Towards a Political Economy of the Hunters and Gatherers: A Study in Historical Materialism

Klaus Hagendorf

May 2009

Towards a Political Economy of the Hunters and Gatherers
A Study in Historical Materialism

Klaus Hagendorf
eurodos@gmail.com

Université Paris Ouest
September 2009

Abstract: *This paper uses the mode of production of the hunter-gatherers as a background to explain basic economic concepts, in particular the meaning of the labour theory of value and it's relationship to optimization of resources. A proof of the marginal value theorem is presented. A new term is introduced to designate labour surplus value, roundabout labour. The analytical expression for socially necessary labour is derived and Adam Smith's paradox between labour value and the adding-up theorem of wages, rent and profit, is resolved. It is shown that far from being limited to the ancient form of society of the hunter-gatherers the labour theory of value holds also in modern times.*

Keywords: historical materialism; labour theory of value; Marxian economics; roundabout labour; Adam Smith; surplus labour; hunter-gatherer society; marginal value theorem; evolutionary ecology; modes of production; productive forces.

JEL classification: B14; B51; D20; D46; Q57; Z10

Introduction

The Theory of Historical Materialism commonly distinguishes 5 different modes of production: 1) the mode of production of the hunters and gatherers or primitive communism, 2) antique society or the mode of production of slave holders, 3) feudalism or the feudal mode of production, 4) capitalism and finally 5) communism. Sometimes are added additional modes like the Asiatic mode of production,

The mode of production of the hunters and gatherers is not only the earliest form of human society, but also the only historical form of a classless society. For the greater part of the existence of our species all humans lived the life of hunters and gatherers. Frederick Engels in his "On the Origin of the Family, Private Property and the State", where he uses a somewhat different classification, describes the hunters and gatherers society as a brotherhood of man.

> "... here we have the opportunity of studying the organization of a society which still has no state.
>
> … That is what men and society were before the division into classes. And when we compare their position with that of the overwhelming majority of civilized men today, an enormous gulf separates the present-day proletarian and small peasant from the free member of the old gentile society." (Engels, 1884, chap. 3).

In this paper we explore the mode of production of the hunters and gatherers in order to clarify basic economic concepts. The model put forward does not pretend to give a realistic interpretation of specific hunter-gatherer societies and we often include remarks concerning the relevance of the economic concepts derived to the modern capitalist economy. However, one may find in the works of anthropologists, archaeologists and ecologists of human behaviour models which are more sophisticated than the one presented here and which try to explain indeed the basic relations of this archaic form of human existence. By emphasising basic economic principles this paper may contribute to a “Political Economy of the Hunters and Gatherers” within the framework of the Theory of Historical Materialism. One should emphasise that there exists a vast literature on hunters and gatherers. The interested reader is referred to “The Cambridge Encyclopedia of the Hunters and Gatherers” (Lee & Daly, 1999) as a starting point. Needless to say there is no “Cambridge Encyclopedia of Historical Materialism.”

Anthropologists and ecologists make use of optimization techniques to study behavioural characteristics of living creatures including humans. This seems to be based on a neo-Darwinian vision of human development. One tries to explain the occurrence of certain behaviours by assuming that living creatures behave in what is in some sense an 'optimal manner'. Evolutionary ecologists and human behavioural ecologists make use of *optimal foraging models* to understand the food gathering methods employed by animals (for example bees) and humans alike. In evolutionary ecology one speaks of the use of neoclassical economic methods. As we shall show here this is not quiet right although neoclassical economics also uses optimization techniques, neoclassical economists generally deny the validity of the labour theory of value. But the use of optimization models in evolutionary ecology and in anthropology implies the validity of the labour theory of value. Labour is the ultimate cost of everything and its application must be optimized. It was for just this reason that Adam Smith used the example of the deer and beaver hunters to show the general validity of the labour theory of value. Referring to Adam Smith's example, the leading neoclassical economist Paul A. Samuelson ridiculed Marxists by applying neo-Ricardian analysis to the mode of production of hunters and gatherers (Samuelson, 1971).

The use of optimization methods in the analysis of evolutionary ecology could more appropriately be called “modern Marxian ecological analysis” as it's analysis follows almost directly from Marx theory.

> L'anthropologie marxienne, qui, reprenant le fil hégélien, conçoit que l'homme se produit en produisant, n'a donc pas nécessairement cet accent prométhéen qu'on lui a prêté. La “valeur-travail”, c'est la loi du moindre effort, le travail en tant que recherche rationnelle d'un résultat utile dans le moindre temps. Cette notion est indissociable de celle d'utilité, utilité que le travail tend à produire... Le travail en général cherche la production d'une utilité dans le moindre temps de dépense. ..."Le concept marxien de "procès de travail social en général" articule donc la question du travail et celle de l'utilité. Et par là ... il gouverne une écologie." (Bidet, Jacques, 1992).

No other than the Soviet mathematician and economist Leonid V. Kantorovich, inventor of linear programming, one of the most important methods of optimization, defended the labour theory of value as naturally linked to the optimization of production.

In other words, the successful application of optimization methods to explaining different modes of production can be a powerful way of exposing the reactionary ideology of neoclassical economics and simultaneously improving the theory of Historical Materialism. This paper is a step towards that goal.

It's principle themes are as follows. First, we present a very simple optimal foraging model of hunters and gatherers. It introduces the production function and shows the negative effect of peripatetic life on productivity since during migrations production is low or zero. But migration is necessary because the productivity of labour falls over the time spent foraging a particular patch (diminishing marginal productivity of labour).

The first question is the optimal period of exploitation of a hunting ground. Next the impact of technological improvement on optimal mobility is examined. The model suggests that the more productive the labour of the hunters and gatherers the less migratory they become. This is a strong support for the hypothesis of Historical Materialism which postulates that with the advancement of the forces of production the mode of production will change and the nomads become settlers. But there is also the counter tendency of reductions in displacement costs. The less costly the displacements the more frequently they occur. In this context it may be important to realize that the wheel was not invented by the hunters and gatherers.

