Optimal Taxation in Dynamic Economies with Increasing Returns

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Abstract

This paper studies optimal taxation in dynamic economies with increasing returns. We show that if there exists a stable open-loop Stackelberg equilibrium, the optimal rate of tax on capital income in the steady state is negative in order to eliminate the wedge between the private and the social rate of return to capital. This result also holds when the government expenditure has a positive effect on production activities of the private agents. In contrast, if the government takes a feedback strategy and if the government budget is balanced in every period, then the optimal capital income taxation rule obtained under the open-loop strategy may be violated. It is, however, shown that if the government can borrow from the public, the negative capital income tax rule may be established even under the feedback policy rule.

JEL classification: D90, H21, H23

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1 Introduction

Ever since the seminal contributions by Chamley (1986) and Judd (1985), numerous authors have studied optimal taxation by using dynamic general equilibrium models. One of the central concerns of this literature is the robustness of the Chamley-Judd proposition which claims that taxing on capital income should be eventually eliminated. This proposition has been examined in a variety of economic environments and many counterexamples have been presented.[1] Formally speaking, the issue of optimal taxation in dynamic economies can be formulated as Stackelberg dynamic games in which private agents behave as followers and the fiscal authority plays the leader's role. A common feature of existing investigations on the dynamic optimal taxation is to fucus on the open-loop solutions of the Stackelberg dynamic games: it is assumed that the government can commit the entire sequence of its strategies. However, it is known that validity of the zero capital income tax proposition is sensitive to this assumption as well. As shown by Kemp, Long and Shimomura (1993) and Benhabib and Rustichini (1996), if the government fails to commit the open-loop policies, the optimal taxation on capital income may not be zero in the long-run equilibrium even if all of the markets are perfectly competitive.

This paper studies optimal taxation in dynamic economies with external increasing returns. The main concern of our discussion is to explore the interaction between the market distortion and the type of policy rule taken by the fiscal authority. Introducing Marshallian externalities into the representative agent model used by Chamley (1986), we first examine the standard open-loop policies. It is shown that if there is a feasible and stable open-loop Stackelberg equilibrium, the optimal rate of tax on capital income is negative in the steady-state. More specifically, under the open-loop policy rule capital should be subsidized in order to internalize the external effects generated by the social level of capital. The rule of negative capital income taxation, however, fails to hold, if the government takes the feedback strategy rather than the open-loop strategy and if the government may not issue interest bearing bonds. We show this result by using an example that is essentially the same as one explored by Xie (1997), Lansing (1999) and Long and Shimomura (1999). We also demonstrate that if the government can issue debt, the negative capital income tax scheme may be established even in the feedback Stackelberg equilibrium.

[1] To name a few, the Chamley-Judd proposition may not hold, if financial markets are incomplete (Aiyagari 1995); if there are untaxed production factors (Correia 1996), or; if commodity markets are impefectly competitive (Judd 1999 and Guo and Lansing 1999).

In addition to the case of Marshallian external increasing returns, we consider the situation in which increasing returns are also generated by the productive public capital. It is revealed that regardless of the presence of government bonds, the open-loop strategy always leads the economy to the steady state in which capital income tax is negative. If the government employs the feedback strategy and if there is no market for government bonds, then the optimal rate of capital income tax is not necessarily negative in the steady state. However, we again show that the negative capital income tax proposition may hold if the government can issue debt.

The remainder of the paper is organized as follows. Section 2 sets up the basic model. Section 3 characterizes the open-loop Stacklberg equilibrium and derives the negative capital income tax proposition. Section 4 considers an example of feedback solution and examines how the main results obtained under the open-loop policy should be modified. By introducing productive public investment into the base model, Section 5 re-examines the open-loop as well as the feedback equilibria. Section 6 concludes the paper.

2 The Base Model

The basic framework of our analysis is the representative agent model used by Chamley (1986). Under a given sequence of fiscal action of the government, the representative household selects its optimal consumption and labor supply over an infinite time horizon. The benevolent government seeks to maximize the household's welfare by controlling fiscal strategies subject to the optimal reaction of the household as well as to the resource and own budget constraints.

2.1. *Household*

The instantaneous felicity of the household depends on consumption, labor supply and public spending of the government. The objective of the household is to maximize a discounted sum of utilities

$$\int_0^\infty e^{-\rho t}\left[u\left(c,n\right)+v\left(g\right)\right]dt, \quad \rho>0$$

subject to the flow budget constraint

$$\dot{k}=\hat{r}k+\hat{w}n-c, \tag{1}$$

and the initial holding of capital stock. In the above, c, n k and g respectively denote consumption, labor supply, capital stock and the government spending per capita. In addition, $\hat{r}$

and $\hat{w}$ denote the after-tax rate of return to capital and the after-tax real wage, respectively. The instantaneous sub-utility function, $u(c,n)$, is assumed to satisfy $u_c > 0$, $u_n < 0$, $u_{cc} < 0$ and $u_{nn} > 0$. Similarly, $v(g)$ is increasing and strictly concave in g.[2] For simplicity, we assume that capital does not depreciate.

For analytical convenience, we first consider an instantaneous optimization problem in which the household maximizes $u(c,n)$ subject to the budget constraint, $c = \hat{w}n + m$. This problem yields the following first-order condition:

$$-\frac{u_c(c,n)}{u_n(c,n)} = \hat{w}. \tag{2}$$

By (2) and the budget constraint, the optimal choices of consumption and labor supply are expressed as $c = c(m,\hat{w})$ and $n = n(m,\hat{w})$. Hence, letting the instantaneous indirect utility function be $\tilde{u}(m,\hat{w}) = u(c(m,\hat{w}), n(m,\hat{w}))$, the intertemporal optimization problem the household solves is rewritten as

$$\max \int_0^\infty e^{-\rho t} \left[\tilde{u}(m,\hat{w}) + v(g)\right] dt$$

subject to

$$\dot{k} = \hat{r}k - m \tag{3}$$

and the initial condition on k. Denoting the costate variable for k by p, the necessary conditions for an optimum are given by the following:

$$\tilde{u}_m(m,\hat{w}) = p, \tag{4}$$

$$\dot{p} = p(\rho - \hat{r}), \tag{5}$$

$$\lim_{t\to\infty} e^{-\rho t} pk = 0. \tag{6}$$

together with (3) and the given initial level of capital stock.

