Contextual algorithm for decision of fuzzy estimation problems with network-like structure of criteria on the basis of fuzzy measures Sugeno

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Abstract

In this article the algorithm for the decision of alternatives' estimation problems for following conditions is considered. Values of alternative's characteristics (properties) are fuzzy. They are formalized as fuzzy sets. The estimation criteria structure is network-like and is formalized as the oriented graph with one source and many drains. The alternative's estimation result is calculated in criterion-source. Connections between criteria are formalized by fuzzy measures Sugeno. Upper-level criteria are considered as contexts for lower-level criteria. Fuzzy integrals Sugeno or Choquet are used as aggregation operator. In article also the properties of fuzzy measure and fuzzy integrals (Sugeno and Choquet) are analyzed. Properties of fuzzy measure and integrals are comparing with properties of other mathematical tools. As example the car's estimation problem is presented.

Keywords: fuzzy measure (Sugeno); fuzzy integral (Sugeno and Choquet); alternatives estimation; criteria structure

1. Introduction

At the present day the need of analytical decisions substantiation in applied problems grows. In practice the majority of analytical problems are alternatives' estimation problems. Often the alternatives' characteristics values cannot be precisely determined. The information which is necessary for this, or is absent, or have high price, or time for her collection is limited. Therefore such problems name as fuzzy problems. The essence of estimation problems consists in the following. On the basis of set of alternative's characteristics values it is necessary to receive a unique estimation of this alternative according to the criteria system. The criteria system is considered as the estimation standard (ideal alternative). The estimation is considered as conformity degree of this alternative to ideal alternative. The criteria system generally can have any acyclic structure. Criteria reflect the specific (partial) concepts and the abstract concepts. For the problem decision it is necessary to transform alternative's estimations at lower-level criterion to her estimations at upper-level criterion. This transformation is performed by means of aggregation (for example, additive, multiplicative or maxmin operators). From properties of aggregation depends, how much precisely the connections between criteria describe expert's opinions which solves a problem. Alternative's characteristics values can be assigned on continuous quantitative or discrete qualitative set of values (for example, colour: {dark blue, red, green}).

The set of criteria, as a rule, is known at the problem formulation. Complexities arise by determination of connections between criteria. Firstly, branches in criteria

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system can be crossed. It means that the alternative's estimation at lower-level criteria can be used for the subsequent aggregation not in one, but in several upper-level criteria. Secondly, the consideration only weights of criteria often is insufficient. The important factor also is semantics of aggregation, for example, realization of logic operations "OR", "And". For example, according to logic "And", the alternative's estimations should aggregate under the scheme: the resultant estimation will be maximal only under condition of the maximal values of entrance estimations by all criteria.

At present moment the decisions techniques of multicriterial problems of estimation and classification (calculation of conformity degree to classes) with use of Sugeno or Choquet integrals (Kima Yeong Min, Kimb Chee Kyeong, 2006; Magyla T., 2002; Pham Tuan, Wagner Michael, 2000; Denguir-Rekik A., Montmain J., Mauris G., 2006; Grabisch M., 2003; Ceberio M. and Modave F., 2004) are known. But in publications mainly the hierarchical problems with non-crossed branches of hierarchy are considered. It is essential restriction for use in practice. Also for decision of estimation's problems are used the methods of neural networks (for example Laukonen E.G., Passino K.M., 1995). However for training neural networks is necessary a plenty of statistical data. Values, which are calculated by means of neural networks, non-always have an explanation. Therefore is topical the creation of universal algorithm for decision of fuzzy estimation problems with network-like criteria structure.

