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15 September 2009

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Abstract: Canonical analysis of the classical general equilibrium model demonstrates the existence of an open and dense subset of standard economies that possess fully-revealing rational expectations equilibria. This paper shows that the analogous result is not true in urban economies. An open subset of economies where none of the rational expectations equilibria fully reveal private information is found. There are two important pieces. First, there can be information about a location known by a consumer who does not live in that location in equilibrium, and thus the equilibrium rent does not reflect this information. Second, if a consumer's utility depends only on information about their (endogenous) location of residence, perturbations of utility naturally do not incorporate information about other locations conditional on their location of residence. Existence of a rational expectations equilibrium is proved. Space can prevent housing prices from transmitting information from informed to uninformed households, resulting in an inefficient outcome. (*JEL Classifications*: D51; D82; R13)

Keywords: Urban Economics; General Equilibrium; Private Information; Rational Expectations

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*We would like to thank John Conlon and Ping Wang for their valuable comments. An early version of this paper was presented at the 2008 North American Regional Science Association International Meeting, the 2009 Midwest Economic Theory Meeting, and the 2009 North American Summer Meeting of the Econometric Society. The second author acknowledges financial support from the Center for Research in Economics and Strategy (CRES) at the Olin Business School, Washington University in St. Louis. The authors retain responsibility for the contents of this paper.

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1 Introduction

1.1 Motivation

People can never fully comprehend the quality and the circumstances of a city until they experience a significant part of their life living in that city. Information on physical amenities of a city (i.e., weather, parks, museums, crime, traffic jams) is easily acquired by both consumers and researchers, so there is institutional and academic work on the quality of life in cities.[1] However, people cannot completely ensure that they choose the right city or location within the city for their family before they start experiencing life there. For example, there could be uncertainty about the quality of schools, congestion of commuting routes contingent on resident and business location, or even major highway closures. Current occupants of the city, or people with friends living in the city, might have information that others don't have. Moreover, even though the current environment of the city can be understood, it is not surprising that the future developments of cities are not known with certainty, but might be known better by current occupants.[2]

On the one hand, information about life in a city is reflected in the demand for and thus the price of housing in the city.[3] Since people are rational in understanding and using the relationship associating a specific state of nature with a specific equilibrium price, depending on what model people have in mind for how equilibrium prices are determined, the price of housing can be a signal for people in choosing a city best suited to their life style. Recall that the concept of rational expectations equilibrium requires agents to use models that are not obviously controverted by their observations of the market. Therefore, the question of whether the price of housing can play

[1] For example, Rosen (1979), Roback (1982), and Blomquist, Berger, and Hoehn (1988) develop a quality of life index for urban areas (QOLI), that measures or implicitly prices the value of local amenities in urban areas.

[2] For example, Cronon (1991) discusses the success of Chicago in surpassing other competitive cities, such as St. Louis, in the early development of the Midwest.

[3] It can also be reflected in wages, but for simplicity we focus on rent.

a significant role in transmitting information from informed people to uninformed people not only addresses the question of the efficiency of housing markets, but is also related to the issue of the existence of rational expectations equilibrium in urban economics.

Available information is utilized by agents in a rational expectations equilibrium, especially the information conveyed by equilibrium prices. Radner (1979) shows that in a particular asset trading model, if the number of states of initial information is finite then, generically, rational expectations equilibria exist where all traders' private initial information is revealed. In contrast to Radner's model, that fixes state-dependent preferences and then focuses on the information concerning traders' conditional probabilities of various events, Allen (1981) considers a space of economies that is defined by state-dependent preferences and confirms Radner's conclusion in that context. When state space is infinite, Allen (1981) shows that the generic existence of fully-revealing rational expectations equilibria depends on the condition that the price space must have at least as high a dimension as the state space. Jordan (1980) considers a model where information revealed by endogenous variables can be affected by expectations, and then characterizes the data that allow the generic existence of rational expectations equilibria. Jordan concludes that unless the public prediction is based on a very narrow class of data, a statistically correct expectation may fail to exist even for otherwise well-behaved economies.

The existence of rational expectations equilibria where prices do not fully reveal the state of nature motivates the development of this paper. As shown in standard general equilibrium models in the literature, fully revealing rational expectations equilibrium demonstrates the efficiency of market prices in information transmission. The cases where the rational expectations equilibrium is not fully revealing are more interesting, for they admit a positive value of private information (that cannot be learned by observing prices) and space for discussing purchases of and strategic behavior using private information. In contrast with standard models, this paper focuses on the existence of non-fully revealing rational expectations equilibrium. In contrast with Allen (1981), who proves the existence of an open and dense subset of economies that possess fully-revealing rational expectations equilibria in the standard general equilibrium model with a finite number of states, this paper shows that the analogous result does not hold in urban economies. An open subset of economies is found, where all the rational expectations equilibria of these economies do not fully reveal private information.

Though in different settings, the common intuition behind these economies is consistent. First of all, households' bid rents reflect their ex ante valuations for housing, and the expected valuations reflect households' information (and their prior distributions) about the states. However, the equilibrium bid rent reveals only the winner's valuation, instead of being determined by all households' valuations. Therefore, in urban economics, *the equilibrium price of land reflects only the ex ante valuation and the information of the household with the highest willingness-to-pay for a location.* In contrast, the standard general equilibrium model has aggregate excess demand that is dependent on every household's demand. This generates complete information revelation in equilibrium generically, if there are enough prices. The difference between the models is due to the standard assumption in urban economics that each person can be in only one place at one time. In this circumstance, the equilibrium price might not fully reveal households' private information, even if there are many prices and few states. *For example, if in equilibrium a household living in one location has information about another location, this information might not be revealed in equilibrium rents.*

The other important component, that yields an open set of economies with not all information revealed in equilibrium, concerns perturbations of utility functions. The set of states affecting utility of households living in one location is assumed to be different from the set of such states in another location; in other words, we use a product structure for the state space.

This is what we mean when we say spatially local perturbations of utility. Thus, when we consider perturbations of utility functions, we do not allow the utility of households living in one location to depend even a little on states belonging to other locations. This is what we mean when we say perturbations are spatially local.

The model that we present covers both within-city locations and the comparison of different cities, though the latter case is the focus of this paper. This paper is organized as follows: Two explicit examples give the intuition behind the non-existence of fully-revealing rational expectations equilibrium in Section 2. For generic results, in Section 3, we find an open subset of economies with no fully-revealing rational expectations equilibrium, provided that perturbations are spatially local. In Section 4, the existence of rational expectations equilibrium is demonstrated. When some household is insensitive (to be defined precisely in this section), there exists a unique non-fully revealing rational expectations equilibrium. When all households are not insensitive, there exists a fully-revealing rational expectations equilibrium. When spatially non-local perturbations are considered, the results are the same as the ones in standard general equilibrium models, namely generic existence of fully revealing rational expectations equilibrium. In this case, generically households are not insensitive. In Section 5, it is shown that the introduction of financial markets into our model can restore the existence of a fully-revealing rational expectations equilibrium, also restoring efficiency of equilibrium allocations. Whether the introduction of financial markets is reasonable is also examined. Section 6 concludes.

2 The Examples

Before stating formally and proving the results, let us examine a few examples. In the first example, one of the households is fully informed, whereas the other has no information. In the second example, both households have partial information about the states of nature in different locations. In both examples, the equilibrium prices are the same in different states, and hence illustrate an economy where the rational expectations equilibria do not fully reveal the private information of households. Examples similar to these appear in the literature on rational expectations in the standard general equilibrium model, though in that literature they belong to the complement of a generic set, and have a very different flavor.

2.1 The Framework

Suppose there are n households indexed by $j \in N \equiv \{1, ..., n\}$ and n locations, $k \in K \equiv \{1, ..., n\}$, each endowed with a fixed land supply of $\bar{s}_k$. We consider the case where consumers obtain different utilities from living in different locations. These could represent either areas within a city or in different cities. There are more than two states in each location, $\omega_k \in \Omega_k$, $k \in K$, representing state-dependent preference differences in our model, each realized with a probability that is common knowledge. Let $\omega \equiv (\omega_k)_{k \in K}$ and $\Omega = \times_{k \in K} \Omega_k$ denote the state and state space of the economy. Beside locations, in state ω, each household j has to choose the lot size of his/her house and the consumption of composite good in k, denoted by $s_{jk}(\omega)$, $z_{jk}(\omega)$, respectively. Since it is impossible to consume a house at the same instant in two locations: $s_{jk}(\omega) > 0$ implies $s_{jk'}(\omega) = 0$, $\forall k' \neq k$. Such a locational indivisibility is one of the unique characteristics of land and houses compared to other commodities. To focus on an exchange economy, standard in both rational expectations general equilibrium and urban economics models, suppose that household j earns a fixed income Y_j of composite good in all states. To placate urban economists, we shall introduce a commuting cost, but all of our arguments hold when commuting cost is set to zero and there is only a utility difference between locations. Consider location 1 as a central business district (CBD) and other locations as suburbs. All job opportunities are located in the CBD. Following Fujita, Krugman, and Venables (2001), there is only commuting from location k, $k > 1$ to the CBD, where the commuting cost from location k to the CBD is denoted by T_k. It is assumed $0 = T_1 < T_2 < ... < T_n < \min(Y_j)_{j \in N}$ to ensure that there is no vacant location.

Each household can consume housing in only one location. Denote household j's consumption plan in k in state ω as $\psi_{jk}(\omega) \equiv (s_{jk}(\omega), z_{jk}(\omega))$ and let $\psi_j(\omega) \equiv (\psi_{jk}(\omega))_{k \in K}$ denote j's consumption plan in state ω in all locations. The ex post utility function of household j living in k in state ω, given $\psi_{jk}(\omega)$, is denoted by $u_{jk}(\psi_{jk}(\omega), \omega)$, $\omega \in \Omega$, and the ex post utility of household j choosing to live in their optimal location is

$$u_j(\psi_j(\omega), \omega) \equiv \max_k \{(u_{jk}(\psi_{jk}(\omega), \omega)_{k \in K}\},\ \omega \in \Omega.$$

Let $p_k(\omega)$ denote the rent per unit of housing in location k in state ω, $k \in K$, $\omega \in \Omega$, and normalize the price of freely mobile composite consumption good to be 1. Let $P_k(\omega) \equiv [p_k(\omega) \ \ 1]$ be the price vector for housing and composite good in k in state ω where the composite good is numeraire. The general optimization problem for household $j \in N$ with n locations, given his/her information structure $\mathcal{F}_j$, is:[4]

$$\begin{aligned}
&\max_{\psi_j(\omega)} \ Eu_j(\psi_j(\omega)|\mathcal{F}_j) \\
&\text{s.t.} \quad \sum_{k \in K} P_k(\omega)\psi_{jk}(\omega) + \sum_{k \in K} \left\lceil \frac{s_{jk}(\omega)}{\sum_{k' \in K} s_{jk'}(\omega)} \right\rceil T_k \leq Y_j, \\
&\qquad \psi_{jk}(\omega) \neq 0 \text{ implies that } \psi_{jk'}(\omega) = 0,\ \forall k, k' \in K, k' \neq k \\
&\qquad \psi_j(\omega) \in \mathbb{R}_+^{2n} \text{ is } \mathcal{F}_j\text{-measurable.}
\end{aligned} \tag{1}$$

Let $P(\omega) \equiv (P_k(\omega))_{k \in K}$ denote the prices in all locations in state ω. The rents are collected and consumed by an absentee landlord L who owns all the housing and whose utility is $u_L((s_{Lk})_{k \in K}, z_L) = z_L$ in all states. The landlord is endowed with an inelastic supply of housing in all locations. Households

[4] The ceiling function, denoted by $\lceil \theta \rceil$, is defined by the smallest integer greater than or equal to θ, i.e., $\lceil \theta \rceil \equiv \min\{n \in \mathbb{Z} | \theta \leq n\}$. Notice that $\lceil \frac{s_{jk}(\omega)}{\sum_{k' \in K} s_{jk'}(\omega)} \rceil$ can be either 1 or 0, depending on whether household j lives in location k or not.

can augment their private information by and only by using the information conveyed by prices.[5] Let $P : \Omega \to \mathbb{R}^{2n}_+$ and $\psi_j : \Omega \to \bigcup_{k \in K} \mathbb{R}^2_+$ be mappings from the state space to the price space and j's consumption space, respectively. The information that prices convey to all agents is denoted by $\sigma(P)$, the sub-σ-field of $\mathcal{F}$ generated by the vector-valued random variable P. Let μ denote a (countably) additive probability measure defined on $(\Omega, \mathcal{F})$, and then $E[u_j(\psi_j(\omega), \omega)|\mathcal{F}_j \vee \sigma(P^*)] \equiv \sum_{\omega \in \Omega} u_j(\psi_j(\omega), \omega)\ \ \mu(\omega|\mathcal{F}_j \vee \sigma(P^*))$ is household j's expected utility of choosing ψ_j,[6] based on private information and the information given by P^*. Following Allen (1981), the concept of rational expectations equilibrium is formally defined as follows.