2.2. *Producers and the Market Equilibrium Condition*

There are identical producers whose number is normalized to unity. The production function of each producer is

$$y = f\left(k, n, \bar{k}, \bar{n}\right), \tag{7}$$

[2] Additive separability between private activities and the public service, g, is assumed for simplicity. It is easy to confirm that the main conclusions of this section still hold when the felicity function is specified as a more general form such that $u = u(c,n,g)$.

where y is output and n is labor input. Here, $\bar{k}$ and $\bar{n}$ represent externalities associated with the aggregate levels of capital and labor.[3] The production function f is assumed to be strictly quasi-concave, increasing and homogeneous of degree one in k and n. Although the external effects could be negative, in this paper we deal with the case of external increasing returns so that $f_{\bar{k}} > 0$ and $f_{\bar{n}} > 0$. Given the external effects, the producers maximize their profits and hence we obtain:

$$r = f_k\left(k, n, \bar{k}, \bar{n}\right), \tag{8}$$

$$w = f_n\left(k, n, \bar{k}, \bar{n}\right), \tag{9}$$

where r and w respectively denote the pre-tax rate of return to capital and the pre-tax real wage.

The equilibrium condition for the product market is

$$y = c + \dot{k} + g. \tag{10}$$

Since the number of producers is assumed to be one, the consistency condition requires that $\bar{k} = k$ and $\bar{n} = n$ for all $t \geq 0$.

2.3. *The Government*

In the base model, we assume that the government does not issue debt so that its flow budget should be balanced in every period. The flow constraint of the government budget is thus given by

$$g = \tau_k r k + \tau_n w n,$$

where τ_k and τ_n denote the rates of tax on capital and wage income. Notice that since the government runs a balanced budget, g is a dependent variable rather than a choice variable of the fiscal authority. Remembering that the production technology satisfies constant returns to scale from the private perspective and that $\hat{r} = (1 - \tau_k)\, r$ and $\hat{w} = (1 - \tau_n)\, w$, the government budget constraint is written as

$$g = f\left(k, n, \bar{k}, \bar{n}\right) - \hat{r}k - \hat{w}n. \tag{11}$$

[3] Specification of production technology in this paper follows Benhabib and Farmer (1994) whose central concern is to study indeterminacy of equilibrium in the representative agent model of growth with social increasing returns. It is also to be noted that if we assume that f is homogenous of degree one in k and $\bar{k}$, then the model becomes a one-sector endogenous growth setting with labor-leisure choice studied by Pelloni and Waldmann (1999).

The government maximizes the discounted sum of indirect utilities of the household by controlling τ_k and τ_n (or $\hat{r}$ and $\hat{w}$). From equation (4), the optimal level of expenditure is expressed as $m = m(p, \hat{w})$. Thus the indirect utility and the optimal labor supply of the household may be expressed as $h(p, \hat{w}) = \tilde{u}(m(p, \hat{w}), \hat{w})$ and $\hat{n}(p, \hat{w}) = n(m(p, \hat{w}), \hat{w})$. Accordingly, in view of (3) and (5), the optimization problem for the government is given by

$$\max \int_0^\infty e^{-\rho t} [h(p, \hat{w}) + v(g)] \, dt$$

subject to

$$\dot{k} = \hat{r}k - m(p, \hat{w}),$$

$$\dot{p} = p(\rho - \hat{r}),$$

$$g = f(k, n(p, \hat{w}), k, n(p, \hat{w})) - \hat{r}k - \hat{w}n(p, \hat{w}),$$

and the transversality condition (6) as well as the initial conditions on k. The control variables of the government, $\hat{r}$ and $\hat{w}$, should be non-negative. We assume that the government takes the external effects into account when choosing the optimal levels of $\hat{r}$ and $\hat{w}$.

3 The Open-Loop Policies

In this section we follow the standard approach to the optimal taxation in which a stable open-loop Stackelberg equilibrium is assumed to exist. To solve the optimization problem of the government given above, set up the Hamiltonian function in such a way that

$$\begin{aligned} H &= h(p, \hat{w}) + v(f(k, n(p, \hat{w}), k, n(p, \hat{w})) - \hat{r}k - \hat{w}n(p, \hat{w})) \\ &\quad + \lambda [\hat{r}k - m(p, \hat{w})] + \mu p(\rho - \hat{r}). \end{aligned}$$

The necessary conditions for an optimum are the following:

$$H_{\hat{r}} = \left[\lambda - v'(g)\right] k - \mu p = 0 \text{ for } \hat{r} > 0 \tag{12}$$

$$\begin{aligned} H_{\hat{w}} &= h_{\hat{w}}(p, \hat{w}) - m_{\hat{w}} - v'(g) [(f_{\hat{n}} + f_{\bar{n}}) n_{\hat{w}} + n - \hat{w}n_{\hat{w}}] \\ &= 0 \text{ for } \hat{w} > 0, \end{aligned} \tag{13}$$

$$\dot{k} = \hat{r}k - m(p, \hat{w}), \tag{14}$$

$$\dot{p} = p(\rho - \hat{r}), \tag{15}$$

$$\dot{\lambda} = \lambda\left(\rho - \hat{r}\right) - v'\left(g\right)\left(f_k + f_{\bar{k}} - \hat{r}\right), \tag{16}$$

$$\dot{\mu} = \mu\hat{r} - h_p + \lambda m_p - v'\left(g\right)\left[\left(f_n + f_{\bar{n}}\right)n_p - \hat{w}n_p\right], \tag{17}$$

$$\mu_0 = 0, \tag{18}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda k = 0; \quad \lim_{t\to\infty} e^{-\rho t}\mu p = 0. \tag{19}$$

Equation (18) is the transversality condition for the costate variable corresponding to p whose initial value should be selected by the government.[4] Provided that the non-negativity conditions on $\hat{r}$ and $\hat{w}$ are not binding, from (12) and (13) the optimal levels of $\hat{r}$ and $\hat{w}$ can be expressed as functions of p, λ, μ and k. Substituting these optimal levels of $\hat{r}$ and $\hat{w}$ into (14), (15), (16) and (17), we may obtain a complete dynamic system with respect to π, λ, μ and k. If there is a solution of this system that satisfies the transversality conditions (18) and (19) as well as the initial condition on capital stock, it represents an open-loop Stackelberg equilibrium. Obviously, without imposing further restrictions on the functional forms involved in the model, it is difficult to check whether or not there exists a feasible and stable solution leading to the steady state equilibrium. In fact, Kemp, Long and Shimomura (1993) demonstrate that in the model of income redistribution by Judd (1985), the open-loop solution may converge to a limit cycle rather than to a stationary equilibrium.[5] In what follows, as many studies in this literature do, we simply assume that the model has a feasible and stable solution.

