# Government's Preference and Timing of Endogenous Wage Setting: Perspectives on Privatization and Mixed Duopoly

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**Abstract**

This study investigates social welfare and privatization depending on the government's preference for tax revenues and the timing of wage setting in either a unionized-mixed or a unionized-privatized duopolistic market. We show that bargaining over wages is always sequential regardless of who decide the timing of endogenous wage setting and market type except for the following cases; (i) there cannot be any sustained equilibrium or (ii) any timing can be sustained as an equilibrium. Moreover, if the government's preference for tax revenues is sufficiently large, the privatization of the public firm is harmful in terms of both social welfare and government's payoff whether the wage setting is simultaneous or not. However, if the government's preference for tax revenues is sufficiently small, there can exist incongruence regarding privatization between the public firm and the government.
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# 1 Introduction

Recently, the economic implications of mixed-oligopoly markets have been an issue of divergent objective functions between the public firm and the government for the market-structure efficiency with respect to the change in competition. It has been argued by Matsumura (1998), White (2001), Barros (1995), Willner (2006), Kato (2008), Saha (2009) and among others in the literature on mixed oligopolies that objective functions differ between public firms and the government[1]. Willner (2006) justified the objective function of the public firm as being the consequence of several assumptions such as the fact that consumers are also taxpayers. Considering partial privatization, Matsumura (1998) assumed that the government puts a larger weight on the consumer surplus than on the producer surplus. More specifically, Kato (2008) showed that the government's privatization of the public firm depends on its preference for tax revenues[2]. Saha (2009) showed that the optimal privatization in a differentiated duopoly when the public firms do not bear the full cost of production and their objective functions differ from the government's objective function. In addition, White (2001) and Barros (1995) discussed the situation in which the government hires managers to manage the public firm. In this case, the preferences of the government and that of the manager of the public firm differ.

These previous works differ from that a public firm, as well as the government, traditionally maximizes social welfare competing with private firms for maximizing their own profits. However, although most previous studies consider different objective function between the public firm and the government, none of these papers have considered the case in which both private and public firms, or the government choose to bargain over wages of the endogenous timing in a unionized mixed duopoly. Hence, we extend Kato (2008)'s framework by assuming that the timing of wage setting is endogenously determined, under which the public firm assigns full weight to social welfare defined as the sum of consumer and producer surpluses, while the government attaches weights to both social welfare and tax revenues. As Kato (2008) pointed out, "the public firm is not a tax collection agency, the public firm does not care about tax revenue but instead cares about the sum of consumer and producer surpluses[3]." Based on this assumption,

[1]Matsumura and Tomaru (2009) demonstrated that introducing shadow cost of public funding changes the results in subsidized mixed oligopoly. Moreover, Capuano and De Feo (2008) for endogenous timing where introduced this cost in a mixed oligopoly. Saha and Sensarma (2008) showed that if the government is producers' profit oriented, it will accommodate the private firm's aggression and cut back the public firm's output through partial privatization.

[2]By introducing taxes (ad valorem or specific) in a mixed oligopoly, Mujumdar and Pal (1998) showed that privatization can increase both social welfare and tax revenues, where an increase in tax does not change the total output but increases the output of the public firm and the tax revenues. Furthermore, by introducing subsidy policy into the Cournot-type model of DeFraja and Delbono (1989), White (1996), Poyago-Theotoky (2001), and Myles (2002) showed that privatization affects neither optimal subsidy rate or improving welfare. However, most papers on mixed oligopolies make a standard assumption on firms' objectives when governmental intervention is incorporated into the mixed oligopoly: private firms are profit-maximizers while the public firm, as well as the government, is a social-welfare maximizer.

[3]Moreover, if there does not exist the government's preference for tax revenue, the government puts the same weight on social welfare and tax revenues. In this case, the government is benevolent since the government's

we investigate incongruities between a public firm and the government, which the present paper seeks to evaluate this assumption in the context of a *unionized* mixed oligopoly.

We present some rationale for discussing objective functions based on government's payoff as follows. For the government, it has been argued in the literature that there is another way to limit the discretionary power of governments when a Leviathan government exists (see Brennan and Buchanan, 1980). For example, Oates (1985) and Zax (1989) found empirical support for a Leviathan government, while Forbes and Zampelli (1989) rejected the assumption of a Leviathan government[4]. Therefore, the literature contains a number of puzzles for fiscal centralization and the size of the public sector (Oates, 1989). These two contrasting views clearly reflect different perceptions of policy-making. Firstly, government is a benevolent maximizer of social welfare. Secondly, it intrinsically is a tax-revenue maximizer. Another argument for this objective function is as follows. Wilson (1989) and Tirole (1994) pointed out that the government consists of different agencies and its mission can be pursued by different officials of the same agencies. Composite missions that reflect the optimization of various goals may not accord with the self interest of officials. Therefore, in a departure from the framework of traditional models that involves a monolithic entity that seeks to maximize social welfare across the public firm and the government, we assume that the public firm gives full weight to the social welfare, which is defined as the sum of the consumer and producer surpluses, while the government attaches weight to both the social welfare and its preference for tax revenues. Some readers may think that this model is not the appropriate one to analyze the issue proposed in the paper since the problem is that tax revenues are not used by the government. However, we follow that, as Brennan and Buchanan (1980, chapter 1 and 2) suggested, "the power to tax, per se, does not carry with it any obligation to use the tax revenues raised in any particular way. The power to tax does not logically imply the nature of spending (Brennan and Buchanan, 1980, p. 8)." This is why we introduce the divergent objective function between the government and the public firm that we model in this paper.

In the literature on unionized (mixed) oligopolies, the bargaining process between the firm and the union has been developed almost independently. For instance, in a spatial context, Brekke and Straume (2004) have analyzed how equilibrium locations in location-price games under Hotelling's model are affected when wage negotiations occur simultaneously. Moreover, theoretical studies that introduce the timing of endogenous wage-setting (i.e., the setting of input costs) into oligopolistic markets include De Fraja (1993a), Corneo (1995), and Barcena-Ruiz and Casado-Izaga (2008), among others. Specifically, Barcena-Ruiz and Casado-Izaga (2008) extend the findings of Brekke and Straume (2004) by introducing the timing of endogenous wage-setting. For non-spatial contexts, De Fraja (1993a) and Corneo (1995) show that in a

payoff represents social welfare itself. See footnote 5 and 6 for more on this point.

[4]For more detailed treatment of the Leviathan government, recent theoretical as well as empirical studies include Keen and Kotsogiannis (2002) and Brülhart and Jametti (2006, 2007).

pure duopoly, when wage bargaining is decentralized at the level of the private firm, unions prefer to play sequentially and vice versa. On the other hand, there have been some attempts, namely, De Fraja (1993b), Haskel and Sanchis (1995), Haskel and Szymanski (1993), Willner (1999), and Ishida and Mastushima (2009), to introduce the union's utility into a model of mixed-duopoly markets. In particular, Ishida and Mastushima (2009) examined the optimal regulatory framework of public firm, focusing on a wage regulation imposed on the public firm by considering comparison of social welfare and Cournot competition in a mixed-duopoly context where outputs are chosen simultaneously after wage settings occur simultaneously.

Considering the divergent functions that exist between a public firm and the government, few studies have been undertaken on how the effect of the timing of wage settings is established by any pair of players (i.e., by two unions; private and public firms (or government); two unions and only one firm (or government), or one union and two firms (where one firm is a public firm and the other is a public firm or government). Therefore, we combine literatures dealing with two separate issues: the mixed-duopoly market with wage setting and a four-player (private and public firms, unions, and the government) market. We consider that the outputs in the mixed duopoly are chosen simultaneously, but we extend previous works by assuming that the timing of wage setting is endogenously determined.

Consequently, the present study differs from the existing literature in at least three important ways. First, comparison of the government's payoff with the social welfare has not been hitherto attempted, while this paper investigates to the literature as it extends the works of Kato (2008) and Choi (2009) who analyzed only the government's perspective on the privatization. Second, the existing studies on mixed oligopolies consider simultaneous wage-setting rather than the effects of different timings of wage setting. Third, our study investigates privatization and social welfare depending on the government's preference for tax revenues when each player chooses the timing of wage setting that is endogenously determined.

Our first main finding shows that bargaining over wages in either unionized-mixed or unionized-privatized duopolies is always sequential regardless of who decides the timing of endogenous wage settings except for the following cases; (i) there cannot be any sustained equilibrium or (ii) any timing can be sustained as an equilibrium. The reasons for this are as follows: (1) each union prefers to decide the wage setting sequentially in either a unionized mixed or privatized duopoly; (2) all the revenues of private firms become zero due to the tax rate in the privatized duopoly, which means that any timing can be sustained as an equilibrium; (3) although the government obtains less output in the simultaneous case, it obtains a higher tax rate, and thus, it obtains a greater governmental payoff in the simultaneous case, while the private firm under the unionized mixed duopoly always prefers to play sequentially, which leads to the results that there cannot be any sustained equilibrium; (4) at the same time, given that the private firm always prefers to play sequentially, the public firm prefers sequential (respectively, simultaneous) wage setting if the government's preference for tax revenues is sufficiently small (respectively, large). Consequently, since the choice of timing of the public firm varies with the government's preference for tax revenues and there is an opposite preference between the government and the private firm, all players prefer to set wages sequentially.

Second, we show that the government never has an incentive to privatize the public firm, while the public firm has an incentive to be privatized depending on the government's preference for tax revenues. If the government's preference for tax revenues is sufficiently large, the interest between the public firm and the government can be coincided. In this case, the privatization of the public firm is harmful, whether or not the wage setting is simultaneous. However, if the government's preference for tax revenues is sufficiently small, there can exist incongruence regarding privatization between the public firm and government because simultaneous wage setting cannot be sustained as an equilibrium. The conflicts between these two views of objective functions typically induce a conflict with regard to the privatization. These results, when the choice of timing of endogenous wage setting is set in a unionized mixed duopoly considering divergent objective function between the government and the public firm, differs from the standard findings of De Fraja (1993a), Corneo (1995), and Barcena-Ruiz and Casado-Izaga (2008). They show that in a pure duopoly, when wage bargaining is decentralized at the level of the private firm, unions prefer to play sequentially and vice versa.

The organization of the paper is as follows. In Section 2, we describe the model. Section 3 presents fixed-timing games regarding the wage setting. Section 4 determines firms' endogenous choices of wage setting and social welfare. Concluding remarks appear in Section 5.

# 2 The Model

The model presented here is based on Choi (2009) and Kato (2008). Consider a mixed-duopoly situation for a homogeneous good that is supplied by a public firm and a private firm. Firm 1 is a profit-maximizing private firm and firm 0 is a public firm that maximizes the social welfare. Assume that the inverse demand is characterized by $p = 1 - x_0 - x_1$, where $p$ is the price of the good, $x_0$ is the output level of the public firm and $x_1$ is the output level of the private firm.