Definition 1 *A rational expectations equilibrium is defined as an equivalence class of $\mathcal{F}$-measurable price functions $P^* : \Omega \to \mathbb{R}^{2n}_+$, and for each $j \in N$, an equivalence class of $\mathcal{F}_j \vee \sigma(P^*)$-measurable allocation functions $\psi^*_j : \Omega \to \bigcup_{k \in K} \mathbb{R}^2_+$ such that*
(i) $P^*_k(\omega) \cdot \psi^*_{jk}(\omega) \leq Y_j - T_k$ for μ-almost every $\omega \in \Omega$;
(ii) If $\psi'_j : \Omega \to \bigcup_{k \in K} \mathbb{R}^2_+$ satisfies the informational constraint that ψ'_j is $\mathcal{F}_j \vee \sigma(P^*)$-measurable and the budget constraint $P^*_k(\omega) \cdot \psi'_{jk}(\omega) \leq Y_j - T_k$, $\forall k \in K$, for μ-almost every $\omega \in \Omega$, then $\forall j \in N$,

$$E[u_j(\psi'_j(\omega), \omega)|\mathcal{F}_j \vee \sigma(P^*)] \leq E[u_j(\psi^*_j(\omega), \omega)|\mathcal{F}_j \vee \sigma(P^*)];$$

(iii) $\sum_{k \in K} \sum_{j \in N} z^*_{jk}(\omega) + z^*_L(\omega) + \sum_{k \in K} \sum_{j \in N} \ulcorner \frac{s^*_{jk}(\omega)}{\sum_{k' \in K} s^*_{jk'}(\omega)} \urcorner T_k = \sum_{j \in N} Y_j$, $\sum_{j \in N} s^*_{jk}(\omega) = \bar{s}_k$, and for each j, $\psi^*_{jk}(\omega) \neq 0$ implies that $\psi^*_{jk'}(\omega) = 0$, $\forall k' \in K, k' \neq k$ for μ-almost every $\omega \in \Omega$.

Condition (i) shows that budget constraint holds for every state that can happen with a positive probability. Condition (ii) represents maximization of expected utility subject to the budget. Condition (iii) represents material balance and restricts each consumer to own housing in one and only one location. This is the minimal perturbation of the standard general equilibrium

[5] When households condition their expectations on additional market variables, the equilibrium concept is defined as a generalized rational expectations equilibrium; see Allen (1998).

[6] Following Aumann (1976), the join $F_j \vee \sigma(P^*)$ denotes the coarsest common refinement of F_j and $\sigma(P^*)$.

model necessary to make it compatible with urban economics, i.e., it is the standard general equilibrium model with a standard assumption in urban economics that restricts each consumer to consume housing in one and only one location. In what follows, we will introduce and solve for a bid rent equilibrium with uncertainty, which is equivalent to the solution of a standard market equilibrium (see Lemma 1 below). This device is common in urban economics, and is used "almost everywhere."

Given a vector of households' utility levels in state ω, $u(\omega) \equiv (u_j(\omega))_{j\in N}$, bid rent $\Psi_{jk}(u_j(\omega), \omega)$ is the maximum rent per unit of housing that the household j is willing to pay for residing in k in state ω while enjoying a given utility level $u_j(\omega)$, $j \in N$, $k \in K$. Similar to that households may learn extra information from P^* in Definition 1, for a given $u^*(\omega) \equiv (u_j^*(\omega))_{j\in N}$, denote $\Psi^*(\omega) \equiv (\Psi_k^*(\omega))_{k\in K}$, where $\Psi_k^*(\omega) \equiv \Psi_k(u^*(\omega), \omega) = \max_j\{\Psi_{jk}(u_j^*(\omega), \omega)\}$, then households form expected utilities based on private information and the information revealed by Ψ^*; however, $\Psi_{jk}(u_j^*(\omega), \omega)$ is determined by households' optimization. Given $u^* : \Omega \to \mathbb{R}_+^n$ and $\Psi^* : \Omega \to \mathbb{R}_+^n$, mappings from the state space to the utility and the bid rent space, respectively, denote $\Psi_{jk}(u_j^*(\omega), \omega) \equiv \max_{\psi_{jk}(\omega)}\{\frac{Y_j - T_k - z_{jk}(\omega)}{s_{jk}(\omega)} | E[u_j(\psi_j(\omega), \omega)|\mathcal{F}_j \vee \sigma(\Psi^*)] = u_j^*(\omega)\}$, a bid rent equilibrium is constituted when the given mappings u^*, Ψ^* and the corresponding $\Psi_{jk}(u_j^*(\omega), \omega)$ are consistent in that $\Psi_k^*(\omega) = \max_j\{\Psi_{jk}(u_j^*(\omega), \omega)\}$, $\forall k \in K$, $\omega \in \Omega$. As shown in Figure 5, $\Psi_{jk}(u_j^*(\omega), \omega)$ is in fact the slope of j's budget line in location k that is tangent to his/her indifference curve with a utility level $u_j^*(\omega)$. When we solve the maximization problem of $\Psi_{jk}(u_j^*(\omega), \omega)$, we obtain the optimal lot size $S_{jk}(u_j^*(\omega), \omega)$. Comparing to $\psi_{jk}(\omega) \equiv (s_{jk}(\omega), z_{jk}(\omega))$ in a standard utility-maximization problem, here we denote $\varphi_{jk}(u_j(\omega), \omega) \equiv (S_{jk}(u_j(\omega), \omega), Z_{jk}(u_j(\omega), \omega))$ to be the optimal consumptions (arg max) in a bid-maximization problem. It can be checked that $S_{jk}(u_j^*(\omega), \omega) = s_{jk}(\omega)$ when $\Psi_{jk}(u_j^*(\omega), \omega) = p_k(\omega)$ when $u_j^*(\omega) = u_{jk}(\psi_{jk}(\omega))$ is given. Furthermore, recall again that in Lemma 1, we will show that the solutions of these two maximization problems are exactly the same. Given u^*, for notational convenience, also denote $S^*_{jk}(\omega) \equiv S_{jk}(u^*_j(\omega), \omega)$, $Z^*_{jk}(\omega) \equiv Z_{jk}(u^*_j(\omega), \omega))$, $\varphi^*_{jk}(\omega) \equiv (S^*_{jk}(\omega), Z^*_{jk}(\omega))$, and $\varphi^*_j(\omega) \equiv (\varphi^*_{jk}(\omega))_{k \in K}$.

Definition 2 *A bid rent equilibrium is defined by an equivalence class of $\mathcal{F}$-measurable house price functions $\Psi^* : \Omega \to \mathbb{R}^n_+$, and for each $j \in N$, an equivalence class of $\mathcal{F}_j \vee \sigma(\Psi^*)$-measurable utility functions $u^*_j : \Omega \to \mathbb{R}_+$ such that for each location $k \in K$, for μ-almost every $\omega \in \Omega$:*

$$\Psi^*_k(\omega) \equiv \Psi_k(u^*(\omega), \omega) = \max_j \{\Psi_{jk}(u^*_j(\omega), \omega)\}; \tag{2}$$

$$\varphi^*_{jk}(\omega) \equiv \varphi_{jk}(u^*_j(\omega), \omega)$$
$$= \begin{cases} \arg\max_{\psi_{jk}(\omega)} \{\frac{Y_j - T_k - z_{jk}(\omega)}{s_{jk}(\omega)} \big| E[u_j(\psi_j(\omega), \omega) | \mathcal{F}_j \vee \sigma(\Psi^*)] = u^*_j(\omega)\} \\ \qquad\qquad\qquad\qquad \textit{if} \;\; j \in \arg\max_j \{\Psi_{jk}(u^*_j(\omega), \omega)\}, \\ (0, 0) \quad \textit{if} \;\; j \notin \arg\max_j \{\Psi_{jk}(u^*_j(\omega), \omega)\}; \end{cases} \tag{3}$$

$$\sum_{j \in N} S^*_{jk}(\omega) = \bar{s}_k,$$
$$\sum_{k \in K} \sum_{j \in N} Z^*_{jk}(\omega) + z^*_L(\omega) + \sum_{k \in K} \sum_{j \in N} \ulcorner \frac{S^*_{jk}(\omega)}{\sum_{k' \in K} S^*_{jk'}(\omega)} \urcorner T_k = \sum_{j \in N} Y_j,$$
$$\textit{and } \varphi^*_{jk}(\omega) \neq 0 \textit{ implies that } \varphi^*_{jk'}(\omega) = 0, \; \forall k' \in K, k' \neq k, \forall j \in N. \tag{4}$$

Here, condition (2) shows that the equilibrium housing price in every location is determined by the highest bid rent among households for the housing there. Condition (3) shows that the equilibrium consumption of the household who lives in k maximizes that household's bid rent in k, given his private information and the information revealed by equilibrium prices. Again, condition (4) represents material balance and the standard urban economics assumption that each consumer lives in one and only one location. That is, for each $u_j(\omega)$, there exists $\Psi_{jk}(u_j(\omega), \omega)$ such that the budget line is tangent to j's indifference curve with utility $u_j(\omega)$ at $\varphi_{jk}(u_j(\omega), \omega)$. The bid rent equilibrium requires that, given the equilibrium utility levels and prices, the corresponding consumptions must satisfy the feasibility and indivisibility conditions.

Since each household can consume housing in at most one location, the consumption set is $\bigcup_{k\in K}\mathbb{R}^2_+$, and the ex post state-dependent preferences of living in k, $k\in K$, can be specified by utilities $u_{jk}:\ \Omega_k\to\kappa_{jk}$, where κ_{jk} is a compact subset of $C^r(\mathbb{R}^2_+,\mathbb{R})$, $r\geq 2$, endowed with the weak C^r compact-open topology. Assume that for μ-almost every $\omega\in\Omega$, $u_{jk}(\varphi,\omega)\in\kappa_{jk}$ satisfies for each $\varphi\in\mathbb{R}^2_+$:

(a) strict (differentiable) monotonicity: $D_\varphi u_{jk}(\varphi,\omega)\in\mathbb{R}_{++}$,

(b) strict (differentiable) concavity: $D_{\varphi\varphi}u_{jk}(\varphi,\omega)$ is negative definite, and

(c) smooth boundary condition: the closure in $\mathbb{R}^2$ of the upper contour set $\{\varphi'\in\mathbb{R}^2_{++}|u_{jk}(\varphi',\omega)\geq u_{jk}(\varphi,\omega)\}$ is contained in $\mathbb{R}^2_{++}$.

These conditions ensure that every household's state-dependent preferences are smooth in the sense of Debreu (1972) so that, conditional on any state with a positive probability, demands are well defined C^{r-1} functions. Our examples satisfy these assumptions.

Although it is well-known that bid-rent and competitive equilibria are closely connected (see for example Fujita, 1989), results in the literature cover only the context of no uncertainty. If the rational expectations equilibria were known to be fully revealing, this result could be applied state by state. We require an equivalence result in the context of uncertainty, especially when the rational expectations equilibrium might not be fully revealing. The proof uses classical duality.

Lemma 1 *Given that all households' preferences are representable by a utility function satisfying conditions (a), (b), and (c),* $(\Psi^*(\omega),u^*(\omega))$ *constitutes a bid rent equilibrium if and only if the corresponding* $(\Psi^*(\omega),(\varphi^*_j(\omega))_{j\in N})$ *constitutes a rational expectations equilibrium in a competitive economy.*

Proof. See Appendix A.