2. The generalized algorithm for the problem decision of alternative's estimation

Let's consider formal representation of criteria's system with network-like structure. We are denoting $C = \{c_i, i = \overline{1, Q^C}\}$ as the set of estimation criteria. The criteria system is formed by means of relations set. Relations can have various sense which depends on a problem. For example, relations can reflect functional dependences of criteria or attributive connections. By analogy to the graphs theory, the criteria system is the acyclic oriented graph with one source cs (the upper-level criterion) and with many drains (the lower-level criteria) $CD = \{cd_j \in C, j = \overline{1, Q^{CD}}\}$ without dangling tops. The condition of acyclicity is natural. It is the condition of tautology absence. The set CD is considered as the set of alternative's characteristics. This set also is universal set in estimation problem. By analogy to the graphs theory, the abstractiveness level of criterion can be described as its length L_i. This length is equal to a maximum quantity of criteria in route: from criterion c_i to any criterion from set CD. Lengths of drains-criteria are equal to zero: $\forall c_i \in CD : L_i = 0$, and the length of source-criterion is maximal: $L_{cs} = \max\limits_{i=\overline{1,Q^C}} L_i$.

For any criterion c_i there are two sets:

$D_i = \{c_j, j = \overline{1, Q^{D_i}}\}$ - is the set of relation values (c_i, c_j) (further - criterion values set);

$S_i = \{c_j, j = \overline{1, Q^{S_i}}\}$ - is the set of relation determination (c_j, c_i) (further - criterion determination set).

For all criteria from the set CD the set of relation values is empty $D_i = \varnothing$. To these criteria the special discrete sets of values $D_i^{CD} = \{d_{ij}^{CD}, j = \overline{1, Q^{D_i}}\}$ are attributed. These sets are considered as the sets of characteristics values of alternative which we estimate in problem. Such structure of criteria system is generalization of hierarchy for which is right: $Q^{S_i} = 1 \forall c_i \in C, c_i \neq cs$. In comparison with hierarchy this structure gives more opportunities for description of expert's preferences system.

The generalized algorithm of estimation problem decision (after the weights assignment for all criteria relations and after assignment of alternative's characteristics values) includes two steps:

1) Transformation of alternative's characteristics values to values which assigned on discrete set D_i^{CD};
2) Consecutive aggregation of these values in criteria. First aggregation is carries out in criteria with $L_i = 0$. The aggregation result is alternative's estimation in criteria with $L_i = 1$ and so on up to criterion with L_{cs}.

The general structure of estimation problem decision does not differ from classical structure of expert systems. It is depicted in figure 1.

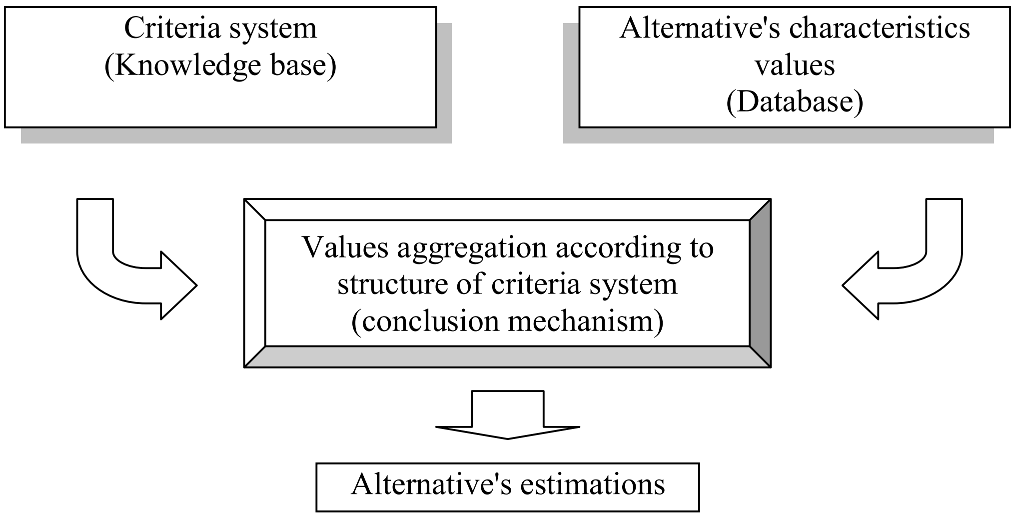

Figure 1 - Structure of estimation problem decision

Adequacy of expert's preferences formalization depends on mathematical properties of aggregation operator. Most effective formalization tool for connections between criteria is the fuzzy measure Sugeno. And most effective aggregation tool is the fuzzy integral.