The model is used to show basic concepts of the process of production, productivity and labour value. These concepts are demonstrated using a Cobb-Douglas production function as well as an S-shaped production function". The S-shaped production function is a typical real form of a production function and can be found in many natural conditions.

Ecologists have developed a "Marginal Value Theorem" which states the optimality condition of the application of labour. It has been used in the analysis of the foraging behaviour of animals, i.e. bees as well as humans. We clarify its relationship to the labour theory of value, in particular we explain the meaning of marginal and average labour values.

Further the possibility of the exploitation of a surplus is shown. There was no exploitation in the mode of production of the hunters and gatherers but its introduction became more likely with the later development of technology and the formation of sedentary communities.

After having shown that all value is indeed labour and that surplus value is the difference between marginal labour value and average labour value we re-examine Adam Smith's example of the deer hunter and show that his example is just a special case of the labour theory of value. We resolve the paradox which is commonly seen in his work: that it asserts the labour theory of value but on the other hand explains the price of a commodity as the sum of labour, rent and profit, the so called 'adding up theorem'. There is no paradox and Smith and Marx's were right to regard rent and profit as surplus labour and neoclassical economists were wrong in denying the existence of surplus labour (Henry, John F., 2000).

A Simple Optimal Foraging Model

An introduction to optimal foraging theory can be found on the Internet site of Dr. Denis O'Neil (O'Neil, Denis, 2009) Another source is the publications and the Internet site of Eric Alden Smith (E. A. Smith, 2009) and another one is the article of Kaplan and Hill (Kaplan, Hillard & Hill, Kim, 1992).

The Formal Model

Hunters and gatherers are living in bands exploiting hunting grounds. After they have exploited one ground they move to another. An economic problem of this mode of production is how long a band has to exploit a hunting ground and consequently how often they have to move to a new ground.

The objective is the maximization of food production per labour unit. To simplify matters, it is assumed that all hunting grounds have equal productivity and that migration time between grounds is equal and constant.

The time path of food production is shown in **Figure 1**. One should observe that in the figure the trend has been eliminated. Instead of leaving the output at the attained level after leaving a hunting ground and migrating to another one, output is reduced to zero. This is done to avoid the impression of some accumulation taking place.

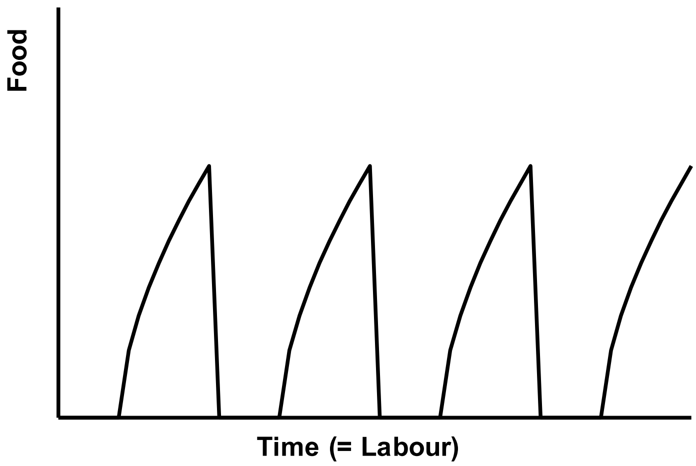

Figure 1: Time Path of Production Function (Trend omitted)

The production function for the exploitation of a hunting ground is

$$\mathbf{Q = f(L_h)}$$

where **Q** – Food, $\mathbf{L_h}$ –labour time used for hunting

Total labour **L** attributed to a hunting ground is

$$\mathbf{L = L_m + L_h}$$

L – total labour, $\mathbf{L_m}$ – migration labour, $\mathbf{L_h}$ – hunting labour

To simplify, we assume that total labour, **L**, to migrate and exploit a hunting ground is a function of time **t**, **L = g(t)**, and more specific **L = t.**

The objective of the hunters and gatherers is to optimize their average productivity of labour

$$\mathbf{Max.\ Q/L}$$

$$\mathbf{subject\ to\ \ Q = f(L_h)\ and\ L = L_m + L_h}$$

$\mathbf{L_m}$ is assumed to be given and constant.

The function of food production including migration time is

$$Q = f(L-L_m)$$

and average food production is

$$Q/L = f(L-L_m) / L$$

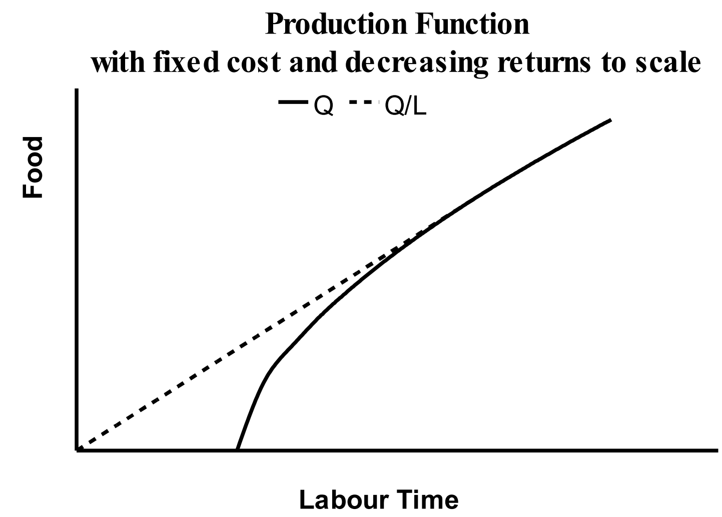

Figure 2: Optimal Foraging Time

The first order condition for its maximum is

$$d(Q/L)/dL = 0,$$

The second order condition is

$$d^2(Q/L)/dL^2 < 0.$$

From **Figure 2** we can also say that optimal productivity of labour or maximal average productivity of labour is there where average labour productivity is equal to marginal labour productivity. This is so, because the line from the origin being tangent to the production function is the maximum labour productivity which is at the point of tangency equal to the marginal labour productivity. In ecology this is known as the Marginal Value Theorem (Charnov, 1976).

At the optimum

$$Q/L = dQ/dL.$$

We take as a first example a production function of the Cobb-Douglas type with only one factor of production, labour.

