Dynamic of Employment and Wages Incentives

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Abstract

This paper studies a dynamic model with efficiency wages and adjustment costs associated with hiring and firing decisions. With linear adjustment costs, the optimal efficiency wage and employment are affected by the real interest rate and adjustment costs. When lumpy costs or convex adjustment costs (symmetric or asymmetric) are taken into account, the interest rate and the adjustment costs do not play any role in determining the equilibrium efficiency wage and level of employment.

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1.**Introduction**

The basic model of input demand assumes that firms instantly adjust their employment of inputs when the economic environment changes. However, in practice, what is observed is that firms' change their demand for inputs more slowly than the shocks to input demand warrant. The typical explanation for this fact relies on the presence of adjustment costs, that is, firms find it is costly to make quick changes.

In dynamic models of investment decisions it is widely recognized that firms face adjustment costs to changing the capital stock [e.g. Lucas, 1967]. The lack of a secondary market for many capital goods makes many investment projects to be irreversible. This may cause firms to be hesitant in investing in capital goods, thus creating substantial costs of adjustment in the capital stock [e.g. Dixit and Pindyck, 1994]. However, the issue of adjustment costs related to changing the employment of workers has been less explored in dynamic models. Many macroeconomic models assume that the labor market is frictionless, as a consequence, employment adjustments are costless and instantaneous [Yashiv, 2000]. When one considers the costs of changing the numbers of employees in the firm, such as the disruptions to production occurring when changing employment causes workers' assignments to be rearranged, search and training costs, severance pay, and the cost of maintaining that part of the personnel function dealing with recruitment and worker outflows, it is easy to see that these costs can have important effects on hiring and/or firing workers, and therefore, affecting the long run investment decisions of the firms.

This paper presents a dynamic model of hiring and firing decisions with adjustment costs associated with them. When adjustment costs are taken into account, it is necessary to distinguish between variable and fixed adjustment costs. Variable adjustment costs depend on the number of workers that the firm is going to hire or fire, while fixed adjustment costs do not depend on them. Empirical evidence suggests that both variable and fixed adjustment costs are important in determining labor demand. When variable adjustment costs are pervasive, employment changes may occur slowly [e.g., Fair, 1985; Fay and Medoff, 1985; Aizcorbe, 1992], while in the case in which fixed adjustment costs are important, the employment can switch suddenly in a discontinuous way [e.g., Nickell, 1984; Shapiro, 1986; Hamermesh, 1989].

In order to examine variable and fixed adjustment costs associated with hiring and firing decisions, the model studies five different types of adjustment costs found in the literature [see Hamermesh and Pfann, 1996]. The adjustment costs considered here are: 1) symmetric linear; 2) symmetric convex; 3) asymmetric convex; 4) piecewise linear and 5) lumpy adjustment costs. Each one of the specifications is appropriate to particular situations. The symmetric linear case is used as the baseline model. The symmetric convex is the most widely used function in the literature. The asymmetric convex case is relevant in empirical studies, as Hamermesh and Pfann (1993) have shown, allowing for asymmetry the apparent importance of variations in the turnover rate disappears. The piecewise linear and lumpy adjustment costs are suitable to analyze discontinuities in optimal decision rules.

In addition, the model instead of assuming an exogenous wage rate, uses the efficiency wage theory to endogenize the wage[1]. The model may be interpreted as one of dynamic demand of labor with endogenous wage. The model allows us to relate wage determination and labor demand with variable and fixed adjustment costs. The results are quite appealing, since for some specifications of the adjustment costs the real wage can differ from the marginal productivity of labor, making the interest rate an important parameter in the model. We can also address whether or not the Solow condition plays a role in the determination of the efficiency wage.

[1] Examples of intertemporal efficiency wage models are Lin and Lai (1994), Faria (2000) and Jellal and Zenou (2000).

The remainder of the paper is organized as follows. The next section presents the baseline model based in which symmetric linear adjustment costs are taken into account. In section three, we compare the results of the baseline model with the ones obtained from each of the others adjustment costs specifications. Section 4 brings forward the concluding remarks.

1. The Baseline Model

There are variable and fixed adjustment costs associated with hiring and/or firing workers, they are represented in general by the function $C(\dot{N})$. As a baseline model, we are going to use the symmetric linear adjustment cost function:

$$C(\dot{N}) = a\left|\dot{N}\right| \qquad (1)$$

where *a* is a positive constant and $\dot{N}$ is the time variation of employment, which is given by:

$$\dot{N} = u \qquad (2)$$

where *u* is the hiring or firing decision.

The representative firm's production function is the following:

$$Y = F(e(w), N) \qquad (3)$$

where *N* is the number of workers employed and *e* is effort, which is assumed to depend on the wage paid by the firm (*w*). The effort function satisfies: $e'(w) > 0; e''(w) < 0$. The function *F (.)* is assumed to satisfy: $F_e > 0, F_N > 0, F_{eN} \geq 0, F_{ee} < 0, F_{NN} < 0$

The representative firm expects the interest rate *r* to be constant into the indefinite future and seeks to choose time paths for hiring and/or firing workers [*u*] and wage [*w*] that maximize its present value, which is:

$$\underset{u,w}{Max}\int_0^{\infty}[F(e(w), N) - wN - C(u)]e^{-rt}\,dt$$

subject to equations (1) and (2).