On the demand side of the market, the representative consumer's utility is a quadratic function given by

$$U = x_0 + x_1 - \frac{1}{2}\left(x_0 + x_1\right)^2.$$

To analyze the union's wage bargaining, we also assume that the public and private firms are unionized and that the wages, $w_i : i = 0, 1$, are determined as a consequence of bargaining between firms and their respective unions. Let $\overline{w}$ and $L_i$ denote the reservation wage and the number of workers who are employed by firm $i$, respectively. The firms are homogeneous with respect to productivity. Each firm adopts a constant returns-to-scale technology where one unit of labor is turned into one unit of the final good; thus, $x_i = L_i$. Taking $\overline{w}$ as a given, the union's optimal wage-setting strategy regarding firm $i$, $w_i$, is defined as

$$\max_{w_i} u_i = (w_i - \overline{w})^{\theta} L_i; i = 0, 1,$$

where $\theta$ is the weight that the union attaches to the wage level. As suggested by Haucap and Wey (2004), Leahy and Montagna (2000) and Lommerud *et al.* (2003), we assume that the union possesses full bargaining power ($\theta = 1$) for the wage level (see also Booth, 1995)[5]; for ease of exposition, we assume that $\overline{w} = 0$ to show our results in a simple way. Thus, we assume that the union sets the wage, while public and private firms unilaterally decide their respective levels of employment.

In what follows, we assume that a specific tax rate is imposed on the public and private firms. This is because the calculations are greatly simplified, without any impact on the implications of our model, if a specific tax rate is imposed on both firms in lieu of an ad valorem tax.

Each firm's profit follows the function

$$\pi_i = (p - w_i)x_i - tx_i, \quad i = 0, 1,$$

where $t$ is the specific tax rate and $i$ indexes the private firms and the public firm. On the other hand, the public firm's objective, $W$, is to maximize welfare, which is defined as the sum of the consumer surplus, the profits of individual firms, and the utilities of unions less the tax revenues. Thus, the public firm aims to maximize social welfare, which is defined as

$$\begin{aligned} SW &= U - \sum_{i=0}^{1} px_i + (\pi_i + u_i) \qquad (1) \\ &= U - T, \end{aligned}$$

where $U - \sum_{i=0}^{1} px_i$ represents the consumer surplus, $T = t(x_0 + x_1)$ denotes the tax revenues, $\pi_i$ is the profit of firm $i$, and $u_i$ is the utility of union $i$[6]. Utilities of unions are included as the part of producer surplus, which is usual in literature. For example, see Barcena-Ruiz and Garzon (2009), and references therein.

In the manner of Kato (2008), we also assume that the government's payoff is given by

$$G = SW + (1 + \alpha)T, \qquad (2)$$

[5] The papers that are closest to our representation of the unions' utilities are Naylor (1998, 1999), Haucap and Wey (2004), Leahy and Montagna (2000), and Lommerud *et al.* (2003). As they suggest, the monopoly union sets the wages but the firm unilaterally decides the level of employment. This is because the wage claims are decided by the elasticity of labor demand rather than the firm's profit. See also Oswald and Turnbull (1985). De Fraja (1993b) also adopted this kind of unions' utilities.

[6] A similar framework is represented by De Fraja (1991), which is assumed that the public firm only cares about the sum of the consumer and producer surpluses. However, De Fraja (1991) assumed that the tax is levied on the quantity of goods by the government in order to finance the public firm's budget losses. Thus, the government does not have preference for tax revenue that is obtained from the market.

where $\alpha$ is the parameter that represents the weight of the government's preference for tax revenues. As Kato (2008) suggested, if $\alpha = 0$, the government puts the same weight on $SW$ and $T$. In this case, the government is benevolent since the government's payoff represents social welfare[7]. Here, $\alpha > 0$, i.e., the government values the tax revenues, $T$, more than the social welfare, $SW$.

Finally, a three-stage game is conducted. The timing of the game is as follows. In the first period, the government sets the specific tax rate. In the second period, either firms or the government or unions simultaneously decide whether to negotiate over wages in either period 1 or period 2. Note that decision of timing of wage setting could be taken in each case by the firms, by the private firm and the government, by the union or a firm (or the government) and its union in lieu of being chosen by firms or the government or unions to decide the production quantities. If the periods of negotiation happen to be identical, the wage-setting process is simultaneous, in which case a Cournot-type game occurs; otherwise, the wage-setting process is sequential. In the third period, firms choose their quantity simultaneously with its counterparts to maximize its respective objective, knowing each union's choice of the wage level.

# 3 Results

Before analyzing the government's payoff and the social welfare, we first consider the respective maximization problems of the public firm, private firm and the government. In this paper, since we focus on a symmetric Nash equilibrium, we assume that all private firms choose the same type of bargaining. Thus, the game is solved by backward induction, i.e., the solution concept used is the subgame perfect Nash equilibrium.

## 3.1 Quantity Competition in a Unionized Mixed Duopoly

In this case, the public firm's objective is to maximize the social welfare, which is defined as the sum of the consumer surplus, individual firms' profits, and unions' utilities less the tax revenues. Thus, given $t$ and given $w_i$ for each firm $i$ $(i = 0, 1)$, the public firm's maximization problem is as:

$$\max_{x_0} \quad SW = U - T \quad \text{s.t.} \quad (p - w_0 - t)x_0 \geq 0. \tag{3}$$

[7] If the public firm cares not only about the sum of consumer and producer surpluses but also about the tax revenues, $T$ is canceled out in equation (1) as in Mujundar and Pal (1988). Under this setting, even if the government puts a larger weight on the tax revenues than on the sum of both surpluses, it never privatizes the public firm. If $\alpha = 0$, the government puts the same weight on $SW$ and $T$. In this case, the government is benevolent since the government's payoff represents social welfare. Without tax revenues and $\alpha = 0$, the detailed computations are available from author upon request; The Appendix B will not be included in the main paper since the inference can be easily verified by putting $\alpha = t = 0$.

As in Ishida and Matsushima (2009), the constraint implies there is some lower-bound restriction on the public firm's profit, i.e., the public firm faces a budget constraint[8].

If the multiplier of the budget constraint is denoted as $\lambda$, the Lagrangian equation can be written as

$$L(x_0, \lambda) = x_1 + x_0 - \frac{(x_1 + x_0)^2}{2} - tx_0 - tx_1 + \lambda(x_0 - x_0^2 - x_1 x_0 - w_0 x_0 - tx_0). \quad (4)$$

Given the specific tax rate, $t$, and the wage-levels, $w_i$, by solving the first-order conditions (4), we obtain

$$\frac{\partial L}{\partial x_0} = 1 - x_1 - x_0 - t + \lambda(1 - 2x_0 - x_1 - w_0 - t) = 0, \quad (5)$$

$$\frac{\partial L}{\partial \lambda} = 1 - x_1 - x_0 - w_0 - t = 0. \quad (6)$$

On the other hand, the optimal output for a private firm is given by

$$\frac{\partial \pi_1}{\partial x_1} = 0 \Leftrightarrow x_1 = \frac{1}{2}(1 - x_0 - w_1 - t). \quad (7)$$

Given these results, we now obtain the output level for each firm. By solving the first-order conditions, (6) and (7), we obtain,

$$x_0 = 1 - t - 2w_0 + w_1, \quad (8)$$

$$x_1 = w_0 - w_1, \quad (9)$$

$$\lambda = \frac{x_1 + x_0 + t - 1}{1 - 2x_0 - x_1 - w_0 - t}. \quad (10)$$

For solving the first-order conditions of the Lagrangian equation, the budget constraint is momentarily treated as binding. We check ex-post whether this omitted constraint is binding.

### 3.2 Wage Setting in a Unionized Mixed Duopoly

**[Simultaneous Wage Setting]**: In the second stage of this case, each wage is set to maximize its firm's union utility: $u_i = x_i w_i$. To do this, the two independent maximization problems should be considered simultaneously. Using (8) and (9), the problem for union $i$ is defined as

$$\max_{w_0} \ u_0 = w_0 x_0 = (1 - t - 2w_0 + w_1)w_0,$$

$$\max_{w_1} \ u_1 = w_1 x_1 = (w_0 - w_1)w_1,$$

respectively. Straightforward computation yields each firm's reaction function as follows:

$$w_0 = \frac{1 - t + w_1}{4}, \quad w_1 = \frac{w_0}{2}. \quad (11)$$

[8]In this model, if the public firm's union does not face the budget constraint with a simple Stone-Geary utility function $u_i = (w_i - \overline{w})^\theta x_i$, the public firm's union can indefinitely raise its wage because the optimal output level of the public firm is independent of the wage.

Then, the equilibrium wages, which are denoted as $w_i^c, i = 0, 1$ are obtained by solving (11); the substitution of each equation in (11) into (8) and (9) yields the respective equilibrium outputs, $x_i^c$. The equilibrium wages and outputs, $w_i^c$ and $x_i^c$, respectively, can be obtained as:

$$w_0^c = \frac{2(1-t)}{7}, \quad w_1^c = \frac{1-t}{7}; \tag{12}$$

$$x_0^c = \frac{4(1-t)}{7}, \quad x_1^c = \frac{1-t}{7}. \tag{13}$$

We now move to the first stage of the game. From (12) and (13), the government's payoff, $G^c$, in the mixed duopoly can be rewritten as:

$$\max_t \ G^c = \frac{5(1-t)[14(1+\alpha t) - 5(1-t)]}{98}.$$

Straightforward computation yields the optimal tax rate as:

$$t^c = \frac{7\alpha - 2}{5 + 14\alpha}. \tag{14}$$

If the weight of the government's preference for tax revenues is sufficiently large in the case of $\alpha > \frac{2}{7}$, the optimal tax rate becomes positive. Conversely, when it is small in the case of $0 < \alpha < \frac{2}{7}$, the optimal tax rate becomes negative, and in the case of $\alpha = \frac{2}{7}$, the optimal tax rate is zero. We find that the greater is the weight of the government's preference for tax revenues, the higher is the tax rate that the government imposes. Thus, by using (14), we have the following result.