In this case we have

$$Q = AL_h^a$$

where **A** is a constant, **a** is the output elasticity of labour $\mathbf{0 < a \leq 1}$.

The function in terms of **L** is

$$Q = A(L-L_m)^a$$

and average productivity of labour is

$$\mathbf{Q/L = A/L(L\text{-}L_m)^a}$$

It's derivative with respect to **L** is

$$\mathbf{d(Q/L)/dL = -AL^{-2}(L\text{-}L_m)^a + aA/L(L\text{-}L_m)^{a-1}}$$

At the optimum this is equal to zero.

$$\mathbf{-AL^{-2}(L\text{-}L_m)^a + aA/L(L\text{-}L_m)^{a-1} = 0}$$

From this follows

$$\mathbf{AL^{-2}(L\text{-}L_m)^a = aA/L(L\text{-}L_m)^{a-1}}$$

Both sides multiplied with **L** gives

$$\mathbf{A/L(L\text{-}L_m)^a = aA(L\text{-}L_m)^{a-1}}$$

But this is the Marginal Value Theorem: **Q/L = dQ/dL**.

The terms on the left side is average productivity, $\mathbf{Q/L = A/L(L\text{-}L_m)^a}$ **,** and the terms on the right side is marginal productivity $\mathbf{dQ/dL = aA(L\text{-}L_m)^{a-1}}$**.**

Both sides divided by **A** is

$$\mathbf{1/L(L\text{-}L_m)^a = a(L\text{-}L_m{}^{)a-1}}$$

$$\mathbf{1/L = a(L\text{-}L_m{}^{)-1}}$$

$$\mathbf{aL = L\text{-}L_m}$$

$$\mathbf{(a\text{-}1)\,L = -L_m}$$

$$\mathbf{L = L_m/(1\text{-}a)}$$

The optimal time allocated to a hunting ground, **L,** is a function of migration time, $\mathbf{L_m}$**,** and the output elasticity of labour, **a**. The greater the productivity (the closer **a** is to **1**) the greater **L. A**nd the greater the migration time, $\mathbf{L_m}$ (cost of migration), the greater the optimal time using a hunting ground.

From $\mathbf{L = L_m + L_h}$ and the result above we get

$$\mathbf{L_m + L_h = L_m/(1\text{-}a)}$$

$$\mathbf{L_h/L_m = 1/(1\text{-}a) - 1}$$

$$\mathbf{L_h/L_m = a/(1\text{-}a)}$$

The ratio of hunting time to migrating time is equal to the ratio of the elasticity of output with respect to labour, **a,** to **1** minus this elasticity.

We may remind the reader here that in the context of a macroeconomic production function of type Cobb-Douglas, $\mathbf{Q = K^aL^{1-a}}$, where **K** is capital, the ratio **a/(1-a)** corresponds to the ratio of profits to wages.

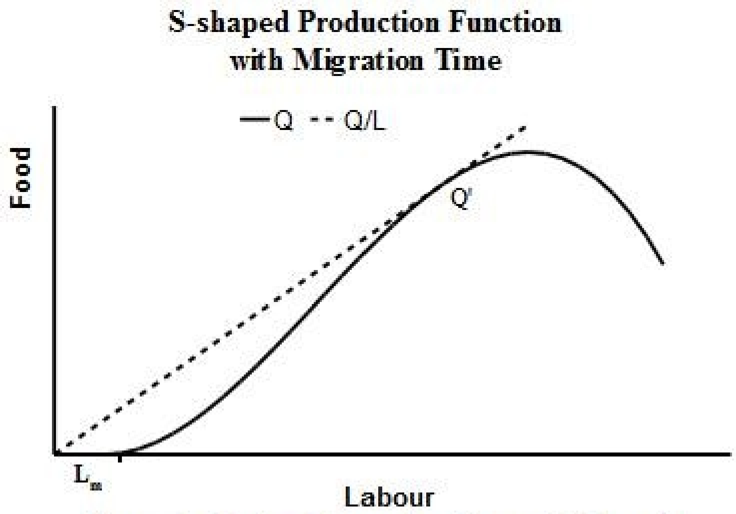

Figure 3: Optimal Foraging Time - S-shaped Production Function

Figure 3 shows the optimal foraging time for the case of an S-shaped production function.

Again one sees the validity of the Marginal Value Theorem **Q/L = dQ/dL** as average productivity is maximal there where the ray from the origin is tangent to the S, at output level **Q'**. But one should notice that this point lies above the point of tangency of the ray originating not at the origin but at the bottom of the S, that is omitting migration time, $\mathbf{L_m}$. The difference in output is potential surplus as we shall see further below.

Proof of the Marginal Value Theorem

The proof of the Marginal Value Theorem is based on a well behaved production function with labour as the only input. The production function is

$$\mathbf{Q = f(L),}$$

The function is concave to the origin and everywhere twice differentiable.

$$\mathbf{dQ/dL > 0}$$

and

$$\mathbf{d^2Q/dL^2 < 0}$$

Total output can also be perceived as average labour productivity times labour.

$$\mathbf{(Q/L)L}$$

Accordingly the marginal productivity of labour **dQ/dL** may also be written as

$$\mathbf{dQ/dL = d((Q/L)L)/dL}$$

Applying the product rule we get

$$\mathbf{L\ d(Q/L)/dL\ \ + Q/L\ dL/dL}$$

As **dL/dL = 1** we get

$$dQ/dL = L\, d(Q/L)/dL + Q/L$$

At the maximum of the average product curve **d(Q/L)/dL** is **zero.**

Therefore, at the maximum of the average product curve the average product is equal to the marginal product of labour.

$$dQ/dL = Q/L$$

which is the Marginal Value Theorem.

The Relationship Between the Degree of Nomadism and Food Production

It is also interesting to investigate into the relationship between mobility and food production. We may take the number of hunting grounds visited per year (360 days), **x,** as an index of mobility or nomadism.

It is **360 = x L** and so **x = 360/L** and for optimal average productivity $L = L_m/(1-a)$. (see above)
From this we get

$$x = (1-a)\ 360/L_m$$

The degree of nomadism, **x**, is a function of productivity as indicated by the elasticity of labour, **a**, and migration labour, L_m.