In the steady-state equilibrium p stays constant over time, and hence from (15) it holds that $\hat{r} = \rho$. In view of (16), this means that $\hat{r} = f_k + f_{\bar{k}}$ holds in the steady state. As a result, the long-run rate of optimal capital income tax is

$$\tau_k = -f_{\bar{k}}/f_k. \tag{20}$$

Namely, the optimal capital income tax is negative in the steady state. If there are no externalities associate with capital ($f_{\bar{k}} = 0$), then the optimal taxation for capital income is

[4] See, for example, Bryson and Ho (1975, pp.56-57) as to the transversality conditions for the costate variables corresponding to the state variables whose initial values are unspecified. If the government reoptimizes at some later date, say $\hat{t}$, under a given level of $k_{\hat{t}}$, then the initial value of μ should satisfy $\mu_{\hat{t}} = 0$. As is well known, this condition is the source of time inconsistency of the open-loop Stackelberg solution, because the optimal value of $\mu_{\hat{t}}$ for the problem solved at the initial time does not generally satisfy $\mu_{\hat{t}} = 0$.

[5] It is, however, to be noted that Judd (1999) claims that the average optimal tax rate on capital income should be zero during the transition even though the economy does not converge to a steady-state equilibrium.

zero in the long-run equilibrium. For example, if the production function is specified as

$$y = k^{\alpha} n^{1-\alpha} \left(\bar{k}^{\alpha} \bar{n}^{1-\alpha}\right)^{\chi}, \quad \chi > 0,$$

where χ represents the scale factor generated by external effects. In this case, the steady-state rate of optimal capital income tax is $\tau_k = -\chi\alpha$. The result of negative tax on capital income derived above corresponds to the finding by Judd (1997) who demonstrates that the optimal capital income tax is negative in the steady state if the commodity market consists of a continuum of imperfectly competitive industries.[6]

It is worth emphasizing that the above conclusion does not depend on the assumption that the government runs a balanced budget in every period. If the government may finance its deficits by issuing bonds, the flow constraint of the government budget becomes

$$\dot{b} = g + \hat{r}a + \hat{w}n - f\left(k, n, \bar{k}, \bar{n}\right), \tag{21}$$

where b is the stock of government debt per capita and a denotes the per-capita total wealth: $a = k + b$. Hence, the budget constraint of the household (3) is replaced with

$$\dot{a} = \hat{r}a - m. \tag{22}$$

In this case, the government can choose the optimal level of g in addition to controlling $\hat{r}$ and $\hat{w}$.

The Hamiltonian function for the government is

$$\begin{aligned} H &= h(p, \hat{w}) + v(g) + \lambda\left[\hat{r}a - m(p, \hat{w})\right] + \mu p(\rho - \hat{r}) + \theta(a - k - b) \\ &\quad + \phi\left[g + \hat{r}a + \hat{w}n(p, \hat{w}) - f(k, n(p, \hat{w}), k, n(p, \hat{w}))\right] \end{aligned}$$

The key first-order conditions that characterize the optimal capital income tax in the steady state are the following:

$$H_{\hat{r}} = (\lambda + \phi)\, a = 0 \text{ for } \hat{r} > 0, \tag{23}$$

$$H_k = -\phi\left(f_k + f_{\bar{k}}\right) - \theta = 0 \text{ for } k > 0, \tag{24}$$

$$H_g = v'(g) + \phi = 0, \tag{25}$$

$$\dot{\lambda} = \lambda(\rho - \hat{r}) - \phi\hat{r} - \theta, \tag{26}$$

[6] See also Judd (1999). Guo and Lansing (1999) generalize the model examined by Judd (1997) and show that the optimal capital income could be positive depending on the way how the government levy taxes on the monopoly profits.

$$\dot{\phi} = \phi\rho + \theta. \tag{27}$$

The steady-state condition, $\dot{\phi} = 0$ in (27) gives $\theta = -\rho\phi$. Thus from (24) we obtain $f_k + f_{\bar{k}} = \rho$. On the other hand, by use of (23) and $\theta = -\rho\phi$, $\dot{\lambda} = 0$ in (26) shows that $\hat{r} = \rho$ holds in the steady state. Consequently, we find that $f_k + f_{\bar{k}} = \hat{r}$ in the steady state, which yields the optimal rate of tax on capital income given by (20).

To sum up, we have shown:

Proposition 1 *If the open-loop Stackelberg game defined above has a feasible and stable solution, then in the presence of positive external effects of capital the optimal rate of capital income tax is negative in the steady-state equilibrium. This result is robust to whether the government can issue debt or must run a balanced-budget in every period.*

The optimal steady-state condition, $f_k + f_{\bar{k}} = \hat{r} = (1 - \tau_k) f_k$, states that the optimal rate on capital income should resolve the divergence between the social and the private rate of return to capital. If capital does not generate any external effect, the social and the private returns to capital are the same so that the optimal policy is eventually to set the tax rate on capital to zero. In the presence of externalities, the capital subsidy will establish the golden rule condition under which the marginal productivity of social level of capital equals to the time discount rate of the private agents. This is nothing but a Pigou-type tax-subsidy scheme in our dynamic context.

4 The Feedback Policies

Following the standard approach, we have assumed that the government can commit its strategy over an entire planning horizon and that such an open-loop policy may have a feasible and stable solution. Xie (1997), Lansing (1999) and Long and Shimomura (1999) focus on the special cases where the open-loop policy of the leader cannot control the dynamic behavior of the costate variables for the followers' problem. The models they discuss may produce the feedback Stackelberg equilibrium in a tractable manner. In general, the feedback Stackelberg solution are hard to obtain except for linear quadratic games or for extremely simple non-linear models. Therefore, the examples presented by those authors are useful to examine analytically how the main results obtained in the open-loop Stackelberg games will be modified in the case of feedback equilibrium. In the Appendix to the paper, we generalize the models used by those authors.