**Lemma 1**: *Suppose that each firm's union is allowed to bargain collectively. Then, the equilibrium wages, output and union's utility levels under a unionized mixed duopoly are given by*

$$w_0^c = \frac{2(1+\alpha)}{5+14\alpha}, \quad w_1^c = \frac{1+\alpha}{5+14\alpha};$$
$$x_0^c = \frac{4(1+\alpha)}{5+14\alpha}, \quad x_1^c = \frac{1+\alpha}{5+14\alpha};$$
$$u_0^c = \frac{8(1+\alpha)^2}{(5+14\alpha)^2}; \quad u_1^c = \frac{(1+\alpha)^2}{(5+14\alpha)^2}.$$

By substituting Lemma 1 into (10), we obtain

$$\lambda = \frac{1}{2} > 0,$$

which shows that the budget constraint is binding. Using Lemma 1 and noting that $G^c = SW^c + (1+\alpha)T^c$, $\pi_1^c$ and $SW^c = U^c - T^c$, we can compute the government's payoff, $G^c$, and the social welfare, $SW^c$, private firm's profit, $\pi_1^c$ as:

$$G^c = \frac{5(1+\alpha)^2}{2(5+14\alpha)}, \quad SW^c = \frac{45(1+\alpha)^2}{2(5+14\alpha)^2}, \quad \pi_1^c = \frac{(1+\alpha)^2}{(5+14\alpha)^2}.$$

[**Sequential Wage Setting: Public Firm's Leader**]: In this case, we discuss that the public firm or its union acts as the leader regarding wage setting. Public firm's union will choose to maximize its utility taking as given the private firm's wage $w_1$ set by private firm's union 1. By solving the first-order condition for private firm's union 1, we have already obtained the best response function to be represent as: $w_1 = \frac{w_0}{2}$. Thus, the problem for public firm's union 0 is defined as

$$\max_{w_0}\ u_0 = w_0 x_0 = \frac{w_0(2 - 2t - 2w_0 + w_0)}{2}.$$

By solving the first-order condition for the public firm's union 0, we have the following result when the rival firms takes wage as given, superscript $l$ stands for the leader and $f$ for the follower[9];

$$w_0^l = \frac{1-t}{3}, \quad w_1^f = \frac{1-t}{6}; \tag{15}$$

$$x_0^l = \frac{1-t}{2}, \quad x_1^f = \frac{1-t}{6}. \tag{16}$$

We now move to the first stage of the game. From (15) and (16), the government's payoff, $G^l$, in the unionized mixed duopoly can be rewritten as:

$$\max_{t}\ G^l = \frac{(1-t)(6+6\alpha t) - 2(1-t)^2}{9}.$$

Straightforward computation yields the optimal tax rate as:

$$t^l = \frac{3\alpha - 1}{2(1+3\alpha)}. \tag{17}$$

As shown in (14), if the weight of the government's preference for tax revenues is sufficiently large in the case of $\alpha > \frac{1}{3}$, the optimal tax rate becomes positive. Conversely, when it is small in the case of $0 < \alpha < \frac{1}{3}$, the optimal tax rate becomes negative, and in the case of $\alpha = \frac{1}{3}$, the optimal tax rate is zero. Thus, by using (17), we have the following result.

**Lemma 2**: *Suppose that each firm's union is allowed to bargain collectively. Then, the equilibrium wages, output and union's utility levels under a unionized mixed duopoly are given by*

$$w_0^l = \frac{1+\alpha}{2(1+3\alpha)}, \quad w_1^f = \frac{1+\alpha}{4(1+3\alpha)};$$

$$x_0^l = \frac{3(1+\alpha)}{4(1+3\alpha)}, \quad x_1^f = \frac{1+\alpha}{4(1+3\alpha)};$$

$$u_0^l = \frac{3(1+\alpha)^2}{8(1+3\alpha)^2}, \quad u_1^f = \frac{(1+\alpha)^2}{16(1+3\alpha)^2}.$$

[9] The superscripts in which wages are bargained first in the private firm are symmetric.

By substituting Lemma 2 into (10), we obtain

$$\lambda = \frac{3}{2} > 0,$$

which shows that the budget constraint is binding. Using Lemma 2 and noting that $G^l = SW^l + (1+\alpha)T^l$, we can compute the government's payoff, $G^l$, the private firm's profit, $\pi_1^f$ and the social welfare, $W^l$, as:

$$G^l = \frac{(1+\alpha)^2}{2(1+3\alpha)}, \quad SW^l = \frac{(1+\alpha)^2}{(1+3\alpha)^2}, \quad \pi_1^f = \frac{(1+\alpha)^2}{16(1+3\alpha)^2}. \tag{18}$$

[**Sequential Wage Setting: Private Firm's Leader**]: Similar to the previous sequential wage setting of public firm's leader, we can directly compute each equilibrium value $w_i^m$, $x_i^m$, $p_i^m$, and $u_i^m$ where $m = l, f; i = 0, 1$ when the private firm or its union acts as leader.

By solving the first-order condition for private firm's union 1, we have already obtained the best response function to be represent as: $w_0 = \frac{1+w_1-t}{4}$. Thus, the problem for private firm's union is defined as

$$\max_{w_1}\ u_1 = w_1 x_1 = \frac{w_1 - tw_1 - 3w_1^2}{4}.$$

By solving the first-order condition for the private firm's union, we have the following result.

$$w_0^f = \frac{7(1-t)}{24}, \quad w_1^l = \frac{1-t}{6}; \tag{19}$$

$$x_0^f = \frac{7(1-t)}{12}, \quad x_1^l = \frac{1-t}{8}. \tag{20}$$

We now move to the first stage of the game. From (19) and (20), the government's payoff, $G^f$, in the unionized mixed duopoly can be rewritten as:

$$\max_t\ G^f = \frac{17(1-t)[48(1+\alpha t) - 17(1-t)]}{1152}.$$

Straightforward computation yields the optimal tax rate as:

$$t^f = \frac{24\alpha - 7}{17 + 48\alpha}. \tag{21}$$

Similar to previous cases, we find that the greater is the weight of the government's preference for tax revenues (i.e., $\alpha > \frac{7}{24}$), the higher is the tax rate that the government imposes. Thus, by using (21), we have the following result.

**Lemma 3**: *Suppose that goods are substitutes and the private firm or its union acts as a leader when each firm's union is allowed to engage in decentralized bargaining. Then, the equilibrium wage, output, and union's utility levels are given by*

$$w_0^f = \frac{7(1+\alpha)}{17+48\alpha}, \quad w_1^l = \frac{4(1+\alpha)}{17+48\alpha};$$

$$x_0^f = \frac{3(1+\alpha)}{17+48\alpha}, \quad x_1^l = \frac{14(1+\alpha)}{17+48\alpha};$$

$$u_0^f = \frac{21(1+\alpha)^2}{(17+48\alpha)^2}, \quad u_1^l = \frac{56(1+\alpha)^2}{(17+48\alpha)^2}.$$

By substituting Lemma 3 into (10), we obtain

$$\lambda = \frac{7}{3} > 0,$$

which shows that the budget constraint is binding. Using Lemma 1, we can compute the government's payoff, $G^f$, and the social welfare, $SW^f$, and private firm's profit, $\pi_1^l$ as:

$$G^f = \frac{17(1+\alpha)^2}{2(17+48\alpha)}, \quad SW^f = \frac{527(1+\alpha)^2}{2(17+48\alpha)^2}, \quad \pi_1^l = \frac{42(1+\alpha)^2}{(17+48\alpha)^2}. \tag{22}$$

### 3.3 Quantity Competition in a Unionized Privatized Duopoly

The previous subsection examined the impact of a unionized mixed duopoly in the case of bargaining. This subsection compares the equilibrium of a unionized mixed duopoly with the equilibrium that would be established in the case of a unionized privatized duopoly under decentralized bargaining processes of unions. As discussed in the basic model, consider the situation of a unionized privatized duopoly for a homogeneous good that is supplied by firm ($k = 1, 2$), which is a profit-maximizing private firm.

In the third stage, given $w_k$ and $t$, the firm $k$'s profit-maximization problem is to maximize $\pi_k = (p - w_k - t)x_k$ where $p = 1 - x_1 - x_2$. Hence, the first-order condition for maximizing $\pi_k$ is

$$x_k = \frac{1 - w_k - x_l - t}{2},$$

when there are two private firms. The symmetry across private firms implies that each output level is given by

$$x_k = \frac{1 - t - 2w_k + w_l}{3}, \quad k \neq l. \tag{23}$$

### 3.4 Wage Setting in a Unionized Mixed Duopoly

[**Simultaneous Wage Setting**]: In the second stage of this case, each wage is set to maximize its firm's union utility: $u_k = x_k w_k$. To do this, the two independent maximization problems should be considered simultaneously. Thus, the problem for union $k$ is defined as

$$\max_{w_k} \ u_k = w_k x_k = \frac{w_k(1 - t - 2w_k + w_l)}{3}.$$

Straightforward computations and symmetry across private firms yield each firm's wage through

$$w_k = \frac{1 - t + w_l}{4}, \quad k \neq l. \tag{24}$$

Therefore, an equilibrium wage for firm $k$, denoted as $w_k^C$, is obtained by solving (24). The substitution of each equation in (24) into (23) yields the equilibrium outputs $x_k^C$. Thus, we have the following result:

$$w_k^C = \frac{1-t}{3}, \quad x_k^C = \frac{2(1-t)}{9}. \tag{25}$$

Turning to the first stage and using the equilibrium outputs and wages, the government's payoff, $G^C$, in a unionized privatized duopoly can be rewritten as:

$$\max_{t}\ G^C = \frac{4(1-t)[9(1+\alpha t)-2(1-t)]}{81}.$$

Straightforward computation yields the optimal tax rate in the unionized privatized duopoly as:

$$t^C = \frac{9\alpha - 5}{2(2+9\alpha)}. \tag{26}$$

If the weight of the government preference for the tax revenues is sufficiently large (in the case when $\alpha > \frac{5}{9}$), the optimal tax rate becomes positive. Conversely, when it is small (in the case when $\alpha < \frac{5}{9}$), the optimal tax rate becomes negative. Further in the case when $\alpha = \frac{5}{9}$, the optimal tax rate is zero. As in the previous analysis, we also find that the greater is the weight of the government's preference for tax revenues, the higher is the tax rate that the government imposes. Similar to the previous subsection, we have the following result.

**Lemma 4**: *Suppose that all the private firms' unions are allowed to bargain collectively. Then, the equilibrium wages, output and union's utility levels under a unionized privatized duopoly are given by*

$$w_k^C = \frac{3(1+\alpha)}{2(2+9\alpha)}, \quad x_k^C = \frac{1+\alpha}{2+9\alpha}, \quad u_k^C = \frac{3(1+\alpha)^2}{2(2+9\alpha)^2}.$$

Similar to the previous subsection, we can compute the government's payoff, $G^C$,the social welfare, $SW^C$ and each private firm's profit, $\pi_k^C$ as;

$$G^C = \frac{(1+\alpha)^2}{2+9\alpha}, \quad SW^C = \frac{7(1+\alpha)^2}{(2+9\alpha)^2}, \quad \pi_k^C = 0. \tag{27}$$

[**Sequential Wage Setting: Private Firm $k$'s Leader**]: In this case, we discuss that the private firm $k$ or its union acts as the leader regarding wage setting. To distinguish notations, let the superscript $L$ (respectively, $F$) denote the equilibrium value in the case of leadership (respectively, followership) wage setting that the rival firm takes as given. Private firm's union $k$ will choose to maximize its utility taking as given the private firm's wage $w_l$ set by private firm's union $l$. By solving the first-order condition for private firm's union $l$, we have already obtained the best response function to be represent as: $w_l = \frac{1-t+w_k}{4}$. Thus, the problem for private firm's union $k$ is defined as

$$\max_{w_k}\ u_k = w_k x_k = \frac{w_k(1-t-2w_k+w_l)}{12}.$$

By solving the first-order condition for the private firm $k$'s union, we have the following result;

$$w_k^L = \frac{5(1-t)}{14}, \quad w_l^F = \frac{19(1-t)}{56}; \tag{28}$$

$$x_k^L = \frac{35(1-t)}{168}, \quad x_l^F = \frac{38(1-t)}{168}. \tag{29}$$

We now move to the first stage of the game. From (28) and (29), the government's payoff, $G^L = G^F$, in the unionized privatized duopoly can be rewritten as:

$$\max_t \ G^L = \frac{24528(1-t)(1+\alpha t) - 5329(1-t)^2}{56448}.$$

Straightforward computation yields the optimal tax rate as:

$$t^L = \frac{168\alpha - 95}{73 + 336\alpha}. \tag{30}$$

We find that the greater is the weight of the government's preference for tax revenues (i.e., $\alpha > \frac{95}{168}$), the higher is the tax rate that the government imposes. Thus, by using (30), we have the following result.

