A Socio-Psychological Theory of Efficiency Wage Growth

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Abstract
This paper provides a socio-psychological theory of efficiency wage growth. The model blends agency theory with the Forced Savings hypothesis by assuming that firms set an increasing wage profile to minimize shirking costs, and that workers’ effort is positively related to the variation of wages. In its simple formulation the model derives some interesting results, such as: i) a positive relationship between the growth rate of efficiency wages and the discount rate; ii) for the case of constant returns of motivation, the growth rate of wages is unrelated with technology and workers’ preferences. The model also allows the analysis of the optimal path of employment. The positive impact of increasing efficiency wage profile on job creation depends only on workers’ returns of motivation and technology.

JEL Classification: D29; J28; J30

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1. Introduction

The relationship between wage growth and seniority is well documented in the literature. There is robust evidence that real wages increase with both age and tenure [see, among others, Medoff and Abraham, 1980; Hutchens, 1989; Altonji and Williams, 1998; and Thornton et al., 1997]. Many economists would argue that this happens because worker productivity raises with experience. However, there is enough psychological evidence to dispute such idea. Many empirical studies show that older workers earn more than younger workers, even holding productivity constant [Hellerstein et al., 1996; Haegeland and Klette, 1999; Abowd et al., 2000].

Among the explanations for this fact two are of interest here. Agency theory models argue that firms set steeper wage profiles to minimize shirking costs [Lazear, 1998]. The steeper wage profile has no relationship with the actual productivity of the workers, since profiles are made steep by paying young workers less than they are worth and by paying old workers more than they are worth\(^1\).

The second explanation is given by the Forced Saving Hypothesis [FSH onwards] which states that workers prefer increasing wages profiles because they prefer increasing consumption stream instead of a flat consumption profile. Despite the possibility of create increasing consumption profiles from a flat or downward-sloping wage profile by saving appropriately, this hypothesis assumes that agents lack self-control and are not able to do so [see Clark, 1999]. The assumption that workers prefer an increasing consumption profile to a flat one with the same present discounted value is derived from a set of behavioral theories.

\(^1\) The applications of this theory vary from analysis on mandatory retirement [Lazear, 1979] to profit sharing schemes [Hashimoto and Raisian, 1985].
such as Prospect theory [e.g., Kahneman and Tversky, 1979], Regret theory [e.g., Bell, 1982] and fairness and reciprocity [e.g., Akerlof, 1982, and Fehr and Gachter, 2000], and Veblen (1899) and Duesenberry (1949) habit formation hypothesis.

As seem above the agency theory puts more emphasis on firms’ decisions on setting an increasing wage profile, while the FSH emphasizes workers’ decisions. This paper presents a model based on both views. It is assumed that workers’ efficiency and effort depend on the actual level of wage and, in line with FSH, on overall job satisfaction, which is positively correlated with the change in workers’ wages [Clark, 1999]. By the same token, firms pay efficiency wages and set increasing wage profiles in order to minimize shirking costs in accordance with the agency theory models. The surprising result of the model is that it provides a theory of efficiency wage growth.

1. The Model

According to the basic efficiency wage model [e.g. Solow, 1979], workers’ effort ($E$) is assumed to be increasing in the actual level of real wage ($w$). In addition, we assume, along the lines of the Forced Saving Hypothesis (FSH), that workers’ effort is a positive function of the change in the worker’s pay $w$:

$$E(w, w), E_1 > 0, E_2 > 0 \quad (1)$$

In accordance with the agency theory, the firm minimizes the incentives to shirk by setting an increasing wage profile, such as:

$$\frac{w}{w} = g \quad (2)$$

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2 See Jellal and Zenou (1999) for a derivation of an effort function dependent on the attributes of the job.
The wage profile represents a further cost for the firm:

\[
\dot{C}(w) = C'(g) = w C'(g), \quad C(0) = 0, C''(g) > 0, C'''(g) \geq 0 \quad (3)
\]

As an additional assumption, one can assume a threshold level for \( g \) such as \( C(g) = 0 \), for \( g \leq g^* \). The threshold level simply postulates that for small values of \( g \), the firm has no additional costs in setting an increasing efficiency wage profile.

Equations (1), (2) and (3) are the new elements borrowed from the FSH and agency theory.

By substituting equation (2) into equation (1) we have:

\[
\dot{E}(w, g) = w^k E(1, g) = w^k e(g), \quad e'(g) > 0 \quad (4)
\]

where the parameter \( k \) indicates the returns of motivation. When \( k = 1 \), there are constant returns, when \( k < 1 \) there are decreasing returns and, finally, in the case of \( k > 1 \), we have increasing returns of motivation.

The representative firm’s problem can be written as:

\[
\begin{align*}
\text{Max}_{\mathcal{g}, N} & \int_0^\infty [F(w^k e(g) N) - wN - wC(g)] e^{-\gamma t} \, dt \\
\text{s.t. } & w = gw \\
& w(0) \text{ given}
\end{align*}
\]

Define \( L = w^k e(g) N \), as the amount of efficient labor, where \( F(L) = F(w^k e(g) N) \) is the well-behaved production function, \( N \) is the amount of labor, and \( r \) is the discount rate.

