Monetary Aggregates and the Business Cycle

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Abstract

In the U.S. business cycle, a monetary aggregate consisting predominantly of sight deposits strongly leads output, time deposits strongly lag output, and a monetary aggregate consisting of both types of deposits tends to be coincident with the cycle. Such movements are observed both before and after the 1979 monetary policy change. Similar dynamics are obtained in a model with multi-stage production and purchase-size heterogeneity when agents optimally choose their mix of cash, checkable, and time deposits used in transactions. The causality in the model runs from real activity to money, rather than the other way around.

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1 Introduction

Explaining the observed positive comovements between nominal money stocks and real activity over the business cycle has been one of the principal challenges to monetary economics. Is the empirical relationship between money and real activity a result of significant causal effects of money on output, or do the movements in nominal money stocks reflect developments in the real economy?

Friedman and Schwartz (1963) represent the most influential study arguing for the former interpretation of the data. A key observation in their study motivating their view is that in the U.S. business cycle movements in the monetary aggregate $M2$ systematically precede movements in real output. Tobin (1970), King and Plosser (1984), and Freeman and Huffman (1991), however, challenge such a view. Specifically, Tobin (1970) makes the point that the observed movements in money likely reflect systematic responses of the Federal Reserve to the economy, while King and Plosser (1984) and Freeman and Huffman (1991) make the case that movements in broad monetary aggregates, such as $M2$, result from the endogenous responses of the banking sector to cyclical fluctuations in the demand for deposits. Thus, according to these researchers the causality primarily runs from real economic activity to money.

However, when Coleman (1996) and Ireland (2003) quantitatively evaluate such 'reverse causality' mechanisms within dynamic general equilibrium models (an otherwise prototypical real business cycle model in the case of Coleman and a New-Keynesian model in the case of Ireland), they find that the reverse causality channels do not account for the cyclical behavior of money data. Coleman (1996) especially points out the inability of reverse causality to generate the lead-lag relationship between money and output documented by Friedman and Schwartz (1963).[1] Our aim is

[1]In line with the data, reverse causality channels generate positive contemporaneous correlations between broad monetary aggregates, such as $M1$ or $M2$, and output, as Coleman (1996) and Freeman and Kydland (2000) show. But, in contrast to the data, such aggregates lag output in their

to revisit this issue. Although our primary focus is on the Friedman-Schwartz observation, we evaluate our general equilibrium model against a broader set of monetary facts that we document here.

Figure 1 presents a version of the empirical regularity highlighted by Friedman and Schwartz (1963). It plots the correlations of real GDP in quarter t with a monetary aggregate called MZM in quarter $t+j$, for $j \in \{-5,-4,-3,-2,-1,0,1,2,3,4,5\}$. Both data series are in logs and filtered with the Christiano and Fitzgerald (2003) band-pass filter.[2] The acronym MZM stands for 'money zero maturity'. This aggregate was proposed by Motley (1988) in response to compositional issues with $M2$ and the label was coined by Poole (1991). The reason for using MZM rather than $M1$ or $M2$ is that, unlike $M1$ or $M2$, MZM is constructed in a consistent way. More precisely, it consists of currency and *all* sight deposits – deposits that are immediately available for withdrawal. In contrast, $M1$ does not contain all sight deposits, while $M2$ contains also time deposits – deposits that are issued in specific maturities and withdrawal of which prior to the maturity date is subject to stiff penalties. The key point of the figure is that MZM, like $M2$ in Friedman and Schwartz (1963), on average leads real GDP – it is more strongly positively correlated with future output than with current and past output. The lead is of three to four quarters.

Figure 1 also plots cross-correlations with real GDP for some other measures of money. Specifically, the monetary base (central bank money given as the sum of currency in circulation and bank reserves), time deposits, and the broadest monetary aggregate published by the Federal Reserve, labeled L. This aggregate is given as the sum of MZM and time deposits.[3] We see that the monetary base slightly lags real

models – the aggregates are more strongly positively correlated with past output than with current and future output.

[2]The data are for the period 1959.Q1-2003.Q4. Detrending the series with the HP filter or taking first differences does not qualitatively change the basic finding presented in the figure.

[3]The measure of time deposits used here – a difference between L and MZM – also includes short-term Treasury securities held by the public, in addition to genuine time deposits issued by

GDP by one quarter, time deposits lag by four to five quarters, and the L aggregate lags by one quarter.

Figure 2 shows the cross-correlations with real GDP when the nominal stocks are deflated with a GDP deflator. We see that MZM still exhibits a strong lead while time deposits still exhibit a strong lag. The only lead-lag relationships that change relative to Figure 1 are that the monetary base and the L aggregate become coincident with the cycle.

It is worth pointing out that the dynamics of MZM and time deposits in relation to output are *qualitatively* unaffected by the 1979 monetary policy change aimed at combating inflation.[4] Figure 3 plots the cross-correlations between real GDP and nominal money stocks for two subsamples, 1959.Q1-1979.Q3 and 1979.Q4-2003.Q4. We see that in both subsamples MZM strongly leads while time deposits strongly lag real GDP. In contrast, the dynamics of the monetary base – an aggregate directly controlled by the Federal Reserve – change not only quantitatively, but also qualitatively. While the monetary base strongly lags output in the first subsample, it slightly leads in the second subsample. Similar findings are also obtained for the real stocks, as Figure 4 shows. (For comparison we also include $M1$ in both figures, whose dynamics change in a similar way as the dynamics of the monetary base.)

We ask if reverse causality can account for the empirical lead-lag relationships documented in Figure 1, and for the lead in MZM in particular. We do so within a calibrated dynamic general equilibrium model with multi-stage production and purchase-size heterogeneity, in which business cycles are set off by technology shocks.

banks. The cyclical behavior of time deposits is unaffected when these securities are stripped out off the measure of time deposits. As the Federal Reserve stopped publishing the L aggregate in 1998, our data for L, and thus also for time deposits, are only for 1959.Q1-1997.Q4.

[4] Gavin and Kydland (1999) document that the cyclical behavior of the monetary base, $M1$, and $M2$ changes in a statistically significant way with the 1979 policy change. Dressler (2007) constructs a dynamic general equilibrium model that accounts for the observed changes in the behavior of the monetary base and $M1$.

We allow for the reverse causality mechanism highlighted by Tobin (1970) as well as for that suggested by King and Plosser (1984) and Freeman and Huffman (1991).

Our way of modeling endogenous responses of deposits to real factors builds on the framework of Freeman and Kydland (2000). In our model agents buy a continuum of goods of different sizes that are used for consumption, capital accumulation, and as intermediate inputs in multi-stage production. They optimally choose their balances of cash and checkable deposits based on the size of purchases made and the nominal rate of return paid by deposits. Tobin's mechanism is captured by assuming that a central monetary authority controls a nominal rate of return on a one-period bond, set according to the Taylor (1993) rule. The authority then endogenously supplies the monetary base in an amount consistent with the nominal interest rate.

As in Coleman (1996) both consumption and investment purchases (and in our case also purchases of intermediate inputs) have to be made with either cash or checkable deposits. While the use of cash in transactions is costless, the use of checkable deposits incurs a fixed cost – a cost independent of the purchase size. Weighting this cost against the nominal rate of return paid by deposits, agents optimally choose checkable deposits for purchases of big-ticket items and cash for purchases of small-ticket items. On *average*, consumption purchases are small, while investment purchases are large (and intermediate input purchases fall somewhere in-between). In addition, similarly to Baumol (1952) and Tobin (1956), agents can use higher-yield time deposits to replenish their balances of cash and checkable deposits within a period. The responses of consumption, investment, and intermediate inputs to technology shocks then induce dynamics of monetary aggregates as the agents optimally rebalance their mix of cash, checkable, and time deposits over the business cycle in response to the purchases made and the rates of return.

We find that when calibrated to meet long-run features of the U.S. economy the model accounts for three key monetary facts documented in this paper: MZM leads real GDP, time deposits lag real GDP, and the L aggregate is broadly coincident with real GDP. In addition to these monetary facts, the model's implications are also consistent with the observed cyclical dynamics of real variables, such as consumption, investment, the change in input inventories, the flow of bank lending, and labor productivity. The key elements of our model generating many of these results are the multi-stage nature of production and purchase-size heterogeneity of consumption, investment, and intermediate goods.

The lead in MZM in our calibrated model is not affected, at least qualitatively, by the way monetary policy is conducted. Specifically, the model predicts MZM leading real GDP even when the monetary authority uses the growth rate of the monetary base as its instrument (and keeps it constant) rather than the nominal interest rate set according to a Taylor rule. The dynamics of MZM are thus primarily determined by the reverse causality mechanism highlighted by King and Plosser (1984) and Freeman and Huffman (1991), rather than by that suggested by Tobin (1970). This robustness of the lead-lag relationship between broad money and real GDP in our model is generally in line with our empirical finding that qualitatively the lead-lag relationship between MZM and output was not affected by the 1979 monetary policy change.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 qualitatively characterizes the optimal choice of the mix of cash, checkable, and time deposits and describes how it relates to movements in monetary aggregates. Section 4 describes calibration. Section 5 presents the quantitative results. Section 6 concludes.

2 The model economy

The model economy consists of an infinitely lived representative consumer, a representative producer, and a representative bank. All three agents take all prices as given. In addition, there is a monetary authority that controls a nominal rate of return on a one-period bond and issues fiat money. Fiat money can be used as 'currency' to buy goods and as 'reserves' held by the representative bank. The role of the representative bank is to create 'inside' money (as opposed to 'outside' fiat money issued by the monetary authority), which can also be used in purchases of goods. In contrast to fiat money, inside money pays a nominal rate of return, but its use in transactions is costly.

The only source of uncertainty are shocks to total factor productivity in an aggregate production function. Each period, a shock is observed by all agents at the beginning of the period before any decisions are made.

2.1 Preferences, technology, and purchase-size heterogeneity

A single technology produces a continuum of goods, indexed by $j \in [0,1]$. These goods can be used for consumption, capital accumulation, and as intermediate inputs. Demand for these different goods is generated by a Leontief-type preferences and technology. The proportions in which these goods are either consumed or used in production determine the 'size' of each good.