4.1. *The Base Model with Logarithmic Utility* .

In what follows, relying on a specific example, we re-examine the optimal capital income taxation scheme discussed in the previous section under an alternative equilibrium concept. In so doing, we need the following lemma, which is a generalization of an example shown by Lansing (1999)[7].

Lemma 1. *Suppose that the instantaneous utility function of the household is*

$$u(c,n) = \log \upsilon(c,n),$$

where $\upsilon(c,n)$ is a homogenous function of c and n. Then the optimal spending m is given by $m = \rho k$.

Proof. The homogeneity assumption means that

$$-\frac{u_n(c,n)}{u_c(c,n)} = -\frac{\upsilon_n(1,n/c)}{\upsilon_c(1,n/c)} = \hat{w}.$$

The above gives the relation between c and n in such a way that $n = \eta(\hat{w})\, c$. Thus the optimal levels of consumption and labor supply are respectively given by

$$c(m,\hat{w}) = \frac{m}{1-\hat{w}\eta(\hat{w})}, \quad n(m,\hat{w}) = \frac{\eta(\hat{w})\, m}{1-\hat{w}\eta(\hat{w})}.$$

As a result, assuming that $\upsilon(c,n)$ is homogenous of degree $\alpha\,(>0)$ in c and n, the indirect sub-utility function becomes

$$\tilde{u}(m,\hat{w}) = \log \upsilon(1,\eta(\hat{w})) - \log(1-\hat{w}\eta(\hat{w})) + \alpha \log m.$$

Since the transversality condition for the household's optimal planning (equation 6) means that the intetemporal budget constraint after t is

$$k_t = \int_t^{\infty} m_s \exp\left(-\int_t^s \hat{r}_{\xi}\, d\xi\right) ds. \tag{28}$$

On the other hand, integration of (5) yields

$$p_s = p_t \exp\left(\int_t^s (\rho - \hat{r}_{\xi})\, d\xi\right). \tag{29}$$

Substituting (29) into (28) and using the first-order condition, $m_s = \alpha / p_s$, we obtain:

$$k_t = \frac{\alpha}{\rho p_t}, \tag{30}$$

[7] See p.447 in Lansing (1999).

which establishes $m_t = \rho k_t$ for all $t \geq 0$. ■

As shown above, p depends on k alone so that the initial value of p is uniquely determined by the initial level of capital in such a way that $p_0 = \alpha/\rho k_0$. As a result, trnsversality condition on the open-loop policy (18) fails to hold. This implies that tax policy of the government cannot directly control the intertemporal decision of the household. It may only control the household's intratemporal decision concerning the optimal choices of consumption and labor supply. Since the household's optimal choice of m_t depends on k_t, the optimal policy selected by the government is time consistent. The optimal levels of c and n are

$$c = \frac{\rho k}{1 - \hat{w}\eta(\hat{w})}, \quad n = \frac{\eta(\hat{w})\rho k}{1 - \hat{w}\eta(\hat{w})}. \tag{31}$$

Furthermore, (5) and (30) imply that the dynamic behavior of capital is

$$\dot{k} = k(\hat{r} - \rho). \tag{32}$$

Accordingly, in view of (31), the optimization problem the government solves is as follows:

$$\max \int_0^\infty e^{-\rho t}\left[\Delta(\hat{w}) + \alpha \log k + v(g)\right] dt$$

subject to (32) and

$$g = f(k, \Lambda(\hat{w})k, k, \Lambda(\hat{w})k) - \hat{r}k - \hat{w}\Lambda(\hat{w})k,$$

where

$$\Delta(\hat{w}) = \log v(1, \eta(\hat{w})) - \log(1 - \hat{w}\eta(\hat{w})) + \alpha \log \rho, .$$

$$\Lambda(\hat{w}) = \frac{\rho\eta(w)}{1 - \hat{w}\eta(\hat{w})}.$$

The optimization problem given above is a standard control problem (a game against the nature). The Hamiltonian function can be set as

$$\begin{aligned} H &= \Delta(\hat{w}) + \alpha \log k + v(f(k, \Lambda(\hat{w})k, k, \Lambda(\hat{w})k) - \hat{r}k - \hat{w}\Lambda(\hat{w})k) \\ &\quad + \lambda k(\hat{r} - \rho). \end{aligned}$$

The necessary conditions for an optimum are (32) and the following:

$$H_{\hat{r}} = \left(\lambda - v'(g)\right)k = 0, \tag{33}$$

$$H_{\hat{w}} = \Delta'(\hat{w}) + v'(g)\,k\left[(f_n + f_{\bar{n}} - \hat{w})\,\Lambda'(\hat{w}) - \Lambda(\hat{w})\right] = 0, \tag{34}$$

$$\dot{\lambda} = \rho\lambda - \lambda(\hat{r} - \rho) - \frac{\alpha}{k} - v'(g)\left[f_k + f_{\bar{k}} - \hat{r} + (f_n + f_{\bar{n}} - \hat{w})\,\Lambda(\hat{w})\right], \tag{35}$$

$$\lim_{t\to\infty} e^{-\rho t} k\lambda = 0. \tag{36}$$

From (33) and (34) the optimal levels of $\hat{r}$ and $\hat{w}$ may be expressed as functions of k and λ. Substituting those functions into (32) and (35) yields a complete dynamic system of k and λ. If this system has a stable solution converging to the stationary equilibrium, we may examine the stead-state conditions in the feedback equilibrium.

In the steady-state equilibrium k stays constant and thus (32) shows that $\hat{r} = \rho$ holds. Therefore, noting that (33) yields $v'(g) = \lambda$, the steady-state condition $\dot{\lambda} = 0$ in (34) means that the following holds in the steady state:

$$f_k + f_{\bar{k}} - \hat{r} = \rho - \frac{\alpha}{\lambda k} - (f_n + f_{\bar{n}} - \hat{w})\,\Lambda(\hat{w}).$$

On the other hand, (35) yields

$$\lambda k = -\frac{\Delta'(\hat{w})}{(f_n + f_{\bar{n}} - \hat{w})\,\Lambda'(\hat{w}) - \Lambda(\hat{w})}.$$

Therefore, we obtain

$$f_k + f_{\bar{k}} - \hat{r} = \rho + \frac{\alpha}{\Delta'(\hat{w})}\left\{(f_n + f_{\bar{n}} - \hat{w})\left[\Lambda'(\hat{w}) - \Lambda(\hat{w})\,\Delta(\hat{w})\right] - \Lambda(\hat{w})\right\}. \tag{37}$$

It is obvious that, under arbitrarily given utility and production functions that satisfy the foregoing assumptions, (37) may not ensure that $\hat{r} = f_k + f_{\bar{k}}$ holds [8]

Consequently, we have shown:

Proposition 2 *If the government takes a feedback strategy and if the government budget is balanced in every period, then the negative capital income tax rule obtained under the open-loop strategy does not necessarily hold.*

In this example, the government can control the intertemporal consumption planning of the household (that is, the decision about m) only in an indirect manner. This fact prevents the government from establishing the golden-rule condition in the steady state equilibrium.