In order to solve this model, consider the following Hamiltonian function:

\[
H = F(L) - wN - wC(g) + \lambda gw \quad (6)
\]

where \( \lambda \) is the co-state variable of \( w \).
Using the Pontryagin maximum principle yields:

\[ F'(L)w^{k-1}e(g) = 1 \]  \hspace{1cm} (7)

\[ F'(L)w^{k-1}Ne'(g) = C'(g) - \dot{\lambda} \]  \hspace{1cm} (8)

\[ \dot{\lambda} = \lambda (r - g) + (1 - k)N + C(g) \]  \hspace{1cm} (9)

plus the transversality condition.

From these optimality conditions we are interested in determining the optimal rate of
growth of efficiency wage \( g^* \).

By defining:

\[ \varepsilon = g \frac{e'(g)}{e(g)} > 0 \]  \hspace{1cm} (10)

and using it into equation (8), taking the logarithms, assuming \( g^* \) constant, and totally
differentiating, we have:

\[ \frac{\dot{N}}{N} = -\frac{\dot{\lambda}}{\lambda} \]  \hspace{1cm} (11)

In the same vein, defining:

\[ \eta = L \frac{F''(L)}{F'(L)} < 0, \text{ and } \gamma = \frac{\dot{N}}{N} \]  \hspace{1cm} (12)

taking the logarithms and totally differentiating equation (7) yields:

\[ \gamma = g \left( 1 - k \right) \frac{1}{\eta} \]  \hspace{1cm} (13)

From equations (7), (8), and (10), we have:

\[ N = \frac{e(g)}{e'(g)}(C'(g) - \dot{\lambda}) = \frac{g}{\varepsilon}(C'(g) - \dot{\lambda}) \]  \hspace{1cm} (14)

Substituting equation (14) into equation (9) yields:
\[ \frac{\dot{\lambda}}{\lambda} = r - g \left[ 1 - \frac{(1-k)(C'(g)-\lambda)}{\varepsilon \lambda} \right] + \frac{C(g)}{\lambda} \]  

(15)

In the steady-state we have:

\[ \frac{\dot{N}}{N} = \gamma = -\frac{\dot{\lambda}}{\lambda} \]

Therefore, by equations (13) and (15) follow:

\[ r - g[1 - (1-k)\frac{(C'(g)-\lambda)}{\varepsilon \lambda}] + \frac{C(g)}{\lambda} = -\frac{g}{\eta} (1-k(1+\eta)) \]  

(16)

Equation (16) is the core of this paper. It determines implicitly the optimal value of the growth rate of efficiency wages, \( g^* \).

As a benchmark model, recall the case for which \( g^* \leq \bar{g} \), so that \( C(g) = 0 \), it yields the following proposition:

**Proposition 1**: The optimal growth of Efficiency wage for the case \( g^* \leq \bar{g} \), is

\[ g^* = \frac{r}{(1-k)(\frac{1}{\varepsilon} - \frac{1}{\eta})+1+k} \]  

(17)

Proposition 1 gives rise to the following results:

i) \( g^*>0 \iff 0<k<\left( \frac{1}{\varepsilon} - \frac{1}{\eta} \right)^{-1} \) or \( k = 1 \). For \( k = 1 \), which characterizes constant returns of motivation, equation (17) reduces to: \( g^* = r/2 \). Thus the optimal growth rate of efficiency wages is independent of \( \varepsilon \) and \( \eta \). As \( \varepsilon \) is the elasticity of motivation relatively to growth pay and \( \eta \) is the magnitude of concavity of the production function, this means that
$g^*$ is independent of firms’ technology and workers’ preferences. In the other case when $k > 0$ and different from one, we must have: $1 - \frac{1}{\varepsilon} > 1$, this condition is satisfied when: $\eta - \varepsilon < \eta \varepsilon < 0$, since $\eta < 0$. In addition notice that this inequality implies that an increase in the returns of motivation leads to an increase in the optimal growth rate of efficiency wages:

$$\frac{dg^*}{dk} > 0$$

ii) $\frac{dg^*}{dr} > 0$, the higher the discount rate induces a higher growth rate of the efficiency wage. The intuition for this result is the following. The higher the discount rate the less important future profits become and the firm puts more importance in present profits. In order to raise profits the firm has to increase its productivity by stimulating workers’ effort. As workers’ effort increases with the growth rate of pay, the firm has an incentive to raise $g$….

ii) For $k < 1$, decreasing returns of motivation, we have, $\frac{dg^*}{d\eta} < 0$ and $\frac{dg^*}{d\varepsilon} > 0$. While for the case of $k > 1$, we have: $\frac{dg^*}{d\varepsilon} < 0$, and $\frac{dg^*}{d\eta} > 0$.