The consumer's preferences are characterized by the utility function[5]

$$E_t \sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \quad \beta \in (0,1), \tag{1}$$

[5]Through out the paper we denote individual-level variables by lower-case letters and their aggregate (per capita) counterparts by corresponding upper-case letters. We also employ the convention that a subscript accompanying a function denotes the derivative of the function with respect to the argument in the subscript. For example, $f_x(x,y)$ denotes the first derivative of a function f with respect to x.

where c_t is given by

$$c_t \equiv \min\left[\frac{c_t(j)}{(1-\omega_C)j^{-\omega_C}}\right], \quad \omega_C \in (-\infty, 0), \tag{2}$$

l_t is leisure, and $u(.,.)$ is increasing and concave in each argument. In (2) $c_t(j)$ denotes consumption of good j. These within-period preferences for the individual goods, borrowed from Freeman and Kydland (2000), induce the consumer to choose consumption of the goods according to the rule

$$c_t(j) = (1-\omega_C)j^{-\omega_C} c_t \tag{3}$$

for some c_t. Integrating (3) from 0 to 1 shows that c_t is total consumption, defined as $\int_0^1 c_t(j)dj$.[6]

Notice that according to (3) consumption of the individual goods increases in j. We therefore interpret j as an index of a purchase size and refer to the right-hand side of equation (3) as a 'purchase-size function'. A small j represents a purchase of a small-ticket item, such as a newspaper, whereas a large j represents a purchase of a big-ticket item, such as a weekly supermarket basked of goods. For $\omega_C > -1$, the function is strictly concave, for $\omega_C = -1$ it is linear, and for $\omega_C < -1$ it is strictly convex. We discuss the meaning of this curvature in Section 4.

Production has two stages of processing. This is captured by assuming that current output depends on both current value added by primary factors of production (capital and labor), as well as on intermediate inputs produced in the previous pe-

[6]As all goods are produced by the same technology, their relative prices are equal to one. Total consumption is therefore given as a simple sum of the $c(j)$'s.

riod.[7] The aggregate production function has the form

$$Y_t = \begin{cases} G_t^{1-\zeta} X_t^{\zeta} & \text{if} \quad \nu = 0 \\ \left[(1-\zeta)\left(G_t\right)^{-\nu} + \zeta\left(X_t\right)^{-\nu}\right]^{-\frac{1}{\nu}} & \text{if} \quad -1 < \nu < 0 \quad \text{or} \quad \nu > 0, \end{cases} \tag{4}$$

where $\zeta \in [0,1)$, Y_t is *gross* output, G_t is value added by primary factors of production, and X_t is a composite intermediate input produced in period $t-1$. G_t is in turn given by the Cobb-Douglas production function

$$G_t = \exp(z_t) K_t^{\theta} H_t^{1-\theta}, \quad \theta \in (0,1)\,, \tag{5}$$

where K_t is a composite capital stock, H_t is labor, and z_t is the log of total factor productivity evolving as

$$z_t = \rho_z z_{t-1} + \varepsilon_t, \quad \rho_z \in (0,1), \quad \varepsilon_t \sim N(0, \sigma_z). \tag{6}$$

As in the case of consumption, the composite intermediate input and capital are given by

$$X_t \equiv \min\left(\frac{X_t(j)}{(1-\omega_X) j^{-\omega_X}}\right), \quad \omega_X \in (-\infty, 0),$$
$$K_t \equiv \min\left(\frac{K_t(j)}{(1-\omega_K) j^{-\omega_K}}\right), \quad \omega_K \in (-\infty, 0).$$

The consumer owns all $K_t(j)$'s and $X_t(j)$'s and rents them out to the producer. For positive rental rates of the individual goods, the producer's cost minimization dictates the following choices, $K_t(j) = (1-\omega_K) j^{-\omega_K} K_t$ and $X_t(j) = (1-\omega_X) j^{-\omega_X} X_t$, for some K_t and X_t. In turn, the consumer optimally accumulates the individual

[7] An extension to more than two stages is straightforward but makes the exposition somewhat cumbersome without changing the main insight. Edge (2007) provides a brief overview of empirical work on multi-stage production.

capital and intermediate goods according to the rules

$$I_{Kt}(j) = (1 - \omega_K) j^{-\omega_K} I_{Kt}, \tag{7}$$

$$I_{Xt}(j) = (1 - \omega_X) j^{-\omega_X} I_{Xt}, \tag{8}$$

for some I_{Kt} and I_{Xt}, which satisfy $I_{Kt} = K_{t+1} - (1 - \delta_K)K_t$ and $I_{Xt} = X_{t+1} - (1 - \delta_X)X_t$. Here δ_K and δ_X are depreciation rates.[8] As in the case of consumption, it is easy to verify that K_t, X_t, I_{Kt}, and I_{Xt} are, respectively, total capital stock, the total stock of intermediate inputs, (gross) total investment in capital, and (gross) total investment in intermediate inputs. The optimality conditions (3), (7), and (8) characterize the purchase sizes of the individual goods, depending on their final use.

2.2 Inside money

Inside money takes the form of 'checkable' deposits – deposits that can be used in purchases of goods. In addition, banks also offer 'time' deposits, which even though cannot be used to purchase goods directly, can be turned into checkable deposits or currency at a cost. We will refer to currency, checkable, and time deposits as the 'means of payment'.

As in the U.S. economy, banks in the model are required to hold a fraction α of checkable deposits in the form of fiat money as reserves; no reserves are required against time deposits. In addition to reserves, checkable deposits are backed by loans financing purchases of capital and intermediate inputs. Time deposits are fully backed by such loans. Inflation is assumed to be always positive so that in equilibrium the real rate of return on loans always exceeds that on fiat money (given by the inverse of the inflation rate). As a result banks do not hold reserves above and beyond the

[8] It is assumed here that the stocks of the individual capital and intermediate goods in period 0 satisfy $K_0(j) = (1 - \omega_K) j^{-\omega_K} K_0$ and $X_0(j) = (1 - \omega_X) j^{-\omega_X} X_0$ for some initial K_0 and X_0.

required minimum. The balance sheet identity of the banking sector is therefore

$$\alpha\left(D_{Ct}/p_t\right) + S_{t+1}\left(K_{t+1} + X_{t+1}\right) = D_{Ct}/p_t + D_{Tt}/p_t, \tag{9}$$

where S_{t+1} is the fraction of the aggregate stock of capital and intermediate inputs financed through bank loans, D_{Ct} is the nominal amount of checkable deposits, D_{Tt} is the nominal amount of time deposits, and p_t is the aggregate price level – the price of goods in dollars. Notice that the amount of new loans (in real terms) extended by banks in period t is given by

$$Q_t \equiv S_{t+1}\left(K_{t+1} + X_{t+1}\right) - S_t\left(K_t + X_t\right). \tag{10}$$

The interest rates offered on deposits and charged for loans are assumed to be contingent on the realization of z_t (the aggregate technology shock). Perfect competition then ensures that in each state of the world the net real rates of return on checkable and time deposits, r_{Ct} and r_{Tt} respectively, are given by

$$r_{Ct} = (1-\alpha)\, r_{Lt} + \alpha\left(\frac{p_{t-1}}{p_t} - 1\right) - \Omega_C, \tag{11}$$

$$r_{Tt} = r_{Lt} - \Omega_T, \tag{12}$$

where r_{Lt} is the net real rate of return on loans, and Ω_C and Ω_T are unit costs of managing checkable and time deposits, respectively. These costs are used here as stand-ins for factors outside of our model affecting the average rates of return on these two types of deposits. Notice that as checkable deposits are partly backed by reserves, their real rate of return depends on the weighted average of the real rates of return on loans and fiat money. In contrast, the real rate of return on time deposits depends only on the real rate of return on loans, which is given by the weighted average of the rates of return on capital r_{Kt} and intermediate inputs r_{Xt}

$$r_{Lt} = r_{Kt}\frac{K_t}{K_t + X_t} + r_{Xt}\frac{X_t}{K_t + X_t}. \tag{13}$$

The rates of return on capital and intermediate inputs are in turn determined by the producer's profit maximization conditions, $r_{Kt} = Y_G(.,.)G_K(.,.)$ and $r_{Xt} = Y_X(.,.)$. Condition (13) again follows from perfect competition and the assumption that the interest rate on loans is state contingent.

2.3 Monetary policy and outside money

In line with much of the literature, the monetary authority controls a nominal rate of return R_t on a one-period bond, set according to a Taylor (1993)-type rule. Specifically,

$$R_t = (1 - \rho_R)\left[R + \omega_y(\ln y_t - \ln y) + \omega_\pi(\ln \pi_t - \ln \pi)\right] + \rho_R R_{t-1}, \tag{14}$$

where y_t is real GDP, $\pi_t \equiv \ln p_t - \ln p_{t-1}$ is the inflation rate, $\rho_R \in (0,1)$ is a smoothing coefficient, and a symbol without a time subscript represents a variable's steady-state value. Notice that real GDP is a narrower measure of aggregate output than gross output Y_t, specified in Subsection 2.1. Real GDP is defined as $y_t \equiv (dY/dG)G_t = Y_t - r_{Xt}X_t$, which in the National Income and Product Accounts (NIPA) corresponds to the product approach to measuring GDP, and where the second equality follows from perfect competition and the constant-returns-to-scale property of the production function (4).

As the monetary authority uses a nominal interest rate to conduct monetary policy, the amount of fiat money in the economy, denoted by MB_t (as for 'monetary base'), is supplied elastically to accommodate the demand for fiat money at that interest rate. Specifically, the monetary base is given as the sum of currency M_t

demanded by consumers and reserves αD_{Ct} held by banks

$$MB_t = M_t + \alpha D_{Ct}. \tag{15}$$

2.4 The consumer's problem

The consumer maximizes the utility function (1) subject to three sets of constraints: transaction, time, and budget.