**Lemma 5**: *Suppose that each firm's union is allowed to bargain collectively. Then, the equilibrium wages, output and union's utility levels under a unionized privatized duopoly are given by*

$$w_k^L = \frac{60(1+\alpha)}{73+336\alpha}, \quad w_k^F = \frac{57(1+\alpha)}{73+336\alpha};$$
$$x_k^L = \frac{35(1+\alpha)}{73+336\alpha}, \quad x_k^F = \frac{38(1+\alpha)}{73+336\alpha};$$
$$u_k^L = \frac{2100(1+\alpha)^2}{(73+336\alpha)^2}, \quad u_k^F = \frac{2166(1+\alpha)^2}{(73+336\alpha)^2}.$$

Using Lemma 5, we can compute the government's payoff, $G^L = G^F$, and the social welfare, $SW^L = SW^F$, and private firm's profit $\pi_k^n; n = F, L$ as:

$$G^L = G^F = \frac{73(1+\alpha)^2}{2(73+336\alpha)}, \quad SW^L = SW^F = \frac{19199(1+\alpha)^2}{2(73+336\alpha)^2}, \quad \pi_k^L = \pi_k^F = 0. \tag{31}$$

# 4 Choice of Wage Setting Timing, Government's Payoff and Social Welfare

## 4.1 Timing of Endogenous Wage Setting

Having derived the equilibrium for three fixed-timing games in the previous section and using the same notation for the timings as before, we will find the Nash equilibrium in the second stage for any given utilities of the unions and the profits of firms under both the mixed and the privatized duopolies[10]. For convenient expression, we call both markets when we do not distinguish the unionized mixed duopoly from the unionized privatized duopolies.

[10] If the private and public firms, unions, government announce in which period they will choose their timing of wage setting, given the specific-tax rate, each player cannot choose its own timing, since depending on each tax rate, the public and private firms' profits, utilities of unions and government's payoff are varied with either Cournot or Stackelberg game. This is why we introduce the fixed timing into our theoretical framework. For more exposition of the backward and forward induction in a simpler setup, see Kreps (1990, pp. 108-110, pp. 174-177).

Let "F" and "S" represent first period and second period with regard to timing choice of wage setting respectively. When agents (the firms or the unions) have chosen "F" or "S", they will play a Cournot-type game of the wage setting in the first stage; when the public firm's agent has chosen "F" while the private firm's agent has chosen "S", a public-leader Stackelberg-type game of the wage setting arises in the second stage; when the private firm's agent has chosen "S" while the public firm's agent has chosen "F", a private-leader Stackelberg-type game of the wage setting arises in the second stage (same notations will be adopted when the unionized privatized duopoly is introduced).

From Lemma 1 to Lemma 5, the reduced endogenous-timing game among unions can be represented by the following payoff Table 1(a) and Table 1(b).

**Table 1: Timing of Wage Setting Among Unions**

Union 1

| | | F | S |
|---|---|---|---|
| Union 0 | F | $u_0^c, u_1^c$ | $u_0^l, u_1^f$ |
| | S | $u_0^f, u_1^l$ | $u_0^c, u_1^c$ |

(a) Unionized Mixed Duopoly

Union 2

| | | F | S |
|---|---|---|---|
| Union 1 | F | $u_1^C, u_2^C$ | $u_1^L, u_2^F$ |
| | S | $u_1^F, u_2^L$ | $u_1^C, u_2^C$ |

(b) Unionized Privatized Duopoly

To find the subgame-perfect Nash equilibrium, we need to compare utilities of unions. All calculations are in the Appendix. Straightforward computations show in both Table 1(a) and (b) that

$$u_0^l > u_0^c > u_0^f, \quad u_1^l > u_1^f > u_1^c, \quad u_k^F > u_k^L > u_k^C; k = 1, 2.$$

These inequalities tell us that regardless of the government's preference for tax revenues, the union of public firm prefers to be leader in bargaining over wages rather than to be follower, while the union of private firm prefers to play sequentially rather than to play simultaneously. Thus, there can be sustained a unique (respectively, multiple) subgame perfect Nash equilibrium (respectively, equilibria) in game of wage setting when the market is the unionized mixed (respectively, privatized) duopoly. Thus, we have the following proposition:

**Proposition 1**: *Suppose that the decision of timing of wage setting is delegated to the unions under both markets. Then, there can be sustained a unique (respectively, multiple) timing of endogenous wage setting when the market is the unionized mixed (respectively, privatized) duopoly:*

*the order(s) is (respectively, are)* $\{F,\ S\}$ *(respectively,* $\{S,\ F\}$, $\{F,\ S\}$*).*

The intuition in the case of the unionized mixed duopoly behind the proposition is as follows. Regardless of the government's preference for raising tax revenues, the fact remains that negotiating wage in the sequential case is the strictly dominant strategy of all unions and plays an important role in the derivation of the result. Since each union independently decides on the timing of wage settings, being a leader is clearly always better than being follower under the unionized mixed duopoly. The leader union of the public firm under the unionized mixed duopoly gets higher wages (i.e., $w_0^l > w_0^f > w_0^c$), and the workers supplied by the leader union are more than those supplied by the follower union (i.e., $x_i^l > x_i^f > x_i^c$). On the other hand, although the workers in the follower union of the private firm supplied are less than those supplied by leader union of the private firm (i.e., $x_i^l > x_i^f > x_i^c$), the follower union of the private firm under the unionized mixed duopoly gets higher wages (i.e., $w_1^f > w_1^l > w_1^c$). This implies that both unions obtain greater utility under the sequential case than under the simultaneous case. Therefore, regardless of the government's preference for tax revenues, each union in the case of a unionized mixed duopoly prefer to decide the wage settings based on a sequential process[11].

Let us now consider the case of a unionized privatized duopoly. In this case, although the leader firm tends to employ fewer workers ($x_k^F > x_k^C > x_k^L$) in a sequential situation, the follower or leader union receives higher wages (i.e., $w_k^L > w_k^F > w_k^C$), and therefore, greater benefits are derived from a sequential situation. This also implies that regardless of the government's preference for tax revenues, both unions under a unionized privatized duopoly tend to acquire greater benefits under a simultaneous situation rather than under a sequential situation.

Similar to the reduced endogenous-timing game among unions, when the decision of timing of wage setting is determined by the government under the unionized mixed duopoly, the reduced endogenous-timing game between the private firm and the government under the unionized mixed duopoly and among private firms under the unionized privatized duopoly can be represented by the following payoff tables.

**Table 2: Timing of Wage Setting between Private Firm and Government or Among Private Firms**

Private Firm 1

| | | $F$ | $S$ |
|---|---|---|---|
| Government | $F$ | $G^c, \pi_1^c$ | $G^l, \pi_1^f$ |
| | $S$ | $G^f, \pi_1^l$ | $G^c, \pi_1^c$ |

[11] As Barcena-Ruiz and Campo (2000) suggested, this result is due to the fact that wages are strategic complements. However, the union of private firm under the unionized mixed duopoly takes best response regarding the union of the public firm without effect of tax rate, they set higher wages in sequential than in simultaneous case. Hence, the public firm pays higher wages than the private firm.

(a) Unionized Mixed Duopoly

Private Firm 2

Private Firm 1

| | $F$ | $S$ |
|---|---|---|
| $F$ | $\pi_1^C, \pi_2^C$ | $\pi_1^L, \pi_2^F$ |
| $S$ | $\pi_1^F, \pi_2^L$ | $\pi_1^C, \pi_2^C$ |

(b) Unionized Privatized Duopoly

Straightforward computations[12] show that

$$\pi_1^l > \pi_1^f > \pi_1^c, \quad G^c > G^f > G^l, \quad \pi_k^C = \pi_k^F = \pi_k^L = 0; k = 1, 2.$$

These inequalities tell us that regardless of the government's preference for tax revenues, the government always prefers to play simultaneously rather than to play sequentially, while the private firm under the unionized mixed duopoly prefers to play sequentially rather than to play simultaneously. On the other hand, each private firm's profit under the unionized privatized duopoly is surprising. The profit profile in each cell of the table is exactly the same (i.e., zero). Thus, we have the following proposition:

**Proposition 2**: *Suppose that the decision regarding the timing of wage settings is not delegated to the unions under both the markets. Then, there cannot be a sustained subgame perfect Nash equilibrium under the unionized mixed duopoly, regardless of the government's preference for tax revenues. However, any timing can be sustained as an equilibrium under the unionized privatized duopoly, regardless of the government's preference for tax revenues.*

Proposition 2 suggests that the leader private firm obtains a higher profit and produces more output from the viewpoint of the private firm and Proposition 1, which means that $\pi_1^l > \pi_1^c$. However, although the government obtains less output in the simultaneous case, it obtains a higher tax rate (i.e., $t^c > t^f > t^l$)[13]; thus, the government obtains a greater payoff in the simultaneous case than in the sequential case, which means that $G^c > G^f > G^l$. Given the private firm's profit, there can not be sustained subgame perfect equilibrium under the unionized mixed duopoly regardless of the government's preference for tax revenues. Second, we find that any timing is possible in a unionized privatized duopoly when the government has a preference for tax revenues, i.e., the government weighs more toward tax revenues than toward social welfare. This is because each firm's revenue becomes zero due to the tax rate regardless of the timing of endogenous wage settings. Hence, any timing is possible. Consequently, Proposition 2 is in contrast to one of the findings in the pure duopoly literature that the owners of firms prefer simultaneous bargaining.

[12] All calculations are in the Appendix.

[13] $t^c > t^l \Leftrightarrow 1 + \alpha > 0, t^c > t^f \Leftrightarrow 1 + \alpha > 0$ and $t^f > t^l \Leftrightarrow 1 + \alpha > 0$.

Alternatively, when the decision of timing of wage setting is determined by the public firm under the unionized mixed duopoly, the reduced endogenous-timing game between the private and public firms under the unionized mixed duopoly can be represented by the following payoff tables.