4. 2. *Government Bonds*

[8] We may find concrete counterexamples to the condition $\hat{r} = f_k + f_{\bar{k}}$, if we specify the utility and the production functions as, for example, $v(c,n) = (c^{\alpha} - n^{\alpha})^{1/\alpha}$ and $y = k^{\beta} n^{1-\beta}\left(\bar{k}^{\beta}\bar{n}^{1-\beta}\right)^{\chi}$.

When the government can issue debt, the optimal choice of m by the household is $m = \rho(k+b) = \rho a$. Considering this relation and $\dot{a}/a = \dot{p}/p = \rho - \hat{r}$, the optimization problem of the government in this case is given by

$$\max \int_0^\infty e^{-\rho t} \left[\Delta(\hat{w}) + \log a + v(g)\right] dt$$

subject to

$$\dot{a} = a(\hat{r} - \rho),$$

$$\dot{b} = g + \hat{r}a + \hat{w}\Lambda(\hat{w})a - f(k, \Lambda(\hat{w})a, k, \Lambda(\hat{w})a),$$

$$a = k + b.$$

The Hamiltonian function for this problem is

$$\begin{aligned} H &= [\Delta(\hat{w}) + \log a + v(g)] + \lambda a(\hat{r} - \rho) + \theta(a - k - b) \\ &\quad + \phi\left[g + \hat{r}a + \hat{w}\Lambda(\hat{w})a - f(k, \Lambda(\hat{w})a, k, \Lambda(\hat{w})a)\right], \end{aligned}$$

The first-order conditions for maximization with respect to control variables yield:

$$H_{\hat{r}} = (\lambda + \phi)a = 0 \text{ for } \hat{r} > 0, \tag{38}$$

$$H_g = v'(g) + \phi = 0, \tag{39}$$

$$H_k = -\theta - \phi(f_k + f_{\bar{k}}) = 0. \tag{40}$$

By use of (38), (39), and (40), we find that the dynamic equations of λ and ϕ are respectively given by

$$\begin{aligned} \dot{\lambda} &= \rho\lambda - \lambda(\rho - \hat{r}) - \frac{\alpha}{a} - \theta - \phi\left[\hat{r} - (f_k + f_{\bar{k}}) - (f_n + f_{\bar{n}} - \hat{w})\Lambda(\hat{w})\right] \\ &= \lambda\hat{r} - \frac{\alpha}{a} - v'(g)\left[f_k + f_{\bar{k}} - \hat{r} + (f_n + f_{\bar{n}} - \hat{w})\Lambda(\hat{w})\right], \end{aligned} \tag{41}$$

$$\dot{\phi} = \rho\phi + \theta = \phi\left[\rho - (f_k + f_{\bar{k}})\right]. \tag{42}$$

Consequently, the steady state conditions $\dot{a} = \dot{\phi} = 0$ means that $f_k + f_{\bar{k}} = \hat{r}$, so that the optimal capital income tax in the steady state is $\tau_k = -f_{\bar{k}}/f_k$.

Proposition 3 *If the government can issue debt, then the negative capital income tax rule in the steady state may hold even in the feedback Stackelberg equilibrium.*

This result corresponds to a proposition in Lansing (1999) who derives the similar result in the context of Judd's (1985) model of optimal income redistribution problem. The dynamic behavior of the costate variable for capital stock given by (41) is the same as (35) in the model without government debt. However, the presence of government bonds provides an additional equation (42), which ensures the golden rule condition in the steady state. This proposition thus demonstrates that the divergence between the time inconsistent, open loop policy and the time consistent feedback policy may be resolved at least in the steady state equilibrium, if the policy maker may have additional control methods.

5 Public Capital Formation

So far, we have assumed that the public expenditure has no effect on the private production activities. In this section, we assume that the government spending has positive effects on the production activities. Since the private technology exhibits constant returns, introducing productive government services yields an additional source of social increasing returns. We first present a brief discussion of the case where public services in the precaution function are flow. We then examine the model in which the production function involves a stock of public capital.

5.1. *The Open-loop Policies*

If the production function involves a flow of public service, it may be specified as

$$y = f\left(k, n, \bar{k}, \bar{n}, g\right), \quad f_g > 0. \tag{43}$$

Hence, even without external effects, the production technology is assumed to exhibit increasing returns to scale in k, n and g.[9] First, assume that the government does not borrow from the public, so that

$$g = f\left(k, n, \bar{k}, \bar{n}, g\right) - \hat{r}k - \hat{w}n. \tag{44}$$

Using $\bar{k} = k$ and $\bar{n} = n$, if (44) has a solution of g, it is expressed as

$$g = G\left(k, n, , \hat{r}, \hat{w}\right),$$

[9] This specification of production technology is close to that of Barro and Sala-i-Martin (1993). However, unlike Barro and Sala-i-Martin (1993), we do not assume that endogenous growth is possible in the long-run equilibrium.

where

$$G_k = \frac{f_k + f_{\bar{k}} - \hat{r}}{1 - f_g}, \quad G_n = \frac{f_n + f_{\bar{n}} - \hat{w}}{1 - f_g}, \quad G_{\hat{r}} = -\frac{k}{1 - f_g}, \quad G_{\hat{w}} = -\frac{n}{1 - f_g}.$$

Here we assume that $0 < f_g < 1$, and thus, other things being equal, an increase in public spending enhances the government deficits.[10]

The Hamiltonian function for the government's optimization problem now becomes

$$H = h(p, \hat{w}) + v\left[G(k, n(p, \hat{w}), \hat{r}, \hat{w})\right] + \lambda\left[\hat{r}k - m(p, \hat{w})\right] + \mu p(\rho - \hat{r}).$$

From the first-order conditions, we obtain

$$H_{\hat{r}} = v'(g) G_{\hat{r}} + \lambda k - \mu p = 0, \tag{45}$$

$$\dot{\lambda} = \lambda(\rho - \hat{r}) - v'(g) G_k. \tag{46}$$

Hence the steady-state conditions $\dot{\lambda} = \dot{p} = 0$ imply that

$$G_k(k, n, \hat{r}, \hat{n}) = \frac{f_k + f_{\bar{k}} - \hat{r}}{1 - f_g} = 0. \tag{47}$$

This condition yields $\hat{r} = f_k + f_{\bar{k}}$, so that the optimal rate of tax on capital income is $\tau_k = -f_{\bar{k}}/f_k$ in the steady state.