3. Fuzzy measure, fuzzy integrals Sugeno and Choquet, their properties and comparison with other aggregation operators

According to (Sugeno M., 1974), the fuzzy measure $g(\cdot)$ is function $g : \boldsymbol{B} \to [0,1]$ ($\boldsymbol{B}$ - power set) which satisfies to following conditions:

1) $g(\varnothing) = 0$;
2) $g(X) = 1$;
3) if $A, B \in \boldsymbol{B}$ and $A \subset B$, then $g(A) \le g(B)$;
4) if $F_n \in B$ and $\{F_n\}$ is monotonous sequences, then $\lim_{n\to\infty} g(F_n) = g(\lim_{n\to\infty} F_n)$.

Expression $g(A)$ is measure which characterizes a fuzzyness degree of judgement $X \in A$. λ - rule is used for construction of fuzzy measures. Let $A, B \in \boldsymbol{B}, A \cap B = \varnothing$. Then $g_\lambda(A \cup B) = g_\lambda(A) + g_\lambda(B) + \lambda \cdot g_\lambda(A) \cdot g_\lambda(B), -1 < \lambda < \infty$.

The parameter $\lambda \in (-1, \infty)$ is called as parameter of normalization for g_λ measures. For discrete set $S = \{s_1, s_2, ..., s_N\}$ the parameter λ determine from equation: $\frac{1}{\lambda}[\prod_{i=1}^{N}(1 + \lambda g_i) - 1] = 1$.

The axiomatics of fuzzy measure give greater opportunities for adequate formalization of connections between criteria. In particular, the fuzzy measure is capable to formalize semantics of expert's preferences.

Generally, the fuzzy measure supposes that the trust's degree of judgement $A (A \ne \varnothing)$ (which is true) not always is equal 1. It means, that sum of trust's degree to judgement A and of degree of its denying can be or more than 1, or equal 1, or less than 1. If to compare axiomatics of fuzzy measure with axiomatics of probability (according to Kolmogorov, as shown (Pospelov D.A., 1986)), it is possible to make conclusion that the probability measure is the shrinking of fuzzy measure. In particular, the fuzzy measure will have the properties of probability for $\lambda = 0$. The probability measure needs additivity of expert's judgement. However these judgements more often are not additive. In estimation problems with hierarchical criteria structure this restriction does not play the big role as branches in hierarchy are not crossed. However in problems with network-like criteria structure this restriction is critical.

Depending on value λ two classes of fuzzy measures are considered: superadditive measures - trust's measures ($\lambda \ge 0$) and subadditive measures - plausibility measures ($-1 < \lambda \le 0$). An outermost case of trust's measures ($\lambda >> 0$) are necessity measures, and of plausibility measures ($\lambda = -1$) - possibility measures. As shown (Pospelov D.A., 1986), the membership of fuzzy set is considered as possibility measure for nesting case of focal elements of power-set.

Let's consider a classical example: toss of coin. From view-point of probability, the subjective estimation for one side of coin will be equal 0.5 (the sum of opposite events for this experience will be equal 1). From view-point of possibility, the subjective estimation for one side of coin will be equal 1 (the sum of opposite events for this experience will be equal 2). The event's possibility is understood as absence of principle restrictions for not-appearance of this event. From view-point of necessity, the subjective estimation for one side of coin will be equal 0 (the sum of opposite events for this experience will be equal also 0). Necessity is understood as presence of restrictions which not guarantee result. These three view-points can be considered as semantics of expert's expectations: pragmatism, optimism and pessimism. Apparently what all three semantics of expert's preferences is formalized by means of one mathematical tool - fuzzy measure.