**Table 3: Timing of Wage Setting Between Private and Public Firms**

Private Firm 1

| | | $F$ | $S$ |
|---|---|---|---|
| Public Firm 0 | $F$ | $SW^c, \pi_1^c$ | $SW^l, \pi_1^f$ |
| | $S$ | $SW^f, \pi_1^l$ | $SW^c, \pi_1^c$ |

From Table 3, comparing social welfare yields that[14]

$$SW^f < SW^c \Leftrightarrow -194\alpha^2 + 170\alpha + 85 < 0, \quad \text{if} \quad \alpha > \hat{\alpha} \fallingdotseq 1.232; \quad \text{otherwise}, \quad SW^f > SW^c.$$
$$SW^l < SW^c \Leftrightarrow -13\alpha^2 + 10\alpha + 5 < 0, \quad \text{if} \quad \alpha > \tilde{\alpha} \fallingdotseq 1.114; \quad \text{otherwise}, \quad SW^l > SW^c.$$

which show that if $0 < \alpha < 1.114$ (respectively, $\alpha > 1.232$), the public firm prefers to play sequentially (respectively, simultaneously) rather than to play simultaneously (respectively, sequentially), while if $1.114 < \alpha < 1.232$, the public firm has a dominant strategy to play in the second opportunity. Given that $\pi_1^l > \pi_1^f > \pi_1^c$, we have the following proposition.

**Proposition 3**: *Suppose that the decision of timing of wage settings is not delegated to the unions under both markets. Then, there are two possible timings for endogenous wage setting depending on the value of* $\alpha$*. If* $0 < \alpha < 1.114$*, the order is either* $\{F, S\}$ *or* $\{S, F\}$*; if* $1.114 < \alpha < 1.232$*, the order is* $\{S, F\}$*; if* $\alpha > 1.232$*, there can be no sustained subgame perfect Nash equilibrium under the unionized mixed duopoly.*

The fact that the public and private firms prefer sequential wage setting if the government's preference for tax revenues is sufficiently small plays an important role in the derivation of this result. In our setting, there are two types of sequential-move equilibria that are always found in the case of endogenous timing in a unionized mixed duopoly if $0 < \alpha < 1.232$, whereas there can be no sustained subgame perfect Nash equilibrium if the government's preference for tax revenues is sufficiently large (i.e., $\alpha > 1.232$). Proposition 3 suggests that the differences in the implementation of leadership depend on the structure of political power with regard to the public firm and the government. In other words, the public firm has an incentive to use the sequential bargaining case when the preference for tax revenue is sufficiently small. There is, however,

[14] When we compare social welfare, by applying each equation to a discriminant and solving for the roots of this equation, we obtain the condition. A negative solution for $\alpha$ is excluded by the assumption that $\alpha > 0$.

congruity for the payoff between the public firm and the government when the preference for tax revenues is sufficiently large.

Similar to Proposition 2, Proposition 3 is in contrast to one of the findings in the pure duopoly literature that owners of firms prefer simultaneous bargaining. In our setting, besides a sequential-move equilibrium that is always found in the case of endogenous wage setting in a unionized mixed duopoly, we find no equilibrium if the government's preference for tax revenues is sufficiently large.

Given Proposition 1, 2 and 3, we obtain the following result[15].

**Proposition 4**: *Bargaining over wages is always sequential regardless of who decides the timing of the endogenous wage setting, except in the case where there cannot be sustained subgame perfect Nash equilibrium or where any timing can be sustained as an equilibrium.*

**Proof**: See Appendix A. ■

Proposition 4 is in contrast to one of the standard findings in both spatial and nonspatial competition literatures that two private firms possess an incentive to set wages simultaneously that can be sustained as an equilibrium outcome (see De Fraja, 1993a; Corneo, 1995; Barcena-ruiz and Casado-Izaga, 2008).

In our setting, since the choice of timing of the public firm is varied with the government's preference for tax revenues and there is an opposite preference between the government and the private firm, all players prefer to set wages sequentially even though the wages are strategic complements under both markets. It therefore does not matter whether the timing of wage settings is determined by which pair of players[16].

### 4.2 Comparison of the Government's Payoff and Social Welfare

Given the timing of each endogenous wage setting, it is instructive to compare both the social welfare and government's payoff in the unionized mixed duopoly with the unionized privatized duopoly.

From Proposition 1, 2 and 3, regardless of who decide endogenous wage negotiation, the government's payoff is determined by either $G^l$ or $G^f$ (respectively, $G^L$ or $G^F$) under unionized mixed (respectively, privatized) duopoly. However, if the public firm choose to decide timing of wage

[15] If $\alpha = 0$, the government puts the same weight on $SW$ and $T$. In this case, the government is benevolent since the government's payoff represents social welfare. Without tax revenues and $\alpha = 0$, the detailed computations are available from author upon request; The Appendix B will not be included in the main paper since the inference can be easily verified by putting $\alpha = t = 0$.

[16] However, Barcena-ruiz and Casado-Izaga (2008) obtained the result that bargaining over wages is simultaneous if and only if two private firms decide the timing of the wage setting, otherwise the negotiation takes place sequentially. Our result differs from their timing of endogenous wage setting due to the fact that there exists the government's preference for tax revenues.

setting, the social welfare is determined by either $SW^l$ or $SW^f$ (respectively, $SW^L = SW^F$) under unionized mixed (respectively, privatized) duopoly given $G^l$ or $G^f$ (respectively, $G^L = G^F$) under each market. Therefore, we immediately have the following proposition.

**Proposition 5**: *Suppose that the government has a preference for tax revenues. Then, each level of government's payoff is determined by*

$$G^c > G^f > G^l > G^C > G^F = G^L,$$

*and each level of social welfare is determined by*

$$\begin{array}{ll}
SW^L = SW^F > SW^C > SW^l > SW^f > SW^c & \textit{if} \quad 0 < \alpha < 0.392. \\
SW^C > SW^L = SW^F > SW^l > SW^f > SW^c & \textit{if} \quad 0.392 < \alpha < 0.594. \\
SW^C > SW^l > SW^L = SW^F > SW^f > SW^c & \textit{if} \quad 0.594 < \alpha < 0.608. \\
SW^l > SW^C > SW^L = SW^F > SW^f > SW^c & \textit{if} \quad 0.608 < \alpha < 0.640. \\
SW^l > SW^C > SW^f > SW^L = SW^F > SW^c & \textit{if} \quad 0.640 < \alpha < 0.647. \\
SW^l > SW^C > SW^f > SW^c > SW^F = SW^L & \textit{if} \quad 0.647 < \alpha < 0.655. \\
SW^l > SW^f > SW^C > SW^c > SW^F = SW^L & \textit{if} \quad 0.655 < \alpha < 0.662. \\
SW^l > SW^f > SW^c > SW^C > SW^F = SW^L & \textit{if} \quad 0.662 < \alpha < 1.099. \\
SW^f > SW^l > SW^c > SW^C > SW^F = SW^L & \textit{if} \quad 1.099 < \alpha < 1.114. \\
SW^f > SW^c > SW^l > SW^C > SW^F = SW^L & \textit{if} \quad 1.114 < \alpha < 1.232. \\
SW^c > SW^f > SW^l > SW^C > SW^F = SW^L & \textit{if} \quad \alpha > 1.232
\end{array}$$

**Proof**: See Appendix A. ■

Proposition 5 suggests that by ignoring simultaneous cases from Proposition 4, the government does not have an incentive to privatize the public firm, while the public firm has an incentive to be privatized depending on the government's preference for tax revenues. If $\alpha$ is sufficiently large (i.e., $\alpha > 0.662$), the interest between the public firm and the government can be coincided. In this case, the privatization is harmful regardless of whether or not the wage setting is simultaneous.

However, if $\alpha$ is sufficiently small (i.e., $0 < \alpha < 0.594$), there can exist incongruence regarding privatization between the public firm and government because simultaneous wage setting cannot be sustained as an equilibrium. In other words, regardless of the government's preference for tax revenues, the government has an incentive to choose either the private leader-public follower or public leader-private follower game, while the public firm does have an incentive to choose privatization when the preference for tax revenues is sufficiently small. The conflict between these two views of objective functions typically induces a conflict with regard to the privatization.

Proposition 5 suggests that differences in the implementation of leadership depend on both the government's preference for tax revenues and who decides the timing of the endogenous wage setting. In other words, all players have incentives to use different leadership game under both markets since the level of social welfare and the government's payoff are obtained as any pair of social welfare and the government's payoff by which the timing of wage setting is established by any pair of players (See these cases at proof of Proposition 5 in Appendix).

On the other hand, given the union's utility, the consumer surplus $CS$ of each market in simultaneous and sequential wage setting cases are represented with same superscripts as Proposition 6:

**Proposition 6**: *Regardless of the government's preference for tax revenues, each level of the consumer surplus is determined by* $CS^c > CS^f > CS^l > CS^C > CS^L = CS^F$.

Proposition 6 suggests that regardless of what competition is introduced in the market, the consumer surplus can not be improved by implementing privatization when the government has a preference for tax revenues. In other words, privatization tends to make consumers worse off even though two private firms under the unionized privatized duopoly decide on the timing of wage setting sequentially. Compared to the social welfare and the government's payoff, Proposition 6 gives us the situation that is the best in terms of consumer surplus if and only if only both the public firm and the union of public firm-follower game take place as long as when bargaining over wages is sequential under the unionized mixed duopoly.

## 5 Concluding Remarks

In this paper, the timing of the endogenous wage settings in a mixed duopoly, with the acceptance of some conflicts of interest between the public firm and the government, has been analyzed, and this study therefore provides new insights into the timing of endogenous wage settings.

We have found that regardless of the government's preference for tax revenues and market type, bargaining over wages is always sequential except for the case where there cannot be a sustained equilibrium or where any timing possible as an equilibrium. These results differs from the standard findings of De Fraja (1993a), Corneo (1995), and Barcena-Ruiz and Casado-Izaga (2008), which showed that in a pure duopoly, unions prefer to play sequentially when wage bargaining is decentralized at the level of the private firm and vice versa. However, the result in the present paper indicates differences in the implementation of endogenous wage settings when the public firm decides to choose the timing of wage settings. Further, we have found that if the government's preference for tax revenues is sufficiently large, the privatization of the public firm is harmful in terms of both social welfare and government's payoff whether the wage setting is simultaneous or not. However, if the government's preference for tax revenues is sufficiently small, there can exist incongruence regarding privatization between the public firm and the government. This result may indicate that differences in the implementation of privatization depend on the political power structure between the public firm and the government.

Finally, we did not extend our results by considering a model where the public firm competes with $n$ private firms or both domestic and foreign private firms, wherein the government seeks to simultaneously maximize tax revenues and social welfare. Also, in this paper, we have used simplifying assumption that each firm's union is allowed to engage in decentralized bargaining and we have limited the policy analysis to privatization. For example, richer policies, such as an ad valorem tax and subsidization policies towards both domestic and international mixed oligopolies, are worth considering in the framework of timing of endogenous wage setting. The extension of our model in these directions remains an agendum for future research.