Food Production and Mobility

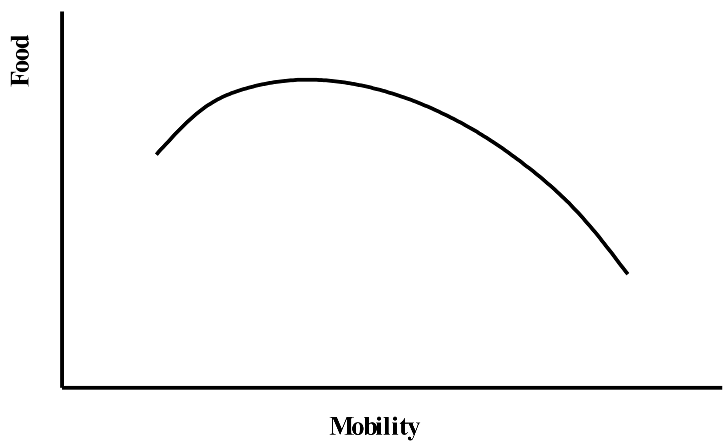

Figure 4 - Food Production and Mobility

Figure 4 shows the relationship between the degree of nomadism, **x**, and yearly food production for a given level of productivity **a**.

The maximum of this curve corresponds to the optimal time of migrating and exploiting a hunting ground, **L***.

The Minimization of the Cost of Labour to Produce Food

We have found the optimal foraging time, **L***, by solving the primal of an optimization problem which was the maximization of average labour productivity. Now we find the same solution by solving its dual, the minimum average labour cost of food production.

From the production function we may derive the average and marginal cost functions and determine the optimal foraging time by minimizing average labour cost. Labour cost is the product of labour units times the wage rate. As we are in a real economy without money, labour cost equals labour value. So average labour cost is equal to *average labour value* and marginal labour cost is equal to *marginal labour value*.

In a money economy average labour cost is equal to *average labour value* times the wage rate and marginal labour cost is equal to *marginal labour value* times the wage rate.

From the production function **Q = f(L)** we can derive the inverse **L = g(Q)** which is the *Demand for Labour*. Then the *function of average labour value* is

$$\mathbf{L/Q = g(Q)/Q}$$

and *the function for marginal labour value* is

$$\mathbf{dL/dQ = d(g(Q))/dQ.}$$

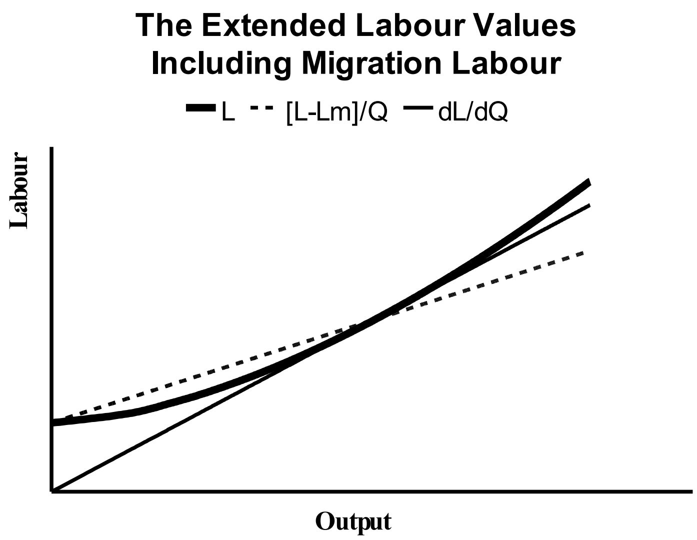

Figure 5: The Extended Labour Values

The **Figure 5** shows the function of labour values (in fact the extended form of it as it contains migration labour, $\mathbf{L_m}$) and **Figure 6** shows the functions of average and marginal labour values.

The function of extended labour values (bold) shows the cost of producing output. The migration time, $\mathbf{L_m}$, can be regarded as some fixed labour cost. This is the amount of labour at zero output. *Average fixed labour cost* are always decreasing with an increase of output. On the other hand, due to diminishing marginal productivity of labour, labour value is increasing as output increases. So, between zero output and the optimal output, where the thin line touches the function of labour values, average total labour value is decreasing as the fixed labour cost of migration is spread over more and more output. The *average labour value* is minimal where the decreasing average fixed labour is just outbalanced by increasing *marginal labour value*. From there onwards the *marginal labour value* of additional output is greater than *average labour value*.

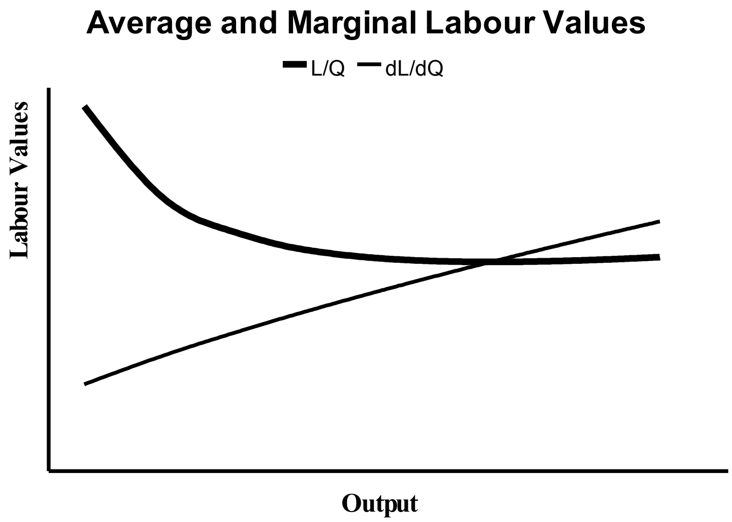

Figure 6: Average and Marginal Labour Values

This can be seen more clearly in **Figure 6** where the curve of marginal labour value cuts the curve of average labour value from below.