If the connections between criteria in estimation problem are formalized by means of fuzzy measures, then for estimation's aggregation is used the fuzzy integral of Sugeno (Sugeno M., 1974) or Choquet (Choquet G., 1953).

Fuzzy integral of Sugeno

The Sugeno integral of function $h: X \to [0,1]$ on discrete set $A \subseteq X, X = \{x_i, i = \overline{1,N}\}$ along the fuzzy measure g is defined as:

$$(s)\int_A h(x) \circ g(\cdot) = \max_{i=1}^{N}(\min(h(x_i), g(H)), H = \{x_j \mid h(x_j) \geq h(x_i), j = \overline{1,N}\} . \qquad (1)$$

Fuzzy integral of Choquet

The Choquet integral of function $h: X \to [0,1]$ on discrete set $A \subseteq X, X = \{x_i, i = \overline{1,N}\}$ along the fuzzy measure g is defined as:

$$(c)\int_A h(x) \circ g(\cdot) = \sum_{i=1}^{N}[h(x_i) - h(x_{i-1})]g(H), h(x_0) = 0 . \qquad (2)$$

From the mathematical view-point the Sugeno and Choquet integrals are in detail considered in (Pham Tuan, Wagner Michael, 2000). Therefore here we shall consider only their most important properties for use on practice. The Sugeno and Choquet integrals provide various properties of aggregation procedure which depend from properties of fuzzy measure. For probability measure ($\lambda = 0$) the fuzzy integral is equivalent to additive aggregation. For possibility measure ($\lambda = -1$) the fuzzy integral is equivalent to maximum of membership (fuzzy logic "OR"). For necessity measure ($\lambda >> 0$) the fuzzy integral is equivalent to minimum of membership (fuzzy logic "And"). Other values λ will determines other aggregation properties.

Without loss of generality we shall accept that membership is arranged on decrease. Then calculation procedures of Sugeno and Choquet integrals can be presented, as it is illustrated in figure 2. In Sugeno integral the basis of logic operators of maximum and minimum is used. The integration result is calculated as crossing of membership and fuzzy measure. In Choquet integral the basis of arithmetic operations of subtraction and multiplication is used. The integration result is calculated as area size.

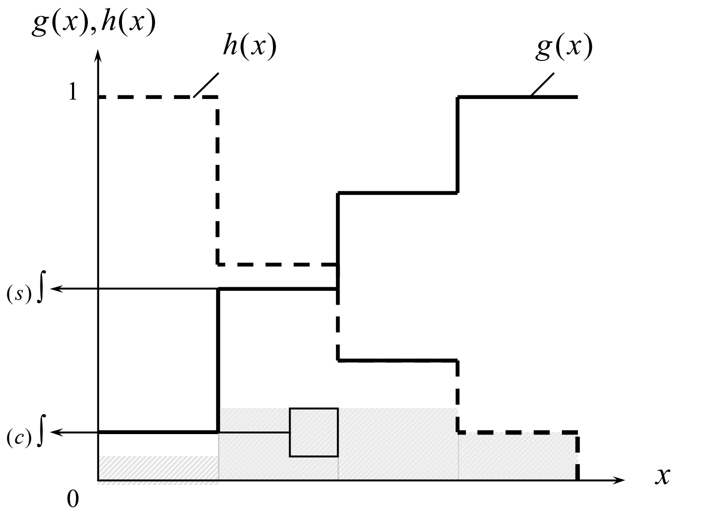

Figure 2 - The calculation procedures of Sugeno and Choquet integrals

However there is one more difference. The result of Sugeno integral takes into account only elements from subset H on which the expression (1) is performed. The result of Choquet integral take into account all elements of set X . Therefore the Sugeno integral allows explaining the received result. It provides the selection of elements of universal set which have influenced and not influenced result.