2.4.1 Transaction constraints

As mentioned above, goods can only be purchased with currency or checkable deposits. Time deposits cannot be exchanged for goods directly but they can be used to replenish the balances of currency and checkable deposits. Within a period, the ratio of checkable deposits to currency after each replenishment is assumed to be the same. Thus, if the consumer wants to replenish his money balances n_t times, he has to hold time deposits in the amount

$$d_{Tt} = n_t(d_{Ct} + m_t). \tag{16}$$

Each replenishment, as well as making the initial deposit d_{Tt} at the start of the period, costs the consumer ϕ units of time.[9]

The use of checkable deposits in purchases incurs a *fixed* cost (i.e., independent of j), normalized by the average size of the type of purchases made (by 'type' we mean whether the purchases are for consumption, intermediate input accumulation, or capital accumulation). We denote this cost by γ_C, γ_X, or γ_K, depending on the type of purchases made and assume that $\gamma_C > \gamma_X > \gamma_K$. This captures the notion

[9]This cost captures the time lost in various activities related to making transfers to and from time deposits, such as trips to the bank, making phone calls to call centers, etc. Making this cost payable in goods rather than time turns out not change our main result.

that the average size of consumption purchases made by households is smaller than the average size of intermediate input purchases made by firms, which in turn is smaller than the average size of capital goods purchases. Or in other words, that consumption goods are on average small-ticket items, whereas capital goods are on average big-ticket items, while intermediate good purchases fall on average somewhere in-between.

As the cost incurred in the use of checkable deposits is independent of j, but the interest earned on the balances used is increasing in j (since the purchase size is increasing in j and therefore so are the required balances), it is profitable to incur the cost in purchases of big-ticket items, but to use cash in purchases of small-ticket items. There is therefore an optimal j_{Ct} such that all consumption purchases with $j < j_{Ct}$ are made with cash while all consumption purchases with $j \geq j_{Ct}$ are made with checkable deposits. Analogously, there are also optimal j_{Kt} and j_{Xt}. This implies the following constraints on the balances of currency and checkable deposits[10]

$$\begin{aligned}
\frac{m_t}{p_t} &= \frac{1}{n_t+1}\left[\int_0^{j_{Ct}} c_t(j)\,dj + \int_0^{j_{Kt}} i_{Kt}(j)dj + \int_0^{j_{Xt}} i_{Xt}(j)dj\right] \\
\frac{d_{Ct}}{p_t} &= \frac{1}{n_t+1}\left[\int_{j_{Ct}}^1 c_t(j)dj + \int_{j_{Kt}}^1 i_{Kt}(j)dj + \int_{j_{Xt}}^1 i_{Xt}(j)dj\right].
\end{aligned}$$

Substituting from the optimality conditions (3), (7), and (8), these two constraints become

$$\frac{m_t}{p_t} = \frac{(j_{Ct})^{1-\omega_C} c_t + (j_{Kt})^{1-\omega_K} i_{Kt} + (j_{Xt})^{1-\omega_X} i_{Xt}}{n_t+1} \tag{17}$$

$$\frac{d_{Ct}}{p_t} = \frac{[1-(j_{Ct})^{1-\omega_C}]c_t + [1-(j_{Kt})^{1-\omega_K}]i_{Kt} + [1-(j_{Xt})^{1-\omega_X}]i_{Xt}}{n_t+1}. \tag{18}$$

[10] We assume that $\Omega_C > \Omega_T > 0$, which means that $r_{Lt} > r_{Tt} > r_{Ct}$. Furthermore, we assume that inflation is always positive. These assumptions together guarantee that the transaction constraints (16)-(18) hold with equality.

2.4.2 Time and budget constraints

The time constraint is

$$l_t + h_t + \phi(n_t + 1) = 1, \tag{19}$$

where h_t is time spent working, and the budget constraint is

$$\begin{aligned} c_t + i_{Kt} + i_{Xt} + \varphi_t + \frac{d_{Ct}}{p_t} + \frac{d_{Tt}}{p_t} + \frac{m_t}{p_t} + \frac{b_t}{p_t(1+R_t)} \\ = w_t h_t + (1 - S_t)(r_{Kt}k_t + r_{Xt}x_t) + Q_t \\ +(1 + r_{Ct})\frac{d_{C,t-1}}{p_{t-1}} + (1 + r_{Tt})\frac{d_{T,t-1}}{p_{t-1}} + \frac{m_{t-1}}{p_t} + \frac{b_{t-1}}{p_t}, \end{aligned} \tag{20}$$

where

$$\varphi_t \equiv (n_t + 1)\left[\gamma_C(1 - j_{Ct}) + \gamma_K(1 - j_{Kt}) + \gamma_X(1 - j_{Xt})\right] \tag{21}$$

is the total cost incurred by using checkable deposits, b_t is holding of the nominal bond, assumed to be in net zero supply, and w_t is the real wage rate given by the profit-maximization condition $w_t = Y_G(.,.)G_H(.,.)$.

Finally, the total stocks of capital and intermediate inputs follow the laws of motion

$$k_{t+1} = (1 - \delta_K)k_t + i_{Kt}, \tag{22}$$

$$x_{t+1} = (1 - \delta_X)x_t + i_{Xt}. \tag{23}$$

It is assumed that the entire stock of intermediate inputs is used up in production in every period, implying $\delta_X = 1$.

2.5 Monetary aggregates

In addition to the monetary base we also define two other monetary aggregates. In line with the U.S. data, we define MZM as the sum of currency and checkable deposits

$$MZM_t \equiv M_t + D_{Ct} \tag{24}$$

and L as the sum of MZM and time deposits

$$L_t \equiv MZM_t + D_{Tt}. \tag{25}$$

Notice that combining constraints (16)-(18), the L aggregate satisfies

$$L_t = M_t + D_{Ct} + D_{Tt} = p_t(C_t + I_{Kt} + I_{Xt}) = p_t Y_t. \tag{26}$$

Equation (26) resembles a standard 'cash-in-advance' constraint, except that 'cash' here includes all three means of payment in our model. For future reference we also rewrite MZM as

$$MZM_t = M_t + D_{Ct} = MB_t + (1-\alpha)D_{Ct} = \left[\frac{1 + (D_{Ct}/M_t)}{1 + \alpha(D_{Ct}/M_t)}\right] MB_t, \tag{27}$$

where the expression in the square brackets is a money multiplier. MZM is thus given as a product of a money multiplier, which in equilibrium is determined by the optimal checkable deposits to currency ratio chosen by the consumer, and the monetary base, determined by the total demand for fiat money at the interest rate set by the monetary authority.

2.6 Equilibrium

The equilibrium is defined in its recursive form. The individual state variables are $\upsilon_{1t} = (k_t, x_t, d_{C,t-1}, d_{T,t-1}, a_{t-1})$, where $a_{t-1} \equiv m_{t-1} + b_{t-1}$, and the corresponding aggregate state variables are $\Upsilon_{1t} = (K_t, X_t, D_{C,t-1}, D_{T,t-1}, A_{t-1})$, where $A_{t-1} \equiv M_{t-1} + B_{t-1}$. In addition, $\Upsilon_{2t} = (z_t, p_{t-1}, R_{t-1})$. The individual decision variables are $\lambda_t = (n_t, j_{Ct}, j_{Kt}, j_{Xt}, h_t, c_t,\ i_{Kt}, i_{Xt}, d_{Ct},\ d_{Tt}, m_t, b_{t+1})$ and their aggregate counterparts are $\Lambda_t = (N_t, J_{Ct}, J_{Kt}, J_{Xt}, H_t, C_t,\ I_{Kt}, I_{Xt}, D_{Ct},\ D_{Tt}, M_t, p_t)$.

A recursive competitive equilibrium of this economy consists of a value function $V(\upsilon_1, \Upsilon_1, \Upsilon_2)$, individual decision rules $\lambda = \lambda(\upsilon_1, \Upsilon_1, \Upsilon_2)$, and aggregate decision rules $\Lambda = \Lambda(\Upsilon_1, \Upsilon_2)$ such that for every $(\upsilon_1, \Upsilon_1, \Upsilon_2)$ these functions satisfy:

1. $V(\upsilon_1, \Upsilon_1, \Upsilon_2) = \max_\lambda \{u(c, 1 - h - \phi(n+1)) + \beta E\left[V(\upsilon_1', \Upsilon_1', \Upsilon_2') | \Upsilon_2\right]\}$

   subject to: (16), (17), (18), (20), (22), and (23);

2. The constraints (16), (17), (18), (20), (22), and (23) are also satisfied at the aggregate level and z_t evolves according to (6);

3. r_{Ct}, r_{Tt}, r_{Lt}, S_t, Q_t, and R_t are given respectively by (11), (12), (13), (9), (10), and (14); and

4. Individual decisions are the same as aggregate decisions and $\upsilon_1 = \Upsilon_1$.

It is easy to verify that the aggregate resource constraint $C_t + I_{Kt} + I_{Xt} + \varphi_t = Y_t$, not used in the definition of the equilibrium, is satisfied by Walras's Law. Notice that we can rewrite the constraint as $C_t + I_{Kt} + (I_{Xt} - r_{Xt}X_t) + \varphi_t = y_t$, where $I_{Xt} - r_{Xt}X_t$ is a change in input inventories. This expression for GDP corresponds to the expenditure approach in the NIPA.

The equilibrium is computed by first approximating the economy with a linear-quadratic economy and then by applying the method described by Hansen and Prescott

(1995).[11]

Notice that as the model has no nominal rigidities, nominal variables affect the dynamics of real variables only through inflation tax effects, which (as the next section shows) affect n_t, and thus time available for work, as well as the optimal j's, and thus output lost in transactions. For plausible calibration, however, these effects are quantitatively small. It is therefore instructive to think of the economy as being block recursive: given the aggregate state (Υ_1, Υ_2), all quantities (including real holdings of the means of payment) and relative prices are determined independently of nominal variables; then, given the equilibrium quantities and relative prices, nominal variables are determined. In addition, among the nominal variables, p_t and R_t are determined first by an Euler equation for bonds and the monetary policy rule (14). The monetary aggregates MB_t, MZM_t, and L_t are then residually determined by equations (15), (26), and (27). Monetary aggregates in our model thus affect neither quantities nor prices (real or nominal). Their movements only reflect the underlying economic activity and the responses of monetary policy to fluctuations in output and inflation, as summarized by the Taylor rule (14).