These curves are known in microeconomic theory as the U-shaped average cost curve which is cut from below by the marginal cost curve. In fact, under conditions of perfect competition the *curve of marginal cost* is equal to the *curve of marginal labour cost*, which as noted above, is equal to marginal labour value times the wage rate.

In **Figure 5** there is a third line (dotted) which represents average labour value omitting migration labour. Notice that the slope of this line is smaller than the slope of the tangential line which is equal to marginal labour value. We may express migration labour, $\mathbf{L_m}$**,** as the difference between marginal labour value, $\mathbf{dL/dQ}$**,** and average labour value without migration time, $\mathbf{L_h/Q}$**,** for the optimal output $\mathbf{Q^*}$.

The formal proof is as follows:

It is

$$\mathbf{L = L_m + L_h}$$

Divided by $\mathbf{Q^*}$

$$\mathbf{L/Q^* = L_m/Q^* + L_h/Q^*}$$

Rearranged we have

$$\mathbf{L_m/Q^* = L/Q^* - L_h/Q^*}$$

$$\mathbf{L_m = [L/Q^* - L_h/Q^*]\ Q^*}$$

But at the optimum, the point of intersection of the functions of average and marginal labour values

$$\mathbf{dL/dQ^* = L/Q^*}$$

Notice that this is the inverse of the Marginal Value Theorem. It expresses the Marginal Value Theorem in terms of average and marginal labour values.

Substituted into the previous expression gives

$$\mathbf{L_m = [dL/dQ^* - L_h/Q^*]\ Q^*}$$

This is a very important result. At the optimum output, **Q***, migration labour, $\mathbf{L_m}$, which represents fixed labour cost, is equal to the difference between marginal labour value and average labour value times output. As $\mathbf{L_m}$ expresses migration labour we name it in honour of Eugen von Böhm-Bawerk[1] roundabout labour. In fact, this amount of labour is potential surplus labour in the Marxian sense. A hunting ground does not yield any surplus as in this case the marginal value theorem holds. The wage, **w**, of the hunters and gatherers can be said to be equal to **w = Q*/L**, the average productivity of labour or in terms of cost all cost consist of labour cost. In terms of Marxian analysis, all cost are equal to the cost of labour power.

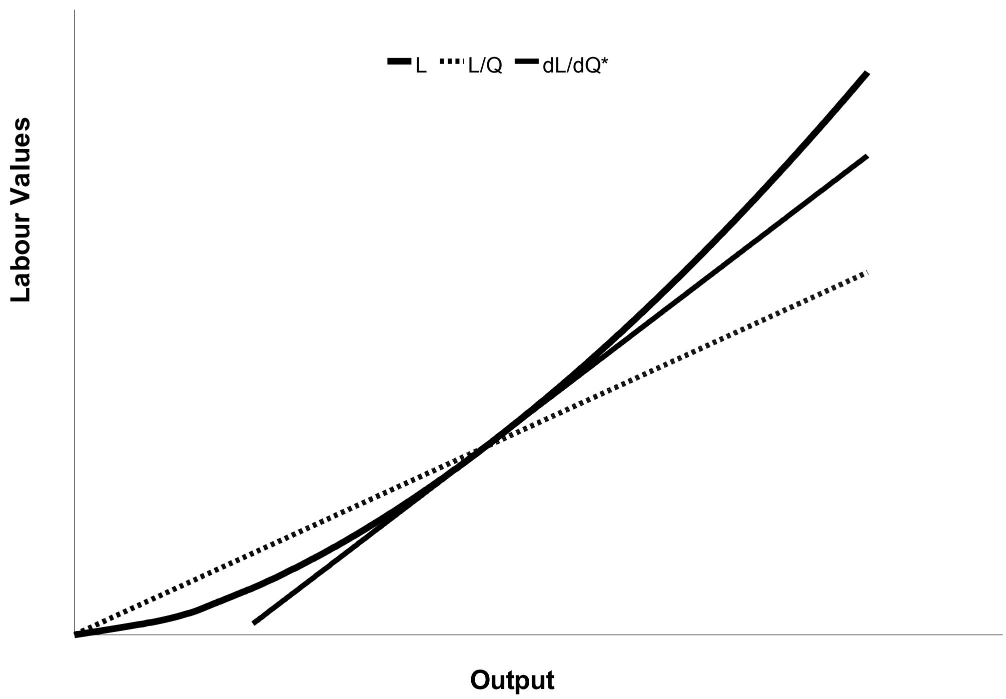

Figure 7: The Functions of Labour Values

More formally, according to Marx, the *value of labour power* is the amount of labour hours times the wage rate, **w**.

1 Eugen von Böhm-Bawerk was strictly opposed to the labour theory of value. For him there was no surplus labour. He developed the concept of roundaboutness of production which is the investment of labour into means of production. According to our interpretation, if there is a roundaboutness of production then there is also roundabout labour.

$$\mathbf{wL = Q^*.}$$

All output goes to labour. Notice that at the optimum $\mathbf{w = Q^*/L}$ and because of the marginal value theorem $\mathbf{Q^*/L = dQ/dL}$ and therefore

$$\mathbf{w = Q^*/L = dQ/dL}$$

This is true for the case of the hunters-gatherers mode of production. In this case there is no exploitation. But if migration labour, or roundabout labour can be saved as in the case of permanent settlements

$$\mathbf{dL/dQ^* > Q^*/L_h}$$

This is the source of surplus value. We shall examine it in detail below.

Figure 7 shows the function of labour values without roundabout labour, $\mathbf{L_m}$. The bold line is *the function of total labour values* originating at the origin. The thin line shows *marginal labour value* for a given roundaboutness of production, $\mathbf{L_m}$. The dotted line showing *average labour value* at the optimum has a slope inferior to the marginal labour value. The difference between them times output equals the roundabout labour, $\mathbf{L_m}$, which can be graphically seen as the distance from the origin to the intersection of the thin line with the ordinate in the fourth quadrant.

Before we explore the origin of surplus value further we examine more closely Adam Smith's case of the prices of deer and beaver.

Adam Smith and the Deer Hunter

We are now in the position to resolve the problem Adam Smith has posed in his “Wealth of Nations”.