2.2 Example 1

Suppose that there are two households ($j\in\{1,2\}$) with the same income ($Y_1=Y_2=Y$), and two locations ($k\in\{x,y\}$) with land endowments $\bar{x}$

and $\bar{y}$, respectively. Household 1's utility is state-dependent but the utility function of household 2 is independent of states. In each location k, there are two states (Low and High) denoted by $\omega_k \in \Omega_k \equiv \{L, H\}$, $k \in \{x, y\}$, which are equally likely to occur and the states in different locations are not correlated. What each agent can observe are events that are subsets of $\Omega \equiv \Omega_x \times \Omega_y$. Denote $\omega \equiv \omega_x \times \omega_y$ as an element of Ω. Furthermore, household 1 has no information, and household 2 knows what the state will be. That is, households' information is represented respectively by $\mathcal{F}_1 = \{\phi, \Omega_x\} \times \{\phi, \Omega_y\}$, $\mathcal{F}_2 = \{\phi, \{H\}, \{L\}, \Omega_x\} \times \{\phi, \{H\}, \{L\}, \Omega_y\}$ which are sub-σ-fields of $\mathcal{F}$, where $\mathcal{F} \equiv \mathcal{F}_1 \vee \mathcal{F}_2$ is the smallest σ-field generated by the class $\mathcal{F}_1 \bigcup \mathcal{F}_2$ of subsets of $\Omega = \{HH, HL, LH, LL\}$. Everything except the true state is common knowledge, so households are assumed to know the relationship between states and prices.

Given information structure $\mathcal{F}_1$, the superscripts on household 1's allocation can be ignored for simplicity until he/she learns something. Utilities will be Cobb-Douglas. The optimization problem for household 1 is to maximize expected utility subject to the budget constraint:

$$\begin{aligned}
\max_{s_{1x}, s_{1y}, z_{1x}, z_{1y}} \quad & Eu_1(s_{1x}, s_{1y}, z_{1x}, z_{1y} | \mathcal{F}_1) \\
& = \max\{E[\alpha_1^\omega \ln(s_{1x}) + \ln(z_{1x}) | \mathcal{F}_1],\ E[\beta_1^\omega \ln(s_{1y}) + \ln(z_{1y}) | \mathcal{F}_1]\} \\
\text{s.t.} \quad & p_x(\omega) s_{1x} + p_y(\omega) s_{1y} + z_{1x} + z_{1y} + \ulcorner \frac{s_{1y}}{s_{1x} + s_{1y}} \urcorner t \leq Y, \\
& s_{1k}\, s_{1l} = 0,\ s_{1k}\, z_{1l} = 0,\ z_{1k}\, z_{1l} = 0, \\
& s_{1k}, z_{1k} \geq 0, \quad \forall k, l = x, y,\ k \neq l,
\end{aligned}$$

where $\alpha_1^\omega, \beta_1^\omega \in \mathbb{R}_{++}$. In contrast, since household 2's utility is state-independent, his/her optimization problem is, for all $\omega \in \Omega$,

$$\begin{aligned}
\max_{s_{2x}(\omega), s_{2y}(\omega), z_{2x}(\omega), z_{2y}(\omega)} & u_2(s_{2x}(\omega), s_{2y}(\omega), z_{2x}(\omega), z_{2y}(\omega), \omega) \\
& = \max\{\alpha_2 \ln(s_{2x}(\omega)) + \ln(z_{2x}(\omega)),\ \beta_2 \ln(s_{2y}(\omega)) + \ln(z_{2y}(\omega))\}
\end{aligned}$$

$$
\begin{aligned}
\text{s.t.} \quad & p_x(\omega)s_{2x}(\omega) + p_y(\omega)s_{2y}(\omega) + z_{2x}(\omega) + z_{2y}(\omega) \\
& \quad + \ulcorner \frac{s_{2y}(\omega)}{s_{2x}(\omega) + s_{2y}(\omega)} \urcorner \, t \leq Y, \\
& s_{2k}(\omega)\, s_{2l}(\omega) = 0, \; s_{2k}(\omega)\, z_{2l}(\omega) = 0, \; z_{2k}(\omega)\, z_{2l}(\omega) = 0, \\
& s_{2k}(\omega), z_{2k}(\omega) \geq 0, \quad \forall k, l = x, y, \; k \neq l,
\end{aligned}
$$

where $\alpha_2, \beta_2 \in \mathbb{R}_{++}$. Suppose that household 1 likes the housing in CBD (x) more than household 2, and household 2 prefers y more than household 1, i.e., $E[\alpha_1^\omega] > \alpha_2$ and $E[\beta_1^\omega] < \beta_2$.

In urban economics, as studied by Alonso (1964), bid rent describes a particular household's willingness to pay for housing in terms of composite commodity, given a fixed utility level. Following Fujita (1989) and our Lemma 1, people live where their bid rents are maximal in equilibrium, and these bid rents constitute equilibrium rents. The bid rent functions of the two households for the housing in x and y are

$$\Psi_{1x}(Eu_1, \omega) = \max_{s_{1x}} \frac{Y - e^{Eu_1}(s_{1x})^{-E[\alpha_1^\omega]}}{s_{1x}}, \tag{5}$$

$$\Psi_{1y}(Eu_1, \omega) = \max_{s_{1y}} \frac{Y - t - e^{Eu_1}(s_{1y})^{-E[\beta_1^\omega]}}{s_{1y}}, \tag{6}$$

$$\Psi_{2x}(u_2(\omega), \omega) = \max_{s_{2x}(\omega)} \frac{Y - e^{u_2(\omega)}(s_{2x}(\omega))^{-\alpha_2}}{s_{2x}(\omega)}, \tag{7}$$

$$\Psi_{2y}(u_2(\omega), \omega) = \max_{s_{2y}(\omega)} \frac{Y - t - e^{u_2(\omega)}(s_{2y}(\omega))^{-\beta_2}}{s_{2y}(\omega)}, \tag{8}$$

where $\omega \in \Omega$. From first and second-order conditions, the optimal land lot sizes for households are

$$S_{1x}^*(Eu_1, \omega) = [\frac{e^{Eu_1}(1 + E[\alpha_1^\omega])}{Y}]^{\frac{1}{E[\alpha_1^\omega]}}, \tag{9}$$

$$S_{1y}^*(Eu_1, \omega) = [\frac{e^{Eu_1}(1 + E[\beta_1^\omega])}{Y - t}]^{\frac{1}{E[\beta_1^\omega]}}, \tag{10}$$

$$S_{2x}^*(u_2(\omega), \omega) = [\frac{e^{u_2(\omega)}(1 + \alpha_2)}{Y}]^{\frac{1}{\alpha_2}}, \tag{11}$$

$$S_{2y}^*(u_2(\omega), \omega) = [\frac{e^{u_2(\omega)}(1 + \beta_2)}{Y - t}]^{\frac{1}{\beta_2}}. \tag{12}$$

From market clearing conditions $S_{jx}^*(\omega) = \bar{x}$ and $S_{jy}^*(\omega) = \bar{y}$, $\forall \omega \in \Omega$, we have

$$Eu_1^* = \begin{cases} \ln[Y] + E[\alpha_1^\omega]\ln[\bar{x}] - \ln[1+E[\alpha_1^\omega]], & \text{if household 1 lives at } x; \\ \ln[Y-t] + E[\beta_1^\omega]\ln[\bar{y}] - \ln[1+E[\beta_1^\omega]], & \text{if household 1 lives at } y, \end{cases} \tag{13}$$

$$u_2^*(\omega) = \begin{cases} \ln[Y] + \alpha_2\ln[\bar{x}] - \ln[1+\alpha_2], & \text{if household 2 lives at } x; \\ \ln[Y-t] + \beta_2\ln[\bar{y}] - \ln[1+\beta_2], & \text{if household 2 lives at } y, \end{cases} \tag{14}$$

for $\omega \in \Omega$. So the equilibrium bid rents of agents in the two locations in two states are

$$\Psi_{1x}^*(\omega) = \frac{E[\alpha_1^\omega]}{1+E[\alpha_1^\omega]}\frac{Y}{\bar{x}}, \tag{15}$$

$$\Psi_{1y}^*(\omega) = \frac{E[\beta_1^\omega]}{1+E[\beta_1^\omega]}\frac{Y-t}{\bar{y}}, \tag{16}$$

$$\Psi_{2x}^*(\omega) = \frac{\alpha_2}{1+\alpha_2}\frac{Y}{\bar{x}}, \tag{17}$$

$$\Psi_{2y}^*(\omega) = \frac{\beta_2}{1+\beta_2}\frac{Y-t}{\bar{y}}, \tag{18}$$

for $\omega \in \Omega$. The equilibrium bid rents are presented in Figure 1, where the horizontal axis represents the location and transportation cost while the vertical axis represents the individuals' bid rents.

Since $\frac{E[\alpha_1^\omega]}{1+E[\alpha_1^\omega]} > \frac{\alpha_2}{1+\alpha_2}$ if and only if $E[\alpha_1^\omega] > \alpha_2$, given $E[\alpha_1^\omega] > \alpha_2$, the bid rent of household 1 for the housing in x is higher than that of household 2 for the housing in x in both states. Similarly, since $\frac{E[\beta_1^\omega]}{1+E[\beta_1^\omega]} < \frac{\beta_2}{1+\beta_2}$ if and only if $E[\beta_1^\omega] < \beta_2$, $E[\beta_1^\omega] < \beta_2$ implies that the bid rent of household 1 for the housing in y is lower than that of household 2 for the housing in y in all states. Therefore, the equilibrium location pattern where household 1 lives at x and household 2 lives in y is verified under the conditions we have assumed.

Notice that there is no equilibrium that fully reveals information. If in equilibrium $\Psi_x^*(HH) = \Psi_x^*(HL) \neq \Psi_x^*(LH) = \Psi_x^*(LL)$, the valuation of household 1 for the housing in x differs in different states (in location x),

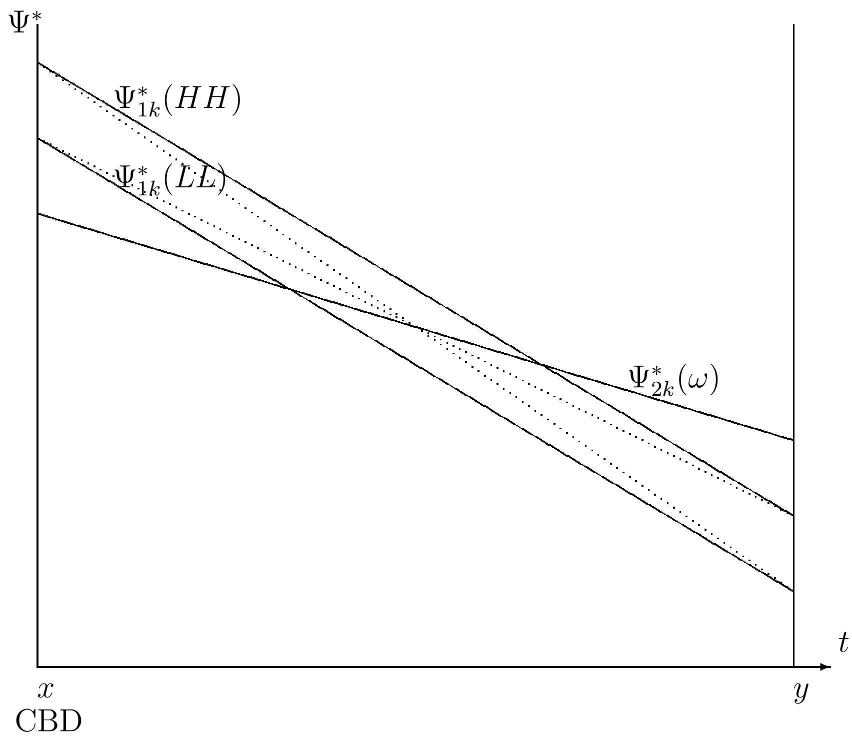

Figure 1: The bid rent functions in Example 1, where the dotted lines represent $\Psi^*_{1k}(HL)$ and $\Psi^*_{1k}(LH)$, respectively.

which conflicts with the assumption that household 1 has no information about the state. Notice also that $\Psi^*_x(\omega)$ and $\Psi^*_y(\omega)$ depend only on the mean of α_1, β_2, and the values of Y, t, $\bar{x}$, and $\bar{y}$. Therefore, the equilibrium rents in the two locations are independent of the realized state, and there exists no fully-revealing rational expectations equilibrium. Even though household 2 knows the state, since household 2 doesn't care about the state, equilibrium prices don't reveal it.