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# Appendix A: Proofs

### Proof of Proposition 1 and 2

(a) Comparison of each union's utility

$$
\begin{aligned}
u_k^L > u_k^C &\Leftrightarrow 813 + 4032\alpha + 1506\alpha^2 > 0,\\
u_k^F > u_k^C &\Leftrightarrow 1341 + 8784\alpha + 12198\alpha^2 > 0,\\
u_k^F > u_k^L &\Leftrightarrow 361 > 350,\\
u_0^c > u_0^f &\Leftrightarrow 1787 + 10116\alpha + 14316\alpha^2 > 0,\\
u_0^l > u_0^c &\Leftrightarrow 11 + 36\alpha + 12\alpha^2 > 0,\\
u_1^f > u_1^c &\Leftrightarrow 9 + 44\alpha + 52\alpha^2 > 0,\\
u_l^l > u_1^c &\Leftrightarrow 1111 + 6208\alpha + 8672\alpha^2 > 0,\\
u_l^l > u_1^f &\Leftrightarrow 607 + 3744\alpha + 5760\alpha^2 > 0.
\end{aligned}
$$

(b) Comparison of each firm's profit

$$
\begin{aligned}
\pi_1^l > \pi_1^c &\Leftrightarrow 761 + 4248\alpha + 5928\alpha^2 > 0,\\
\pi_1^f > \pi_1^c &\Leftrightarrow 9 + 44\alpha + 52\alpha^2 > 0,\\
\pi_1^l > \pi_1^f &\Leftrightarrow 383 + 2400\alpha + 3744\alpha^2 > 0.
\end{aligned}
$$

(c) Comparison of each output

$$
\begin{aligned}
&x_0^c > x_0^l \Leftrightarrow 1 + 6\alpha > 0, \quad x_0^c > x_0^f \Leftrightarrow 53 + 150\alpha > 0, \quad x_0^l > x_0^f \Leftrightarrow 13 + 36\alpha > 0,\\
&x_1^l > x_1^c \Leftrightarrow 53 + 1486\alpha > 0, \quad x_1^f > x_1^c \Leftrightarrow 1 + 2\alpha > 0, \quad x_1^l > x_1^f \Leftrightarrow 39 + 120\alpha > 0,\\
&x_k^F > x_k^C \Leftrightarrow 1 + 11\alpha > 0, \quad x_k^C > x_k^L \Leftrightarrow 3 + 21\alpha > 0.
\end{aligned}
$$

(d) Comparison of each wage level

$$
\begin{aligned}
&w_k^L > w_k^C \Leftrightarrow 7 + 24\alpha > 0, \quad w_k^F > w_k^C \Leftrightarrow 1 + 2\alpha > 0, \quad w_k^L > w_k^F \Leftrightarrow 3 > 0,\\
&w_0^f > w_0^c \Leftrightarrow 1 + 2\alpha > 0, \quad w_0^l > w_0^c \Leftrightarrow 1 + 2\alpha > 0 \quad w_0^l > w_0^f \Leftrightarrow 1 + 2\alpha > 0,\\
&w_1^l > w_1^c \Leftrightarrow 3 + 8\alpha > 0, \quad w_1^f > w_1^c \Leftrightarrow 1 + 2\alpha > 0, \quad w_1^f > w_1^l \Leftrightarrow 1 > 0,
\end{aligned}
$$

(e) Comparing each government's payoff under the unionized mixed duopoly with each consumer surplus under the unionized privatized duopoly, straightforward computations show that

$$
G^c > G^f \Leftrightarrow 240 > 238, \quad G^c > G^l \Leftrightarrow 15 > 14, \quad G^l > G^f \Leftrightarrow 102 > 96. \qquad \blacksquare
$$

**Proof of Proposition 4**

We provide eighteen cases where the different wage setting games can take place.

1 When the market is under the unionized mixed duopoly, the timing of wage setting is established only by the two unions, by the union of the public firm and the government (or public firm), by one union and the government (or public firm) or by all four possible players (the government (or public firm) the private firm and two unions). Note that when the decision of timing of wage setting is determined by the public firm, depending on the critical value of $\alpha$, the timing of wage setting is established by each case ($\alpha \in (0, 1.114), \alpha \in (1.114, 1.232), \alpha > 1.232$), while when the government decides to choose timing of endogenous wage setting, the timing of wage setting is established regardless of $\alpha$ as shown in the main text. Let $g$ (respectively, $m$) denote the case in where the decision of timing of wage setting is determined under the unionized mixed duopoly with government (respectively, public firm), and unions.

2 When the market is under the unionized privatized duopoly, the timing of wage setting is established only by the two unions, by the union of the private firm and only one the private firm, by one union and the private firm or by all four possible players (two private firms and two unions). Let $p$ denote the case in where the decision of timing of wage setting is determined under the unionized privatized duopoly by the private firm and unions.

[$g$-**1**]**:** Consider that government and the private firm decide the timing of wage setting. Then, the government prefers to play simultaneously rather than to play sequentially. Given this, the private firm to play sequentially rather than to play simultaneously. Thus, there cannot be sustained as an equilibrium.

[$g$-**2**]**:** Consider that government and the union of private firm decide the timing of wage setting. Then, the government prefers to play simultaneously rather than to play sequentially. Given this, the private firm to play sequentially rather than to play simultaneously. Thus, there cannot be sustained as an equilibrium.

[$g$-**3**]**:** Consider that the union of public firm and the union of private firm under the unionized mixed duopoly decide the timing of wage setting. In this case, the union of the public firm has a dominant strategy, which is to bargain in the first opportunity, and the union of private firm thus chooses to play to be a follower. The only equilibrium is that in which the union of the public firm acts as a leader and the union of private firm acts as a follower. [$g$-**4**]: The same result can be obtained if the union of public firm and the private firm under the unionized mixed duopoly decide the timing of wage setting.

[$g$-**5**]**:** Imagine that the game is played by all four possible players: the government the private firm and two unions under the unionized mixed duopoly. In this case, the union of the public firm has a dominant strategy, which is bargain to in the first opportunity. Given this, the union of the private firm always prefers to play sequentially. Thus, the sequential bargaining can be taken place between the union of the public firm and the union of the private firm. However, the government always prefers to play simultaneously rather than to play sequentially, which means that there cannot be sustained as an equilibrium among four players.

[$m$-**1**]**:** Consider that the public firm and the private firm decide the timing of wage setting. Then, the public firm prefers to play simultaneously rather than to play sequentially if the government's preference for tax revenue is in the range of $\alpha > 1.232$. Given this, the private firm to play sequentially rather than to play simultaneously. Thus, there can not be sustained as an equilibrium. [$m$-**2**]**:** The same result can be obtained if the union of public firm and the union of private firm under the unionized mixed duopoly decide the timing of wage setting if $\alpha > 1.232$.

[$m$-**3**]**:** Consider that the union of public firm and the union of private firm under the unionized mixed duopoly decide the timing of wage setting. In this case the union of the public firm has a dominant strategy, which is to bargain in the first opportunity, and the union of private firm thus chooses to play to be a follower. The only equilibrium is that in which the union of the public firm is the leader and the union of private firm acts as a follower. This is same as [$g$-**3**]. [$m$-**4**]: The same result can be obtained if the union of public firm and the private firm under the unionized mixed duopoly decide the timing of wage setting. This is same as [$g$-**4**].

[$m$-**5**]**:** Consider that the public firm and the private firm decide the timing of wage setting. Then, the public firm prefers to play sequentially rather than to play simultaneously if the government's preference for tax revenue is in the range of $0 < \alpha < 1.114$. Given this, the private firm to play sequentially rather than to play simultaneously. Given that the public and private firms prefer to play sequentially, since the social welfare and private firm's profit obtained by both firms is greater in that case, there are two equilibria: one firm acts as a leader and the other firm acts as a follower if the government's preference for tax revenue is in the range of $0 < \alpha < 1.114$. [$m$-**6**]**:** The same result can be obtained if the public firm and the union of private firm under the unionized mixed duopoly decide the timing of wage setting and the government's preference for tax revenue is in the range of $0 < \alpha < 1.114$.

[$m$-**7**]**:** Consider that the public firm and the private firm decide the timing of wage setting if he government's preference for tax revenue is in the range of $1.114 < \alpha < 1.232$. In this case, the public firm has a dominant strategy, which is to bargain in the second opportunity, and the private firm prefers to play sequentially rather than to play simultaneously. The only equilibrium is that in which the public firm is the follower and the private firm acts as a leader. [$m$-**8**]**:** The same result can be obtained if the public firm and the union of private firm under the unionized mixed duopoly decide the timing of wage setting if he government's preference for tax revenue is in the range of $1.114 < \alpha < 1.232$.

[$m$-**9**]**:** Imagine that the game is played by all four possible players: both firms and unions under the unionized mixed duopoly. The same result can be obtained if the public and private firms and their two unions under the unionized mixed duopoly decide the timing of wage setting if he government's preference for tax revenue is in the range of $\alpha > 1.232$. Thus, there cannot be sustained as an equilibrium among four players.

[$m$-**10**]**:** Imagine that the game is played by all four possible players: both firms and unions under the unionized privatized duopoly. If the government's preference is in $\alpha \in (0, 1.114)$, there is not conflict of interest among four players. The union of the public firm has a dominant strategy, which is to bargain in the first opportunity to avoid becoming a follower. Given this, the union of the private firm and the private firm prefer to play sequentially rather than to play simultaneously. Thus, only equilibrium is that in which the union of private firm, the public and private firms act as follower because they cannot push the union of the public firm to behave as a follower.

[$m$-**11**]**:** Imagine that the game is played by all four possible players: both firms and unions under the unionized privatized duopoly. Suppose that the government's preference is in $\alpha \in (1.114, 1.232)$. In this case, the public firm has a dominant strategy, which is to bargain in the second opportunity. Given this, two unions and the private firms prefer to play sequentially rather than to play simultaneously. The only equilibrium is that in which two unions and the private firm act as leader because they cannot push the public firm to behave as a leader.

[$p$-**1**]**:** Consider that both private firms under the unionized privatized duopoly decide the timing of wage setting. Then, the profits of both firm becomes zero whether they play to sequentially or not. Given this, any timing is possible as we provided by Proposition 3.

[$p$-**2**]**:** Consider that both unions under the unionized privatized duopoly decide the timing of wage setting. Given that both unions prefer to play sequentially, there are two equilibria: one union acts as a leader and the other union acts as a follower.

[$p$-**3**]**:** Consider that the union and the private firm under the unionized privatized duopoly decide the timing of wage setting. As shown above, the private firm is indifferent to choose the timing whether sequential case or not. Given this, the union prefers to play sequentially. In this case, there are two equilibria: one firm acts as a leader and the other union acts as a follower and vice versa.

[$p$-**4**]**:** Imagine that the game is played by all four possible players: both firms and unions under the unionized privatized duopoly. Since both unions under the unionized privatized duopoly prefer to play sequentially, each union does not have incentives to play simultaneously. Thus, only equilibrium is to set wage sequentially because neither firm can push its union to behave as a simultaneous union. ■

**Proof of Proposition 5**

Given the comparing each government's payoff in the proof of Proposition 1 and 2, straightforward computations show that

$$G^C > G^L = G^F \Leftrightarrow 672 > 657, \quad G^l > G^C \Leftrightarrow 9 > 6.$$

Comparison of social welfare: Note that $SW^L = SW^F$.