3 The optimal choice of the means of payment

Characterizing the optimal choice of the means of payment will help us understand the dynamics of monetary aggregates presented in Section 5. Notice from equations (16)-(18) that, for given c_t, i_{Kt}, and i_{Xt}, the optimal m_t/p_t, d_{Ct}/p_t, and d_{Tt}/p_t are determined by the optimal choice of n_t, j_{Ct}, j_{Kt}, and j_{Xt}. After substituting from the transaction constraints (16)-(18) for m_t/p_t, d_{Ct}/p_t, and d_{Tt}/p_t in the budget constraint

[11] Before computing the equilibrium, the model is transformed so that all nominal variables are stationary.

(20), the first-order condition for n_t becomes

$$(n_t+1)\left\{u_{Lt}\phi+\frac{u_{Lt}}{w_t}\left[\gamma_C(1-j_{Ct})+\gamma_K(1-j_{Kt})+\gamma_X(1-j_{Xt})\right]\right\} \tag{28}$$
$$=\beta E_t\frac{u_{L,t+1}}{w_{t+1}}\left\{\left[r_{T,t+1}-\left(\frac{p_t}{p_{t+1}}-1\right)\right]\frac{m_t}{p_t}+(r_{T,t+1}-r_{C,t+1})\frac{d_{Ct}}{p_t}\right\},$$

while the first-order conditions for j_{Ct}, j_{Kt}, and j_{Xt}, become, respectively,

$$(n_t+1)\frac{u_{Lt}}{w_t}\gamma_C = \Xi_t\frac{c_t(1-\omega_C)(j_{Ct})^{-\omega_C}}{n_t+1}, \tag{29}$$
$$(n_t+1)\frac{u_{Lt}}{w_t}\gamma_K = \Xi_t\frac{i_{Kt}(1-\omega_K)(j_{Kt})^{-\omega_K}}{n_t+1}, \tag{30}$$
$$(n_t+1)\frac{u_{Lt}}{w_t}\gamma_X = \Xi_t\frac{i_{Xt}(1-\omega_X)(j_{Xt})^{-\omega_X}}{n_t+1}. \tag{31}$$

Here

$$\Xi_t=\beta E_t\frac{u_{L,t+1}}{w_{t+1}}\left[r_{C,t+1}-\left(\frac{p_t}{p_{t+1}}-1\right)\right]=\beta E_t\frac{u_{L,t+1}}{w_{t+1}}R_{C,t+1},$$

where $R_{C,t+1}\equiv r_{C,t+1}+\pi_{t+1}/(1+\pi_{t+1})\approx r_{C,t+1}+\pi_{t+1}$ is the nominal interest rate for checkable deposits.

Although these optimality conditions look complicated, they have simple interpretations. The expression on the left-hand side of equation (28) is the cost of using time deposits – the cost of withdrawal ϕ, incurred $n+1$ times, and the total costs of using checkable deposits in purchases after each withdrawal (both types of costs are expressed in the utility of leisure). The expression on the right-hand side of equation (28) is the cost of holding currency and checkable deposits – the expected extra interest that could be earned if the balances were held instead as time deposits. After substituting for m_t/p_t and d_{Ct}/p_t in (28) from (17) and (18) we see that, other things being equal, an increase in c_t, i_{Kt}, or i_{Xt} increases n_t. Notice also that an increase in $r_{T,t+1}$ increases n_t, whereas an increase in p_t/p_{t+1} reduces it.

The expression on the left-hand side of equations (29)-(31) is the marginal cost of increasing the number of goods purchased with checkable deposits – the respective fixed cost paid $n_t + 1$ times. The right-hand side of that equation is the marginal benefit – the interest earned on the balances used in purchases of the additional good. Focusing, for example, on equation (29), other things being equal, an increase in c_t leads to a lower j_{Ct}; i.e., more goods are purchased with checkable deposits when total expenditures on consumption increase. A fall in j_{Ct} leads to an increase in the checkable deposits to currency ratio and thus to an increase in the MZM multiplier. An increase in the nominal interest rate also decreases j_{Ct}; i.e., the size of the marginal purchase for which it is profitable to use checkable deposits decreases as the opportunity cost of holding currency increases. Notice also that a decline in n_t leads to a lower j_{Ct}. This is because a lower n_t increases the purchase-size after each withdrawal across all goods; i.e., the consumer makes less frequent but larger purchases of all goods. A similar argument also applies to equations (30) and (31).

Finally, notice from equations (17) and (18) that an increase in c_t, i_{Kt}, or i_{Xt} mechanically increases both m_t/p_t and d_{Ct}/p_t for a given set of n_t, j_{Ct}, j_{Kt}, and j_{Xt}. How this affects the checkable deposits to currency ratio, and thus the MZM multiplier, depends on whether the j's are closer to zero or closer to one. If, for instance, j_C is close to zero (i.e., most consumption purchases are made with checkable deposits) an increase in c_t increases the ratio and thus the MZM multiplier.

4 Calibration

Baseline parameter values are summarized in Table 1. Most of these values are based on U.S. postwar averages. In the next section we also experiment with alternative parameterizations.

To start, the length of the period in our model corresponds to one quarter. The parameter θ in the production function equals the model's steady-state capital share of GDP and is set equal to 0.36 on the basis of the average capital share in NIPA. We set ν equal to 4, which implies an elasticity of substitution between value added and intermediate inputs of 0.2. This elasticity is sufficient to generate a delayed full response of real GDP to a technology shock. The depreciation rate δ_K is set equal to 0.025, which together with the long-run share of investment in GDP equal to 0.25, implies a steady-state capital to (quarterly) GDP ratio equal to 10. As mentioned above, the depreciation rate for intermediate inputs is set equal to one. The weight on intermediate goods in production ζ is chosen together with the discount factor β so that the model generates the steady-state capital to GDP ratio equal to 10, and the ratio of the stock of intermediate inputs to GDP equal to 0.31 – an estimate obtained when intermediate inputs are measured as the sum of material and supplies, goods in progress, and unfinished construction. In line with the RBC literature, the autocorrelation coefficient ρ_z in the process for technology shocks is set equal to 0.95 and the standard deviation of its innovations to 0.0076.

As in Freeman and Kydland (2000), the reserve ratio α is set equal to 0.1. Ω_T is set equal to 0.0571, which implies a steady-state real rate of return on time deposits equal to the average real rate of return on 3-month certificates of deposit. The value of Ω_C is set equal to 0.0581, which implies a steady-state annual real rate of return on checkable deposits equal to 0.1%. This is somewhat higher than the weighted average of the quoted rates for MZM deposits in order to take account of implicit returns from services that come with checkable deposits (such as direct debits or payroll management in the case of firms).

As mentioned above, the parameters ω_C, ω_K, and ω_X govern the curvature of the purchase-size function and thus the dispersion of $c_t(j)$, $i_{Kt}(j)$, and $i_{Xt}(j)$. A $\omega \in (-1, 0)$ means that the purchase-size function is strictly concave and thus that the dispersion of purchase sizes is relatively small. In contrast, a $\omega < -1$ means that the purchase-size function is strictly convex and thus that the purchase-size dispersion is relatively large. We set ω_C equal to -0.5 and ω_K equal to -2 based on the notion that the differences is the sizes of capital goods are larger than the differences in the sizes of consumption goods.[12] The purchase-size dispersion of intermediate goods probably falls somewhere in-between. We therefore set ω_{Xt} equal to -1. The purchase-size functions for $C_t(j)$, $I_{Kt}(j)$, and $I_{Xt}(j)$ are plotted in Figure 5.

The costs of using checkable deposits, γ_C, γ_K, and γ_X, are calibrated on the basis of the average use of currency in purchases of consumption, intermediate, and capital goods. When currency held abroad (usually estimated to be around 70% of the outstanding stock) is excluded, currency makes up on average about 1.74% of the L aggregate. We distribute this stock across purchase types in the following way, using information on sectoral money balances in the United Kingdom (we are not aware of similar publicly available data for the United States). In the United Kingdom, about 90% of all currency is held by households, with the remaining part held by private nonfinancial corporations (PNFCs). We use the holdings of currency by households as a proxy for the amount of currency used for consumption purchases. We further assume that PNFCs use 90% of their currency balances for intermediate input purchases. This distribution of the use of cash, together with the average fraction of currency in the L aggregate, implies the following values for the costs of use of checkable deposits: γ_C equal to 0.0014, γ_K equal to 0.0000416, and γ_X equal to 0.0002579.

The cost of replenishment ϕ is calibrated to match the average MZM to L ratio. In the United States, about a half of MZM consists of saving deposits. Although, unlike time deposits, saving deposits can be easily withdrawn, it is plausible that at

[12]The difference between expenditures on an aircraft and a TV set (a 'large' vs. a 'small' capital good) is likely to be bigger than the difference between expenditures on weekly supermarket shopping and a coffee (a 'large' vs. a 'small' consumption good).

least part of the stock is not used for transaction purposes on a regular basis. In our baseline calibration we assume that only 50% of saving deposits are held for their transaction services. This implies a MZM to L ratio of 0.36 and ϕ equal to 0.0001577. This value of ϕ gives the total replenishment cost, $\phi(n+1)$, equal to 0.0415% of the time endowment, which is about 36 minutes per quarter, or 0.085% of the model's GDP [calculated as $w\phi(n+1)$]. The sum of the total replenishment and transaction costs, $w\phi(n+1)+\varphi$, in turn amounts to about 0.56% of the model's GDP. This is close to the estimated cost of 0.5% of GDP incurred by banks in providing checkable deposits and credit services, obtained by Aiyagari, Braun, and Eckstein (1998).

The final set of parameters that need to be assigned values are the parameters of the utility function and the monetary policy rule. Given that most of the movements in hours worked over the business cycle are due to the movements in the number of people employed, rather than hours worked per person, we use the specification of the utility function due to Hansen (1985). Specifically, $u(.,.) = \ln(c) + bh$ with b equal to 3.028. As for the parameters of the monetary policy rule, we set ω_y equal to 0.125, which is equal to 0.5 when inflation and interest rates are annualized, ω_π equal to 1.5, and ρ_R equal to 0.75. These are relatively standard in the literature (see, for instance, Woodford, 2003, Chapter 1).