2.3 Example 2

Follow the same setting as in the previous example, but suppose that household 1 knows the state in location y, but has no information about location x. On the other hand, household 2 knows only the state in x, but not the state in y. Let $\Omega \equiv \Omega_x \times \Omega_y$, where $\Omega_x = \Omega_y \equiv \{H, L\}$ represent the state spaces in locations x and y. $\mathcal{F}_1 = \{\phi, \Omega_x\} \times \{\phi, \Omega_y, \{H\}, \{L\}\}$,

$\mathcal{F}_2 = \{\phi, \Omega_x, \{H\}, \{L\}\} \times \{\phi, \Omega_y\} \subseteq \mathcal{F}$ are sub-σ-fields representing private information. Again, the relationship between states and prices is common knowledge.

Each household chooses to live in one and only one location. Moreover, households make their decisions simultaneously. Given an event $\omega \in \Omega$, both households' utilities are state-dependent, so their optimization problems are

$$
\begin{aligned}
\max_{s_{1x}, s_{1y}(\omega), z_{1x}, z_{1y}(\omega)} \quad & Eu_1(s_{1x}, s_{1y}(\omega), z_{1x}, z_{1y}(\omega)|\mathcal{F}_1) \\
& = \max\{E[\alpha_1^\omega \ln(s_{1x}) + \ln(z_{1x})|\mathcal{F}_1],\ \beta_1^\omega \ln(s_{1y}(\omega)) + \ln(z_{1y}(\omega))\} \\
\text{s.t.} \quad & p_x(\omega)s_{1x} + p_y(\omega)s_{1y}(\omega) + z_{1x} + z_{1y}(\omega) + \ulcorner \frac{s_{1y}(\omega)}{s_{1x}(\omega) + s_{1y}(\omega)} \urcorner\, t \leq Y, \\
& s_{1x}\, s_{1y}(\omega) = 0,\ s_{1x}\, z_{1y}(\omega) = 0,\ z_{1x}\, s_{1y}(\omega) = 0,\ z_{1x}\, z_{1y}(\omega) = 0, \\
& s_{1x}, s_{1y}(\omega), z_{1x}, z_{1y}(\omega) \geq 0;
\end{aligned}
$$

$$
\begin{aligned}
\max_{s_{2x}(\omega), s_{2y}, z_{2x}(\omega), z_{2y}} \quad & Eu_2(s_{2x}(\omega), s_{2y}, z_{2x}(\omega), z_{2y}|\mathcal{F}_2) \\
& = \max\{\alpha_2^\omega \ln(s_{2x}(\omega)) + \ln(z_{2x}(\omega)),\ E[\beta_2^\omega \ln(s_{2y}) + \ln(z_{2y})|\mathcal{F}_2]\} \\
\text{s.t.} \quad & p_x(\omega)s_{2x}(\omega) + p_y(\omega)s_{2y} + z_{2x}(\omega) + z_{2y} + \ulcorner \frac{s_{2y}(\omega)}{s_{2x}(\omega) + s_{2y}(\omega)} \urcorner\, t \leq Y, \\
& s_{2x}(\omega)\, s_{2y} = 0,\ s_{2x}(\omega)\, z_{2y} = 0,\ z_{2x}(\omega)\, s_{2y} = 0,\ z_{2x}(\omega)\, z_{2y} = 0, \\
& s_{2x}(\omega), s_{2y}, z_{2x}(\omega), z_{2y} \geq 0;
\end{aligned}
$$

Note that, in fact, the optimized utility of household 1 is state-dependent at y, denoted by $u_{1y}^*(\omega)$, and state-independent at x, denoted by Eu_{1x}^*; $u_{2x}^*(\omega)$ and Eu_{2y}^* are similarly defined. To present a rational expectations equilibrium without revelation of private information, suppose that $E[\alpha_1^\omega] > \alpha_2^\omega$ and $E[\beta_2^\omega] > \beta_1^\omega$, for all $\omega \in \Omega$.

Given these conditions, suppose that households 1 and 2 choose to live in locations x and y, respectively. Their bid rent functions are, $\forall \omega \in \Omega$,

$$\Psi_{1x}(Eu_1, \omega) = \max_{s_{1x}} \frac{Y - e^{Eu_1} s_{1x}^{-E[\alpha_1^\omega]}}{s_{1x}}, \tag{19}$$

$$\Psi_{1y}(u_1(\omega),\omega) = \max_{s_{1y}} \frac{Y - t - e^{u_1(\omega)} s_{1y}^{-\beta_1^\omega}}{s_{1y}}, \tag{20}$$

$$\Psi_{2x}(u_2(\omega),\omega) = \max_{s_{2x}} \frac{Y - e^{u_2(\omega)} s_{2x}^{-\alpha_2^\omega}}{s_{2x}}, \tag{21}$$

$$\Psi_{2y}(Eu_2,\omega) = \max_{s_{2y}} \frac{Y - t - e^{Eu_2} s_{2y}^{-E[\beta_2^\omega]}}{s_{2y}}. \tag{22}$$

Thus, the optimal lot sizes for household 1 and 2 are, $\forall \omega \in \Omega$,

$$S_{1x}^*(Eu_1,\omega) = [\frac{e^{Eu_1}(1 + E[\alpha_1^\omega])}{Y}]^{\frac{1}{E[\alpha_1^\omega]}}, \tag{23}$$

$$S_{1y}^*(u_1(\omega),\omega) = [\frac{e^{u_1(\omega)}(1 + \beta_1^\omega)}{Y - t}]^{\frac{1}{\beta_1^\omega}}, \tag{24}$$

$$S_{2x}^*(u_2(\omega),\omega) = [\frac{e^{u_2(\omega)}(1 + \alpha_2^\omega)}{Y}]^{\frac{1}{\alpha_2^\omega}}, \tag{25}$$

$$S_{2y}^*(Eu_2,\omega) = [\frac{e^{Eu_2}(1 + E[\beta_2^\omega])}{Y - t}]^{\frac{1}{E[\beta_2^\omega]}}. \tag{26}$$

From $S_{jx}^*(\omega) = \bar{x}$ and $S_{jy}^*(\omega) = \bar{y}$, $\forall \omega \in \Omega$, we have

$$Eu_1^*(\cdot|\mathcal{F}_1) = \begin{cases} \ln[Y] + E[\alpha_1^\omega]\ln[\bar{x}] - \ln[1 + E[\alpha_1^\omega]], & \text{if household 1 lives at } x; \\ \ln[Y - t] + \beta_1^\omega \ln[\bar{y}] - \ln[1 + \beta_1^\omega], & \text{if household 1 lives at } y, \end{cases} \tag{27}$$

$$Eu_2^*(\cdot|\mathcal{F}_2) = \begin{cases} \ln[Y] + \alpha_2^\omega \ln[\bar{x}] - \ln[1 + \alpha_2^\omega], & \text{if household 2 lives at } x; \\ \ln[Y - t] + E[\beta_2^\omega]\ln[\bar{y}] - \ln[1 + E[\beta_2^\omega]], & \text{if household 2 lives at } y. \end{cases} \tag{28}$$

Again, households' equilibrium bid rents are

$$\Psi_{1x}^*(\omega) = \frac{E[\alpha_1^\omega]}{1 + E[\alpha_1^\omega]} \frac{Y}{\bar{x}}, \tag{29}$$

$$\Psi_{1y}^*(\omega) = \frac{\beta_1^\omega}{1 + \beta_1^\omega} \frac{Y - t}{\bar{y}}, \tag{30}$$

$$\Psi_{2x}^*(\omega) = \frac{\alpha_2^\omega}{1 + \alpha_2^\omega} \frac{Y}{\bar{x}}, \tag{31}$$

$$\Psi_{2y}^*(\omega) = \frac{E[\beta_2^\omega]}{1 + E[\beta_2^\omega]} \frac{Y - t}{\bar{y}}, \tag{32}$$

where $\omega \in \Omega$. The equilibrium bid rents are drawn in Figure 2, where the horizontal axis represents the location and transportation cost, whereas the individual bid rents are represented by the vertical axis.

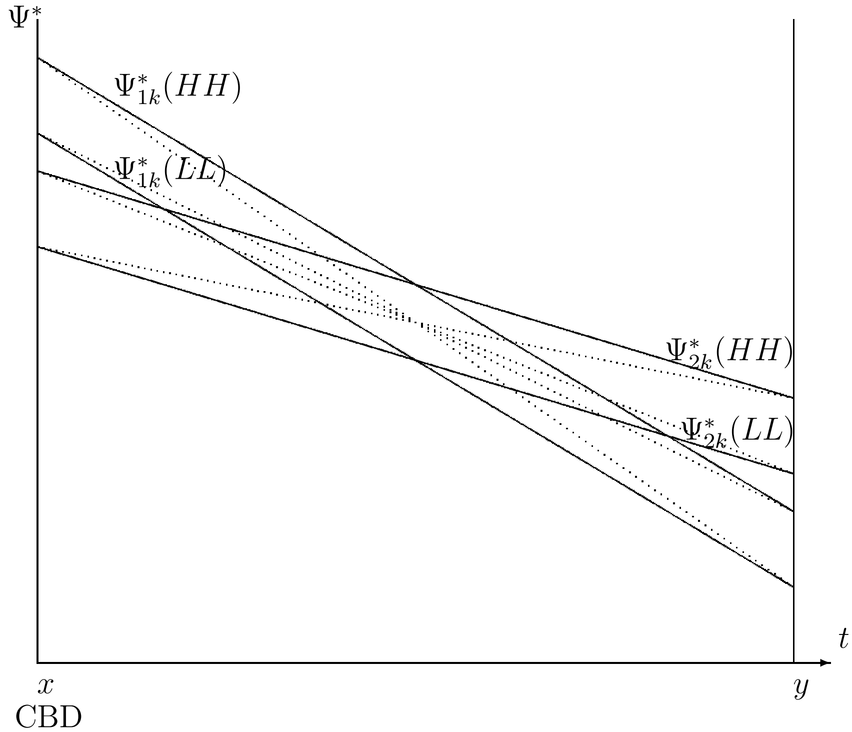

Figure 2: The bid rent functions in Example 2, where the dotted lines represent $\Psi^*_{1k}(HL)$, $\Psi^*_{1k}(LH)$, $\Psi^*_{2k}(HL)$, and $\Psi^*_{2k}(LH)$, respectively.

Inequalities $E[\alpha_1^\omega] > \alpha_2^\omega$ and $E[\beta_2^\omega] > \beta_1^\omega$, $\forall\omega$, imply that the bid rent of household 1 (household 2) for the housing in x (y) is always higher than that of household 2 (household 1). So the equilibrium location pattern where household 1 lives at x and household 2 lives at y is verified.[7]

Again, there is no fully revealing equilibrium in this example. Since Ψ^*_x and Ψ^*_y depend only on Y, t, the mean of the preference parameters and the endowments of land in each location, the equilibrium bid rents are the same in all the realized states. That is, the mapping from prices to preferences is not injective, so fully-revealing rational expectations equilibrium does not exist.[8]

[7] Even when households can observe other households' consumption (of housing and composite good), given that the states in two locations are not correlated, the non-existence of fully-revealing generalized rational expectations equilibria (GREE) still holds in this example.

[8] In these two examples, each household has either full information or no information about the state of a location. We can consider another example where each household has partial information about the state of a location, i.e.,$\Omega_x = \Omega_y = \{H, M, L\}$, $\mathcal{F}_1 = \{\phi, \{H, M\}, \{L\}, \Omega_x\} \times \{\phi, \{H\}, \{M, L\}, \Omega_y\}$, and $\mathcal{F}_2 = \{\phi, \{H\}, \{M, L\}, \Omega_x\} \times$

These examples illustrate different causes for the equilibrium not fully revealing private information: The first example arises because the informed household doesn't care about different states. The second one arises due to the mismatch between informed households and their locations. In the next section, we show that these unfortunate circumstances can persist under small perturbations.

3 An Open Subset of Economies without Fully Revealing Equilibria

The examples represent two points in the space of economies with no fully revealing rational expectations equilibrium. In this section, we generalize the examples and show that, in economies under uncertainty where there is no market for contingent claims or financial contracts, fully revealing rational expectations equilibrium is not present for an open set of economies. But for all parameters satisfying a condition, there exists a rational expectations equilibrium (that might not be fully revealing). This will be proved in the next section.