When g denotes a stock of public capital rather than a flow of public service, the government budget constraint presents the dynamic equation of g as follows:[11]

$$\dot{g} = f\left(k, n, \bar{k}, \bar{n}, g\right) - \hat{r}k - \hat{w}n. \tag{48}$$

The Hamiltonian function is set as

$$\begin{aligned} H &= h(p, \hat{w}) + v(g) + \lambda\left[\hat{r}k - m(p, \hat{w})\right] + \mu p(\rho - \hat{r}) \\ &\quad + \phi\left[f\left(k, n, \bar{k}, \bar{n}, g\right) - \hat{r}k - \hat{w}n\right], \end{aligned}$$

where ϕ is a costate variable for the public capital. The first-order for an optimum include:

$$(\lambda - \phi) k - \mu p = 0 \quad \text{for } \hat{r}. > 0, \tag{49}$$

[10] The equilibrium level of g satisfying (44) may not be uniquely determined. For example, if $f\left(k, n, \bar{k}, \bar{n}, g\right) = \hat{f}\left(k, n, \bar{k}, \bar{n}\right) g^{\eta} \quad (0 < \eta < 1)$, there are dual values of g that fulfills (44). In this case we assume that the government selects the higher level of g at which it holds that $1 - f_g = 1 - \eta \hat{f} g^{\eta - 1} > 0$.

[11] The discussion given below presents optimal fiscal policy in growth models with public capital accumulation studied by, for example, Futagami, Morita and Shibata (1993) and Zhang (2000).

$$\dot{\lambda} = \lambda\left(\rho - \hat{r}\right) - \phi\left(\hat{r} - f_k - f_{\bar{k}}\right), \tag{50}$$

$$\dot{\phi} = \phi\left(\rho - f_g\right) - v'\left(g\right). \tag{51}$$

Again, the steady state condition $\hat{r} = \rho$ means that from (50) $\hat{r} = f_k + f_{\bar{k}}$. Note that condition (51) gives

$$f_g + \frac{v'\left(g\right)}{\phi} = \rho = f_k + f_{\bar{k}}. \tag{52}$$

This is the steady-state representation of the non-arbitrage condition between private and public capital, which states that the social rate of return to the private capital is equal to the return to the public capital that involves its marginal contribution to the household's felicity, $v'\left(g\right)/\phi$.

Proposition 4 *If the public spending has a positive effect on the production activities, the optimum taxation on capital income is negative in the steady state. This conclusion is robust to whether the public services are flow or they are generated by the stock of public capital.*

To sum up, as far as the open-loop equilibrium is concerned, introducing productive public spending into the base model does not yield an essential change as to the optimal capital income taxation in the long-run equilibrium.

5.2. *The Feedback Policies*

In this sub section, we restrict our attention to the model with public capital formation. First, assume that the government must runs a balanced budget in every period. In the case of a specific feedback equilibrium discussed in the previous section, the Hamiltonian function for the government's optimization problem is given by

$$\begin{aligned} H &= \Delta\left(\hat{w}\right) + \alpha \log k + v\left(g\right) + \lambda k\left(\hat{r} - \rho\right) \\ &\quad + \phi\left[f\left(k, \Lambda\left(\hat{w}\right)k, k, \Lambda\left(\hat{w}\right)k, g\right) - \hat{r}k - \hat{w}\Lambda\left(\hat{w}\right)k\right]. \end{aligned}$$

The critical first-order conditions for finding out the optimal capital taxation in the steady state are the following:

$$H_{\hat{r}} = \left(\lambda - \phi\right)k = 0 \ \text{ for } \hat{r} > 0, \tag{53}$$

$$H_{\hat{w}} = \Delta'\left(\hat{w}\right) + \phi\left[\left(f_n + f_{\bar{n}} - \hat{w}\right)\Lambda'\left(\hat{w}\right) - \Lambda\left(\hat{w}\right)\right]k = 0 \text{ for } \hat{w} > 0, \tag{54}$$

$$\dot{\lambda} = \lambda\rho - \lambda\left(\hat{r} - \rho\right) - \frac{\alpha}{k} - \phi\left[\left(f_k + f_{\bar{k}} - \hat{r}\right) + \left(f_n + f_{\bar{n}} - \hat{w}\right)\Lambda\left(\hat{w}\right)\right]. \tag{55}$$

Using the steady-state condition, $\hat{r}=\rho$, (53), (54) and (55) reveal that in the steady state it holds that

$$f_k + f_{\bar{k}} - \hat{r} = \rho + \frac{\alpha}{\Delta'(\hat{w})}\left\{(f_n + f_{\bar{n}} - \hat{w})\left[\Lambda'(\hat{w}) - \Lambda(\hat{w})\Delta(\hat{w})\right] - \Lambda(\hat{w})\right\}.$$

The above equation is the same as (37) in the model without public capital. Therefore, as before, the negative capital income tax rule obtained in the open-loop equilibrium is not necessarily established.