5 Quantitative analysis

This Section first reports the cross-correlations of the model's variables with output. It then displays impulse-response functions in order to bring out the mechanism behind the cross-correlations. Finally, it reports cross-correlations for alternative parameterizations. It also links our results with the previous literature by comparing our model's implications with those of Freeman and Kydland's (2000) model.

5.1 The baseline experiment

Figure 6 displays the main results for our baseline calibration. As Figure 1 it plots correlations between y_t (real GDP) and MB_{t+j}, MZM_{t+j}, and $D_{T,t+j}$ for $j = \{-5, ..., 0, ..., 5\}$.[13] The correlations are based on 100 draws of sequences of ε_t, with the length of each sequence the same as the length of the actual data. As in the case of the actual data, in each draw the artificial time series were filtered with the Christiano-Fitzgerald (2003) band-pass filter and the cross-correlations were recorded. Figure 6 plots the average cross-correlations for the 100 repetitions.

We see that as in the U.S. economy, MZM in the model leads real GDP. In addition, although in the model time deposits do not lag real GDP, they are more positively correlated with past GDP than with future GDP. In contrast to the data, however, the model generates a strongly lagging monetary base.

Table 2 reports cross-correlations between real GDP and some additional macro variables for the United States and the model economy. As in the U.S. economy, consumption, investment, and hours worked in the model are strongly procyclical.[14] Although our model has a two-stage production process, these results are in line with those of a prototypical business cycle model set off by technology shocks (e.g., Cooley and Prescott, 1995).

In addition, in line with the U.S. data, the model generates movements in labor productivity (real GDP per hour worked) that lead movements in output. This dynamics of labor productivity has been highlighted as an important feature of the U.S. business cycle by, among others, Christiano and Todd (1996). Also in line with

[13] L_t, which is constrained according to equation (26) by movements in Y_t and p_t, is in all experiments strongly procyclical with no phase shift, and is therefore omitted from this and the following figures.

[14] Consumption in the data is measured as the sum of nondurables, services, and government consumption. Investment is constructed as the sum of durables, private fixed investment, government fixed investment, and the change in inventories. In the model, investment is defined accordingly as the sum of I_{Kt} and a change in input inventories.

the U.S. data, the change in input inventories in the model ($dX_t = I_{Xt} - r_{Xt}X_t$), as well as bank lending Q, lead real GDP.[15] Finally, the price level is countercyclical both in the data and in the model, but the model does not generate the observed lead-lag relationship between the price level and real GDP.

As the movements in monetary aggregates in our model are primarily driven by real economic activity, and by the dynamics of expenditures on consumption, capital accumulation, and intermediate inputs in particular, it is important that the model generates dynamics of these variables that are broadly in line with the U.S. data. Furthermore, as the multi-stage feature of the production process is a crucial element of our model (as the next subsection shows), it is also desirable that it implies dynamics of labor productivity that are not out of line with respect to the U.S. experience.

5.2 Impulse-responses

Figure 7 plots the responses of key variables to a 1% increase in total factor productivity in period 1 in order to demonstrate the mechanism generating the results in Figure 6 and Table 2. The top panel shows responses of expenditures on consumption C_t, capital accumulation I_{Kt}, and intermediate inputs I_{Xt}, together with the response of real GDP y_t. On impact, real GDP somewhat increases but it does not reach its peak until period 2 due to the two-stage production process. Consumption increases on impact due to consumption smoothing and continues to slightly increase for several periods afterwards. Expenditures on intermediate inputs also increase on impact. This is because intermediate inputs accumulated in period 1 increase the marginal product of primary factors of production, capital and labor, in period 2. After period 1, I_{Xt} returns slowly back to its steady state. Finally, expenditures on

[15] Defining the change in input inventories as $I_{Xt} - X_t$ does not change this result.

investment in capital I_{Kt} somewhat fall on impact but increase sharply in period 2, after which they slowly return back to their steady state.

The mid-left panel shows the responses of the optimal cut-off j's. As expenditures on consumption and intermediate inputs increase in period 1, the sizes of these types of purchases increase across all j's. As discussed in Section 3, this makes it profitable to use checkable deposits for more goods. Both j_{Ct} and j_{Xt} therefore fall. As expenditures on investment increase in period 2, j_{Kt} also falls in that period. The movements in j_{Ct} and j_{Xt} in period 1, and in j_{Kt} in period 2, are reflected in the changes in the balances of checkable deposits used for expenditures on C_t, I_{Kt}, and I_{Xt}, plotted in the mid-right panel. In particular, the declines in j_{Ct} and j_{Xt} in period 1 are reflected in the pick-ups in that period in the real balances of checkable deposits used for consumption and intermediate input purchases, defined as $D_{C,C,t} \equiv [1-(j_{Ct})^{1-\omega_C}]C_t/(n_t+1)$ and $D_{C,X,t} \equiv [1-(j_{Xt})^{1-\omega_X}]I_{Xt}/(n_t+1)$, respectively. And the decline in j_{Kt} in period 2 is reflected in the increase in that period in the real balances of checkable deposits used for expenditures on investment, defined as $D_{C,K,t} \equiv [1-(j_{Kt})^{1-\omega_K}]I_{Kt}/(n_t+1)$.

The bottom left panel plots the responses of time deposits D_{Tt} and the frequency of withdrawals n_t. Notice that both n_t and D_{Tt} increase in period 2 as I_{Kt} increases. This is because, as discussed in Section 3 in relation to equation (28), increases in C_t, I_{Xt}, or I_{Kt}, increase n_T. As all three types of expenditures are above their steady states in period 2, n_t increases in that period (in period 1 instead the increase in C and I_X is somewhat counterbalanced by the decline in I_{Kt}). The increase in n_t in turn reduces the size of all purchase sizes per withdrawal, making j_{Ct} and j_{Xt} to increase in period 2. These increases in j_{Ct} and j_{Xt} occur because the average sizes of consumption and intermediate input expenditures are smaller than the average size of investment expenditures. A given increase in n_t thus has a bigger effect on j_{Ct}

and j_{Xt} than on j_{Kt}. As a result, the real balances of checkable deposits used for purchases of consumption and intermediate inputs fall in period 2 and the consumer ends up using checkable deposits predominantly for expenditures on investment.

The overall effect of this rebalancing on MZM is summarized in the response of this aggregate plotted in the bottom-right panel. The initial increase in $D_{C,C,t}$ and $D_{C,X,t}$ in period 1 increases MZM on impact, before the full rise in real GDP in period 2. And as the effect of the increase in $D_{C,K,t}$ in period 2 on MZM is smaller than the effect of the declines in $D_{C,C,t}$ and $D_{C,X,t}$ in that period, MZM peaks in period 1, rather than in period 2. MZM thus leads real GDP.

The shift away from currency to checkable deposits in period 1 also causes a sharp fall in MB on impact. This is because a dollar fall in the demand for currency is compensated by an increase in the demand for reserves of only α dollars. Subsequently, MB recovers and starts to increase, as the size of purchases, and thus the use of checkable deposits in transactions, begins to fall.

5.3 Alternative parameterizations

Figure 8 reports the cross-correlations for MZM_t, D_{Tt}, and MB_t under alternative parameterizations in order to further investigate the mechanism and the model's ability to account for the data. In particular, we consider alternative steady-state checkable deposits to currency ratios as well as equal treatment of the three purchase types (i.e., $\omega_C = \omega_K = \omega_X = -1$ and $j_C = j_K = j_X$). Finally, we also compare the predictions of our model with those of Freeman and Kydland (2000) model, hereafter referred to as the FK model. As mentioned in the Introduction, the FK model is one of the few models in the literature incorporating different monetary aggregates into a dynamic general equilibrium. And as their modeling approach is the closest to ours, their model is a natural benchmark with which to compare our results.

The top-left panel of Figure 8 plots the cross-correlations when the steady-state checkable deposits to currency ratio is equal to 12.5 (in the baseline case the ratio is 20.8). This ratio is obtained when all saving deposits in the United States are excluded from our measure of checkable deposits. As a result the implied costs of using checkable deposits are higher than in the baseline case, as the cash intensity is higher. Under this alternative calibration the phase shift of MZM is much stronger than in the baseline case. In fact, MZM now leads real GDP by about as much as in the data (in both cases by about three quarters). Furthermore, time deposits in the model lag real GDP almost as much as in the data (three compared to four quarters).

The top-right panel shows the results for the opposite case when all saving deposits are included in our measure of checkable deposits. The average checkable deposits to currency ratio is now equal to 29.17 and the implied costs of using checkable deposits are smaller than in the baseline case. In this case both MZM and time deposits become more coincident with the cycle.

In both cases, the dynamics of the monetary base, which is out of line with respect to the data in the baseline case, becomes largely unaffected. This is further discussed at the end of this subsection.

The bottom panel of Figure 8 shows the results when we make the steady-state cash intensity, as well as the purchase-size function, identical across the three purchase types. Specifically, the purchase-size function is now linear. We see that in this case the lead in MZM largely disappears. This is because demand for checkable deposits now reaches its peak at the same time as real GDP. Essentially, the rise in $D_{C,C,t}$ and $D_{C,X,t}$ in period 1 is not big enough and MZM reaches its peak with the rise in $D_{C,K,t}$ in period 2.

Finally, Figure 9 compares the cross-correlations for MZM in our model with those in the FK model. Freeman and Kydland (2000), however, assume that the monetary authority follows a constant money growth rule, $\ln(MB_t) - \ln(MB_{t-1}) = \mu$. In order to make our model comparable with theirs, we replace the Taylor rule with this constant money growth rule. This way we also investigate the extent to which the results are driven by the endogeneity of monetary base that occurs due to the Taylor rule, and thus the quantitative importance of Tobin's channel of reverse causality in our model.

When we assume a constant money growth rule, our model differs from the FK model only by the multi-stage feature of the production process, the requirement that all three types of purchases are made with cash or checkable deposits (only consumption is subject to such a constraint in the FK model), and the presence of time deposits.[16]

When MB_t is exogenous, the price level p_t is determined in a market for fiat money by equating the supply of fiat money with the demand for its real balances so that the money market clearing condition (15), when expressed in real terms, is satisfied.