Suppose there are two households ($j \in N \equiv \{1,2\}$), and two locations ($k \in K \equiv \{x,y\}$). Let $\Omega \equiv \Omega_x \times \Omega_y = \{H,L\} \times \{H,L\}$ be the finite payoff-relevant state space of the economy. Households are assumed to maximize their conditional expected utilities, where the ex post state-dependent preferences of living in location k are specified by $u_{jk} : \Omega_k \to \kappa_{jk}$, where κ_{jk} is a compact subset of $C^r(\mathbb{R}^2_+, \mathbb{R})$ functions, $r \geq 2$, which is endowed with the weak C^r compact-open topology. For each state ω, the economy $(Y, u_j(\psi_j(\omega), \omega)_{j \in N})$ is a smooth economy as defined by Debreu (1972). It is important to notice that u_{jk} is payoff-relevant to only Ω_k, that is, we assume that people living in location k care only about the state in k. Later, we

$\{\phi, \{H,M\}, \{L\}, \Omega_y\}$. Then if household 1 (2) lives in x (y) in equilibrium, except in state LL, states of two locations are not fully revealed by equilibrium bid rents. So there does not exist a fully revealing rational expectations equilibrium.

consider the perturbations that maintain this property.

Before we prove the results, some characteristics of equilibrium must be defined. In a rational expectations equilibrium, the information can be fully revealing, which means that all households can learn the state of nature by observing the equilibrium price and using their private information. Alternatively, the information can be non-fully revealing in a rational expectations equilibrium, where at least one household cannot tell the state of nature from the equilibrium price and their private information. Their formal definitions are as follows.

Definition 3 *A fully-revealing rational expectations equilibrium is a rational expectations equilibrium such that*

$$\mathcal{F}_j \vee \sigma(\Psi^*) = \mathcal{F}, \quad \forall j \in N. \tag{33}$$

When there is at least one j such that the above equality does not hold, we say it is a non-fully-revealing rational expectations equilibrium.

In other words, conditioning on a fully revealing equilibrium price function is equivalent to knowing the pooled information of all households in the economy. Though Allen (1981) proves the existence of an open and dense subset of economies with fully-revealing rational expectations equilibrium in the classical framework, when perturbations location-by-location are considered, Theorem 1 will show that the same statement does not hold in urban economics. Utility functions defined location-by-location are formally called local utilities.[9] We have been using them in this paper up to this point.

Definition 4 (Local Utilities)
Households' preferences are called local when their preferences satisfy $\forall j \in N, k \in K$, $u_{jk} : \Omega_k \to \kappa_{jk}$. If for some j, k, there exists k', $k' \neq k$ such that $u_{jk} : \Omega_k \times \Omega_{k'} \to \kappa_{jk}$ is not constant for some $\omega_{k'}, \omega'_{k'} \in \Omega_{k'}$, then it is called non-local.

[9] Throughout this paper, only preference perturbations are considered since endowment perturbations give households more information if they are state-dependent, and perturbations of ex ante information are not smooth.

That is, saying that utilities are local requires that each household's utility at location k is measurable with respect to only Ω_k when they live in location k. We shall require that when utility functions are perturbed, if they start local, they remain local. We call this a "spatially local perturbation." *Spatially local perturbation* means that if people living in a location care only about the state in the location where they live, then when their utility function is perturbed, it continues to have this property. Spatially local perturbations are more realistic than non-local perturbations in urban economics, since it is not persuasive to say that the perturbed preferences conditional on residence in location k depend on the state in another location. For example, when preference perturbations are considered, in most cases, the state of commuting congestion or crime (or the quality of schools) in Chicago is irrelevant to that in New York. Therefore, in urban economics, it doesn't make sense to consider spatially non-local perturbations as used in standard models. Throughout this paper, to highlight the distinct essence of urban economics, we focus on spatially local perturbations.

It is possible to add other kinds of perturbations to the model, for example national or regional uncertainty, but this would only complicate notation.

Theorem 1 *Given the discrete state space Ω, consider local perturbations of households' preferences. There exists an open subset of economies that possess no fully-revealing rational expectations equilibrium.*

Proof. Consider example 1 first. Notice that in equilibrium, household 1's marginal rate of substitution for housing in terms of composite commodity in location x is $\frac{E[\alpha_1^\omega]}{1+E[\alpha_1^\omega]}\frac{Y}{\bar{x}}$. On the other hand, household 2's marginal rate of substitution for housing in x is $\frac{\alpha_2^\omega}{1+\alpha_2^\omega}\frac{Y}{\bar{x}}$. Let $\alpha_1^{HH} = \alpha_1^{HL} > \alpha_1^{LH} = \alpha_1^{LL}$ and $\beta_1^{HH} = \beta_1^{LH} > \beta_1^{HL} = \beta_1^{LL}$.

Since in the example $E[\alpha_1^\omega] > \alpha_2$ and $E[\beta_1^\omega] < \beta_2$, we can choose $\epsilon^\alpha = \frac{E[\alpha_1^\omega]-\alpha_2}{(E[\alpha_1^\omega]+\alpha_2)Y+(2+E[\alpha_1^\omega]+\alpha_2)\bar{x}} > 0$, $\epsilon^\beta = \frac{\beta_2-E[\beta_1^\omega]}{(E[\beta_1^\omega]+\beta_2)Y+(2+E[\beta_1^\omega]+\beta_2)\bar{y}} > 0$, and $\epsilon =$

$\min\{\epsilon^{\alpha}, \epsilon^{\beta}\}$. Recall that the equilibrium marginal utilities in example 1 are

$$v^* \equiv (D_{s_{1x}} Eu_1^*, D_{s_{1y}} Eu_1^*, D_{z_{1x}} Eu_1^*, D_{z_{1y}} Eu_1^*, D_{s_{2x}} u_2^*(\omega), D_{s_{2y}} u_2^*(\omega), D_{z_{2x}} u_2^*(\omega), D_{z_{2y}} u_2^*(\omega)). \tag{34}$$

Centered at v^*, consider all spatially local perturbations of utility functions within an open set in the weak C^r topology such that

$$D_{s_{1k}} Eu_1 \in (D_{s_{1k}} Eu_1^* - \epsilon, D_{s_{1k}} Eu_1^* + \epsilon), \tag{35}$$

$$D_{z_{1k}} Eu_1 \in (D_{z_{1k}} Eu_1^* - \epsilon, D_{z_{1k}} Eu_1^* + \epsilon), \tag{36}$$

$$D_{s_{2k}} u_2(\omega) \in (D_{s_{2k}} u_2^*(\omega) - \epsilon, D_{s_{2k}} u_2^*(\omega) + \epsilon), \tag{37}$$

$$D_{z_{2k}} u_2(\omega) \in (D_{z_{2k}} u_2^*(\omega) - \epsilon, D_{z_{2k}} u_2^*(\omega) + \epsilon), \qquad k \in K. \tag{38}$$

These perturbations are evaluated in k, $k \in K$, individually, and are thus spatially local perturbations. Then it can be checked that all utilities within this neighborhood generate bid rents that are within ϵ of the equilibrium bid rents in example 1. Furthermore, household 1's realized marginal rate of substitution for housing in terms of composite good in location x is always higher than the marginal rate of substitution of household 2; household 2's marginal rate of substitution for housing in location y is always higher than that of household 1.[10]

Now we can prove the non-existence of fully revealing rational expectations equilibrium for all economies in this neighborhood. Suppose for any set of preferences within these spatially local perturbations, there exists a fully revealing rational expectations equilibrium $(\varphi_1^*, \varphi_2^*, \Psi^*)$. Then the uninformed household (household 1) can infer the state of nature by observing Ψ^*. However, within the perturbations, the equilibrium bid rents are the same across states, contradicting that Ψ^* is a fully-revealing rational expectations equilibrium price.

[10]In location x, for example, since the lowest MRS for household 1 is $\frac{E[\alpha_1^{\omega}]/\bar{x}-\epsilon}{(1+E[\alpha_1^{\omega}])/Y+\epsilon}$, and the highest MRS for household 2 is $\frac{\alpha_2/\bar{x}+\epsilon}{(1+\alpha_2)/Y-\epsilon}$, household 1's MRS is greater than household 2's MRS if and only if $\epsilon < \epsilon^{\alpha} = \frac{E[\alpha_1^{\omega}]-\alpha_2}{(E[\alpha_1^{\omega}]+\alpha_2)Y+(2+E[\alpha_1^{\omega}]+\alpha_2)\bar{x}}$. Similarly, household 2's MRS in location y is greater than that of household 1 if and only if $\epsilon < \epsilon^{\beta} = \frac{\beta_2-E[\beta_1^{\omega}]}{(E[\beta_1^{\omega}]+\beta_2)Y+(2+E[\beta_1^{\omega}]+\beta_2)\bar{y}}$.

Obviously, a similar argument works for the cases with more than 2 states and example 2. *Q.E.D.*

This paper shows that if one household has the information about a specific location, if he doesn't live there in equilibrium, the housing price in that location will not reveal his information. If a household lives in the location about which he is informed, there is an information gain (in that he can maximize ex post utility instead of expected utility), but also a information spillover to all other households in that they can learn private information about that location by observing the equilibrium housing price. When local utility and spatially local perturbations are considered, the information spillover plays no role for the households living in other locations. However, when spatially non-local perturbations are considered, a small perturbation makes the states of all locations relevant to the utility of living in k. So, as shown in Allen (1981), there exists an open and dense set of economies possessing fully revealing rational expectations equilibrium.

Finally, we make a remark here: If there is no fully revealing rational expectations equilibrium, an equilibrium allocation can fail to be Pareto optimal. Consider a variation of Example 1 shown in Figure 3. When the probability is quite evenly distributed over states in Ω_k, $k = 1, 2$, household 1's bid rent for the CBD is larger than that of household 2, and household 2's bid rent for location 2 is larger than that of household 1. So in equilibrium, household j lives in location j, $j = 1, 2$ in both states. However, in a Pareto optimum, household j lives in $3 - j$, $j = 1, 2$ when $\omega = LH$. Therefore, we have an example with an equilibrium allocation that is ex ante but not ex post efficient.

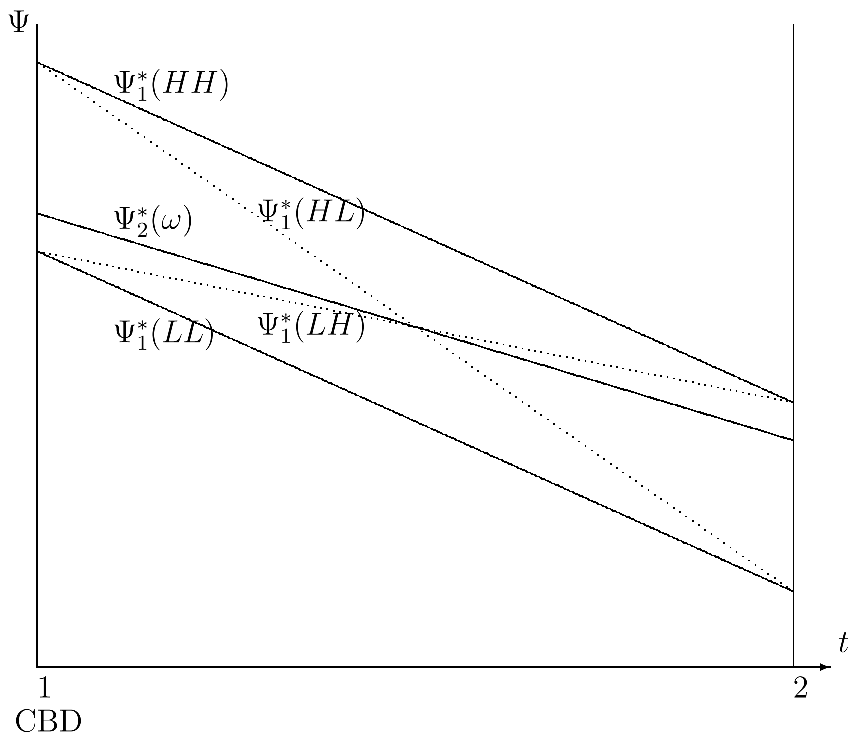

Figure 3: The non-fully revealing rational expectations equilibrium allocation can fail to be Pareto optimal.