He asserts:

> “In that early and rude state of society which precedes both the accumulation of stock and the appropriation of land, the proportion between the quantities of labour necessary for acquiring different objects seems to be the only circumstance which can afford any rule for exchanging them for one another. If among a nation of hunters, for example, it usually costs twice the labour to kill a beaver which it does to kill a deer, one beaver should naturally exchange for or be worth two deer. It is natural that what is usually the produce of two days’ or two hours’ labour, should be worth double of what is usually the produce of one day’s or one hour’s labour.”
>
> (A. Smith, 1976, chap. VI)

In fact, he assumes that there are linear production functions for deer and beaver. But this assumption is too restrictive. We shall instead use production functions with a higher degree of empirical validity. We assume production functions with diminishing marginal productivities of labour as in the above analysis.

$$Q_D = A_D L_D^a \quad \text{and} \quad Q_B = A_B L_B^b$$

$$\mathbf{0 < a, b \ \leq 1}$$

$\mathbf{Q_D}$ – amount of deer, $\mathbf{A_D}$ – a constant, **a** – elasticity of labour to hunt deer
$\mathbf{Q_B}$ – amount of beaver, $\mathbf{A_B}$ – a constant, **b** – elasticity of labour to hunt beaver

Furthermore:

$$\mathbf{L = L_D + L_B}$$

L – total labour power can be employed either to hunt deer $\mathbf{L_D}$ or beaver $\mathbf{L_B}$

We may create from the 2 production functions as well as the constraint on labour power a production possibility frontier.

$$\mathbf{Q_B = f(Q_D)}$$

Production Possibility Frontier (PPF)
Deer and Beaver Hunting

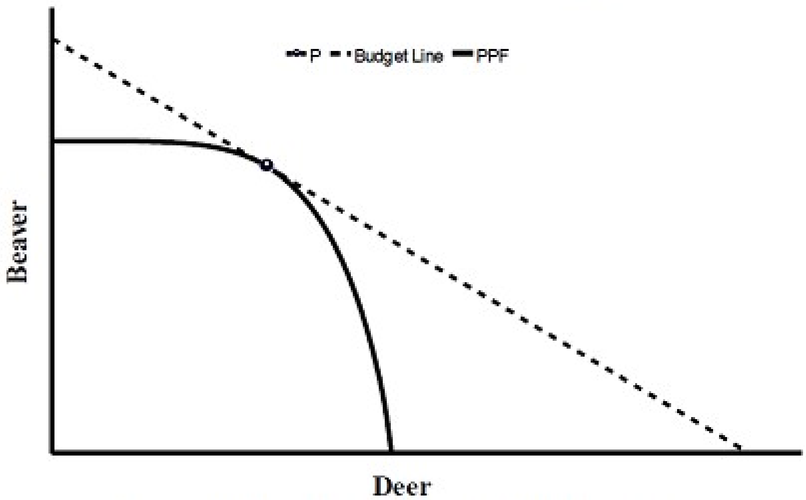

Figure 8: Production Possibility Frontier

Along this frontier all labour power is used either for hunting deer or beaver but in different quantities. However, at any point of that curve the cost of an extra unit of deer in terms of beaver or vice versa can be determined. This cost is labour value. In the Marxian sense this cost represents socially necessary labour. We are now deriving the proper analytical expression of this concept.

Along the production possibility frontier, whatever the choice of amount of deer or beaver, labour power is fully employed. This can be expressed with the total differential of labour

$$dL = \frac{\delta L}{\delta Q_D} dQ_D + \frac{\delta L}{\delta Q_B} dQ_B = 0$$

The total differential is equal to zero because the total amount of labour power employed, **L**, does not change. What changes is the proportion of labour directed towards deer hunting, $\mathbf{L_D}$, or beaver hunting, $\mathbf{L_B}$.

From this follows

$$\frac{\frac{\delta L}{\delta Q_D}}{\frac{\delta L}{\delta Q_B}} = -\frac{d\,Q_B}{d\,Q_D}$$

The ratio of the marginal labour values of deer production and beaver production is equal to the negative of the slope of the production possibility frontier. Notice that the slope of $\mathbf{f(Q_D)}$ is negative, so $\mathbf{-dQ_B/dQ_D}$ is positive.

In microeconomic theory, the negative of the slope of the production possibility function, $\mathbf{-dQ_B/dQ_D}$, is called *the marginal rate of transformation (MRT).* This marginal rate of transformation is equal to the reciprocal of relative prices $\mathbf{p_D/p_B}$.

We prove this with the help of the budget constraint. A budget, **T**, can be used either buying beaver, $\mathbf{Q_B}$ or deer, $\mathbf{Q_D}$.

$$\mathbf{T = p_B Q_B + p_D Q_D}$$

T – budget, $\mathbf{p_B}$ – price of beaver, $\mathbf{p_D}$ – price of deer

$$\mathbf{Q_B = T/p_B - p_D/p_B\ Q_D}$$

$$\mathbf{dQ_B/dQ_D = -p_D/p_B}$$

alternatively

$$\mathbf{-dQ_B/dQ_D = p_D/p_B}$$

The reciprocal of the ratio of relative prices equals the marginal rate of transformation, $\mathbf{-dQ_B/dQ_D}$,

Substituting the marginal rate of transformation by the ratio of relative prices, we have the extremely important result:

$$\frac{\frac{\delta L}{\delta Q_D}}{\frac{\delta L}{\delta Q_B}} = \frac{p_D}{p_B}$$

The ratio of marginal labour values is equal to the ratio of prices. Marginal labour values are proportional to prices.[2] In the case of the hunters and gatherers, marginal labour values equal prices as there is no money. The marginal labour values, $\mathbf{\delta L/\delta Q_D}$ and $\mathbf{\delta L/\delta Q_B}$**,** are the analytic expressions of the *socially necessary labour* in Marx. These marginal labour values varie depending on demand!

Graphically this can be seen in **Figures 5** and **7**. The price of food is equal to the marginal labour value which is indicated by the slopes of the thin lines.

