{\pi(\omega,t)\,[l_m(t)+\gamma(\omega,t)]}{\left[\frac{X(\omega,t)}{A(\omega)} r(t) + (1-\mu)(L_I^e(t)+\varepsilon(\omega,t))\right]^2} = k_1$$

where k_1 is a real constant. Now we solve the expression above for $\varepsilon(\omega, t)$ and obtain the *reaction function* of "Nature" to the agent's decision as

$$\varepsilon(\omega, t) = \sqrt{\frac{\pi(\omega,t)\,[l_m(t)+\gamma(\omega,t)]}{k_1(1-\mu)}} - \frac{X(\omega,t)}{A(\omega)(1-\mu)} r(t) - L_I^e(t). \tag{13}$$

We can now plug it into the maximization problem $(P_{\max})$ and solve for γ:

$$\max_{\gamma(.)} \int_0^1 [l_m(t)+\gamma(\omega,t)] \frac{\pi(\omega,t)}{\sqrt{\frac{\pi(\omega,t)\,[l_m(t)+\gamma(\omega,t)]\,(1-\mu)}{k_1}}} d\omega$$

$$\text{sub} \int_0^1 \gamma(\omega,t) d\omega = 0.$$

Rearranging, this problem becomes

$$\max_{\gamma(.)} \int_0^1 [l_m(t)+\gamma(\omega,t)]^{\frac{1}{2}} \left(\pi(\omega,t) k_1/(1-\mu)\right)^{\frac{1}{2}} d\omega$$

$$\text{sub} \int_0^1 \gamma(\omega,t) d\omega = 0.$$

Again, we solve $P_{\max}$ as a problem of calculus of variations. By setting $c(\omega,t) = \int_0^\omega \gamma(s,t)ds$, so that $c'(\omega,t) = \gamma(\omega,t)$, $P_{\max}$ becomes

$$\max_{c'} \int_0^1 F(c')d\omega$$

$$\text{sub} \quad c(0) = 0; c(1) = 0$$

where $F(c') \equiv F(\gamma) = [l_m(t)+\gamma(\omega,t)]^{\frac{1}{2}}\left[\pi(\omega,t)k_1\right]^{\frac{1}{2}}$. With the same reasoning as before, the Euler theorem, $F_c - \frac{d}{d\omega}F_{c'} = 0$, implies

$$F_{c'} \equiv F_\gamma = -\frac{(\pi(\omega,t)k_1)^{\frac{1}{2}}}{2[l_m + \gamma(\omega,t)]^{\frac{1}{2}}} = -k_2$$

where $k_2 \in R_+$. From F_γ we can derive the expression for $\gamma(\omega,t)$ as

$$\gamma(\omega,t) = \frac{\pi(\omega,t)k_1}{4k_2^2} - l_m. \tag{14}$$

Plugging it into (13), we obtain

$$\begin{aligned} \varepsilon(\omega,t) &= \sqrt{\frac{\pi(\omega,t)\left[l_m(t) + \frac{\pi(\omega,t)k_1}{4k_2^2} - l_m(t)\right]}{k_1}} - \frac{X(\omega,t)}{A(\omega)(1-\mu)}r(t) - L_I^e(t) = \\ &= \frac{\pi(\omega,t)}{2k_2} - \frac{X(\omega,t)}{A(\omega)(1-\mu)}r(t) - L_I^e(t). \end{aligned} \tag{15}$$

