A Theory of Gender Wage Gap

Jellal, Mohamed and Nordman, Christophe

Al Makrîzî Institut d'Economie

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Mohamed Jellal

Al Makrîzi Institut D'économie

Christophe Nordman

Dial , IRD- Paris

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Abstract

In this paper, we introduce uncertainty of the labour productivity of women in a competitive model of wage determination. We demonstrate that more qualified women are then offered much lower wages than men at the equilibrium. This result is consistent with the glass ceiling hypothesis according to which there exist larger gender wage gaps at the upper tail of the wage distribution.

JEL Classification: J24

Keywords: Gender wage gap; Glass ceiling; Productivity; Uncertainty

1. Introduction

In many countries, either developed or developing, it is well known that men and women with identical productive characteristics receive different levels of wage (see Blau and Kahn, 2000). Among the various explanations which have been suggested, economists have proposed theoretical models which most often focus on either qualifications or labour market treatment of similarly qualified individuals. On an empirical side, recent studies have evidence larger gender wage gaps at the upper tail of the wage distribution, so that it concerns in most cases the more skilled workers. This is the so-called glass ceiling effect above women in the labour market.
The seminal paper is due to Albrecht et alii (2003) using Swedish data. They show that the gender wage gap is empirically increasing throughout the conditional wage distribution and accelerating at the top during the nineties. Using data for Spain, De la Rica et alii (2005) stratify their sample by education group and find that the gender wage gap is expanding over the wage distribution only for the group with tertiary education, meaning that there is a glass ceiling only for the more educated[1]. Using the European Community Household Panel data set, similar conclusions are reached by Arulampalam et alii (2004) for their ten European countries s, both in the public and private sectors, with a gender wage gap typically wider at the top.

Clearly, this finding seems challenging with respect to the existing arguments which seek to formally explain the gender wage gap. For instance, in models of statistical discrimination, differences in the treatment of men and women arise from average differences between the two groups in the expected value of productivity or in the reliability with which productivity may be predicted, which lead employers to discriminate on the basis of that average. Discriminatory exclusion of women from 'male' jobs can also result in an excess supply of labour in 'female' occupations, depressing wages there for otherwise equally productive workers. However, there is no reason to expect larger gaps at the upper tail of the wage distribution.

An exception is the contribution of de la Rica et alii (2005). Since high-educated women have participation rates which are only slightly lower than male participation rates, women's and men's wages should not be very different in the lower part of the income distribution. Conversely, in the upper tail of the distribution, employees are most often reluctant to invest in women's training. This occurs because women have more favourable outside opportunities than men

[1] Conversely, for the less educated groups, the gender wage gap is wider at the bottom than the top (see de la Rica et alii, 2005).

within the household, for domestic work or child care, and hence are more likely to quit their job. Our purpose in this paper is to further investigate the idea that there is more uncertainty on female jobs than on male jobs.

Specifically, we consider a competitive model of wage determination with uncertainty on the women's productivity. We assume that employers do as if male and female employees have equal productivity, but they attach more uncertainty to the women's careers. This is the case if they face greater incertitude towards females' employment duration over time, for instance as a result of their more discontinuous work participation. Wee demonstrate that firms are expected to offer lower wages to women, since they pass the risk of variability in women's production on female wages. Furthermore, the negative risk premium increases as women are more qualified.
The remainder of the paper is organized as follows. In Section 2, we present a competitive model of wage determination with uncertainty on female productivity. In section, we derive the optimal wage policy. We show that employers set a negative wage premium on the female wage which is a convex function of the female level of human capital. Finally, Section 4 concludes.

2. The basic model

We consider a representative firm which produces a good Y^t at date t. This good is sold on a competitive market and is treated as the numeraire ($p=1$). To produce that good, the firm hires two types of workers, men and women. We denote respectively by h_1 and h_2 the levels of human capital for a man and for a woman, h_1 being not necessarily equal to h_2. For the sake of simplicity, there is no on-the-job training in our model, so that the level of human capital remains constant over time for each employee. Let n_1 and n_2 be respectively the numbers of men and women who are currently working in the representative firm.