- $SW^L < SW^C \Leftrightarrow -25425\alpha^2 + 4380\alpha + 2190 < 0$ if $\alpha > 0.392 \fallingdotseq \alpha_1^c$; otherwise, $SW^L > SW^C$.
- $SW^L < SW^l \Leftrightarrow -53001\alpha^2 + 17082\alpha + 8541 < 0$ if $\alpha > 0.594 \fallingdotseq \alpha_2^c$; otherwise, $SW^L > SW^l$.
- $SW^C < SW^l \Leftrightarrow -6\alpha^2 + 2\alpha + 1 < 0$, if $\alpha > 0.608 \fallingdotseq \alpha_3^c$; otherwise, $SW^C > SW^l$.
- $SW^L < SW^f \Leftrightarrow -496928\alpha^2 + 171258\alpha + 85629 < 0$ if $\alpha > 0.640 \fallingdotseq \alpha_4^c$; otherwise, $SW^L > SW^f$.
- $SW^L < SW^c \Leftrightarrow -658658\alpha^2 + 240170\alpha + 120085 < 0$ if $\alpha > 0.647 \fallingdotseq \alpha_5^c$; otherwise, $SW^L > SW^c$.
- $SW^C < SW^f \Leftrightarrow -10431\alpha^2 + 3876\alpha + 1938 < 0$ if $\alpha > 0.655 \fallingdotseq \alpha_6^c$; otherwise, $SW^C > SW^f$.
- $SW^C < SW^c \Leftrightarrow -901\alpha^2 + 340\alpha + 170 < 0$ if $\alpha > 0.662 \fallingdotseq \alpha_7^c$; otherwise, $SW^C > SW^c$.
- $SW^l < SW^f \Leftrightarrow -135\alpha^2 + 102\alpha + 51 < 0$ if $\alpha > 1.099 \fallingdotseq \alpha_8^c$; otherwise, $SW^l > SW^f$.
- $SW^l < SW^c \Leftrightarrow -13\alpha^2 + 10\alpha + 5 < 0$ if $\alpha > 1.114 \fallingdotseq \alpha_9^c$; otherwise, $SW^l > SW^c$.
- $SW^f < SW^c \Leftrightarrow -194\alpha^2 + 170\alpha + 85 < 0$ if $\alpha > 1.232 \fallingdotseq \alpha_{10}^c$; otherwise, $SW^f > SW^c$.

Therefore, we get the relations as follows:

0   0.392   0.594   0.608   0.640

$SW^L = SW^F > SW^C > SW^l > SW^f > SW^c$

$SW^C > SW^L = SW^F > SW^l > SW^f > SW^c$

$SW^C > SW^l > SW^L = SW^F > SW^f > SW^c$

$SW^l > SW^C > SW^L = SW^F > SW^f > SW^c$

0.640   0.647   0.655   0.662   1.099

$SW^l > SW^C > SW^f > SW^L = SW^F > SW^c$

$SW^l > SW^C > SW^f > SW^c > SW^F = SW^L$

$SW^l > SW^f > SW^C > SW^c > SW^F = SW^L$

$SW^l > SW^f > SW^c > SW^C > SW^F = SW^L$

1.099   1.114   1.232

$SW^f > SW^l > SW^c > SW^C > SW^F = SW^L$

$SW^f > SW^c > SW^l > SW^C > SW^F = SW^L$

$SW^c > SW^f > SW^l > SW^C > SW^F = SW^L$ ■

**Proof of Proposition 6**

Using Lemma 1-5, we get the consumer surplus of each market in both simultaneous and sequential cases. These calculations are as follows:

$$CS^c = \frac{25(1+\alpha)^2}{2(5+14\alpha)^2}, \quad CS^l = \frac{(1+\alpha)^2}{2(1+3\alpha)^2}, \quad CS^f = \frac{289(1+\alpha)^2}{2(17+48\alpha)^2},$$
$$CS^C = \frac{2(1+\alpha)^2}{(2+9\alpha)^2}, \quad CS^L = CS^F = \frac{5329(1+\alpha)^2}{2(73+336\alpha)^2}.$$

Comparing each consumer surplus under the unionized mixed duopoly with each consumer surplus under the unionized privatized duopoly yields that

$$\begin{aligned} CS^c > CS^f &\Leftrightarrow 340\alpha + 956\alpha^2 > 0, \\ CS^c > CS^l &\Leftrightarrow 10\alpha + 29\alpha^2 > 0, \\ CS^f > CS^l &\Leftrightarrow 102\alpha + 297\alpha^2 > 0, \\ CS^C > CS^L = CS^F &\Leftrightarrow 4380\alpha + 19935\alpha^2 > 0, \\ CS^l > CS^C &\Leftrightarrow 12\alpha + 45\alpha^2 > 0. \end{aligned}$$ ■

## It is available from author upon request

## Appendix B: Case of $\alpha = t = 0$

*For the reviewers and editor, this appendix will not be included in the main paper. However, this is only available for the reviewers and editor: the case of $\alpha = t = 0$. In this case where we have been abbreviated, we present on separate page.*

### 5.1 Quantity Competition in a Unionized Mixed Duopoly when $\alpha = t = 0$

In the present stage, the public firm's objective is to maximize welfare which is defined as the sum of consumer surplus, each firm's profit, and each union's utility:

$$SW = U - px_0 - px_1 + \pi_1 + u_1 + \pi_0 + u_0 = U. \tag{32}$$

Given $w_i$ for each firm, the public firm's maximization problem is as follows:

$$\max_{x_0} \quad SW = U \quad \text{s.t.} \quad (p - w_0)x_0 \geq 0$$

Denoting the multiplier of the budget constraint $\lambda$, the Lagrangian equation can be written as

$$L(x_0, \lambda) = x_1 + x_0 - \frac{(x_1 + x_0)^2}{2} + \lambda(x_0 - x_0^2 - x_1 x_0 - w_0 x_0)$$

Taking $w_i$ as given, the first-order conditions are given by

$$\frac{\partial L}{\partial x_0} = 1 - x_1 - x_0 + \lambda(1 - 2x_0 - x_1 - w_0) = 0 \tag{33}$$

$$\frac{\partial L}{\partial \lambda} = 1 - x_1 - x_0 - w_0 = 0 \tag{34}$$

On the other hand, the optimal output for the private firm is given by

$$\frac{\partial \pi_1}{\partial x_1} = 0 \Leftrightarrow x_1 = \frac{1}{2}(1 - x_0 - w_1) \tag{35}$$

Given these results, we now obtain the output level for each firm. Solving the first-order conditions (34) and (35), we obtain,

$$x_0 = 1 + w_1 - 2w_0 \tag{36}$$

$$x_1 = w_0 - w_1, \tag{37}$$

$$\lambda = \frac{x_1 + x_0 - 1}{1 - 2x_0 - x_1 - w_0} \tag{38}$$

To solve for Lagrangian equation, the budget constraint is momentarily treated as binding. We check ex-post that the omitted this constraint is binding.

## 5.2 Wage Setting in a Unionized Mixed Duopoly when $\alpha = t = 0$

[**Simultaneous Wage Setting**]: A case where each union's wage is determined as a result of collective bargaining between the firm and the union is considered. To do this, the two independent maximization problems should be considered simultaneously as follows;

$$\max_{w_0}\ u_0 = w_0 x_0 = (1 + w_1 - 2w_0)w_0, \tag{39}$$

$$\max_{w_1}\ u_1 = w_1 x_1 = (w_0 - w_1)w_1, \tag{40}$$

respectively. Straightforward computation yields each firm's reaction function as follows:

$$w_0 = \frac{1 + w_1}{4}, \quad w_1 = \frac{w_0}{2}.$$

Straightforward computation yields that

**Lemma A-1**: *Suppose that each firm's union is allowed to bargain collectively. Then, the equilibrium wages, output union's utility and private firm's profit levels are given by*

$$w_0^c = \frac{2}{7}, \quad w_1^c = \frac{1}{7}, \quad x_0^c = \frac{4}{7}, \quad x_1^c = \frac{1}{7}, \quad u_0^c = \frac{8}{49}, \quad u_1^c = \frac{1}{49}, \quad \pi_1^c = \frac{1}{49}.$$

Substituting Lemma A-1 into (38) then we have

$$\lambda^c = 3 > 0$$

which shows that the public firm sets the output that yields zero profit in equilibrium.

Noting that $SW^c = U^c$, we can compute the social welfare $SW^c$ and consumer surplus $CS^c$ as follows;

$$SW^c = \frac{45}{98}, \quad CS^c = \frac{25}{98}. \tag{41}$$

[**Sequential Wage Setting: Public Firm's Leader**]: In this case, we discuss that the public firm acts as the leader. Public firm's union 0 will choose to maximize its utility taking as given the private firm's wage $w_1$ set by private firm's union 1. By solving the first-order condition for private firm's union 1, we already obtain the best response function to be represent as: $w_1 = \frac{w_0}{2}$. Thus, the problems for public firm's union 0 are defined as

$$\max_{w_0}\ u_0 = w_0 x_0 = \frac{w_0(2 - 3w_0)}{2}$$

By solving the first-order condition for the public firm's union 0, we obtain

**Lemma A-2**: *Suppose that each firm's union is allowed to bargain collectively. Then, the equilibrium wages, output and union's utility and private firm's profit levels are given by*

$$w_0^l = \frac{1}{3}, \quad w_1^f = \frac{1}{6}, \quad x_1^f = \frac{1}{6}, \quad x_0^l = \frac{1}{2}, \quad u_0^l = \frac{1}{6}, \quad u_1^f = \frac{1}{36}, \quad \pi_1^f = \frac{1}{36} \tag{42}$$

By substituting Lemma A-2 into (38), we obtain

$$\lambda^l = \frac{2}{3} > 0,$$

which shows that the budget constraint is binding. Using equilibrium values, we can compute the social welfare, $SW^l$ and consumer surplus $CS^l$ as follows;

$$SW^l = \frac{4}{9}, \quad CS^l = \frac{2}{9} \tag{43}$$

[**Sequential Wage Setting: Private Firm's Leader**]: Similar to the previous sequential wage setting of public firm's leader, we can directly compute each equilibrium value $w_i^m$, $x_i^m$, $u_i^m; m = l, f$, $\pi_1^l$ and the social welfare $SW^f$ as;

**Lemma A-3**: *Suppose that each firm's union is allowed to bargain collectively. Then, the equilibrium wages, output and union's utility and private firm's profit levels are given by*

$$w_0^f = \frac{7}{24}, \quad w_1^l = \frac{1}{6}, \quad x_0^f = \frac{7}{12}, \quad x_1^l = \frac{1}{8}, \quad \pi_1^l = \frac{1}{64}, \tag{44}$$

$$u_0^f = \frac{49}{288}, \quad u_1^l = \frac{1}{48}, \quad SW^f = \frac{527}{1152}, \quad CS^f = \frac{289}{1152}. \tag{45}$$

By substituting Lemma A-3 into (38), we obtain

$$\lambda^f = \frac{1}{2} > 0,$$

which shows that the budget constraint is binding.