As the monetary base is now exogenous, it is uncorrelated with real GDP. The amount of monetary base used as currency, however, is still endogenous. We therefore plot cross-correlations for M_t in Figure 9 instead of those for MB_t. We see that even under a constant money growth rule MZM leads real GDP in our model, although less so than under the Taylor rule. In contrast, in the FK model it slightly lags. The fact that MZM leads output under both the Taylor rule and the constant money growth rule implies that, at least in our setup, the Friedman-Schwartz observation is driven by endogenous creation of deposits in response to real activity, rather than the endogenous nature of fiat money.

Notice that in both models currency is countercyclical and its cross-correlations

[16]Freeman and Kydland (2000) refer to the aggregate in their model consisting of currency and checkable deposits as $M1$.

with real GDP are a mirror image of the cross-correlations of MZM. This behavior of currency also drives the lagging character of the monetary base in our model. In order to overturn this result and bring the movements in the monetary base closer to the data, perhaps more complicated purchase-size functions would be required. Such functions need to generate a rise in the checkable deposits to currency ratio when expenditures on consumption and intermediate inputs increase, in order to generate the lead in MZM, but at the same time also increase demand for currency.[17]

6 Conclusion

The empirical regularity that over the business cycle movements in monetary aggregates, such as MZM, precede movements in output has led to two broad lines of work. One line of work develops theories based on important nominal rigidities that make movements in money cause movements in output. The other line of work explores the possibility that the observed money-output correlations are due to systematic responses of the Federal Reserve to the economy or due to the endogenous nature of deposit creation. Previous studies, however, have found that when incorporated into quantitative dynamic general equilibrium models, reverse causality channels do not, by themselves, generate the observed lead-lag relationship between money and output. Indeed, in contrast to the data, monetary aggregates, such as $M2$ or MZM, tend to lag output in such models.

We revisit the question if reverse causality can account for the observed dynamics of money data in relation to output. Incorporating an optimal choice of the mix of cash, checkable, and time deposits into an equilibrium business cycle model with purchase-size heterogeneity and multi-stage production, we find that when calibrated

[17] We have experimented with different parameterizations of our purchase-size functions but did not find a set of parameter values that would generate the observed behavior of both MZM and MB at the same time.

to the U.S. economy, the model produces movements in MZM, time deposits, and the broad aggregate L in relation to output that are consistent with the post-war U.S. experience – MZM leads output, time deposits lag output, and the L aggregate is broadly coincident with output.

A key mechanism generating this result is that, following a positive aggregate technology shock, but before output reaches its peak, agents start making purchases for which it is optimal to use checkable deposits. Although the central bank in our model responds to the economy by following a Taylor rule, the dynamics of monetary aggregates are primarily determined by endogenous creation of deposits in response to changes in the demand for different means of payment. This result is broadly in line with the empirical finding documented in this paper that both before and after the 1979 monetary policy change the lead-lag empirical relationship between MZM and output, and between time deposits and output, stayed qualitatively the same.

We have kept the model economy intentionally simple along many dimensions in order to focus squarely on the reverse causality channels, and to transparently bring out their mechanisms. We have thus abstracted from various sources of nominal rigidities, such as sticky prices and wages, and various forms of limited participation in money markets that give rise to money nonneutralities. We do not claim that such frictions play no role in the dynamics of real or nominal variables. Our analysis simply points out that the endogenous nature of deposit creation in response to developments in real economic activity, thought by previous studies to be unable to explain the empirical lead-lag relationships between money and output, can go a long way in accounting for such empirical regularities once the multi-stage nature of the production process and purchase-size heterogeneity of consumption, investment, and intermediate goods are taken into account.

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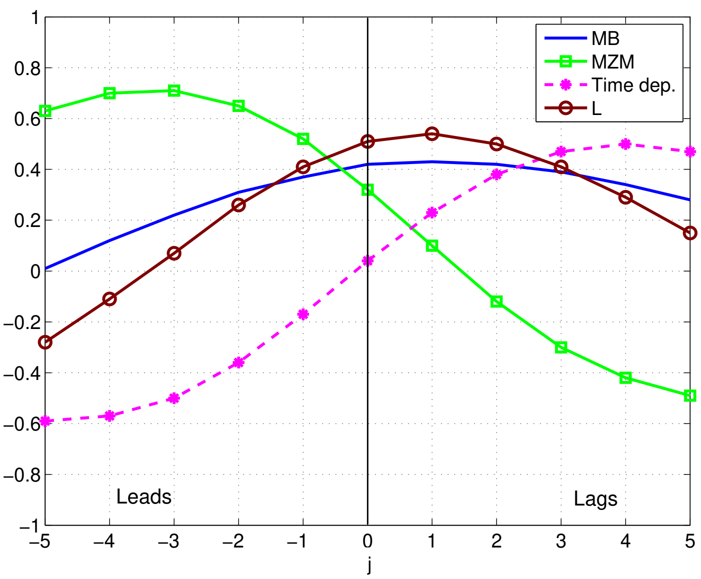


Figure 1: Correlations between monetary aggregates in period $t+j$ and real GDP in period t for logged data filtered with Christiano-Fitzgerald (2003) band-pass filter, 1959.Q1-2003.Q4.

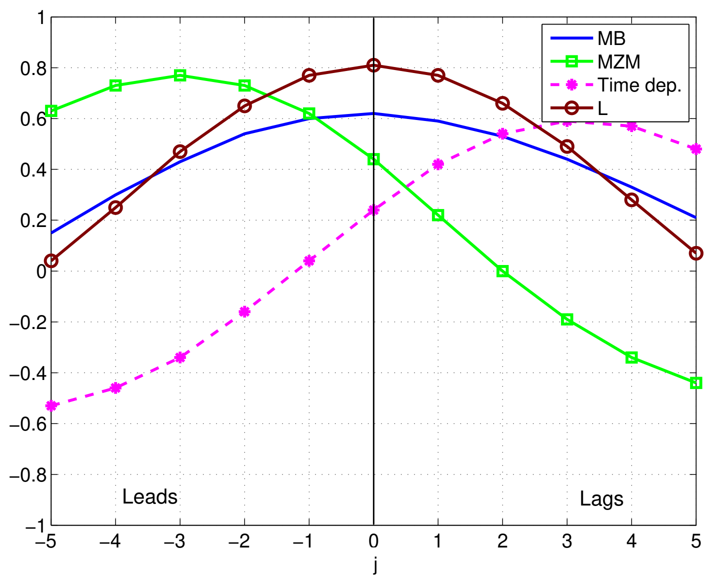


Figure 2: Correlations between deflated monetary aggregates in period $t + j$ and real GDP in period t for logged data filtered with Christiano-Fitzgerald (2003) band-pass filter, 1959.Q1-2003.Q4.

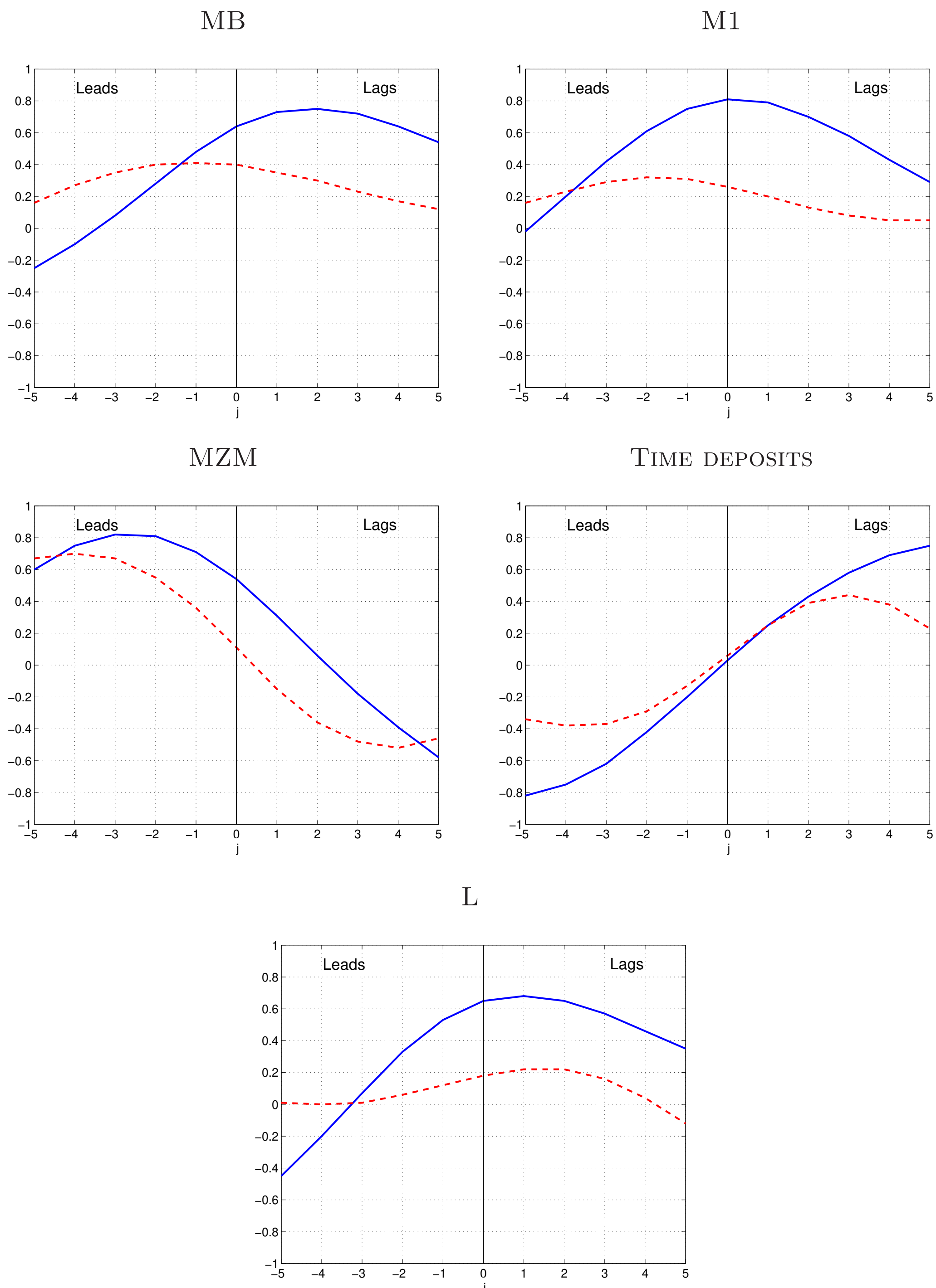


Figure 3: Cross-correlations between monetary aggregates and real GDP in two subsamples. Solid line – 1959.Q1-1979.Q3; dashed line – 1979.Q4-2003.Q4 (1979.Q4-1997.Q4 in the case of time deposits and the L aggregate).