In order to solve this simple optimal control problem consider the following Hamiltonian function:

$$H = F(e(w), N) - wN - C(u) + \lambda u$$

where λ is the co-state variable associated with hiring and/or firing decisions.

The first order conditions are:

$$\lambda = a \qquad (4)$$

$$e'(w)\, F_e = N \qquad (5)$$

$$\dot{\lambda} - r\lambda = -[F_N - w] \qquad (6)$$

plus the transversality condition.

In the steady state [$\dot{\lambda} = \dot{N} = 0$] we have from equations (4) and (6):

$$F_N = w + ra \qquad (7)$$

Notice that in equation (7) the wage rate differs from the marginal productivity of labor due to the presence of adjustment costs, and because of these adjustment costs the interest rate affects wage and employment in this model.

Equations (5) and (7) determine the equilibrium wage and employment in the model. However, by defining the following elasticities functions:

$$\varepsilon(w) = \frac{w e'(w)}{e(w)}$$

$$\eta_e = \frac{e(w) F_e}{F(e(w), N)}$$

$$\eta_N = \frac{N F_N}{F(e(w), N)}$$

where $\varepsilon(w)$ is the standard elasticity of effort with respect to wage and η_e, η_N are, respectively, the elasticities of efficiency with respect to effort and to employed workers. One can rewrite equations (5) and (7) to get:

$$\varepsilon(w) = \frac{\eta_N}{\eta_e} \frac{w}{w + ra} \qquad (8)$$

equation (8) determines the optimal efficiency wage. Given the equilibrium efficiency wage one can find the equilibrium level of employment by equations (5) or (7). It is clear that the optimal demand for labor is negatively related to the efficiency wage. The higher is the equilibrium efficiency wage, the lower is the equilibrium level of employment.

The comparative statics derived from equations (8) and (5) or (7), shows us that an increase in the real interest rate or in the adjustment costs lead to a rise in the efficiency wage, and, as a consequence, to a fall in the level of employment. In the same vein, an increase in the elasticity of efficiency with respect to effort leads to an increase in the efficiency wage, and to a fall in the level of employment, while an increase in the elasticity of efficiency with respect to employment, generates a fall in the efficiency wage and an increase in the level of employment.

It is important to stress that for the case of low rate of interest [close or equal to zero] it follows from equation (8) that :

$$\varepsilon(w) = \frac{\eta_N}{\eta_e} \qquad (9)$$

which is the result of Rasmawamyand and Rowthorn (1991). Notice that in this case the efficiency wage does not depend on the adjustment costs, whatever the size of these costs. This shows how important is the magnitude of the interest rate to amplify the impact of adjustment costs.

By supposing $\frac{\eta_N}{\eta_e}=1$, it follows that the Solow condition $[\varepsilon(w)=1]$ can hold if and only if the interest rate is zero, since $a > 0$ by assumption. If $\frac{\eta_N}{\eta_e}=1$ and $r > 0$, the elasticity of effort with respect to wage is $\varepsilon(w)<1$, that is the equilibrium efficiency wage is higher than the one given by the Solow condition. This happens because firms have an incentive to pay a higher wage to minimize turnover costs [which affect the adjustment costs in this model].

In general, however, the following equivalence holds: $\varepsilon(w)\leq 1$ if and only if $w\leq \hat{w}$ and $\varepsilon(w)>1$ if and only if $w>\hat{w}$, where: $\hat{w}=ra\frac{\eta_e}{\eta_N-\eta_e}$. This formalizes the statement of Stiglitz (1987), he argues that many of the results of efficiency wage theory depend crucially on the existence of some region(s) where an increase in wage leads to more than proportionate increases in work effort.

2. Other Specifications of Adjustment Costs

This section considers the alternative specifications of adjustment costs in the model and compares their results with the baseline model presented last section.

By considering symmetric convex adjustment costs such as:

$$C(\dot{N})=\frac{b}{2}(\dot{N})^2 \qquad (10)$$

or asymmetric convex adjustment costs such as:

$$C(\dot{N})=\frac{b}{2}(\dot{N})^2-c\dot{N}+\exp(c\dot{N})-1 \qquad (11)$$

and solving the model- and noticing that in steady state we have from equation (2) $\dot{N}=u=0$- we obtain the same solution [see Appendix for derivation]:

$$\varepsilon(w)=\frac{\eta_N}{\eta_e} \qquad (12)$$

$$F_N = w \qquad (13)$$

that is, when convex adjustment costs are taken into account, no matter if they are symmetric or asymmetric, the equilibrium efficiency wage is determined by the rate between the elasticity of efficiency with respect to employment and the elasticity of efficiency with respect to effort, and the equilibrium level of employment is determined by the equality between the efficiency wage and the marginal productivity of labor.