The model also allows us to assess job creation. By substituting equation (17) into equation (13) we derive the evolution of employment, or the optimal path of employment with efficiency wages given by:

$$\frac{N^*}{N} = \gamma^* = \frac{r}{\eta} \frac{1 - k(1 + \eta)}{(1-k)(\frac{1}{\varepsilon} - \frac{1}{\eta}) + 1 + k}$$

From equation (13) it is clear that:

$$\frac{d\gamma}{dg^*} = \left(1 - k(1 + \eta)\right) \geq 0 \Leftrightarrow 1 \leq k(1 + \eta)$$
As a consequence, an increasing efficiency wage profile will lead to increasing employment if and only if and only\( k \) and \( \eta \) satisfy condition (19)\(^3\).

Now we can address the case in which \( g^* \) is above the threshold level \( \bar{g} \). We have two cases of interest, one in which the adjustment costs are linear and another when they are quadratic. When the adjustment costs are linear: \( C(g) = cg \), it follows from equation (16):

\[
g^* = \frac{r}{1 + k - \frac{c}{\lambda} - (1 - k)[\frac{(c - \lambda)}{\varepsilon \lambda} + \frac{1}{\eta}]} \tag{20}
\]

From equation (20) we obtain:

\[
\frac{dg^*}{dc} < 0 \iff \frac{1}{\lambda} < -\frac{(1 - k)}{\varepsilon \lambda} \tag{21}
\]

which holds true for \( \lambda > 0 \) and \( k > 1 + \varepsilon \), or \( \lambda < 0 \) and \( k < 1 + \varepsilon \).

In the case of quadratic adjustment costs, assume: \( C(g) = \frac{c}{2} g^2 \), equation (16) is rewritten as:

\[
g^2 \left[ \frac{c(1 - k)}{\varepsilon \lambda} + \frac{c}{2} \right] - g \left\{ \frac{(1 - k)}{\varepsilon} - \frac{1}{\eta} [1 - k (1 + \eta)] \right\} + r = 0 \tag{17}
\]

which is a quadratic equation allowing for two different values of \( g^* \):

\[
g^*_1 = \frac{-B + \sqrt{B^2 - 4Ar}}{2A}
\]

\[
g^*_2 = \frac{-B - \sqrt{B^2 - 4Ar}}{2A}
\]

where:

\( ^3 \) Job creation could be treated in a more sophisticated way by following Phelps and Hoon (1992) and Faria (2000) by considering the dynamics of labor hiring explicitly. In this case we could consider the link between job satisfaction and quits as in Clark et al. (1998).
In the case of a positive \( g^*_1 \) and a negative \( g^*_2 \) the firm rules out \( g^*_2 \). In the case where both are positive, both satisfy the second order condition: \( H_{gg} (g)<0 \). And the firm decides what is the optimal growth rate of efficiency wage by calculating the net cash flow for each wage profile and choosing the one that yields the higher net cash flow\(^4\).

In the case of adjustment costs it becomes more difficult to assess the marginal impact of workers’ preferences, technology, discount rate, and marginal adjustment costs. The difficulty arises from the fact that the comparative statics depends on the chosen value of \( g^* \). However, one simple way to look at this is by investigating the case of \( k = 1 \), which yields: \( A = c/2 \) and \( B = -1 \), therefore we have:

\[
g^*_{1,2} = \frac{1 \pm \sqrt{1 - 2 r c}}{c}
\]

restricting the analysis for real solutions: \( 1 > 2 r c \), the positive root yields the following comparative statics result:

\[
\frac{d g^*_1}{d r} = \frac{-1}{\sqrt{1 - 2 r c}} < 0
\]

\[^4\text{An alternative criteria would be to choose the value of } g^* \text{ that guarantees the stability of the model: } \frac{d g}{d g} < 0, \text{ however it has been assumed that } g \text{ is constant.}\]
Therefore the presence of quadratic adjustment costs imposes a further opportunity cost which is evaluated at the discount rate $r$, then increasing $g$ induces an implicit cost $r c$ for the firm.

2. Conclusion

This paper presents a model that assumes, along the lines of agency theory, that firms set an increasing wage profile to minimize shirking costs. In addition, it is assumed that workers’ effort increases with real wages, which is consistent with the efficiency wage hypothesis, and that workers’ effort is also positively related to overall job satisfaction which is strongly positively correlated with the rate of growth of wages. This idea follows from the Forced Savings hypothesis, which postulates that workers’ prefer an increasing wage profile in order to satisfy their preferences towards increasing consumption profile.

The surprising result of the model is that it provides a theory of efficiency wage growth. In its simple and intuitive version the model derives a positive relationship between the growth rate of efficiency wages and the discount rate. Furthermore, in the case of constant returns of motivation, the growth rate of wages is unrelated with technology or workers’ preferences. The model also allows the analysis of the optimal path of employment. The positive impact of increasing efficiency wage profile on job creation depends only on workers’ returns of motivation and technology.
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