Now we can use the two conditions imposed by the constraints

$$\int_0^1 \gamma(\omega,t)d\omega = 0 \iff \int_0^1 \left[\frac{\pi(\omega,t)k_1}{4k_2^2} - l_m(t)\right]d\omega = 0,$$

$$\int_0^1 \varepsilon(\omega,t)d\omega = 0 \iff \int_0^1 \left[\frac{\pi(\omega,t)}{2(1-\mu)k_2} - \frac{X(\omega,t)}{A(\omega)(1-\mu)}r(t) - L_I^e(t)\right]d\omega = 0$$

to find the constants

$$k_1 = \frac{4k_2^2(1-\mu)l_m(t)}{\int_0^1 \pi(\omega,t)d\omega} \tag{16}$$

and

$$k_2 = \frac{\int_0^1 \pi(\omega,t)d\omega}{2(1-\mu)\left[\frac{r(t)}{(1-\mu)}\int_0^1 \frac{X(\omega,t)}{A(\omega)}d\omega + L_I^e(t)\right]}. \tag{17}$$

Substituting (17) into (16), we obtain

$$k_1 = \frac{l_m(t)\int_0^1 \pi(\omega,t)d\omega}{(1-\mu)\left[\frac{r(t)}{(1-\mu)}\int\limits_0^1 \frac{X(\omega,t)}{A(\omega)}d\omega + L_I^e(t)\right]^2}. \tag{18}$$

Finally we can plug (17) and (18) into (14) and (15) in order to obtain the optimal pair $\gamma^*(\omega,t)$, $\varepsilon^*(\omega,t)$ as

$$\gamma^*(\omega,t) = \frac{\pi(\omega,t)k_1}{4k_2^2} - l_m(t) = l_m(t)\left[\frac{\pi(\omega,t)}{\int_0^1 \pi(\omega,t)d\omega} - 1\right]$$

and

$$\begin{aligned}
\varepsilon^*(\omega,t) &= \frac{\pi(\omega,t)}{\int\limits_0^1 \pi(\omega,t)d\omega}\left[\frac{r(t)}{(1-\mu)}\int\limits_0^1 \frac{X(\omega,t)}{A(\omega)}d\omega + L_I^e(t)\right] - \frac{r(t)}{(1-\mu)}\frac{X(\omega,t)}{A(\omega)} - L_I^e(t) = \\
&= L_I^e(t)\left[\frac{\pi(\omega,t)}{\int\limits_0^1 \pi(\omega,t)d\omega} - 1\right] + \frac{\pi(\omega,t)}{\int\limits_0^1 \pi(\omega,t)d\omega}\frac{r(t)}{(1-\mu)}\int\limits_0^1 \frac{X(\omega,t)}{A(\omega)}d\omega - \frac{r(t)}{(1-\mu)}\frac{X(\omega,t)}{A(\omega)}
\end{aligned}$$

from which we can easily obtain expressions (10) and (11).

B Characterization of the Steady State and Proof of Proposition 1

Expressions (10) and (11) prove to be relevant as soon as we turn to the *steady-state* equilibrium. Then

$$\frac{\dot{X}(\omega,t)}{X(\omega,t)} \equiv \mu i(\omega) = n$$

and, as $\dot{E}(t)/E(t) = 0$, it is $r(t) = \rho$. It is easy to show, by substituting for $L_I^e(t) + \varepsilon(\omega,t)$ (as given in (11)) into $v(\omega,t)$, that the R&D returns $(v(\omega,t)A(\omega)/X(\omega))$ are equalized across industries. In fact

$$\begin{aligned}\frac{A(\omega)v(\omega)}{X(\omega)} &= \frac{\pi(\omega,t)A(\omega)}{X(\omega,t)\rho+\dfrac{A(\omega)L_I^e(t)+\varepsilon(\omega,t)}{X(\omega,t)}(1-\mu)} = \\ &= \frac{\pi(\omega,t)}{\frac{X(\omega,t)}{A(\omega)}\rho+(1-\mu)\left(\frac{\pi(\omega,t)}{\int_0^1\pi(\omega,t)d\omega}\left(L_I^e(t)+\frac{\rho}{(1-\mu)}\int_0^1\frac{X(\omega,t)}{A(\omega)}d\omega\right)-\frac{\rho}{(1-\mu)}\frac{X(\omega,t)}{A(\omega)}\right)} = \\ &= \frac{\pi(\omega,t)}{\frac{\mu}{n}\rho\big(L_I^e(t)+\varepsilon(\omega,t)\big)+(1-\mu)\left(\frac{\pi(\omega,t)}{\int_0^1\pi(\omega,t)d\omega}\left(L_I^e(t)+\frac{\rho}{(1-\mu)}\frac{\mu}{n}L_I^e(t)\right)-\frac{\rho}{(1-\mu)}\frac{\mu}{n}\big(L_I^e(t)+\varepsilon(\omega,t)\big)\right)}.\end{aligned}$$