The Sugeno integral also has advantage from the view-point of adequacy. It is not linear function and consequently it allows to model threshold reactions which are peculiar to many natural systems. In complex analytical problems it is important. For example, it is necessary to make model of system which has following reaction:

- the first entrance parameter is increased - the resultant estimation does not change;
- the second entrance parameter also is increased - the resultant estimation also does not change;
- the third entrance parameter is increased - the resultant estimation is changed.

In complex systems the synergetic effects are modelling by means of this integral.

4. Contextual algorithm for decision of fuzzy estimation problems with network-like criteria structure

Using fuzzy measures and fuzzy integrals, it is possible to construct the general algorithm for estimation problem decision. This algorithm will consist of following steps.

1) Transformation of alternative's characteristics values to values which are assigned on discrete set D_i^{CD} .

Transformation is performed for formation of membership h_i which is used for integration in quantitative criterion c_i. Transformation is performed differently for qualitative and quantitative characteristics of alternative.

For qualitative characteristics of alternative. The assignment of qualitative characteristics is performed directly on discrete set. Estimations of alternative's qualitative characteristics are represented as membership: $h_i : D_i^{CD} \to [0,1]$.

For quantitative characteristics of alternative. For quantitative characteristics this transformation is performed by means of linguistic variable, for example as shown in (T. Magyla, 2002). To each quantitative criterion c_i the linguistic variable is attributed:

$$T_i = \{(d_{ij}^{CD}, t_{ij}), j = \overline{1, Q^{D_i}}; t_{ij} : [R_{ij}^{\min}, R_{ij}^{\max}] \to [0,1]\},$$

where $[R_{ij}^{\min}, R_{ij}^{\max}]$ - is numerical interval which can be various for different pairs (d_{ij}^{CD}, t_{ij}).

The linguistic variable T_i is composed of functions which correspond to values set elements of drain-criterion. These functions are assigned on numerical intervals.

The membership for integration is determined as the conformity degree of characteristic's value to linguistic descriptions d_{ij}^{CD}:

$$h_i = \{e_{ij} = \max_{[R_{ij}^{\min}, R_{ij}^{\max}]} \min(t_{ij}, r_i), j = \overline{1, Q^{D_i}}\}.$$

The transformation algorithm of quantitative characteristics is illustrated on figure 3.

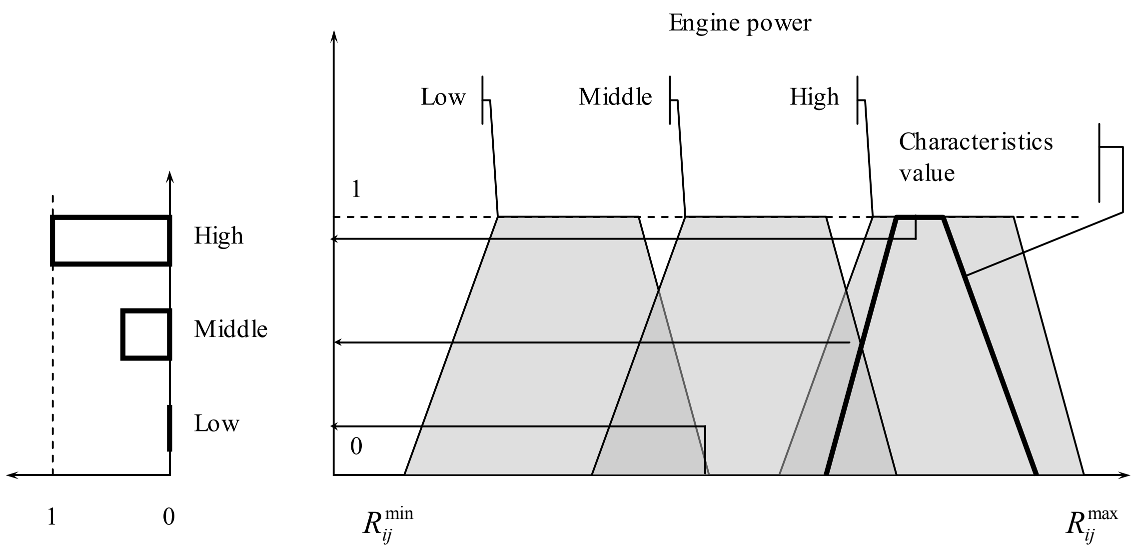

Figure 3 - The transformation algorithm of quantitative characteristics of alternative