4 The Existence of Rational Expectations Equilibrium

After presenting an open subset of economies that possess non-fully-revealing rational expectations equilibrium, it is natural to ask: Can a rational expectations equilibrium fail to exist in urban economies? This can undermine the minimal requirement for further analysis in urban economics with uncertainty. In this section, the existence of (not necessarily fully-revealing) rational expectations equilibrium is examined, given the assumption of ordered relative steepness of bid-rents. First we describe how the existence of equilibrium depends on the number of locations relative to the number of households.

When the number of locations is greater than the number of households, since each household can consume housing in at most one location, there must exist at least one location where no household lives. In these abandoned locations, by Walras' Law, the price of housing is zero. Therefore, unless the commuting cost is very high and these locations are far away from the CBD, households have an incentive to move into these locations to enjoy a higher utility. In this case, there is no equilibrium.

When the number of locations is the same as the number of households, the assumption of ordered relative steepness of bid rents ensures that every location is occupied by exactly one household in equilibrium. Therefore, we can settle households one-by-one from the core to periphery in the order of the slopes of their bid rents, constituting an equilibrium allocation.[11] Thus, we know ex ante what information will be revealed by equilibrium prices, so we can add this information to the consumer's optimization problem. The case when the number of households is larger than the number of locations is left to future work. This case is difficult because we don't know ex ante (due to an endogenous lot size) where consumers will reside in equilibrium, so we don't know what information will be revealed by equilibrium prices. This would be the case, for example, if there were a continuum of consumers.

Suppose there are n households and n locations. Before proving a theorem on the existence of equilibrium, we need to make following assumptions on households' bid rents. These assumptions are standard in urban economics; see for example Fujita (1985, 1989).[12] To avoid abuse of notation, let $\tilde{s}_j(t,\omega)$ and $\tilde{z}_j(t,\omega)$ denote the consumptions of lot size and composite good at a distance t from the CBD in state ω. Given a specific state ω and a utility level u, denote $\tilde{\Psi}_j(t,u,\omega) \equiv \max_{\tilde{s}_j(t,\omega),\tilde{z}_j(t,\omega)}\{\frac{Y_j-t-\tilde{z}_j(t,\omega)}{\tilde{s}_j(t,\omega)}|u_j(t,\omega)=u\}$ household j's bid rent for housing at distance t from the CBD.[13]

[11]Without the assumption of ordered relative steepness of bid rents, we must find a fixed point in the information structure, which is hard.

[12]In fact, in standard urban economics, the assumption of ordered relative steepness relates to only the uniqueness of equilibrium and makes the proof easier, but existence of equilibrium in urban economics can be proved without this assumption when there is no uncertainty; see Fujita and Smith (1987).

[13]Notice that though locations are discrete points on the line representing distance to the CBD, households' bid rents are in fact continuous functions of the distance from core.

Assumption 1 (Ordered Relative Steepness of Bid Rent) *Households' bid rent functions are ordered by their relative steepnesses. That is, given $j < j' \leq n$, $\tilde{\Psi}_j(t, u_j, \omega)$ is steeper than $\tilde{\Psi}_{j'}(t, u_j, \omega)$: Given $\omega \in \Omega$, whenever $\tilde{\Psi}_j(\bar{t}, u_j, \omega) = \tilde{\Psi}_{j'}(\bar{t}, u_{j'}, \omega) > 0$ for some $\bar{t}$, u_j and $u_{j'}$, then*

$$\tilde{\Psi}_j(t, u_j, \omega) > \tilde{\Psi}_{j'}(t, u_{j'}, \omega) \quad \forall\, 0 \leq t < \bar{t}, \tag{39}$$

$$\tilde{\Psi}_j(t, u_j, \omega) < \tilde{\Psi}_{j'}(t, u_{j'}, \omega) \quad \forall\, t > \bar{t} \text{ wherever } \tilde{\Psi}_j(t, u_j, \omega) > 0. \tag{40}$$

When households have the same utility function but different incomes, and when housing is a normal good, ordered relative steepness of bid rents is naturally satisfied.[14] However, when households have different utilities but the same income, ordered relative steepness of bid rent is not implied. The assumption of ordered relative steepness of bid rents ensures that given arbitrary levels of utilities for two agents, for each state, their bid rents must cross at (no more than) one point as shown in Figure 4, where the bid rent curves shift down as the utility levels increase. For example, the Cobb-Douglas utilities in Examples 1 and 2 satisfy the assumption of ordered relative steepness of bid rents, and so do quasi-linear utilities. In what follows, we prove the existence of rational expectations equilibrium, given the assumption of ordered relative steepness of bid rents.

4.1 When households are insensitive

To begin, given ordered steepness of bid rents and the same number of consumers and locations, use Assumption 1 to order consumers so that consumer 1 has the steepest bid rent, consumer 2 the next steepest, and so forth. Since the examples in Section 2 highlight the condition required for the existence of non-fully revealing rational expectations equilibrium, in what follows we focus on the case where households present insensitivity. Recall that the utility of household j in state ω from living in location j is denoted by $u_{jj}(\psi_{jj}(\omega), \omega)$.

[14] See Fujita (1989), pages 28-29.

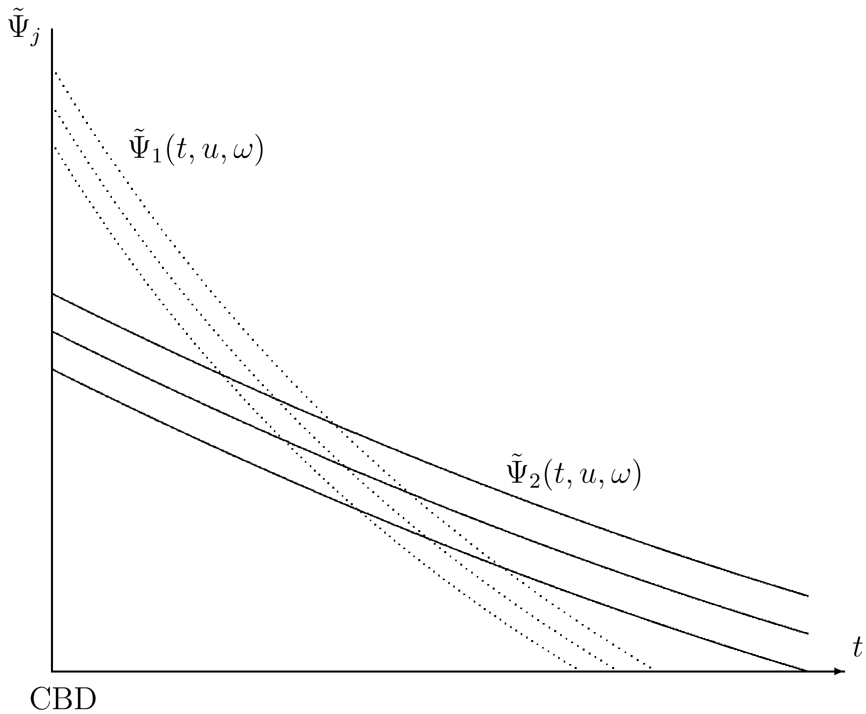

Figure 4: Example where households' bid rents satisfy ordered relative steepness of bid rents.

Definition 5 (Insensitivity)
There exist states $(\omega, \omega') \in \Omega \times \Omega$ such that for each household $j \in N$ such that ω and ω' are in different partition elements of $\mathcal{F}_j$ and[15]

$$\left.\frac{D_{s_{jj}(\omega)} u_{jj}(\psi_{jj}(\omega), \omega)}{D_{z_{jj}(\omega)} u_{jj}(\psi_{jj}(\omega), \omega)}\right|_{\varphi^*_{jj}(\omega)} = \left.\frac{D_{s_{jj}(\omega')} u_{jj}(\psi_{jj}(\omega'), \omega')}{D_{z_{jj}(\omega')} u_{jj}(\psi_{jj}(\omega'), \omega')}\right|_{\varphi^*_{jj}(\omega')}, \tag{41}$$

but there exists $j' \in N$ for whom ω and ω' are in the same element of $\mathcal{F}_{j'}$ (with a positive probability), $u^*_{j'j'}(\psi_{jj}(\omega), \omega) \neq u^*_{j'j'}(\psi_{jj}(\omega'), \omega')$.

Given that housing is a normal good, we will show below that equilibrium always exists and is unique in our model, and the question then becomes whether it is fully revealing or not. We will show that insensitivity is a necessary and sufficient condition for existence of a non-fully revealing rational expectations equilibrium.

The intuition for the first part of the definition of insensitivity is that for any household who has information in distinguishing two states, his/her

[15]States that nobody can distinguish and that do not matter to anyone can be combined.

marginal rate of substitution in k is independent of these realized states. However, to ensure that the household's information is not trivial, we need the second part of the definition which implies that his/her information about location k does matter for another household. Insensitivity can result from one or more of several sources: utility could be quasi-linear, or information about conditions in one location can be irrelevant to the consumer living there, or some information is irrelevant to all consumers.

Let $\mathcal{P}(\Omega)$ be the power set of Ω. Now, consider a public partitional information function $\mathcal{I} : \Omega \to \mathcal{P}(\Omega)\backslash\{\phi\}$ such that for every $\omega \in \Omega$, a nonempty subset $\mathcal{I}(\omega)$ of Ω is assigned, where: (1) for every $\omega \in \Omega$, $\omega \in \mathcal{I}(\omega)$; (2) $\omega' \in \mathcal{I}(\omega)$ implies $\mathcal{I}(\omega') = \mathcal{I}(\omega)$. Moreover, for every (ω, ω') satisfying insensitivity, $\mathcal{I}(\omega') = \mathcal{I}(\omega)$. This condition implies that when ω and ω' are insensitive, and ω' and ω'' are insensitive, then $\mathcal{I}(\omega) = \mathcal{I}(\omega') = \mathcal{I}(\omega'')$. So it can be checked that

$$\mathcal{I}(\omega) = \{\omega' \big| \mathcal{I}(\omega') = \mathcal{I}(\omega)\} \tag{42}$$

In other words, $\mathcal{I}(\omega)$ is a partition element collecting states that are directly or transitively insensitive with ω. Intuitively, for all states in $\mathcal{I}(\omega)$, either households have no information to distinguish them, or the informed household cannot reflect its information by differences in its marginal rate of substitution. The non-fully revealing rational expectations equilibrium is supported by the σ-algebra generated by the public partitional information function.

Theorem 2 *Given Assumption 1 and that housing consumption is a normal good, under insensitivity, for $j = 1, ..., n$, there is an equivalence class of $\sigma(\mathcal{I})$-measurable bid rent functions $\Psi^* : \Omega \to \mathbb{R}_+^{2n}$ and $\mathcal{F}_j \vee \sigma(\Psi^*)$-measurable consumption functions $\varphi_j^* : \Omega \to \mathbb{R}_+^2 \cup \mathbb{R}_+^2$ that constitute a unique non-fully* *revealing rational expectations equilibrium such that, for $k \in K$,*

$$\Psi_k^*(\omega) \equiv \Psi_{kk}(u_k^*(\omega), \omega)$$
$$= \max_{s_{kk}(\omega), z_{kk}(\omega)} \{ \frac{Y_k - T_k - z_{kk}(\omega)}{s_{kk}(\omega)} \Big| E[u_{kk}(\psi_{kk}(\omega), \omega) | \mathcal{F}_k \vee \sigma(\Psi^*)] = u_k^*(\omega) \}; \tag{43}$$

$$\varphi_{jk}^*(\omega) \equiv \varphi_{jk}(u_k^*(\omega), \omega) = \begin{cases} (\bar{s}_k,\ Y_k - T_k - \Psi_k^*(\omega)\, \bar{s}_k), & \text{if } j = k, \\ (0, 0), & \text{if } j \neq k; \end{cases} \tag{44}$$

and the unique equilibrium utility level $u_k^*(\omega)$, $k \in K$, satisfies

$$\Psi_k(u_k^*(\omega), \omega) = \frac{D_{s_{kk}(\omega)} E[u_{kk}(\psi_{kk}(\omega), \omega) | \mathcal{F}_k \vee \sigma(\Psi^*)]}{D_{z_{kk}(\omega)} E[u_{kk}(\psi_{kk}(\omega), \omega) | \mathcal{F}_k \vee \sigma(\Psi^*)]} \Bigg|_{\varphi_{kk}^*(\omega)}. \tag{45}$$

Proof. First, we use the implication from Lemma 1 that bid rent equilibrium is a rational expectations equilibrium. Next, a bid rent equilibrium will be constructed, and the existence and uniqueness of the equilibrium will be proved. Finally, it will be shown that the unique bid rent (rational expectations) equilibrium is non-fully revealing.