Far from being true only for the ancient times of the hunters and gatherers this remains valid for the capitalist mode of production also, if there would be a state of perfect competition in the capitalist

2 This proof has been put forward in a somewhat disguised form by Henderson and Quandt (Henderson & Quandt, 1980, p. 92 ff.). There the $\mathbf{h_1}$ and $\mathbf{h_2}$ are the marginal labour values.

mode of production. Many economists, bourgeois, orthodox, Ricardian, Keynesian or Marxist deny that capitalism can be in a state of perfect competition. Nevertheless, under perfect competition prices are proportional to marginal labour values and therefore the labour theory of value holds. Adam Smith's case is a special case where marginal labour values equal average labour values. This is just the case of the Marginal Value Theorem.

We shall now explore more closely the fact that surplus labour is the difference between marginal labour value and average labour value.

The Origin of Exploitation

Above we examined the labour value function and found that there is a discrepancy between average labour and marginal labour value which is due to the labour value of migration or roundabout labour, $\mathbf{L_m}$. In **Figure 7** this can be seen, as the slope of the dotted line representing average labour value, $\mathbf{L_h/Q^*}$, is smaller than the slope of the thin line representing marginal labour value.

We have shown that roundabout labour, $\mathbf{L_m}$ is

$$\mathbf{L_m = [dL/dQ^* - L_h/Q^*]\ Q^*}$$

We have also shown, that marginal labour value $\mathbf{dL/dQ^*}$ is the price of food for the hunters and gatherers. If there are improvements in agricultural food production and food storage, then this roundabout labour can be saved. Engels suggests that the improvements in food storage through the introduction of pottery may have been one of the major factors improving general productivity and leading to permanent settlements (Engels, 1884). If the settlers can dispose of that amount of labour for other things than food production, they accumulate wealth. Indeed it is this surplus labour, with which the pyramids have been built!

To gain a surplus a settler has to pay a wage to the labourer which is equal to the wage of the hunters-gatherers, $\mathbf{w = Q^*/L}$. But his total cost are less than total production as he has to pay only $\mathbf{wL_h}$ instead of $\mathbf{wL}$.

In order to achieve a surplus

1) The average subsistence wage must be paid which is $\mathbf{w = Q^*/L}$ and as $\mathbf{Q^*/L = dQ/dL}$, $\mathbf{w = dQ/dL}$
2) The average productivity of labour employed, $\mathbf{Q^*/L_h}$ must be above this wage, $\mathbf{Q^*/L_h > w}$.

In terms of output surplus is

$$\mathbf{Q^*_R = [Q^*/L_h - dQ^*/dL]\ L}$$

In terms of value surplus is:

$$\mathbf{L^*_R = [dL/dQ^* - L_h/Q^*]\ Q^*}$$

In microeconomic theory, a system in perfect competition and constant returns to scale has the property that the money value of the surplus labour, $\mathbf{wL_R}$, is equal to the return on capital, $\mathbf{rK}$ ($\mathbf{r}$ – rate of interest, $\mathbf{K}$ – capital), for a profit maximising firm.

From our analysis it is obvious that social progress can be achieved by economizing on migration labour. With the introduction of settlements the accumulation of wealth becomes also possible. One should notice however that exploitation is not possible, if there is enough settlement land for every one. But in any case settlements open up the way for the exploitation of slave labour where the slaves have been acquired by war.

In Historical Materialism the mode of production of the slave holders is based on the exploitation of slaves. One may say that the slaves are prevented from migrating and that the migration labour or roundabout labour is directed to the creation of wealth for the slave holders.

The Sraffian Model and Roundabout Labour

We shall not leave the hunting ground without gaining a particularly precious trophy, Piero Sraffa.

Piero Sraffa, in order to examine fundamental value relationships of a system of production, has developed a model of production (Sraffa, 1960), which, like the Leontief model, is based on linear production functions. He assumes, that the system is totally static, there is no change. He makes this assumption in order to avoid any marginal reasoning. As there is no change there are no marginal magnitudes. However, at some point he introduces a rate of interest and assumes that a part of the value added of the production process would go to profits.

From our analysis we know that profits presupposes roundabout labour. But if there is roundabout labour this is incompatible with a static state. The roundaboutness is an expression of the dynamics of a system. So, if the production system is truly static there cannot be any roundabout labour and consequently no profits as the difference between marginal labour value and average labour value is zero.

But if we introduce some roundaboutness and consequently roundabout labour this also means that the system becomes dynamic. With some roundaboutness the production function is a straight line originating somewhere from the abscissae to the right of the origin and the function of labour values originates at a point on the ordinate where labour is greater zero. For the case of the function of labour values we can calculate for each point of the function average fixed cost of labour which is the amount of roundabout labour divided by output. This average fixed cost would always be positive but declining with increased output. So it would always be advantageous to increase output which gives rise to dynamics. Hence there is no static equilibrium in this case.

Now considering Sraffa's model, there is capital which represents some roundaboutness. So his system can never be in a truly static state. This means also that the rate of interest is always greater zero. In fact, Sraffa's assumption contradicts Schumpeter's proposition that in a truly static state the rate of interest must be zero (Schumpeter, 1982; Clark, 1912). Sraffa's analysis is the analysis of a system in equilibrium but not of a system in a static equilibrium. His proposition of a perfect static state is not compatible with a model which includes capital as this introduces some roundaboutness of production. Notice, that Schumpeter assumes for his static circular flow that there is no capital as a stock.

However, Sraffa's notion of dated quantities of labour is a proper definition of the labour value of commodities, the socially necessary labour which is the sum of the value of labour power plus surplus labour (Hagendorf, 2008).

Optimization and the Labour Theory of Value

Finally we shall apply our insights against the most ardent opponents of the labour theory of value. In the beginning of this paper we underlined the importance of optimization methods to the labour theory of value. One cannot discuss the subject of the labour theory of value without referring to optimization methods and their use in economics. So there cannot be a serious discussion of the labour theory of value without referring to the introducer of these methods into economics. We refer here to Jevons' and his treatment of labour and the subsequent discussion on value by authors like Alfred Marshall and John Bates Clark, which led to the creation of the American Economic Association and the British Royal Economic Society. The most famous Marxists Maurice Dobb and Ronald Meek have not gone properly into the details of these discussions (Dobb, 1975) and Ronald Meek (1973). They are only examples of a whole army of left economists who try to justify Marx but who do not do the essential work of Marxists which is the criticism of bourgeois political economy. The importance of the relationship between optimization methods and the labour theory of value becomes particularly clear in the work of Kantorovich which can be regarded as a milestone in modern Marxian economics.