2) Consecutive aggregation of alternative's characteristics values in criteria.

To each criterion the set of fuzzy measures according to elements quantity of criterion determination set is attributed: $M_i = \{\mu_{ij}(\cdot): 2^{D_i} \to [0,1]; j = \overline{1, Q^{S_i}}\}$. This set we name as criterion contexts. For source-criterion the context is the self criterion. Fuzzy measures are determined on values set of criterion. Contexts set and the set of criterion values are illustrated in figure 4.

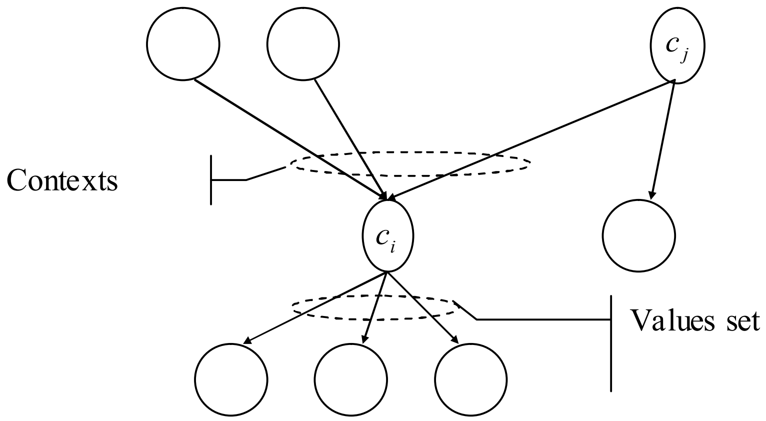

Figure 4 - Criterion connections

Such structure provides calculation of several contextual estimations of alternative in one criterion. It most adequately describes structure of estimation problem decision. Really, for example, the car's estimation by criterion "Engine power" can be considered differently from several view-points: safety, prestigiousness and fuel consumption of the car.

The fuzzy integral of membership along fuzzy measure μ_{ij} provides calculation of an alternative's estimation in criterion c_i for context j:

$$e_{ij} = (s)\int_{D_i} h_i \circ \mu_{ij}(\cdot); j = \overline{1, Q^{S_i}} ,$$

where membership is composed of alternative's estimations in criteria from D_i. Integration is performed consistently in all tops of criteria structure, since criteria of length $L_i = 0 \ldots L_{CS}$.

The algorithm is used by authors in software Expro Master.

5. Example of car's estimation in according to structure of consumer preferences

In figure 5 the simplified structure of criteria system in car's estimation problem is illustrated. Such problem arises at preferences determination of potential customers for creation of company marketing strategy. In practice the structure of consumers' preferences is much more complex. Therefore simplification only demonstrates functioning of the considered algorithm. In figure 5 the arrows depict criteria subordination. Apparently, four criteria are considered in several contexts. Two criteria ("Cost" and "Engine power") are quantitative.