Substituting for the steady state value of $X(\omega,t)=\frac{\mu}{n}A\left(L_I^e(t)+\varepsilon(\omega,t)\right))$ we finally obtain

$$\begin{aligned}\frac{A(\omega)v(\omega)}{X(\omega)} &= \frac{\pi(\omega,t)}{(1-\mu)\dfrac{\pi(\omega,t)}{\int_0^1\pi(\omega,t)d\omega}L_I^e(t)+\dfrac{\rho\mu}{n}\dfrac{\pi(\omega,t)}{\int_0^1\pi(\omega,t)d\omega}L_I^e(t)} = \\ &= \left(\frac{L_I^e(t)}{EL(t)}\left(1-\mu+\frac{\rho\mu}{n}\right)\right)^{-1}.\end{aligned}$$

Now, by using the arbitrage equation for any industry (equation (7)), we can solve for $L_I^e(t)$ and obtain

$$L_I^e(t)=\frac{EL(t)\left(1-\frac{\alpha(\omega)}{\lambda(\omega)}\right)}{\frac{\mu}{n}\rho+1-\mu},$$

or, in per capita terms,

$$l_m^e=\frac{E\left(1-\frac{\alpha(\omega)}{\lambda(\omega)}\right)}{\frac{\mu}{n}\rho+1-\mu} \tag{19}$$

Dividing the market-clearing condition

$$L(t)=\int_0^1\frac{\alpha(\omega)EL(t)}{\lambda(\omega)}d\omega+L(t)\int_0^1\left[l_m+\gamma(\omega,t)\right]d\omega.$$

by $L(t)$, we can write

$$1=E\int_0^1\frac{\alpha(\omega)}{\lambda(\omega)}d\omega+l_m \tag{20}$$

Given the absence of uncertainty on aggregate, and average, expected amount of research, then $l_m^e(t) = l_m(t)$. The steady-state resource (20) and arbitrage (19) equations allow us to find the equilibrium values of l_m and E as

$$E = \frac{\frac{\mu}{n}\rho + 1 - \mu}{\left(\frac{\mu}{n}\rho - \mu\right)\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega + 1}$$

and

$$l_m = \frac{1 - \int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega}{\left(\frac{\mu}{n}\rho - \mu\right)\int_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega + 1}.$$

The proof of proposition 1 is now straightforward. By plugging the mean value l_m into expression (10) we obtain

$$l_m + \gamma(\omega, t) = l_m \frac{\pi(\omega, t)}{\int\limits_0^1 \pi(\omega, t)d\omega} = \frac{1 - \int\limits_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega}{\left(\frac{\mu}{n}\rho - \mu\right)\int\limits_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega + 1} \cdot \frac{\alpha(\omega) - \frac{\alpha(\omega)}{\lambda(\omega)}}{1 - \int\limits_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega} = \frac{\alpha(\omega) - \frac{\alpha(\omega)}{\lambda(\omega)}}{\left(\frac{\mu}{n}\rho - \mu\right)\int\limits_0^1 \frac{\alpha(\omega)}{\lambda(\omega)}d\omega + 1}$$

Since by definition $L_I^*(\omega, t) \equiv L(t)\left[l_m + \gamma(\omega, t)\right]$, steady state R&D investments coincide with those given in (9).