Now suppose that the government may issue debt. The flow budget constraint of the government becomes

$$\dot{b} = I + \hat{r}a + \hat{w}n - f\left(k, n, \bar{k}, \bar{n}, g\right),$$

where $I=\dot{g}$ and $a=k+b$. We assume that the investment for public capital is nonnegative. The Hamiltonian function for the government's problem may be set as

$$\begin{aligned} H &= \Delta(\hat{w}) + \alpha \log a + v(g) + \lambda a(\hat{r}-\rho) \\ &\quad + \phi\left[I + \hat{r}a + \hat{w}\Lambda(\hat{w})\rho a - f(k, \Lambda(\hat{w})\rho a, k, \Lambda(\hat{w})\rho a, g)\right] \\ &\quad + \zeta I + \theta(a-k-b). \end{aligned}$$

The control variables for the government are $\hat{r}$, $\hat{w}$ and I. The necessary conditions for an optimum should involve the following:

$$H_{\hat{r}} = a(\lambda+\phi) = 0 \text{ for } \hat{r} > 0, \tag{56}$$

$$H_I = \phi + \zeta = 0 \text{ for } I > 0, \tag{57}$$

$$H_k = -\phi(f_k + f_{\bar{k}}) - \theta = 0, \tag{58}$$

$$\dot{\lambda} = \rho\lambda - \lambda(\hat{r}-\rho) - \phi\left[\hat{r} + (\hat{w} - f_n - f_{\bar{n}})\Lambda(\hat{w})\rho\right] - \theta, \tag{59}$$

$$\dot{\phi} = \rho\phi + \theta, \tag{60}$$

$$\dot{\zeta} = \rho\zeta + \phi f_g - v'(g). \tag{61}$$

Notice that the steady state condition for ϕ in (60) is $\rho\phi+\theta=0$. Thus, from (58) we find that $\hat{r} = f_k + f_{\bar{k}}$. This means that if there is no external effect of private capital, it holds that $\hat{r}=f_k$ so that $\tau_k = 0$ in the steady state. In other words, when external increasing returns are associated only with public capital formation, the public investment should be eventually financed by wage income taxation alone. Finally, (56), (57) and $\dot{\zeta} = \dot{\phi} = 0$ in (60) and (61) present

$$f_g + \frac{v'(g)}{\zeta} = \rho = f_k + f_{\bar{k}}.$$

This is equivalent to the non-arbitrage condition between the private and the public capital shown by (52) in the case of the open-loop equilibrium.

We may summarize the above discussion:

Proposition 5 *If there is a productive public capital and if the government runs a balanced budget in every period, in the feedback equilibrium the optimal capital income tax rule established in the open-loop equilibrium fails to hold. However, if the market for government bond exists, the optimal capital income taxation in the steady state in the open-loop equilibrium may hold even if the government takes a feedback strategy.*

6 Conclusion

This paper has studied optimal taxation in dynamic economies with external increasing returns. We have paid attention to the issue of how the optimal capital taxation changes according to the presence of market imperfection and to the type of policy rules the fiscal authority can take. We have shown that when the stable open-loop Stackelberg equilibrium can be established, the optimal rate of tax on capital income is negative in the steady state in order to internalize external effects generated by the social level of capital stock. This conclusion also holds when increasing returns are generated by the productive public capital. In analyzing an example of feedback equilibrium, we have confirmed that the negative capital income tax rule obtained under the open-loop policies may not hold, if the government does not issue interest bearing bonds. However, if the market for the government bonds exists, the negative capital income tax rule is established even under the feedback policy rule.

The main results of this paper suggest that capital income taxation should not be used unless the source of market distortion is the external effect generated by capital stock. In other words, the capital income taxation in the steady state should be zero if there are no externalities associated with the aggregate level of capital. Although such an optimal rule of capital income taxation may not be held in the feedback equilibrium, introduction of additional strategies of the government such as issuing debt can establish the proposition. In this sense, the results derived in this paper would support a rather strong advocate of the zero-capital income taxation claimed by Atkenson, Chari and Kehoe (1999) in the context of models with perfect markets.[12]

[12] See also Chari and Kehoe (1999), Coleman II (2000), and Jones, Manuelli and Rossi (1997).

Appendix

Xie (1997), Long and Shimomura (1999) and Lansing (1999) deal with a class of Staceklberg-type differential games in which the open-loop solutions coincide with the corresponding feedback solutions. Although these authors present a variety of examples with different economic implications, the models they examine have a common feature: the trajectories of the costate variables for the follower's problem are independent of the control variables of the leader. In this appendix we develop a general discussion that involves the examples examined by those studies as special cases.

For expositional simplicity, we assume that both the leader and the follower have the same state variables and the same dynamic constraints. These simplifications are not essential for the main results discussed below. First, assume that the follower solves the following optimization problem:

$$\max \int_0^\infty u\left(x_t, c_t, z_t\right) e^{-\rho t} dt$$

subject to

$$\dot{x}_t = g\left(x_t, c_t, z_t\right), \tag{A1}$$

where x is the vector of state variables for the follower, and c and z respectively denote vectors of control variables of the follower and the leader. The initial value of x is also given for the follower. In the case of open-loop Stackelberg game, the leader announces the whole sequence of its control variables, $\{z_t\}_{t=0}^\infty$, which are the functions of time. The follower takes this sequence as given and selects the optimal strategy, $\{c_t\}_{t=0}^\infty$.

To characterize the optimization conditions for the follower, set up the Hamiltonian function such that

$$\mathcal{H}\left(x_t, c_t, \psi_t, z_t\right) = u\left(x_t, c_t, z_t\right) + p_t g\left(x_t, c_t, z_t\right).$$

The necessary conditions for an optimum are:

$$u_c\left(x_t, c_t, z_t\right) + p_t g_c\left(x_t, c_t, z_t\right) = 0, \tag{A2}$$

$$\dot{p}_t = p_t\left(\rho - g_x\left(x_t, c_t, z_t\right)\right) - u_x\left(x_t, c_t, z_t\right), \tag{A3}$$

together with (A1) and the transversality conditions:

$$\lim_{t\to\infty} e^{-\rho t} p_t x_t = 0, \tag{A4}$$

Since (A2) yields $c_t = c(x_t, p_t, z_t)$, the canonical equations for the follower's problem can be summarized as

$$\dot{x}_t = g(x_t, c(x_t, p_t, z_t), z_t) \tag{A5}$$

$$\dot{p}_t = p_t [\rho - g_x (x_t, c(x_t, p_t, z_t), z_t)] - u_x (x_t, c(x_t, p_t, z_t), z_t) \tag{A6}$$