Following a standard argument in urban economics, given Assumption 1, every location is occupied by exactly one household. Since household 1 has the steepest bid rent, from equation (2) in Definition 2, he/she must occupy the housing in location 1 in equilibrium. After settling household 1, we can consider the problem as the one with $n-1$ households ($j \in \{2, ..., n\}$) and $n-1$ locations ($k \in \{2, ..., n\}$). Then, household 2 has a steeper bid rents than remaining households, so his/her equilibrium bid rent for the housing in location 2 is higher than that of other households. Therefore, in equilibrium, household 2 occupies the housing in location 2. Following the same logic, in equilibrium all households are arranged so that household j lives in location j, $j \in N$, or say that location k is occupied by household k, $k \in K$.

Given that household k is located in location k, as shown in Figure 5, the intercept of budget line $Y_k - T_k$ and the housing supply $\bar{s}_k$ are determined by parameters. Now, given arbitrary u, the slope of budget line $\Psi_k(u, \omega)$ and the corresponding $\varphi_{kk}(u, \omega)$ are uniquely determined (by the cross point of the budget line and the vertical line $\bar{s}_k$). Furthermore, given consumption point $\varphi_{kk}(u,\omega)$, since households' preferences are smooth, the slope of the indifference curve passing through $\varphi^*_{kk}(u,\omega)$ is uniquely determined. Letting $\Phi_{kk}(u,\omega) \equiv \frac{D_{s_{kk}(\omega)}E[u_{kk}(\psi_{kk}(\omega),\omega)|\mathcal{F}_k \vee \sigma(\Psi^*)]}{D_{z_{kk}(\omega)}E[u_{kk}(\psi_{kk}(\omega),\omega)|\mathcal{F}_k \vee \sigma(\Psi^*)]}\Big|_{\varphi_{kk}(u,\omega)}$, the equilibrium utility level (and the equilibrium housing price in location k) is given by solving $\Psi_k(u,\omega) = \Phi_{kk}(u,\omega)$, $\omega \in \Omega$, as shown in Figure 5. Let $f_{kk}(u,\omega) \equiv \Psi_k(u,\omega) - \Phi_{kk}(u,\omega)$, since Ψ_k and Φ_{kk} are continuous in u, f_{kk} is continuous in u. At $\bar{E}$, $f_{kk}(u,\omega) < 0$ since $\Psi_k(u,\omega) = 0$ and $\Phi_{kk}(u,\omega) > 0$ at $\bar{E}$ by monotonicity. Given $\bar{s}_k > 0$, $\Psi_k(u,\omega)$ is increasing as $z_{kk}(\omega)$ decreases and, by the smooth boundary condition, $\Phi_{kk}(u,\omega) \to 0$ as $z_{kk}(\omega) \to 0$, which implies that $\exists \underline{u}$ such that $f_{kk}(u,\omega) > 0, \forall u \le \underline{u}$. Therefore, by the intermediate value theorem, there exists a $u^*_k(\omega)$ solving $f_{kk}(u,\omega) = 0$, $\omega \in \Omega$, and thus, there exists a rational expectations equilibrium. The uniqueness of equilibrium can be guaranteed by the condition that $\Phi_{kk}(u,\omega)$ is increasing in u, which is true when the consumption of housing is a normal good as shown in Berliant and Fujita (1992).

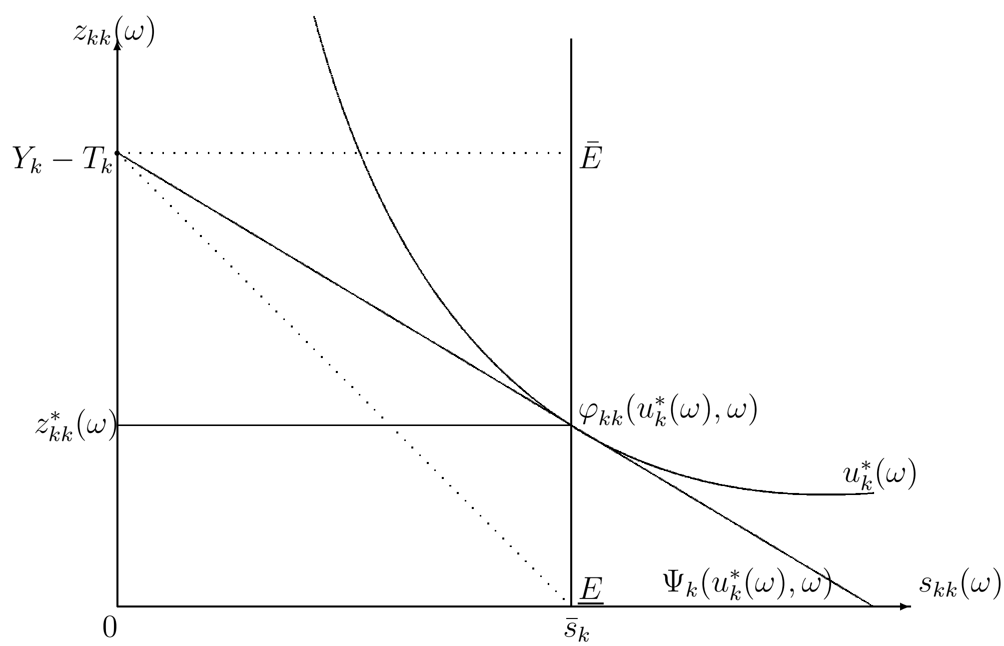

Figure 5: The determination of equilibrium housing price and equilibrium utility for household k in location k in state ω, $k \in K$.

Under insensitivity, we want to prove that the unique rational expectations equilibrium is non-fully revealing. Suppose on the contrary that the equilibrium is fully-revealing, then choosing arbitrary k, we have

$$\Psi_k^*(\omega) = \Psi_{kk}(u^*(\omega), \omega) \neq \Psi_{kk}(u^*(\omega), \omega') = \Psi_k^*(\omega'), \;\; \forall \omega, \omega' \in \Omega. \tag{46}$$

First, for household k (living in location k in equilibrium), any such pair (ω, ω') must be in different partition elements. That is, $\mathcal{F}_k \vee \sigma(\Psi^*) = \mathcal{F}$. Second, from (43) and (45), we have $\Psi_k(u, \omega) \neq \Psi_k(u, \omega')$ which implies, $\forall \omega, \omega' \in \Omega$,

$$\frac{D_{s_{kk}(\omega)} u_{kk}(\psi_{kk}(\omega), \omega)}{D_{z_{kk}(\omega)} u_{kk}(\psi_{kk}(\omega), \omega)} \Big|_{\varphi_{kk}^*(\omega)} \neq \frac{D_{s_{kk}(\omega')} u_{kk}(\psi_{kk}(\omega'), \omega')}{D_{z_{kk}(\omega')} u_{kk}(\psi_{kk}(\omega'), \omega')} \Big|_{\varphi_{kk}^*(\omega')}. \tag{47}$$

However, from insensitivity, there exist ω, $\omega' \in \Omega$ such that

$$\frac{D_{s_{kk}(\omega)} u_{kk}(\psi_{kk}(\omega), \omega)}{D_{z_{kk}(\omega)} u_{kk}(\psi_{kk}(\omega), \omega)} \Big|_{\varphi_{kk}^*(\omega)} = \frac{D_{s_{kk}(\omega')} u_{kk}(\psi_{kk}(\omega'), \omega')}{D_{z_{kk}(\omega')} u_{kk}(\psi_{kk}(\omega'), \omega')} \Big|_{\varphi_{kk}^*(\omega')}, \tag{48}$$

a contradiction with (47). In fact, these non-fully revealing equilibrium prices reveal nothing beyond $\sigma(P(\omega))_{\omega \in \Omega}$ in equilibrium. *Q.E.D.*

4.2 When households are not insensitive

Insensitivity is necessary and sufficient for the existence of a non-fully revealing rational expectations equilibrium. Since, with insensitivity, there is some useful information that is not transmitted from informed to uninformed households, the rational expectations equilibrium is non-fully revealing. Let $\tilde{\sigma}_k \equiv \sigma(\Omega_k) \times (\times_{k' \neq k}\{\phi, \Omega_{k'}\})$, which is the σ-algebra indicating that only the state in k is known, whereas all states in other locations are completely unknown. Without insensitivity, the equilibrium can only be fully-revealing.

Theorem 3 *Given Assumption 1 and housing consumption is a normal good, under no insensitivity, there exists a unique rational expectations equilibrium that is fully revealing.*

Proof. From Assumption 1 and Lemma 1, as in the proof of Theorem 2, there exists a rational expectations equilibrium which corresponds to the bid rent equilibrium. When the insensitivity condition is violated, the realized marginal rates of substitution are different $\forall \omega_k \in \Omega_k$, in k where he/she lives. Furthermore, with no insensitivity, $\tilde{\sigma}_k \subseteq \mathcal{F}_k$ for the household k living in k; otherwise, from footnote 15, there exist $\omega_k, \omega'_k \in \Omega_k$ that can be distinguished by $j' \neq k$ who does not live in k, a contradiction with no insensitivity. Since this is true for all $k \in K$, and the equilibrium bid rent in k is equal to the marginal rate of substitution of household living in k, so the equilibrium bid rents are different in each state, implying that the rational expectations equilibrium is fully revealing. *Q.E.D.*

In the literature, an open and dense subset of standard economies with fully revealing rational expectations equilibrium is found. However, under the natural assumption of spatially local perturbations of utility functions, as shown in the previous section, an open subset of urban economies with only a non-fully revealing equilibrium is found. Recall that, consistent with what is shown in standard general equilibrium models, there is also an open subset of urban economies with only fully revealing equilibria: The easiest way to present this is to exchange the information given to households 1 and 2 in our examples and use spatially local perturbations of utility functions. Then within these perturbations, the rational expectations equilibrium can only be fully revealing (since there is no mismatch between the information known by households and their locations). Therefore, neither the set of fully revealing nor the set of non-fully revealing economies can be dense under the structure of urban economics. Non-fully revealing equilibrium is more interesting in highlighting the potential positive value and the strategic use of information. When non-local perturbations are considered, though they are not so reasonable in urban economics, the results are the same as the ones in standard general equilibrium models. That is, there is an open and dense subset of economies that possess a fully revealing rational expectations equilibrium.

As shown in the comparison in Table 1, the inefficiency in information transmission in a housing/land market rests on two key assumptions: spatially local utility perturbations and the standard setting in urban economics that every household can consume housing in only one place. When either of them is violated, the result in standard models is restored. That is, in economic circumstances where there is no location structure or no spatially local property of utility, generically, the efficiency of prices in information transmission is attained in a rational expectations equilibrium. We conclude that geographic structure, together with spatially local utility properties, can play a role in distorting the efficiency of the market in transmitting information from informed to uninformed households.

	Households can consume housing in only one place	Ordinary consumption set
Spatially local utility perturbations	Open subsets of economies with fully revealing and non-fully revealing equilibria (Urban economics)	An open and dense subset of economies with fully revealing equilibria
Spatially non-local utility perturbations	An open and dense subset of economies with fully revealing equilibria	An open and dense subset of economies with fully revealing equilibria (Standard model)

Table 1: A comparison of the results in this paper with the results in the literature.

If households can be redistributed so that location is coincident with information, then we can create a fully-revealing rational expectations equilibrium. However, this idea seems impractical since in most cases, unless the households are very risk averse, households' subjective preferences for location do not necessarily depend on the information that they have. A classical way to induce households to reveal their private information, as shown in Debreu (1959) Chapter 7 and Arrow (1964), is to consider contingent claim or financial markets. This idea is discussed in the next section.

5 Adding Financial Markets

When contingent claims or financial markets are included, do our examples with no fully-revealing rational expectations equilibrium survive? This interesting question is examined here.