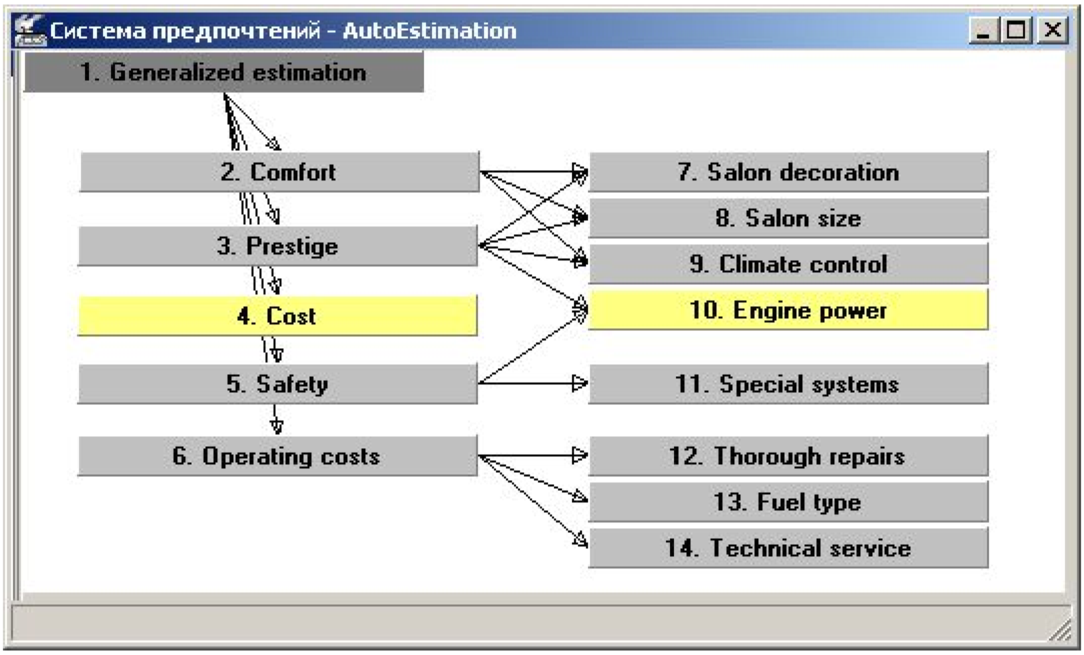

Figure 5 - Criteria system structure

In table 1 contextual estimations of hypothetical car in criteria are represented. Criteria numbers correspond to numbers in figure 5. In left column of table the criteria are listed. In upper line of table the criteria contexts are listed.

Table 1 - Contextual estimations of hypothetical car in criteria

	1	2	3	4	5	6	7…14
1	0,408						
2	0,493						
3	0,309						
4	0,75						
5	0,385						
6	0,611						
7		0,493	0,287				
8		0,72	0,512				
9		1	1				
10			0,15		0,206		
11					0,781		
12						0,69	
13						0,383	
14						0,63	

6. Conclusions

The considered algorithm provides the problem decision of alternative's estimation which has network-like estimation criteria structure. Criteria can be quantitative and qualitative. Quantitative criteria correspond to alternative's characteristics which are measured in quantitative scale. Qualitative criteria correspond to alternative's characteristics which are measured in qualitative discrete scales. The algorithm estimates alternative in several criterion contexts. Use of contexts provides the description of network-like criteria structure. Criterion contexts formalize the different view-points on estimation value. Connections in criteria structure are formalized by means of fuzzy measures Sugeno. The aggregation tool is fuzzy integral of Sugeno or Choquet. Use of additive measures (for example, probability measures) in network-like criteria structures causes the repeated account of same criterion and leads to systematic error at estimation. Non-additive fuzzy measures of Sugeno allow avoiding this imperfection. The offered algorithm can be used in any estimation problems without restrictions.

By means of algorithm the classification problem can be solved as estimation problem of object's conformity to earlier formed classes. The algorithm also can be used in clusterization problem for distance measurement between objects. In this case the distance is measured in context of criteria structure. Really, same objects can be united into classes depending on consideration context. For example, the dinner fork can be united both with tableware's, and with murder instruments. Besides by means of offered algorithm it is possible to formulate and solve problem of non-statistical forecast: connections of criteria structure and alternative's characteristics can change under events influence in future. We plan to publish a decision method of this problem in the near future.

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