Now let us define the value function of the follower's problem:

$$V(x_t, Z_t) = \max_{\{c_s\}_{s=t}^{\infty}} \left\{ \int_t^{\infty} e^{-\rho(s-t)} u(x_s, c_s, z_s)\, ds \text{ s.t. (A1), } x_t = \text{given} \right\},$$

where Z_t denotes a given sequence of the leader's strategies after t, that is, $Z_t = \{z_s\}_{s=t}^{\infty}$. As is well known, when the value function is differentiable with respect to x_t, we obtain $p_t = V_x(x_t, Z_t)$. Therefore, the follower's optimal choice satisfying (A2) may be written as

$$c_t = c(x_t, V_x(x_t, Z_t), z_t). \tag{A7}$$

This means that the optimal level of the control variable for the follower depends not only on the levels of the state variables and the control variables of the leader at t but also on the entire sequence of the leader's policy variables after t, $\{z_s\}_{s=t}^{\infty}$. However, if V_x is independent of Z_t, then we obtain $\psi_t = V_x(x_t)$, so that the costate variable for the follower depends on the x_t alone. As a result, (A5) becomes

$$c_t = c(x_t, V_x(x_t), z_t). \tag{A8}$$

Namely, the optimal value of c_t is completely specifies by x_t and z_t.

If the above holds, denoting the leader's instantaneous objective function by $U(x, c, z)$, the optimization problem of the leader becomes

$$\max \int_0^{\infty} e^{-\rho t} U(x_t, c(x_t, V_x(x_t), z_t), z_t)\, dt$$

subject to

$$\dot{x}_t = g(x_t, c(x_t, V_x(x_t), z_t), z_t)$$

and the initial condition on x. Since this problem is a 'game against the nature', the open-loop solution for this problem coincides with the feedback solution that satisfies the Hamilton-Jacobi-Bellman equation such that

$$\rho V^*(x_t) = \max_{z_t} \{u(x_t, c(x_t, \psi(x_t), z_t), z_t) + V_x^*(x_t)\, g(x_t, c(x_t, \psi(x_t), z_t), z_t)\}, \tag{A9}$$

where $V^*(x_t)$ is the value function for the leader's problem defined by

$$V^*(x_t) = \max_{\{z_s\}_{s=t}^{\infty}} \left\{ \int_t^{\infty} e^{-\rho(s-t)} U\left(x_t, c\left(x_t, V_x\left(x_t\right), z_t\right), z_t\right), \text{s.t. (A1), } x_t = \text{given} \right\}$$

The resulting optimal choice of z_t satisfies

$$u_c c_z + u_z + V_x^*\left(x_t\right)\left(c_z + g_z\right) = 0. \tag{A10}$$

(A8) gives the feedback solution $z_t = z(x_t)$ in which the leader's control variables depend only on the current level of the state variables. Naturally, this solution coincides with the open loop solution obtained by solving the leader's problem by use of the maximum principle.

An obvious condition for that V_x is independent of Z_t is that V function can be written as additively separable function between x_t and Z_t.

$$V\left(x_t, Z_t\right) = V^1\left(x_t\right) + V^2\left(Z_t\right). \tag{A11}$$

As shown in Lemma 1, the value function of the representative family in our example is

$$V\left(k_t, \{\hat{w}_s\}_{s=t}^{\infty}, \{\hat{r}_s\}_{s=t}^{\infty}, \{g_s\}_{s=t}^{\infty}\right) = \max \int_t^{\infty} e^{-\rho(s-t)} \left[\Delta\left(\hat{w}_s\right) + \alpha \log k_s + v\left(g_s\right)\right] ds,$$

Noting that the solution of (32) is $k_s = k_t \exp\left(\int_t^s \left(\hat{r}_\xi - \rho\right) d\xi\right)$, we find that the value function is rewritten as

$$\begin{aligned} & V\left(k_t, \{w_s\}_t^{\infty}, \{\hat{r}_s\}_{s=t}^{\infty}, \{g_s\}_{s=t}^{\infty}\right) \\ = & \frac{\alpha}{\rho} \log k_t + \int_t^{\infty} e^{-\rho(s-t)} \left\{ \alpha \left(\int_t^s \left(\hat{r}_\xi - \rho\right) d\xi \right) + \Delta\left(\hat{w}_s\right) + v\left(g_s\right) \right\} ds, \end{aligned}$$

so that the separability condition is satisfied.

Similarly, in the models used by Lansing (1999) and Long and Shimomura (1999), x is a scalar and the followers dynamic equations (A6) and (A7) are written as

$$\begin{aligned} \dot{x}_t &= h\left(z_t\right) x_t - 1/p_t, \\ \dot{p}_t &= p_t\left(\rho - h\left(z_t\right)\right). \end{aligned}$$

As shown by Lemma 1, the solutions of x_t and p_t that fulfill the transversality condition, $\lim_{t\to\infty} e^{-\rho t} p_t x_t = 0$, should satisfy

$$p_t x_t = 1/\rho \text{ for all } t \geq 0.$$

Therefore, the costate variable of the follower is independent of the sequence of z_t in every period. [13] The models examined by Xie (1997) assumes that income is uniformly taxed. Using our notation, the felicity function of the follower is $u = \left(c_t^{1-\sigma} - 1\right) / (1-\sigma) + v(z_t)$ and the dynamic constraint for the follower is $\dot{x}_t = h(z_t)\, x_t^{\beta} - c_t$. The canonical equations of the follower's problem is

$$\begin{aligned} \dot{x}_t &= h(z_t)\, x_t^{\beta} - p_t^{-1/\sigma}, \\ \dot{p}_t &= p_t \left(\rho - \beta h(z_t)\, x_t^{\beta-1}\right). \end{aligned}$$

It is easy to see that the solution of the above satisfying the transversality condition holds the relation $p_t^{1/\sigma} x_t = 1/\rho$, either if $\beta = \sigma$ or if $\beta = 1$. (When $\beta = 1$, the model is becomes an "Ak" model and hence the transition process does not exist.), Again, p_t depends on x_t alone so that the leader cannot directly control the behavior of p_t. Xie's (1997) examples show that the assumption of a log utility function of the follower may not be necessary to establish the case where the future values of the leader's control variables fail to affect the costate variables of the follower's problem.[14]

[13] Kemp, Long ans Shimomura (1993) analyze the feedback solution of the model in Judd (1985). A specific example they use belongs to the calss of models discussed above.

[14] Mino (2000) presents a more detailed discussion on the conditions under which the solutions in open-loop Stackelberg differential games satisfy dynamic consistency.

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