We consider that the firm does not really know how long a worker will stay in the firm. This does not seem unrealistic a priori. Assuming that the expected duration of a job in a specific firm is given by $E(T_1)$ and $E(T_2)$ respectively for a man and a woman, then $q_1 = 1/E(T_1)$ and $q_2 = 1/E(T_2)$ are the probabilities respectively for a man and for a woman to quit their current job. We suppose that the probability to quit a job is higher for a woman, i.e. $q_2 > q_1$. It is well acknowledged that there exist gender differences in the labour force participation. Women are less likely to have a paid job than men, they also most often interrupt their formal activities. This may occur because of births or other family events, women being for instance more likely to care for their elderly parents or to spend time educating their children.

Hence, firm are likely to suffer from a higher uncertainty when evaluating the long-term productivity of their female workers with respect to male workers. To formalize this uncertainty, we introduce into the definition of the level of employment a random term on the female productivity. The quantity of efficient labour is then a random variable denoted by $\tilde{N}^t$:

$$\tilde{N}^t = h_1 n_1^t + \tilde{\varepsilon} h_2 n_2^t \tag{1}$$

where $h_1 n_1^t$ and $\tilde{\varepsilon} h_2 n_2^t$ are respectively the male and female levels of employment. Importantly, we assume that the mean level of productivity is equal for male and female workers. However, there is more uncertainty on the female labour force participation., so that the term $\tilde{\varepsilon}$ may be described by a random variable such that $E(\tilde{\varepsilon}) = 1$ and $Var(\tilde{\varepsilon}) = \sigma_\varepsilon^2$. To get closed-form solutions, we make the following assumption concerning $\tilde{\varepsilon}$.

Assumption 1. *The parameter* $\tilde{\varepsilon}$ *follows a Normal distribution* $N(1, \sigma_\varepsilon^2)$.

Without loss of generality, we neglect the role of the capital factor in the production process. The production function for the representative firm may be expressed as:

$$\tilde{Y}^t = F(h_1 n_1^t + \tilde{\varepsilon} h_2 n_2^t) \tag{2}$$

$F(.)$ being a continuous function with decreasing returns ($F'(.) > 0$, $F''(.) < 0$). We account for turnover costs in the model. Let l_1^t and l_2^t be the numbers of men and women hired each year by the firm. Hiring more qualified workers is more costly for the firm. We rely on linear specification for the turnover costs $c_1^t(l_1^t)$ and $c_2^t(l_2^t)$, so that $c_1^t(l_1^t) = c(h_1) l_1^t$ and $c_2^t(l_2^t) = c(h_2) l_2^t$ ($c'(.) > 0$, $c''(.) < 0$). As there are entry and exit of workers in our setting, the following equations fully characterize the dynamics of employment within the representative firm respectively for men and women:

$$dn_1^t / dt = l_1^t - q_1 n_1^t \tag{3}$$

$$dn_2^t / dt = l_2^t - q_2 n_2^t \tag{4}$$

At date t, the total level of employment either for men or women is given by the number of employees at date $t-1$ plus the difference between the number of hiring workers and the number of voluntary exits between $t-1$ and t. Finally, we define the profit function for the firm at date t. Let w_1 and w_2 be the levels of wage for men and women, workers being remunerated at their marginal productivity. Hence, the firm's expected profit $E\tilde{\Pi}^t$ is :

$$E\tilde{\Pi}^t = E[F(h_1 n_1^t + \tilde{\varepsilon} h_2 n_2^t)] - w_1 n_1^t - w_2 n_2^t - c(h_1) l_1^t - c(h_2) l_2^t \tag{5}$$