Similar to Hirshleifer's (1971) conclusion in cases with technological uncertainty, speculative profits from price revaluation give individuals incentives to disseminate their information. We show that when there is market uncertainty, the same incentives exist and thus all households' information is revealed in equilibrium.

Following the setting of our Example 1 and Magill and Quinzii (1996), consider that before consuming composite good and housing, the two households can buy and sell state-contingent financial securities in financial markets. That is, consider a one-period, two-stage model as follows. At the beginning of the first stage, households are endowed with e_j^0 units of numeraire (composite consumer good), $j = 1, 2$. Household 2 has complete information about the states in the two locations, whereas household 1 has no information. The financial markets are opened in stage 1, where the two households can buy and sell securities. Assume that the financial markets are complete in that the number of securities is the same as the number of states, so we can use the same index for securities and states. Specifically, the security ω, $\omega \in \Omega$, is a contract promising to deliver one unit of numeraire (income) in state ω, and 0 in other states, in the second stage. All securities are perfectly monitored and perfectly enforced. After closing the financial markets and the end of the first stage, the state is realized and all security returns are paid at the beginning of the second stage. Then an absentee landlord trades with households in spot housing markets. The game is complete when the housing markets are closed. We want to know whether or not there is a fully-revealing rational expectations equilibrium under the new setting.

Let e_j^ω be household j's endowment in state ω in the second stage, and let the row vector $\nu_j \equiv (\nu_j(\omega))_{\omega\in\Omega} \in \mathbb{R}^4$ be household j's portfolio. Let $q \equiv (q(\omega))_{\omega\in\Omega} \in \mathbb{R}^4$ and $V \equiv (V(\omega))_{\omega\in\Omega} \in \mathbb{R}^{16}$ where $q(\omega) \in \mathbb{R}$ and $V(\omega) \in \mathbb{R}^4$ represent the price vector of security ω and the payoff matrix of securities in state ω, respectively. That is, $V(\omega)$ is a row vector of zeros except that the element representing state ω is 1, and $V(\omega) \neq V(\omega')$, for all $\omega \neq \omega'$. The fully revealing rational expectations equilibrium under the new setting can be solved by backward induction as follows.

Suppose there exists a fully revealing rational expectations equilibrium. From Section 2.2, given $Y_j(\omega)$, households' indirect utility functions with optimization in stage 2 are

$$U_1(\omega) = \alpha_1^\omega \ \ln \bar{x} - \ln(1+\alpha_1^\omega) + \ln Y_1(\omega),$$
$$U_2(\omega) = \beta_2^\omega \ \ln \bar{y} - \ln(1+\beta_2^\omega) + \ln Y_2(\omega).$$

Through monotonic transformations of these indirect utility functions, household j's optimization problem in stage 1 can be written as

$$\begin{aligned} \max_{\nu_j} \ & \tilde{U}_j(\omega) \equiv \ln Y_j(\omega) \\ \text{s.t.} \ & q \cdot \nu_j^T = e_j^0, \\ & Y_j(\omega) - e_j^\omega = V(\omega)\, \nu_j^T, \ \ \omega \in \Omega, \end{aligned}$$

where ν_j^T denotes the transpose of j's portfolio vector. Denoting the true state as $\hat{\omega}$, since households learn the true state by observing prices in a fully revealing rational expectations equilibrium, it is obvious that the equilibrium security prices must satisfy $q(\hat{\omega}) = 1$ and $q(\omega) = 0$, $\forall \omega \neq \hat{\omega}$. Since for arbitrary different $\hat{\omega}$, $\hat{\omega}'$, the corresponding equilibrium price vectors are not the same, each q^* reveals a unique $\hat{\omega}$. Therefore, it follows that q^* supports a fully-revealing rational expectations equilibrium.

Though we show that adding financial markets helps to reveal the informed household's private information, there are some issues with this idea. Grossman and Stiglitz (1980) argue that the informed household can use their private information to take advantage of uninformed households. Thus, if the financial markets and the corresponding fully-revealing equilibrium prices make private information publicly available to every household, the informed household could not earn an information rent (coming from asymmetric information) and has an incentive to hide his/her private information (by pretending to be uninformed). Therefore, though adding financial markets can restore the existence of a fully-revealing rational expectations equilibrium, there are reasons why these financial markets might not function. Of course, if financial asset markets are incomplete for whatever reason, the problems we have discussed return.

6 Conclusions

Radner (1979), Jordan (1980), and Allen (1981) prove the existence of an open and dense subset of standard economies that possess fully-revealing rational expectations equilibria. Since in urban economies there is an open subset of economies without fully-revealing rational expectations equilibrium, Allen's theorem about the existence of a dense subset of economies possessing fully-revealing rational expectations equilibrium does not extend to urban economies when spatially local perturbations of utilities are considered. These perturbations retain the property that the utility of living at a location depends only on the consumption bundle at that location and the resolution of uncertainty about local variables only. Furthermore, since an open subset of economies with fully revealing rational expectations equilibria can easily be constructed, we cannot challenge the existence of an open subset of economies that possess fully-revealing rational expectations equilibria in the context of urban economies. Therefore, neither the set of fully revealing nor the set of non-fully revealing economies can be dense under the structure of urban economics.

This paper highlights the importance of "local conditions" for the existence of rational expectations equilibria in urban economies. The existence of a unique rational expectations equilibrium is proved with the assumption of ordered relative steepness of bid rents. Whether the rational expectations equilibrium is fully revealing or non-fully revealing depends on the insensitivity condition: When insensitivity is satisfied, the unique rational expectations equilibrium is non-fully revealing; otherwise, the equilibrium is fully revealing. Though introducing financial markets can restore the existence of fully-revealing rational expectations equilibrium, many provisos also accompany it. In summary, geography can play a role in undermining the efficiency of market prices in transmitting information from informed to uninformed households.

One potential extension of this paper is to consider a continuum of households; however, the intuition that the mismatch of locally-informed households and their corresponding equilibrium locations is likely to yield an open subset of economies possessing only non-fully revealing rational expectations equilibrium seems robust. Other topics for future research are to extend the intuition behind our results to other models. For example, in an overlapping generations model, time may play a role similar to the spatial structure in preventing information transmission. Moreover, when search/matching models are considered, stable equilibrium may also pick only the best of all potential matches. In either of these cases, we conjecture that there exists an open subset of economies with no fully-revealing rational expectations equilibrium, since agents with information about states in other lifetimes (in the overlapping generations framework) or in other equilibrium matches (in the search framework) might not have their information reflected in equilibrium prices.

Appendix A. Proof of Lemma 1

Comparing Definition 1 and Definition 2, since condition (iii) is the same as equations (4), for μ-almost every $\omega \in \Omega$, we only need to prove that $((\psi_j^*(\omega))_{j\in N}, P^*(\omega))$ satisfies (i) and (ii) if and only if $((\varphi_j^*(\omega))_{j\in N}, \Psi^*(\omega))$ satisfies (2) and (3), given $\varphi_j^*(\omega) = \psi_j^*(\omega)$, $\Psi_k^*(\omega) = p_k^*(\omega)$, and $u_j^*(\omega) = u_{jk}(\psi_{jk}^*(\omega), \omega)$, $\forall j \in N$, $k \in K$.

First, to prove this, given that (2) and (3) are satisfied but either (i) or (ii) is not true, we want to show contradictions. If (i) is not true, there exists $\Omega_0 \subseteq \Omega$ with $\mu(\Omega_0) > 0$ such that $P_k^*(\omega) \cdot \psi_{jk}^*(\omega) > Y_k - T_k$, $\forall \omega \in \Omega_0$. Then for $\omega \in \Omega_0$, we have $p_k^*(\omega)s_{jk}^*(\omega) + z_{jk}^*(\omega) > Y_k - T_k$, which together with $\Psi_k^*(\omega) = p_k^*(\omega)$ implies

$$\Psi_k^*(\omega) > \frac{Y_k - T_k - z_{jk}^*(\omega)}{s_{jk}^*(\omega)}, \quad \forall \omega \in \Omega_0,$$

a contradiction with (2) and (3), given that the utility level is the same as the optimalized level in Definition 1, i.e., $u_j^*(\omega) = u_{jk}(\psi_{jk}^*(\omega), \omega)$.

On the other hand, if (ii) is not true, then $\exists j \in N$ and $\psi_j'(\omega)$ within the budget constraint such that

$$E[u_j(\psi_j'(\omega), \omega) \big| \mathcal{F}_j \vee \sigma(\Psi^*)] > E[u_j(\psi_j^*(\omega), \omega) \big| \mathcal{F}_j \vee \sigma(\Psi^*)]. \tag{49}$$

For this household j and for location k where he/she lives in equilibrium, we can choose $u_j^*(\omega) = E[u_j(\psi_j^*(\omega), \omega) \big| \mathcal{F}_j \vee \sigma(\Psi^*)]$, and then by strict concavity and strict monotonicity, there exists $\epsilon > 0$ and $\psi_j''(\omega) \equiv \frac{\psi_j'(\omega)+\psi_j^*(\omega)}{2} - \epsilon$ such that $E[u_j(\psi_j''(\omega), \omega) | \mathcal{F}_j \vee \sigma(\Psi^*)] = u_j^*(\omega)$. Since $[\Psi_k^*(\omega) \;\; 1] \cdot \psi_{jk}''(\omega) < Y_k - T_k$ implies $\Psi_k^*(\omega) < \frac{Y_k - T_k - z_{jk}''(\omega)}{s_{jk}''(\omega)}$,[16] letting $p_k''(\omega) \equiv \frac{Y_k - T_k - z_{jk}''(\omega)}{s_{jk}''(\omega)}$, we have $p_k''(\omega) > \Psi_k^*(\omega)$, though $\psi_{jk}''(\omega)$ and $\psi_{jk}^*(\omega)$ yield the same expected utility level $u_j^*(\omega)$. That is, given $u_j^*(\omega)$, $\psi_{jk}''(\omega)$ supports a higher $p_k''(\omega)$ than $\Psi_k^*(\omega)$. Therefore, $\varphi_{jk}^*(\omega) = \psi_{jk}^*(\omega)$ does not maximize $\Psi_{jk}(u_j^*(\omega), \omega)$, a contradiction with equation (3).

[16] Recall that $\Psi_k^*(\omega) \equiv \Psi_k(u^*(\omega), \omega) = \max_j\{\Psi_{jk}^*(\omega)\}$.

Secondly, supposing that (i) and (ii) hold, but either (2) or (3) is not satisfied, we want to prove that there is a contradiction. If (2) does not hold, there exists $k \in K$, $j \in N$, and $\Omega_0 \subseteq \Omega$ with $\mu(\Omega_0) > 0$ such that $\forall \omega \in \Omega_0$, $\Psi^*_{jk}(\omega) > \Psi^*_k(\omega)$ but j does not live in location k. Suppose that j lives in location $k' \neq k$. Then for this household j, since he/she can pay less for the housing in k than the price that makes he/she indifferent between the housing in k and k', household j has an incentive to move from k' into location k to increase his/her utility in all for all $\omega \in \Omega_0$, a contradiction with condition (ii) that φ^*_j maximizes j's conditional expected utility.

If (3) does not hold, since the budget line with $\Psi^*_{jk}(\omega)$ is not tangent to the indifference curve for a given u for some states $\omega \in \Omega_0$, where $\mu(\Omega_0) > 0$. By strict concavity, there exists $\psi^{'}_{jk}(\omega) \neq \varphi^*_{jk}(\omega)$ such that $u_{jk}(\psi^{'}_{jk}(\omega), \omega) = u_{jk}(\varphi^*_{jk}(\omega), \omega)$, and thus $E[u_j(\psi'_j(\omega), \omega) | \mathcal{F}_j \vee \sigma(\Psi^*)] = Eu^*_j$, where Eu^*_j is the optimal utility level solved from Definition 1. Choosing $\psi^{''}_{jk}(\omega) \equiv \frac{\varphi^*_{jk}(\omega) + \psi^{'}_{jk}(\omega)}{2}$, then by strict concavity, $\psi^{''}_{jk}(\omega)$ is available for household j in achieving a higher utility level, i.e., $E[u_j(\psi''_j(\omega), \omega) | \mathcal{F}_j \vee \sigma(\Psi^*)] > Eu^*_j$, a contradiction with (ii) that φ^*_j maximizes household j's expected utility conditional on the private information and the information revealed by equilibrium prices.

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