The important result here in contrast with the model with linear adjustment costs is that the equilibrium efficiency wage and employment level do not depend on adjustment costs or on the interest rate. Concerning the Solow condition, it may hold when $\frac{\eta_N}{\eta_e}=1$. In the case of the traditional specification of the production function [Solow, 1979] as:

$$Y = F(e(w), N) = f(e(w)N)$$

$\frac{\eta_N}{\eta_e}=1$ implies that the Solow condition holds and it determines the equilibrium efficiency wage.

When one considers piecewise linear costs, defined as:

$$C(\dot{N})=\begin{pmatrix} b_1\dot{N}, & b_1>0, iff\ \dot{N}\geq 0 \\ b_2\dot{N}, & b_2<0, iff\ \dot{N}<0 \end{pmatrix}$$

the model is equivalent to the baseline model for the case of $b_1 > 0$. For the case of $b_2 < 0$ we have the following inequality [provided that $w + rb_2 > 0$] :

$$\varepsilon(w)=\frac{\eta_N}{\eta_e}\frac{w}{w+ra}<\frac{\eta_N}{\eta_e}\frac{w}{w+rb_2}=\varepsilon(w_2)$$

that is, the efficiency wage set for the case of $b_2 < 0$ is less than the one that holds for the case of $b_1 > 0$ or $a > 0$.

Finally, when one considers the case of lumpy costs, defined as:

$$C(\dot{N}) = k_1 I_1(\dot{N}) + k_2 I_2(\dot{N})$$

where the $I_j(\dot{N})$ are indicator functions, with $I_1(\dot{N}) = 1$ if $\dot{N} > 0$, and 0 otherwise; and $I_2(\dot{N}) = 1$ if $\dot{N} < 0$, and 0 otherwise. The $k_j > 0$ indicate the sizes of the lumpy costs. We obtain the same results as in the convex case.

3. Concluding Remarks

This paper studies a dynamic model with efficiency wages and adjustment costs associated with hiring and firing decisions. Therefore it provides a framework to analyze the impact of adjustment costs on employment and wages. In the baseline model, characterized by linear adjustment costs, the optimal efficiency wage and employment are affected by the real interest rate and adjustment costs. In particular, an increase in the real interest rate or in the adjustment costs lead to a rise in the efficiency wage, and, as a consequence, to a fall in the level of employment. Moreover, the efficiency wage is not determined by the Solow condition which holds as a very special case in which the interest rate is zero and the elasticity of efficiency with respect to employment is equal to the elasticity of efficiency with respect to effort.

The paper also analyzes alternative specifications of adjustment costs. It is shown that when piecewise linear adjustment costs are considered, the results are the qualitatively the same as in the baseline case. However, when lumpy costs or convex adjustment costs (symmetric or asymmetric) are taken into account, the equilibrium efficiency wage is determined by the rate between the elasticity of efficiency with respect to employment and the elasticity of efficiency with respect to effort, and the equilibrium level of employment is determined by the equality between the efficiency wage and the marginal productivity of labor. Therefore the interest rate and the adjustment costs do not play any role in determining the equilibrium efficiency wage and level of employment.

The lack of robustness in these results show how sensitive the determination of wages and employment can be in relation to the specification of the adjustment costs. However, this shortcomings can have a positive side since one can infer what type of adjustment costs prevail by investigating empirically whether wages and employment are affected by adjustment costs and the interest rate. By our model, if these factors do not affect wages and employment, it is indicative that convex or lumpy adjustment costs prevail, and, alternatively, if wages and employment reflect the interest rate and or adjustment costs, it suggests that the adjustment costs are linear.

Appendix

Considering the case of quadratic adjustment costs: $C(\dot{N}) = \frac{b}{2}(\dot{N})^2$. The first order conditions are:

$$\lambda = bu \qquad \text{(A1)}$$

$$e'(w)\,F_e = N \qquad \text{(A2)}$$

$$\dot{\lambda} - r\,\lambda = -[F_N - w] \qquad \text{(A3)}$$

In steady state we have from equation (2): $\dot{N} = u = 0$, it is easy to see that equations (12) and (13) are the equilibrium conditions:

$$\varepsilon(w) = \frac{\eta_N}{\eta_e}\frac{w}{w + rbu} = \frac{\eta_N}{\eta_e}\frac{w}{w} = \frac{\eta_N}{\eta_e}$$

$$F_N = w + r\lambda = w + r\,bu = w$$

In the same vein, by solving the model for asymmetric adjustment costs:

$C(\dot{N})=\frac{b}{2}(\dot{N})^2-c\dot{N}+\exp(c\dot{N})-1$, we have as first order conditions:

$$\lambda=bu-c+c\exp cu \qquad \text{(A4)}$$

$$e'(w)F_e=N \qquad \text{(A5)}$$

$$\dot{\lambda}-r\lambda=-[F_N-w] \qquad \text{(A6)}$$

In steady state we have: $\dot{N}=u=0$, therefore we have equations (12) and (13) as equilibrium solutions:

$$\varepsilon(w)=\frac{\eta_N}{\eta_e}\frac{w}{w+r[bu-c+c\exp cu]}=\frac{\eta_N}{\eta_e}\frac{w}{w}=\frac{\eta_N}{\eta_e}$$

$$F_N=w+r\lambda=w+r[bu-c+c\exp cu]=w$$

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