The problem for the firm is to maximize its expected profit discounted at the interest rate r subject to the constraints which characterize the dynamics of employment over time :

$$\begin{aligned} &\max_{l_1^t, l_2^t} \int_{\Re+} \left\{ E[F(h_1 n_1^t + \tilde{\varepsilon} h_2 n_2^t)] - w_1 n_1^t - w_2 n_2^t - c(h_1) l_1^t - c(h_2) l_2^t \right\} e^{-rt} dt \\ &s.t. \quad \begin{cases} dn_1^t / dt = l_1^t - q_1 n_1^t \\ dn_2^t / dt = l_2^t - q_2 n_2^t \end{cases} \end{aligned} \tag{6}$$

3. The optimal wage policy

We now turn to the optimal wage policy. It may easily be found by solving the previous problem of optimal control. Let us define the current value of the Hamiltonian H such that:

$$H = E[F(h_1 n_1 + \tilde{\varepsilon} h_2 n_2)] - w_1 n_1 - w_2 n_2 - c(l_1) - c(l_2) + \lambda_1 (l_1 - q_1 n_1) + \lambda_2 (l_2 - q_2 n_2) \tag{7}$$

where λ_1 and λ_2 are the co-state variables associated respectively to the constraints on the levels of employment n_1 and n_2. The conditions of optimality for this problem are $\partial H / \partial l_1 = 0$, $\partial H / \partial l_2 = 0$, $d\lambda_1 / dt = r\lambda_1 - \partial H / \partial n_1$ and $d\lambda_2 / dt = r\lambda_2 - \partial H / \partial n_2$. Hence, we get:

$$-c(h_1) + \lambda_1 = 0 \tag{8}$$

$$-c(h_2) + \lambda_2 = 0 \tag{9}$$

$$d\lambda_1 / dt = r\lambda_1 - E[h_1 F'(\tilde{N})] + w_1 + \lambda_1 q_1 \tag{10}$$

$$d\lambda_2 / dt = r\lambda_2 - E[\tilde{\varepsilon} h_1 F'(\tilde{N})] + w_2 + \lambda_2 q_2 \tag{11}$$

Since the marginal cost $c(.)$ is fixed by assumption (education levels are fixed), this implies that λ_1 and λ_2 are constant, so that $d\lambda_1 / dt = 0$ and $d\lambda_2 / dt = 0$. From (10) and (11), it follows that $\lambda_1 = (E[h_1 F'(\tilde{N})] - w_1)/(r + q_1)$ and $\lambda_2 = (E[\tilde{\varepsilon} h_2 F'(\tilde{N})] - w_2)/(r + q_2)$. Since $c(h_1) = \lambda_1$ and $c(h_2) = \lambda_2$, we obtain the following optimal wages for men and for women:

$$w_1 = h_1 E[F'(\tilde{N})] - (r + q_1)c(h_1) \tag{12}$$

$$w_2 = h_2 E[\tilde{\varepsilon} F'(\tilde{N})] - (r + q_2)c(h_2) \tag{13}$$

At the equilibrium, the optimal male wage is given by the difference between the expected marginal productivity $h_1 E[F'(\tilde{N})]$ and the weighted turnover costs $(r + q_1)c(h_1)$. A similar reasoning applies for (13), but we note that there is an additional random term $\tilde{\varepsilon}$ when defining the woman's marginal productivity $h_2 E[\tilde{\varepsilon} F'(\tilde{N})]$. The normality assumption for the random perturbation $\tilde{\varepsilon}$ allows us to further specify the optimal wage policy for a competitive firm.

Proposition 1. *The optimal wage policy for the firm is such that:*

$$w_1 = h_1 E[F'(\tilde{N})] - (r + q_1)c(h_1) \tag{14}$$

$$w_2 = h_2 E[F'(\tilde{N})] + \upsilon E[F''(\tilde{N})] h_2^2 - (r + q_2)c(h_2) \tag{15}$$

where $\upsilon = Var(\tilde{\varepsilon} n_2) / E(\tilde{\varepsilon} n_2)$ is the coefficient of variation associated to the female productivity.