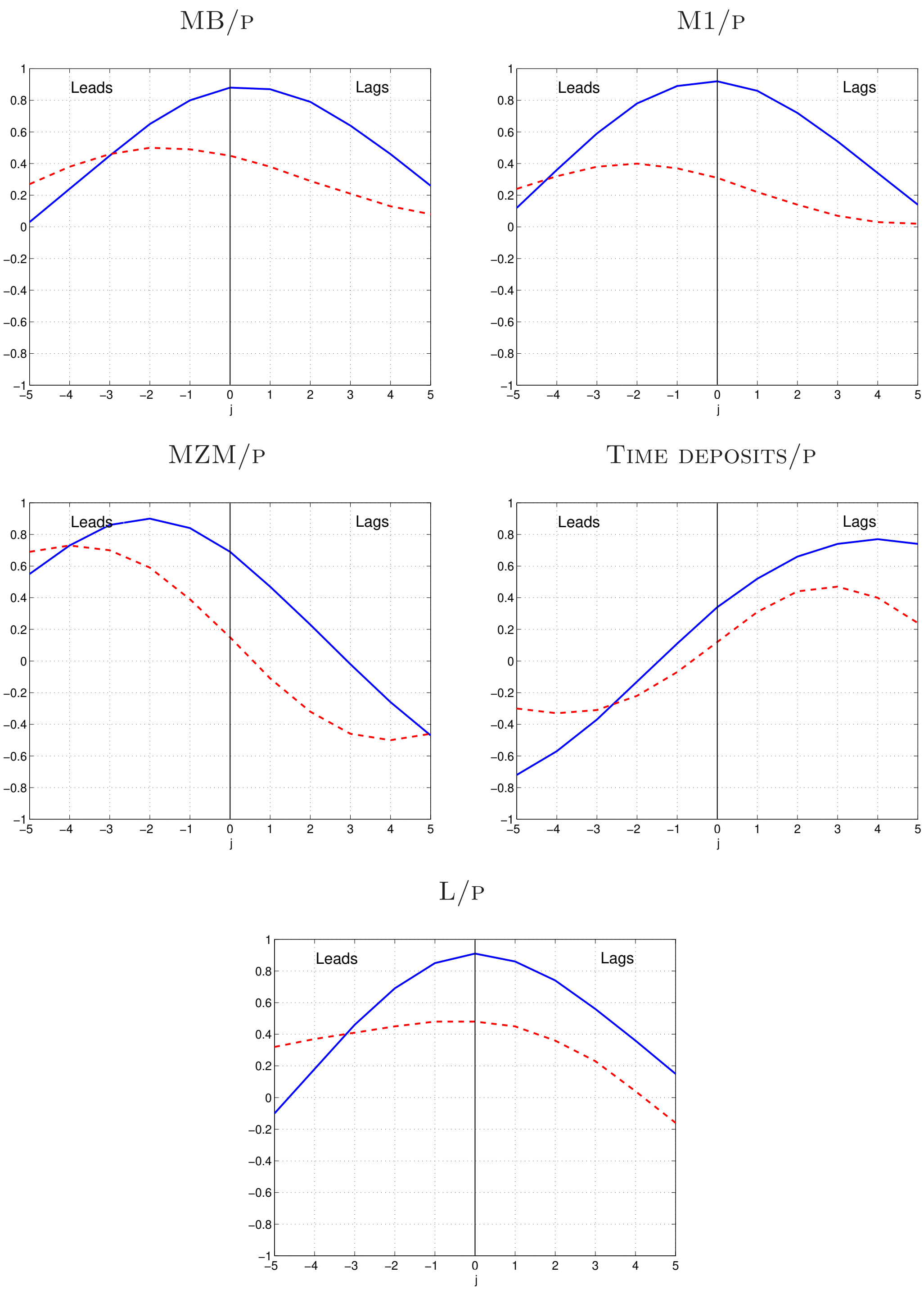


Figure 4: Cross-correlations between deflated monetary aggregates and real GDP in two subsamples. Solid line – 1959.Q1-1979.Q3; dashed line – 1979.Q4-2003.Q4 (1979.Q4-1997.Q4 in the case of time deposits and the L aggregate).

Table 1: Baseline calibration

Symbol	Value	Definition
Preferences		
β	0.994	Discount factor
b	3.028	Weight on leisure
Production		
θ	0.36	Capital share in GDP
ν	4.0	Parameter governing elasticity of substitution
ζ	$9.026 * 10^{-4}$	Weight on intermediate goods
δ_K	0.025	Depreciation rate for capital
δ_X	1.0	Depreciation rate for intermediate goods
ρ_z	0.95	Persistence of the technology shock
σ_z	0.0076	Standard deviation of ε_z
Banks		
α	0.1	Required reserves ratio
		Unit costs for:
Ω_T	0.0571	time deposits
Ω_C	0.0581	checkable deposits
Transaction technology		
		Curvature of the purchase-size function for:
ω_C	-0.5	consumption goods
ω_K	-2.0	capital goods
ω_X	-1.0	intermediate goods
ϕ	$1.577 * 10^{-4}$	Cost of withdrawal from time deposits
		Cost of use of checkable deposits in purchases of:
γ_C	0.0014	consumption goods
γ_K	$4.16 * 10^{-5}$	capital goods
γ_X	$2.579 * 10^{-4}$	intermediate goods
Monetary policy		
π	0.014	Steady-state inflation
ω_y	0.125	Fed's reaction to output
ω_π	1.5	Fed's reaction to inflation
ρ_R	0.75	Smoothing coefficient

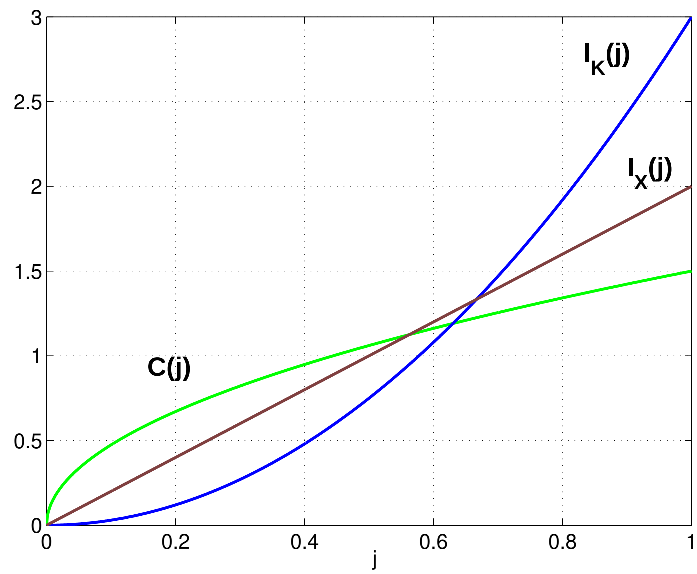


Figure 5: Purchase-size functions for baseline parameterizations: $C(j) = 1.5j^{0.5}$, $I_X(j) = 2j$, $I_K = 3j^2$.

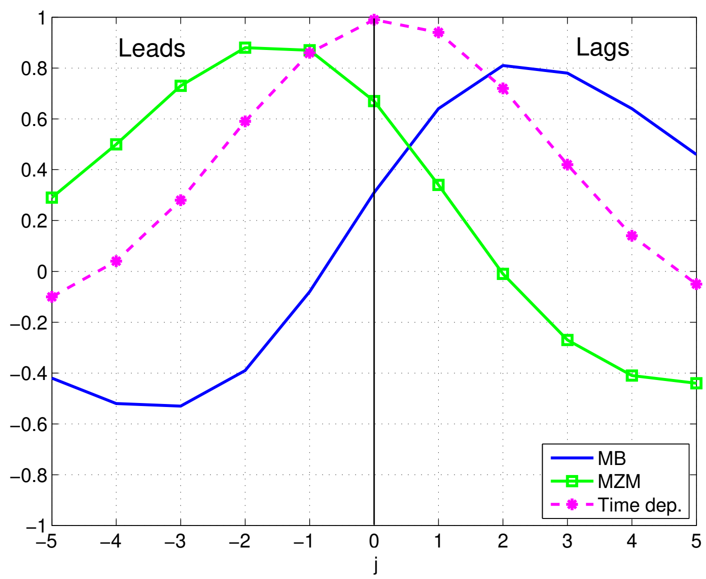


Figure 6: Results for baseline calibration.