### 5.3 Quantity Competition in a Unionized Privatized Duopoly when $\alpha = t = 0$

As discussed in the basic model, consider a privatized-duopoly situation for a homogeneous good that is supplied by firm $k = 1, 2$. Firm $k$ ($k = 1, 2$) is a private firm, profit-maximizing firm. Thus, the inverse demand is assumed by $p = 1 - x_1 - x_2$. Similar to the previous subsection, we discuss one Cournot- and two Stackelberg-type privatized-duopoly models of fixed timing.

In the present stage, taking as $w_k$ and solving the private firms of first-order conditions, we obtain,

$$x_k = \frac{1 - w_k - x_l}{2}, \quad k \neq l.$$

Straightforward computation yields each private firm's reaction function as follows:

$$x_k = \frac{1 - 2w_k + w_l}{3}, \quad k \neq l. \tag{46}$$

Similar to the case of unionized mixed duopoly, the private firm $k$ is increased when the private firm $l$'s wage is increased. The wage $w_k$ is decreasing in the output $x_k$.

## 5.4 Wage Setting in a Unionized Privatized Duopoly when $\alpha = t = 0$

[**Simultaneous Wage Setting**]: Turning to the first stage, we consider a case where each union's wage is determined as a result of collective bargaining between the firm and the union. Thus, problem for union $k$ is defined as

$$\max_{w_k} \; u_k = w_k x_k = \frac{w_k(1 - 2w_k + w_l)}{3}$$

Straightforward computation and symmetry across private firms yield each firm's wage;

$$w_k = \frac{1 + w_l}{4}, \quad k \neq l. \tag{47}$$

Thus, we have the following result:

**Lemma A-4**: *Suppose that the all private firms' union is allowed to bargain collectively. Then, the equilibrium wages, output, union's utility and private firms levels are given by*

$$w_k^C = \frac{1}{3}, \quad x_k^C = \frac{2}{9}, \quad u_k^C = \frac{2}{27}, \quad \pi_k^C = \frac{4}{81}.$$

Noting that $SW^C = U^C$, we can compute the social welfare $SW^C$ and consumer surplus $CS^C$ as follows;

$$SW^C = \frac{28}{81}, \quad CS^C = \frac{8}{81} \tag{48}$$

[**Stackelberg-Type Game**]: Consider the game where the private firm 1 is the leader. To solve for the backwards-induction quantity of this game, we use the private firm 2's union reaction function $w_2 = (1 + w_1)/4$ as in the simultaneous-move games. To distinguish notations, the superscript $L$ is defined when the private firm 1 acts as the leader and $F$ is defined when the private firm 2 acts as the leader. The private firm 1's best response that maximizes

$$\max_{w_1} \; u_1 = w_1 x_1 = \frac{w_1(5 - 7w_1)}{12}$$

Straightforward computation and symmetry across private firms yield each firm's wage;

$$w_1^L = \frac{5}{14}, \quad w_2^F = \frac{19}{56}. \tag{49}$$

Therefore, we have the following result:

**Lemma A-5**: *Suppose that the all private firms' union is allowed to bargain collectively. Then, the equilibrium wages, output, union's utility and private firms levels are given by*

$$\begin{aligned} w_k^L &= \frac{5}{14}, \quad x_k^L = \frac{35}{168}, \quad u_k^L = \frac{175}{2352}, \quad \pi_k^L = \frac{25}{576}, \\ w_k^F &= \frac{19}{56}, \quad x_k^F = \frac{38}{168}, \quad u_k^F = \frac{361}{4704}, \quad \pi_k^F = \frac{361}{7056}. \end{aligned}$$

Noting that $SW^L = SW^F = U^L = U^F$, we can compute the social welfare $SW^L = SW^F$ and consumer surplus $CS^L = CS^F$ as follows;

$$SW^L = SW^F = \frac{19199}{56448}, \quad CS^L = CS^F = \frac{5329}{56448}.$$

# 6 Timing of Endogenous Wage Setting when $\alpha = t = 0$

In this section, we will find the Nash equilibria in the first stage for any given utilities of the unions and the profits of firms under both the mixed and the privatized duopolies. The reduced endogenous-timing game among unions can be represented by the following payoff tables.

**Table A-1: Timing of Wage Setting Among Unions**

Union 1

| Union 0 | | $F$ | $S$ |
|---|---|---|---|
| | $F$ | $u_0^c, u_1^c$ | $u_0^l, u_1^f$ |
| | $S$ | $u_0^f, u_1^l$ | $u_0^c, u_1^c$ |

(a) Unionized Mixed Duopoly

Union 2

| Union 1 | | $F$ | $S$ |
|---|---|---|---|
| | $F$ | $u_1^C, u_2^C$ | $u_1^L, u_2^F$ |
| | $S$ | $u_1^F, u_2^L$ | $u_1^C, u_2^C$ |

(b) Unionized Privatized Duopoly

To find the subgame-perfect Nash equilibrium, we need to compare utilities of unions. Straightforward computations shows in both Table A-1(a) and (b) that

$$u_i^c < u_i^f, \quad u_i^c < u_i^l; \quad i = 0, 1,$$
$$u_k^C < u_k^F, \quad u_k^C < u_k^L; \quad k = 1, 2.$$

These inequalities tell us that regardless of what type of competition is introduced, each union prefers sequential wage setting when each union can decide the timing of wage setting. This is because except for the case with $w_1^f = w_1^l$, the wages paid by firms and outputs in the sequential case are higher than those of the simultaneous case (i.e., $w_k^C < w_k^F < w_l^L$, $w_i^c < w_i^f < w_i^l$, $x_k^C < x_k^L < x_l^L$, and $x_i^c < x_i^l < x_i^f$). So there are multiple subgame perfect Nash equilibria in the observable delay game of wage setting: Union acts as either a follower or a leader: (S, F), (F, S). Thus, we have the following proposition:

**Proposition A-1**: *Suppose that the decision of timing of wage setting is delegated to the unions under both markets. Then, there are two possible endogenous orders of moves in each competition type: the order is either (F, S) or (S, F).*

Regardless of the type of market, the fact that negotiating the wages in the sequential move is the strictly dominant strategy of each union plays an important role in the derivation of the result. Since it is each union that is making timing decisions of wage setting, being a follower is clearly always better than moving simultaneously. Hence $u_i^f > u_i^c$ and $u_k^F > u_k^C$; all workers employed by a leader union get higher wages in the sequential move while the follower union obtains more employment in sequential move. As a result, the follower union gets greater utility than the leader union although a higher wage set by the leader union. This is because the wage claims are decided by the elasticity of labor demand rather than the firm's profit. Therefore, regardless of the type of market, both unions prefer to decide the wage setting sequentially.

Similar to the timing of wage setting among unions, the reduced endogenous-timing game between the public and private firms can be represented by the following payoff tables.

**Table A-2: Timing of Wage Setting Among Firms**

Private Firm 1

| Public Firm 0 | | $F$ | $S$ |
|---|---|---|---|
| | $F$ | $SW^c, \pi_1^c$ | $SW^l, \pi_1^f$ |
| | $S$ | $SW^f, \pi_1^l$ | $SW^c, \pi_1^c$ |

(a) Unionized Mixed Duopoly

Private Firm 2

| Private Firm 1 | | $F$ | $S$ |
|---|---|---|---|
| | $F$ | $\pi_1^C, \pi_2^C$ | $\pi_1^L, \pi_2^F$ |
| | $S$ | $\pi_1^F, \pi_2^L$ | $\pi_1^C, \pi_2^C$ |

(b) Unionized Privatized Duopoly

Straightforward computations show that

$$SW^c > SW^f, \quad SW^c > SW^f, \quad \pi_1^c > \pi_1^l, \quad \pi_1^c > \pi_1^f \quad \text{under unionized mixed duopoly,}$$
$$\pi_k^C > \pi_k^F, \quad \pi_k^C > \pi_k^L; \quad k = 1, 2.$$

These inequalities tell us that each firm prefers simultaneous wage setting to sequential wage setting when each firm can decide the timing of wage setting under both the unionized mixed and unionized privatized duopolies. So there are multiple subgame perfect Nash equilibria in the observable delay game of wage setting: (S, S) or (F, F). Thus, we have the following proposition:

**Proposition A-2**: *Suppose that the decision of timing of wage setting is not delegated to the unions under both markets. Then, there are two possible endogenous orders of moves for each market; the order is either the first opportunity or the second opportunity.*

Contrast to Proposition A-1, regardless of what type of market, the fact that negotiating wage in the simultaneous move is the strictly dominant strategy of all firms plays an important role in the derivation of the result. From proposition A-1, we have considered that both the leader and follower firms pay for higher wages and obtain the lower outputs under sequential wage setting than those under simultaneous wage setting. These effects are reversed for the firms' profits when the decision of timing of wage setting is not delegated to the unions under both markets. As a result, the follower firm under unionized privatized (respectively, mixed) duopoly gets greater either profit (respectively, social welfare) than the leader firm since a lower wage is set and a higher output is obtained by the follower firm. Regardless of type of market, Proposition A-2 indicates that follower firm obtains higher either profit or social welfare in the simultaneous wage setting under respective market than in the sequential wage setting under respective market. Thus, each firm prefers being follower to being leader, which both firms prefer to decide the wage setting simultaneously. Thus, we get the unique subgame perfect equilibrium stated in Proposition A-2.

Given Propositions A-1 and A-2, we obtain the following result.

**Proposition A-3**: *Bargaining over wages is simultaneous if and only if all the firms decide on the timing of endogenous wage-setting, regardless of the market type. Otherwise, wage setting takes place sequentially.*

**Proof**: The proof is the same as that provided by Barcena-Ruiz and Casado-Izaga (2008, pp. 155-157). ■

From the derived observations of the social welfare in both unionized mixed and privatized duopolies, it is instructive to compare social welfare under unionization structures in the unionized mixed duopoly with the unionized privatized duopoly. Comparing each social welfare give us a situation that social welfare in a unionized mixed duopoly is improved regarding the privatization. Thus, the results of this comparison are summarized in the following proposition.

**Proposition A-4**: *Each level of social welfare is determined by*

$$SW^C < SW^L = SW^F < SW^l < SW^f < SW^c.$$

Proposition A-4 suggests that when the decentralization mode is determined under both markets, social welfare in a unionized privatized duopoly can be improved regarding the privatization.

On the other hand, given the union's utility, the consumer surplus $CS$ of each market in simultaneous and sequential wage setting cases are represented with same superscripts as Proposition A-5:

**Proposition A-5**: *Each level of the consumer surplus is determined by*

$$CS^C < CS^L = CS^F < CS^l < CS^f < CS^c.$$

Proposition A-5 suggests that regardless of what type of competitions in the markets, the consumer surplus can not be improved regarding the privatization. This result gives us the situation that is the best in terms of both social welfare and consumer surplus when both the firms under the unionized mixed duopoly do not delegate the wage setting to the unions.