Leonid Kantorovich, the inventor of linear programming in his book "The Best Use of Economic Resources"(1965) does not only explain the application of linear programming to economic problems but he defends also the labour theory of value. The publication of the book (in Russian 1958) forced the most famous American economists Dorfman, Samuelson and Solow to publish their "Linear Programming and Economic Analysis" (Dorfman, Samuelson, & Solow, 1958) in which Kantorovich is never mentioned. And of course they deny the validity of the labour theory of value categorically. They are hypocritical, because they know the works of Jevons, Marshall, John Bates Clark and Irwing Fisher very well – which means that they know that those authors did not simply deny the validity of the labour theory of value but were aware of its dangers to capitalism and therefore had disguised it.

The Harvard professor Robert Dorfman took up the task of separating Kantorovich and optimization theory in general from the Marxists in his review of the English translation of Kantorovich's book which was published by Harvard University Press and is full of printing errors especially in the mathematical part, where it becomes almost unintelligible at some places. He writes "Linear programming became a powerful analytic tool for perfecting the Walrasian theory of general equilibrium in the hands of L. W. McKenzie, H. W. Kuhn, and a number of others, culminating in G. Debreu's Theory of Value"(Dorfman, 1966). After a discussion of the relationship between Kantorovich's concept of shadow prices (objectively determined valuations) and the value of marginal products, Dorfman concludes:

> "Simply as a matter of doctrine, the concept that the value of anything is the amount of labor time that its possession enables the owner to save is not to be found in either Ricardo or Marx. It is much closer, indeed, to Adam Smith's concept that value is equivalent to a command over labor. It is closer still to the modern bourgeois, neoclassical view that the value of any productive instrument is equal to its marginal productivity, the only difference being the purely verbal one that Kantorovich has chosen to use labor time as the numéraire in terms of which to express the value of everything else."(Dorfman, 1966)

We see here when he speaks of "the amount of labor time that its possession enables the owner to save" he refers to what Marxists call 'value of labour power'. But one should notice that the labour value of a commodity, **dL/dQ,** is higher than the value of labour power expended on its production, **L/Q,** the difference we have called roundabout labour. But everyone should realize that it is exactly the roundabout labour that the slave holder exploits from the slave. The slave is hold captive and instead of migrating is forced to work. If Dorfman, in front of the pyramids, would say "But there is no roundabout labour!" he should acknowledge that the people of Israel proves it's existence. The money value of roundabout labour or surplus labour is indeed the difference between labour commanded and wages in Adam Smith.

Université Paris Ouest, 22.9.2009

Klaus Hagendorf

Bibliography

Bidet, Jacques. (1992). Y a-t-il une écologie marxiste? In Bidet, Jacques & Texier, Jacques (Eds.), *L'écologie, ce matérialisme historique*, Actuel Marx (Vol. 12, pp. 96-112). Paris: Presses universitaires de France.

Charnov, E. L. (1976). Optimal foraging, the marginal value theorem. *Theoretical Population Biology*, *9*(2), 129-136.

Clark, J. B. (1912). Review: [untitled]. *The American Economic Review*, *2*(4), 873-875.

Dobb, M. (1975). *Theories of value and distribution since Adam Smith : ideology and economic theory*. London: Cambridge university press.

Dorfman, R. (1966). Review: [untitled]. *The American Economic Review*, *56*(3), 592-597.

Dorfman, R., Samuelson, P. A., & Solow, R. M. (1958). *Linear Programming and Economic Analysis*. New York, Toronto, London: McGraw-Hill.

Engels, F. (1884). *The Origin of the Family, Private Property and the state : In the Light of the Researches of Lewis H. Morgan*. New York: International publishers. http://www.marxists.org/archive/marx/works/1884/origin-family/index.htm.

Hagendorf, K. (2008). The Labour Theory of Value: A Marginal Analysis. *Munich Personal RePEc Archive*, (10202), http://mpra.ub.uni-muenchen.de/10202/

Henderson, J. M., & Quandt, R. E. (1980). *Microeconomic theory: a mathematical approach* (3nd ed.). New York: MacGraw Hill.

Henry, John F. (2000). Adam Smith and the Theory of Value: chapter VI Reconsidered. *History of Economics Review*, *31*(Winter 2000).

Kantorovich, L. V. (1965). *The Best Use of Economic Resources*. Cambridge (Mass.): Harvard University Press.

Kaplan, Hillard, & Hill, Kim. (1992). The Evolutionary Ecology of Food Acquisition. In E. A. Smith & B. Winterhalder (Eds.), *Evolutionary Ecology and Human Behavior* (pp. 167-201). New York: Aldine de Gruyter.

Lee, R., & Daly, R. H. (1999). *The Cambridge encyclopedia of hunters and gatherers*. New York: Cambridge University Press.

Meek, R. L. (1973). *Studies in the labour theory of value* (2nd ed.). London: Lawrence and Wishart.

O'Neil, Denis. (2009). Patterns of Subsistence: Menu of Topics. *Paterns of Subsistence*. http://anthro.palomar.edu/subsistence/Default.htm.

Samuelson, P. A. (1971). Understanding the Marxian Notion of Exploitation: A Summary of the So-Called Transformation Problem Between Marxian Values and Competitive Prices. *Journal of Economic Literature*, *9*(2), 399-431.

Schumpeter, Joseph Alois. (1974). *Theory of Economic Development*. London: Oxford University Press.

Smith, A. (1976). *An Enquiry into the Nature and Causes of the Wealth of Nations. The Glasgow edition of the works and correspondence of Adam Smith*. Oxford: Clarendon Press.

Smith, E. A. (2009). Eric Alden Smith's Home Page. http://faculty.washington.edu/easmith/.