Proof. Given the normality assumption for $\tilde{\varepsilon}$, we can use the lemma of Stein (Rubinstein, 1976). Let us consider two variables X and Y which are bivariate normally distributed. If the function $f(Y)$ is continuously differentiable, then $\mathrm{cov}(X, f(Y)) = E(f'(Y))\mathrm{cov}(X, Y)$. By definition, we have $E[\tilde{\varepsilon} F'(\tilde{N})] = E(\tilde{\varepsilon})E[F'(\tilde{N})] + \mathrm{cov}(\tilde{\varepsilon}, F'(\tilde{N}))$. Now, applying the Stein's lemma to our problem, we get $\mathrm{cov}(\tilde{\varepsilon}, F'(\tilde{N})) = E[F''(\tilde{N})]\mathrm{cov}(\tilde{\varepsilon}, h_1 n_1 + h_2 n_2 \tilde{\varepsilon})$. Provided that $E(\tilde{\varepsilon}) = 1$ and $Var(\tilde{\varepsilon}) = \sigma_\varepsilon^2$, we deduce $\mathrm{cov}(\tilde{\varepsilon}, h_1 n_1 + h_2 n_2 \tilde{\varepsilon}) = h_2 n_2 \sigma_\varepsilon^2$. Using (13), we finally obtain the following female wage $w_2 = h_2 E[F'(\tilde{N})] + h_2^2 \sigma_\varepsilon^2 n_2 E[F''(\tilde{N})] - (r + q_2)c(h_2)$.

Let us further expand the term $h_2^2 \sigma_\varepsilon^2 n_2 E[F''(\tilde{N})]$. We know that $Var(\tilde{\varepsilon} n_2) = n_2^2 \sigma_\varepsilon^2$ and $E(\tilde{\varepsilon} n_2) = n_2$ since $E(\tilde{\varepsilon}) = 1$. Hence, given the definition of $\upsilon = Var(\tilde{\varepsilon} n_2) / E(\tilde{\varepsilon} n_2)$, it follows that $h_2^2 \sigma_\varepsilon^2 n_2 E[F''(\tilde{N})] = \upsilon h_2^2 E[F''(\tilde{N})]$. **QED**

We now find that there is an additional term in the definition of the optimal female wage. It is given by the sum of the marginal expected productivity $h_2 E[F'(\tilde{N})]$ and a negative term

$\upsilon E[F''(\tilde{N})]h_2^2$, minus the opportunity cost in terms of turnover $(r+q_2)c(h_2)$. Interestingly, the additional term $\upsilon h_2^2 E[F''(\tilde{N})]$ is a risk premium due to uncertainty on female productivity. As F'' is negative, this risk premium is negative. It depends on the shape of the technology F, on the coefficient of variation for the female productivity υ, and also on the squared level of the woman's skill level h_2^2. As the gender wage gap is a convex positive function of h_2, one expects a significantly higher difference between male and female wages at the top of the income distribution, where workers are characterized by high education levels. This is exactly the core of the glass ceiling effect.

4. Conclusion

In this paper, we have attempted to explain why the gender wage gap may vary along the wage gap distribution. For that purpose, we have introduced in a competitive labour market model uncertainty on the female productivity, as women have more frequently interrupted careers and may choose to quit the labour force either to spend time with their children, to care for elderly parents, or to move with their husband when the latter is promoted in a new location.

We demonstrate that accounting for uncertainty on the female productivity has important implications on the gender wage gap. Our main results are that firms are expected to set lower wage for women given uncertainty and that the underlying negative risk premium is higher for high-skilled women. Hence, in our theoretical framework, a larger gender wage gap is expected at the top of the wage distribution, as recently evidenced in European countries (Albrecht et alii, 2003, De la Rica et alii, 2005, Arulampalam et alii, 2004). A question worth would be to assess the relevance of our argument dealing with uncertainty on female labour participation, as there may exist alternative theoretical explanations to rationalize the glass ceiling effect, and we leave this issue for future research.

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