Table 2: Cross-correlations for additional variables

	Correlation of Real GDP with:										
Variable x	x(t-5)	x(t-4)	x(t-3)	x(t-2)	x(t-1)	x(t)	x(t+1)	x(t+2)	x(t+3)	x(t+4)	x(t+5)
					U.S. ECONOMY, 1959.Q1-2003.Q4						
GDP	0.03	0.26	0.52	0.76	0.94	**1.00**	0.94	0.76	0.52	0.26	0.03
Consumption	-0.17	0.06	0.29	0.51	0.69	**0.80**	0.83	0.78	0.68	0.54	0.38
Investment	0.11	0.31	0.55	0.77	0.92	**0.96**	0.86	0.66	0.39	0.12	-0.11
Change in inp. inventories	0.42	0.58	0.70	0.73	0.62	**0.40**	0.12	-0.16	-0.37	-0.50	-0.54
Hours[a]	-0.31	-0.09	0.18	0.46	0.71	**0.88**	0.94	0.89	0.75	0.56	0.34
Labor productivity	0.74	0.81	0.81	0.73	0.58	**0.36**	0.12	-0.12	-0.33	-0.47	-0.54
Bank lending	0.22	0.37	0.53	0.64	0.68	**0.63**	0.50	0.31	0.11	-0.09	-0.25
Price level	-0.40	-0.58	-0.73	-0.81	-0.84	**-0.79**	-0.68	-0.53	-0.34	-0.14	0.06
					BASELINE CALIBRATION						
GDP (y)	-0.09	0.07	0.33	0.64	0.90	**1.00**	0.90	0.64	0.33	0.07	-0.09
Consumption (c)	-0.23	-0.04	0.25	0.57	0.82	**0.92**	0.85	0.67	0.46	0.29	0.18
Investment ($i_K + dX$)	-0.05	0.11	0.37	0.67	0.91	**0.99**	0.87	0.58	0.25	-0.02	-0.18
Change in inp. inventories (dX)	0.29	0.52	0.67	0.63	0.35	**-0.08**	-0.47	-0.67	-0.63	-0.43	-0.21
Hours (H)	-0.10	-0.02	0.17	0.46	0.76	**0.96**	0.95	0.73	0.40	0.08	-0.14
Labor productivity (y/H)	-0.05	0.24	0.59	0.85	0.93	**0.81**	0.54	0.27	0.09	0.03	0.04
Bank lending (Q)	0.17	0.36	0.58	0.69	0.57	**0.23**	-0.21	-0.56	-0.69	-0.59	-0.37
Price level	0.54	0.52	0.45	0.31	0.12	**-0.09**	-0.28	-0.42	-0.49	-0.51	-0.50

Note: Except for the change in input inventories, which is measured relative to the stock, all series are in logs. Before computing the statistics, the data were filtered with the Christiano-Fitzgerald (2003) band-pass filter. The entries for 'baseline calibration' are averages for 100 runs of the model, with each run of the length of 168 periods. Like the data, the artificial series in each run were filtered with the Christiano-Fitzgerald (2003) band-pass filter.

[a] Hours are from the Establishment Survey, 1964.Q1-2000.Q4.

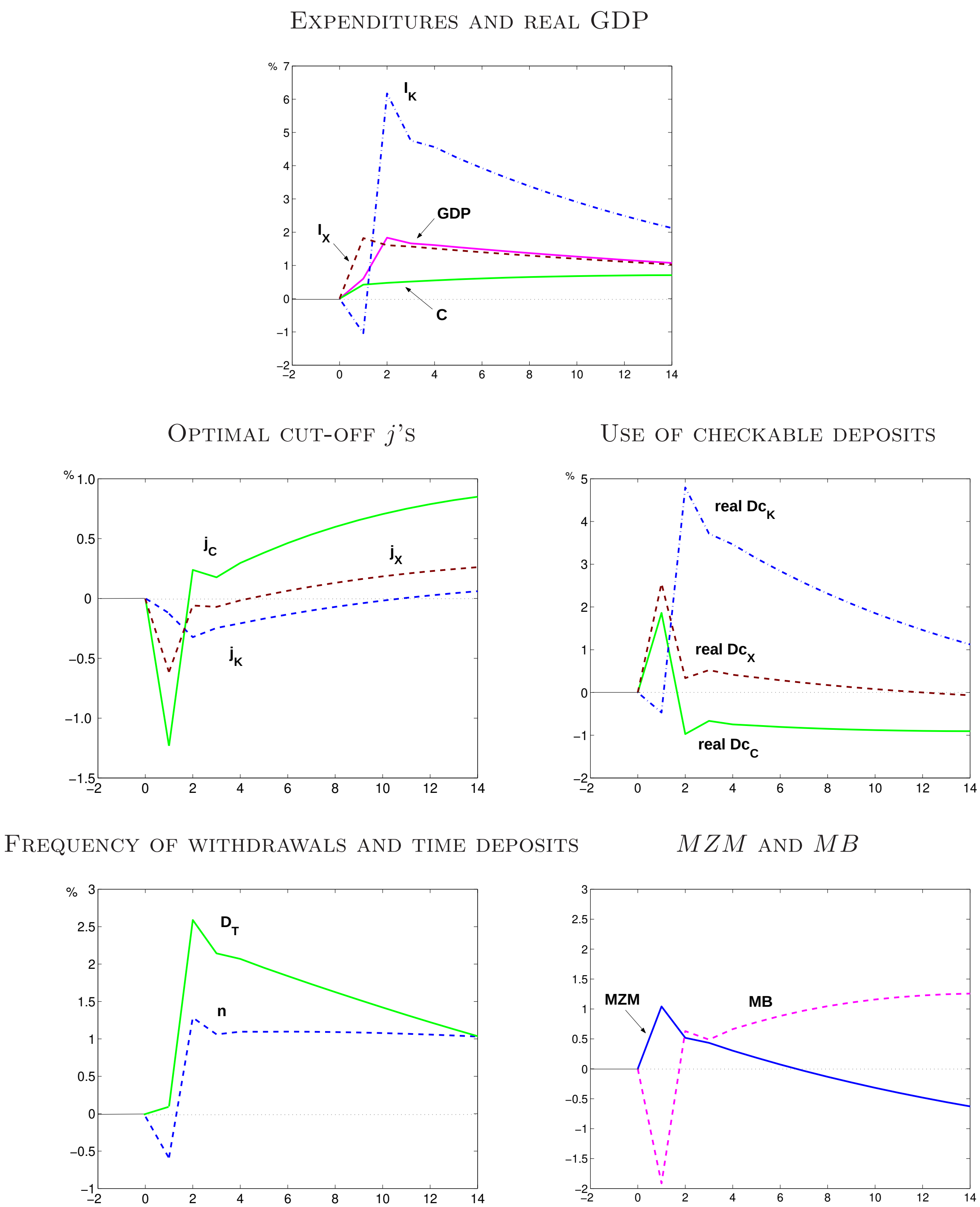
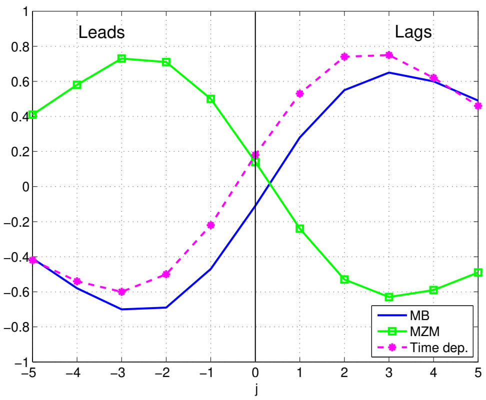
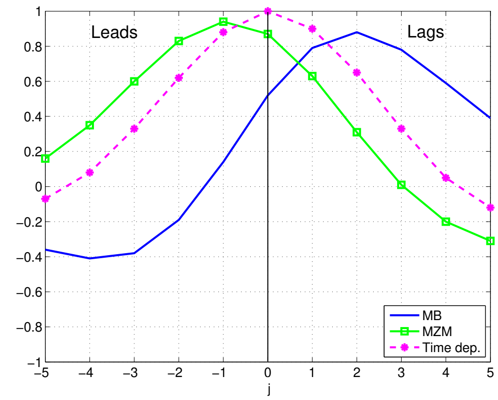


Figure 7: Responses to a 1% increase in total factor productivity, baseline calibration.

Smaller steady-state D_C/M than in baseline ($D_C/M = 12.5$)



Larger steady-state D_C/M than in baseline ($D_C/M = 29.17$)



Equal treatment of purchase types: $\omega_C = \omega_K = \omega_X = -1$ and $j_C = j_K = j_X$

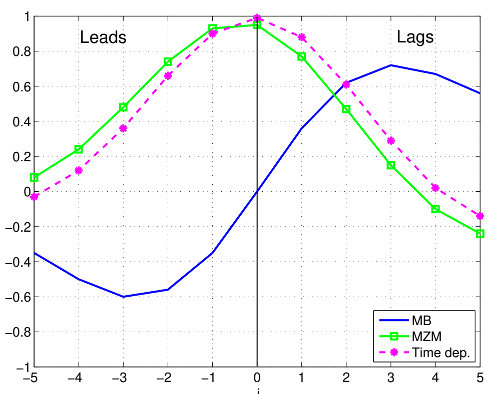


Figure 8: Alternative parameterizations of the transaction technology.

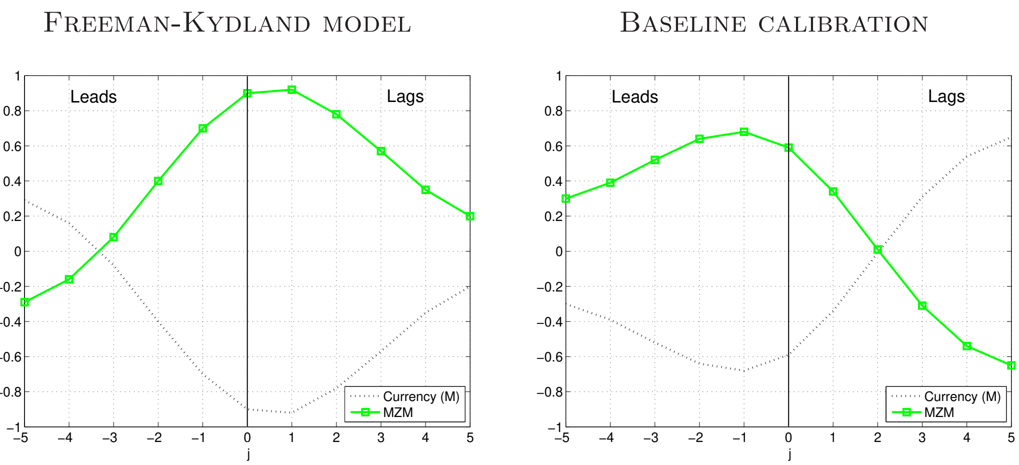


Figure 9: The model under a money growth rule and the Freeman and Kydland (2000) model; in both cases $\log(MB_t) - \log(MB_{t-1}) = \mu$.